ALGEBRAIZATIONS WITH MINIMAL CLASS GROUP

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1. Introduction

The following question has been posed in [12]:

Question. Let R be a complete normal local ring with coefficient field $\mathbf{C}$. Does there exist a local ring A, essentially of finite type over $\mathbf{C}$, such that the class group $Cl(A)$ of A is generated by the canonical module ω_A of A and its completion $\hat{A} \simeq R$?

In general one knows that $Cl(A) \to Cl(R)$ is injective (see [1]) and the question of how small one can make $Cl(A)$ arises. Srinivas has constructed UFD's (i.e., $Cl(A) = 0$) with arbitrary rational double point singularities in his study of the K-theory of these singularities (see [15]). Kollár conjectured that any isolated hypersurface singularity would have an UFD globalization and some partial results were obtained by Buium (see [2]). In [12] the first author and Srinivas settled the above question in the affirmative for isolated complete intersection singularities.

Recently, Heitmann (see [6]) has constructed, for any complete local ring R over $\mathbf{C}$ of depth at least two, UFD's with completion R. But these rings are not geometric in general and they do not have dualizing modules. Indeed, a theorem of Murthy asserts that a geometric Cohen-Macaulay UFD is Gorenstein (see [11]). So for non-Gorenstein R it seems more natural to look for geometric A's with class group generated by the canonical module.

In this note we will prove that the above question has an affirmative answer in the case of *normal surface singularities*:

Theorem 1.1. *Given an analytic normal surface singularity* $\mathrm{Spec}(R) = (X, 0)$, *there is an affine algebraic surface* $X = \mathrm{Spec}(A)$ *and a closed point* $0 \in X$ *which represents the same germ* $(X, 0)$ *and the class group of* X *is generated by the canonical divisor* ω_X.

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The proof follows by projecting the singularity into $(\mathbf{C}^3, 0)$ and studying a suitable equisingular algebraic family via the monodromy of Lefschhetz pencils and a variant of the Noether-Lefschetz theorem, as in [12].

2. Construction of the Family

Let $(X,0) = \mathrm{Spec}(R)$ be the given analytic germ and $(X,0) \subset (\mathbf{C}^N, 0)$ be an embedding of it. Let $L : \mathbf{C}^N \to \mathbf{C}^3$ be a generic linear projection and $\nu : (X,0) \to (Y,0)$ be the restriction of L to X, and let $(Y,0)$ be the image. Then the singular locus $(\Sigma, 0)$ of $(Y,0)$ is a curve, possibly singular at 0 and generically $(Y,0)$ has A_∞ singularities, i.e., locally defined by the equation $x^2 + y^2 = 0$. Moreover, in this situation $\nu : X \to Y$ can be identified with the normalization and hence one can reconstruct $(X,0)$ out of $(Y,0)$. As $(Y,0)$ is a hypersurface germ in $(\mathbf{C}^3,0)$ it is defined by an analytic function $f \in \mathbf{C}\{x,y,z\}$. Let I be the reduced ideal of $(\Sigma,0)$. Then we have the following:

Theorem 2.1. (cf. [13]) *The function f is finitely I-determined and is right-equivalent to a polynomial. Moreover given an integer r, there exists an integer $k = k(r,f)$ such that whenever $g \in \mathcal{M}^k \cap I^{(2)}$, f and $f+g$ are right-equivalent via an automorphism which is identity modulo $\mathcal{M}^r$, where $\mathcal{M}$ is the maximal ideal of* $\mathbf{C}\{x,y,z\}$.

Though the second part of the theorem is not explicity stated there, it can be easily obtained by multiplying by a power of the maximal ideal on both sides of the basic inclusion of Pellikaan $\mathcal{M}^{k+1} I^{(2)} \subset \mathcal{M}^* \tau^*(F)$ (cf. [13], page 375, line 12).

Let Σ be a compactification of $(\Sigma,0)$ in $\mathbf{P}^3$, which is smooth outside 0. Then by a result of de Jong (cf. [7]), there exists a homogenous polynomial $F \in \mathbf{C}[x,y,z,t]$ such that $\{F = 0, 0\} \simeq (Y,0)$ and $Y := \{F = 0\}$ is smooth outside Σ and has only A_∞ and D_∞ points on $\Sigma - 0$.

Let $\pi : \widetilde{\mathbf{P}^3} \to \mathbf{P}^3$ be an embedded resolution of Y. It can be assumed to be the blow up of a coherent sheaf of ideals $\mathcal{I}$ supported on Σ (cf. [5], Theorem 7.17). Generically $\mathcal{I}$ can be assumed to be the reduced ideal I of Σ. By the Artin-Rees lemma there is an integer r such that $\mathcal{I} \cap \mathcal{M}^r \subset \mathcal{M}.\mathcal{I}$. Let $k = k(r,F)$ be as in Theorem 2.1. Let $d_0 \in \mathbf{N}$ be an integer such that $d_0 > l := \deg F$ and for all $d > d_0$ we have

(i) The restriction map $r : H^0(\mathbf{P}^3, \mathcal{M}^k \cap I^{(2)}(d)) \to H^0(\mathbf{P}, I^{(2)}/\mathcal{M}_p^4.I^{(2)})$ is surjective for all $p \in \Sigma - 0$ where $\mathcal{M}_p$ is the maximal of $\mathcal{O}_{\mathbf{P}^3,p}$.

(ii) $V := \mathbf{C}.t^{d-l}F + H^0(\mathbf{P}^3, \mathcal{M}^k \cap I^{(2)}(d))$ is very ample on $\mathbf{P}^3 - \Sigma$.

(iii) $A_d := (d^3 - 6d^2 + 11d - 6)/6 > h^0(\mathcal{O}_\Sigma(d-4)) + p_g(X,0)$ where $p_g(X,0)$ is the geometric genus of the singularity $(X,0$ and h^i of a sheaf is the dimension of H^i. This is possible because $p_g(X,0)$ is a constant and $h^0(\mathcal{O}_\Sigma(d-4))$ is a linear function in d by the theorem of Riemann-Roch, while the left-hand side is a cubic polynomial in d.

Let $\mathbf{P} \subset \mathbf{P}(V^*)$ be the hyperplane defined by the subspace $\mathcal{M}^k \cap I^{(2)}(d)$ and S be the complement of $\mathbf{P}$ in $\mathbf{P}(V^*)$. For each $s \in S$ let Y_s denote the subscheme of $\mathbf{P}^3$ defined by $s = 0$ and Z_s be the strict transform of Y_s in $\widetilde{\mathbf{P}^3}$. Consider the families,

$$\mathcal{Y} := \{(x,s) \in \mathbf{P}^3 \times S \mid x \in Y_s\}$$
$$\mathcal{Z} := \{(x,s) \in \widetilde{\mathbf{P}^3} \times S \mid x \in Z_s\} .$$

Let $f : \mathcal{Z} \to S$ be the second projection.

3. Elementary Properties of the Family

Recall that $\pi : \widetilde{\mathbf{P}^3} \to \mathbf{P}^3$ was the embedded resolution of Y. Let $\tilde{Y}$ be the strict transform of Y in $\widetilde{\mathbf{P}^3}$ and put

$$E_0 := \pi^{-1}(0) \cap \tilde{Y} .$$

For each $s \in S$, we let $\tilde{\Sigma}_s \subset Z_s$ be the "strict transform" of Σ, i.e.,

$$\tilde{\Sigma}_s := \overline{\pi^{-1}(\Sigma - 0) \cap Z_s} .$$

Lemma 3.1. *In the above situation we have*

(i) *For every $s \in S, Z_s \cap \pi^{-1}(0) = E_0$ and Z_s is non-singular along E_0.*
(ii) *$f : \mathcal{Z} \to S$ is a submersion along $E_0 \times S \subset \mathcal{Z}$.*
(iii) *there exists a codimension 2 subset T of S such that for all $s \in S - T, Z_s$ is smooth along $\tilde{\Sigma}_s$.*

Proof. Fix an $s \in S$. By the theorem of Pellikaan there is an automorphism of $(\mathbf{C}^3, 0)$ which is identity modulo $\mathcal{M}^r$ and defines an isomorphism of $(Y_s, 0)$ with $(Y, 0)$. Since $\mathcal{M}^r \cap \mathcal{I} \subset \mathcal{M}.\mathcal{I}$ it follows that this automorphism extends to the blow up of $\mathcal{I}$ in a neighbourhood of 0 and acts trivially on the fibre $\pi^{-1}(0)$, because it acts trivially on $\mathcal{I}/\mathcal{M}^r \cap \mathcal{I}$ which maps onto $\mathcal{I}/\mathcal{M}.\mathcal{I}$ and hence acts trivially on $\oplus \mathcal{I}^m/\mathcal{M}.\mathcal{I}^m$. Hence it fixes E_0 and defines an isomorphism of (Z_s, E_0) with $(\tilde{Y}, E_0)$. This proves (i). As (ii) is a local assertion, it follows from (i).

By the classification of line singularities (cf. [14], table on page 488), there is a subspace $T_p \subset H^0(\mathbf{P}^3, I^{(2)}/\mathcal{M}_p^4.I^{(2)})$ of codimension 3 for each $p \in \Sigma - 0$ such that all functions in $H^0(\mathbf{P}^3, \mathcal{M}^k \cap I^{(2)}(d)) - r^{-1}(T_p)$ has singularities of type A_∞, D_∞ or J_∞ at p. By assumption (i) it follows that $r^{-1}(T_p)$ has codimension 3 in $H^0(\mathbf{P}^3, \mathcal{M}^k \cap I^{(2)}(d))$. Define T' to be the closure of $\cup_{p\in\Sigma-0} r^{-1}(T_p)$. Then T' has codimension ≥ 2 and is invariant under scalar multiplication. Let T be the image of T' in S. Then T has codimension ≥ 2 in S. Hence for every $s \in S - T, Y_s$ has only singularities of the above-mentioned types. It is easy to prove by local computations that A_∞, D_∞ and J_∞ are resolved by the blow up of reduced singular locus. Hence Z_s is smooth along $\tilde{\Sigma}_s$ for $s \in S - T$. This proves (iii). □

Let C and D be the critical and the discriminant locus of f, i.e., $C := \{(x,s) \in \mathcal{Z} \mid Z_s$ is singular at $x\}$ and $D := f(C)$.

Corollary 3.2. *Outside the set $T \subset S$, we have*

(i) $C - f^{-1}(T)$ *is smooth and irreducible of dimension* $\dim S - 1$.
(ii) $f : C - f^{-1}(T) \to D - T$ *is birational.*
(iii) *For general* $s \in D - T, Z_s$ *has an ordinary double point.*
(iv) $\mathcal{Y} \to S$ *is an admissible family of surfaces over* $S - T$.

Proof. Since V is very ample, it gives rise to an embedding of $\mathbf{P}^3 - \Sigma$ in $\mathbf{P}(V)$. Let $\pi' : C \to \mathbf{P}^3$ be the projection. Then $C - \pi'^{-1}(\Sigma) \to \mathbf{P}^3 - \Sigma$ is the projective normal bundle of $\mathbf{P}^3 - \Sigma$ in $\mathbf{P}(V)$, by [9]. Moreover it is also proved there that $C - \pi'^{-1}(\Sigma) \to D$ is birational with the general point corresponds to an ordinary double point on Z_s. By Lemma 3.1 Z_s is non-singular along $Z_s \cap \pi^{-1}(\Sigma)$ for all $s \in S - T$. Hence the discriminant of $f : \mathcal{Z} - f^{-1}(S - T) \to S - T$ is $D - T$. This proves (i), (ii) and (iii) of the corollary. The assertion (iv) follows from the definition of an admissible deformation (cf. [8]). Hence by normalizing $\mathcal{Y}|_{S-T}$ we obtain a family of normal surfaces $\mathcal{X} \to S - T$ with a section σ such that each $X_s \to Y_s$ is the normalization and the singularities $(X_s, \sigma(s))$ are all isomorphic. □

Lemma 3.3. *For general* $s \in S - D$ *one has*
(i) $\pi_1(Z_s) = 0$, *hence* $H^2(Z_s, \mathbf{Z})$ *is torsion free*
(ii) $H^2(Z_s, \mathcal{O}_{Z_s}) \neq 0$.

Proof. (i) By stratified Morse theory (cf. [4], Part II, Theorem 1.1(1), pp. 150–151) it follows that $Y_s - \Sigma$ is simply connected for general $s \in S$, because V is very ample on $\mathbf{P}^3 - \Sigma$, which is simply connected. Since Z_s is smooth and contains $Y_s - \Sigma$ as a dense open subset, it is simply connected.

(ii) Choose an $s \in S - D$ and write X, Y and Z for X_s, Y_s and Z_s respectively. Then we have the following exact sequence:

$$0 \to \mathcal{C} \to \mathcal{O}_Y \to \mathcal{O}_\Sigma \to 0 .$$

Here $\mathcal{C} = I$ is the conductor $\mathcal{H}om_{\mathcal{O}_Y}(\mathcal{C}, \mathcal{O}_Y)$ and is also the ideal sheaf of Σ. Also note that $\nu_* \mathcal{O}_X = \mathcal{H}om_{\mathcal{O}_Y}(\mathcal{C}, \mathcal{O}_Y)$, as $\mathcal{O}_Y$-modules. If we take $\mathcal{H}om_{\mathcal{O}_Y}(-, \mathcal{O}_Y)$ to the above exact sequence, we get

$$0 - \mathcal{H}om_{\mathcal{O}_Y}(\mathcal{O}_Y, \mathcal{O}_Y) \to \mathcal{H}om_{\mathcal{O}_Y}(\mathcal{C}, \mathcal{O}_Y) \to \mathcal{E}xt^1_{\mathcal{O}_Y}(\mathcal{O}_\Sigma, \mathcal{O}_Y) \to 0 .$$

Hence we obtain an exact sequence:

$$0 \to \mathcal{O}_Y \to \nu_* \mathcal{O}_X \to \mathcal{E}xt^1_{\mathcal{O}_Y}(\mathcal{O}_\Sigma, \mathcal{O}_Y) \to 0$$

because the map $\mathcal{H}om_{\mathcal{O}_Y}(\mathcal{O}_Y, \mathcal{O}_Y) \to \mathcal{H}om_{\mathcal{O}_Y}(\mathcal{C}, \mathcal{O}_Y)$ is the natural map $\mathcal{O}_Y \to \nu_* \mathcal{O}_X$. From the long exact cohomology sequence of this exact sequence, we obtain,

$$H^1(Y, \mathcal{E}xt^1_{\mathcal{O}_Y}(\mathcal{O}_\Sigma, \mathcal{O}_Y)) \to H^2(Y, \mathcal{O}_Y) \to H^2(Y, \nu_* \mathcal{O}_X) \to 0 .$$

Also note that,

$$\mathcal{E}xt^1_{\mathcal{O}_Y}(\mathcal{O}_\Sigma, \mathcal{O}_Y) = \omega_\Sigma \otimes \omega_Y^{-1} = \omega_\Sigma(4 - d) .$$

Hence $h^2(Y, \nu_* \mathcal{O}_X) = h^2(X, \mathcal{O}_X) \geq h^2(Y, \mathcal{O}_Y) - h^1(Y, \mathcal{E}xt^1_{\mathcal{O}_Y}(\mathcal{O}_\Sigma, \mathcal{O}_Y)) = A_d - h^1(\Sigma, \mathcal{E}xt^1_{\mathcal{O}_Y}(\mathcal{O}_\Sigma, \mathcal{O}_Y)) = A_d - h^1(\Sigma, \omega_\Sigma(4-d)) = A_d - h^0(\Sigma, \mathcal{O}_\Sigma(d-4)) > p_g(X, 0)$.

From the Leray spectral sequence applied to $p : Z \to X$, we obtain an exact sequence,

$$H^0(X, R^1 p_* \mathcal{O}_Z) \to H^2(X, p_* \mathcal{O}_Z) \to H^2(Z, \mathcal{O}_Z) \ .$$

Hence $h^2(Z, \mathcal{O}_Z) \geq h^2(X, p_*, \mathcal{O}_Z) - p_g(X, 0) = h^2(X, \mathcal{O}_X) - p_g(X, 0) > 0$. Hence $H^2(Z, \mathcal{O}_Z) \neq 0$, which proves (ii). □

4. A Noether-Lefshetz Theorem

Here we prove that for a general $s \in S, Z_s$ has Pic generated by the exceptional cycles (the reduced irreducible components of E_0), $f^* \mathcal{O}_{Y_s}(1)$ and $\tilde{\Sigma}_s$. The representation of $\pi_1(S - D, s)$ on $H^2(Z_s, \mathbf{Q})$ gives rise to a local system $\mathcal{H}$ on $S - D$. Let $\mathcal{H}^\pi$ be the space of invariants of this representation and $\mathcal{P}$ be its orthogonal complement with respect to the intersection form. Note that the restriction of the intersection form is non-degenerate on $\mathcal{H}^\pi$ as it contains an ample divisor. Hence we have an orthogonal direct sum decomposition,

$$\mathcal{H} = \mathcal{H}^\pi \oplus \mathcal{P} \ .$$

Let us denote by $A_s \subset H^2(Z_s, \mathbf{Z})$ the subgroup generated by the irreducible components of $E_0, \tilde{\Sigma}_s$ and $f^*(\mathcal{O}_{Y_s}(1))$.

Lemma 4.1. *In the above situation we have*

(i) $\mathcal{H}^\pi_s = A_s \otimes \mathbf{Q}$.

(ii) *The local sub-system $\mathcal{P}$ is irreducible.*

Proof. The theorem of the fixed part of Deligne (cf. [3]) states that

$$\mathcal{H}^\pi_s = \mathrm{Im}(H^2(\mathcal{Z}, \mathbf{Q}) \to H^2(Z_s, \mathbf{Q})) \ .$$

Now choose a line $L \subset S$ that intersects D transversely. Then $L \cap D \subset D - T$ and Z_s has exactly one quadratic singularity for each $s \in L \cap D$. Let $L' = L - D$ and $Z_{L'} = f^{-1}(L')$. By a theorem of Zariski $\pi_1(L', s) \to \pi_1(S - D, s)$ is surjective. Hence

$$\mathrm{Im}(H^2(\mathcal{Z}, \mathbf{Q}) \to H^2(Z_s, \mathbf{Q})) = \mathrm{Im}(H^2(Z_{L'}, \mathbf{Q}) \to H^2(Z_s, \mathbf{Q})) = \mathcal{H}^\pi_s \ .$$

Since $Z_{L'}$ is a pencil of hypersurfaces in P^3, it is smooth and rational. Let $\overline{Z}_{L'}$ be a smooth compactification of $Z_{L'}$, with a morphism $\bar{\pi} : \overline{Z}_{L'} \to \mathbf{P}^3$ which restrict to π on $Z_{L'}$. Then $\overline{Z}_{L'}$ is also smooth rational and complete, hence the cycle map $\bar{c} : \mathrm{Pic}(\overline{Z}_{L'}) \to H^2(\overline{Z}_{L'}, \mathbf{Z})$ is an isomorphism. Now look at the diagram (with exact top row):

$$\begin{array}{ccccc} \mathrm{Pic}(\overline{Z}_{L'}) & \longrightarrow & \mathrm{Pic}(Z_{L'}) & \longrightarrow & 0 \\ \bar{c} \downarrow & & c \downarrow & & \\ H^2(\overline{Z}_{L'}, \mathbf{Z}) & \longrightarrow & H^2(Z_{L'}, \mathbf{Z}) & \xrightarrow{j} & H^2(Z_s, \mathbf{Z}) \end{array}$$

Hence it suffices to compute the image of $\mathrm{Pic}(\overline{Z}_{L'})$. Since $E_0 \times S \subset \mathcal{Z}$, it follows that each reduced irreducible component of E_0 is in the image. The complement of all irreducible components of the exceptional divisor of $\overline{Z}_{L'} \to \mathbf{P}^3$ is isomorphic to $\mathbf{P}^3 - \Sigma$. Hence its Picard group is generated by $\overline{\pi}^* \mathcal{O}_{\mathbf{P}^3}(1)$. It is also clear that $\tilde{\Sigma}_s$ is in the image as it is $\overline{Z_s \cap \pi^{-1}(\Sigma - 0)}$. This proves (i).

Now each point of $L \cap D$ defines a vanishing cycle and the space of invariants is precisely the orthogonal complement to the span of vanishing cycles, by the Picard-Lefschetz formula (cf. [9]). Hence the span of vanishing cycles is the stalk $\mathcal{P}_s$ of $\mathcal{P}$ at each point. Since the smooth points of D form a connected subset of S, it follows that the vanishing cycles form a single conjugacy class and hence $\mathcal{P}$ is irreducible. This proves (ii). □

Lemma 4.2. *For $s \in S_U$, where S_U is the complement of countably many analytic subsets in S, we have:*

$$NS(Z_s) \otimes \mathbf{Q} = A_s \otimes \mathbf{Q} \, .$$

Proof. By Hodge theory, the map of sheaves $\mathcal{P} \to R^2 f_* \mathcal{O}_{\mathcal{Z}}|_{(S-D)}$ is surjective after tensoring with $\mathbf{C}$. Since $\mathcal{P}$ is irreducible and the kernel of this map is a local sub-system, it has to be injective. If $s \in S - D$, then in some open neighbourhood U of s (in the Euclidean topology), $\mathcal{P}$ can be trivialised as a local system, and $R^2 f_* \mathcal{O}_{\mathcal{Z}}|_{S-D}$ as an $\mathcal{O}_U$-module, so that a non-zero element $v \in \mathcal{P}_s$ yields a holomorphic function of several variables on U which is not identically zero. Hence the zero set of this function is a closed analytic subset $Z_v \subset U$ of smaller dimension, and the collection of such v is countable as $\mathcal{P}_s \subset H^2(Z_s, \mathbf{Q})$ is countable. Hence for $s \in \{U - \cup_{v \neq 0} Z_v\}$, the map $\mathcal{P}_s \to H^2(Z_s, \mathcal{O}_{Z_s})$ is injective. But $S - D$ can be covered by a countable collection of such sets U. Then for any $s \in S_U := \cup_U \{U - \cup_{v \neq 0} Z_v\}$ the map $\mathcal{P}_s \to H^2(Z_s, \mathcal{O}_{Z_s})$ is injective. By the exponential sequence and GAGA we have,

$$NS(Z_s) \otimes \mathbf{Q} = \mathrm{Ker}(H^2(Z_s, \mathbf{Q}) \to H^2(Z_s, \mathcal{O}_{Z_s})) \, .$$

Hence $\mathcal{P}_s$ is orthogonal to $NS(Z_s) \otimes \mathbf{Q}$, i.e., the cycles representing $\mathcal{P}_s$ are not algebraic. Hence the statement. □

Corollary 4.3. *For $s \in S_U$ one has*

$$NS(Z_s) = A_s \, .$$

Proof. One clearly has

$$A_s \subset NS(Z_s) \subset H^2(Z_s, \mathbf{Z}) \, .$$

As by Lemma 4.2 the result is true over $\mathbf{Q}$ and by Lemma 3.3 (i) we know that $H^2(Z_s, \mathbf{Z})$ is torsion free, it is sufficient to show that A_s is a primitive lattice in $H^2(Z_s, \mathbf{Z})$. Now take a look at the diagram used in the proof of Lemma 4.1. From the Leray spectral sequence of the map $Z_{L'} \longrightarrow L'$ one gets that the map j is injective, with as image the invariants of the monodromy. Hence $H^2(Z_{L'}, \mathbf{Z})$ is primitive in $H^2(Z_s, \mathbf{Z})$. It follows from the exponential sequence that the cokernel of $c : \mathrm{Pic}(Z_{L'}) \longrightarrow H^2(Z_{L'}, \mathbf{Z})$ injects into a $\mathbf{C}$-vectorspace, hence its image also must be primitive (cf. [12], Lemma 12 for a proof that works for algebraic Pic). Hence, A_s as the image of $\mathrm{Pic}(\overline{Z}_{L'})$ in $H^2(Z_s, \mathbf{Z})$ is primitive. □

Proof of Theorem 1.1. Choose an $s \in \cup S_U$ and write X, Y and Z for X_s, Y_s and Z_s respectively. Let $\Sigma' \subset X$ be the inverse image $\nu^{-1}(\Sigma)$ of Σ. By corollary 4.3, it follows that the class group of X is generated by Σ' and $\nu^*\mathcal{O}_Y(1)$. So it only remains to prove that the class of Σ' represents the dualizing module of $(X, 0)$. Duality for finite maps applied to ν gives:

$$\nu_* \mathcal{H}om_{\mathcal{O}_X}(\mathcal{O}_X, \omega_X) = \mathcal{H}om_{\mathcal{O}_Y}(\nu_*\mathcal{O}_X, \omega_Y) .$$

Since ω_Y is locally free as Y is a hypersurface, we have

$$\nu_*\omega_X = \mathcal{H}om_{\mathcal{O}_Y}(\nu_*\mathcal{O}_X, \omega_Y) = \mathcal{H}om_{\mathcal{O}_Y}(\nu_*\mathcal{O}_X, \mathcal{O}_Y) \otimes \omega_Y = \mathcal{C} \otimes \omega_Y .$$

Hence the class of $\mathcal{C}$ represents the dualizing module as $\omega_Y = \mathcal{O}_Y(d-4)$ is locally free. □

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