Gutenberg School of Management and Economics

Discussion Paper Series

Systemic Risk, Banking and Sovereign Debt in the Euro Area

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April 2012

Discussion paper number 1207

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Systemic Risk, Banking and Sovereign Debt in the Euro Area*

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April 30, 2012

Abstract

We introduce a new systemic risk measure, the change in conditional joint probability of default ($\Delta CoJPoD$), that assesses the effects of interdependence within the financial system on the general financial system default risk. We apply our measure to examine the fragility of the European financial system during the ongoing sovereign debt crisis, encompassing 10 euro area sovereigns and 44 European Union banks in the period 01.01.2008 to 31.12.2011. Our results show that *joint* distress risk has increased since the end of 2009, parallel to decoupling of investors' perceptions about *individual* sovereign default risk. Overall, a default of Germany would have the highest contribution to systemic risk, while the effect of Greek default is limited. Regarding the effect of the sovereign debt crisis on the EU banking system, we find evidence for "too-big-to-save", riskiness-of-business and asset quality considerations when investors assess the banking system's vulnerability to sovereign risk. Leverage seems to be less informative in that respect. Our model could be an integral part of a policy makers' tool set to evaluate the usefulness and feasibility of bailout measures.

Keywords: Banking Stability, Financial Distress, Tail Risk, Contagion

JEL-Classification: C16, C61, G01, G21.

*This project was initiated during my research stay at the Financial Stability Surveillance Division of the European Central Bank. I would like to thank Carsten Detken, Paul Hiebert, Ivan Alves, Frank Betz, Paola Donati, Silviu Oprica and the rest of the FSS division staff for the valuable discussions and recommendations. I would also like to thank Isabel Schnabel for valuable advice and support. This paper benefited from the suggestions and comments of Charles Goodhart, Andreas Barth and the remaining participants of the 1st Workshop in Financial Economics at the Johannes Gutenberg University Mainz. All remaining errors are my own.

Contents

1 Introduction

The recent problems of Greece, Ireland, Italy, Portugal and Spain have turned the attention to the question of what would happen within the common currency area if one of the mentioned governments actually defaulted on its obligations to the creditors. In this context, an accurate measurement of the feedback effects among sovereign debtors and between sovereigns and the European banking system is warranted. Thus, a central motivation for this paper is the need for euro area-wide systemic risk measures for exact quantification of the effects of default risk on the financial stability of the eurozone.

Zhou (2010) points out that when assessing the systemic role of a financial institution, we should consider whether its distress co-occurs with distress of other institutions - the so called "too-many-to-fail" problem, investigated by Acharya and Yorulmazer (2007). Zhou (2010) argues that this effect is more relevant for financial crises than the popular "too-big-to-fail" argument (Boyd and Gertler, 1994; Kaufman, 2002). The author points out that any future macroprudential regulation should take into account the whole system as a complex entity, in order to secure financial stability. Such regulation should investigate the influence of systemically important institutions on the stability of the system.

We propose a new systemic risk measure, the change in conditional joint probability of default ($\Delta CoJPoD$) that represents the contribution of the interdependence of an entity (a sovereign or a bank) with the financial system to the overall default risk of the system. Our methodology views the financial system as a joint distribution of its constituents and incorporates market information on individual default risk, derived from CDS spreads. This allows us to capture the market perceptions about future systemic events in the debt market and how they affect distress expectations in the financial system.

The procedure we implement includes three steps. First, we recover probabilities of default from each entity's CDS spread series, using a bootstrapping procedure that follows Hull and White (2000). Since joint default risk is not traded, we need to impose some flexible structure on the interdependence between the individual entities under investigation. Thus, as a second step, we apply the recently developed Consistent Information Multivariate Density Optimizing (CIMDO) methodology[1] to recover the euro area mul-

[1]Put in perspective, the CIMDO methodology has the advantage over many Merton-based methods, most prominently

tivariate probability distribution. Third, we calculate the new systemic risk measure, the Conditional Joint Probability of Default (CoJPoD) using the recovered multivariate density, and analyze its properties.

Conceptually, our approach is related to the CoVaR (Adrian and Brunnermeier, 2010) and the Shapley value (Tarashev et al., 2010), which view systemic risk contributions as the difference in the value-at-risk (VaR) of the system when an entity defaults, compared to the case when no default occurs in the system. Since we focus on the tail region of the euro area asset joint distribution, the methodology can also be described as an extension of the quantile-based risk measure development, initiated by the CoVaR (Adrian and Brunnermeier, 2010). The main difference to those two concepts is that while they focus on conditional value-at-risk or conditional expected shortfall, the objects of our analysis are conditional *probabilities of default*.

Zhou (2010) stresses the inability of the CoVaR to account for multivariate interactions, as it focuses on bilateral relationships either between two institutions or between an aggregated system index and an individual institution. Another major difference of our approach to the CoVaR is that we use information regarding market expectations on default, while the CoVaR uses historical stock market data. Giglio (2011) points out that reduced-form approaches, recovering return distributions from historical data, as the CoVaR, suffer from the low number of extreme events in market data. In contrast, approaches like ours try to circumvent this issue, by recovering default probabilities from derivatives which are more sensitive to default risk, such as CDS or option contracts.

The reason to choose the CIMDO approach to model joint probabilities, is that this methodology has solid conceptual underpinnings, allowing us to focus on the market beliefs of the performance of an institution or a sovereign, while avoiding a direct investigation of their capital structure. This makes the approach suitable for analyzing the systemic risk between both financial institutions and sovereign states. With respect to sovereign default risk estimation, this appears to be a more attractive alternative to the Sovereign Contingent Claims Analysis of Gray, Merton and Bodie (2008) and Gray (2011),

the Contingent Claims Analysis (CCA) by Gray, Merton and Bodie (2008) and the approach of Lehar (2005), due to its departure from normality and the intrinsically dynamic dependence structure, represented by the CIMDO copula. The CIMDO approach has also been shown to perform exceptionally well in the default region of the system's joint distribution, compared to standard and mixture distributions that are usually used to model market comovement (see Goodhart and Segoviano (2009) for further information and discussions).

where the authors try to sort the capital structure of a sovereign in a particular way, depending on its maturity, in order to fit it in a Merton Model's framework (Merton, 1974). This requires the assets and liabilities to be assigned to a category at every given point in time, making the method relatively cumbersome. Applying the CIMDO methodology, we avoid this procedure by focusing directly on probabilities of default derived from market data and assuming a standardized distribution for our initial beliefs about the individual entity's assets. Notwithstanding, we still rely on the intuition of the Merton Model, that an entity (in our case - a bank or a sovereign) defaults on its debt, once its assets can no longer cover its liabilities.

We apply our methodology to a set of 10 euro area (EA) sovereigns and 44 European Union (EU) banks in the period 01.01.2008 to 31.12.2011. Initially, we focus on default risk feedbacks between sovereigns, followed by a study of how sovereign risk affects the EU banking system. Our results show that *joint* sovereign distress risk has increased since the end of 2009, parallel to a decoupling of investors' perceptions about *individual* sovereign default risk. We find that Germany and Netherlands would have the highest contribution to the systemic risk of the euro area in case of default, while the effect of Greece is marginal, at best. The latter fact might be explained by the low correlation of Greek sovereign assets with the rest of the euro area. Considering the effect of sovereign default on the EU banking system, we find that large banks are more vulnerable to sovereign risk, compared to medium-size and small ones. This might hint at "too-big-to-save" (Hellwig, 1998; Hüpkes, 2005; Demirgüç-Kunt and Huizinga, 2010; Völz and Wedow, 2011) considerations among investors. With respect to financial gearing, we could not confirm a straightforward relationship between leverage level and default vulnerability. On the other hand, we find that higher-performing banks are expected to be more vulnerable to sovereign default, which might be explained with market perceptions that the higher returns of the banks in question come from riskier activities. Considering asset quality, we find evidence that the amount of doubtful loans in banks' loan portfolios affects investors' perceptions of how vulnerable the banking system is to sovereign default. We also find that banks with potentially higher exposure to the debt of the GIIPS tend to be considered as more vulnerable to sovereign default. The latter result might hint that the market considers not only the *current* asset quality (the doubtful loans ratio), but also the *expected* deterioration of assets in forming its joint default expectations about the EA banking system.

Our contribution to the existing literature is twofold. First, as usually the literature on sovereign debt crises focuses on individual defaults, we are among the first ones to analyze the joint default behavior of countries. In addition, our study is among the few in financial stability literature to concentrate on the feedback effects between sovereigns and the banking system. Second, on methodological level, we propose a procedure that alleviates the "curse of dimensionality," inherent in multivariate distribution modeling, based on sorting on banking financial characteristics.

This paper is organized as follows. In section 2, we introduce the $\Delta CoJPoD$ measure and propose a procedure to derive it. Section 3 introduces our dataset and empirical strategy, while section 4 presents the estimation results. Section 5 concludes.

2 CoJPoD

2.1 Derivation

Our starting point in calculating the $\Delta CoJPoD$ is to derive the joint probability of default ($JPoD$) of the system, which can be interpreted as the *system's fragility to default events*. Let the system be described by a n-dimensional joint distribution, $P(x_1, x_2, ..., x_n)$, with density $p(x_1, x_2, ..., x_n)$, where x_1, x_2, ...,x_n are the logarithmic assets of the respective institution X_1, X_2, , X_n.

We define Joint Probability of Default ($JPoD$) as follows:

$$JPoD_{x_1,x_2,...,x_n} = \int_{\overline{\mathbf{x}}_1}^{+\infty} \int_{\overline{\mathbf{x}}_2}^{+\infty} ... \int_{\overline{\mathbf{x}}_n}^{+\infty} p(x_1, x_2, ..., x_n) dx_1 dx_2 ... dx_n \tag{1}$$

where $\overline{\mathbf{x}}_1$, $\overline{\mathbf{x}}_2$, ..., $\overline{\mathbf{x}}_n$ are the individual default thresholds[2] of the respective entities.[3]

Then, applying Bayes rule, we derive the Conditional Joint Probability of Default of the system of n entities, conditional on entity k defaulting:

[2]The default thresholds are defined in the sense of the classical structural model (Merton, 1974).

[3]Note that, following Segoviano (2006) and Gorea and Radev (2012), our default region is in the right tail of the distribution. This caveat does not affect our results, but significantly simplifies our estimation procedure.

$$
\begin{aligned}
CoJPoD_{system_{-k}|x_k>\overline{\mathbf{x}}_k} &= JPoD_{x_1,x_2,\ldots,x_{k-1},x_{k+1},\ldots,x_n|x_k>\overline{\mathbf{x}}_k} \\
&= \frac{JPoD_{x_1,x_2,\ldots,x_n}}{PoD^k} \\
&= \frac{JPoD_{system}}{PoD^k}
\end{aligned} \tag{2}
$$

where PoD^k is the individual default probability of entity k.

To calculate the contribution of entity k's default on system's default risk, we need to subtract the *JPoD* of the system constituents excluding the entity in question. Our $\Delta CoJPoD$ measure is then

$$
\Delta CoJPoD_{system_{-k}|x_k>\overline{\mathbf{x}}_k} = CoJPoD_{system_{-k}|x_k>\overline{\mathbf{x}}_k} - JPoD_{system_{-k}} \tag{3}
$$

In essence, this is a multivariate extension of the difference between conditional and unconditional probability of default. We compare the risk of the system when entity k is included and defaults, to the situation where entity k is excluded, or otherwise said - independent from the system. So defined, $\Delta CoJPoD$ is the probabilistic alternative to the CoVaR (Adrian and Brunnermeier, 2010).

Suppose $JPoD'_{system}$ is the joint probability of default of the system, if entity k is independent of the rest of the system, other things equal. Applying Bayes rule, we can reformulate $JPoD'_{system}$ as

$$
\begin{aligned}
JPoD'_{system} &= JPoD'_{x_1,x_2,\ldots,x_{k-1},x_k,x_{k+1},\ldots,x_n} \\
&= JPoD'_{x_1,x_2,\ldots,x_{k-1},x_{k+1},\ldots,x_n|x_k>\overline{\mathbf{x}}_k} \cdot PoD^k \\
&= JPoD'_{x_1,x_2,\ldots,x_{k-1},x_{k+1},\ldots,x_n} \cdot PoD^k \\
&= JPoD_{system_{-k}} \cdot PoD^k
\end{aligned} \tag{4}
$$

Then, $JPoD_{system_{-k}}$ can also be represented in the following way:

$$
\begin{aligned}
JPoD_{system_{-k}} &= \frac{JPoD'_{x_1,x_2,\ldots,x_n}}{PoD^k} \\
&= CoJPoD'_{system_{-k}|x_k>\overline{\mathbf{x}}_k}
\end{aligned}
\tag{5}
$$

where $CoJPoD'_{system_{-k}|x_k>\overline{\mathbf{x}}_k}$ is the conditional counterpart of $JPoD'_{system}$ with respect to entity k. Thus, our systemic risk contribution from equation 3, $\Delta CoJPoD_{system_{-k}|x_k>\overline{\mathbf{x}}_k}$, transforms to

$$
\Delta CoJPoD_{system_{-k}|x_k>\overline{\mathbf{x}}_k} = CoJPoD_{system_{-k}|x_k>\overline{\mathbf{x}}_k} - CoJPoD'_{system_{-k}|x_k>\overline{\mathbf{x}}_k} \tag{6}
$$

The measure can be viewed hence as the difference between the effects of default on systemic fragility when the system is dependent or independent of the respective entity. Thus, $\Delta CoJPoD_{system_{-k}|x_k>\overline{\mathbf{x}}_k}$ measures the contribution to the systemic default risk due to the system's interconnectedness with entity k.

There are numerous ways to calculate the individual and joint probabilities of default to derive $\Delta CoJPoD_{system_{-k}|x_k}$. To calculate individual probabilities of default (PoD), we choose a bootstrapping procedure that incorporates all available CDS contracts of an entity up to 5-year horizon. Then we transform the individual PoDs to multivariate PoDs using the CIMDO procedure introduced by Segoviano (2006).

2.2 Marginal Probability of Default Recovery

The usual method for estimating probabilities of default from CDS spreads used in the literature is to use the most liquid contracts on the market, 5-year CDS spreads, to estimate one-year probabilities of default, applying the simple formula

$$
PoD_t = \frac{CDS_t * 0.0001}{1 - RecoveryRate}, \tag{7}
$$

where CDS_t is the 5-year CDS spread at time t, PoD_t is the resulting probability of default estimate and $RecoveryRate$ is an assumed recovery rate of the face value of the underlying bond in case of default. As only the first of the five annual premia is used in the formula, it is believed that the resulting series reflect accurately the one-year probabilities of default.

In this exposition, we use a refined way of estimating probabilities of default (PoD), the CDS bootstrapping. The procedure follows Hull and White (2000) and is based on a basic cumulative probability model, which incorporates recovery rates, risk-free refinancing rates and cumulative compounding. The model uses CDS contracts of different maturities to calibrate hazard rates of particular time horizons to estimate cumulative probabilities of default. This method could be used for both sovereign and corporate probability of default estimation. The resulting risk measures are risk-neutral probabilities of default and satisfy the no-arbitrage condition in financial markets.

We propose using all available maturities from 1 to 5 year of CDS spreads to recover the PoD of an entity. The CDS contracts have quarterly premium payments as a general rule, so we adjust the procedure accordingly. We also correct for accrual interest, as suggested by Adelson et al. (2004). As risk-free rates, required as inputs, we use all available maturities of AAA Euro Area bond yields from 1 to 5 years. The recovery rate is uniformly set at 40 %, as this is the prevailing assumption in literature and practice.[4] The resulting series are 5-year cumulative probabilities of default and to accommodate the one-year horizon of interest to policy makers, they have to be annualized, using the formula:

$$PoD_t^{annual} = 1 - (1 - PoD_t^{cum})^{\frac{1}{T}}, \tag{8}$$

where T is the respective time horizon (T=5 for 5-year PoD) and PoD_t^{annual} is the annualized version of the cumulative PoD_t^{cum}.

Figure 1 presents the results from both procedures for a distressed sovereign, namely Greece, for the period 01.01.2008 to 31.12.2011. We notice the main drawback of the simple calculation method (in red) - while the series generally overlap in tranquil times, they diverge during the distress period starting from May 2010. The margin increases

[4] Sturzenegger and Zettelmeyer (2005) find that the historical sovereign recovery rates are usually between 30 and 70%. Zhang, Schwaab, and Lucas (2012) use those results as motivation to choose 50% recovery rate for their default estimations. We decide to be more conservative with regard to the loss given default assumption, as the recent negotiations for the Private Sector Involvement (PSI) in the Greek bailout packages suggest haircuts between 50 and 70%. As non-institutional investors are the main participants in the CDS markets, we argue that their expectations of default risk are what the CDS spreads reflect, thus we remain with the usual recovery rate convention in financial literature. For a discussion on how different recovery rates affect the PoD estimates, please refer to Gorea and Radev (2012).

rapidly with the rise of CDS spreads, leading to results higher than unity at the end of the period, which we truncate at 1 to match the definition of probability. The bootstrapped probabilities, on the other hand, have fairly reasonable annualized values in the distress period, peaking in the region from 45 to 50%. The reason for this misalignment is that the simple formula can be seen as a linear approximation of the more elaborate bootstrapping procedure, and does not account for all its caveats. The formula performs well at low levels of CDS spreads (Germany, France, Deutsche Bank), but fails for distressed sovereigns or corporates (Greece, Dexia).

2.3 Multivariate Probability Density Recovery

We base our methodology on the CIMDO approach, introduced in Segoviano (2006). It builds on the minimum cross-entropy procedure by Kullback (1959) and consists in recovering an unknown multivariate asset distribution using empirical information about its constituting marginal distributions. In essence, the CIMDO approach is related to the structural credit model by Merton (1974) and Black and Cox (1976), where an entity defaults if it crosses a predefined default threshold. The difference of the CIMDO model to the structural model comes from the fact, that in the former the threshold is fixed, while in the latter, it is allowed to vary. With the default threshold fixed, the CIMDO approach transfers mass from the center of an *ex ante* (or *prior*) joint asset distribution to the tails in such a way that it matched the empirically observed market expectations about the probability of default of each individual entity. The resulting *posterior* joint distribution, or CIMDO distribution, has two main properties: first, it matches the market consensus views about the default region of the unobserved asset distribution of the system, and second, it allows possesses fat tails, even if our starting assumption is a joint normal distribution. The latter property reflects the well-documented fact that financial markets are characterized by a higher number of crashes, than predicted by the normal distribution. Furthermore, regardless of the *ex ante* joint distribution assumption (a joint normal or a fatter-tailed distribution) the posterior CIMDO distribution is consistent with the observed data.

To start with, we define the financial system as a portfolio of debt issuers. We observe

n issuers, namely the X_1, X_2 to X_n entities defined in the previous section, with their logarithmic assets represented by n random variables x_1, x_2, to x_n. The cross-entropy objective function is then:

$$\chi(p,q) = \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\cdots\int\limits_{-\infty}^{+\infty} p(x_1,x_2,...,x_n)\ln\left[\frac{p(x_1,x_2,...,x_n)}{q(x_1,x_2,...,x_n)}\right]dx_1\cdots dx_{n-1}dx_n \tag{9}$$

where $p(x_1, x_2, ..., x_n)$, $q(x_1, x_2, ..., x_n) \in \mathbb{R}^n$ are the posterior and the prior distributions respectively. The primary objective of the minimum cross-entropy approach is to minimize the probabilistic difference $\chi(p, q)$ between our *ex ante* joint distribution $q(\cdot)$ and the *ex post* joint distribution $p(\cdot)$, given that the latter fulfills a set of constraints on the tail mass of the underlying marginal distributions. This set of consistency constraints should relate the *posterior* distribution to empirical data:

$$\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\cdots\int\limits_{-\infty}^{+\infty} p(x_1,x_2,...,x_n)\mathbf{I}_{[\overline{\mathbf{x}}_1,\infty)}dx_1\cdots dx_{n-1}dx_n = PoD_t^1 \tag{10}$$

$$\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\cdots\int\limits_{-\infty}^{+\infty} p(x_1,x_2,...,x_n)\mathbf{I}_{[\overline{\mathbf{x}}_{x_2},\infty)}dx_1\cdots dx_{n-1}dx_n = PoD_t^2 \tag{11}$$

$\cdots$

$$\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\cdots\int\limits_{-\infty}^{+\infty} p(x_1,x_2,...,x_n)\mathbf{I}_{[\overline{\mathbf{x}}_{x_n},\infty)}dx_1\cdots dx_{n-1}dx_n = PoD_t^n \tag{12}$$

with PoD_t^1, PoD_t^2 to PoD_t^n representing the CDS-derived expected probabilities of default of X_1, X_2, ..., X_n. $\mathbf{I}_{[\overline{\mathbf{x}}_1,\infty)}$, $\mathbf{I}_{[\overline{\mathbf{x}}_2,\infty)}$ to $\mathbf{I}_{[\overline{\mathbf{x}}_n,\infty)}$ are binary functions incorporating the default thresholds $\overline{\mathbf{x}}_1$, $\overline{\mathbf{x}}_2$ to $\overline{\mathbf{x}}_n$[5] of the respective institution. Whenever the logarithmic assets of an entity are above the respective threshold, the entity's binary function takes the value of one, and zero otherwise. As explained above, the moment consistency

[5]Each default threshold is derived by inverting a univariate standard normal cumulative density function at the sample average value of the individual entity probabilities of default.

constraints should ensure that the region of default of the "posterior" distribution is consistent with the market consensus default expectations for each sovereign or bank. In addition, in order to qualify as a density, $p(\cdot)$ needs to satisfy the additivity constraint $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1, x_2, ..., x_n)dx_1 \cdots dx_{n-1}dx_n = 1$.

Taking this set of constraints into account, the Lagrangian function to be minimized is:

$$
\begin{aligned}
L(p,q) = & \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1, x_2, ..., x_n) \ln\left[\frac{p(x_1, x_2, ..., x_n)}{q(x_1, x_2, ..., x_n)}\right] dx_1 \cdots dx_{n-1}dx_n \\
& + \lambda_1 \left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1, x_2, ..., x_n)\mathbf{I}_{[\overline{\mathbf{x}}_1,\infty)} dx_1 \cdots dx_{n-1}dx_n - PoD_t^1\right] \\
& + \lambda_2 \left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1, x_2, ..., x_n)\mathbf{I}_{[\overline{\mathbf{x}}_2,\infty)} dx_1 \cdots dx_{n-1}dx_n - PoD_t^2\right] \\
& + \cdots \\
& + \lambda_n \left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1, x_2, ..., x_n)\mathbf{I}_{[\overline{\mathbf{x}}_n,\infty)} dx_1 \cdots dx_{n-1}dx_n - PoD_t^n\right] \\
& + \mu \left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} p(x_1, x_2, ..., x_n) dx_1 \cdots dx_{n-1}dx_n - 1\right]
\end{aligned}
\tag{13}
$$

where μ, λ_1, λ_2 to λ_n are the Lagrange multipliers of the respective constraints. The optimal solution for the *posterior* density then reads:[6]

$$
p^*(x_1, x_2, ..., x_n) = q(x_1, x_2, ..., x_n)exp\left\{-\left[1 + \mu + \sum_{i=1}^{n}\lambda_i \mathbf{I}_{[\overline{\mathbf{x}}_i,\infty)}\right]\right\} \tag{14}
$$

Hence, in order to derive the optimal posterior distribution, all we need is the prior distribution (multivariate standard normal in our case), the optimal Lagrange multipliers and the individual default thresholds. Furthermore, the posterior possesses two important properties: first, as stated above, regardless of the *prior* assumption, the *ex post* distribution possesses fat tails, and second, due to the dynamic updating through the individual

[6]Appendix A.1 contains a detailed solution of the minimum cross-entropy optimization problem in CIMDO context.

empirical information, the posterior joint distribution is time-varying by construction.

Segoviano (2006) and Gorea and Radev (2012) present detailed robustness checks with respect to some of the main parameters underlying the CIMDO approach: prior distribution and dependence structure assumptions, and performance in the default region of the joint distribution.

A commonly overlooked property of the CIMDO model is that if independence is assumed for the prior distribution (e.g. by assuming zero-correlation structure for the prior distribution, as in Segoviano, 2006),[7] it transfers to the posterior distribution as well. Appendix A.2 provides a multivariate proof of this caveat when multivariate joint normal distribution is assumed as a prior. In a recent study, Peña and Rodriguez-Moreno (2010) compare the predictions of several systemic risk models, including the CIMDO-derived Banking Stability Index (BSI), but assume zero-correlation structure for the CIMDO's initial distribution guess. If this assumption proves wrong, which most likely is the case for the bank assets investigated in the mentioned study, that would lead to significant underestimation of the joint default risk between the considered entities. Even more, due to the independence of the posterior distribution, any conditional measures derived using it will be identical to their unconditional counterparts. The later fact has a huge effect on our $\Delta CoJPoD$ measure, as it is exactly the difference between the conditional JPoD and its unconditional alternative. If we elaborate on the way it is defined, and especially what transformations lead to Equation 6, we can easily show that this measure will be exactly 0 at any point of time, despite any dynamics in the individual PoDs. Empirical evidence for this analytical result is provided in Subsection 4.1.2.[8] Since the initial correlation structure assumption is crucial for the CIMDO approach, we rely on market estimates to explicitly allow it to differ from the identity matrix.

[7]In general, zero correlation does not imply independence and simple analytical examples are readily available. However, if zero correlation is assumed for a joint normal asset distribution, the resulting joint probabilities of default are a product of the individual entity probabilities of default. Hence, any systemic probability measure that conditions on particular entities defaulting, will be equal to the product of the PoDs of the remaining entities. Otherwise said, the conditioning on some entity defaulting, we do not get additional information about the default of the remaining entities, apart from the one already contained in their individual probabilities of default. The latter fact exactly complies with the probabilistic definition of independence.

[8]For further empirical arguments, see Gorea and Radev (2012).

3 Data and Estimation Strategy

3.1 Data

We recover marginal probabilities of default using CDS premia for contracts with maturities from 1 to 5 years for the period 01.01.2008 and 31.12.2011. The probabilities of default bootstrapping procedure that we employ (O'Kane and Turnbull, 2004; Nomura, 2004; and Gorea and Radev, 2012), requires as additional inputs refinancing interest rates, which we choose to be the AAA euro area government bond yields for maturities from 1 to 5 years. The CDS spreads and the government bond yields are at daily frequency, which is also the frequency of the resulting probabilities of default. Our analysis covers 10 euro area (EA) sovereigns (Austria, Belgium, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal and Spain) and 44 European Union (EU) banks. The banks dataset includes 34 euro area banks. The euro area banks, as well as the additional 10 non-euro area EU banks are chosen to represent 50 to 70 % of the assets of their respective banking system. The list of banks in our analysis is presented in Table 2. For consistency and asset-pricing purposes,[9] the CDS contracts are denominated in euro.[10] To alleviate the "curse of dimensionality" inherent in our estimation, we choose to split our sample in portfolios, according to a set of criteria: total assets (TA), return on equity (ROE), return on assets (ROA), net interest margin (NIM), efficiency ratio (ER), deposits-to-funding (DF) ratio, assets-to-equity (AE) ratio, loan-loss-provisions-to-net-interest-income ratio, non-performing-loans-to-total-loans ratio ("doubtful loans", DL), net-loans-to-total-assets ratio. The sources of the CDS data are Datastream and Bloomberg. In addition, the government bond yields are downloaded from Datastream, while the raw data for the individual bank characteristics for the analysed period are provided by Bloomberg and Bankscope.

[9] In order to arrive at compareable CDS-derived probabilities of default, all components in the calculation should be under a common currency measure.

[10] For many of the sovereigns both euro and US-dollar-denominated CDS contracts are traded. In an unreported analysis, we came to the conclusion that the difference in the absolute levels of the series cannot be explained solely by exchange rate dynamics. As CDS contracts are usually traded over the market, it is difficult to find information on the exact volumes traded of each type. After additional talks with professionals, we were assured that in the case of sovereigns, the US-dollar-denominated contracts are more liquid. For this reason, when available, those were chosen in our analysis and the data was transformed using euro-dollar exchange rates, downloaded from Bloomberg.

3.2 Financial Characteristics Selection and Portfolios Construction

To reduce the effect of the curse of dimensionality, we choose to form equal-size portfolios within our banking sample. That would not only help us reduce the dimensions of our problem, but would also make our results comparable across portfolios. We select 10 financial statement indicators, singled out in the financial literature as important systemic risk factors. Those factors form five broad groups: size, financial gearing, asset quality, performance, liquidity and funding.

- **Size**

  Total assets. Brunnermeier and Pedersen (2009) identify size as the major driver of systemic risk, according to their theory of "margin spiral". The authors provide evidence that banks adjust their assets, such that leverage is high in upturns and low in downturns of the economic cycle, making leverage a procyclical characteristic. Sorting by size should provide us with insights whether bigger banks were exposed to higher default risk stemming from sovereign difficulties in the indicated period.

- **Leverage**

  Adrian and Brunnermeier (2011) propose the *assets-to-equity ratio* as a measure of financial gearing. The intuition behind this sorting is that banks with higher leverage should be more susceptible to adverse credit events in the financial markets. Moreover, many large European Union banks invested heavily in EA sovereign bonds before the subprime crisis and could become insolvent in case of a sudden drop in the value of their assets.

- **Asset quality**

  Loan loss provisions to net interest income. This ratio reflects whether the lending risk undertaken by the banks is appropriately remunerated by higher interest margins. Hence, this measure should be as low as possible.

  Doubtful loans. The non-performing-loans-to-total-loans ratio is another measure for the quality of a bank's portfolio of assets. An increase of this ratio should make banks more vulnerable to credit events that further impair its loan quality.

- **Performance**

  Return on equity. The return-on-equity ratio is a standard measure of corporate efficiency. The main benefit of the measure is that it shows the profitability of the funds invested or reinvested in the company's equity. The main drawback comes from the fact that high-leverage companies could have artificially high ROE ratios, which might reflect the company's excessive risk-taking, rather than its growth potential.

  Return on assets. As an alternative, we propose the ROA ratio, which is the profit from every euro of assets that the bank controls. A probable weakness of this accounting measure is that balance sheet value of assets may differ from the market value of assets, making it difficult to compare across industries. Within the banking industry this is a lesser issue, due to the regular marking to market of assets.

  Net interest margin. The NIM is calculated as interest income minus interest expenses over average earning assets. It indicates how successful the bank's investment decisions were compared to the interest-bearing assets. A negative value could indicate non-optimal banking credit policy or fast deterioration in the quality of assets.

  Efficiency ratio. This ratio is also sometimes referred to as cost-to-income ratio and compares the overhead costs of running the bank to the revenues from the bank's business. The higher the ratio, the less efficient the bank's operations are.

- **Liquidity and Funding**

  Deposits to funding. The DF ratio is calculated by dividing the total deposits by the sum of total deposits, short- and long-term borrowing and repurchase agreements. This measure reflects the share of stable funding (deposits) to the total amount of a bank's funding. The less a bank relies on wholesale funding, the less exposed it is to global volatility and credit crunches during global crises. The higher this ratio, the better insured is a bank to global market fluctuations.[11]

  Net loans to total assets. This liquidity measure reflects the share of loans less loan loss provisions to total assets. An increase in that ratio might signal liquidity shortages.

[11] Of course, this measure is only meaningful when there are no runs on the bank. Since bank runs will affect not only the deposits, but also the general funding availability, the information content of this liquidity measure is reduced during such periods.

Our set of banks is sorted by the time average of each of those characteristics, and for comparability reasons we construct portfolios of equal size. We divide the 44 banks in 11 subsets, resulting in four banks per portfolio. In each bank portfolio, we add a sovereign as a trigger for default risk considerations. Thus, we reduce the joint density modeling to a 5-dimensional problem. For each portfolio within each characteristic, the $\Delta CoJPoD$ in case of Spain's default is of our primary interest, resulting in 110 time series for further analysis. The frequency of these financial characteristics is quarterly for Bloomberg and annual for Bankscope data. In Table 4, we present the ranking of the banks according to the 10 factors.

4 Empirical Results

Our empirical analysis is organized as follows. First, we investigate the default risk contributions among 10 euro area sovereigns. With the sovereign debt crisis at its peak, it is important for us to examine the dynamics of our systemic risk measures and identify possible trends, as well as major regulatory interventions and their effects. Second, we focus on the influence of sovereign default risk on the European banking system. We select both euro area and non-euro area EU banks for our analysis, as the recent events show that the high interconnectedness of the EU banking system facilitates spillover effects from the distressed euro area sovereigns. Furthermore, our representative set of banks makes the current analysis a highly representative study of the fragility of the European Union banking system.

4.1 Euro Area Sovereign Default Risk

4.1.1 Marginal Probability of Default Results

Figure 2 depicts the CDS-implied annualized probabilities of default for the 10 sovereigns in our analysis. We observe very similar values in the beginning of our sample period, pointing at investors' confidence in the individual EA members' ability to service their debt. We observe a peak in the individual PoDs during the global recession after Lehman Brother's collapse, but the individual default risk gradually subsides throughout 2009.

A major decoupling occurs in November 2009, after the announcement of the newly-elected Greek government that the previously reported data on the government deficit was strongly misleading. The divergence thereafter of market expectations about individual sovereign default risk might be due not only to doubts in the individual governments ability to service their debt, but also in the potential of the euro area as a whole to support its members in need. What can also be noticed is that PoD level of Greece rises throughout the whole period, while the default risk perceptions with regard to the rest of the distressed countries - Ireland, Portugal, Spain and Italy - seem to stabilize in the second half of 2011. Nonetheless, the recent credit rating downgrades of several euro area members signal that the segmentation in the market default risk perceptions might be a lasting phenomenon.

4.1.2 Conditional Joint Probability of Default Results

In this sub-section, we present the $\Delta CoJPoD$ results for our set of euro area sovereigns. Table 1 contains the dependence structure that we employ in our euro area sovereign analysis. At the end of the subsection, we confirm graphically the analytical argument in Section 2.3 that assuming independence among entities is not suitable for the analysis of conditional probabilities of default, and especially impractical when trying to derive $\Delta CoJPoD$.

Let us first investigate the ingredients of the $\Delta CoJPoD$ measure. Figure 3 shows the results for the correction term $JPoD_{system_{-k}}$ in Equation 3. The general vulnerability of the reduced system rises during throughout the period and reaches 0.25 % by the end of 2011. What might seem surprising at first glance, is that apparently excluding Greece increases the vulnerability of the rest of the system. This result can be explained after a closer examination of Table 1. Due to the already mentioned decoupling in investors' perceptions about individual sovereign risk, especially with regard to Greece, Greek assets seem to be less correlated with the rest of the system. Hence, if Greece is included in $JPoD_{system_{-k}}$ (all 9 cases where Greece is not the entity k), and another, much highly correlated sovereign, is excluded (that is - assumed to be independent from the rest of the system), this intuitively reduces the $JPoD_{system_{-k}}$. And conversely, if Greece is the particular entity k, the correlation between the remaining entities in $system_{-k}$ is higher, leading to a higher probability of them to jointly default (purple line).

Figure 4 provides the results for the conditional joint probability of default of the system, given a particular sovereign defaults. We notice that the ordering is now inverted, compared to the individual PoDs depiction. The highest CoJPoD is in the case of default of Germany, narrowly traced by that of Netherlands. This is quite understandable, considering the definition of the *CoJPoD* measure and the basic logic that if countries, perceived to be the safest in a system, actually default that should affect greatly the default risk of the remaining, riskier countries.

In Figure 5, we present the $\Delta CoJPoD$ results for the 10 euro area sovereigns. As expected from the analysis of $CoJPoD$, Germany and Netherlands have the highest perceived contribution to the euro area default risk, given their own default. We observe that before Lehman Brothers' file for bankruptcy in September 2008 the perceptions for the systemic risk contribution of a country's default were practically non-existent. This derives directly from the fact that a *joint* sovereign default within the euro area was perceived as a highly unlikely event. The contribution rises during the turmoil period after Lehman's default, and peeks between January and April 2009, gradually subsiding afterwards. The $\Delta CoJPoD$ measure starts rising again after the announcement of the Greek government budget problems in November 2009 and peeking at nearly 10 percentage points for Germany at the end of November 2011.

A more elaborate interpretation of the $\Delta CoJPoD$ is that its first part, the $CoJPoD$ reflects the relative dynamics of systemic fragility, represented by the $JPoD_{system}$, to individual entity's default risk. For the case of Germany, although German perceived individual risk has been increasing slightly but steadily throughout the sample period, obviously the systemic fragility has risen with faster (or fallen with slower) pace. At the other end of the spectrum is Greece, where the individual risk dynamics has outpaced the system's one, both in terms of growth and in magnitude, resulting in lower risk contribution due to interdependence. A positive result for the risk contribution $\Delta CoJPoD$ means that due to the interconnectedness of the respective sovereign to the rest of the euro area the fragility of the system rises by more than if the country default is an independent event. Overall, the results for $\Delta CoJPoD$ mean that for Germany this difference is much higher than the respective effect of a default of any other country.

The reader should notice that there is a second effect contributing to the final results, apart from pure dependence, namely the level effect of the systemic and individual default risks. As the individual level of default risk of Greece is high compared to the systemic default risk level, $CoJPoD$ will be low, leading to low results for $\Delta CoJPoD$, as well. The benefit of our model is that it takes into account the interaction of both those effects when evaluating the effects of interdependence on systemic default risk.

Comparing the results for $CoJPoD$ and $\Delta CoJPoD$, we do not see much to have changed, especially for Germany and Netherlands. This stems from the relatively low magnitude of the unconditional adjustment term $JPoD_{system_{-k}}$ for those countries. The lower $CoJPoD$ is though, the higher the relative contribution of the adjustment term to $\Delta CoJPoD$. This emanates to highest degree for Greece, where after the adjustment, the relative contribution to the systemic default risk is practically wiped out. We can relate this fact to our observation that $JPoD_{system_{-k}}$ for Greece is higher than for any other country, due to its low correlation to the rest of the system.

The effects of low correlation are taken to their extreme in Figure 6, where we present the results for $\Delta CoJPoD$ if the countries are assumed to be independent. As argued in Subsection 2.3, the default contribution of any of the sovereigns is not different from 0, due to the fact that under independence, the conditional and unconditional JPoD are the same. This has major repercussions for our analysis, if we assume that the entities under examination are not correlated,[12] given that in reality they are.

With regard to policy decision-making, we must note that estimating $CoJPoD$ should not be the final step in evaluating whether a bailout package to prevent a sovereign from defaulting is preferable to monetary interventions to address the effects of letting the sovereign default. To come up with meaningful regulatory suggestions pro and con a bailout package, the $CoJPoD$ should be coupled with an estimate of the losses to the system given the respective sovereign defaults. The resulting expected loss estimate should be used to determine the size of the considered bailout package. This expected size should

[12] An example for such an assumption in CIMDO related context can be seen in Peña and Rodriguez-Moreno (2010).

be then compared to the welfare costs of alternative instruments in the regulatory tool kit. Note that even if a sovereign bailout package turns out to be optimal in order to minimize social costs, it might not be feasible even with the broadest possible international cooperation. Policy makers should then resort to their remaining tools to address the consequences of a sovereign default. In either case, *CoJPoD* is an indispensable ingredient of the decision-making process.

4.2 Effect of Sovereign Default on the EU Banking System

In this subsection, we shift the focus of the financial crises from a purely EA-sovereign-related perspective, and study the perceived effects of sovereign default on the EU banking system. The topic of whether and how a sovereign default could affect the EU financial system is a major concern for regulators, as EU banks hold most of the debt generated by euro area countries and this debt is a sizable part of banks' assets portfolia.

We choose a particular sovereign, Spain, to be the trigger of default risk in the banking system.[13] Due to their small relative size, it is safe to assume that Greece, Ireland and Portugal could be bailed-out if needed and hence the resulting default risk within the EU banking system could be relatively easily defused. That leaves Spain and Italy as the main concern among the GIIPS (Greece, Ireland, Italy, Portugal and Spain). The debt level of these two countries might make a default event infeasible to prevent if they meet difficulties to service their payments (e.g. due to short-term illiquidity issues). For that reason, the ECB has continuously intervened on the debt market once Spain and Italy announced that they would issue new debt to cover their short-term funding needs.

Figure 7 depicts the $\Delta CoJPoD$ results given a default of Spain for 11 portfolios sorted by *size*. We notice a clear split of our portfolios in two groups, with portfolios 1-4, hence the biggest banks in our sample, reacting much more intensively to increases in Spanish default risk. The spikes occur throughout 2008 up to the end of the global recession in mid-2009. After relatively stable 9 months, the conditional fragility of the biggest banks rises again in March-April 2010, and in mid-2011 it surpasses the levels during Lehman Brothers' turmoil. The higher level after July 2011 could be attributed to increased at-

[13] We present and interpret the results for several financial factor groups. The rest of the results are available upon request.

tention of markets to the problems of Italy and Spain. Our results could be explained not only by the sizeable EA sovereign debt holdings on the balance sheets of the biggest banks, but also by the uncertainty about the economic conditions in the European Union during the sample period. The high susceptibility of big banks to sovereign default risk might be related to "too-big-to-save" considerations by international investors. The recent experience with the prolonged political process of bailout-packages ratification might explain why investors could be skeptical about multilateral governments' cooperation to support these international conglomerates.

With regard to *leverage*, Figure 8 provides a mixed picture. There are significant peaks during the sample period, especially in the second half of 2011, but the most vulnerable banks groups turn out to be those with relatively modest level of leverage. This indicates that the financial gearing level might not be a good indicator for the reaction of banks to sovereign debt problems. An argument why leverage can provide misleading results is the fact that during crises financial institutions tend to procyclically reduce their leverage level, sometimes at high cost, which makes them highly vulnerable to financial markets volatility. Further insights into this issue could be provides when instead of average leverage, we consider its dynamics.

Interestingly enough, the sorting by return on equity (Figure 9) reveals that the market perceptions of the default risk of the *highest-performing* banks tend to react more intensively to sovereign default risk. The top three portfolia appear to have four to six times higher $\Delta CoJPoD$ than the remaining, especially in the periods around the Bear Stearns episode, the bankruptcy of Lehman Brothers and the following global recession, as well as during the more recent events, related to the sovereign debt crisis. A possible explanation might be that in international investors' view, the higher performance might signal that the banks in question are involved in too risky activities.

We now turn our attention to the sorting by *asset quality*, measured by the doubtful loans ratio. Figure 10 provides evidence that international investors do take asset quality into account when assessing default risk - default expectations with regard to banks with high non-performing-loans-to-total-loans ratio tend to react to a greater extent to sovereign debt risk increase than expectations related to banks with modest values of this indicator.

The ordering by net loans to total assets (Figure 11) seems to be relatively uninformative on whether international investors take the banks' *liquidity and funding* situation into account when assessing their vulnerability to sovereign default. The most vulnerable banks appear to have a moderate net loans to total assets levels (portfolia 4, 5 and 6). A deeper investigation into the composition of those three portfolia (Table 4, last column) reveals that they include mainly Greek, Irish, Italian, Spanish and Portuguese banks. Hence, they are composed of banks that are most likely to have large exposures to the debt of the countries that are the most harshly hit by the sovereign debt crisis. Our results, therefore, provide a different angle to the asset quality on the banks' balance sheets. While in Figure 10 we found evidence for investors' concerns about the *actual* non-performing loans in the banks' loans portfolia, here we find that the market might be worried about the *future* loan performance. This might signal that loans portfolio composition (and especially - the share of loans to distressed sovereigns) plays a crucial role in forming the market consensus with regard to expected joint default in the banking system, stemming from an increase in sovereign default risk.

5 Conclusion

We introduce a new systemic risk measure, the $\Delta CoJPoD$, that assesses the effects of interdependence within the financial system on the general systemic default risk. The measure is related to the CoVaR and the Shapley value and captures the relationship between overall systemic fragility and individual default risk. We then apply our procedure to estimate the effect of sovereign default risk, first among euro area countries, and then between sovereigns and the European Union banking system.

Our results show that before Lehman Brothers' collapse, the euro area countries were viewed as relatively riskless, with low probability of joint default. The rise of systemic fragility after November 2009 led to decoupling of investors' perceptions about the effects in case of default of any of those sovereigns. Overall, a default of Germany would have the highest contribution to systemic default risk, while Greece appears to have the lowest influence. Regarding the effects of the sovereign debt crisis on the EU banking system, we find evidence for “too-big-to-save”, riskiness-of-business, and asset quality considerations when investors assess the banking system’s vulnerability to sovereign risk. Leverage seems to be less informative in that respect.

Our model could be an integral part of a policy makers’ tool set to evaluate the usefulness and feasibility of bailout measures. The project contributes to the ongoing debate on default risk measures and will improve our understanding of the effects of the regulatory interventions and economic reforms needed to strengthen the euro area and prepare it for the new global financial and economic challenges.

References

[1] Acharya, V., and T. Yorulmazer, 2007, "Too Many to Fail - An Analysis of Time-Inconsistency in Bank Closure Policies," *Journal of Financial Intermediation*, 16 (1): 1-31.

[2] Adrian, T., and M. Brunnermeier, 2010, "CoVaR," Staff Report No. 348, Federal Reserve Bank of New York, New York, NY.

[3] Botev, Z, and D. Kroese, 2011, "The Generalized Cross Entropy Method, with Applications to Probability Density Estimation," Methodology and Computing in Applied Probability, 13 (1): 1–27.

[4] Boyd, J., and M. Gertler, 1994, "The Role of Large Banks in the Recent U.S. Banking Crisis," *Federal Reserve Bank of Minneapolis Quarterly Review*, 18(1): 2–21.

Demirgüç-Kunt, A. and H. Huizinga, 2010, "Are Banks Too Big to Fail or Too Big to Save? International Evidence from Equity Prices and CDS Spreads," Policy Research Working Paper No. 5360, World Bank.

[5] Giglio, S., 2011, "Credit default swap spreads and systemic financial risk," Proceedings, Federal Reserve Bank of Chicago: 104–141, May 2011.

[6] Goodhart, C., and M. Segoviano, 2009, "Banking Stability Measures," IMF Working Paper 09/4.

[7] Gorea, D., and D. Radev, 2012, "Determinants of Joint Sovereign Default Risk in the Euro Area," Working Paper, GSEFM Frankfurt-Mainz-Darmstadt, May 2012.

[8] Gray, D., 2011, "Enhanced Bank Stress Testing Incorporating Sovereign Risk, Funding Cost, and Liquidity Risk Using Contingent Claims Analysis (CCA)," ESRB Advisory Technical Committee Workshop, Background Paper, August 2011.

[9] Gray, D., R. Merton, and Z. Bodie, 2007, "Contingent Claims Approach to Measuring and Managing Sovereign Credit Risk," *Journal of Investment Management*, 5(4): 5-28.

[10] Hellwig, M., 1998, "Too big to be rescued?," *Schweizer Bank* 11: 1-35.

[11] Hull, J., and A. White, 2000, "Valuing credit default swaps I: No counterparty default risk," *Journal of Derivatives*, 8: 29-40

[12] Hüpkes, E. H. (2005): ""Too Big to Save" - Towards a Functional Approach to Resolving Crises in Global Financial Institutions," in *Systemic Financial Crisis: Resolving large bank insolvencies*: 193–215, Douglas Evanoff and George Kaufman, eds.

[13] Kaufman, G., 2002, "Too big to fail in banking: What remains?," *The Quarterly Review of Economics and Finance*, 42(3): 423–436.

[14] Kullback, S., 1959, "Information Theory and Statistics," John Wiley, New York.

[15] Lehar, A, 2005, "Measuring Systemic Risk: A Risk Management Approach," *Journal of Banking and Finance*, 29(10): 2577-2603.

[16] Merton, R., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *The Journal of Finance*, 29: 449-470.

[17] O'Kane, D., and S. Turnbull, 2003, "Valuation of Credit Default Swaps," Lehman Brothers, Fixed Income Quantitative Credit Research.

[18] Peña, J., and M. Rodriguez-Moreno, 2010, "Systemic Risk Measures: The Simpler the Better?" Working Paper.

[19] Rime, B., 2005, "Do "too big to fail" expectations boost large banks issuer ratings?," Swiss National Bank Working Paper.

[20] Segoviano, M., 2006, "Consistent Information Multivariate Density Optimizing Methodology," FMG Discussion Papers dp557, Financial Markets Group.

[21] Sturzenegger, F. and J. Zettelmeyer, 2005, "Haircuts: Estimating Investor Losses in Sovereign Debt Restructurings, 1998-2005 ," IMF Working Paper, WP/05/137.

[22] Tarashev, N, C. Borio, and K. Tsatsaronis, 2010, "The systemic importance of financial institutions," BIS Working Papers, 308.

[23] Völz, M., and M. Wedow (2009): "Market discipline and too-big-to-fail in the CDS market: Does banks' size reduce market discipline? Does banks' size distort market

prices? Evidence for too-big-to-fail in the CDS market," *Journal of Empirical Finance*, 18: 195–210.

[24] Zhang, X., B. Schwaab, and A. Lucas, 2012, "Conditional probabilities and contagion measures for Euro Area sovereign default risk," Working Paper, VU University Amsterdam, January 12, 2012.

[25] Zhou, C., 2010, "Are Banks Too Big to Fail? Measuring Systemic Importance of Financial Institutions," *International Journal of Central Banking*, 6(4): 205-250.

A Solutions and Proofs

A.1 Solution of Minimum Cross Entropy

The minimum cross entropy procedure can be viewed as a part of an iterative algorithm to approximate a target probability density f, using empirical data describing its underlying unknown process.[14] In this procedure, an a-priori (or prior) density q is updated to a posterior density p, given the following **Cross Entropy Postulate**:

1. Conditional on a prior density q of a set $\mathfrak{X} \subset \Re^d$,
2. we minimize the Csiszár Cross Entropy measure [15]

$$\mathcal{D}(p \to q) = \int\limits_{\mathfrak{X}} q(\mathbf{x}) \cdot \psi\left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right) d\mathbf{x} \tag{15}$$

   with respect to $p(\mathbf{x})$, with $\mathbf{x}$ being a column vector and $\mathbf{x} \in \Re^d$,

3. given the moment constraints

$$\mathbb{E}_p K_i(\mathbf{X}) = \int\limits_{\mathfrak{X}} p(\mathbf{x}) \cdot K_i(\mathbf{x}) d\mathbf{x} = \hat{\kappa}_i, i = 0, ..., n. \tag{16}$$

   where $\{K_i(\mathbf{x})\}_{i=1}^n$ is a set of suitably chosen functions and $\hat{\kappa}_i$ is empirical information describing the behaviour of the system, $\mathbb{E}_f K_i(\mathbf{X})$.

The *Minimum Cross Entropy Problem* is then defined as

$$\min_p \mathcal{D}(p \to q) \tag{17}$$

subject to the constraints

$$\int\limits_{\mathfrak{X}} p(\mathbf{x}) \cdot K_i(\mathbf{x}) d\mathbf{x} = \hat{\kappa}_i, i = 0, ..., n. \tag{18}$$

and

[14] For further details on the cross-entropy method and its generalizations, please consult with e.g. Botev and Kroese (2011).

[15] The Csiszár Cross Entropy measure is a measure of directed divergence between probability densities (Botev and Kroese, 2011).

$$\int p(\mathbf{x})d\mathbf{x} = 1 \tag{19}$$

The corresponding Lagrangian is then

$$\begin{aligned}\mathcal{L}(p;\boldsymbol{\lambda},\lambda_0) =& \\ &= \int q(\mathbf{x})\cdot\psi\left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right)d\mathbf{x} + \lambda_0\left(1-\int p(\mathbf{x})d\mathbf{x}\right) + \sum_{i=1}^{n}\lambda_i\left(\hat{\kappa}_i - \int p(\mathbf{x})\cdot K_i(\mathbf{x})d\mathbf{x}\right) \\ &= \sum_{i=0}^{n}\lambda_i\cdot\hat{\kappa}_i + \int\left(q(\mathbf{x})\cdot\psi\left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right) + p(\mathbf{x})\cdot\sum_{i=0}^{n}\lambda_i\cdot K_i(\mathbf{x})\right)d\mathbf{x},\end{aligned} \tag{20}$$

where $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, ..., \lambda_n]^T$, $\hat{\kappa}_0 = 1$, and $K_0(\cdot) = 1$.

Let us assume that $\{K_i(\mathbf{x})\}_{i=0}^n = \{I_i(\mathbf{x})\}_{i=0}^n$, where I_i, $i = 1, 2, ..., n$ are binary functions taking values of unity when the respective x_i satisfies some condition, and zero otherwise, and $I_0 = 1$. The first order condition with respect to $p(\mathbf{x})$ is then

$$\frac{\partial\left(q(\mathbf{x})\cdot\psi\left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right) + p(\mathbf{x})\cdot\sum_{i=0}^{n}\lambda_i\cdot\mathbf{I}_i\right)}{\partial p(\mathbf{x})} \stackrel{!}{=} 0 \tag{21}$$

the latter can be further simplified as follows:

$$q(\mathbf{x})\cdot(q(\mathbf{x}))^{-1}\cdot\psi'\left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right) + \sum_{i=0}^{n}\lambda_i\cdot\mathbf{I}_i = 0 \tag{22}$$

$$\psi'\left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right) = -\sum_{i=0}^{n}\lambda_i\cdot\mathbf{I}_i \tag{23}$$

Assume $\psi(\mathbf{x}) = \mathbf{x}\cdot\ln(\mathbf{x})$, which is referred to in the literature as the Kullback-Leibler distance[16]. The Csiszár Cross Entropy measure can then me transformed as

$$\begin{aligned}\int q(\mathbf{x})\cdot\psi\left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right) &= \int q(\mathbf{x})\cdot\frac{p(\mathbf{x})}{q(\mathbf{x})}\cdot\ln\left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right) \\ &= \int p(\mathbf{x})\cdot\ln\left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right),\end{aligned} \tag{24}$$

while our $\psi'\left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right)$ takes the form

[16] The Kullback-Leibler distance is a usual assumption that allows us to avoid setting additional constraints to secure the non-negativity of $p(\mathbf{x})$.

$$\psi'\left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right) = \left(\frac{p(\mathbf{x})}{q(\mathbf{x})} \cdot \ln\left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right)\right)'$$
$$= \ln\left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right) + \frac{p(\mathbf{x})}{q(\mathbf{x})} \cdot \left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right)^{-1} \tag{25}$$
$$= \ln\left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right) + 1,$$

Substituting in our first order condition Equation (23) and simplifying further yields

$$\ln\left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right) + 1 = -\sum_{i=0}^{n} \lambda_i \cdot \mathbf{I}_i \tag{26}$$

$$\ln\left(\frac{p(\mathbf{x})}{q(\mathbf{x})}\right) = -1 - \sum_{i=0}^{n} \lambda_i \cdot \mathbf{I}_i. \tag{27}$$

The solution to the Minimum Cross Entropy problem is then

$$p(\mathbf{x}) = q(\mathbf{x}) \cdot exp\left\{-\left[1 + \sum_{i=0}^{n} \lambda_i \mathbf{I}_i\right]\right\} \tag{28}$$

Changing the notation of the Lagrange multiplier of the additivity constraint to μ, we arrive at

$$p(\mathbf{x}) = q(\mathbf{x}) \cdot exp\left\{-\left[1 + \mu + \sum_{i=1}^{n} \lambda_i \mathbf{I}_i\right]\right\}, \tag{29}$$

which is the general form of the solution to the CIMDO minimization problem.

A.2 Proof of independence within the joint default region of the CIMDO distribution

To prove independence, we want to show that using standard normal distribution as a prior, the following holds for the posterior CIMDO distribution and its marginals:

$$
\begin{aligned}
P(x_1 > \overline{\mathbf{x}}_1, x_2 > \overline{\mathbf{x}}_2, ..., x_n > \overline{\mathbf{x}}_n) &= P(x_1 > \overline{\mathbf{x}}_1) \cdot P(x_2 > \overline{\mathbf{x}}_2) \cdots P(x_n > \overline{\mathbf{x}}_n) \\
&= \prod_{i=1}^{n} P(x_i > \overline{\mathbf{x}}_i),
\end{aligned} \tag{30}
$$

where $P(x_1 > \overline{\mathbf{x}}_1)$, $P(x_2 > \overline{\mathbf{x}}_2)$,..., $P(x_n > \overline{\mathbf{x}}_n)$ and $P(x_1 > \overline{\mathbf{x}}_1, x_2 > \overline{\mathbf{x}}_2, ..., x_n > \overline{\mathbf{x}}_n)$ are the cumulative marginal and joint CIMDO probabilities.

Proof:

We present a *direct* proof of the statement above. We start by expressing $P(x_n > \overline{\mathbf{x}}_n)$, $P(x_1 > \overline{\mathbf{x}}_1, x_2 > \overline{\mathbf{x}}_2, ..., x_{n-1} > \overline{\mathbf{x}}_{n-1})$ and $P(x_1 > \overline{\mathbf{x}}_1, x_2 > \overline{\mathbf{x}}_2, ..., x_n > \overline{\mathbf{x}}_n)$ in terms of the prior (multivariate standard normal) distribution and the thresholds $\overline{\mathbf{x}}_1$, $\overline{\mathbf{x}}_2$ to $\overline{\mathbf{x}}_n$:

$$
\begin{aligned}
&P(x_n > \overline{\mathbf{x}}_n) = PoD^n \\
&= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} p(x_1, x_2, ..., x_n) \mathbf{I}_{[\overline{\mathbf{x}}_n, \infty)} dx_1 \cdots dx_{n-1} dx_n \\
&= \int_{\overline{\mathbf{x}}_n}^{+\infty} \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} (2\pi)^{\frac{n}{2}} e^{\left(-\frac{\sum_{i=1}^{n} x_i^2}{2}\right)} e^{(-(1+\mu+\sum_{i=1}^{n-1} \lambda_i \mathbf{I}_{[\overline{\mathbf{x}}_i, \infty)} + \lambda_n))} dx_1 \cdots dx_{n-1} dx_n,
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
&P(x_1 > \overline{\mathbf{x}}_1, x_2 > \overline{\mathbf{x}}_2, ..., x_{n-1} > \overline{\mathbf{x}}_{n-1}) = \\
&= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} p(x_1, x_2, ..., x_n) \mathbf{I}_{[\overline{\mathbf{x}}_1, \infty)} \cdot \mathbf{I}_{[\overline{\mathbf{x}}_2, \infty)} \cdots \mathbf{I}_{[\overline{\mathbf{x}}_{n-1}, \infty)} dx_1 \cdots dx_{n-1} dx_n \\
&= \int_{-\infty}^{+\infty} \int_{\overline{\mathbf{x}}_{n-1}}^{+\infty} \cdots \int_{\overline{\mathbf{x}}_1}^{+\infty} (2\pi)^{\frac{n}{2}} e^{\left(-\frac{\sum_{i=1}^{n} x_i^2}{2}\right)} e^{(-(1+\mu+\sum_{i=1}^{n-1} \lambda_i + \lambda_n \mathbf{I}_{[\overline{\mathbf{x}}_n, \infty)}))} dx_1 \cdots dx_{n-1} dx_n,
\end{aligned} \tag{32}
$$

where $\mathbf{I}_{[\overline{\mathbf{x}}_1, \infty)}$, $\mathbf{I}_{[\overline{\mathbf{x}}_2, \infty)}$ to $\mathbf{I}_{[\overline{\mathbf{x}}_n, \infty)}$ are indicator functions that take the value of one in the cases where the assets of X_1, X_2 to X_n are beyond their individual thresholds, respectively. Then, the joint probability of distress is as follows:

$$
\begin{aligned}
&P(x_1 > \overline{\mathbf{x}}_1, x_2 > \overline{\mathbf{x}}_2, ..., x_n > \overline{\mathbf{x}}_n) = \\
&= \int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \cdots \int\limits_{-\infty}^{+\infty} p(x_1, x_2, ..., x_n) \mathbf{I}_{[\overline{\mathbf{x}}_1, \infty)} \cdot \mathbf{I}_{[\overline{\mathbf{x}}_2, \infty)} \cdots \mathbf{I}_{[\overline{\mathbf{x}}_n, \infty)} dx_1 \cdots dx_{n-1} dx_n \\
&= \int\limits_{\overline{\mathbf{x}}_n}^{+\infty} \int\limits_{\overline{\mathbf{x}}_{n-1}}^{+\infty} \cdots \int\limits_{\overline{\mathbf{x}}_1}^{+\infty} (2\pi)^{\frac{n}{2}} e^{\left(-\frac{\sum_{i=1}^{n} x_i^2}{2}\right)} e^{(-(1+\mu+\sum_{i=1}^{n} \lambda_i))} dx_1 \cdots dx_{n-1} dx_n,
\end{aligned}
\tag{33}
$$

Rearranging $P(x_n > \overline{\mathbf{x}}_n)$, we get

$$
\begin{aligned}
&P(x_n > \overline{\mathbf{x}}_n) = \\
&= \int\limits_{\overline{\mathbf{x}}_n}^{+\infty} (2\pi)^{-\frac{1}{2}} e^{-\frac{x_n^2}{2}} e^{-\lambda_n} dx_n \int\limits_{-\infty}^{+\infty} \cdots \int\limits_{-\infty}^{+\infty} (2\pi)^{\frac{n-1}{2}} e^{\left(-\frac{\sum_{i=1}^{n-1} x_i^2}{2}\right)} e^{(-(1+\mu+\sum_{i=1}^{n-1} \lambda_i \mathbf{I}_{[\overline{\mathbf{x}}_i, \infty)}))} dx_1 \cdots dx_{n-1},
\end{aligned}
\tag{34}
$$

Analogously, for $P(x_1 > \overline{\mathbf{x}}_1, x_2 > \overline{\mathbf{x}}_2, ..., x_{n-1} > \overline{\mathbf{x}}_{n-1})$, we come at:

$$
\begin{aligned}
&P(x_1 > \overline{\mathbf{x}}_1, x_2 > \overline{\mathbf{x}}_2, ..., x_{n-1} > \overline{\mathbf{x}}_{n-1}) = \\
&= \int\limits_{\overline{\mathbf{x}}_{n-1}}^{+\infty} \cdots \int\limits_{\overline{\mathbf{x}}_1}^{+\infty} (2\pi)^{\frac{n-1}{2}} e^{\left(-\frac{\sum_{i=1}^{n-1} x_i^2}{2}\right)} e^{(-\sum_{i=1}^{n-1} \lambda_i)} dx_1 \cdots dx_{n-1} \int\limits_{-\infty}^{+\infty} (2\pi)^{-\frac{1}{2}} e^{-\frac{x_n^2}{2}} e^{(-(1+\mu+\lambda_n \mathbf{I}_{[\overline{\mathbf{x}}_n, \infty)}))} dx_n,
\end{aligned}
\tag{35}
$$

Hence, for the product of the latter probabilities, we have:

$$
\begin{aligned}
&P(x_1 > \overline{\mathbf{x}}_1, x_2 > \overline{\mathbf{x}}_2, ..., x_{n-1} > \overline{\mathbf{x}}_{n-1}) \cdot P(x_n > \overline{\mathbf{x}}_n) = \\
&= \left[\int\limits_{\overline{\mathbf{x}}_{n-1}}^{+\infty} \cdots \int\limits_{\overline{\mathbf{x}}_1}^{+\infty} (2\pi)^{\frac{n-1}{2}} e^{\left(-\frac{\sum_{i=1}^{n-1} x_i^2}{2}\right)} e^{(-\sum_{i=1}^{n-1} \lambda_i)} dx_1 \cdots dx_{n-1} \int\limits_{-\infty}^{+\infty} (2\pi)^{-\frac{1}{2}} e^{-\frac{x_n^2}{2}} e^{(-(1+\mu+\lambda_n \mathbf{I}_{[\overline{\mathbf{x}}_n, \infty)}))} dx_n \right] \\
&\cdot \left[\int\limits_{\overline{\mathbf{x}}_n}^{+\infty} (2\pi)^{-\frac{1}{2}} e^{-\frac{x_n^2}{2}} e^{-\lambda_n} dx_n \int\limits_{-\infty}^{+\infty} \cdots \int\limits_{-\infty}^{+\infty} (2\pi)^{\frac{n-1}{2}} e^{\left(-\frac{\sum_{i=1}^{n-1} x_i^2}{2}\right)} e^{(-(1+\mu+\sum_{i=1}^{n-1} \lambda_i \mathbf{I}_{[\overline{\mathbf{x}}_i, \infty)}))} dx_1 \cdots dx_{n-1} \right] \\
&= \left[\int\limits_{\overline{\mathbf{x}}_n}^{+\infty} \int\limits_{\overline{\mathbf{x}}_{n-1}}^{+\infty} \cdots \int\limits_{\overline{\mathbf{x}}_1}^{+\infty} (2\pi)^{\frac{n}{2}} e^{\left(-\frac{\sum_{i=1}^{n} x_i^2}{2}\right)} e^{(-(1+\mu+\sum_{i=1}^{n} \lambda_i))} dx_1 \cdots dx_{n-1} dx_n \right] \\
&\cdot \left[\int\limits_{-\infty}^{+\infty} \int\limits_{-\infty}^{+\infty} \cdots \int\limits_{-\infty}^{+\infty} (2\pi)^{\frac{n}{2}} e^{\left(-\frac{\sum_{i=1}^{n} x_i^2}{2}\right)} e^{(-(1+\mu+\sum_{i=1}^{n} \lambda_i \mathbf{I}_{[\overline{\mathbf{x}}_i, \infty)}))} dx_1 \cdots dx_{n-1} dx_n \right].
\end{aligned}
\tag{36}
$$

As the integral in the last square brackets is in fact the additivity constraint in our optimization problem, it equals 1 by definition. The remaining term equals our definition for the joint probability $P(x_1 > \overline{\mathbf{x}}_1, x_2 > \overline{\mathbf{x}}_2, ..., x_n > \overline{\mathbf{x}}_n)$. If we repeat the procedure iteratively for the joint distributions $P(x_1 > \overline{\mathbf{x}}_1, x_2 > \overline{\mathbf{x}}_2, ..., x_i > \overline{\mathbf{x}}_i)$, for $i = n-1,\ ...,\ 2$, we arrive at the following decomposition:

$$\begin{aligned} P(x_1 > \overline{\mathbf{x}}_1, x_2 > \overline{\mathbf{x}}_2, ..., x_n > \overline{\mathbf{x}}_n) &= P(x_1 > \overline{\mathbf{x}}_1) \cdot P(x_2 > \overline{\mathbf{x}}_2) \cdots P(x_n > \overline{\mathbf{x}}_n) \\ &= \prod_{i=1}^{n} P(x_i > \overline{\mathbf{x}}_i), \end{aligned} \tag{37}$$

Hence, the product of the marginal probabilities of distress $P(x_1 > \overline{\mathbf{x}}_1)$, $P(x_2 > \overline{\mathbf{x}}_2)$, ..., and $P(x_n > \overline{\mathbf{x}}_n)$ equals the joint probability of distress, meaning that within the joint distress region, the entities X_1, X_2 and X_n are independent. ■

B Figures

Figure 1: 5-year annualized CDS-implied probabilities of default of Greece, using the simple formula 7 (GR(simple)) and the bootstrapping procedure (GR(boot)). The 5-year annualized CDS-implied bootstrapped probabilities of default are derived from the respective cumulative ones using formula 8. Euro-denominated CDS spreads are used. Period: 01.01.2008 - 31.12.2011. Source: own calculations.

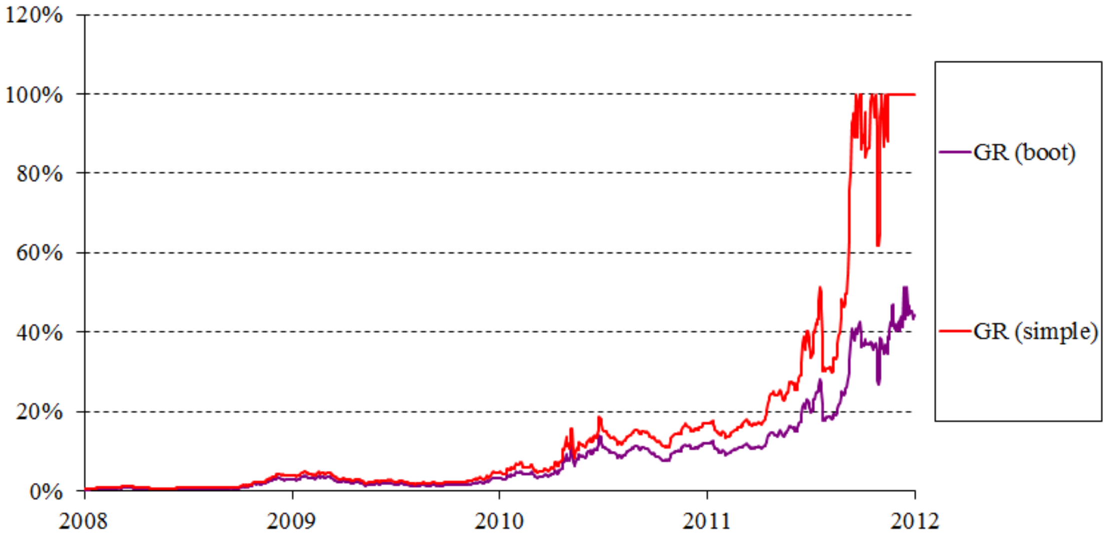


Figure 2: 5-year annualized CDS-implied bootstrapped probabilities of default for 10 sovereigns: Austria (AT), Belgium (BE), France (FR), Germany (GE), Greece (GR), Ireland (IE), Italy (IT), Netherlands (NL), Portugal (PT), Spain (ES). Euro-denominated CDS spreads are used. Period: 01.01.2008 - 31.12.2011. Source: own calculations.

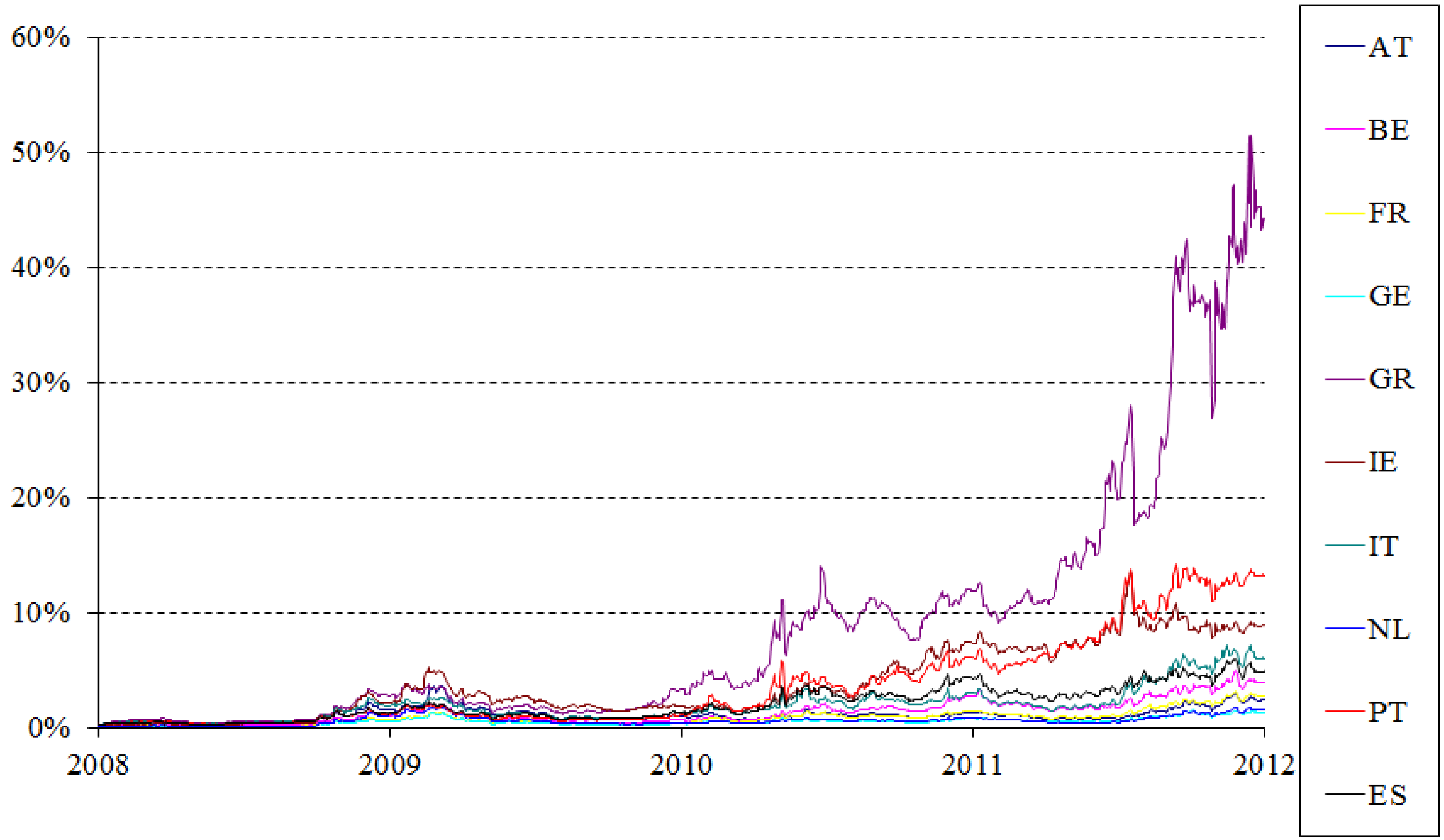


Figure 3: Joint probabilities of default (JPoD) involving 10 sovereigns for the period 01.01.2008 - 31.12.2011. “System” is the full set of sovereigns, listed in Figure 2 and the text. For each series plotted, a particular sovereign is not included in the calculation, reducing the problem to 9 dimensions. E.g. “System-AT”, means that Austria is excluded from the calculations. The abbreviations of the sovereigns are analogous to those in Figure 2. Non-zero correlation structure is used in defining the prior distribution. The actual correlation matrix used in deriving the current figure excludes the respective sovereign, listed in the legend. The full system correlation matrix is presented in Table 1. The correlation structure calculation is explained in the text and in Table 1.

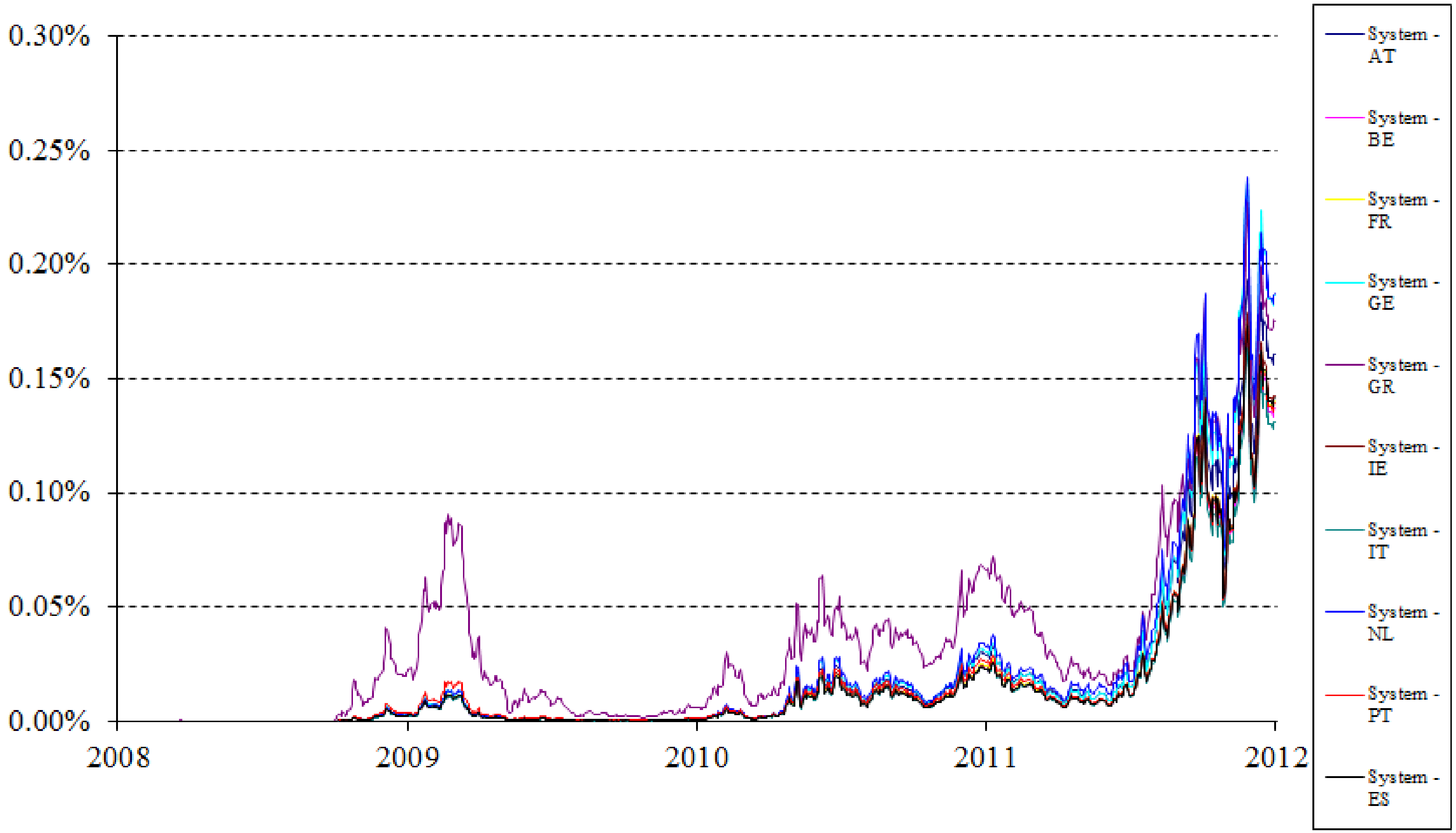


Figure 4: Conditional joint probabilities of default (CoJPoD) involving 10 sovereigns for the period 01.01.2008 - 31.12.2011. Non-zero correlation structure is used in defining the prior distribution. The correlation matrix is presented in Table 1. The correlation structure calculation is explained in the text. The legend details the country that we condition our 10-entity- system joint probability of default on to derive each series. For explanation of the abbreviations, see Figure 2. **Note:** In contrast to Figure 3, here we use 10-dimensional joint probabilities of default.

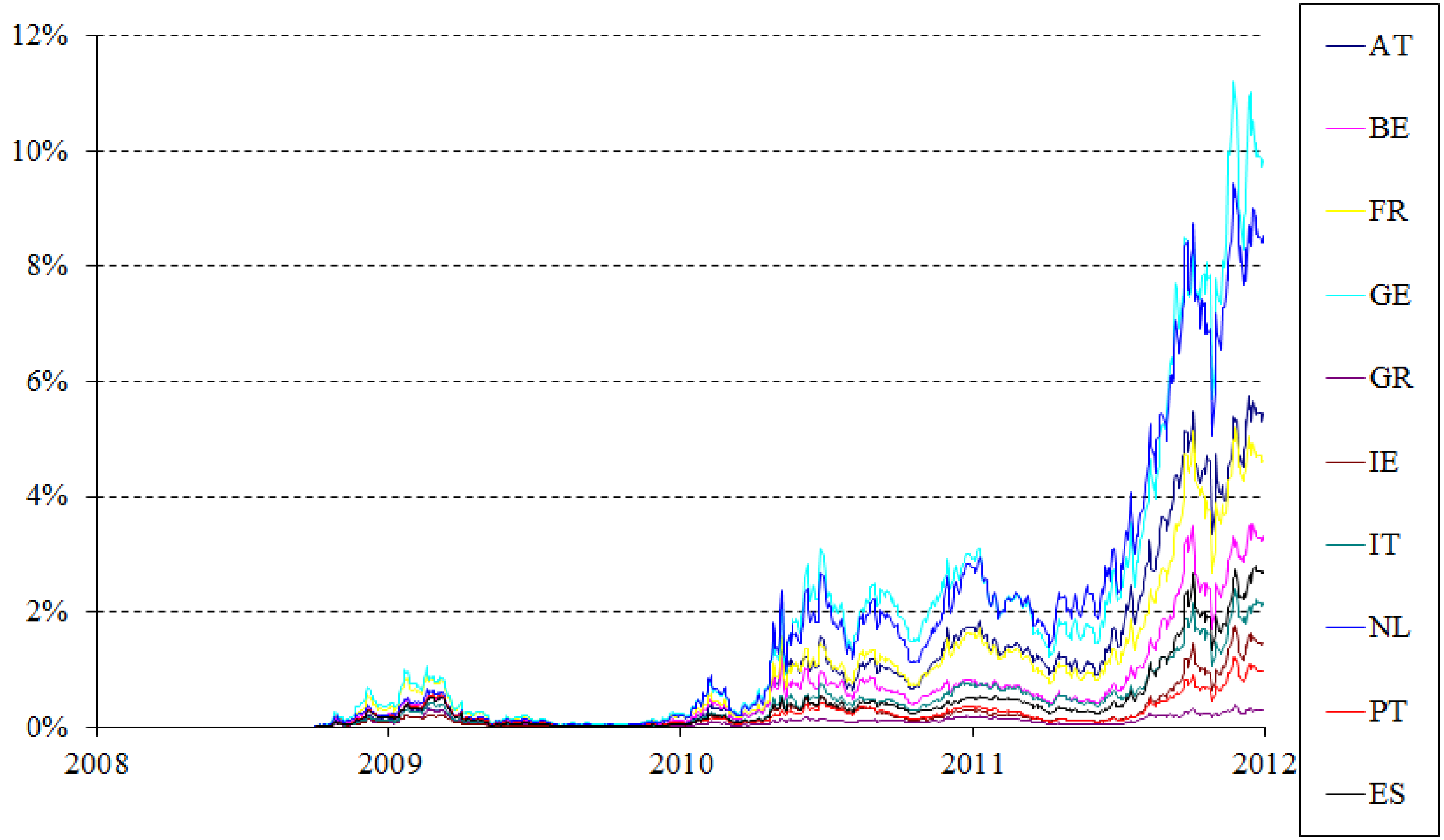


Figure 5: Change in conditional joint probabilities of default ($\Delta CoJPoD$) involving 10 sovereigns for the period 01.01.2008 - 31.12.2011. Non-zero correlation structure is used in defining the prior distribution. The correlation matrix is presented in Table 1. The correlation structure calculation is explained in the text and in Table 1. For explanation of the abbreviations, see Figure 2. $\Delta CoJPoD$ is the difference between the respective series presented in Figures 4 and 3.

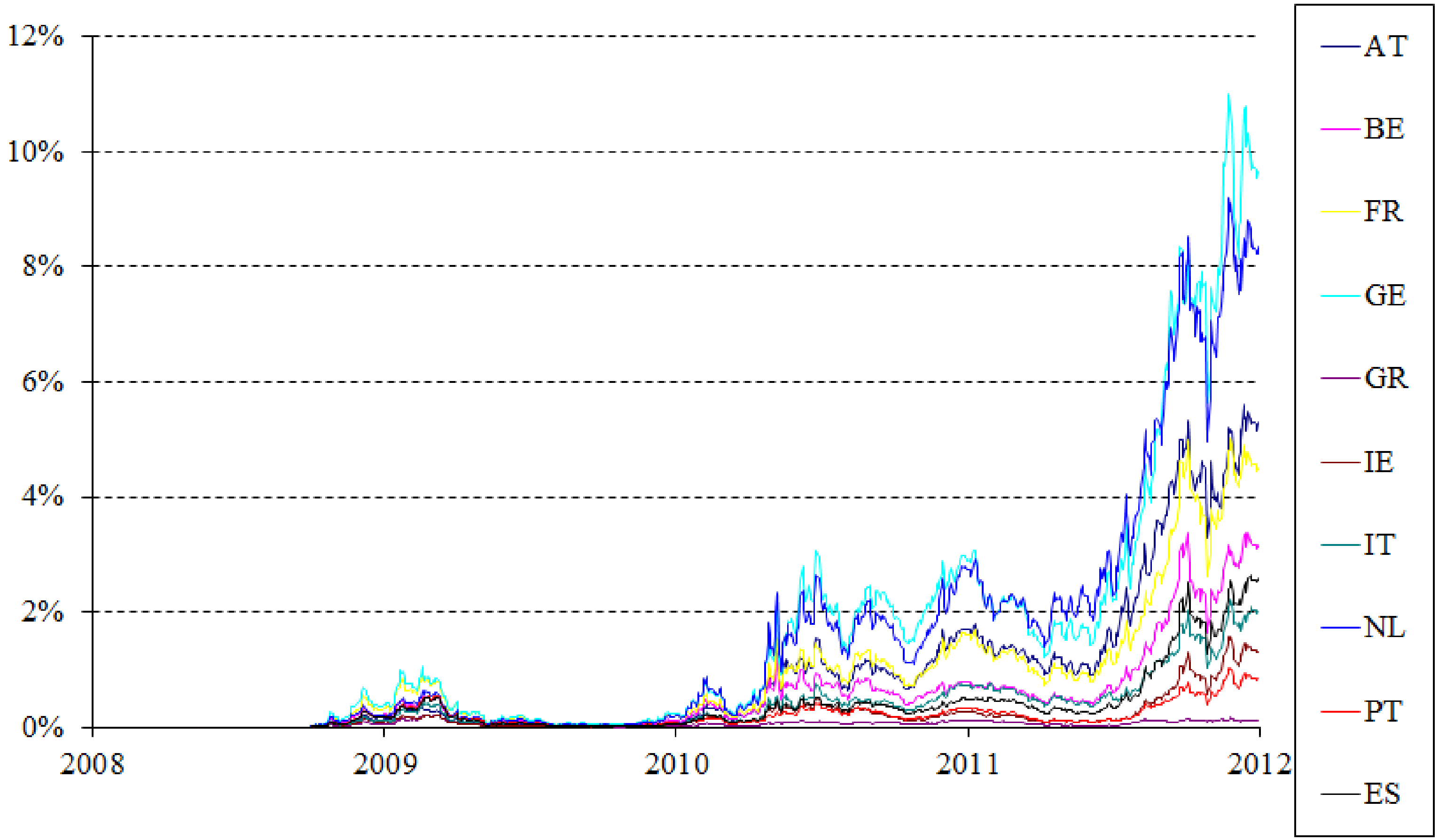


Figure 6: Change in conditional joint probabilities of default ($\Delta CoJPoD$) involving 10 sovereigns for the period 01.01.2008 - 31.12.2011. Zero correlation between all sovereigns involved is assumed in defining our prior distribution. $\Delta CoJPoD$ is the difference between zero-correlation analogues of the respective series presented in Figures 4 and 3. This plot confirms the analytical result that if independence is assumed for the prior distribution, it transfers to the CIMDO posterior distribution, yielding $\Delta CoJPoD = 0$. For explanation of the abbreviations, see Figure 2.

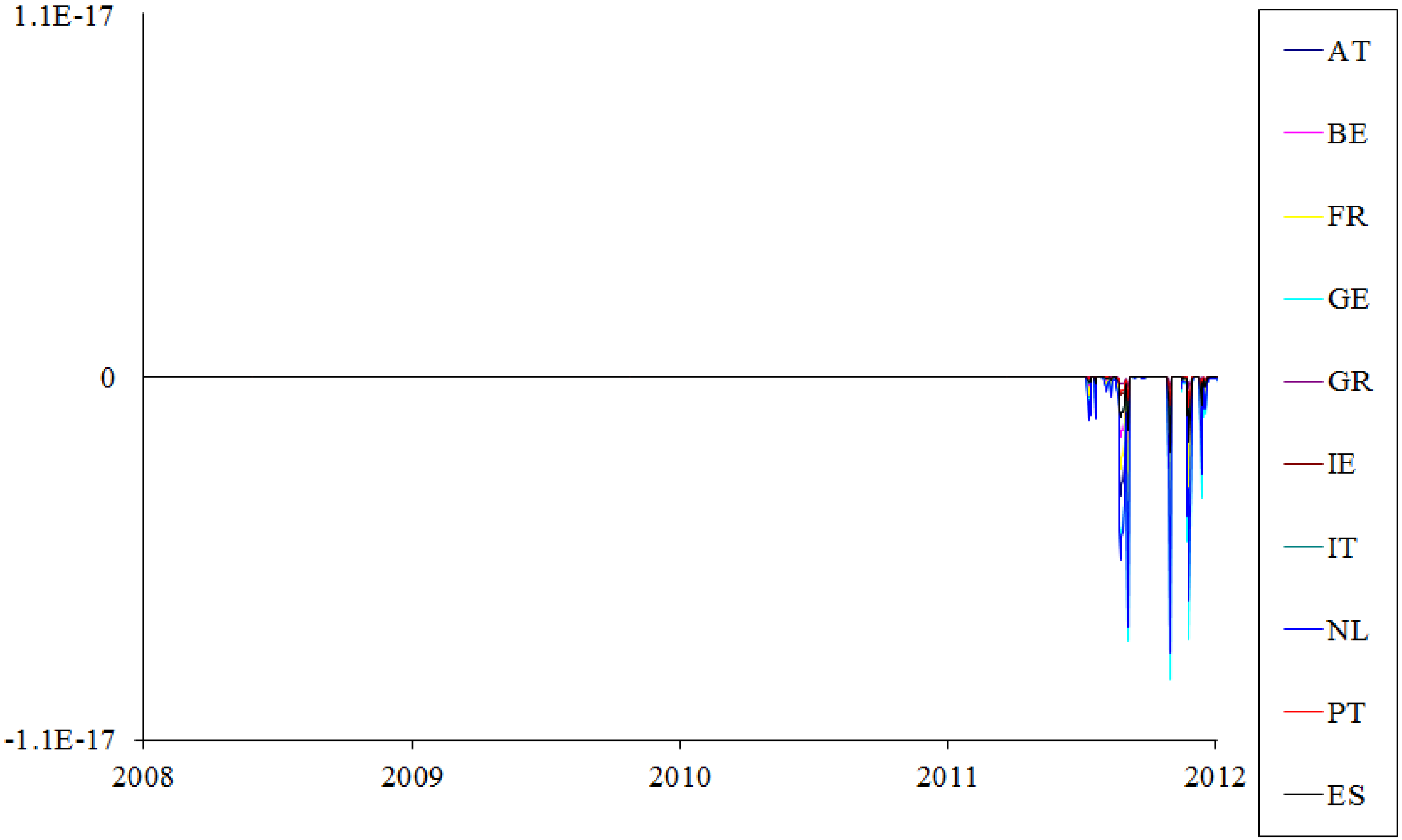


Figure 7: Change in conditional joint probabilities of default ($\Delta CoJPoD$) involving 11 portfolios of 44 banks and 1 sovereign for the period 01.01.2008 - 31.12.2011. Each portfolio includes 4 banks and 1 sovereign. Spain is considered uniformly as a default-risk-triggering sovereign. Only the series conditioning on default of Spain are presented for each portfolio. The assignment of a bank to a portfolio is governed by its **size**, compared to the rest, sorted in descending order. E.g. Portfolio 1 (PF 1) includes the four banks with highest value of *total assets*. The set of banks is presented in Tables 2 and 3. The complete portfolio assignment is presented in Table 4, column TA. Non-zero correlation structure is used in defining the prior distribution. The correlation structure calculation is explained in the text and in Table 1. $\Delta CoJPoD$ is calculated analogously to the sovereign case. The total assets are denominated in euro.

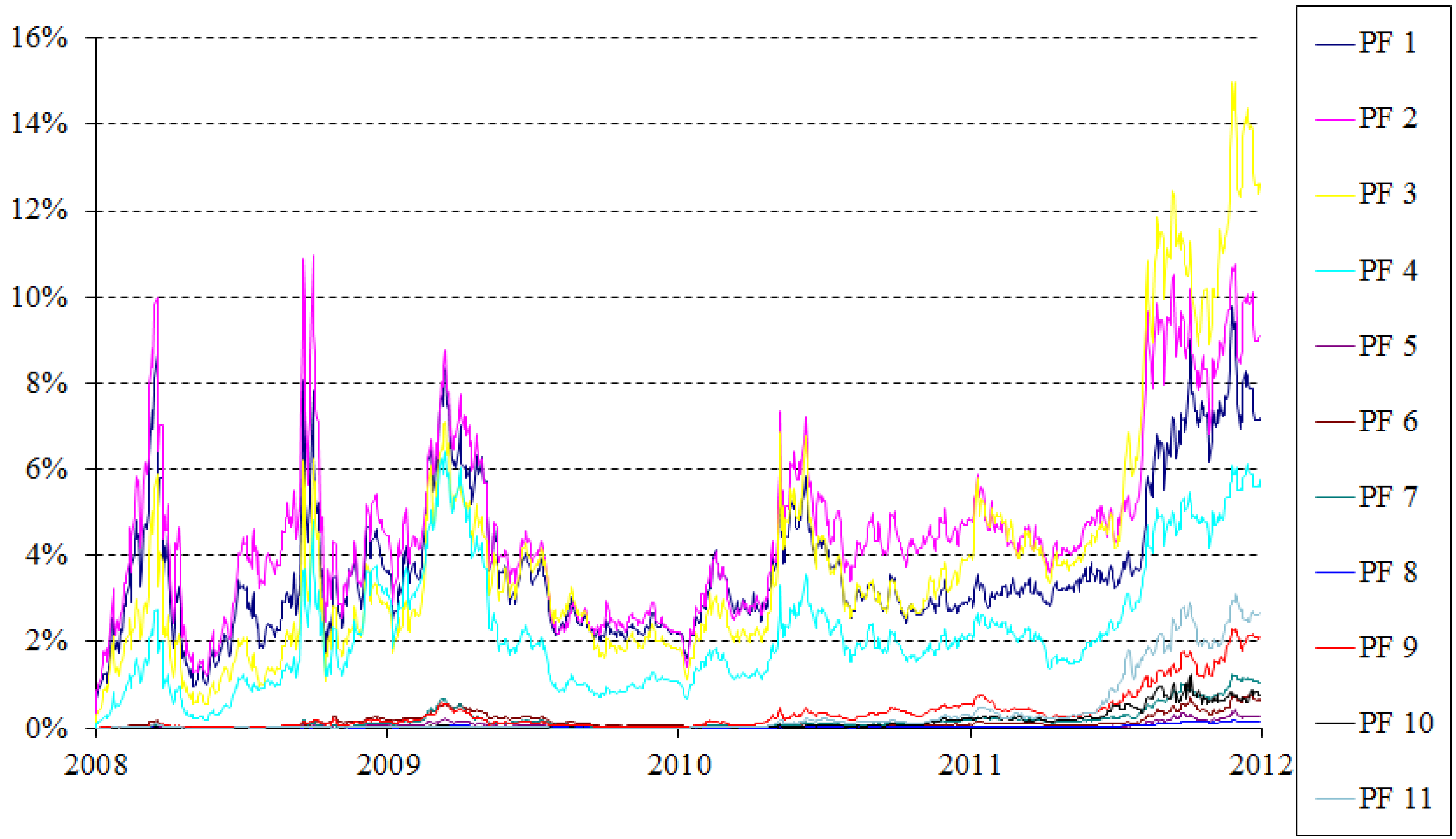


Figure 8: Change in conditional joint probabilities of default ($\Delta CoJPoD$) involving 11 portfolios of 44 banks and 1 sovereign for the period 01.01.2008 - 31.12.2011. Each portfolio includes 4 banks and 1 sovereign. Spain is considered uniformly as a default-risk-triggering sovereign. Only the series conditioning on default of Spain are presented for each portfolio. The assignment of a bank to a portfolio is governed by its **leverage**, compared to the rest, sorted in descending order. E.g. Portfolio 1 (PF 1) includes the four banks with highest value of *assets to equity*. The set of banks is presented in Tables 2 and 3. The complete portfolio assignment is presented in Table 4, column AE. Non-zero correlation structure is used in defining the prior distribution. The correlation structure calculation is explained in the text and in Table 1. $\Delta CoJPoD$ is calculated analogously to the sovereign case. The financial data used to calculate the leverage are denominated in euro.

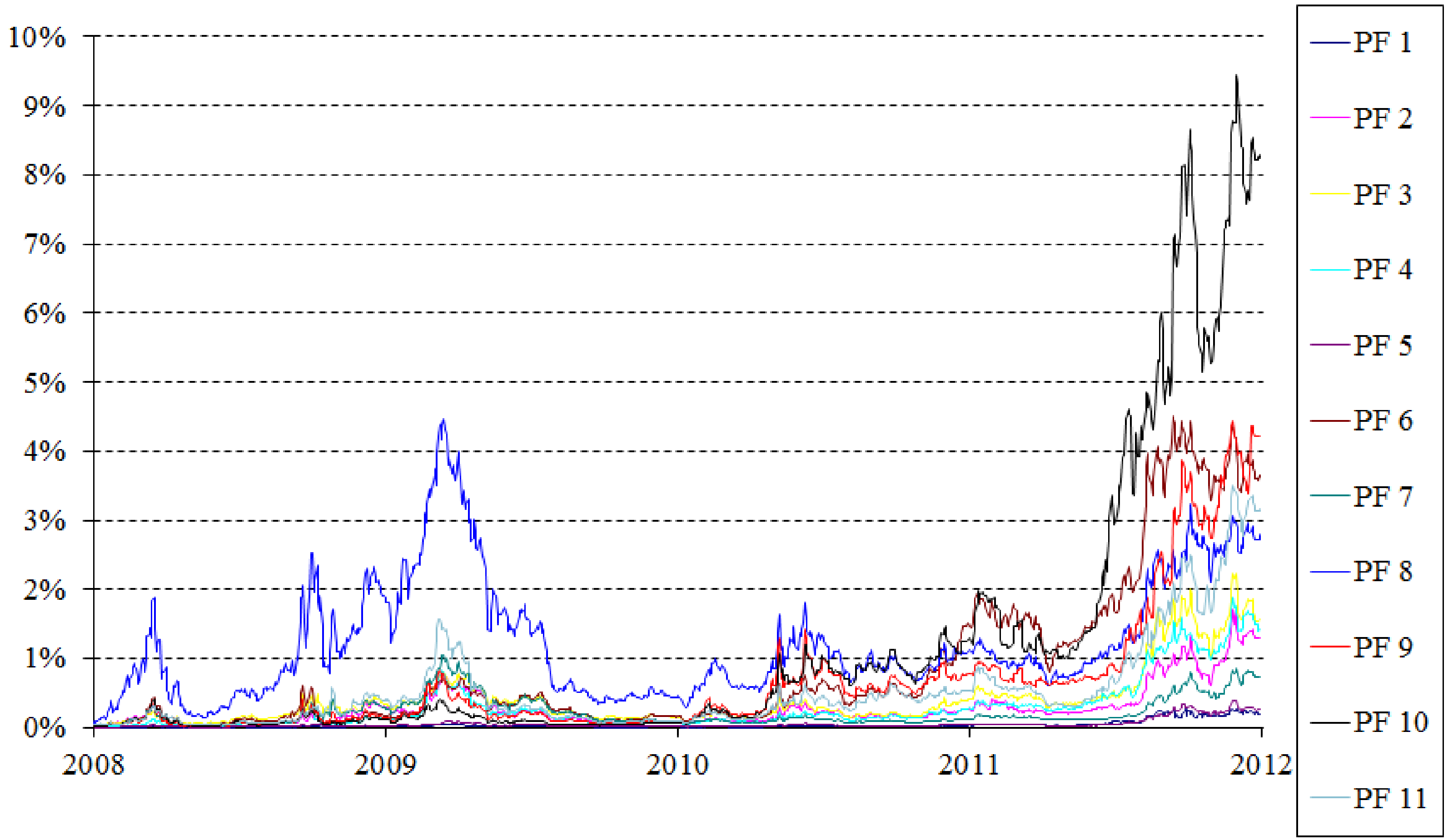


Figure 9: Change in conditional joint probabilities of default ($\Delta CoJPoD$) involving 11 portfolios of 44 banks and 1 sovereign for the period 01.01.2008 - 31.12.2011. Each portfolio includes 4 banks and 1 sovereign. Spain is considered uniformly as a default-risk-triggering sovereign. Only the series conditioning on default of Spain are presented for each portfolio. The assignment of a bank to a portfolio is governed by its **performance**, compared to the rest, sorted in descending order. E.g. Portfolio 1 (PF 1) includes the four banks with highest value of *return on equity*. The set of banks is presented in Tables 2 and 3. The complete portfolio assignment is presented in Table 4, column AE. Non-zero correlation structure is used in defining the prior distribution. The correlation structure calculation is explained in the text and in Table 1. $\Delta CoJPoD$ is calculated analogously to the sovereign case. The financial data used to calculate the leverage are denominated in euro.

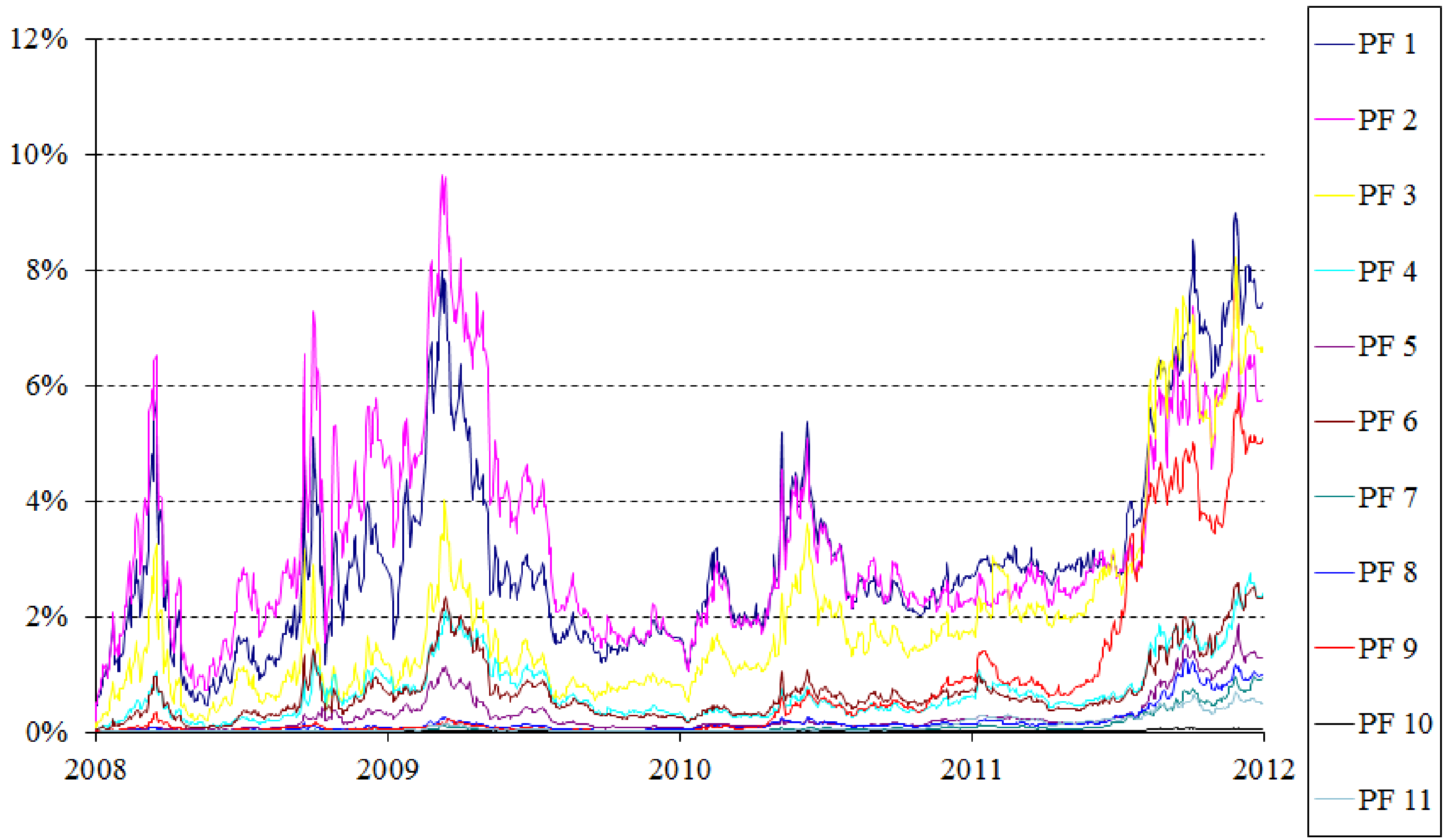


Figure 10: Change in conditional joint probabilities of default ($\Delta CoJPoD$) involving 11 portfolios of 44 banks and 1 sovereign for the period 01.01.2008 - 31.12.2011. Each portfolio includes 4 banks and 1 sovereign. Spain is considered uniformly as a default-risk-triggering sovereign. Only the series conditioning on default of Spain are presented for each portfolio. The assignment of a bank to a portfolio is governed by its **asset quality**, compared to the rest, sorted in descending order. E.g. Portfolio 1 (PF 1) includes the four banks with highest value of *doubtful loans*. The set of banks is presented in Tables 2 and 3. The complete portfolio assignment is presented in Table 4, column DL. Non-zero correlation structure is used in defining the prior distribution. The correlation structure calculation is explained in the text and in Table 1. $\Delta CoJPoD$ is calculated analogously to the sovereign case. The financial data used to calculate the doubtful loans ratio are denominated in euro.

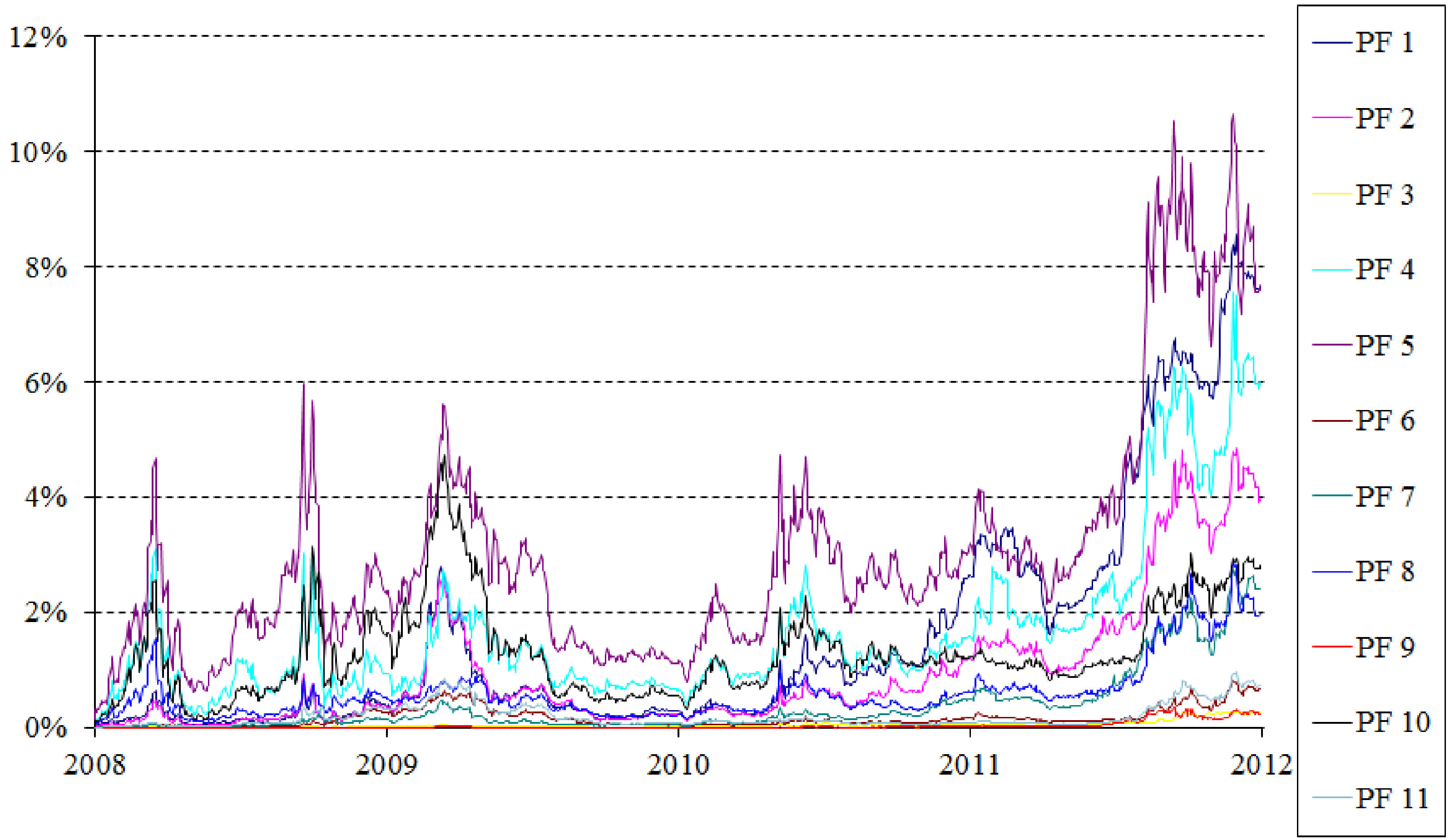
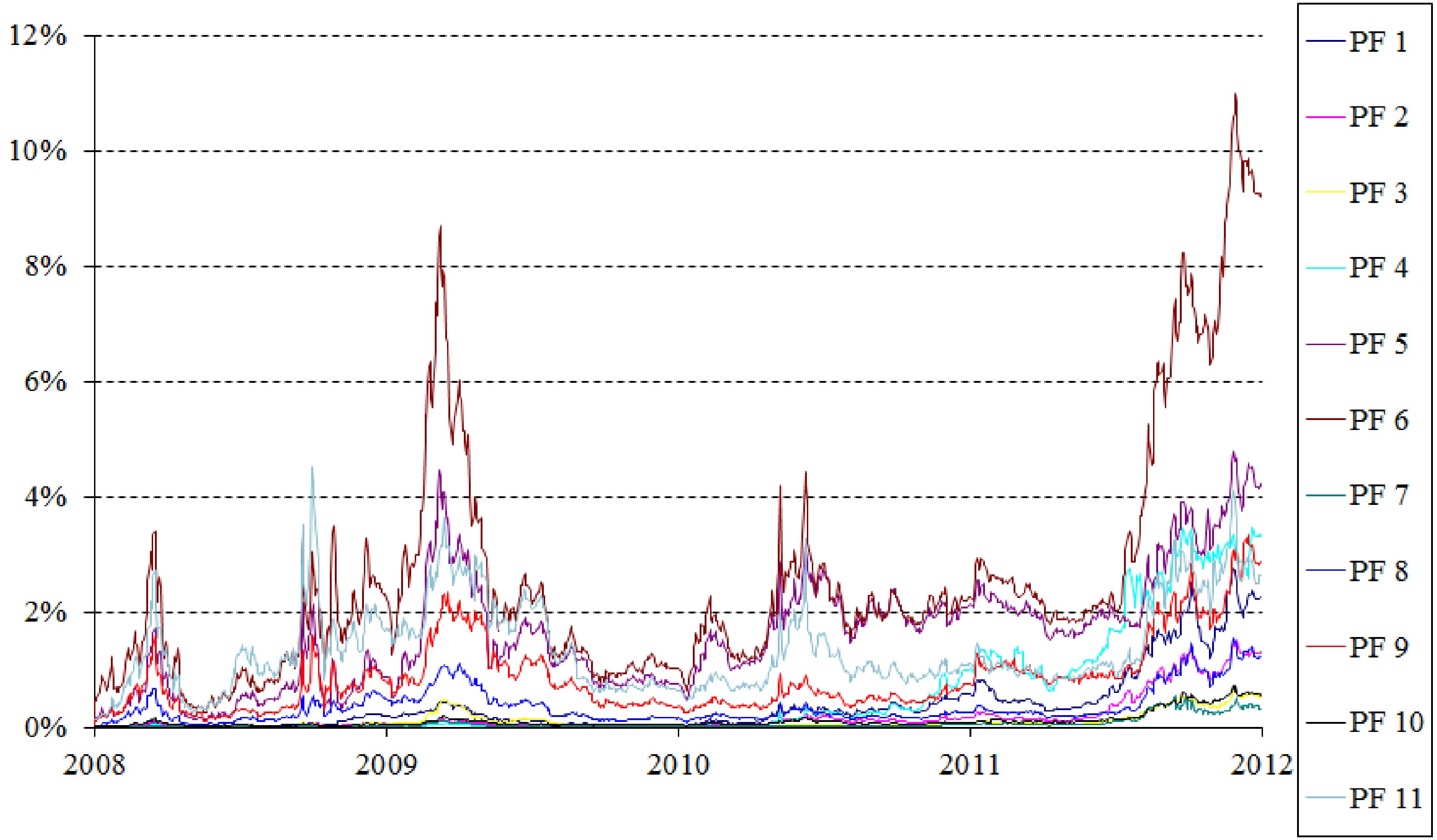


Figure 11: Change in conditional joint probabilities of default ($\Delta CoJPoD$) involving 11 portfolios of 44 banks and 1 sovereign for the period 01.01.2008 - 31.12.2011. Each portfolio includes 4 banks and 1 sovereign. Spain is considered uniformly as a default-risk-triggering sovereign. Only the series conditioning on default of Spain are presented for each portfolio. The assignment of a bank to a portfolio is governed by its **liquidity and funding**, compared to the rest, sorted in descending order. E.g. Portfolio 1 (PF 1) includes the four banks with highest value of *net loans to total assets*. The set of banks is presented in Tables 2 and 3. The complete portfolio assignment is presented in Table 4, column DL. Non-zero correlation structure is used in defining the prior distribution. The correlation structure calculation is explained in the text and in Table 1. $\Delta CoJPoD$ is calculated analogously to the sovereign case. The financial data used to calculate the doubtful loans ratio are denominated in euro.



C Tables

Table 1: Correlation structure between 10 sovereigns: Austria (AT), Belgium (BE), France (FR), Germany (GE), Greece (GR), Ireland (IE), Italy (IT), Netherlands (NL), Portugal (PT), Spain (ES). Period: 01.01.2008 - 31.12.2011. The correlations are calculated between changes in the 5-year CDS spreads of the sovereigns in the respective column and row.

	AT	BE	FR	GE	GR	IE	IT	NL	PT	ES
AT	1.00	0.70	0.73	0.75	0.13	0.54	0.66	0.79	0.46	0.62
BE		1.00	0.82	0.74	0.16	0.69	0.83	0.72	0.65	0.81
FR			1.00	0.82	0.22	0.63	0.81	0.73	0.60	0.76
GE				1.00	0.19	0.59	0.72	0.75	0.56	0.69
GR					1.00	0.19	0.21	0.16	0.17	0.17
IE						1.00	0.71	0.55	0.77	0.74
IT							1.00	0.68	0.71	0.90
NL								1.00	0.49	0.63
PT									1.00	0.73
ES										1.00

Table 2: List of euro area banks used in our analysis. The set of banks is chosen to cumulatively represent minimum 50 % of the banking industry in the respective country.

Euro Area Banks		
	Country code	Name
1	AT	Erste Group Bank AG
2	AT	Raiffeisen Bank International Austria
3	BE	Dexia SA
4	BE	KBC Groep NV
5	DE	Bayerische Landesbank
6	DE	Commerzbank AG
7	DE	Deutsche Bank AG
8	DE	Landesbank Berlin Holding AG
9	ES	Banco Bilbao Vizcaya Argentaria
10	ES	Banco de Sabadell SA
11	ES	Banco Santander SA
12	ES	Bankinter SA
13	FR	Banque Federative du Credit Mutuel
14	FR	BNP Paribas
15	FR	Credit Agricole SA
16	FR	Natixis
17	FR	Societe Generale
18	GR	Alpha Bank AE
19	GR	EFG Eurobank Ergasias SA
20	IE	Allied Irish Banks PLC
21	IE	Governor & Co of the Bank of Ireland
22	IE	Irish Life and Permanent
23	IT	Banca Monte dei Paschi di Siena
24	IT	Banca Popolare di Milano
25	IT	Banco Popolare SC
26	IT	Intesa Sanpaolo SpA
27	IT	UniCredit SpA
28	IT	Unione di Banche Italiane SCPA
29	NL	ING Groep NV
30	NL	Rabobank
31	NL	SNS Bank Netherlands
32	PT	Banco BPI SA
33	PT	Banco Comercial Portugues SA
34	PT	Espirito Santo Financial Group

Table 3: List of additional non-euro area European Union banks used in our analysis. The set of banks is chosen to cumulatively represent minimum 50 % of the banking industry in the respective country.

Other European Union Banks		
	Country code	Name
1	DK	Danske Bank A/S
2	GB	Barclays PLC
3	GB	HSBC Holdings PLC
4	GB	Lloyds Banking Group PLC
5	GB	Royal Bank of Scotland Group
6	GB	Standard Chartered PLC
7	SE	Nordea Bank AB
8	SE	Skandinaviska Enskilda Banken
9	SE	Svenska Handelsbanken AB
10	SE	Swedbank AB

Table 4: Ranking assignment in descending order of the 44 banks used in our analysis with respect to 10 financial characteristics. The numbers in the columns for the financial characteristics come from the ordering in Tables 2 and 3. The values from 35 to 44 are given to the non-euro area European Union banks in the same order as in Table 3. PF 1 to PF 11 list the 4 banks that are included in the final 5-entity portfolios for each financial characteristic. The abbreviations stand for, as follows: total assets (TA), return on equity (ROE), return on assets (ROA), net interest margin (NIM), efficiency ratio (ER), deposits-to-funding (DF) ratio, assets-to-equity (AE) ratio, loan-loss-provisions-to-net-interest-income (LLP-to-NII) ratio, non-performing-loans-to-total-loans ("doubtful loans", DL) ratio, net-loans-to-total-assets (NL-to-TA) ratio.

		Financial Characteristics									
Ranking	Portfolios	TA	ROE	ROA	NIM	ER	DF	AE	LLP-to-NII	DL	NL-to-TA
1		39	11	2	2	4	37	3	20	20	31
2	PF 1	14	27	8	32	20	40	8	21	25	28
3		7	39	11	19	16	22	7	22	23	33
4		37	37	10	18	32	18	22	38	27	10
5		36	7	9	1	35	29	31	3	21	18
6	PF 2	15	9	43	9	8	1	13	31	26	12
7		29	40	41	11	6	32	29	39	1	30
8		11	36	26	40	24	7	6	5	38	24
9		17	38	12	24	28	27	15	19	18	21
10	PF 3	27	26	23	28	38	23	35	33	2	44
11		38	14	32	23	25	4	36	37	8	43
12		6	13	18	37	23	2	5	16	19	25
13		26	30	1	26	39	19	14	18	6	32
14	PF 4	30	17	42	10	15	30	38	36	17	23
15		3	6	19	27	1	38	21	29	39	19
16		9	4	44	25	26	33	43	25	13	20
17		41	1	36	20	29	11	16	6	36	34
18	PF 5	16	41	24	4	22	9	42	17	14	2
19		35	35	37	33	17	24	32	1	15	9
20		13	23	40	34	30	10	41	27	10	38
21		5	2	28	38	13	28	17	2	4	1
22	PF 6	4	3	33	21	33	20	12	10	31	11
23		40	15	14	30	31	21	20	44	9	27
24		42	29	29	36	27	39	39	14	16	26
25		43	28	17	39	34	42	44	11	35	35
26	PF 7	23	19	34	12	14	25	4	9	33	3
27		1	43	27	41	42	31	37	34	11	41
28		21	25	6	35	36	41	9	23	28	22
29		44	42	25	44	12	6	30	12	37	29
30	PF 8	20	44	7	31	2	26	10	13	5	42
31		8	18	30	43	40	36	11	15	7	5
32		25	33	35	14	37	15	40	28	12	40
33		28	10	15	29	44	34	27	26	24	6
34	PF 9	2	34	13	15	41	12	19	24	22	4
35		33	32	38	17	10	35	33	35	44	37
36		10	24	16	42	18	44	1	42	32	39
37		31	8	22	6	5	17	23	32	40	13
38	PF 10	34	12	5	13	19	8	34	4	30	17
39		19	16	31	7	43	43	18	41	29	8
40		22	22	21	3	9	5	24	30	34	14
41		18	31	4	8	11	13	2	40	42	15
42	PF 11	12	21	3	5	21	14	26	8	41	36
43		24	5	39	16	7	3	25	7	3	16
44		32	20	20	22	3	16	28	43	43	7