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Capital Regulation with Heterogeneous Banks
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Capital Regulation with Heterogeneous Banks*

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Abstract

We study the impact of capital regulation on the quality of the banking sector in the presence of heterogeneous banks. Closely related to Morrison and White (2005), we provide a general equilibrium framework with heterogeneous individuals that differ in their ability of successfully completing a risky investment project. In addition to the classical moral hazard problem, we identify an additional countervailing selection problem of a stricter capital regulation. More regulatory capital decreases the deposit rate and mitigates the severity of the moral hazard problem. This decrease in the deposit rate, however, comes at the cost of a worsening of the selection problem. We show that rising heterogeneity in the banking sector increases the allocation effect and thus, improves the selection among individuals.

Keywords: bank regulation, risk-taking, financial stability.

JEL-Classification: G21, G28.

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1 Introduction

Hardly any issue has been discussed as intensively in the most recent years as the appropriate amount of bank equity. Most arguments refer to a quantitative effect of too strict regulation,\footnote{For example, in 2009, Josef Ackermann, former CEO of Deutsche Bank, stated in an interview that “more equity might increase the stability of banks. At the same time however, it would restrict their ability to provide loans to the rest of the economy”, see Ackermann (2009). On the other hand, Admati, DeMarzo, Hellwig, and Pfleiderer (2011) point out that the common arguments against too much equity are either fallacies, irrelevant facts, or even myths. In particular, they argue that higher capital requirements do not force banks to reduce lending activities since higher regulatory equity does not require to set capital aside or to hold additional reserves.} and only little is said about potential qualitative effects. While Admati, DeMarzo, Hellwig, and Pfleiderer (2011) argue that higher capital requirements improve the quality of bank lending decisions, we provide a general equilibrium framework coming along with a second countervailing effect of capital regulation on the quality of the banking system.

We argue that an economy with heterogeneous banks may suffer simultaneously from a moral hazard problem and a selection problem among individuals. As the analysis of Admati, DeMarzo, Hellwig, and Pfleiderer (2011) proposes, we restrict the tools of the regulator to minimum capital requirements. While stricter capital requirements make banks to have more ‘skin in the game’ and thus reduce the moral hazard problem, they also increase the selection problem. Individuals who are best able to run banks are not allowed to absorb the entire supply of debt when they cannot immediately raise new equity. Hence, qualitatively worse agents become banks. In addition, the size of the banking sector is an important factor, as the arguments of the banking lobby and the literature related to the problem of a credit crunch suggest. In our model, the agents’ endogenous decision whether to open a bank or to become a debtor implies the efficient size of the banking sector. Only very strict capital requirements may also reduce the volume of invested funds, which forces individuals to invest inefficiently into a risk-free asset instead of depositing with a bank. We conclude that the regulator must try to balance the three effects in order to maximize the efficiency of the economy. We therefore incorporate the view of Admati, DeMarzo, Hellwig, and Pfleiderer (2011) that strict capital requirements may be adequate to solve moral hazard problems, but we present a channel beside the common arguments how capital requirements may affect negatively the efficiency of the economy. Particularly, it may reduce the size of the banking sector and introduce a severe...
selection problem.

Our work enters a large list of papers dealing with the relationship of banking regulation and bank risk-taking behavior. This strand of the literature, however, has not concluded to a clear evidence so far. Within the theoretical literature, a positive relationship between regulation and risk-taking has been found in, e.g., Koehn and Santomero (1980), Kim and Santomero (1988), Rochet (1992), Gennotte and Pyle (1991), Blum (1999), Hakenes and Schnabel (2011), while the contrary result is present in, e.g., Furlong and Keeley (1989) and Hellmann, Murdock, and Stiglitz (2000). Empirically, Shrieves and Dahl (1992), Aggarwal and Jacques (2001), and Rime (2001) identify a positive relationship between regulation and risk-taking, while Jacques and Nigro (1997) find lower risk levels in response to an increase of banks’ capital. Our results relate to the findings in the theoretical work of Calem and Rob (1999) and Diamond and Rajan (2000) and the empirical result of Heid, Porath, and Stolz (2004), who show that the effect of regulation on bank risk-taking is ambiguous. As most of the existing theoretical work relies on homogeneous banks, we argue similar to VanHoose (2007) that a homogeneous banking sector is inconsistent with reality and that there are many dimensions along which banks differ.² Within a world of heterogeneous banks, there are some agents replying to stricter regulation with a lower risk-taking while other agents increase their risk structure.

Regarding the theoretical setup, our paper is closely related to the work of Morrison and White (2005). In their paper, agents make their decision about collecting deposits from other individuals and opening a bank, investing only their own funding resources into a risky project, or depositing their initial endowment with a bank. Individuals receive some fee per unit deposits when they decide to open a bank, but only a portion of individuals has the ability to monitor an investment project and thus, has a higher probability to run this project successfully. In order to examine the role of banking regulation, they include a welfare maximizing regulator armed with three policy instruments. She can define minimum capital requirements, she awards licenses for running a bank, and since screening applicants is not perfectly possible, the regulator can close a bank if there is some indication that it is managed by a low-ability manager. One key assumption is that the regulator always assigns as many licenses as there are sound individuals. The goal of her actions is to incentivize high-ability individuals to run a bank and monitor the risky

²For example, Demirgüç-Kunt and Huizinga (2010) show that banks are heterogeneous with respect to their fee income share and that those banks with large non-traditional banking activities on the one hand tend to have a higher return on assets, but on the other hand tend to be individually more risky.
investment projects while the low-ability individuals deposit their funds with a bank. So the regulator tries to solve a moral hazard problem and an adverse selection problem, but also controls the size of the banking sector. Morrison and White (2005) show that if the costs for monitoring are low, there is no need for a regulator, since the high-ability individuals will run banks and monitor their risky projects while the low-ability individuals will deposit their endowment with a bank. For higher values of monitoring costs, however, there is a need for a regulator since on the one hand, high-ability individuals may not have incentives to monitor any longer (moral hazard) and on the other hand, low-ability individuals want to run a bank (adverse selection). It follows that a stricter regulation in terms of auditing improves the moral hazard problem and tighter capital requirements mitigate the adverse selection problem since it pushes the low-ability banks out of the market. In addition, it decreases the size of the banking sector, leading to an inefficiently small amount of monitored investment projects.

However, by slightly changing the framework of Morrison and White (2005), we demonstrate that the selection effect might be countervailing to the moral hazard effect. Particularly, we endogenize the deposit rate and introduce an outside option to the agents in a way that they can also invest into a risk-free asset (costless storage technology). Moreover, the regulator’s single tool is to define a minimum capital adequacy ratio. It appears that the deposit market allocates, for a given regulation, the individuals into banks and depositors in a welfare maximizing way. In our model, auditing as well as predefining a fixed number of banks are not instruments of the regulator’s toolbox, contrary to Morrison and White (2005). We abstract from those tasks since they do not provide a realistic reflection of current banking regulation. First, although the current regulation demands a large set of information and requirements before opening a bank, they do not limit the licenses up to a particular number, see Board of Governors of the Federal Reserve System (2013). Second, to our opinion, auditing in terms of closing a bank before the realization of any return is rather unrealistic. In Morrison and White (2005), auditing is modeled as bank closing after evaluating its business strategy in terms of investment and monitoring choice. In general, however, the regulator has the power to close a bank only in response to an malfunction which is hardly detected ex ante a project return has realized.

While in Morrison and White (2005) the number of banks is fixed and hence, for every ‘bad’ bank a ‘good’ bank must disappear, this effect is a classical adverse selection. In our model, however, market entry by a ‘bad’ bank does not force a ‘good’ bank to disappear. Therefore we call the adjustment of the average ability of bankers selection effect.
More precisely, we consider a continuum of individuals, which are heterogeneous with respect to an unobservable ability to successfully complete investment projects, drawn from the unit interval. The individuals have the same decision set as in Morrison and White (2005) with an additional outside option. They can invest their initial endowment into a risk-free asset, deposit it with another individual or open a bank by taking deposits and investing into a risky project. It turns out that individuals with a high success probability decide endogenously to invest into a risky investment project, and individuals with a low success probability prefer to lend their funds in the deposit market to ‘better’ individuals. The deposit rate is endogenized and tries to balance demand and supply.

We then show that the deposit market solves the selection problem, since it serves as a vehicle to transfer funding resources to those individuals with highest abilities to successfully run investment projects. We further introduce in our model the classical Myers and Majluf (1984) moral hazard problem, which arises due to limited liability. Giving banks the possibility to blow up their balance sheet by increasing the amount of deposits taken, banks increase project risk to mitigate their success probability in order to decrease the expected value of depositors’ claims against the bank. In order to illustrate the impact of various levels of capital requirements on the size, the composition and the riskiness of the banking sector, we assume an exogenous regulator whose only tool is to set the minimum capital requirements for banks.\textsuperscript{4}

It turns out that the selection problem and the moral hazard problem have countervailing effects on banks’ riskiness. The economic argument is straightforward. If the regulator strengthens regulation by demanding higher capital requirements, she decreases ceteris paribus the demand for deposits. The resulting decline of the deposit rate incentivizes some depositors to open a bank, hence increases the selection problem. Since those banks unambiguously have a lower ability than the already existing banks, the average ability of bankers decreases, increasing average riskiness of banks. However, a lower leverage and a decrease in funding costs leads banks to decrease project risk, mitigating the moral hazard problem. Hence, the overall effect of regulatory changes on aggregate project risk is ambiguous. The result holds as long as capital requirements are not too strict, so that all individuals prefer running a bank or depositing their funds rather than investing into the risk-free asset. If regulation is very strict, demand in the deposit market is too low to imply an equilibrium interest

\textsuperscript{4}One could endogenize the role of the regulator, e.g., by giving him the goal to maximize aggregate payoff of the economy and at the same time minimize the potential negative spillover effects to depositors. Weighting these goals differently, one would obtain different levels of capital requirements.
rate for which all individuals want to participate in the deposit market. Hence, the volume of managed funds in the banking sector shrinks, imposing a size effect. In addition, the moral hazard effect and the allocation effect are no longer countervailing. Although stricter regulation then mitigates both selection problem and moral hazard problem, the size effect is dominant so that the benefits of a larger banking sector always outweigh the costs of a more pronounced moral hazard behavior and a lower average ability of the banking sector.

Our model suggests that the degree of heterogeneity plays a crucial role in determining the optimal minimum capital requirements. Since the strength of the allocation effect increases and the moral hazard effect diminishes for higher dispersion of the individuals' ability, rising heterogeneity implies decreasing optimal capital requirements. Differences in the quality of bank lending decisions give rise to gains of specialization that improve the quality of banks and decreases the interest rate on deposits.

We conclude that, in a general equilibrium framework, the deposit market provides an important channel through which banking regulation can control the selection and the moral hazard problem in the banking sector. In particular, we show that the regulator faces the challenge to balance gains from a beneficial allocation of funding resources and costs from moral hazard behavior appropriately for relatively loose capital requirements. For very strict requirements, however, she must consider an additional size effect, although the problems of selection and moral hazard can be solved simultaneously. By endowing the regulator with a more realistic toolbox and endogenizing the deposit rate as well as the number of banks in the banking sector, we therefore challenge the results of Morrison and White (2005) who argue that, under the assumption of an exogenously given number of banks, stricter capital requirements solve the selection and moral hazard problem simultaneously, coming however at the cost of a smaller banking sector. Our model coincides with this conclusion only for very strict capital requirements and only due to the storage technology opportunity. However, relaxing the regulatory standard shows diametrical results. If regulation is appropriately designed so that the size of the banking sector is maximized, the selection problem and the moral hazard problem are countervailing. Higher capital requirements solve the moral hazard problem but make the selection problem more severe. Hence, the regulator must try to balance the two effects in order to achieve the maximum aggregate payoff in the economy.

The paper is organized as follows. In section 2, we first introduce the basic setup of
the theoretical model as well as the decision structure of all agents. Section 3 describes
the payoffs and business opportunities, illustrates the allocation effect, and introduces
the agents’ outside options. We then present the equilibrium outcome in section 4 and
discuss the impact of capital requirements on the simultaneous problem of selection among
individuals, moral hazard and the size of the banking sector. In section 5, we analyze
the effect of different degrees of heterogeneity on the optimal level of minimum capital
requirements. Section 6 concludes.

2 Model Setup

We follow Morrison and White (2005) with the basic setup of the model and consider a one
period economy with a continuum of risk-neutral agents with mass 1, denoted by \( i \).
All individuals are heterogeneous with respect to an unobservable ability \( a_i \), \( a_i \sim U(0,1) \).
We interpret this competence as different levels of efficiency in monitoring and project
screening. Each agent is endowed with capital \( C \).
She may use this amount for one of
three different investment opportunities and she consumes the endowment plus returns
at the end of the period. The three investment opportunities are as follows: First, she
can run an investment project chosen from a whole set of projects, \( y_i \in [0,1] \) with a
risk-return structure à la Allen and Gale (2004), i.e., the success probability is decreasing
in the return of the project, \( \left( \frac{\partial p(y_i, a_i)}{\partial y_i} \leq 0 \right) \).
We assume the success probability to be
increasing in the unobservable ability of the agent \( \left( \frac{\partial p(y_i, a_i)}{\partial a_i} \geq 0 \right) \).
Particularly, we take
the functional form \( p(y_i, a_i) = (1 - y_i) a_i \). The investment pays a return \( x \cdot y_i \) in case of
success and zero otherwise, where \( x \) is a constant scaling factor.
Thus, individuals with
different abilities have different expected returns from investing into the risky projects.

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5 In contrast to Morrison and White (2005), we consider a continuum of agents for the reason of
computational convenience.
6 In section 5, we abstract from the unit interval and analyze different degrees of heterogeneity by
generalizing the uniform distribution. Note that the results of the paper qualitatively do not depend on
the distribution of \( a_i \).
7 Since the continuum of agents is normalized to mass 1, the total endowment in the economy is \( C \).
8 In order to rule out any hedging motives, we restrict individuals to choose only one investment
project.
9 We assume \( x \) to be sufficiently large such that the expected return from investing into the risky
project is high enough to ensure that not all agents will invest only into the risk-free asset. Assuming,
e.g., \( x = 8 \) would ensure that half of the population has a non-negative NPV from investing only its
endowment in the efficient project.
The invested amount is not restricted to the own endowment, but agents are allowed to collect further funds from other agents. We call the agents collecting deposits banks. Thus, a second investment opportunity is to lend an amount $D_i$ of the own endowment to other agents. This lending pays an interest $r$ per unit of $D_i$, which is completely determined by supply and demand. However, deposits can only be repaid if banks have enough cash flow, i.e., only in case of a successful completion of the project. We assume that depositors can verify banks’ investment and returns, which prevents banks from mimicking being insolvent. We further assume a zero correlation structure regarding the return of two different investment projects, $corr(x_{yi},xy_{yj}) = 0, \forall i, j \in I$ in order to rule out any hedging motive for lending agents. Moreover, we make depositors to anticipate the moral hazard behavior of banks and to be able to price risks accordingly in the deposit rate.\footnote{Note that with the existence of a deposit insurance, one could also interpret the banks’ payments to depositors as the sum of payments to the depositors and a deposit insurance where depositors receive a fixed amount corresponding to the expected repayment without a deposit insurance.}

The third investment opportunity is to store the endowment in a risk-free asset, which pays a gross interest rate $r_f = 1$.

We introduce a regulator as an additional agent who has the power to put a minimum capital adequacy on banks. In contrast to Morrison and White (2005), we restrict the toolbox to set minimum capital requirements since it represents the cornerstone of the Basel regulatory framework.\footnote{We abstract from the possibility to close a bank before returns are realized and predefining an exogenous number of banks, respectively. First, although a regulator might in principle have the power to close a bank, she will not do so before any misbehavior has occurred and she is hardly able to detect any malfunction ex ante. Second, a regulator does hardly limit licenses up to a particular number, but allows to open a bank when all obligations are fulfilled.}

The regulator’s adjustment screw implies a leverage ratio, which is defined as the ratio of equity capital $C$ over total assets, $C + D_i$, i.e., $\frac{C}{C+D_i}$. Since $C$ is fixed, she has to define the maximum amount of funds banks can raise from individuals, $D^{max}$. The timing and sequence of events in the model are as follows: First, the regulator defines the minimum capital adequacy banks are required to hold. Second, individuals decide about the choice of investing into the risky project, depositing at a bank, or investing into the risk-free asset. If the risky project was selected, individuals decide simultaneously about becoming a bank, choose the volume of deposits they want to take, as well as the risk-return structure of the risky investment project. Finally, returns are realized and deposits are paid back.
3 Payoffs and Investment Choice

The expected profit of a bank that invests into the risky project consists of the return from the investment project and the costs from borrowing in the deposit market.\textsuperscript{12} Thus, it is given by

\[ E(\pi^y_i) = (C + D_i)(1 - y_i)a_i x y_i - (1 - y_i)a_i r D_i - C. \]

We assume that there will be no bailout of deposits so that individuals lending in the deposit market will only be repaid if the borrowing counterparts can generate enough positive cash flow, which is always the case when they succeed in the investment project. Thus, the individual’s expected profit from lending his own endowment takes the expected ability of banks into account and reads

\[ E(\pi^D_i) = \frac{1}{\int 1_{jbor} dj} \int a_j (1 - y_j) 1_{jbor} dj C r - C, \]

where \( \frac{1}{\int 1_{jbor} dj} \int a_j (1 - y_j) 1_{jbor} dj C r \) denotes the average expected success probability of the borrowing counterparts. Thus, \( \frac{1}{\int 1_{jbor} dj} \int a_j (1 - y_j) 1_{jbor} dj C r \) describes the expected repayment.

All individuals also have the option to invest their funding resources into a risk-free asset. Since the deposit rate has to exceed the risk-free rate, an individual will never borrow for this investment. This gives the expected profit

\[ E(\pi^{RF}_i) = Cr_f - C = 0. \]

In order to find the optimal investment decision as well as the optimal volume in the

\textsuperscript{12}We will later show that there exists only a pooling equilibrium in the deposit market. Therefore, in order to simplify notation, we will never use a subscript \( i \) for the interest rate.
deposit market, agents solve the maximization problem

\[
\max_{\alpha_i, \beta_i, D_i, y_i} E(\pi_i) = \alpha_i \left\{ (C + D_i)(1 - y_i)a_i y_i - (1 - y_i)a_i r D_i - C \right\} \\
+ \beta_i \left\{ \frac{1}{1 - y_i} \int_a \left( (1 - y_j) 1_{j \text{bor}}dj \right) C r - C \right\} \\
+ (1 - \alpha_i - \beta_i) \{ C r_f - C \}
\]  
(1)

s.t.

\[
0 \leq \alpha_i + \beta_i \leq 1 \\
D_i \leq D^{\text{max}} \\
y_i \in [0, 1].
\]

Note that, due to the linearity of the profit functions, agents will only choose one of the possible investment opportunities, i.e., they will either invest into the risky project, become a depositor, or invest into the risk-free asset.

An individual \( i \) will decide to invest into the risky project, i.e., \( \alpha_i = 1 \), if and only if

\[
\frac{\partial E(\pi_i)}{\partial \alpha_i} > 0 \quad \text{and} \quad \frac{\partial E(\pi_i)}{\partial \beta_i} > \frac{\partial E(\pi_i)}{\partial \alpha_i},
\]

it will lend its endowment to a bank (\( \beta_i = 1 \)) if and only if

\[
\frac{\partial E(\pi_i)}{\partial \beta_i} > 0 \quad \text{and} \quad \frac{\partial E(\pi_i)}{\partial \beta_i} > \frac{\partial E(\pi_i)}{\partial \alpha_i},
\]

and it is indifferent between investing and lending if and only if

\[
\frac{\partial E(\pi_i)}{\partial \alpha_i} = \frac{\partial E(\pi_i)}{\partial \beta_i} > 0.
\]

If non of these conditions hold, agents will neither provide deposits, nor act with regard to an investment into the risky project, but only invest into the risk-free asset.

Given agents choose to finance a risky investment project, i.e., \( \alpha_i = 1 \), they face the decision whether or not they want to collect deposits and hence, become a bank as well as about the risk-return structure of the project.
Proposition 1. All agents with \( \alpha_i = 1 \) choose, due to limited liability, an inefficient high project risk \( y_i^* \), which is increasing in the leverage and the deposit rate.

**Proof.** See Appendix A.A. ■

Proposition 2. All agents with \( \alpha_i = 1 \) want to, for a given deposit rate \( r \), either take as much funds as possible, or they want to take no deposits.

**Proof.** See Appendix A.B. ■

It is important to note that, since we consider a continuum of banks, they do not have the market power to influence the interest rate on the deposit market and take it therefore as given.

One might think that the existence of a moral hazard problem results in an interior solution for the optimal amount of deposits for a sufficiently high value of \( D^{\text{max}} \). The reason could be the deterioration of the expected return per unit invested for an increasing deposit volume, which could decrease expected profits large enough to incentivize banks not to collect as much deposits as possible. However, since banks take the deposit rate as given, the decline in expected return is not strong enough to outweigh the benefit from a larger investment volume.

Both the decision about the project choice and the decision about the optimal debt level do not depend on the agent’s unobservable ability. All individuals for whom it is beneficial to choose the investment project will decide for the same project \( y_i^* = y^* \), independent of \( a_i \). Thus, since agents’ ability affects the success probability, they have different expected returns from investing into the risky projects.

### 3.1 Allocation Effect

The intersection of both the expected return of the investment project and the expected return from depositing at a bank for various ability \( a_i \), depicted in Figure 2, ensures that there exists a critical level of ability \( a^* \), above which agents decide not to deposit their
funds with a bank. The remaining fraction of agents with ability \( a_i \in [0, a^*] \) will either deposit their complete endowment with a bank or invest into the risk-free asset. Ignoring any participation constraint and assuming that no individual chooses to invest into the risk-free asset, all agents whose ability exceeds \( a^* \) will open a bank and all remaining agents deposit their funding resources with a bank. Thus, depositors and the regulator know the expected ability of banks to be \( \frac{1}{2}(1 + a^*) \).

**Figure 2:** Expected profit for investing into a risky project or depositing at a bank for a given interest rate \( r \), deposit volume \( D \), and project return \( y_i \).

The market clearing condition for the deposit market allows us to identify the critical ability \( a^* \),

\[
\int_0^{a^*} C\,di = \int_{a^*}^1 D_{\text{max}}\,di \\
\Leftrightarrow \quad a^* = \frac{D_{\text{max}}}{C + D_{\text{max}}}.
\]

Thus, the ability level, above which individuals decide to choose the risky investment project, depends positively on the regulatory maximum of depositor lending \( D_{\text{max}} \).\(^{14}\)

\(^{13}\)Technically, \( \frac{\partial E(\pi^y_i)}{\partial a_i} > 0 \) and \( \frac{\partial E(\pi^D_i)}{\partial a_i} = 0 \) as well as \( E(\pi^y_i|a_i = 0) < E(\pi^D_i|a_i = 0) \) ensures the intersection of both expected return functions.

\(^{14}\)\( \frac{\partial a^*}{\partial D_{\text{max}}} = -\frac{C}{(C + D_{\text{max}})^2} > 0 \).
Equation (2) and Figure 2 provide an illustrative intuition of the selection problem we intend to highlight in our model. For a stricter regulation in the sense of higher capital requirements (decrease of $D_{\text{max}}$), the critical ability $a^*$ decreases, implying that the banking sector might become less stable since the average ability of banks decreases.

### 3.2 Participation Constraints

So far, we have solved the individuals’ maximization problem but neglected any participation constraints. Therefore, we will now implement the natural constraints the agents are facing in order to derive a necessary and sufficient condition for the existence of a debt market.

First, all individuals have the possibility to use only their own endowment for investing into the risky project. This outside option pays the expected profit

$$E(\pi_i^{O1}) = C(1 - y_i)a_ixy_i - C,$$

being maximized for the project $y = \frac{1}{2}$.

Second, individuals are allowed to invest their initial endowment into the risk-free asset, which pays

$$E(\pi_i^{O2}) = Cr_f - C.$$

Thus, in order to set up the constraints for participating in the banking sector for both opening a bank or depositing funds with a bank, we compare the two outside options with the expected profit for individuals being a bank as well as with the expected profit for depositing initial funding resources.

Since all banks choose the maximum possible amount of deposits, $D_i = D_{\text{max}}$, and using the optimal project choice $y^* = \frac{1}{2} + \frac{rd_{\text{max}}}{2(C+D_{\text{max}})x}$, the expected profit for deposit taking and running a bank reads

$$E(\pi_i|y^*) = (C + D_{\text{max}})\left(\frac{1}{2} - \psi\right)a_i\left(\frac{1}{2} + \psi\right)x - \left(\frac{1}{2} - \psi\right)a_irD_{\text{max}} - C$$

with \(\psi = \frac{rd_{\text{max}}}{2(C+D_{\text{max}})x}\).
Thus, the participation constraints for agents to open a bank read

\[
(C + D^{max}) \left( \frac{1}{2} - \psi \right) a_i \left( \frac{1}{2} + \psi \right) x - \left( \frac{1}{2} - \psi \right) a_i r D^{max} - \frac{1}{4} C x a_i \geq 0 \quad \text{(BOR1)}
\]

and

\[
(C + D^{max}) \left( \frac{1}{2} - \psi \right) a_i \left( \frac{1}{2} + \psi \right) x - \left( \frac{1}{2} - \psi \right) a_i r D^{max} - Cr_f \geq 0, \quad \text{(BOR2)}
\]

which, solving for \( r \), give two boundaries for agents investing into the risky project to taking deposits, \( r_{bor}^{1}(D^{max}) \) and \( r_{bor}^{2}(a_i, D^{max}) \).\(^{15}\)

Since the expected profit from banking is decreasing in \( r \), banks are willing to demand additional funding resources in the deposit market if the equilibrium interest rate is below both boundaries. Note that the first boundary does not depend on \( a_i \), implying that either no or all agents are willing to obtain deposits for investing into the risky investment project. However, the second boundary depends positively on \( a_i \).\(^{16}\) Economically, agents with a higher ability can expect a higher return from investing into the risky project and thus, funding costs have to be higher in order to incentivize those agents to invest into the risk-free asset instead of collecting deposits and opening a bank.

In contrast to the participation constraints for being a banker, which are independent from the ability of the depositors, the constraints for lending the endowment to a bank depend on the expected success probability of the depositing bank. Since the ability of bankers is not observable, depositors form expectations about their counterparts’ ability\(^{17}\) as well as their optimal project choice, taking into account that all banks choose the maximum amount of deposits, \( D_i = D^{max} \). Thus, the participation constraints for depositors with respect to a risky investment of their endowment by their own and with respect to investing into the risk-free asset then read

\[
\frac{1}{2} (1 + a^*) \left( \frac{1}{2} - \psi \right) r C - \frac{1}{4} C x a_i \geq 0 \quad \text{(LEND1)}
\]

\(^{15}\)Both participation constraints are quadratic functions in \( r \), so that there exist in both cases two interest rates that fulfill the constraints with equality. However, we can rule out those interest rates that would generate optimal projects \( y_i \notin [0, 1] \).

\(^{16}\)In the following, we will evaluate the second borrowing constraint (BOR2) always at the critical bank ability \( a^* \). Since this constraint is increasing in \( a_i \) and \( a^* \) is the lowest ability level for borrowing banks, the minimum binding participation constraint (BOR2) can only be at \( a^* \).

\(^{17}\)They use the expected value of the bankers’ ability, \( \frac{1}{2}(1 + a^*) \).
and
\[ \frac{1}{2}(1 + a^*)(\psi - \psi) r C - C r_f \geq 0. \] (LEND2)

Solving for \( r \) again delivers two boundaries for individuals to participate as a lender in the deposit market, \( r_{\text{lend}}^1(a_i, D_{\text{max}}) \) and \( r_{\text{lend}}^2(a_i) \). In contrast to the negative relationship of expected profit and funding costs \( r \) for banks, the expected profit for depositors is increasing in \( r \), so that depositing is incentive compatible if the equilibrium interest rate lies above both boundaries. Moreover, the effect of \( D_{\text{max}} \) on the participation constraints is ambiguous. First, the participation constraints depend positively on \( D_{\text{max}} \), which arises since \( a^* \) is an increasing function in \( D \). The intuition is that an increasing \( a^* \) ceteris paribus increases the average success probability of investing banks, hence increasing the expected payoff for depositors. Second, both constraints are negatively depending on the moral hazard effect, which is also increasing in \( D_{\text{max}} \).

Again, one of the participation constraints, i.e., the decision to either deposit or invest into the risk-free asset, does not depend on the individuals’ ability, implying that either all agents or no agent are willing to deposit their endowment rather than investing into the risk-free asset. In contrast, the participation constraint forming the decision about depositing or investing initial capital into a risky project depends positively on the individual abilities \( a_i \). Obviously, since high-ability agents expect higher project returns than low-ability agents, they require a higher interest rate in order to offer additional funding resources in the form of deposits.\(^{18}\)

We know that individuals want to lever up their equity for investing into the risky project and become a bank if the equilibrium interest rate lies below both boundaries for borrowing banks. We further know that there are individuals that are willing to lend if the equilibrium interest rate is above both boundary rates for depositing.

**Proposition 3.** There exists a deposit market if and only if \( r_{\text{bor}}^{1i} \geq r_{\text{eq}} \geq r_{\text{lend}}^{1j} \) and \( r_{\text{bor}}^{2i} \geq r_{\text{eq}} \geq r_{\text{lend}}^{2j} \) are satisfied for at least some individuals \( i, j \in I \) with ability \( a_i \neq a_j \).

**Proof.** See Appendix A.C. ■

\(^{18}\)Similar as borrowing constraint (BOR2), we will evaluate the first lending constraint (LEND1) always at bank \( a^* \). Since this constraint is decreasing in \( a_i \) and \( a^* \) is the highest ability level for depositors, the maximum binding participation constraint (LEND1) can only be at \( a^* \).
4 Equilibrium

Taking the participation constraints into account, we now characterize the equilibrium outcome where either all funds of the economy will be invested into the investment project or a part of the endowments will be tied up in the risk-free asset. Since the deposit market is at the heart of our model, any assumption regarding its mechanism is essential. We suppose that all participating individuals enter the market at the same time, and the matching of banks and depositors is purely random. Note that depositing agents are indifferent between lending to as few banks as possible or to fully diversify their deposits. This is caused by the risk neutrality of individuals, the identical expected ability of the bankers, and the zero correlation between returns of the projects of banks. For the same reason, we can exclude bargaining power for any agent.

In the previous chapter, we have identified situations in which a deposit market exists. Since we are interested in the effect of changes in the minimum capital requirements on banks’ behavior, we first concentrate the analysis on the case in which the capital regulation is binding in the sense that banks want to, but are not allowed to take further deposits.\footnote{The case of an unconstrained equilibrium is analyzed in Proposition 8.} For binding capital regulation we claim that there exists only a pooling equilibrium in the deposit market.

**Proposition 4.** There exists a pooling equilibrium in the deposit market in the sense that every bank gets the same expected volume of deposits $D^{\text{max}}$ at the same market clearing interest rate $r^{eq}$.

**Proof.** See Appendix A.D. \hfill \blacksquare

We define the pooling equilibrium in the following definition:

**Definition 1.** A pooling equilibrium is a set of allocations $\{D_i, y_i, \alpha_i, \beta_i\}, i \in [0, 1], y_i \in [0, 1]$ and a deposit market interest rate $r^{eq}$, such that

- given the deposit rate, the allocation solves each agent’s maximization problem
- the deposit market clears.
Proposition 5. There exists no separating equilibrium, in which banks with different abilities $a_i$ prefer different contracts, i.e., contracts specifying different deposit rates and volumes.

Proof. See Appendix A.E.

The intuition why there exists only a pooling equilibrium is straightforward. First, the individual ability $a_i$ is not observable. Second, the relevant participation constraint for running a bank, BOR1, is independent of $a_i$, which in addition is just a scaling factor for the expected profit from investing into the risky project. Hence, if the borrowing banks have the choice between two or more contracts, different banks always prefer the same contract. Finally, depositors can not distinguish between different abilities of bankers in order to claim ability-dependent deposit rates.

However, the pooling equilibrium differs for various levels of capital regulation with respect to the fund volume invested in the risky project. If the regulator demands a very high leverage ratio, the expected profit of the investment project does not exceed the expected profit from the risk-free asset for some agents. Thus, some part of the endowment of the economy will not be invested in investment projects. Note that depositing those funds is not possible due to the regulatory constraint. We call this situation limited participation equilibrium.

4.1 Limited Participation Equilibrium

The limited participation equilibrium is characterized by a situation in which demand and supply of funding resources can not be equalized by an equilibrium interest rate that fulfills the participation constraints of all individuals. For very strict capital requirements, some agents with a too low success probability can only generate a low expected return from the investment project and thus prefer not to switch to become a bank (see Figure 3). However, since BOR2 is increasing in the ability level, there are still individuals with $a_i > a^*$ for which their individual participation constraint BOR2 is above LEND2 evaluated at $a^*$. The excess supply of funds drives down the equilibrium interest rate to $r_{eq} = r_{lend}^2$ due to a Bertrand price competition argument and will rather be invested into the risk-free asset instead of the investment project. Hence, as in Morrison and White (2005), the
adverse selection problem and the moral hazard problem can be solved simultaneously for very strict capital requirements, but it comes at the cost of an inefficiently high investment volume into the risk-free asset. Moreover, note that the limited participation equilibrium exists only due to the assumption of the additional outside option to invest into the risk-free asset. It would disappear if we drop this investment opportunity, as it is done in Morrison and White (2005). Note further that limited participation does not necessarily require some individuals not to participate in the banking sector, but only that not all agents can deposit their complete funding resources.

Figure 3: Participation constraints and equilibrium interest rate for parameter values $x = 12$ and $C = 1$.

4.2 Full Participation Equilibrium

For any regulatory capital requirement below the very strict ones that result in the limited participation equilibrium, all funding resources of the economy are invested in the risky project. We claim the existence of the full participation equilibrium in the following proposition:

Proposition 6. For $x$ sufficiently large, there exists a full participation equilibrium in the sense that all agents participate by either running a bank or depositing their endowment with a bank.

Proof. See Appendix A.F.
Proposition 7. The allocation that solves the problem is given by:

- \( \forall i \) with \( a_i \in \left[0, \frac{D_{\text{max}}}{C + D_{\text{max}}} \right] : D_i = 0, \alpha_i = 0, \beta_i = 1 \)
- \( \forall i \) with \( a_i \in \left[\frac{D_{\text{max}}}{C + D_{\text{max}}}, 1\right] : D_i = D_{\text{max}}, y_i = \frac{1}{2} + \frac{rD_{\text{max}}}{2(C + D_{\text{max}})}, \alpha_i = 1, \beta_i = 0 \).

The equilibrium interest rate is given by \( r^{eq} = \frac{D_{\text{max}}}{(C + D_{\text{max}})} \).

**Proof.** See Appendix A.G. \( \blacksquare \)

The intuition for the full participation equilibrium interest rate is as follows. Remember that there exists a certain ability level \( a^* = \frac{D_{\text{max}}}{C + D_{\text{max}}} \) for which individuals with a higher ability want to take as many deposits as possible and invest into the risky project and individuals with a lower ability will act as depositors. Remember further that there exists an equilibrium deposit market interest rate that is below the borrowing constraints for all agents with ability above \( a^* \) and above the lending constraints for all agents with ability below \( a^* \). Since the equilibrium interest rate serves as a market clearing price, individuals with exact the critical ability level \( a^* \) must be just indifferent between opening a bank and depositing. We illustrate the equilibrium interest rate for different levels of \( D_{\text{max}} \) in Figure 3.

Note that so far, we have analyzed the case in which the minimum capital requirements are binding. However, there also exists an unconstrained equilibrium for high values of the maximum level of deposits \( D_{\text{max}} \).

Proposition 8. There exists an unconstrained equilibrium with \( D_{\text{uc}}^{\text{max}} \) defined by the intersection of \( r_{1}^{\text{bor}}(D_{\text{uc}}^{\text{max}}) \) and \( r_{1}^{\text{lend}}(D_{\text{uc}}^{\text{max}}, a_i|a_i = a^*) \).

**Proof.** See Appendix A.H. \( \blacksquare \)

Intuitively, the existence of the unconstrained equilibrium stems from the following effect: For high values of \( D_{\text{max}} \), the agent that is indifferent between lending and borrowing has a high ability \( a_i \). This individual requires a high interest rate in order to offer its endowment as deposits. However, this high payment on debt decreases the expected profit for banks and incentivizes them to invest only equity into the investment project. Hence, having an endogenous price for deposits, there exists a natural boundary for deposit taking although the regulator would allow a higher leverage.
4.3 Comparative Statics

After having developed the equilibrium for different levels of capital regulation, we will now analyze in more detail the role of the deposit market and the effects emanating with respect to regulation.

Formally, we define the allocation effect stemming from the selection problem as the difference between the average ability of the pool of banks and the average ability of all individuals:

$$AE = \frac{1}{2}(1 + a^*) - \frac{1}{2} = \frac{D_{\text{max}}}{2(C + D_{\text{max}})}.$$

Thus, a stricter regulation leads to a decline in $a^*$, implying that some agents with a lower success probability decide to become a bank instead of being a depositor. Hence, ceteris paribus, a stricter regulation decreases the average success probability of banks.

It is convenient to define the moral hazard effect in our model as the exceedance of the project risk over the efficient project,

$$MHE = \frac{rD_{\text{max}}}{2(C + D_{\text{max}})x}.$$

Decomposing the effect of a stricter regulation on the moral hazard effect demonstrates the second effect imposed by the deposit market on banks’ risk-taking.

$$\frac{\partial MHE}{\partial D_{\text{max}}} = \frac{rC}{2(C + D_{\text{max}})^2x} + \frac{\partial r}{\partial D_{\text{max}}} \frac{D_{\text{max}}}{2(C + D_{\text{max}})x} > 0.$$

Change of moral hazard effect
due to deposit market interest rate

Since a stricter regulation decreases the demand for deposits, some individuals have to switch from depositing to being a bank, which, ceteris paribus, decreases the average success probability in the pool of depositors. Since the ‘best’ agent in the pool of depositors has now a lower success probability and hence a lower expected return from its outside option to run a bank, depositors are now willing to offer their funding resources at a
lower interest rate, which implies a reduction of the moral hazard effect.\textsuperscript{20} Therefore, this second effect is countervailing to the allocation effect and mitigates the negative impact of moral hazard behavior on the aggregate payoff in the economy.

Note that the remaining reaction of the moral hazard effect on a stricter regulation is due to the classical problem of limited liability. If the regulator strengthens regulation by demanding a larger share of equity (decrease in $D_{\text{max}}$), banks choose less risky projects since they have more ‘skin in the game’.

Thus, the regulatory austerity leads first to a worsening of the pool of banks, increasing the selection problem. Second, a lower deposit rate decreases the moral hazard problem in addition to its reduction through a lower leverage.

The first result we want to highlight is the fact that changes in capital requirements in the region of full participation do not affect the volume of managed funds in the banking sector. One concern of Morrison and White (2005) with regard to tighter capital requirements is a welfare mitigating decrease of the whole banking sector. However, endogenizing the deposit market interest rate and reducing the instruments of the regulator to minimum capital requirements implies that the deposit market takes the role of controlling the number of banks and the volume of managed funds in the banking sector. It appears that the number of banks and the size of the banking sector are disentangled. This is one of the key differences to the model of Morrison and White (2005), where the number of banks is fixed by the number of licenses and hence, stricter capital requirements directly decrease the volume of managed funds. Moreover, the introduction of a deposit rate that equalizes demand and supply implies that the selection problem is controlled by the deposit market. It incentivizes agents with a low ability to deposit their funds with a bank and high-ability agents to open a bank. However, a regulatory change in terms of stricter capital requirements affect the average success probability of the pool of banks in three different ways. First, a stricter regulation has an immediate effect on the banks’ project choice. Since banks have ‘more skin in the game’, their incentive to take excessive risks diminishes. Second, the average ability of the pool of banks gets worse. Low ability agents decide to open a bank which increases the number of banks, but decreases the average success probability of all investing agents and thus, ceteris paribus, the expected return to depositors. The third effect of stricter regulation again affects the moral hazard

\textsuperscript{20}See Appendix B for a analytical derivation of the dependence between the equilibrium interest rate and $D_{\text{max}}$.\hfill
behavior of banks. After some agents changed from being a depositor to being a bank, the remaining supply side of debt has a lower ability and thus accepts a lower deposit rate. This lower debt rate translates into a decline of the optimal project risk chosen by banks. Hence, our result of a countervailing moral hazard and selection problem with a constant volume of bank-financed investment projects is contrary to the findings of Morrison and White (2005).

Our second result shows that the effects of stricter capital requirements in a situation of limited participation are different to the full participation case. In such a situation, the effect of higher capital requirements on the selection problem has the opposite direction and hence, works in the same course as the moral hazard effect. Since banks are forced to reduce their total assets, they require a higher margin to run the bank. Since banks with a low ability cannot generate this margin, they will drop out of the market. Thus, after a regulatory austerity, the banking sector has a higher average ability. However, the size of the banking sector and hence, the amount that is invested into the risky project decreases. In some sense, these results are similar to the findings of Morrison and White (2005), but note that the limited participation case only occurs because we introduce the risk-free asset as an additional investment opportunity.

Figure 4: Average success probability and Aggregated expected profits for parameter values $x = 12$ and $C = 1$.

Figure 4 shows the average success probability of the risky investment and the expected total profits in the economy. In the limited participation case, the average success probability is decreasing in the deposit volume, since both the selection and the moral hazard
problem get more pronounced. However, in the area of full participation, where the selection and the moral hazard problem are countervailing, the concavity of the average success probability demonstrates that the sensitivity of the allocation effect with respect to $D^{\text{max}}$ may be relatively strong for higher levels of capital requirements, but may be outweighed by the sensitivity of the moral hazard effect for a looser regulation.\(^{21}\) The intuition is that for high values of $D^{\text{max}}$ only some agents become a bank and borrow in the deposit market in order to invest into the risky asset. If the regulation gets looser, these few banks increase demand in the deposit market only by a small amount. Hence, only a small number of agents switches from being a depositor to being a bank, which translates into a small change of the allocation effect.

The implementation of a tighter capital regulation has heterogeneous effects on agent’s riskiness if we consider a full participation equilibrium. Note that the decision about the risk-return structure implies a differentiated picture. First, those agents for whom it was optimal to be a bank already before the regulatory reform are only prone to the moral hazard effect. Since a stricter regulation incentivizes these banks to choose less risky projects, their riskiness unambiguously decreases.

Second, those agents who have been depositors before the reform, but now switch to collect deposits and invest into the risky project are affected by both the allocation and the moral hazard effect. They replace their counterparty risk from lending their endowment to a bank by their individual project risk. On the one hand, the pool of banks ceteris paribus has a lower average success probability than the former pool. On the other hand, all banks choose less risky projects. Hence, for this group, the effect of a stricter regulation on the riskiness is ambiguous.

Finally, agents still lending in the deposit market face an ambiguous effect regarding the riskiness of their portfolio. Banks choose less risky projects, but the pool of banks gets worse. Thus, this group of agents is prone to a mitigation of the moral hazard problem and a worsening of the allocation of funding resources, too. The relative impact of the two effects depends on the level of regulation, $D^{\text{max}}$.

Note that, although the group of agents which have been banks before and after the reform is not affected by the allocation effect, this group’s relationship between regulation and risk-taking is also influenced by the deposit market since they choose the riskiness of their own projects.
projects partly according to the deposit market interest rate. Tighter capital requirements
decrease the deposit rate, which motivates banks to finance less risky projects.

In the limited participation equilibrium, we have a decline in risk-taking for all banks,
since a stricter capital regulation decreases the moral hazard effect and increases the
allocation effect. Intuitively, banks choose less risky projects and the pool of banks gets better. Therefore, the quality of lending agents’ portfolios of deposits unambiguously improves.

5 Degree of Heterogeneity

In this chapter, we analyze the effect of different degrees of heterogeneity on the optimal
level of minimum capital requirements. For this purpose, we will weaken the assumption
that the individual ability is distributed on a unit interval and introduce a mean preserving
spread as a more general notation. More precisely, all agents are different with respect
to an unobservable ability \( a_i \), \( a_i \sim U(a, \bar{a}) \), but the mean of the distribution is still assumed to be \( a^{mean} = 0.5 \). We define \( \Delta_a = \bar{a} - a \) as the degree of heterogeneity. Note
that the representation of the payoffs and the investment choices of section 3 still apply.

As in section 3.1, there exists a critical ability \( a^* \), below which agents will either deposit
their endowment at a bank or invest into the risk-free asset. The remaining fraction
of agents with ability \( a_i \in [a^*, \bar{a}] \) will decide in favor of the investment project. Thus,
depositors as well as the regulator know the expected ability of banks to be \( \frac{1}{2}(a^* + \bar{a}) \). Using the market clearing condition for the deposit market, we find the critical ability for
any level of heterogeneity

\[
a^* = \frac{aC + \pi D_{max}}{C + D_{max}}.
\]

Thus, the critical ability depends positively on the regulatory maximum of deposit bor-
rowing, \( D_{max} \), and the strength of the effect is increasing in the degree of heterogeneity.

The three effects of stricter regulation, derived in section 4.3, differ in their reliance on
the degree of heterogeneity. First, the direct moral hazard effect resulting from banks

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22 In order to focus only on the heterogeneity of ability in terms of variance, we fix the mean of the
distribution at the same level as in the previous chapters. We thus keep the results comparable and
avoid a level effect introduced by differences in the mean. For the reason of simplicity, we still assume a
uniform distribution.

23 \( \frac{\partial a^*}{\partial D_{max}} = \frac{\Delta_a C}{C + D_{max}} > 0 \).
having ‘more skin in the game’ does not depend on the overall distribution of the ability level. In the maximization problem, banks make their decision regarding the investment project only based on their own ability.

Second, the selection effect does depend directly on the heterogeneity of the banking sector. Due to the mean preserving spread, the average ability of banks has changed, while the average ability of all individuals remains constant. With the lower bound \( \underline{a} \) and the upper bound \( \overline{a} \) of the distribution of the ability, the allocation effect reads

\[
AE = \frac{(2\overline{a} - 1)D_{max}^2}{2(C + D_{max})}.
\]

Similar to section 4.3, stricter regulation implies a decline in the critical ability. Thus, agents with a lower success probability decide to become a bank instead of lending their endowment to better agents. Hence, ceteris paribus, a stricter regulation decreases the success probability of banks. However, the strength of the allocation effect depends on the degree of heterogeneity. More precisely, the change of the allocation effect originating from a stricter regulation is the stronger the higher the degree of heterogeneity. The economic intuition is straightforward. The loss in average ability of banks through a stricter regulation gets stronger with a larger dispersion in the individuals’ ability.

Finally, the additional moral hazard effect though the equilibrium interest rate on debt is also shown to be sensitive to the heterogeneity of agents’ ability, since it affects the level of the critical ability \( a^* \). With a more general degree of heterogeneity, the equilibrium interest rate reads \( r^{eq} = \frac{\pi(D_{max}^2 - C) + C}{(C + \pi D_{max})^2} x \). This interest rate on debt is still decreasing in the stringency of regulation, which, ceteris paribus, increases the success probability of banks. The strength of this effect is increasing in the degree of heterogeneity. Moreover, the interest rate declines in the dispersion of the agents’ ability.\(^{24}\) Intuitively, this result stems from the fact that the average ability of those agents who decide to become a bank unambiguously increases with increasing heterogeneity. Hence, due to the increase in the average success probability of banks, depositors accept a lower compensation rate for the riskiness of their deposits.

\(^{24}\)\( \frac{\partial r^{eq}}{\partial D_{max}} = \frac{\pi^2 C}{(C + \pi D_{max})} x > 0, \frac{\partial^2 r^{eq}}{\partial D_{max}^2} = \frac{2\pi C^2}{(C + \pi D_{max})^3} x > 0, \) and \( \frac{\partial r^{eq}}{\partial \overline{a}} = -\frac{C^2 x}{(C + \pi D_{max})} < 0. \)
Analytically, the moral hazard effect can be expressed in the more general case as

\[
MHE = \frac{D_{max}(\bar{a}(D_{max} - C) + C)}{2(C + \bar{a}D_{max})(C + D_{max})}.
\]

It is increasing in the level of debt, and this increase depends on the degree of heterogeneity as well as the ratio \( \frac{C}{D_{max}} \).

A natural objective for the regulator is the maximization of expected aggregated profits in the economy,

\[
AP = \frac{1}{\bar{a} - a^*} \int_{a_i}^{\bar{a}} [(C + D_{max})(1 - y)a_iyx - (1 - y)a_irD_{max}] da_i - C
= \frac{\bar{a} - a^*}{\bar{a} - a} \left[ (C + D_{max}) \frac{1}{2}(\bar{a} + a^*)(1 - y)yx - \frac{1}{2}(\bar{a} + a^*)(1 - y)rD_{max} \right] - C.
\]

Figure 5: Optimal regulatory maximum amount of deposits \( D_{max} \) for parameter values \( x = 12 \) and \( C = 1 \).

In our model, expected profits are entirely generated by banks and then distributed among all individuals via the deposit market. Since the characteristic polynomial for the optimal regulatory maximum amount of deposits is of degree 5 and there does not exist a general analytical solution, we have solved the problem numerically, illustrated in Figure 5 for a maximum ability \( \bar{a} \in [0.5, 1] \).\(^{25}\) Not surprisingly, the optimal regulation for a decrease

\(^{25}\)See Appendix D for a derivation of the characteristic polynomial.
in the degree of heterogeneity approaches towards $D^{max} = 0$. In the extreme case where all agents have the same success probability, they would be indifferent between investing and lending, and since there would be no market for deposits, there would be no moral hazard behavior. For increasing heterogeneity, however, Figure 5 indicates that optimal regulation should allow debt financing up to a certain degree. Moreover, the optimal level of debt is increasing in the degree of heterogeneity. Economically, differences in the quality of lending decisions imply positive effects of specialization. High-ability agents can generate the highest expected payoff from investment projects and depositors can benefit from that by providing funding to them. However, debt financed investment projects imply a moral hazard behavior that undermines the positive effects of specialization.

6 Conclusion

This paper has analyzed the impact of a simultaneous problem of moral hazard and selection among individuals on the relationship between banking regulation and bank risk. To this end, we modify the model of Morrison and White (2005) by endogenizing the deposit market interest rate and restricting the regulator’s responsibility to the policy of a minimum capital adequacy. We remove the task to predefine exogenously a fixed number of banks and to exert auditing in terms of closing a bank before any return has realized. To our view, both additional tasks in Morrison and White (2005) hardly reflect a realistic representation of banking regulation. First, there is no fixed number of banking institutes, but a bank license is granted to any applicant who is able to meet all relevant requirements. Second, a regulator is hardly able to know project returns before those projects mature and she can merely close a bank just because of its predictions.

We argue in a general equilibrium framework that, contrary to Morrison and White (2005), a stricter regulation does not necessarily lead to a more stable banking system. An economy where banks are heterogeneous with respect to a project success probability may suffer from a simultaneous moral hazard and selection problem when banks are forced to increase their leverage ratio. In particular, there are three effects arising from a stricter regulation. First, the moral hazard problem weakens since banks have more ‘skin in the game’. Due to a diminished problem of limited liability, banks choose less risky investment projects. Second, there is a countervailing problem of selection among individuals. Banks are not allowed to absorb the excess supply of debt funding when a regulator demands a
larger capital adequacy and additional equity cannot be raised. Therefore, some agents with a lower success probability start doing bank business such that the average quality of the banking sector deteriorates. This allocation effect, however, leads to a decline in the interest rate for debt which again mitigates the moral hazard problem. Thus, the overall effect of stricter banking regulation remains ambiguous.

In terms of risk exposure, the effect of stricter capital requirements is heterogeneous across agents. On the one hand, those agents with a high success probability are not affected by the allocation effect and thus, reduce project risk. On the other hand, the change in the regulatory framework incentivizes agents with a lower ability to engage in risky projects instead of lending in the deposit market. This behavior might increase their own risk exposure, which then spills over to the portfolio of depositors.

We further show that the effect of regulatory changes differ in the degree of heterogeneity. While the reduction of moral hazard due to ‘more skin in the game’ is independent of the distribution of agents’ ability in the economy, both the selection problem as well as the moral hazard channel due to the price of debt do vary for an altering degree of heterogeneity. While stricter regulation exacerbate the problem of selection among individuals the more the larger the degree of heterogeneity, it also lowers the interest rate on debt to a larger extent if the difference between the highest and lowest ability agent increase.

We provide a theoretical framework showing a novel aspect of how stricter capital regulation might translate into a riskier banking system. Therefore, the new regulatory framework should have in mind that a stricter capital regulation potentially mitigates banks’ malfunction due to moral hazard, but it might also have countervailing effects of either an increased project failure due to a lower quality in the banking system or a too small banking sector. These should especially be taken into account when there are large differences in banks’ ability due to, e.g., specialization.
References


Appendix A: Proofs

A  Proof of Proposition 1

Since \( \alpha_i = 1 \), the expected profit is given by

\[
E(\pi_i) = (C + D_i)(1 - y_i)a_i xy_i - (1 - y_i)a_i r D_i - C.
\]

The first-order condition with respect to the risk-return structure yields

\[
\frac{\partial E(\pi_i)}{\partial y_i} = 0 \iff y_i^*(D_i, r) = \frac{1}{2} + \frac{r D_i}{2(C + D_i)x},
\]

with \( \frac{1}{2} \) being the efficient project, and the second term indicating a moral hazard effect due to limited liability. Obviously, the project risk increases in the moral hazard effect. This distortion from the efficient project depends not only on the level of deposits \( D_i \), but also on the deposit rate \( r \) (see Appendix C).

B  Proof of Proposition 2

Since \( \alpha_i = 1 \), the expected profit is given by

\[
E(\pi_i) = (C + D_i)(1 - y_i)a_i xy_i - (1 - y_i)a_i r D_i - C.
\]

Plugging in for the optimal project choice \( y_i^* \) gives

\[
E(\pi_i^{y_i^*}) = (C + D_i) \left( \frac{1}{4} - \frac{r^2 D_i^2}{4(C + D_i)^2x^2} \right) a_i x - \left( \frac{1}{2} - \frac{r D_i}{2(C + D_i)x} \right) a_i r D_i - C.
\]
Differentiating w.r.t. $D_i$:

$$\frac{\partial E(\pi_i^{eq})}{\partial D_i} = a_i x \left( \frac{1}{4} \frac{r^2D_i^2}{4(C + D_i)^2x^2} - (C + D_i) \frac{\partial^2 r^2D_i^2}{\partial D_i^2} \right) - a_i r \left( \frac{1}{2} \frac{r D_i}{2(C + D_i)x} - D_i \frac{\partial \pi_i^{eq}}{\partial D_i} \right)$$

$$= a_i \left( \frac{(C + D_i)^2x^2 - r^2D_i^2 - 2r^2C - 2(C + D_i)^2x + 2(C + D_i)r^2D_i + 2r^2D_iC}{4(C + D_i)^2x} \right)$$

$$= \frac{(C + D_i)^2x(x - 2r) + r^2(D_i^2 + 2CD_i)}{4(C + D_i)^2x}.$$

Case 1 ($x > \frac{2C + D_i}{C + D_i}r$): $\frac{\partial E(\pi_i^{eq})}{\partial D_i} > 0$.

Case 2 ($x = \frac{2C + D_i}{C + D_i}r$): $\frac{\partial E(\pi_i^{eq})}{\partial D_i} = 0$.

$$\frac{\partial^2 E(\pi_i^{eq})}{\partial D_i^2} = \frac{r^2(2D_i + 2C)(C + D_i)^2}{16(C + D_i)^4x^2} - \frac{r^2 C^2}{2(C + D_i)^3x} > 0.$$

Note, that the first derivative has a root for some positive $D_i$. Since the second derivative is strictly positive, it indicates that we have a (global) minimum. Hence, we have a corner solution such that for $x = \frac{2C + D_i}{C + D_i}r$, the agent takes either deposits $D_i = D_{\text{max}}$ or $D_i = 0$.

Case 3 ($x < \frac{2C + D_i}{C + D_i}r$): $\frac{\partial E(\pi_i^{eq})}{\partial D_i} < 0$.

## C Proof of Proposition 3

If a deposit market exists, there must be at least one agent depositing and one agent borrowing at some equilibrium interest rate. According to the participation constraints for banking, we know that $\forall r^{eq}$ with $r^{eq} \leq r_1^{bor}(D_i|D_i = D_{\text{max}})$ and $r^{eq} \leq r_2^{bor}(a_i, D_i|D_i = D_{\text{max}})$, banks want to borrow in the deposit market $D_{\text{max}}$ and invest $(C + D_{\text{max}})$ into the risky project instead of investing only $C$ into the risky project or the risk-free asset.

According to the participation constraints of lending agents, we know that $\forall r^{eq}$ with $r^{eq} \geq r_1^{lend}(a_j, D_j|D_j = D_{\text{max}})$ and $r^{eq} \geq r_2^{lend}(D_j|D_j = D_{\text{max}})$, agents want to deposit the volume $C$ with a bank instead of investing $C$ into the risky project or the risk-free asset. Hence, $r_1^{bor}(D_i|D_i = D_{\text{max}}) \geq r^{eq} \geq r_2^{lend}(a_j, D_j|D_j = D_{\text{max}})$ and $r_2^{bor}(a_i, D_i|D_i = D_{\text{max}}) \geq r^{eq} \geq r_2^{lend}(D_j|D_j = D_{\text{max}}).$
by contradiction. Assume no deposit market. Then there is

(i) either no demand for deposits, i.e., \( \forall i \in I \), either \( r_{eq} > r_{bor}^{1}(D_i|D_i = D^{max}) \) or \( r_{eq} > r_{bor}^{2}(a_i, D_i|D_i = D^{max}) \),

(ii) or no supply of deposits, i.e., \( \forall j \in I \), either \( r_{lend}^{1}(a_j, D_j|D_j = D^{max}) > r_{eq} \) or \( r_{lend}^{2}(D_j|D_j = D^{max}) > r_{eq} \),

(iii) or both.

D Proof of Proposition 4

Denote the maximum regulatory credit volume by \( D^{max} \). Consider a situation in which all agents with an ability \( a_i \geq a^{*}(D^{max}) \) borrow \( D^{max} \) at the same interest rate \( r_{eq} \). We know from the participation constraints that banks have no incentive to demand a lower deposit volume. Obviously, they also have no incentive to offer a higher interest rate.

Suppose some bank \( i \in I \) only accepts an interest rate \( r < r_{eq} \). Since \( r_{eq} \) makes the agent with ability \( a^{*}(D^{max}) - \epsilon \) indifferent between depositing and borrowing in the deposit market, agent \((a^{*}(D^{max}) - \epsilon)\) can be incentivized by an interest rate \( r < (r_{eq} - \nu) < r_{eq} \) to switch from depositing to borrowing. Hence, it is beneficial for depositors of bank \( i \) to deposit at bank \((a^{*}(D^{max}) - \epsilon)\) the volume \( D^{max} \) with an interest rate \((r_{eq} - \nu)\). Since investing \( C \) or depositing \( C \) is less worth for agent \( i \in I \) than borrowing \( D^{max} \) and investing \((C + D^{max})\) at interest rate \( r_{eq} \), she has no incentive to deviate.

Suppose some lending agent \( j \in I \) only accepts an interest rate \( r > r_{eq} \). Since there exists some agent \((a^{*}(D^{max}) + \epsilon)\), which can be incentivized to switch from borrowing to depositing for an interest rate \((r_{eq} + \nu)\), \( r_{eq} < (r_{eq} + \nu) < r \), it is beneficial for the borrowing partner of agent \( j \in I \) to borrow at agent \((a^{*}(D^{max}) + \epsilon)\) the volume \( D^{max} \) at the interest rate \((r_{eq} + \nu)\). Since investing \( C \) or borrowing \( D^{max} \) and investing \((C + D^{max})\) are less worth than depositing \( C \) at interest rate \( r_{eq} \), agent \( j \in I \) has no incentive to deviate. Offering a lower interest rate \( r < r_{eq} \) or supplying a lower volume than \( C \) is never beneficial. Hence, also depositing agents have no incentive to deviate.
E Proof of Proposition 5

Suppose two different contracts \((r_1, D_1)\) and \((r_2, D_2)\) with \(D_1, D_2 \leq D^{\text{max}}\), so that we have for each bank \(i \in I\):

\[
(C + D_1) a_i y - a_i r_1 D_1 - C \geq (C + D_2) a_i y - a_i r_2 D_2 - C
\]

or

\[
(C + D_1) a_i y - a_i r_1 D_1 - C \leq (C + D_2) a_i y - a_i r_2 D_2 - C.
\]

None of these inequalities do depend on \(a_i\). Hence, all borrowing banks prefer the same contract.

F Proof of Proposition 6

In order to have a full participation equilibrium, it is required that the participation constraints of all agents are fulfilled. We show that there exists some area where the binding constraint for borrowing agents lies above the binding constraint for lending agents.

Claim: For some \(D\), the binding constraint for borrowing agents is (BOR1).

\[
(BOR2) > (BOR1) \iff \frac{1}{4} C x a^* > C r_f \\
\iff D > \frac{4C}{x - 4}.
\]

Note that we use the lower bound of (BOR1) with \(a_i = a^*\).

Claim: For some \(D\), the binding constraint for lending agents is (LEND1).

\[
(LEND1) > (LEND2) \iff \frac{1}{4} C x a^* > C r_f \\
\iff D > \frac{4C}{x - 4}.
\]
Note that we use the upper bound of (LEND1) with $a_i = a^*$. 

Claim: For some $D_{\text{max}}$, the binding constraint for lending agents, (LEND1), is below the binding constraint for borrowing agents, (BOR1).

\[
\text{(BOR1)} \geq \text{(LEND1)}
\]
\[
\Leftrightarrow (C + D_{\text{max}}) \left( \frac{1}{2} - \psi \right) a^* \left( \frac{1}{2} + \psi \right) x - \left( \frac{1}{2} - \psi \right) a^* r D_{\text{max}} - \frac{1}{4} C x a^* \geq \frac{1}{2} (1 + a^*) \left( \frac{1}{2} - \psi \right) r C - \frac{1}{4} C x a^*
\]
\[
\Leftrightarrow (C + D_{\text{max}}) a^* \left( \frac{1}{2} + \psi \right) x - a^* r D_{\text{max}} \geq \frac{1}{2} (1 + a^*) r C
\]
\[
\Leftrightarrow a^* (C + D_{\text{max}}) x - a^* r D_{\text{max}} \geq r C + a^* r C
\]
\[
\Leftrightarrow a^* (C + D_{\text{max}}) (x - r) \geq r C
\]
\[
\Leftrightarrow D_{\text{max}} (x - r) \geq r C
\]
\[
\Leftrightarrow r \leq \frac{x D_{\text{max}}}{C + D_{\text{max}}}
\]

Note that we use both the lower bound of (BOR1) and the upper bound of (LEND1) with $a_i = a^*$.

Claim: Denote $\tilde{r} = \frac{x D_{\text{max}}}{C + D_{\text{max}}}$. The level of debt $D_{\text{max}}$, where the relevant participation constraints of borrowing and lending agents evaluated at the interest rate $\tilde{r}$ is zero, is larger than the interception of (BOR1) and (BOR2) (and (LEND1) and (LEND2), respectively).

\[
\text{(LEND1)}(\tilde{r}) = 0 (= \text{(BOR1)}(\tilde{r}))
\]
\[
\Leftrightarrow \frac{1}{2} (1 + a^*) \left( \frac{1}{2} - \psi \right) \tilde{r} C - \frac{1}{4} C x a^* = 0
\]
\[
\Leftrightarrow \frac{1}{2} (1 + a^*) \frac{C + D_{\text{max}} - a^* D_{\text{max}}}{2 (C + D_{\text{max}})} a^* x C - \frac{1}{4} C x a^* = 0
\]
\[
\Leftrightarrow \frac{a^* C x (C - a^* D_{\text{max}})}{4 (C + D_{\text{max}})} = 0
\]
\[
\Leftrightarrow (D_{\text{max}})^2 - D_{\text{max}} C - C^2 = 0.
\]

Since D is assumed to be positive, we find the relevant participation constraints evaluated at the equilibrium interest rate to be zero at

\[
D_{\text{max}} = \frac{1 + \sqrt{5}}{2} C.
\]
Thus, in order to be (BOR1) and (LEND1) the binding constraints,

\[ D_{\text{max}} = \frac{1 + \sqrt{5}}{2} C > \frac{4C}{x - 4} \]
\[ \iff x > 4 + \frac{8}{1 + \sqrt{5}}. \]

G Proof of Proposition 7

Suppose \( r_{eq} = \frac{D_{\text{max}}x}{(C + D_{\text{max}})} \). Note that \( \frac{D}{C + D} < \frac{2C + D}{C + D} \). Hence, according to Appendix A.B, the borrowing decision is \( D_i = D_{\text{max}} \).

Consider some bank \( a' \) for which \( E(\pi y_i | a_i = a', y_i = y^*) > E(\pi D | a_i = a') \). Since \( \frac{\partial E(\pi y_i)}{\partial a_i} > 0 \) and \( \frac{\partial E(\pi D)}{\partial a_i} = 0 \), \( \forall a'' > a' \) we have \( E(\pi y_i | a_i = a'', y_i = y^*) > E(\pi D | a_i = a'') \). Similar to this, \( \forall a''' < a'' \) with \( E(\pi y_i | a_i = a''' , y_i = y^*) < E(\pi D | a_i = a''') \) we have \( E(\pi y_i | a_i = a''' , y_i = y^*) < E(\pi D | a_i = a''') \).

For bank \( a^* = \frac{D_{\text{max}}}{C + D_{\text{max}}} \), the interest rate \( r_{eq} \) solves \( E(\pi y_i | a_i = a^*, y_i = y^*) = E(\pi D | a_i = a^*) \):

\[
(C + D_{\text{max}}) a^*(1 - y^*) y^* - r_{eq} a^*(1 - y^*) D_{\text{max}} - C = \frac{1}{2} (1 + a^*)(1 - y^*) C r_{eq} - C
\]
\[
\iff D_{\text{max}} \left( \frac{1}{2} + \frac{D_{\text{max}} r_{eq}}{2(C + D_{\text{max}}) x} \right) x - \frac{r_{eq} (D_{\text{max}})^2}{(C + D_{\text{max}})^2} = \frac{1}{2} (1 + a^*) (1 - y^*) C r_{eq} - C
\]
\[
\iff \frac{1}{2} D_{\text{max}} x + \frac{(D_{\text{max}})^2 x}{2(C + D_{\text{max}}) x} = \frac{(D_{\text{max}})^2 x}{(C + D_{\text{max}})^2} = \frac{1}{2} \frac{1}{(C + D_{\text{max}})^2} (C + D_{\text{max}})^2 x
\]
\[
\iff \frac{1}{2} D_{\text{max}} x + \frac{(D_{\text{max}})^3 x}{2(C + D_{\text{max}})^2} = \frac{(D_{\text{max}})^3 x}{2(C + D_{\text{max}})^2} = \frac{1}{2} (C + D_{\text{max}})^3 x
\]
\[
\iff D_{\text{max}} (C + D_{\text{max}})^2 - (D_{\text{max}})^3 x = C^2 D_{\text{max}} x + 2C (D_{\text{max}})^2 x.
\]

According to the argumentation above, demand in the deposit market is then given by \( \int_0^1 D_{\text{max}} di = (1 - a^*) D_{\text{max}} = \left( \frac{C}{C + D_{\text{max}}} \right) D_{\text{max}} \) and supply by \( \int_0^{a^*} C di = a^* C = \frac{D_{\text{max}}}{C + D_{\text{max}}} C \). Hence, the allocation solves the agents’ problem and the deposit market clears.

H Proof of Proposition 8

We have shown the existence of an intersection of \( r_{1}^{\text{lend}} \) and \( r_{1}^{\text{bor}} \) in the proof of Proposition 6 (Appendix A.F). We now show that this intersection defines the equilibrium interest rate \( r_{eq} = r_{1}^{\text{bor}}(D_i|D_i = D_{\text{max}}) = r_{1}^{\text{lend}}(a_i, D_i|a_i = a^*, D_i = D_{\text{max}}) \):
∀ r^{eq} \leq r^{bor}_1(D_i|D_i = D^{max})$, the demand in the deposit market is given by \( \lim_{D^{max} \to \infty} (1 - a^*) \cdot D^{max} \) and 0 otherwise. The supply is given by \( a^* C \forall r^{eq} \geq r^{lend}_1 \) and 0 otherwise. Denote the individual lending volume at which \( r^{bor}_1(D_i|D_i = D^{max}) \) and \( r^{lend}_1(a_i, D^{max}|a_i = a^*, D_i = D^{max}) \) intersect by \( D^{uc} \). Since \( r^{lend}_1(a_i, D^{max}|a_i = a^*, D_i = D^{max}) > r^{bor}_1(D_i|D_i = D^{max}) \) \( \forall D^{max} > D^{uc} \), maximum supply in the deposit market is given by \( a^*_\text{crit} = a^*(D^{uc})C \). Since demand for debt financing \( \to \infty \) if \( D^{max} \to \infty \) (due to \( \frac{\partial (1 - a^*) \cdot D^{max}}{\partial D^{max}} > 0 \)) and the equilibrium interest rate tries to balance demand and supply, \( r^{eq} = r^{bor}_1(D_i|D_i = D^{uc}) = r^{lend}_1(a_i, D^{max}|a_i = a^*_\text{crit}, D_i = D^{uc}) \).

Banks have no incentive to offer a higher interest rate \( r_i > r^{eq} = r^{bor}_1(D_i|D_i = D^{max}) \) since for \( r_i > r^{bor}_1(D_i|D_i = D^{uc}) \), it is optimal to invest only \( (C + D^{max}) \) into the risky project. Since there is excess demand, accepting only a lower interest rate \( r_i < r^{eq} \) by some bank \( i \in I \) leads to \( D_i = 0 \). This is equivalent to investing \( (C + D^{max}) \) into the risky project, which is not optimal. Hence, banks have no incentive to deviate.

Suppose some lending agent \( j \in I \). Accepting only some interest rate \( r_j > r^{eq} \) leads to a demand of 0, because of \( r_j > r^{bor}_1(D_i|D_i = D^{uc}) = r^{eq} \). By construction, investing \( (C + D^{max}) \) at an equilibrium interest rate \( r^{eq} \) is not optimal for agent \( j \). Offering an interest rate \( r_j < r^{bor}_1(D_i|D_i = D^{uc}) \) can also not be optimal because profits from lending in the deposit market are increasing in \( r_j \). Hence, lending agents have no incentive to deviate.

Obviously, the market equilibrium as the equilibrium in the deposit market does also hold for all finite \( D^{max} > D^{uc} \).

**Appendix B: Equilibrium Interest Rate**

Solve for equilibrium interest rate \( r^{eq} \):

\[
(C + D) \left( \frac{1}{2} - \psi \right) a^* \left( \frac{1}{2} + \psi \right) x - \left( \frac{1}{2} - \psi \right) a^* rD - C = \frac{1}{2} (1 + a^*) \left( \frac{1}{2} - \psi \right) rC - C
\]

\[
\Leftrightarrow \quad (C + D) a^* \left( \frac{1}{2} + \psi \right) x - a^* rD = \frac{1}{2} (1 + a^*) rC
\]

\[
\Leftrightarrow \quad (C + D) \frac{D}{C + D} \left( \frac{1}{2} + \frac{rD}{2(C + D)x} \right) x - \frac{D}{C + D} rD = \frac{1}{2} \left( 1 + \frac{D}{C + D} \right) rC
\]

\[
\Leftrightarrow \quad \frac{D(C + D)x + rD}{2(C + D)x} x - \frac{rD^2}{C + D} = \frac{1}{2} \left( \frac{1}{2} + \frac{D}{C + D} \right) rC
\]

\[
\Leftrightarrow \quad Dx(C + D) + rD^2 - 2rD^2 = (C + 2D)rC
\]

\[
\Leftrightarrow \quad r^{eq} = \frac{Dx}{(C + D)}.
\]
\[
\frac{\partial r^{eq}}{\partial D} = \frac{x(C + D) - Dx}{(C + D)^2} = \frac{xC}{(C + D)^2}.
\]

⇒ for any \( D \), we have \( \frac{\partial r^{eq}}{\partial D} > 0 \).

**Appendix C: Moral Hazard and Allocation Effect**

From the expected profit from deposit lending,
\[
E(\pi^D) = \frac{1}{2}(1 + a^*(D^{max}))(1 - y(D^{max}, r))rC,
\]
we get the allocation effect:
\[
AE = \frac{1}{2}a^*(D^{max}) = \frac{D^{max}}{2(C + D^{max})}.
\]

Since equity capital is exogenously given, the allocation effect depends only on \( D^{max} \) according to
\[
\frac{\partial AE}{\partial D^{max}} = \frac{C}{2(C + D^{max})^2} > 0.
\]
As the comparative statics points out, an increase in the volume of additional funds from depositors \( D^{max} \) leads to a stronger allocation effect.

From optimal project choice
\[
y^* = \frac{1}{2} + \frac{rD^{max}}{2(C + D^{max})x},
\]
we get the moral hazard effect:
\[
MHE = \frac{rD^{max}}{2(C + D^{max})x}.
\]

The moral hazard effect depends not only on \( D^{max} \), but also on the interest rate that has to be paid for deposits. The equilibrium interest rate, however, is also depending on \( D^{max} \) (see Appendix B). Then the moral hazard effect depends on \( D^{max} \) according to
\[
\frac{\partial MHE}{\partial D^{max}} = C \left[ \frac{\partial r^{eq}}{\partial D^{max}} \cdot D^{max} + r^{eq} \right] + \frac{\partial r^{eq}}{\partial D^{max}} \cdot (D^{max})^2 \frac{1}{2(C + D^{max})^2x} > 0.
\]
Since the first derivative of $r$ with respect to $D_{\text{max}}$ is positive, we find an increasing moral hazard effect if the volume of additional funds from depositors $D_{\text{max}}$ increases. Thus, the decrease of $D_{\text{max}}$ (equivalent to a stricter regulation) on the one hand weakens the allocation effect, which is negative for the aggregate expected profits, but on the other hand also decreases the moral hazard effect.

### Appendix D: Various degrees of heterogeneity

#### A Moral Hazard Effect

\[
MHE = \frac{D_{\text{max}}}{2(C + D_{\text{max}})x} \cdot r_{eq} = \frac{D_{\text{max}}(\bar{a}(D_{\text{max}} - C) + C)}{2(C + \bar{a}D_{\text{max}})(C + D_{\text{max}})}
\]

with

\[
r_{eq} = \frac{\bar{a}(D_{\text{max}} - C) + C}{(C + \bar{a}D_{\text{max}})} \cdot x.
\]

\[
\frac{\partial MHE}{\partial D_{\text{max}}} = \frac{C}{(C + D_{\text{max}})^2} \cdot r_{eq} + \frac{D_{\text{max}}}{2(C + D_{\text{max}})x} \cdot \frac{\partial r_{eq}}{\partial D_{\text{max}}} > 0.
\]

\[
\frac{\partial^2 MHE}{\partial D_{\text{max}} \partial \bar{a}} = \frac{C}{(C + D_{\text{max}})^2} \cdot \frac{\partial r_{eq}}{\partial \bar{a}} + \frac{D_{\text{max}}}{2(C + D_{\text{max}})x} \cdot \frac{\partial^2 r_{eq}}{\partial D_{\text{max}} \partial \bar{a}}
\]

\[
= \frac{C^2}{(C + D)^2(C + \bar{a}D)^3} \left( \sqrt{\bar{a}}D_{\text{max}} + C \right) \left( \sqrt{\bar{a}}D_{\text{max}} - C \right).
\]

Thus,

\[
\frac{\partial^2 MHE}{\partial D_{\text{max}} \partial \bar{a}} < 0, \text{if} \sqrt{\bar{a}} < \frac{C}{D_{\text{max}}};
\]

\[
\frac{\partial^2 MHE}{\partial D_{\text{max}} \partial \bar{a}} = 0, \text{if} \sqrt{\bar{a}} = \frac{C}{D_{\text{max}}};
\]

\[
\frac{\partial^2 MHE}{\partial D_{\text{max}} \partial \bar{a}} > 0, \text{if} \sqrt{\bar{a}} > \frac{C}{D_{\text{max}}};
\]
B Optimal Capital Regulation

\[ AP = \frac{1}{\bar{a} - a} \int_{\bar{a}}^{a} [(C + D_{\text{max}})(1 - y)a, yx] da - C \]

\[ = \frac{\bar{a} - a^*}{\bar{a} - a} \left( (C + D_{\text{max}}) \left( \frac{1}{2} (\bar{a} + a^*) (1 - y)yx \right) - C \right) \]

Using \( y = \frac{1}{2} - \frac{r_{eq} D_{\text{max}}}{2(C + D_{\text{max}})} \), \( a^* = \frac{\pi (D_{\text{max}} - C) + C}{C + \pi D_{\text{max}}} \) and \( r_{eq} = \frac{\pi (D_{\text{max}} - C) + C}{C + \pi D_{\text{max}}} \), and defining \( \widetilde{\text{MHE}} \) as the moral hazard effect without the interest rate (i.e., the pure 'skin in the game effect') and \( AA \) as the average ability of banks, we find

\[ AP = Cx \left[ \left( \frac{1}{4} - \frac{(D_{\text{max}})^2}{4(D_{\text{max}} + C)^2 x^2} \frac{x^2(\bar{a}(D_{\text{max}} - C) + C)^2}{(\bar{a}D_{\text{max}} + C)^2} \right) \frac{1}{2} \left( \frac{2\pi D_{\text{max}} + C}{(D_{\text{max}} + C)} \right) \right] - C \]

\[ = \frac{1}{4} Cx \cdot AA - Cx \cdot AA \cdot \widetilde{\text{MHE}}^2 \cdot (r_{eq})^2 - C. \]

Denote \( D_{\text{max}} = D \). Taking the first derivative wrt \( D \):

\[ \frac{\partial AP}{\partial D} = Cx \left[ \frac{1}{4} \frac{\partial AA}{\partial D} - \frac{\partial AA}{\partial D} \frac{\widetilde{\text{MHE}}^2 \cdot (r_{eq})^2 - AA \cdot \widetilde{\text{MHE}}^2 \cdot (r_{eq})^2 - AA \cdot \widetilde{\text{MHE}}^2 \cdot (r_{eq})^2 - D}{D} \right] \]

\[ = Cx \left[ \frac{C(2\bar{a} - 1)}{8(C + D)^2} - \frac{C(2\bar{a} - 1)D^2x^2(\bar{a}(D - C) + C)^2}{8x^2(C + D)^4(\pi D + C)^2} - \frac{(2\pi D + C)D^2x^2(\bar{a}(D - C) + C)^2}{4x^2(C + D)^4(\pi D + C)^2} \right] . \]

The characteristic equation is then given by:

\[ (\pi D + C)^3(C + D)^2(2\bar{a} - 1) = (\pi(D - C) + C)^2(\pi D + C)D^2(2\bar{a} - 1) \]

\[ + (\pi(D - C) + C)^2(\pi D + C)2D(2\pi D + C) \]

\[ + (\pi(D - C) + C)2D^2\bar{a}^2(2\pi D + C)(C + D). \]