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Foreign vs. domestic public debt and the composition of government expenditure: A political-economy approach

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Abstract

We consider an economy in which a political-support maximizing government takes into account different generations’ preferences when deciding how much to borrow abroad and how much to spend on public consumption and investment. We show that, in equilibrium, the government’s choices are shaped by three parameters: the effect of government spending on output, the expected output costs of a default, and the relative political weight of the generations currently alive. We conclude that the joint dependence on these parameters establishes a strong, but non-linear relationship between the share of foreign debt in total public debt and the share of investment in total government spending.
1 Introduction

In the context of the recent financial turmoil, a lot of attention has been devoted to the level and sustainability of public debt. But while the economic literature on the topic is growing rapidly, most studies ignore the fact that almost every country borrows money abroad as well as from its residents. In fact, a casual look at the evidence reveals that governments differ a great deal with respect to the structure of their debt.\(^1\) As documented by the histogram in Figure 1, which presents country averages for the period 1996 to 2000, the median in that time periods was 49.5 percent, with external-debt shares ranging from close to 0 percent (Switzerland) to more than 90 percent (Tajikistan). However, large external-debt shares are no developing-country-specific phenomenon: while the average share of foreign debt in total government debt for the United Kingdom was 15 percent between 1996 and 2000, this share was 39 percent for Australia, 45 percent for Denmark, and 86 percent for Estonia.

![Figure 1: Share of external public debt in total public debt. Average, 1996 - 2000 (in percent). Data source: Panizza (2008)](image)

Are these differences related to governments’ spending decisions, specifically the share of investment in total public expenditure? It could be argued that a large external debt burden disciplines governments and induces them to use the borrowed funds more productively. Conversely, the option to default on foreign liabilities might run in the opposite direction and raise the share

\(^1\) A large dataset that decomposes public debt into its domestic and foreign components has been compiled by Panizza (2008). We are indebted to Ugo Panizza for making this dataset available to us.
of non-productive government spending. A first look at the data suggests that there is a positive, though weak correlation (53.25 percent) between a government’s financing and spending decisions: Figure 2 shows that countries issuing relatively more debt abroad choose higher investment shares.

Figure 2: The structure of government debt and the composition of public spending. Averages: 1996-2000 (in percent). The vertical axis depicts the ratio of public investment over public transfers and investment – the ratio we will focus on in our subsequent analysis. Data source: Panizza (2008), OECD and World Bank (WDI)

In this paper, we argue that the structure of government debt and the composition of public spending are indeed closely related – however, not through a simple causal tie, but due to the fact that the two magnitudes are driven by the same fundamental forces. To support our claim, we present an overlapping generations model of a small open economy whose government decides on the level of public investment and the volume of transfers in every period. Public spending is financed by raising public debt, and, for a given domestic demand for government bonds, the government has to decide how much money to raise on international capital markets. When making these decisions, the government considers the preferences of the two generations currently alive, with the weight of old and young agents in the government’s objective function reflecting the relative political influence of these generations. We show that the government’s spending and financing decisions are intertwined in various ways: for example, public investment raises domestic income and thus the supply of domestic savings, thereby reducing the share of foreign liabilities in total government debt. Conversely, a higher stock of foreign debt raises the likelihood of default and thus the effective costs of foreign borrowing. This, in turn, reduces the government’s generosity when it comes to handing out transfers to its population.
We use our model to identify the following *exogenous* factors that determine the government’s financing and spending decisions: the effect of public investment on aggregate productivity, the political weight of different generations, and the expected output cost of sovereign default: the stronger the impact of current public investment on future output, the higher the incentive to use government resources for investment purposes. Moreover, higher public investment raises domestic incomes and savings, which, in turn, reduces the need to tap international capital markets. The relative political weights of the two generations matter since old agents put a much higher emphasis on public transfers than on public investment – an obvious preference, since the benefits of public investment only accrue to *future* cohorts. This effect pulls down income and aggregate savings, and thus raises the need to borrow abroad. Finally, the larger the income losses that are expected in case of default, the greater the share of foreign debt in total public debt, since a lower likelihood of default reduces the expected costs of foreign debt. The same mechanism raises public transfers and thus lowers the share of public investment in total government expenditure. These findings suggest that there is, indeed, a positive relationship between the structure of public debt and the composition of government spending.

The remainder of this paper is organized as follows. The next section reviews related studies on the composition of public spending, the structure of government debt, and on sovereign default. In chapter 3 we present and solve our model and derive the effects of varying crucial economic parameters. Section 4 summarizes and concludes.

2 Literature

Our paper is related to three large strands of literature: on the composition of public spending, on the level and structure of government debt, and on the determinants and consequences of sovereign default. While the first two aspects are the topic of numerous welfare-oriented analyses, our brief – and admittedly non-exhaustive – literature review focuses on those studies that explore the political economy of the government’s spending and financing

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Bassetto and Sargent (2006) analyze the political mechanisms that shape the decision on public investment and find theoretical support for the implementation of investment enhancing rules when finitely-lived agents do not take into account the welfare of future generations and vote for a sub-optimal level of public capital formation. The role of inter-generational conflict is further emphasized in the OLG models of Bassetto (2008) and Kaas (2003). The latter contribution shows how a government’s policy on tax-financed public investment is shaped by the voting behavior of generations that differ with respect to the gains and losses from taxes and public investment. However, since the population growth rate is positive, government policy essentially reflects the preferences of the young generation. Bassetto (2008) adds the notion that – via the joint determination of taxes, transfers and public goods – each generation’s strategic position for the next bargaining round is determined. Poutvaara (2006) shows how tax-financed public investment in education and social security can be sustained in an environment with repeated voting.

The political economy of public debt goes back to the seminal contribution of Cukierman and Meltzer (1989) who relate the government’s borrowing decision to an intragenerational distributional conflict. More recently, Battaglini and Coate (2008) and Yared (2010) have analyzed the economic and political forces that determine the level of public debt in equilibrium. Unlike in our paper, however, government revenue is devoted to public goods and rents, and government bonds are exclusively held by domestic residents. The latter assumption is relaxed in Song et al. (2012) who analyze the determinants of government spending in a multi-country world. However, in their model, all international debt contracts are perfectly enforceable, and default is excluded by assumption.

There are few papers that link the government’s spending decision to the composition of its debt. Exceptions are the works by Mahdavi (2004) and Alper et al. (2008). The latter investigate how a government’s preferences over different spending types affect welfare in an OLG framework and show that the way of financing is crucial. However, their model only distinguishes between credit and seignorage as government’s sources of revenue, and not between different types of debt as we do. Mahdavi (2004) provides an empirical analysis on how the level of foreign government debt influences the composition of public spending. Using a sample of developing countries he finds rather mixed evidence on the effect of increasing external credit on pro-
ductive and nonproductive government expenditure categories.

The voluminous literature on sovereign default has recently been surveyed by Augiar and Amador (2013) and Tomz and Wright (2013). Highlighting the large levels of domestic public debt, Reinhart and Rogoff (2011) argue that the literature’s fixation on foreign debt may miss an important trigger of defaults and inflationary episodes. Broner and Ventura (2010) also question the prominent role of foreign debt by showing that the existence of secondary markets essentially erases the difference between external and internal public debt. By contrast, Drazen (1998) presents a model where differences between the costs of foreign and domestic debt reflect the respective political weights of creditors. While domestic residents influence the government’s choices through voting, external lenders are constrained to imposing a penalty in case of default. Drazen (1998) shows how the composition of debt, the budget deficit as well as the volume of government spending are determined by the political standing of the two types of creditors. However, he does not relate the government’s financing decision to its decision on different types of spending.

3 The model

3.1 Households

In this section, we present a two-generation OLG model of a small open economy to explore how the profile of public expenses – i.e. the share of public investment and transfers – is related to the share of foreign debt in total public debt. In each period $t$, a young generation (born in period $t$) and an equally-sized old generation (born in period $t-1$) coexist. Individuals maximize their utility by choosing a consumption path for their remaining lifetime. The utility of an individual $j \in [0, 1]$ living in period $t$ and $t+1$ is given by

$$U_t^j(j) = C_t^y(j) + E_t C_{t+1}^o(j)$$

with $C_t^y$ and $C_{t+1}^o$ denoting consumption at young and old age, respectively, and $E_t$ denoting conditional expectations. The use of a utility function that is linear in consumption and the assumption that the subjective discount factor equals one substantially simplifies the analysis.

DiGiacchino et al. (2005) follow a similar approach, but in their model countries issue debt in a monetary union.

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3 DiGiacchino et al. (2005) follow a similar approach, but in their model countries issue debt in a monetary union.
When choosing the utility-maximizing consumption path the individual has to consider the constraints:

\[ C_t^y(j) = Y_t + T_y^t - D_{t+1}^H(j) - (1 - \varphi_t)\zeta_t \]  (2)

and

\[ C_{t+1}^o(j) = Y_{t+1} + T_{t+1}^o + \varphi_{t+1} R_{t+1} D_{t+1}^H(j) - (1 - \varphi_{t+1})\zeta_{t+1}. \]  (3)

In both periods, agents earn an income \((Y_t, Y_{t+1})\) and receive a transfer \((T_y^t, T_{t+1}^o)\) from the government. Note that we assume that neither income nor transfers differ across members of a generation.

To implement his optimal consumption path, individual \(j\) spends the amount \(D_{t+1}^H(j)\) on domestic government bonds, which represent the only store of value available to domestic agents. The real return on these bonds is given by \(R_{t+1}\). Apart from selling bonds to domestic savers, the government may also borrow from foreign creditors. Every government bond has a maturity of just one period, i.e. has to be repaid in the next period.

We explicitly account for the possibility that the government defaults on its – domestic and foreign – debt. The binary variable \(\varphi_t\) captures this by taking the value of 1 if the government opts for repayment, and 0 if it decides not to honor its obligations. The latter case is associated with costs to all private agents currently alive, which are reflected by the parameter \(\zeta_t\).\(^4\) We denote the probability of a default in period \(t\) by \(p_t\).

Due to our assumption that utility is linear in consumption and that agents take the interest rate, the probability of default, and the government’s transfer-schedule as given, expected returns on the market for government bonds have to satisfy the following condition:

\[ (1 - p_{t+1})R_{t+1} = 1 \]  (4)

If this condition holds members of the young generation are willing to save any amount up to their first-period disposable income. In what follows, we assume that they will, indeed, save the maximum amount, i.e.

\(^4\) These output losses could stem from a reduction in trade or financial market turmoil after a default. Borensztein and Panizza (2009) discuss the various costs that may be associated with a default.
\[ D_{t+1}^H(j) = Y_t + T^y_t - (1 - \varphi_t) \zeta_t. \] (5)

### 3.2 The Government

In every period, the government has to decide whether to default on its existing debt, how to finance government expenditure, and how to spend the funds it raised. Its objective is to maximize a political support function that takes into account the interests of both generations currently alive. While the default decision takes place at the beginning of the period, the financing and spending decisions take place simultaneously thereafter.

Whether the government decides to default on its debt in a given period depends on a comparison between benefits and costs. Of course, a default positively affects the government’s budget, because the outstanding debt is not repaid. On the other hand, a default lowers the income of young and old residents in the same period. The extent of this reduction is given by the realization of the “default cost” variable \( \zeta_t \), which is drawn from a uniform distribution with support \([0, b]\) in every period. We assume that \( b \leq Y_{t+j} \) for \( j \geq 0 \), which implies that the cost of default cannot exceed a generation’s income. The setting of our model implies that the maximum default costs for the whole economy in one period are \( 2b \). Hence, \( b \) reflects the unconditional mean of economy-wide default costs. Note that, unlike in Eaton and Gersovitz (1981), a default has no consequences for a government’s future access to international financial markets.

At the time the government decides on the volume and composition of its debt, it also chooses an expenditure profile. It can either spend its funds on transfer payments to the residents currently alive \( (T^y_t \text{ and } T^o_t) \), or invest in the public capital stock \( K_{t+1}^G \). Since we assume that public capital depreciates within one period, the capital stock in period \( t+1 \), i.e. \( K_{t+1}^G \) coincides with public investment in period \( t \). The latter enhances the income of both generations living in the following period, i.e. \( Y_{t+1} = Y_{t+1}(K_{t+1}^G) \). Finally, we assume that the government is not discriminating any generation via its transfer payments, i.e. \( T^y_t = T^o_t = T_t \). Note that these transfers can also be negative.

Combining the assumptions described above, the period-\( t \) budget constraint
of the government is given by

\[ 2T_t + K^G_{t+1} + \varphi_t R_t(D^H_t + D^F_t) = D^H_{t+1} + D^F_{t+1} \]  \hspace{1cm} (6)

The LHS of this equation represents public expenses, which consist of public transfers \( T_t \), public investment \( K^G_{t+1} \) and (possibly) the repayment of existing public debt \( R_t(D^H_t + D^F_t) \), where \( D^F_t \) represents the stock of foreign debt at the start of period \( t \). The RHS represents the financing of these expenses via domestic and foreign debt issued at the end of period \( t \) and to be repaid in period \( t+1 \).

Combining (2), the government’s budget constraint (6), and the assumption that young agents save their entire disposable income (5), we can derive

\[ D^H_{t+1} = D^F_{t+1} + 2Y_t - K^G_{t+1} - 2(1 - \varphi_t)\zeta_t - \varphi_t R_t(D^H_t + D^F_t) \] \hspace{1cm} (7)

and

\[ T_t = D^F_{t+1} + Y_t - K^G_{t+1} - (1 - \varphi_t)\zeta_t - \varphi_t R_t(D^H_t + D^F_t) \] \hspace{1cm} (8)

Expression (8) reflects an important finding: if the government repays its debt, this reduces the transfer a generation receives by the same amount although each generation gets only 50 percent of the government’s total spending on transfers. This is due to a “multiplier”-like effect: debt repayment lowers the transfer to the young generation. The resulting decrease in income and savings reduces domestic demand for government bonds, which – through the government’s budget constraint – further lowers the volume of transfers paid to both generations.

The government’s objective function is a weighted sum of the utility of old and young residents alive. These weights reflect the relative influence of the old (\( \omega \)) and the young (\( 1 - \omega \)) cohort in the domestic political process.\(^5\) Thus, the government in period \( t \) maximizes

\[ V^G_t = \omega C^o_t + (1 - \omega)U^p_t \] \hspace{1cm} (9)

\(^5\) In defining the government’s objective function, we thus follow the same approach as Song et al. (2012). A probabilistic-voting motivation of such a “political support function” is given in Coughlin and Murrell (1990) and further elaborated in Persson and Tabellini (2000).
subject to the constraint (6).\(^6\)

We start our analysis by considering the government’s choice between default \((\varphi_t = 0)\) and repayment \((\varphi_t = 1)\). Having observed the realization of (potential) default costs \(\zeta_t\) in period \(t\), the government reneges on its debt if its objective function (9) without repayment takes a higher value than in case of repayment.

To analyze the government’s decision, we substitute (1) – (3) into (9). Using the fact that \(C^g_t = 0\) and substituting (8) into (3) we can formulate the condition that has to be satisfied for a government to strictly prefer default:

\[
\omega \left[ 2Y_t + D_{t+1}^{F,d} - K_{t+1}^{G,d} - 2\zeta_t \right] + (1 - \omega)E_t \left[ 2Y_{t+1}(K_{t+1}^{G,d}) + D_{t+2}^F - K_{t+2}^{G} \right.
- \varphi_{t+1}R_{t+1}D_{t+1}^{F,d} - 2(1 - \varphi_{t+1})\zeta_{t+1} \left| \varphi_t = 0 \right. \bigg]
>
\omega \left[ 2Y_t + D_{t+1}^{F,nd} - K_{t+1}^{G,nd} - R_t D_t^F \right] + (1 - \omega)E_t \left[ 2Y_{t+1}(K_{t+1}^{G,nd}) + D_{t+2}^F - K_{t+2}^{G} \right.
- \varphi_{t+1}R_{t+1}D_{t+1}^{F,nd} - 2(1 - \varphi_{t+1})\zeta_{t+1} \left| \varphi_t = 1 \right. \bigg]
\]

In (10), the subscripts \(d\) and \(nd\) refer to the decision to default \((d)\) or not to default \((nd)\). By using these subscripts, we allow for the possibility that the government’s investment and borrowing choices in period \(t\) depend on its default decision. However, as we state in the following Lemma, such a relationship does not exist, and the choice between default and repayment in period \(t\) merely depends on a comparison between the exogenous costs of default \((\zeta_t)\) and the repayment burden on foreign debt.

**Lemma 1** The government decides to default on its debt if \(\zeta_t < \hat{\zeta}_t\), with \(\hat{\zeta}_t = \frac{1}{2}R_t D_t^F\)

The proof can be found in the appendix.

The key to understanding this result lies in the fact that the present and future volumes of domestic debt \((D_t^H\text{ and } D_{t+1}^H)\) appear on neither side of the inequality (10). This is because – as expressed by (7) and (8) – the repayment of domestic debt determines the size of the transfer that accrues to both old agents’ utility is simply driven by their consumption level in the second (and last) period of their lives.

\(^6\) Obviously,
generations: if the government chooses repayment, this lowers the transfers. If it defaults, transfers increase. Due to the “multiplier” effect through which any reduction in young agents’ income is magnified, repayments and transfers completely cancel out, and the default decision becomes irrelevant for old agents’ consumption.

To some extent, this extreme result is, of course, driven by our modelling choices – in particular the linearity of all payoffs. However, it highlights one of the key differences between domestic and foreign debt, which would also characterize models with more sophisticated specifications: domestic creditors are directly affected by the government’s choice of tax and transfer schemes. Through the government budget constraint, these taxes and transfers are ultimately linked to the default vs. repayment decision. By contrast, foreign creditors are entitled to receive their repayment in full, and there is no way for the government to alter its net obligations unless it decides to default on its debt. Note that this result stands in contrast to the argument of Reinhart and Rogoff (2011) who claim that domestic debt is an important driver of outright or creeping default.

As stated in Lemma 1, the threshold value $\zeta_t$ depends on the burden of repayment in period $t$. If the sum of principal and interest on (foreign) debt is high, the likelihood that $\zeta_t$ falls below the critical threshold is high as well, i.e. it is less likely that the government decides to repay its debt. Using the result from Lemma 1 and the assumption that $\zeta_t$ follows a uniform distribution on the interval $[0,b]$, we can derive the default probability $p$. For period $t+1$ this probability is given by

$$p_{t+1} = \frac{R_{t+1} D_{t+1}^F}{2b}.$$  \hspace{1cm} (11)

The expression in (11) allows us to endogenize the interest rate on government debt $R_{t+1}$. External creditors are assumed to maximize the same utility function as domestic residents. This implies that – like domestic agents – they are willing to lend as long as the expected yield on government debt (accounting for the possibility of default) equals the constant risk-free return offered by international financial markets $R^* = 1$. Hence, the following condition has to be satisfied:
The equilibrium interest rate can be computed by solving the equation $R_{t+1} = \frac{1}{1 - pt_{t+1}} = \left(1 - \frac{R_{t+1}^{F}}{2b}\right)$. Equation (12) is depicted in Figure 3, with the LHS being represented by the 45-degree line and the RHS by a parabola. Note that, for a solution to exist, we have to assume that $b \geq 2D_{t+1}^{F}$, i.e. the expected costs of default must be large enough. Moreover, the interest rate that solves (12) may not be unique. There may be an equilibrium that is characterized by a high probability of default and a high interest rate (point A) and another equilibrium that is characterized by a low probability of default and a low interest rate (point B). In what follows, we assume that lenders are able to coordinate on the low-interest rate equilibrium. This implies that the equilibrium value of $R_{t+1}$ increases in $D_{t+1}^{F}$, i.e. the more the government borrows abroad, the higher will be the price of debt, since the risk of default – the probability of the realisation $\zeta_{t+1}$ being smaller than the threshold $\hat{\zeta}_{t+1}$ – is rising.

The solution to this equation is given by $R_{t+1} = \frac{b - \sqrt{b^2 - 2D_{t+1}^{F}b}}{D_{t+1}^{F}}$.

Note also that the equilibrium in point B is (locally) stable, i.e. starting from a point in the neighborhood of point B, the interest rate will converge to that equilibrium.
We now turn to the government’s financing and spending decisions. Using the above results and the decision rule on default, the government’s objective function can be expressed as follows:

\[ V^G_t = \omega \left[ 2y_t(K^G_t) + D^F_{t+1} - K^G_{t+1} - \varphi_t R_t D^F_t - 2(1 - \varphi_t)\zeta_t \right] + (1 - \omega) \left[ 2y_{t+1}(K^G_{t+1}) + D^F_{t+2} - K^G_{t+2} - D^F_{t+1} \right. \\
\left. \quad - 2 \int_0^{\hat{\zeta}_{t+1}} \zeta_{t+1} dF(\zeta_{t+1}) \right] \tag{13} \]

where \( F(\zeta_{t+1}) = \frac{\zeta_{t+1}}{b} \) is the cumulative distribution function of \( \zeta_{t+1} \). The optimal values for public investment \( K^G_{t+1} \) and external debt \( D^F_{t+1} \) are implicitly characterized by the following first-order conditions:

\[ \frac{\partial V^G_t}{\partial K^G_{t+1}} = -\omega + (1 - \omega)2 \frac{\partial y_{t+1}}{\partial K^G_{t+1}} = 0 \tag{14} \]

and

\[ \frac{\partial V^G_t}{\partial D^F_{t+1}} = \omega - (1 - \omega) \left[ 1 + \frac{2\hat{\zeta}_{t+1} \frac{\partial \hat{\zeta}_{t+1}}{\partial D^F_{t+1}}}{b} \right] \]
\[ \quad = \omega - (1 - \omega) \left[ 1 + \frac{b - \sqrt{b^2 - 2D^F_{t+1}b}}{2\sqrt{b^2 - 2D^F_{t+1}b}} \right] = 0 \tag{15} \]

The conditions in (14) and (15) have straightforward interpretations: members of the old generation would like the government to increase transfers by reducing its investment expenditure and raising its foreign borrowing. By contrast, young agents are in favor of higher investment spending, since this raises their future income. Moreover, they realize that higher foreign borrowing raises future repayment obligations, which hurts members of the young generation since it reduces expected future transfers. This is also because higher foreign debt increases the likelihood of a default and thus expected future income losses. The weights \( \omega \) and \((1 - \omega)\) determine how strongly these conflicting interests affect the government’s decision.
3.3 Government spending and public debt in equilibrium

We assume that agents’ income is related to public investment through the following function \( Y_{t+1}(K^G_{t+1}) = A(K^G_{t+1})^\alpha \), with \( A \) representing total factor productivity (TFP), which is assumed to be constant. We can substitute this functional form into (14) to derive the government’s optimal investment expenditure:

\[
\tilde{K}^G_{t+1} = \tilde{K}^G = \left( \frac{2^{1-\omega}}{\omega} \alpha A \right)^{\frac{1}{1-\alpha}}
\]  

Equation (16) demonstrates that public investment \( \tilde{K}^G \) is a constant. This is due to the stationarity of our model environment – there are no time-variant exogenous variables like, e.g., population size – and to our specific assumptions on the costs associated with default, namely, the fact that there is no persistent punishment of defaulting governments. Due to the young and old generations’ distinct interests, \( \tilde{K}^G \) is decreasing in \( \omega \), but increasing in \( 1 - \omega \). Moreover, it is higher for higher values of the TFP parameter \( A \).

Substituting this expression into the production function yields:

\[
\tilde{Y}_{t+1} = \tilde{Y} = A^{\frac{1}{1-\alpha}} \left( \frac{2^{1-\omega}}{\omega} \alpha A \right)^{\frac{\alpha}{1-\alpha}}
\]  

The optimal value of foreign debt can be derived by solving (15) for \( D^F_{t+1} \):

\[
\tilde{D}^F_{t+1} = \tilde{D}^F = \frac{b}{2} \left( 1 - \frac{1}{z^2} \right)
\]  

where \( z \equiv \frac{2^{1-\omega}}{1-\omega} - 1 \). To guarantee that \( \tilde{D}^F \) is strictly positive, we assume that \( \frac{\omega}{1-\omega} > 1 \), i.e. \( \omega > 0.5 \). Obviously, \( \tilde{D}^F \) increases in \( b \): if the expected costs of default are high, the probability of default is low even for high levels of foreign debt. This reduces expected income losses for the young generation and provides the government with an incentive to increase its foreign debt. It is also easy to show that \( \tilde{D}^F \) is higher for higher values of \( z \) and thus \( \omega \): if the weight of the old generation in the political process gets higher, the government emphasizes current benefits at the expense of future costs and thus increases foreign borrowing.
Combining (12) and (18), we can derive the equilibrium interest rate on government debt $\tilde{R}_{t+1}$:

$$
\tilde{R}_{t+1} = \tilde{R} = \frac{2z}{z + 1}
$$

(19)

In equilibrium, the interest rate charged by the government’s creditors increases in $z$, which, in turn, increases in $\omega$: a larger weight of the old generation in the government’s objective function raises foreign borrowing, makes default more likely, and thus raises the interest rate charged by international and domestic lenders.

Since both $\tilde{R}_{t+1}$ and $\tilde{D}_{t+1}^F$ are constant, the threshold value $\hat{\zeta}_{t+1}$ is constant, too, and we have $\hat{\zeta} = \frac{z}{2}(1 - \frac{1}{z})$. Using (11), (18) and (19) we can eventually derive the (constant) probability of default, which is given by:

$$
\tilde{p} = \frac{z - 1}{2z}
$$

(20)

Using the above results and (7) we can calculate the volume of domestic debt in period $t+1$ for the case of default and for the case of repayment in period $t$:

$$
\tilde{D}_{t+1}^H(\varphi_t = 0) = 2\tilde{Y} + \tilde{D}^F - \tilde{K}^G - 2\tilde{\zeta}_t
$$

(21)

$$
= \Lambda - 2\tilde{\zeta}_t
$$

with $\Lambda \equiv 2\tilde{Y} + \tilde{D}^F - \tilde{K}^G$ being determined by (16) – (18).\footnote{Specifically, $\Lambda \equiv (2A) \frac{1}{2\alpha} \left( \frac{2\alpha}{2\alpha + 1} \right) \left( 1 - \frac{1/\alpha}{2\alpha} \right) + \frac{b}{2} \left( 1 - \frac{1}{2z} \right)$.} Note that $z > 0$ implies that $\Lambda > 0$.

$$
\tilde{D}_{t+1}^H(\varphi_t = 1) = 2\tilde{Y} + \tilde{D}^F - \tilde{K}^G - \tilde{R}(\tilde{D}^F + \tilde{D}_{t+1}^H)
$$

(22)

$$
= \Lambda - 2\hat{\zeta} - \tilde{R}\tilde{D}_{t+1}^H
$$

Note that $\tilde{D}_{t+1}^H(\varphi_t = 0) > \tilde{D}_{t+1}^H(\varphi_t = 1)$ since $\zeta_t < \hat{\zeta}$ for the government to
choose default and since $D^H_t \geq 0$. Hence, the volume of domestic borrowing in case of default is higher than in case of repayment.\footnote{For a given value of $D^H_t$, the expected value of $D^H_{t+1}$ is $E(D^H_{t+1}) = \Delta - D^H_t$, with $\Delta = \Lambda - (1 - \frac{1}{2}) \tilde{R}D^F_t$. Assuming that $D^H_0 = 0$, the expected level of domestic debt oscillates between 0 and $\Delta$.}

In equilibrium public transfers for both cases are given by:

\begin{align}
\tilde{T}_t(\varphi_t = 0) &= \tilde{Y} + \tilde{D}^F - \tilde{K}^G - \zeta_t \\
&= \Pi - \zeta_t \\
\tilde{T}_t(\varphi_t = 1) &= \tilde{Y} + \tilde{D}^F - \tilde{K}^G - \tilde{R}(\tilde{D}^F + D^H_t) \\
&= \Pi - 2\hat{\zeta} - \tilde{R}D^H_t
\end{align}

with $\Pi = \Lambda - \tilde{Y}$. Following the same reasoning as above, one can easily see that $\tilde{T}_t(\varphi_t = 0) > \tilde{T}_t(\varphi_t = 1)$, i.e. by opting for default the government chooses a higher level of public transfers.

Using these results, we can compute the share of external debt in the total debt a government incurs at the end of period $t$. Denoting this share by $\tilde{s}_t$ and distinguishing between periods of default and repayment, we get

\begin{align}
\tilde{s}_t(\varphi_t = 0) &= \frac{\tilde{D}^F}{D^F + D^H_{t+1}(\varphi_t = 0)} := \frac{\tilde{D}^F}{\Omega_t(\varphi_t = 0)} \tag{25}
\end{align}

and

\begin{align}
\tilde{s}_t(\varphi_t = 1) &= \frac{\tilde{D}^F}{D^F + D^H_{t+1}(\varphi_t = 1)} := \frac{\tilde{D}^F}{\Omega_t(\varphi_t = 1)} \tag{26}
\end{align}

where $\Omega_t(\varphi_t = 0) = \tilde{D}^F + \Lambda - 2\zeta$ and $\Omega_t(\varphi_t = 1) = \tilde{D}^F + \Lambda - 2\hat{\zeta} - \tilde{R}D^H_t$. Since $\zeta_t < \hat{\zeta}$ for the government to choose default and since $D^H_t > 0$, the share of foreign debt in total debt is smaller in default periods than in repayment periods, i.e. $\tilde{s}_t(\varphi_t = 0) < \tilde{s}_t(\varphi_t = 1)$.
In a similar way, we calculate the share of public investment in total government expenditure in period $t$.\footnote{Note that if the government honors its debt, the repayment sum $\tilde{R}(D^F + D^H)$ is also part of its expenditures.} This variable, which we denote by $\tilde{q}_t$, is given by

$$\tilde{q}_t(\varphi_t = 0) = \frac{\tilde{K}^G}{\tilde{K}^G + 2\tilde{T}_t(\varphi_t = 0) := \frac{\tilde{K}^G}{\Omega_t(\varphi_t = 0)} (27)$$

and

$$\tilde{q}_t(\varphi_t = 1) = \frac{\tilde{K}^G}{\tilde{K}^G + 2\tilde{T}_t(\varphi_t = 1) + \tilde{R}(D^F + D^H)} := \frac{\tilde{K}^G}{\Omega_t(\varphi_t = 1)} (28)$$

with $\Omega_t(\varphi_t = 0)$ and $\Omega_t(\varphi_t = 1)$ being defined above. Since $\Omega_t(\varphi_t = 0) > \Omega_t(\varphi_t = 1)$, the share of investment in total public spending is lower in periods of default.

Combining (25) and (27) as well as (26) and (28), it is easy to show that, while the ratios $\tilde{s}_t$ and $\tilde{q}_t$ depend on a government’s default decision and initial domestic debt in period $t$, the ratio $\gamma \equiv \frac{\tilde{q}_t}{\tilde{s}_t}$ does not. In fact, we have

$$\gamma(A, \omega, b) = \frac{\tilde{K}^G}{\tilde{D}^F} (29)$$

Equation (29) suggests a positive relationship between $\tilde{q}_t$ and $\tilde{s}_t$, with the factor of proportionality ($\gamma$) depending on the parameters $A$, $b$, and $\omega$. Apparently, $\gamma$ increases in $A$ since a higher productivity of government spending raises $\tilde{K}^G$. The factor $\gamma$ decreases in $b$ since higher expected costs of default make it more attractive to borrow abroad and thus raise $\tilde{D}^F$. Finally, $\gamma$ decreases in $\omega$, which represents the political weight of the old generation: this weight has a negative impact on investment spending $\tilde{K}^G$ and a positive effect on foreign borrowing $\tilde{D}^F$, and thus makes the ratio $\gamma$ decrease.

This result has two important implications: First, it delivers a theoretical explanation for the positive correlation suggested by Figure 2. Governments that spend relatively more of their budget on public investment tend to issue
relatively more of their debt abroad and vice versa. The reason is that both decisions are shaped by the three economic and political fundamentals we identify in our model and that the parameter which links those decisions ($\gamma$) is strictly positive. Second, the expression in (29) also explains why the relationship between the composition of government debt and the structure of public spending is not very strong: a change in any of the parameters that determine $s$ and $q$ also affects $\gamma$, i.e. the factor of proportionality that links the two shares. An empirical test of our theory would have to account for this non-linearity. Moreover, it would have to control for other factors that potentially determine the real-world counterparts of $q$ and $s$. This, however, would be beyond the scope of the current analysis.

4 Conclusion

Our goal was to explore the relationship between the structure of government debt and the composition of public expenditure. This project was based on the notion that the spending decisions affect the financing decision through its influence on the availability of domestic savings, while the financing decision determines the potential volume of transfers, but also the risk of sovereign default.

To achieve this aim, we developed a simple model that incorporated some of the important distributional trade-offs faced by a political-support maximizing government. In particular, by explicitly allowing for the possibility of government default, we captured one of the key differences between domestic and foreign debt: unlike foreign creditors, domestic holders of public debt are directly affected by the government’s choice of taxes and transfers, which may partly (or completely) undo the losses inflicted by debt repudiation. While our extreme result that domestic debt is completely irrelevant for the government’s default decision is certainly due to our modelling strategy – in particular, the linearity of agents’ objective functions – we observe that this fact reflects a fundamental difference between internal and external debt that is relevant even if a defaulting government cannot discriminate between domestic and foreign creditors. The special role of foreign debt makes the government’s financing decision non-trivial: while increasing foreign borrowing allows to increase transfers to the generations currently alive, it also raises the future repayment burden. Moreover, a higher debt level raises the likelihood of default and thus expected future income losses. The latter effect is the stronger the lower the maximal cost of a default. As a consequence, countries in which a government default would potentially cause huge output
losses and in which the old generation has a strong political weight should see their governments incur rather high levels of external debt. As far as the spending decision is concerned, the old generation favors spending on transfers while the young generation emphasizes the positive effects of public spending on future income. Again, the government’s decision ultimately depends on the relative political weight of the two generations. The contribution of our paper is to highlight the mutual dependence between the government’s spending and financing decisions and to detect the fundamental forces that shape the government’s simultaneous choices.

Our analysis offered a theoretical explanation for the positive correlation we observed in Figure 2 by identifying a – certainly non-exhaustive! – list of parameters that affect the government’s spending and financing choices. Furthermore these parameters also account for at least a part of the observed cross-country heterogeneity in government’s decisions. While our aim was to provide a theoretical idea why public expenditure composition and debt structure are linked, we leave the empirical test of other influential factors to another paper. We believe, however, that our analysis paves the road for a deeper understanding of the forces that shape government borrowing and spending behavior and, ultimately, financial stability.
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A Proof of Lemma 1

In this appendix, we show that the government’s default condition (10) can be simplified to the relation pinned down by Lemma 1. To save on notation, we denote variables that are chosen in period $t$ after a default has occurred by using the superscript $'$. Conversely, we use the superscript $-$ if no default took place. Accordingly, in the following period this leads to the superscript $''$ under a defaulting regime in $t+1$ and $\equiv$ for no default.

We claim that there is a the threshold level $\hat{\zeta}_t$ that indicates the turning point at which a government would “switch” from repayment to default. Using (10), this threshold turns out to be

$$\hat{\zeta}_t = \frac{1}{2}R_t^F D_t^F + \frac{1}{2}(D_{t+1}^{F'} - D_{t+1}^-) + \frac{1}{2}(K_{t+1}^{G'} - K_{t+1}^-)$$

$$+ \frac{1 - \omega}{2\omega}E_t\left[2(Y_{t+1}(K_{t+1}^G) - Y_{t+1}(K_{t+1}^G^-)) + (D_{t+2}^{F''} - D_{t+2}^{F''-}) + (K_{t+2}^{G''} - K_{t+2}^{G''-})\right]$$

Note, that we assume that a default takes place in period $t+1$ ($D_{t+2}^{F''} \text{ and } K_{t+2}^{G''}$). The alternative assumption of repayment in period $t+1$ ($D_{t+2}^{F''-} \text{ and } K_{t+2}^{G''-}$) would be inconsequential for our results.

We now have to prove, that $k = l = m = 0 \text{ and } n = o = 0$.

Starting with $n$ and $o$, i.e showing the independence of the government’s decisions about foreign debt and investment expenses in $t+1$ from the default decision in $t$, we compute the derivation of the government’s utility function (9) in period $t+1$ if default took place in $t$:

$$V_{t+1}^{G''} = 2\omega(Y_{t+1}(K_{t+1}^{G''}) - \zeta_{t+1}) + \omega(D_{t+2}^{F''} - K_{t+2}^{G''})$$

$$+ (1 - \omega)\beta\left[2Y_{t+2}(K_{t+2}^{G''}) + \int_{\hat{\zeta}_{t+2}}^{\tilde{\zeta}_{t+2}} \left(D_{t+3}^{F''} - K_{t+3}^{G''} - 2\zeta_{t+2}\right) dF(\zeta_{t+2})

+ \int_{\hat{\zeta}_{t+2}}^{\tilde{\zeta}_{t+2}} \left(D_{t+3}^{F''} - K_{t+3}^{G''} - K_{t+2}^{G''} D_{t+2}^{F''}\right) dF(\zeta_{t+2})\right]$$

Equivalently, for no default in period $t$ the objective function of the government in $t+1$ is:

22
\[ V_{t+1}^{G''} = 2\omega(Y_{t+1}(K_{t+1}^{G''}) - \zeta_{t+1}) + \omega(D_{t+2}^{F''} - K_{t+2}^{G''}) \\
+ (1 - \omega) \beta \left[ 2Y_{t+2}(K_{t+2}^{G''}) + \int_{\hat{\zeta}_{t+2}}^{\hat{\zeta}_{t+1}} (D_{t+3}^{F''} - K_{t+3}^{G''} - 2\zeta_{t+2}) dF(\zeta_{t+2}) \\
+ \int_{\hat{\zeta}_{t+1}}^{b} (D_{t+2}^{F''} - K_{t+2}^{G''} - R_{t+1}^{F''} D_{t+1}^{F''}) dF(\zeta_{t+2}) \right]. \]

Since the only difference in these objective functions is the appearance of \( K_{t+1}^{G'} \) and \( K_{t+1}^{G''} \), which due to the linearity of the functions do not matter for the choice of optimal values of foreign debt \( D_{t+2}^{F''} \) and investment \( K_{t+2}^{G''} \), this is a proof for independence of future governments’ choices of today’s default decision, in particular \( D_{t+2}^{F''} = D_{t+2}^{F''} \) and \( K_{t+2}^{G''} = K_{t+2}^{G''} \).\(^{12}\)

Next, we show that in \((\text{??})\) \( k = l = m = 0 \).

The government’s utility \((9)\) in period \( t \) for the case of default is

\[ V_t^{G'} = 2\omega(Y_t - \zeta_t) + \omega(D_{t+1}^{F'} - K_{t+1}^{G'}) \\
+ (1 - \omega) \beta \left[ 2Y_{t+1}(K_{t+1}^{G'}) + \int_{\hat{\zeta}_{t+1}}^{\hat{\zeta}_{t+2}} (D_{t+2}^{F'} - K_{t+2}^{G'} - 2\zeta_{t+1}) dF(\zeta_{t+1}) \\
+ \int_{\hat{\zeta}_{t+1}}^{b} (D_{t+2}^{F'} - K_{t+2}^{G'} - R_{t+1}^{F'} D_{t+1}^{F'}) dF(\zeta_{t+1}) \right]. \]

If the policymaker opts for repayment of foreign debt in period \( t \), we get

\[ V_t^{G'} = 2\omega(Y_t - \frac{1}{2} R_t D_t^F) + \omega(D_{t+1}^{F'} - K_{t+1}^{G'}) \\
+ (1 - \omega) \beta \left[ 2Y_{t+1}(K_{t+1}^{G'}) + \int_{\hat{\zeta}_{t+1}}^{\hat{\zeta}_{t+2}} (D_{t+2}^{F'} - K_{t+2}^{G'} - 2\zeta_{t+1}) dF(\zeta_{t+1}) \\
+ \int_{\hat{\zeta}_{t+1}}^{b} (D_{t+2}^{F'} - K_{t+2}^{G'} - R_{t+1}^{F'} D_{t+1}^{F'}) dF(\zeta_{t+1}) \right], \]

It is obvious, that the optimal values for investment and external debt are.

\(^{12}\) Note, that this not necessarily implies identical levels for internal debt \((D_{t+2}^{F''} = D_{t+2}^{F''})\) or transfers \((R_{t+1}' = R_{t+1}')\).
the same in both cases, i.e. $\tilde{K}_{t+1} = \tilde{K}_{t+1}$ and $\tilde{D}_{t+1} = \tilde{D}_{t+1}$. Since $R$ is a function of $D^F$, the same holds for $\tilde{R}_{t+1}$ and $\tilde{R}_{t+1}$. Therewith, it is proven, that $k = l = m = 0$ and that the government’s default decision is determined by the condition stated in Lemma 1.