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Pareto-Improving Redistribution of Wealth - The Case of the NLSY 1979 Cohort

Klaus Wälde

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Johannes Gutenberg University Mainz
Gutenberg School of Management and Economics
Jakob-Welder-Weg 9
55128 Mainz
Germany
wiwi.uni-mainz.de

Contact details

Klaus Wälde
Chair in Macroeconomics
Johannes Gutenberg-University Mainz
Jakob-Welder-Weg 4
55128 Mainz

waelde@uni-mainz.de

Pareto-Improving Redistribution of Wealth – The Case of the NLSY 1979 Cohort

Klaus Wälde[1]
Johannes-Gutenberg University Mainz and CESifo

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We quantify a model of idiosyncratic labour income and idiosyncratic interest rates to perfectly match the evolution of the wealth distribution of the NLSY 79 cohort from 1986 to 2008. Given a simple and plausible labour income process with a positive growth rate of labour income, the crucial feature is an ex-ante heterogeneity in interest rate distributions and an interest rate that fluctuates between two values. One value lies below and one above the threshold level that implies a stationary long-run wealth distribution. We propose an 'investment insurance' which, if implemented in 1986, would have reduced wealth inequality of the NLSY 79 cohort in 2008 by 23%. It would also have increased ex-ante welfare and would even have increased average wealth in 2008 by 38%. The economic rationale behind this surprising finding that everybody becomes richer through an inequality-reducing policy intervention is explained.

JEL Codes: C02, D31, E21
Keywords: dynamics of wealth distributions, NLSY 1979 cohort, capital income risk, redistribution, insurance, Fokker-Planck equations

1 Introduction

[Motivation] There is a huge discrepancy between the actual distribution of wealth, the believed distribution of wealth and the desired distribution of wealth in the US. Norton and Ariely (2011) asked a representative sample of the population of the United States how they believed the wealth distribution in the US looks like and what they would find the most appropriate wealth distribution for their country. While the desired wealth distribution (with a Gini coefficient of 0.2) was less unequal than the believed wealth distribution (0.51), the actual distribution was still much more unequal (0.76) than what respondents believed. This suggests that understanding the determinants of the wealth distribution, its evolution over time and how easily and how quickly policy could make distributions more equal is of enormous public interest.

[The open issue] While there is quite some research on the distribution of wealth (see below), very little is known about how quickly it changes and how fast policy could have an impact on its properties. This paper therefore asks two questions: (i) What are the necessary building blocks of an explanation of the dynamics of wealth? To make this question precise, we ask, under which conditions a relatively standard model of idiosyncratic risk with standard parameter values can perfectly match the evolution of the NLSY 79 wealth distribution from

[1]Klaus Wälde, Johannes-Gutenberg University Mainz, Mainz School of Management and Economics, Jakob-Welder-Weg 4, 55131 Mainz, Germany, fax + 49.6131.39-23827, phone + 49.6131.39-20143, waelde@uni-mainz.de. I would like to thank Jess Benhabib, Michael Graber, Andrey Launov, Jean-Baptiste Michau, Benjamin Moll, Shenghao Zhu and seminar participants at the University of Basel, the CRD and at the National University of Singapore for comments and discussions. Excellent research assistance was provided by Paul Strasser and, especially, Tobias Heilmann (né Nagel).

1986 to 2008. Given that both the 1986 and the 2008 wealth distribution (with Gini values of 0.41 and 0.54, respectively) are too unequal according to the Norton-Ariely findings, we then ask (ii) what the effects of an insurance mechanism against interest rate uncertainty are for inequality and average wealth. We will refer to this insurance mechanism as 'investment insurance'. More precisely, we study how far the Gini coefficient would have approached the desired value if a redistribution mechanism had been in place since 1986. We also study how the wealth distribution changes under an investment insurance if it is implemented only as of 2008.

[The setup] Individuals face uncertain labour income as they stochastically move back and forth between employment and unemployment. Individuals can self-insure by accumulating wealth. When unemployed, the individual's maximum debt level is given by a natural borrowing constraint. Individuals face uncertain returns on their wealth. The return fluctuates randomly between two values.[2] The transition rates between these values can differ between individuals, which we describe as an individual's 'financial type'. Each individual draws its type before entering the labour market. Individuals with a high financial ability experience high returns more often than individuals with a lower financial ability. As we want to understand one cohort of the US population, we work with a partial equilibrium model. The wage, unemployment benefits, the growth rate of the wage and benefits and the realization of interest rates (but not their probabilities) are exogenous.

The quantitative analysis is facilitated by the use of Fokker-Planck equations. Treating wealth as a continuous variable, they are partial differential equations who describe the evolution of the wealth distribution over time. They can be derived from the fundamentals of the model, taking optimal consumption behaviour of agents into account.

In our calibration, we compute the average wage, wage growth, the arrival rates of jobs and the separation rate from the data. In our baseline calibration, we set the time preference rate to 1% and the degree of risk aversion to 1.01 (to just lie above the logarithmic case). We perform robustness analyses below.

[Findings] The distribution of wealth in 2008 has more probability mass to the right and to the left than the original distribution in 1986 when individuals entered the labour market. We match the 2008 distribution, using the 1986 distribution as the initial condition when solving the Fokker-Planck equations. We set the low realization of the return (at 3.5%) to lie below the threshold level which implies a stationary distribution.[3] Joint with a parameter determining the minimum consumption level at the natural borrowing limit, this allows us to assure that the left tail of the wealth distribution converges in 22 years from the initial distribution to the density in 2008. The high realization (set at 4.5%) lies above this threshold level. This is essential to converge to the fat right tail in the distribution of wealth in 2008.

The convergence after 22 years to the 2008 density is (close to) perfect as we allow for 20 different financial types times two initial conditions. We fit the model density for 2008 to the data density by identifying the share of individuals in the NLSY cohort that belong to different financial types. Technically, we compute the weights of a mixture of densities from different types in the model such that the joint density fits the data density. Increasing the number of different types further would yield a perfect fit.

We "test" our calibration by comparing the standard deviation of the idiosyncratic interest rate in our setup with empirical standard deviations reported in the literature. It seems that empirical standard deviations are one or two orders of magnitude *larger* than those needed

[2] To simplify the numerical analysis, we assume individuals are myopic with respect to changes in the interest rate. See below for further discussion.

[3] In models with idiosyncratic income risk without economic growth, there is a stationary distribution if the interest rate lies below the time preference rate, $r < \rho$. As we allow for growth, there is a stationary distribution in our setup if the interest rate lies below the time preference rate plus the product of risk aversion and the growth rate, $r < \rho + \sigma g$.

in our model to match the dynamics of the wealth distribution. Interest rate uncertainty is therefore almost "too successful" in explaining wealth inequality.

Given the public concern about the inequality of wealth, the question arises whether there are policy instruments which are both desirable from an individual perspective and which could help reduce inequality of wealth. As individuals do not know their financial abilities before becoming economically active, they live behind a "veil of ignorance" when young.[4] This suggests that individuals would gain from an insurance against bad draws from the distribution of financial abilities.

The insurance mechanism we suggest requires individuals that experience a high return to pay an insurance fee. Those that experience a low return receive a subsidy. We analyse the effects of a self-financing insurance scheme where income and expenditure is identical at each moment in time. This corresponds to what is often called a 'fair' insurance scheme that could be provided by a competitive industry.

In the absence of an insurance scheme, the Gini coefficient in 2008 is given by 0.54.[5] If such an insurance scheme had been introduced from the moment individuals enter the labour market in 1986, we find that a fee of 2% (to be paid on the high return) would have reduced the Gini coefficient for 2008 by 19% to 0.43.

Most surprisingly, such an insurance scheme is not only ex-ante welfare improving but increases wealth even of those that experience a high return, i.e. that pay the fee. This is due to the assumption that high returns lie above the threshold level for stationary long-run distributions. In this sense, if understanding fat right tails requires fluctuating returns which sometimes exceed the threshold level (as emphasized by Benhabib et al., 2015), then redistribution can be Pareto-improving and makes everybody richer. Individuals with high returns become richer even though the insurance scheme reduces their returns as lower returns imply lower consumption growth and therefore more wealth accumulation. This effect overcompensates the direct channel of the lower returns on wealth accumulation for our calibrated model. In the absence of an insurance scheme, average wealth in 2008 amounts to approx. 111,000 US$ (in prices of 1986). Under an investment insurance, average wealth rises to 154,000 US$, i.e. by 38%.

[Related literature] The analysis in this paper builds on the literature analysing the determinants of skewness of and thick tails in wealth distributions (see Benhabib and Bisin, 2016, for a survey and introduction). Our paper takes its crucial inspiration for the quantitative fit from Benhabib et al. (2011, 2014, 2015) by borrowing the idea of interest rate uncertainty from them. Quantitatively, this leads to the perfect fit of the dynamics of the wealth distribution. As the Solow residual in growth accounting makes sure that the growth rate is accounted for 100%, interest rate uncertainty makes sure that empirical wealth distributions can be fitted 100% by theoretical models. At the same time, our estimated distribution of interest rates is very consistent, as will be discussed below, with empirical distributions of idiosyncratic interest rates (Flavin and Yamashita, 2002).

Our labour income process is inspired by the search and matching literature starting with Diamond (1982), Mortensen (1982) and Pissarides (1985). We let labour income fluctuate between a wage when employed and unemployment benefits when unemployed. Corresponding transition rates will then quantified by average durations in employment and unemployment, respectively, in the NLSY. We agree that any realistic income process would need much more structure (see e.g. Blundell et al., 2015, and the references therein). An outstanding example for an empirically more convincing income process is the precautionary saving and on-the-job search model by Lise (2013). Yet, he assumes a constant interest rate and focuses on one cross-

[4] In terms of the first wave in the NLSY with wealth information in 1986, individuals are 21 to 28 years old.

[5] This is lower than the 2007 value of 0.82 in Díaz-Gímenez et al. (2011, table 2). Apart from different data sources, we assume that this is mainly due to the fact that this paper looks at one NLSY cohort and not at the entire population.

section of wealth. An argument in favour of our simple income structure is the well-known finding that the empirical skewness in the earnings distribution is not enough to generate sufficiently skewed and thick-tailed wealth distributions (Benhabib and Bisin, 2016, sect. 3.1). We therefore demonstrate that even with such a simple process, we can perfectly match the dynamics of the distribution of wealth.[6]

Turning to our first main contribution, previous analyses of wealth distributions have mostly focused on a particular cross-section of wealth at one point in time. Very recently, Gabaix et al. (2015) or Kaymak and Poschke (2015) are interested in the changes of properties of wealth distributions over time. Gabaix et al. provide a fascinating analysis of the potential of continuous-time setups. We share with them the belief in the usefulness of Fokker-Planck equations.[7] Our work differs from theirs in many respects, but mainly in our more elaborate micro-foundation of the decision process and the quantitative focus.[8] Kaymak and Poschke (2015) provide a detailed analysis of the US tax system. They find that "changes in taxes and transfer account for nearly half of the rise in wealth concentration". Despite the obvious importance of the transfer system, we abstract from it. We also abstract from an "awesome state" in the labour income process which can be argued to contradict standard skewness properties of earnings distributions. We rather focus on stochastic asset returns. Finally, we calibrate our model by fitting densities and not individual moments.

Our analysis of the effect of an investment insurance is our second main contribution and further distinguishes our work from studies just mentioned. By contrast, other papers are related to our analysis of the insurance mechanism. Most broadly speaking, all papers studying the redistribution of wealth via policy intervention share the same idea of "correcting the wealth distribution". The most visible statement of this type reaching beyond academic analysis is probably made by Piketty (2014).[9] Benhabib et al. (2011) show that a capital income tax and an estate tax can reduce wealth inequality. Benhabib et al. (2015) propose an accounting framework for wealth dynamics. They find that skewed distributions of earnings, savings and bequest rates that differ across wealth levels and capital income risk are all necessary for matching wealth dynamics and also social mobility. Kindermann and Krueger (2015) argue that redistribution should be grounded in normative considerations. They find that a marginal tax rate of close to 90% for the top 1% earners is optimal in a model where the earnings and wealth distribution has properties as the empirical distributions in the US. Brüggemann (2016), in a model with entrepreneurs, also finds that welfare maximizing top marginal tax rates lie above 80%.[10]

The distinguishing idea of our policy proposal is simple: If interest rate uncertainty is at the basis of wealth inequality and these interest rates are idiosyncratic, it is a straightforward question to ask how an insurance mechanism against this idiosyncratic interest rate risk would

[6] In a dynamic wealth-inequality accounting analysis, one would ask how changes of the wealth distribution over time can be attributed to the labour income process, to the distribution of capital returns, to the role of bequests and the fiscal system. We have to leave such a detailed empirical analysis for future work. See below for further discussion.

[7] We use the approach described in Bayer and Wälde (2010, 2015) to derive the Fokker-Planck equations for our case.

[8] While around a handful of papers have used Fokker-Planck (aka Kolmogorov forward) equations in economic theory (see Bayer and Wälde, 2015, for more references), the numerical potential in terms of accuracy and speed of these equations has not yet been exploited as much as in the current paper.

[9] For a discussion of Piketty's book from an academic perspective, see Krusell and Smith (2015) for theoretical aspects and Blume and Durlauf (2015) for a more empirical focus.

[10] Analyses in the Mirrlees tradition to optimal taxation also advocate large marginal tax rates for top earners. Diamond and Saez (2011) suggest marginal income tax rates of up to 76% (for a broadened tax base as compared to the current US system). Going beyond a static structure, Michau (2014) and Golosov et al. (2016), among others, study optimal redistribution in a life-cycle setting. No attempt is made to investigate the wealth distribution in this context.

affect the wealth distribution. This question has not been asked so far. The surprising quantitative finding that "everybody gets happier and richer" while inequality is reduced is therefore new as well.

[Table of contents] The next section describes an individual facing idiosyncratic risk of a stationary interest rate and idiosyncratic risk of deterministically growing labour income. Given this growth process, section 3 derives a stationary representation of our dynamic economy and defines equilibrium. Section 4 analyses the dynamics of distributions for detrended variables and shows how distributions evolve for variables with trend. Section 5 demonstrates the empirical fit of the evolution of the distributions for wealth (with trend) and section 5 undertakes our policy analysis.

2 The model

2.1 The individual

Our individual owns wealth $a(t)$ that increases in a deterministic fashion when capital income $r(t)a(t)$ plus labour income $z(t)$ exceeds consumption $c(t)$,

$$da(t) = \{r(t)a(t) + z(t) - c(t)\} dt. \tag{1}$$

The instantaneous interest rate is denoted by $r(t)$. It fluctuates between two values $\{r^{\text{int}}, r^{\text{low}}\}$ as described by

$$dr(t) = \left[r^{\text{int}} - r^{\text{low}}\right] dq_{\lambda^{\text{low}}}(t) + \left[r^{\text{low}} - r^{\text{int}}\right] dq_{\lambda^{\text{int}}}(t). \tag{2}$$

The arrival rates of the corresponding Poisson processes are λ^{low} and λ^{int}. They are state-dependent, i.e. there is no upward jump when the interest rate is high and no downward jump when the interest rate is low, $\lambda^{\text{low}}\left(r^{\text{int}}\right) = \lambda^{\text{int}}\left(r^{\text{low}}\right) = 0$.

The arrival rates are heterogenous across individuals. This captures the idea that individuals differ in the financial abilities. Formally, each individual draws arrival rates λ^{low} and λ^{int} from a two-dimensional distribution before becoming economically active. Once drawn, the arrival rates remain constant throughout life. When an individual draws a high λ^{low} it leaves the state with a low return relatively quickly. When the individual has a high λ^{int}, it leaves the state with high returns relatively quickly.

To make the model parsimonious, we make λ^{int} a falling function of λ^{low}. This makes sure that an individual that draws a high λ^{low} has a low λ^{int}. The individual will therefore spend more time in expectation in the regime with the intermediate return than in the regime with the low return. A high λ^{low} therefore stands for a high financial ability. To fix ideas, let us assume there are n different ability types i, i.e. n different levels λ_i^{low} (and therefore n implied levels λ_i^{int}) from which the individual draws. The probability to be an individual of type i is denoted by p_i.

Labour income $z(t)$ grows and jumps according to

$$dz(t) = gz(t)dt + [w(\Gamma(t)) - b(\Gamma(t))] dq_\mu(t) + [b(\Gamma(t)) - w(\Gamma(t))] dq_s(t). \tag{3}$$

The deterministic growth rate of labour is given by the constant g as visible in the deterministic dt-part of this equation. For simplicity, we let labour income fluctuate between two states only. This could represent the wage while working and unemployment benefits while unemployed. These jumps are described by the increments $dq_i(t)$ of Poisson processes with arrival rates $\mu(z(t))$ moving the individual from unemployment to employment and with arrival rate $s(z(t))$ moving the individual from employment to unemployment. The arrival rates are state-dependent such that no employed worker can be matched into a job ($\mu(w) = 0$) and that no unemployed worker can lose a job ($s(b) = 0$). The arrival rates $\mu(b)$ and $s(w)$ are positive and constant. The growth process of labour income implies that labour income is a function of past growth rates. We denote this trend component by

$$\Gamma(t) = \Gamma(0)\, e^{gt} \tag{4}$$

and express wages and benefits as as a function of the trend, i.e. $z(t) \in \{w(\Gamma(t)), b(\Gamma(t))\}$.

We assume individuals maximize the present value of their utility stream. Her intertemporal utility function reads

$$U(t) = E \int_t^{\infty} e^{-\rho[\tau - t]} u(c(\tau))\, d\tau. \tag{5}$$

Instantaneous utility $u(c(\tau))$ is a function of consumption and is assumed to reflect constant relative risk aversion (CRRA) with risk aversion parameter σ, i.e.

$$u(c(t)) = \begin{cases} \frac{c(t)^{1-\sigma}-1}{1-\sigma}, & \sigma > 0, \sigma \neq 1, \\ \ln(c), & \sigma = 1. \end{cases} \tag{6}$$

We allow workers to borrow up to their natural borrowing limit. It is given by the amount of debt that they can pay back with probability one, i.e. in all possible states of the world. We assume that all debt contracts are such that the current personal interest is paid (in the case of positive wealth) or has to be paid (in the case of debt). The worst possible state for a person in debt is therefore the high interest rate r^{int}. We require consumption to be sufficiently large at all wealth levels to guarantee survival of the individual, i.e. $c^z(a) \geq c^{\min}$. For notational simplicity, we relate this minimum consumption level to unemployment benefits by

$$c^{\min}(t) = \xi b(t) \tag{7}$$

where $0 < \xi < 1$. This implies (see web appendix) that the natural borrowing limit is

$$a^{\text{nat}}(t) = -\frac{(1-\xi)\, b(t)}{r^{\text{int}} - g}. \tag{8}$$

The debt level is the higher, the larger the growth rate g of unemployment benefits. It falls when the minimum consumption level requires a larger share ξ of unemployment benefits and when the interest rate r rises.[11]

2.2 Optimal behaviour

Optimal consumption is a function of the wealth level, the labour market status, the trend and the interest rate. When our individual maximizes utility, she takes her current wealth level and the uncertainty from labour income growth and transitions into account. Individuals are assumed to be myopic with respect to the interest rate. Changes in individual returns come as a surprise and are not anticipated.[12] Optimal consumption is therefore denoted by $c(a, w, \Gamma)$ or $c(a, b, \Gamma)$ and is described by a Keynes-Ramsey rule for the employed worker that reads

$$\begin{aligned} -\frac{u''(c(a,w,\Gamma))}{u'(c(a,w,\Gamma))} dc(a,w,\Gamma) = &\left\{ r - \rho + s\left[\frac{u'(c(a,b,\Gamma))}{u'(c(a,w,\Gamma))} - 1\right]\right\} dt \\ &- \frac{u''(c(a,w,\Gamma))}{u'(c(a,w,\Gamma))}\left[c(a,b,\Gamma) - c(a,w,\Gamma)\right] dq_s. \end{aligned} \tag{9a}$$

[11] The borrowing limit would not exist (it would be minus infinity) if the growth rate g is higher than the interest rate r^{int}. In our quantitative analysis below, $r^{\text{int}} > g$ holds.

[12] This assumption allows us to avoid unnecessary complexities. With anticipation of uncertain interest rates, we would have four (coupled) Keynes-Ramsey rules and a system of four (coupled) Fokker-Planck equations instead of two. While this would be theoretically straightforward, we leave the numerical analysis of such a more elaborated system for future work. Such an additional complexity could be reduced if one allowed for a continuum of wage incomes as in Lise (2013).

To understand this stochastic differential equation, consider first the case of employment as an absorbing state, i.e. the case of a separation rate of zero. Consumption would grow if the interest rate r exceeds the time preference rate ρ. With a positive arrival rate, we see that consumption grows faster: The term in squared brackets is positive as consumption when employed at a wage w is larger than when unemployed when receiving benefits $b < w$ (which implies higher marginal utility from consumption when unemployed). As a consequence, consumption growth tends to be faster. This is obviously the effect of precautionary saving. As individuals anticipate the risk of experiencing lower labour income, they reduce the consumption level, accumulate wealth faster and thereby experience faster consumption growth.

For the unemployed worker, the Keynes-Ramsey rule reads

$$-\frac{u''(c(a,b,\Gamma))}{u'(c(a,b,\Gamma))}dc(a,b,\Gamma) = \left\{r - \rho - \mu\left[1 - \frac{u'(c(a,w,\Gamma))}{u'(c(a,b,\Gamma))}\right]\right\}dt - \frac{u''(c(a,b,\Gamma))}{u'(c(a,b,\Gamma))}\left[c(a,w,\Gamma) - c(a,b,\Gamma)\right]dq_\mu. \tag{9b}$$

Here, the effect of labour income uncertainty is reversed. Again, the term is squared brackets is positive such that overall consumption growth is smaller as compared to a situation where unemployment lasts forever. Individuals anticipate that at some point in the future labour income will be high again such that they increase their consumption level and thereby save less.

This completes the description of this partial equilibrium model. Equation (1) describes the evolution of wealth conditional on optimal consumption (9), (3) describes the evolution of income $z(t)$ and the growth process is described by (4).

3 Detrending and equilibrium

3.1 Detrending

Before we can define equilibrium in our economy, we derive a stationary version of the model by detrending the above equations. Rising labour income comes from technological progress that increases at the rate g as used in the income equation (3). Given the trend (4), we obtain the following detrended variables,

$$\hat{z}(t) \equiv \frac{z(t)}{\Gamma(t)},\ \hat{a}(t) \equiv \frac{a(t)}{\Gamma(t)},\ \hat{c}^z(\hat{a}(t)) \equiv \frac{c^z(a(t),\Gamma(t))}{\Gamma(t)}. \tag{10}$$

This implies (see app. D.2.1) from (A.14) and (A.15) that

$$\hat{w} = \frac{w(\Gamma(t))}{\Gamma(t)},\quad \hat{b} = \frac{b(\Gamma(t))}{\Gamma(t)}. \tag{11}$$

The detrended natural borrowing limit with the corresponding minimum consumption level can be obtained from (7) and (8) as

$$\hat{c}^{\min} = \xi\hat{b},\quad \hat{a}^{\text{nat}} = \frac{a^{\text{nat}}(t)}{\Gamma(t)} = -\frac{(1-\xi)\hat{b}}{r^{\text{int}} - g}. \tag{12}$$

Given that the natural borrowing limit is a function of r^{int} in both regimes, the borrowing limit does not change when the individual interest rate changes.

These auxiliary variables evolve over time as well. Using the laws of motion for the underlying variables, the detrended income process follows (see app. D.2.2)

$$d\hat{z}(t) = \left[\hat{w} - \hat{b}\right]dq_\mu(t) - \left[\hat{b} - \hat{w}\right]dq_s(t). \tag{13}$$

The evolution of detrended wealth $\hat{a}(t)$ reads

$$d\hat{a}(t) = \{(r-g)\hat{a}(t) + \hat{z}(t) - \hat{c}^z(\hat{a}(t))\}\,dt. \tag{14}$$

We can express the evolution of detrended consumption as a function of detrended wealth for the time in between transitions on the labour market as

$$\frac{d\hat{c}^w(\hat{a})}{d\hat{a}} = \frac{\frac{r-\rho}{\sigma} - g + \frac{s}{\sigma}\left[\left(\frac{\hat{c}^w(\hat{a})}{\hat{c}^b(\hat{a})}\right)^\sigma - 1\right]}{(r-g)\hat{a} + \hat{w} - \hat{c}^w(\hat{a})}\hat{c}^w(\hat{a}), \tag{15a}$$

$$\frac{d\hat{c}^b(\hat{a})}{d\hat{a}} = \frac{\frac{r-\rho}{\sigma} - g - \frac{\mu}{\sigma}\left[1 - \left(\frac{\hat{c}^b(\hat{a})}{\hat{c}^w(\hat{a})}\right)^\sigma\right]}{(r-g)\hat{a} + \hat{b} - \hat{c}^b(\hat{a})}\hat{c}^b(\hat{a}). \tag{15b}$$

Equations (12) to (15b) form the basis of our analysis of the dynamics of distribution and of our definition of equilibrium. Before we define the latter, let us gain some intuition for the distribution of wealth.

3.2 Consumption and wealth dynamics

To understand the dynamics of consumption and wealth, it is crucial to distinguish between three "regimes". They are jointly determined by the level of the interest rate relative to preference, growth and job-market parameters. The low-interest-rate regime holds for

$$r < \rho + \sigma g. \tag{16}$$

This condition follows from analysing the Keynes-Ramsey rule (15a), see appendix. This is the standard regime, the precautionary savings literature has looked at so far.

Our individual finds herself in the intermediate regime when the interest rate satisfies

$$\rho + \sigma g < r < \rho + \sigma g + \mu \tag{17}$$

and the high-interest rate regime holds for r sufficiently large, i.e. for $\rho + \sigma g + \mu < r$.[13] We do not expect that the high-interest rate regime will be empirically relevant: The arrival rate μ for jobs is of an order of magnitude larger than any real world interest rate. As a consequence, we do not expect that individual interest rates r are ever larger than $\rho + \sigma g + \mu$. We will therefore now analyse the low-interest-rate and the intermediate-interest-rate regime.[14]

- Low-interest-rate regime

When the interest rate is sufficiently low as in (16), (detrended) consumption $\hat{c}^w(\hat{a})$ of employed workers rises over time for $a < a_w^*$, i.e. as long as the employed worker is sufficiently poor. Detrended consumption $\hat{c}^b(\hat{a})$ of unemployed workers falls. This is illustrated in the following figure.

Wealth is fixed by a lower bound from (12) and an upper bound $\hat{a}_w^*$ which is the support

$$\hat{a} \in \left[\hat{a}^{\text{nat}}, \hat{a}_w^*\right[\tag{18}$$

[13] We do not consider the singular cases where r lies on the boundaries as they do not play any significant role for our analysis. If we understand r to be a continuous random variable, the probability to lie on the boundaries is zero plus zero and can therefore be neglected.

[14] These conditions illustrate that one could obtain similar quantitative findings for a stochastic time preference rate (as in Krusell and Smith, 1998) and a fixed interest rate. Stochastic interest rates have the advantage that they can be observed more easily than stochastic time preference rates. This allows to test the model more easily (see below).

of our wealth variable $\hat{a}$. It can be shown that relative consumption at the upper bound needs to satisfy (see appendix or the web appendix of Bayer Wälde, 2015, for the case of $g = 0$)

$$\frac{\hat{c}^b(\hat{a}_w^*)}{\hat{c}^w(\hat{a}_w^*)} = \left(1 - \frac{r - \rho - \sigma g}{s}\right)^{-1/\sigma} \tag{19}$$

and that obviously at the TSS where $da = 0$,

$$\hat{c}^w(\hat{a}_w^*) = r\hat{a}_w^* + w \tag{20}$$

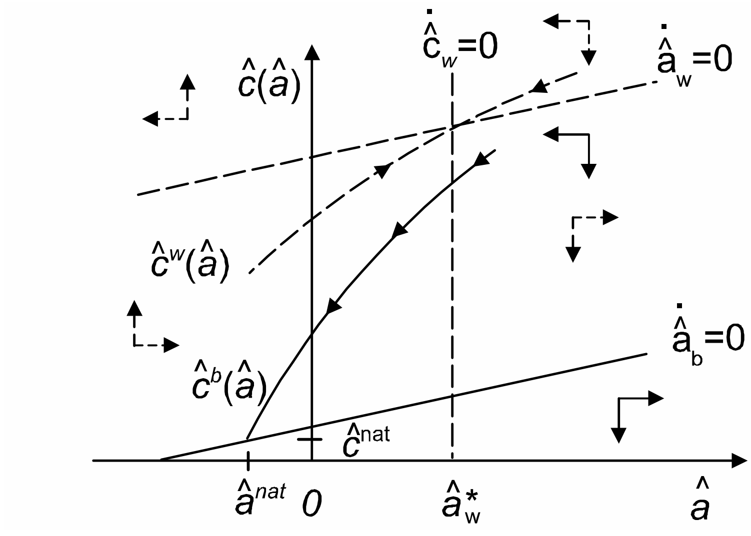

Figure 1 *Consumption and wealth dynamics in the low-interest rate regime*

- Intermediate interest rates

When the interest rate is at intermediate levels from (17), (detrended) consumption $\hat{c}^w(\hat{a})$ of employed workers rises for any wealth level. By contrast, (detrended) consumption $\hat{c}^b(\hat{a})$ of unemployed workers falls only for $a < a_b^*$, i.e. if the unemployed worker is sufficiently poor (see app. D.2.5 for details). This is illustrated in figure 2.

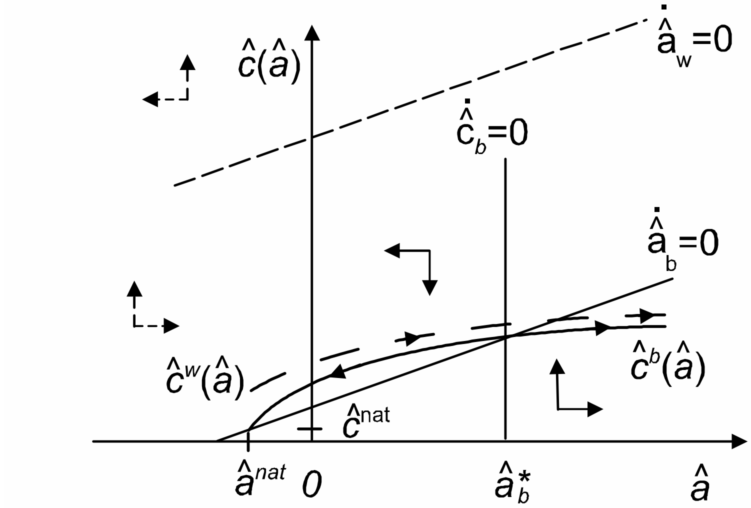

Figure 2 *Consumption and wealth dynamics for an intermediate interest rate r from (17)*

The consumption level at the temporary steady state in this setup is given by

$$\hat{c}^b(\hat{a}_b^*) = r\hat{a}_b^* + b \tag{21}$$

and relative consumption is given by

$$\frac{\hat{c}^b(\hat{a}_b^*)}{\hat{c}^w(\hat{a}_b^*)} = \left(1 - \frac{r-\rho-\sigma g}{\mu}\right)^{1/\sigma}. \tag{22}$$

The natural borrowing limit and the minimum consumption level remain unchanged at the values in (12).

- Consumption and wealth distributions

We are now in a position to gain some intuition about the distribution of wealth and consumption in our model. If the interest rate stayed always in the low-interest rate regime, we would be in a standard Huggett-Aiyagari model. The wealth level of an individual would then be bounded between 0 and a_w^* in figure 1. If we think about sufficiently many individuals in the low regime, there would be a density of wealth with this support. If we were only in the intermediate regime, there would be no upper bound and the wealth distribution would probably not be characterised by a long-run stationary distribution.

In our setup, we let the interest rate take values both in the low and in the intermediate regime. Wealth will be accumulated very quickly when in the intermediate regime which is the basic mechanism through which we obtain a wealth distribution with enough probability mass in the right tail.

3.3 Equilibrium

We are now in a position to define equilibrium in our economy. An individual behaves optimally when following the Keynes-Ramsey rule (15a) when employed and the rule (15b) when unemployed. Consumption jumps from one equilibrium path to the other when the individual loses a job or finds one. The two boundary conditions for the Keynes-Ramsey rule are given by the consumption level $\hat{c}^w(\hat{a}_w^*)$ from (20) and the consumption level $\hat{c}^b(\hat{a}_w^*)$ following from (19) at the temporary steady state. The wealth level a_w^* at the temporary steady state is then defined such that consumption in the state of unemployment at the natural borrowing limit is given by the minimum consumption level, i.e. $\hat{c}^b(\hat{a}^{\text{nat}}) = \hat{c}^{\min}$.[15]

When the interest rate jumps, the boundary conditions at the temporary steady state change to the values $\hat{c}^b(\hat{a}_b^*)$ from (21) and the consumption level $\hat{c}^w(\hat{a}_b^*)$ following from (22). The wealth level a_b^* also changes but is chosen according to the same logic, i.e. such that $\hat{c}^b(\hat{a}^{\text{nat}}) = \hat{c}^{\min}$.

4 Distributional dynamics of detrended variables

4.1 Densities and subdensities of wealth

We are interested in the distribution of detrended wealth over time. This is a continuous random variable depending on optimal detrended consumption (which we assume to be known) and the detrended income $\hat{z}(t)$. Let $p(\hat{a},t)$ be the probability density function of detrended wealth. This density can be split into two "sub-densities" $p^w(\hat{a},t)$ and $p^b(\hat{a},t)$ according to

$$p(\hat{a},t) = p^w(\hat{a},t) + p^b(\hat{a},t).$$

[15] This also illustrates the idea behind the numerical solution: Guess $\hat{a}_w^*$ and check whether $\hat{c}^b(\hat{a}^{\text{nat}}) = \hat{c}^{\min}$ holds. If not, adjust the guess.

Those sub-densities can be seen as a product of a conditional density $p(\hat{a}, t|\hat{z})$ with the probability of being in state $\hat{z}$ (see Bayer and Wälde, 2010, 2015 for more discussion). A description of the evolution of $p(a,t)$ is given by the Fokker-Planck equations (FPEs). They read (see app. D.2.7)

$$\frac{\partial}{\partial t}p^{w}(\hat{a},t)+[(r-g)a+\hat{w}-\hat{c}^{w}(\hat{a})]\frac{\partial}{\partial \hat{a}}p^{w}(\hat{a},t)=\left[\frac{\partial}{\partial \hat{a}}c^{w}(\hat{a})-(r-g)-s\right]p^{w}(\hat{a},t)+\mu p^{b}(\hat{a},t), \tag{23a}$$

$$\frac{\partial}{\partial t}p^{b}(\hat{a},t)+\left[(r-g)a+\hat{b}-\hat{c}^{b}(\hat{a})\right]\frac{\partial}{\partial \hat{a}}p^{b}(\hat{a},t)=sp^{w}(\hat{a},t)+\left[\frac{\partial}{\partial \hat{a}}c^{b}(\hat{a})-(r-g)-\mu\right]p^{b}(\hat{a},t). \tag{23b}$$

Note that the evolution of the wealth density has a direct link to the optimal consumption-saving paths as these equations display optimal consumption levels $\hat{c}^{z}(\hat{a})$ and its derivatives.

Note that we are not interested in any stationary distributions here but always work with the evolution of the distribution. We start at some (empirically) given initial distribution and then proceed to match the final (empirically given) distribution.

4.2 Distributional dynamics of variables with trend

Once we have computed (i) the policy functions using (15a) and (15b) for the two interest rate regimes and (ii) the densities and their evolution over time from (23a) and (23b), we need to transform these findings for detrended variables back into levels before we can compare them with data. Going back to levels for "normal" variables is straightforward by inversion of (10), $z(t)=\hat{z}(t)\Gamma(t)$, $c(a,z,\Gamma)=\hat{c}(a,z)\Gamma(t)$ and $a(t)=\hat{a}(t)\Gamma(t)$. The densities $p^{z}(\hat{a},t)$ and $p(\hat{a},t)$ can be retransformed by Edgeworth's method of translation (Benhabib and Bisin, 2016, sect. 1.2, Wälde, 2012, theorem 7.3.2). This translation describes the link between a random variable ($\hat{a}$ in our case) and its transformation ($a(\tau)=\hat{a}(\tau)\Gamma(\tau)$ here). For our stationary support (18) for detrended wealth and using trend (4), this transformation implies a support for wealth $a(\tau)$ that evolves over time τ,

$$a(\tau)\in\left[-\frac{\hat{b}}{r-g}\Gamma(\tau),\hat{a}^{*}\Gamma(\tau)\right[.$$

The density $g(a,\tau)$ of wealth with trend is then given by (see app. D.3.3)

$$g(a,\tau)=p\left(\frac{a(\tau)}{\Gamma(\tau)},\tau\right)\frac{1}{\Gamma(\tau)}. \tag{24}$$

5 The empirical fit

5.1 Data and some descriptive statistics

We extract the wealth distribution from the NLSY70 for all waves that provides information on wealth. A visual impression of the fairly equal distribution of wealth when individuals are young in 1986 and the steady increase in inequality as the cohort becomes older is provided by the left figure in fig. 3. The figure uses the entire sample, i.e. we do not distinguish by ethnic groups, sex or educational background.[16] The right figure shows the density as predicted by our model with the (close to perfect) fit for 2008. We will discuss this figure in detail below.

[16] In earlier calibrations, we divided the sample into 12 observationally distinguishable groups. In the absence of interest rate uncertainty, we were unable to match the upper tail of the wealth distribution. It is well-known that, unless one assumes an "awesome state", this would be the case even if we allowed for more labour income states than just two. See Kaplan et al. (2016, footnote 32) for a similar finding.

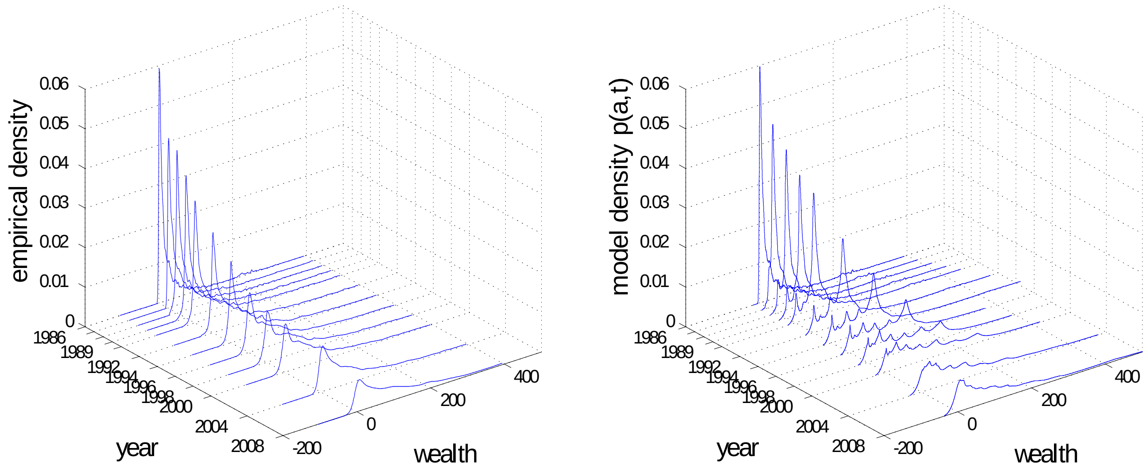

Figure 3 *The dynamics of the wealth distribution in the data and in the model*

The NLSY data is also used for computing various parameters in our model. An overview of those parameters and also of exogenously fixed parameters is in table 1.

μ	s	$\hat{w}$	g	$\hat{b}/\hat{w}$	ξ	ρ	σ	r^{low}	r^{int}
21.99%	1.19%	2280.8$	3.4%	30%	97%	1%	1.01	3.5%	4.5%

Table 1 *Parameter values*

We match the average duration in employment and unemployment for the 79 cohort and thereby their average unemployment rate of 5.2% by the job arrival rate μ and the separation rate s. The average monthly wage in 1986 is $\hat{w}$ and the growth rate of labour income is g. Before computing the growth rate, all wages were expressed using the 1986 price level. (All other variables and especially the wealth levels are also expressed in 1986 prices.) We infer unemployment benefits by assuming a replacement rate of 30%. This is a compromise between the higher statutory replacement rate and the fact that benefits are not paid forever in the US (but are done so in the paper).

The share for consumption ξ is chosen such that the natural borrowing limit (8) corresponds to the smallest (perceivable) wealth level in the data. The time preference rate ρ, risk aversion σ and the interest rates r^{low} and r^{int} are exogenously fixed. Robustness analysis is undertaken below in section 6.3.

5.2 Empirical fit

So far, we only talked about distributions of wealth for one individual that is looking into the future. To obtain cross-sectional distributions requires us to use a law of large numbers. When we assume that the number of individuals is sufficiently large within our cohort, the individual probability to own wealth below a certain threshold is then the same as the share in the population of individuals holding this threshold or less. We can therefore fit the aggregate wealth distributions to our individual densities, as they also represent cross-sectional densities in our model.

5.2.1 Targeting the wealth distribution in 2008

We fit the model distribution to the wealth distribution in 2008 by first solving for optimal consumption paths (15a, b) for both the high and the low interest rate. This gives consumption paths $\hat{c}^z(\hat{a})$ as illustrated in figures 1 and 2. We then solve the Fokker-Planck equations in (23a, b) for the (expected) interest rate path that results for the different financial types. As initial conditions we take the densities of wealth in 1986 for employed and unemployed workers. We solve the equations over a period of 22 years.[17] We use $n = 20$ financial types with two initial conditions each (starting with high or with a low interest rate). This gives $2n = 40$ densities of wealth for 2008.

We obtain the probabilities p_i to be of a certain financial type by minimizing the sum of squared distances between the density provided by the model and the data at around 8000 equidistant wealth levels between $\hat{a}^{\text{nat}}$ and a_b^*. The probability-p_i weighted sum of the the 40 densities is the density predicted by the model. With our cross-sectional interpretation, the probabilities p_i to be of a certain financial type $i = 1...n$ is equal to the share of individuals of that type. Fitting the wealth distribution in 2008 therefore means making a statement on how financial ability is distributed in our NLSY cohort.

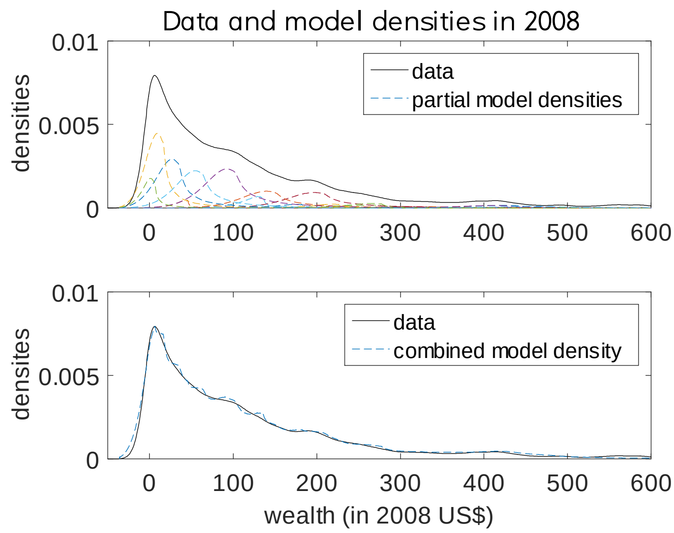

Figure 4 *The wealth distribution in 2008 in the data and in the model*

An illustration of the empirical fit is in fig. 4. The above figure shows the empirical density and 40 partial densities, i.e. one density for each type. Only around 8 or 9 are visible as the other receive very low weights. Summing these partial densities up, the lower figure shows that the fit is almost perfect. If we used more financial types, if we chose the arrival rates or the minimum consumption parameter ξ in an optimal way (to further minimize the distance between the model and data density), the fit must be perfect at some point. This points to a general property of models with interest rate uncertainty: The role of interest rate uncertainty is identical to the role of the Solow growth residual in empirical growth analyses. In the latter, everything that is not explained by capital growth, labour growth, maybe human capital growth, composition of the labour force more generally (and potentially other measures) is captured by the Solow growth residual. With the latter, an empirical growth rate can always be perfectly explained by observables (factors of production) and unobservables (the residual).

[17] As our Fokker-Planck equations are linear, we solve them by employing the method of characteristics (see Nagel, 2013, ch. 5 for details). Consumption paths are obtained by a shooting algorithm. The matlab code is available at waelde.com/pub.

Interest rate uncertainty also captures everything which is not explained for by the income process, observable heterogeneity in the cohort, the policy system (taxation and transfers) or inheritances. One contribution of this paper therefore lies in showing that any wealth dynamics can easily be captured by reasonable models. The empirical challenge lying ahead now consists in modelling all measurable heterogeneity to end up with some distribution-accounting that splits the evolution of distributions into observable and unobservable characteristics.

5.2.2 The dynamics of the distribution

When we chose p_i such that the fit in 2008 is perfect, the fit between the initial distribution and 2008 is bound not to be perfect.[18] The right figure in fig. 3 provides a visual impression of the fit between 1986 and 2008. As we would like to understand the fit also from a quantitative perspective, we suggest the following measure for how well the model densities fit the empirical densities,

$$F(t) = 1 - \frac{\int_{-\infty}^{\infty} \left| g^{\text{model}}(a,t) - g^{\text{data}}(a,t) \right| da}{2}.$$

The model densities are given by $g^{\text{model}}(a,t) = g(a,t)$ as described in (24). The density obtained from the data is described by $g^{\text{data}}(a,t)$. We consider their absolute distance as indicated by $|.|$. Image the densities do not have any overlap (like two uniform distributions one ending at x and the other one starting at $y > x$). We would then obtain $F(t) = 0$ as the integral over the densities would yield 2. The value of 0 would indicating no fit at all. By contrast, when the model density is identical to the data density, we would obtain $F(t) = 1$, indicating a perfect fit.

Applied to the densities illustrated in fig. 3, we get the values displayed in the following table.

1986	1987	1988	1989	1990	1992	1994	1996	1998	2000	2004	2008
1.00	.70	.56	.52	.51	.56	.62	.68	.73	.76	.82	.95

Table 2 *The quantitative fit of the model judged by the fit of densities*

The fit is perfect by construction for 1986 as we use the density from the data as initial distribution for the model. Between 1986 and 2008, the fit becomes worse. This is not a surprise as it was not targeted by the calibration. Finally, in 2008, the fit is close to perfect again.

An obvious generalization of the calibration approach would not choose an interest rate distribution that fits the density in 2008 only. It would look for distributions for, say, 1990, 2000 and 2008. This would make sure that the fit is close to 1 for these years as well.

5.2.3 The distribution of idiosyncratic interest rates

As our method guarantees a perfect fit in 2008 (and could guarantee perfect fits in other years as well as just discussed), an empirical test or at least plausibility check consists in looking at the quantitative distribution of the interest rate in our model. In our setup, the realization of the interest rate can be 3.5% and 4.5%. We let the individual draw from a distribution of arrival rates λ^{low} and λ^{int}. We compute the probability to be a certain financial type such that we match the 2008 distribution. From the perspective of t, this gives a certain probability $\text{Prob}(r = r^{high}, \tau)$ for any individual at any point in time to have a high interest rate.

[18] By letting the numerical code run m times, one could easily make the fit perfect at m points in time.

Comparing this time-moving probability distribution to empirical idiosyncratic distributions would be a second approach to judge the empirical fit. The probabilities for our values of p_i range from 13% to 87%. The highest standard deviation such a setup can generate is 0.05%.

While a more thorough econometric test would be desirable, possibly building on distributions of interest rates in the model which have more than two realisations, we see that the predicted distributions of interest rates is empirically plausible. Flavin and Yamashita (2002) report that mean returns to housing are 6.6% with a standard deviation of 14.2%. They obtain this finding from PSID data but report that similar findings hold for the data provided by Case and Shiller (1989). Standard deviations for other assets in Flavin and Yamashita (2002, table 1A) are all higher than the 0.05% needed here to perfectly match the empirical wealth distribution. This clearly shows that one can generate wealth densities by a model with interest rate uncertainty which is much less volatile than what is empirically plausible. Summarizing, it seems that empirically plausible specifications for idiosyncratic interest rate "overexplain" wealth inequality – just as empirically plausible specifications of idiosyncratic income processes seem to "underexplain" wealth inequality.[19]

6 Investment insurance and wealth inequality

We now turn to our policy application. It is well-known that expected utility maximizers desire full insurance against risk when there is no moral hazard. Individuals would therefore want to fully insure against fluctuations in labour income due to unemployment. As the latter does imply the risk of moral hazard (the unemployed do not search in an socially optimal way), unemployment insurance has to balance the trade-off between providing insurance and providing incentives.

We make the same argument in this paper for investment decisions. Some individuals are lucky with their investments while others have bad luck. Clearly, returns on investment also depend on knowledge, prior information collection, experience and the like which are not idiosyncratic. The same is of course true for employment and unemployment. We focus here on the purely idiosyncratic nature of returns to investment.

In our model, the interest rate can take two values, the value of r^{low} that satisfies (16) and the intermediate level r^{int} satisfying (17). We now study the effect of an investment insurance on the evolution of wealth. We want to understand how large this investment insurance would have to be such that the desired distribution of wealth would result. We also measure the benefits of this insurance in terms of aggregate capital stock (see Angeletos, 2007).

6.1 The insurance mechanism

When the probability for a path i is given by $p(i)$ and there are sufficiently many individuals in the cohort, then the share of individuals that have drawn path i is given by $p(i)$. The share of individuals at a point t in time that have a high interest rate is then given by

$$s(t) = \Sigma_{\{i \text{ with } r(t)=r^{\text{int}}\}} p(i),$$

in words by the sum of the probabilities of those paths where the interest rate $r(t)$ in year t is high, i.e. equals r^{int}. The insurance mechanism we consider is a combination of an insurance fee ϕ and a subsidy β. Net returns are given by

$$r_{\text{net}}^{\text{int}} = (1 - \phi(t))\, r^{\text{int}} \tag{25}$$

$$r_{\text{net}}^{\text{low}} = (1 + \beta(t))\, r^{\text{low}} \tag{26}$$

[19] A next step in the analysis of this conjecture would work with a theoretical structure that is rich enough to allow households to invest in as many assets as reported by Flavin and Yamashita (2002).

We assume that this is a fair insurance scheme in the sense that income and expenditure from the scheme balance at each instant. One can imagine that such a scheme is organized by public institutions or by a competitive insurance company. Denoting the share of individuals with a high return in t by $s^{\text{high}}(t)$,[20] it would hold for both cases that

$$\phi(t)\, r^{\text{int}} s^{\text{high}}(t) \int a p^{w}(a,t)\, da = \beta(t)\, r^{\text{low}} \left[1 - s^{\text{high}}(t)\right] \int a p^{w}(a,t)\, da. \tag{27}$$

In words, a share $s^{\text{high}}(t)$ of the (average) capital stock of the cohort under consideration gets a high return r^{int}. With a fee of $\phi(t)$, the left-hand side is the income of the insurance scheme. The right-hand side then measures total expenditure for the share $1 - s^{\text{high}}(t)$ of individuals that get a low return. This equality can be rewritten such that we can express the subsidy $\beta(t)$ as a function of the fee

$$\beta(t) = \frac{s^{\text{high}}(t)}{1 - s^{\text{high}}(t)} \phi(t) \frac{r^{\text{int}}}{r^{\text{low}}}. \tag{28}$$

The (path of the) fee $\phi(t)$ can then be fixed to achieve the desired distribution of wealth.

6.2 Quantifying the insurance mechanism

We work with a constant insurance fee ϕ. Given the calibrated shares $s^{\text{high}}(t)$ of individuals with a high return from sect. 5.2.1, we compute time paths of the insurance subsidy $\beta(t)$ for fees ϕ of 0.02 and 0.5. Given the paths of $\beta(t)$ for a given ϕ, we can compute optimal consumption and saving behaviour and the resulting evolution of wealth. Note that the net interest rate for the intermediate case in (25) is constant for a constant ϕ while the net interest rate for the low-interest rate case is time-dependent. This then tells us how the wealth distribution would have looked like in 2008 if an insurance mechanism had been in place since 1986. It also tells us how large ϕ needs to be such that the desired wealth distribution would result.

6.2.1 A modest fee

We start with a constant fee of $\phi = 0.02$. The implied interest rates are displayed in the following figure.

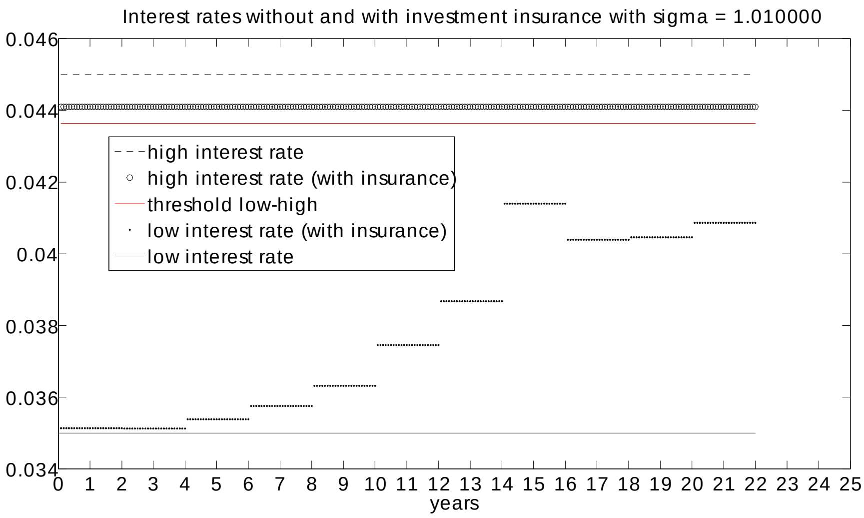

Figure 5 *The interest rates without and with the insurance scheme*

[20]Given the share p_i of the population to be endowed with financial ability i and the resulting time paths of interest rates, we can compute the share $s^{\text{high}}(t)$ of the population which has a high return at t.

The figure shows the low and the intermediate interest rate before the insurance scheme over the 22 years we observe our cohort. The figure also displays the high return after the fee and the corresponding low returns increased by the subsidy $\beta(t)$ from (28) such that the insurance constraint (27) holds.

6.2.2 The effect on the distribution of wealth

When we implement the above scheme, we obviously have lower net returns for good realizations of the interest rate. Given a constant fee ϕ, the high return is by a constant percentage of 2% lower than in the case without investment insurance. For low returns, however, returns are higher. The tend to be higher for the 90s than for earlier times. The question is, which of these opposing effects dominates.

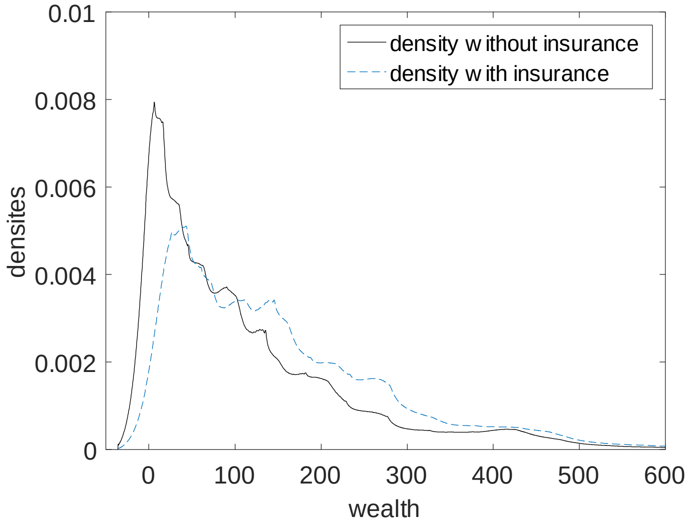

Figure 6 *The density of wealth in 2008 under an investment insurance of* $\phi = 0.02$

Figure 6 shows that the wealth distribution moves to the right. Average wealth increases from approx. 111,450 \$ to 153,720\$ by more than 37%. The Gini coefficient falls by 19% from 0.54 without an investment insurance to 0.43 under an investment insurance.

6.2.3 The mechanics behind this finding

Previous analyses have shown that the support of wealth of the stationary distribution approaches infinity when the time preference rate approaches the interest rate from below. Even though we do not consider stationary distributions but focus on the dynamic change of some empirically given initial distribution, we have a similar quantitative finding. The upper limit a_w^* and thereby the support of wealth becomes larger, the closer the interest rate approaches $\rho + \sigma g$, i.e. the time preference rate adjusted for the growth rate (see (16) and (17)), from below. Obviously, as there is a finite time horizon of 22 years, there is an upper limit to a_w^* when $r \nearrow \rho + \sigma g$.[21]

As the right panel of the next figure shows, we also find that average wealth becomes larger as r approaches $\rho + \sigma g$ from below. This does not seem surprising as with a larger support $]a^{\text{nat}}, a_w^*[$ one would expect that the mean rises. Most interestingly, however, average wealth falls when the interest rises above the threshold level. More concisely, our quantitative findings indicate that

$$\frac{d\bar{a}}{dr} \gtrless 0 \Leftrightarrow r \lesseqgtr \rho + \sigma g. \tag{29}$$

[21] When the interest rate exceeds the threshold level from (16) and (17), i.e. when $r > \rho + \sigma g$, the support extends to infinity. The temporary steady state a_b^* also grows further.

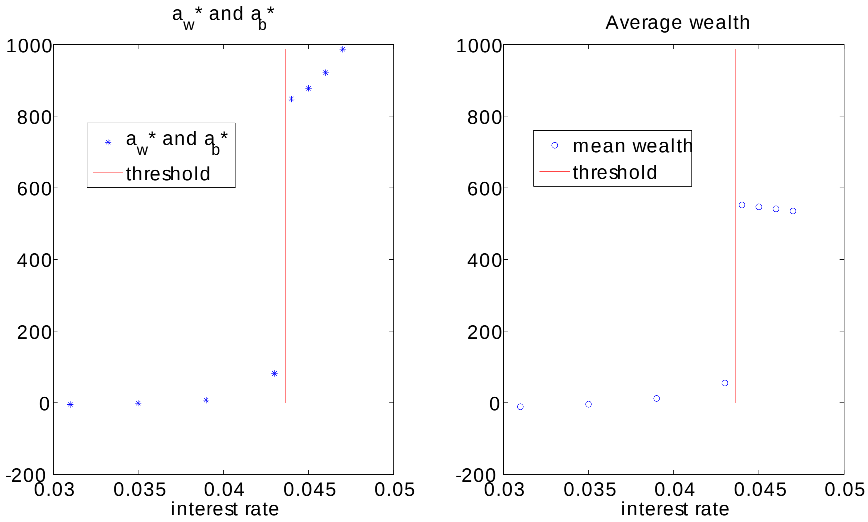

Figure 7 *Average wealth as a function of the interest rate after 22 years*

We see this finding for average wealth also for the entire densities. As fig. 8 shows, the rise in the mean follows from an smooth increase and decrease, respectively, of the densities. While the increase is faster for low r, the decrease is slow but visible for large r.

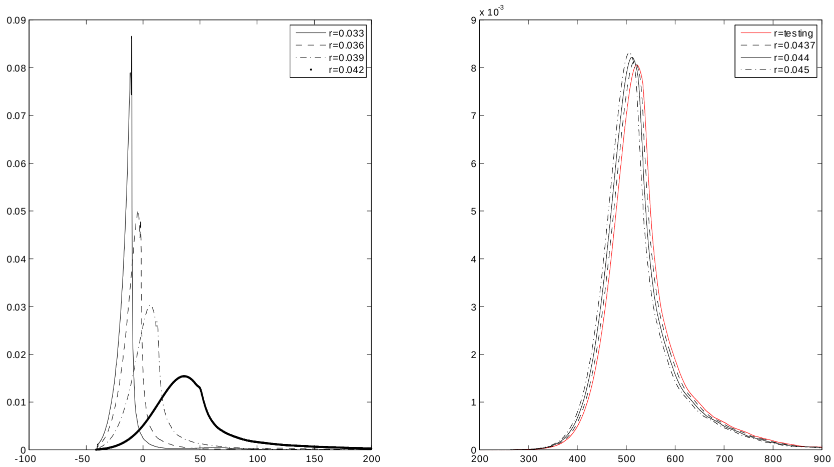

Figure 8 *Densities after 22 years for different interest rates (left panel: $r < \rho + \sigma g$, right panel: $r > \rho + \sigma g$)*

These two figures provide the mechanical explanation why our investment insurance implies that everybody becomes richer. Those that benefit from the insurance, i.e. those with a low return, experience a shift of their wealth distribution to the right, they become richer. Those that pay the insurance fee, i.e. those with a high return, *also* experience a shift of their wealth distribution to the right, i.e. the become richer as well. The central question from these findings is: why do individuals that experience a lower return have a higher wealth after 22 years?

6.2.4 Intuition

To understand the mechanism, consider again fig. 8. Each of the densities result from one fixed interest rate and fluctuations of labour income between w and b. To gain intuition why mean wealth after 22 years rises below $r+\sigma g$ and falls above this value, we switch of the uncertainty in labour income and study the remaining deterministic model. In such a model, we ask whether the wealth level $a(\tau)$ at some future point in time τ (set at 22 years as well) is larger when the interest rate r is larger. Formally, we study the condition under which $da(\tau)/dr > 0$ for an invariant initial wealth level $a(t)$ (and of course interest rate r and preference parameters). We also set the growth rate g equal to zero such that w is constant as well as this already yields the essential insight.[22]

With an intertemporal budget constraint $\int_t^\infty e^{-r[\tau-t]}c(\tau)\,d\tau = a(t) + \int_t^\infty e^{-r[\tau-t]}w d\tau$ and preferences as in (5), optimal consumption is given by $c(\tau) = \frac{\rho-(1-\sigma)r}{\sigma}\left(a(\tau)+\frac{w}{r}\right)$.[23] This expression gives us two channels via which the interest rate affects consumption (and thereby growth of wealth). First, the interest affects the present value of labour income. The higher the interest rate, the lower this present value w/r. Second, the interest rate affects the share $\frac{\rho-(1-\sigma)r}{\sigma}$ out of the present value of total wealth $a(\tau)+w/r$ the household consumes. As is well-known, the consumption level falls in the interest rate only if the degree σ of risk aversion is smaller than one. Plugging optimal consumption into the dynamic budget constraint and solving for $a(\tau)$ for $\tau \geq t$ yields

$$a(\tau) = \left(a(t)+\frac{w}{r}\right)e^{\frac{r-\rho}{\sigma}[\tau-t]} - \frac{w}{r}. \tag{30}$$

This shows us a third channel – the growth rate $(r-\rho)/\sigma$ of consumption.

When we ask whether an individual is richer in τ when he receives a higher interest rate as of t, we find

$$\frac{da(\tau)}{dr} > 0 \Leftrightarrow \left(a(t)+\frac{w}{r}\right)\frac{\tau-t}{\sigma} > \frac{w}{r^2}\left(1-e^{-\frac{r-\rho}{\sigma}[\tau-t]}\right). \tag{31}$$

This condition immediately shows that it always holds for $r \leq \rho$. The right-hand side is non-positive for any $\tau > t$ and the left-hand side is strictly positive as long as $a(t)+\frac{w}{r} > 0$, which is satisfied if we let a natural borrowing limit as in (8) hold here as well. Hence, we see that wealth $a(\tau)$ rises the higher the interest rate r if $r < \rho$, just as in the left part of fig. 7.

For $r > \rho$, one can easily construct an example where we see that future wealth falls in the interest rate: when $a(t)$ is sufficiently negative, the slope with which the left-hand side rises over time (as $\tau - t$ increases) is small. The right-hand side is concave in $\tau - t$, however. If the slope is sufficiently small, there is a range $]0, \bar{\tau}-t[$ within which $da(\tau)/dr < 0$. This is illustrated in the next figure.

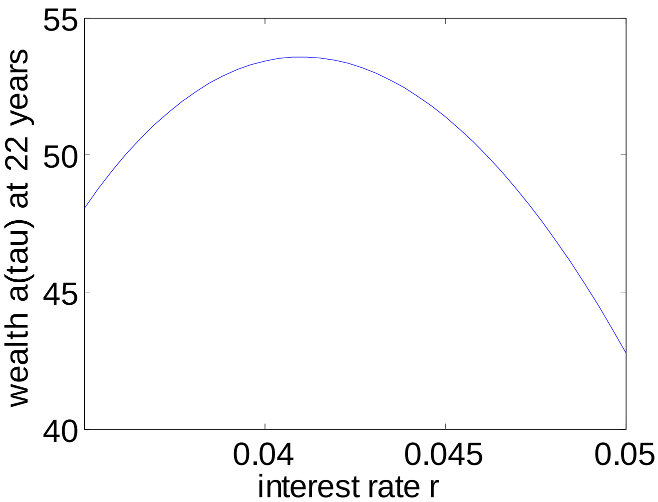

Figure 9 *Non-monotonic link between wealth at $\tau > t$ as a function of the interest rate*

[22]See the note by Wälde (2016) on more details.

[23]For a textbook treatment, see e.g. Wälde (2012, ch. 5.6.1).

This figure provides the intuition for our finding why an investment insurance makes everybody richer. As the investment insurance increases returns if they lie below the threshold level (which in this figure is ρ rather than $\rho + \sigma g$ in the stochastic model), those that experience higher returns below the threshold level become richer. As lower returns due to the insurance fee are paid by those that have returns above the threshold level and those lower returns imply higher wealth, those that pay the insurance fee become richer as well. This is why the overall wealth distribution moves to the right as shown in figure 6.

Coming back to the three channels discussed before and after (30), one channel obviously implies that a higher interest rate should lead to more wealth at some future point in time: A higher interest rate means higher capital income ra. A higher interest rate with a risk-aversion of $\sigma = 1.01$ implies a higher consumption level and would therefore reduce wealth. The third channel is consumption growth: it rises in the interest rate and reduces wealth for future points in time. As the derivative in (31) shows, risk-aversion does not play a role in fixing its sign. It is the growth rate of consumption which implies that wealth can fall in the interest rate. Our robustness analysis, to which we now turn, will confirm this. Even with a risk aversion below unity average wealth falls when the interest rate, once beyond the threshold, increases.

6.3 Robustness

What is the effect of the degree σ of risk aversion on the insurance effect of an investment insurance? Setting σ at .99 instead of at 1.01 as so far produces the following result.

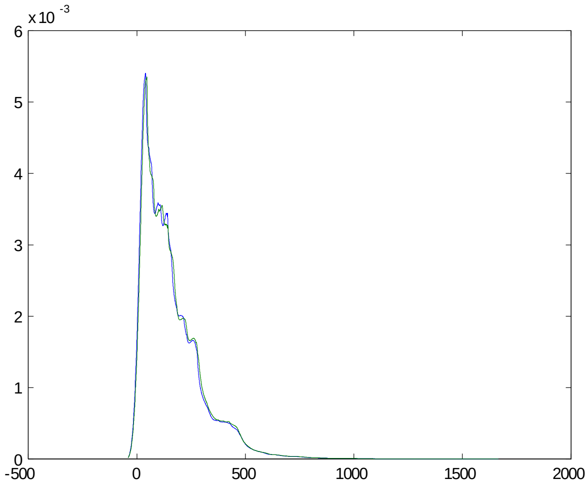

Figure 10 *The effect of an investment insurance on the distribution of wealth in 2008 with σ above and below unity*

The figure shows that the degree of risk aversion above or below unity does not have a major impact on the resulting distributional effects.

We also worked with risk-aversion that was more distant from unity. This moves the threshold level $\rho + \sigma g$ temporary steady states a_w^* and a_b^* considerably and thereby also the distributions of wealth. When increasing the time preference rate from currently 1% to 2%, the threshold level and temporary steady states move as well. In some cases, the interest rates need to be adjusted to obtain a match of the dynamics of the wealth distribution. There were no fundamental differences, however, in the fit of the model. We are therefore confident that our findings do not depend on the specific quantitative outcomes of our calibration procedure.

7 Conclusion

The paper starts out by matching the dynamics of the wealth distribution of the NLSY 79 cohort from 1986 to 2008 by the evolution of a density of wealth resulting from an idiosyncratic risk model with optimal saving. The crucial feature for the quantitative match is idiosyncratic interest rate uncertainty. We exploit the strength of Fokker-Planck equations in describing the dynamics of distributions to quantitatively perfectly match the 2008 density, using the 1986 empirical density as initial condition for solving our Fokker-Planck equations. We argue that any model with interest rate uncertainty must be capable of perfectly matching a wealth distribution as the distribution of idiosyncratic interest rates gives as much flexibility as the Solow growth residual for matching empirical growth rates.

Given our calibration of preferences parameters, parameters of the labour income process and of the capital market, we argue that an insurance mechanism, called an 'investment insurance' is a mechanism that would be desirable for individuals that do not know their financial type, i.e. that cannot predict future returns of their investments.

When we quantify the effect of an investment insurance, we work with a modest insurance fee of 2% applied to high returns. We found that this not only reduces wealth inequality but even increases average wealth by more than one third. We explain the intuition behind this result theoretically and thereby support the claim that an investment insurance seems a highly promising policy tool to reduce wealth inequality.

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