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Simple-majority rule and the size of the Bundestag

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Abstract

How should an excessively large parliament be effectively reduced in size without violating constitutional principles? This is a question that the German Bundestag discussed since introducing the 2013 electoral reform until today.

Facing a Bundestag consisting of 709 members and facing some public dissatisfaction, a reform to decrease the parliament's size was adopted in 2020. With the 2017 elections taking place under the new electoral rule, the size would have been 686 instead of 709. However, the opposition filed a lawsuit against the new electoral law with the German Federal Constitutional Court. Aside from legal considerations, the adherence to plurality rule has to be criticised from a social-choice perspective.

This paper aims to determine if the size and composition of the Bundestag change. In particular, whether the size is reduced when the German parliament's directly-elected members are elected using the simple-majority rule. Thus, a statistical simulation is carried out. We show that the targeted size of the Bundestag of 686 MP can be achieved by using the simple-majority rule to select the directly-elected members of parliament. Though, as we find indications that even Condorcet losers were elected into parliament, applying the simple-majority rule would ensure that only Condorcet winner would be elected directly into the Bundestag.

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Contents

List of Figures

List of Tables

1. Introduction

Although the size of the Bundestag is set at 598 members[1], the 19^{th} German Bundestag, elected in 2017, consists of 709 members, which is the largest Bundestag in German history to date (Bundeswahlleiter, 2017). This excessive expansion increases costs and reduces the ability to work (Bundestag, 2020). Facing significant public pressure, a reform to decrease the parliament's size was passed in October 2020. However, the opposition filed a lawsuit against the new electoral law with the German Federal Constitutional Court (BVerfG) in February 2021, as they consider it to be unconstitutional (see Section 2.2). The verdict is still pending.

Furthermore, the new electoral law still adheres to selecting the directly-elected members of the Bundestag by plurality rule, although it is widely known that a plurality winner may well be defeated in a head-to-head majority vote by several other candidates (Sen, 2017; Saari and Newenhizen, 1988). Moreover, even a Condorcet loser, the candidate whom all other candidates defeat, might be elected under plurality voting. The winner of this kind of election is not necessarily the candidate who is preferred over all other candidates from the majority of voters' point of view. It instead is the one who encounters a favourable candidate environment, which Maskin and Sen (2016) have vividly illustrated. Yet, democratic elections shall be independent of irrelevant alternatives, a principle that follows immediately from the method of majority decision (Sen, 2017, Ch. 10*) and was firstly emphasized by Arrow (1963) in his famous work. A rule respecting this and a set of other sound conditions is the simple-majority rule, as it has been shown in various works from May (1952) to Dasgupta and Maskin (2008); Dasgutpa and Maskin (2014). Not much would even need to be changed in German federal election law to implement the simple-majority rule instead. Essentially, voters must be allowed to disclose their order of preferences regarding the candidates, instead of only their first preference, as is currently the case due to the plurality rule. Note that we are focusing on the *first* vote here alone, thus the election of the directly-elected members.

Although the simple majority is the only voting method that can fulfil the requirements of the principle of majority decision, the fundamental insight of Arrow's impossibility theorem remains, according to which every possible voting rule must have some flaw. Sometimes, even the simple-majority rule still fails to work well due to its indeterminacy if preference profiles lead to a full-majority cycle (see Section 3.1). In such a case, there is no Condorcet winner, and only an (instant) run-off might bring forth an election winner (Dasgupta and Maskin, 2004; Barbaro, 2021). However, by running the simple-majority rule on all 299 electoral districts, we do not find an example of indeterminacy, i.e., we found a Condorcet winner in each district.

This paper aims to determine if the size and composition of the Bundestag change. In particular, does the size reduce when the German parliament's directly-elected members are voted for with the help of the simple-majority rule? The anticipated outcome of the simulation is based on these considerations:

[1] According to Section 1 (1) sentence 1 BWahlG.

Most of the directly-elected members of the Bundestag presently belong to the majority party, which leads to an excessive need for compensatory mandates. However, our data in Section 4.1 shows us that the two parties, Christlich Demokratische Union Deutschlands (CDU) and Christlich-Soziale Union in Bayern (CSU), henceforth referred to as the Union parties, greatly benefit from the violation of the condition **I**, the independence of irrelevant alternatives (see Section 3.1). Changing the voting method to the simple-majority rule, which does not violate **I**, would most probably increase the heterogeneity of directly-elected representatives and thus lead to a lower need for compensation. Ceteris paribus, a reduction in the size of the Bundestag is anticipated.

It is, however, essential to note in advance that this paper's expected resizing of the Bundestag does not equal a final result close to 598 seats, as this would be unrealistic given the mechanism of overhang and compensatory mandates in combination with the current voter preferences.[2] Given these circumstances, such a vast size change could be attained by a significant reduction in the number of electoral districts, similar to what was proposed by the opposition (Bundestag, 2019). Nonetheless, this paper focuses primarily on changing the voting method for the election of the directly-elected members to the simple-majority rule.

2. Correlating Electoral Reforms

2.1. Electoral Reform 2013

In 2013, the German Bundestag introduced the first fundamental electoral law reform in decades. The German Federal Constitutional Court ruled that the previously existing federal election law was contrary to the constitution[3]: In certain cases, a *negative weighting of votes* occurred. This means that „more second votes for a party in certain constellations could mean fewer seats in the Bundestag and, vice versa, fewer second votes could mean more seats“(Seils, 2013). This paradox effect results from the combination of two phenomena: On the one hand, there are overhang mandates, which can arise from the two-vote system that is enshrined in Basic Law (see the ruling of the Constitutional Court 1992). Using the plurality rule, the first vote goes to a specific electoral-district candidate of a party (or partially non-partisan). The second vote is given to the party in order to ensure proportional representation. Overhang mandates arise when a party wins a large number of directly-elected mandates with the help of the first vote – more than the number of seats it would be entitled to according to the number of second votes.

Every directly-elected politician is entitled to one Bundestag seat, regardless of the number of second votes their party receives. The seats, which exceed the percentage of

[2]This statement holds as long as there are parties in the Bundestag that win none or only very few electoral districts relative to its (second) votes. The figure of 598 can only be reached if each party's shares of first- and second-votes are equal.

[3]Ruling of July 25, 2012, on the constitutional review of the Nineteenth Act Amending the Federal Election Law of November 25, 2011

second votes, are called overhang seats. The second phenomenon is presumably less well known: The state lists of a party used to be treated as linked (before the reform in 2013). In this way, it was avoided that all votes in each state, which did not result in a complete mandate, were lost. For this reason, the first step in seat allocation was to determine how many seats a party had won nation-wide. This quota of seats was then allocated to the individual state lists in a second step. However, this process could result in the paradoxical effect of a *negative voting weight* when combined with the phenomenon of overhang mandates: Additional second votes for a party that did not have the effect of increasing seats due to overhang mandates in one federal state could lead to the same party being allocated fewer list mandates in another federal state, therefore resulting in fewer seats altogether. Conversely, this also means that a party, which gained overhang mandates in a federal state due to its strength in the first-votes, received more seats in the Bundestag in total, the fewer second votes it received in the respective federal state. (Seils, 2009).

The core of the electoral law reform 2013, which was adopted to prevent the negative weighting of votes, is the introduction of *compensatory mandates*. All overhang mandates that arise in the allocation of seats are compensated for. If a party receives one or more overhang mandates due to the election, the total number of seats in the Bundestag is increased until the size ratio of the political parties in the Bundestag reflects the proportion of second votes in the election. However, this process generally holds an inflation tendency for the parliament's size. Thorough simulations by Weinmann (2013) have shown that under this election law, it would practically be impossible to meet the standard size of 598 seats, as the lowest number of seats in the entire simulation amounts to 610 - even in the case of election results favourable to the size of the Bundestag.

Consequently, it was scientifically anticipated that there would eventually be an excessive increase in the number of seats of the Bundestag under the electoral reform of 2013. Several possibilities have been discussed by judicial and political scientists as to how the 2013 electoral-law reform could increase the size of the Bundestag. It is considered possible that the interplay of overhang and compensatory mandates plays a large part in increasing the size of the parliament. Nevertheless, a need for compensatory mandates can also be caused, e.g., by rounding errors, varying voting participation rates in the federal states, or a high number of second votes for any other parties not involved in allocating seats (Dehmel and Jesse, 2013).

2.2. Current Electoral Reform

As already mentioned in Section 1, the 19^{th} German Bundestag elected in 2017 consists of 709 members, which is the largest Bundestag in German history to date. Since this expansion in the parliament's size increases costs and reduces the ability to work, an electoral reform to decrease its size was adopted on October 8, 2020. The size of the German Bundestag is planned to be reduced with the help of three approaches. Firstly, overhang mandates within a party are off-set against mandates from other state lists of the same party.[4] Secondly, the compensation of overhang mandates is only started after the third overhang mandate. The third step of the reform, reducing the number of electoral districts from the current 299 to 280, is not to be taken until the federal election in 2025. The entire reform is questioned in terms of its effectiveness, which is also indicated by a calculation of the political scientist Behnke (2020). According to Behnke's calculations, the overall effect of the reduction in the Bundestag size that the electoral reform will exert is very small, on average around ten seats. When applying the actual voting results from 2017 to the voting law from the newly-adopted electoral reform[5], a total of 686 seats is obtained. In either case, the resulting reduction is due to the three uncompensated overhang mandates. The effect of the first approach, offsetting with list mandates, is almost negligible under the given circumstances. The third approach, reducing the number of electoral districts, would have a genuinely noticeable effect - even more significant than the second one - if the decrease in districts were to take place on a much larger scale than what the reform proposes. Thus, the only step of the current reform that might lead to a moderate reduction in the Bundestag size is the second one. However, this particular approach is questionable from a constitutional point of view due to its inability to ensure the equality of success values (BVerfG, 2012). Because if there are overhang mandates that are not compensated for, it may again lead to precisely the constitutionally questionable effect that the electoral law reform in 2013 was intended to prevent - the negative weighting of votes (cf. Subsection 2.1).

3. The Simple-Majority Rule

3.1. Axiomatic Superiority of the Simple-Majority Rule

In terms of social-choice theory, the optimality of a collective-decision rule is evaluated by examining which axioms it satisfies (Sen, 1995; Dasgupta and Maskin, 2008). According to the *Impossibility Theorem* by Arrow (1963), there is no ideal aggregation method since no voting rule can satisfy Arrow's well-known set of rational and democratic axioms. Nevertheless, just like Sen (2017) has stated, the only voting method that fulfils the requirements of the principle of majority decision is the *simple-majority rule* (SMR). Moreover, as shown by Dasgupta and Maskin (2008) in detail, when focusing on the fundamental democratic axioms, robustness of the simple-majority rule prevails - even for the broadest class of domains.

This can be explained based on the following set of principles: Pareto principle, anonymity and neutrality. The three most prominent voting rules, which satisfy all of the three just-mentioned principles are:

1. The *Simple-Majority Rule* - candidate x wins if, for all other candidates y in the given set, more voters prefer x to y than y to x (*Condorcet winner*). To create such social orders, voters must express a ranking of all candidates.

[4] However, CSU and CDU are treated as different parties, which reduces the effect of this first approach.
[5] We make use of the programme *Mandatsrechner*.

2. *Borda Count* - each candidate gets one point for every voter who ranks them first, two points for every voter who ranks them second, and so on. Candidate x wins if x's total points are lowest among all candidates in the given set.

3. *Plurality Rule* - candidate x wins if more voters rank x first than they do any other candidate (although when implemented in reality, there is no ranking option; rather, each vote counts as first preference under plurality rule).

However, a fourth standard axiom cannot be satisfied by all of the mentioned voting methods; in fact, only the simple majority rule can (Dasgupta and Maskin, 2008). This so-called *independence of irrelevant alternatives* (condition **I**) has gained significant attention since having been emphasized by Nash (1950) and Arrow. Arrow has defined **I** this way: „The social preferences between alternatives x and y depend only on the individual preferences between x and y."Meaning that adding another alternative should not alter the preference ordering between x and y. Borda count, also called rank-order voting, as well as the plurality rule, fail to satisfy this requirement. Straightforward examples are provided, e.g. by Dasgupta and Maskin (2008); Maskin (2014).

Under simple-majority voting the choice between x and y depends only on how many voters prefer x to y and how many prefer y to x - not on whether other candidates are also up for election. On the other hand, it is often shown that even the Condorcet loser (the candidate who is defeated in any head-to-head match-up) can become the plurality winner (Sen, 2017; Saari and Newenhizen, 1988) (and we have the indication that this exactly has been the case in at least three electoral districts in Saxony at the 2017 elections, cf. Section 4.2).

Nevertheless, as mentioned in this chapter's beginning, the fundamental insight of Arrow's impossibility theorem remains. According to this, every possible voting rule must have some flaw. Sometimes, even the simple-majority rule still fails to work well due to its indeterminacy of a winner if preference profiles lead to a full-majority cycle—the so-called *Condorcet paradox.* In other words, the simple-majority rule may not satisfy a fifth axiom called *decisiveness* - at least in some cases.

It is, however, important to note that such a scenario did not even once occur in the simulation conducted for this paper. Thus, for each constituency, a Condorcet winner could be determined. This is because the voter preferences for the Bundestag election do not follow an arbitrary pattern. Instead, they are ideologically driven and often fulfil the requirement of value restriction of which single-peakedness is a special case (Sen, 1966).

The insights from this chapter so far can be summarized as such: Neither of the three prominent voting rules, nor any other voting rule can satisfy all axioms (Arrow, 1963) or even these five crucial axioms - Pareto, anonymity, neutrality, **I**, and decisiveness - on *all* domains (Dasgupta and Maskin, 2008). However, while it makes sense to be inclusive of all domains in theory, this is not always necessary in practice. Because some preferences may actually be highly unlikely, as Black (1948) notes. According to his findings, the average voter's preference for the top candidates is primarily determined by how far they are from their own position in the left-right ideological space. Thus, in most elections, especially in federal elections (cf. Section 4.1), voting preferences are *single-peaked* because the candidates are usually arranged ideologically on a horizontal axis. This can be illustrated with a simple example in reference to the 2017 Bundestag election. The parties currently represented in the Bundestag are: Die Linke (LINKE), Bündnis 90/Die Grünen (GRÜNE), Sozialdemokratische Partei Deutschlands (SPD), Christlich Demokratische Union Deutschlands (CDU) and Christlich-Soziale Union in Bayern (CSU) combined (UNION), Freie Demokratische Partei (FDP) and the Alternative für Deutschland, abbreviated as AFD.

Regarding the last federal election, the voter preferences of a LINKE voter are statistically very likely to be in the same order as listed above (cf. Figure 1 on page 14). There is a moderate probability that instead of the GRÜNE, the SPD might be in second place and with less probability, perhaps even the UNION. However, according to the Politikbarometer 2017, a cumulative data set of the German federal election in 2017, the probability that a voter who lists the LINKE as his first preference will prefer the FDP or AFD in second place is vanishingly small (Forschungsgruppe Wahlen, Mannheim, 2018).

3.2. Legislative Situation

According to the Federal Constitutional Court's ruling of July 25, 2012, on the constitutional review of the Nineteenth Act Amending the Federal Election Law of November 25, 2011, the legislator is free to determine and design an electoral system. Thus, the constitution does not impose a particular voting scheme, but basic constitutional requirements must be met, such as the principle of electoral equality. According to Article 38 (3) of the constitution, the legislator remains up to whether he organizes the German Bundestag's voting procedure as a majority election or a proportional representation election.

Summarizing this information, it would be permissible to determine the directly-elected mandates of the Bundestag using the simple-majority rule instead of the plurality rule. Since the simple-majority rule is regarded as a majority voting method from a scientific point of view, the legal position regarding the admissible voting procedure for the German Bundestag would even facilitate its implementation. Something worth highlighting in this context is the court's definition of majority voting. As the ruling also states that „the election must provide democratic legitimacy to the representatives", it raises the question of whether the design of the electoral system allows for a candidate to be declared the winner who was not even elected by the majority—a potential scenario in case of using the plurality rule as a voting method[6] (see Subsection 3.1). After all,

[6] A recent attempt to abolish the run-off elections to select the lord mayors and the district chief executives in North Rhine-Westphalia failed in court. The amended election law that aimed to impose plurality voting did not meet the constitution's requirements, according to the ruling of the state court in 2019. The government justified the targeted abolishment in view of low voter turnouts. The (directly) elected incumbents may lack legitimacy if up to two-thirds of total voters abstain from casting a vote in run-offs, as it is often observed. Plurality voting, thus, yields more legitimacy as the turnout tends to be significantly higher in the first-round elections. The constitutional court

according to democratic understanding, this would call into question the candidate's legitimacy to represent the population, since „the will of the majority, not that of the minority, should be decisive“ (Sen, 2017).

Moreover, the 1992 Federal Constitutional Court's ruling states that voters need to see who is running for a mandate before casting their vote and how their vote will affect the candidate's success or failure. This previous sentence refers to the effect of the negative voting weight, which could occur again due to the new reform (cf. Section 2). For this reason in particular and other constitutionally questionable aspects such as ambiguity of the election procedure, the opposition has filed a lawsuit with the Federal Constitutional Court currently being reviewed.

3.3. Application to the Federal Election

What simulation results can be expected when applying the simple-majority rule to the directly-elected Bundestag members' election? Before answering this question, it is essential to highlight that this paper's approach merely involves a change in the electoral procedure of the "first-vote" election of the Bundestag, which determines the direct mandates. The electoral procedure of the "second-vote" election shall remain with a proportional representation. The point of interest for the simulation is the 2017 election of the 19^{th} Bundestag and the electoral law prevailing at the time.

By changing the voting method to the simple-majority rule, we anticipate an increased heterogeneity of directly-elected members and less compensatory mandates. This is because most of the direct mandates are presently obtained by the majority party—the Union (cf. Subsection 3.1)—which leads to a great need for compensation. The reason for their high number of direct mandates can be attributed to the fact that under plurality rule, the Union greatly benefits from the violation of condition **I**. Since the simple-majority rule does not violate **I**, applying it to the federal election would most probably increase the heterogeneity of directly-elected representatives and thus lead to a lower need for compensatory mandates. This, in turn, would also argue for an accompanying reduction in the size of the Bundestag.

Furthermore, we expect the electoral outcome to be decisive, i.e. finding a Condorcet winner for each constituency since the preferences of voters of the Bundestag election are largely value-restricted (see Sections 3.1 and 4.2).

rejected this view. It states that voter turnout is *one* indicator among others to judge whether a voting scheme satisfies the "principle of majority". This principle may also be violated if a candidate takes office with only about 20 to 30 per cent of votes. In this respect, the distance to the majority (obviously, the court defines the majority as a strict majority of votes) should be considered when assessing the appropriateness of a voting rule. Cf. Barbaro (2021).

4. Simulating the Size of the Bundestag

4.1. Data Description and Approach

In order to simulate the size of the Bundestag under the simple-majority rule for electing the directly-elected members of parliament, we carried out a calculation for the size of the current (i.e. the 19^{th}) Bundestag by using data from a nation-wide cumulative data set consisting of 36,689 observations. Those were collected by the Forschungsgruppe Wahlen, Mannheim (2018) with the help of a longitudinal section, more specifically, a cross-section that has been repeated regularly over the entire year 2017 (trend). We furthermore included data from the last Bundestag election's actual voting results, which were obtained from the Federal Election Commissioner (Bundeswahlleiter).

Incidentally, since our simulation is primarily concerned with the resulting size of the $19^{t}h$ Bundestag, we only examine the parties represented therein.

We compiled preference profiles as well as the corresponding relative probabilities of the different orderings with the help of the statistical package `R` (R Core Team, 2020). The data set by Forschungsgruppe Wahlen, Mannheim (2018) involved the variables named V61, V62, and V63, for which the survey participants disclosed what party they rank first, second, and third. This information we then utilized to generate the individual orderings of the ranks one to three. For the ranking of the fourth to sixth places (as there are six parties represented in the 19^{th} Bundestag), we used the variables V49 to V55. Here, participants gave each party a rating, ranging from -5 to $+5$, where -5 is the worst rating they could give and $+5$ the best. The subdivision into different variables was necessary to obtain accurate preference orderings. The rankings (variables V61, V62, and V63) correspond to the votes that would be cast in an election under the simple majority rule. However, they partially contradict the ratings collected through the variables V49 to V55. Therefore, the more accurate rankings were used to generate the individual orderings from the first to the third preference since they are given only up to that number. We then used the ratings for computing the fourth preference and higher.

In this context, it is also necessary to emphasize that we assume a correspondence between the voters' general party orderings, i.e. their second-vote preferences, and their orderings concerning the district candidates. We have to use the second-vote data as a proxy for the first-vote preferences due to missing data. So if a voter indicates that they (generally) rank the Union party first and the SPD second, we assume that their first-vote preferences are precisely the same, meaning they prefer the district Union candidate and rank the SPD competitor second, and so on. There is considerable evidence that this is an admissible assumption, as the data by Forschungsgruppe Wahlen, Mannheim (2018) shows that there is more than 90% correspondence between the two votes cast in the last federal election (for Union and SPD voters). This correspondence is weaker for small-party voters since they are assumed to split their votes strategically. The reason for this is that in some elections, voters may cast their ballots strategically and contrary to their actual party preferences. One may think that the splitting of votes occurs due to the electorate's opportunity to vote for individuals with their first-vote.

However, this is not true for Bundestag elections (Rudzio, 2019). Rather, people who strategically place their votes in federal elections do so out of fear of *losing* their first-vote to disfavoured candidates (Sen, 2020). As a consequence, voters split their first- and second-vote. Even though such voters may prefer party x (a small party) over all other parties, they would give their first-vote to a large party candidate, or rather to their second-ranked party. This specific placement of strategic votes is largely attributable to the current voting method of electing the district winners, the plurality rule (Sen, 2020). Since plurality rule violates condition **I**, plurality rule give voters an incentive to vote strategically (Gibbard, 1973; Satterthwaite, 1975; Dasgupta and Maskin, 2008). We, however, neglect such vote splitting in our simulations since there is no incentive to vote strategically under simple-majority rule[7]. There is an additional possibility that vote splitting in Bundestag elections may occur due to the 5%-hurdle (Rudzio, 2019). Since our simulation only includes parties that actually managed to enter the Bundestag, this factor is negligible when running the simulation.

Using the data set as just explained, we generated a preference ordering for each survey participant and calculated how often each particular ordering (e.g., Union first, GRÜNE second, SPD third, ..., AfD last) occurred. In another variable, the survey participants disclose the party they (were planning to) vote for at the upcoming Bundestag election.

Thus, we were able to generate the percentage distribution of the different types of voters to create subsets of particular voters in particular states or state-groups. For instance, we calculate a list for the state Bavaria consisting of six data frames (UNI-voters, SPD-voters, ..., AFD-voters). We then calculate any ordering's frequency for each type of voter. It calculated, for instance, how often Bavarian GRÜNE-voters have an ordering such as Union 4 - SPD 2 - Left party 3 - Green party 1 - FDP 5 - ADF 6 and so forth. Summarizing this technical information, it means that green-party voters in Bavaria can be described as the group of voters within the scope of our simulation, which is entitled to cast a vote in Bavaria and prefers the green party over all other parties - as depicted in Figure 1. Now, if, for instance, 15% of the Green-voters in Bavaria have an ordering such as

$$\text{GRÜNE} \succ \text{CSU} \sim \text{SPD} \succ \text{FDP} \succ \text{LINKE} \succ \text{AFD},$$

then, such an ordering we assign to 15% of green-party voters observed in Bavaria's actual Bundestag election. Once again, the issue of vote splitting[8] needs to be addressed in this context. In the previous section, we explained why we did not consider vote splitting in the simulation using the simple-majority rule. However, this explanation does not hold for the simulation steps in which we incorporated the actual data. It may remain unclear why we did not modify the actual data under the plurality rule so that vote splitting would not occur - as it would be the case under the simple-majority rule. However, since

[7] This holds for all cases of decisiveness.

[8] Note that the term vote splitting is sometimes used in the context of the **I** condition, see Maskin (2020) and should not be confused with the split of votes in a two-votes system like the German one.

we have only approximate values for such a data change[9], we decided to leave the actual data unaltered, which leads to a strengthening of our overall result. This is because, based on the aforementioned findings, it is to be expected that in an actual Bundestag election under the simple-majority rule, the vote splitting would be significantly reduced. Taking into account the reduced vote splitting, our simulation results would likely lead to an even smaller Bundestag, since the higher share of first-votes for smaller parties would increase the heterogeneity in the Bundestag even further and thus reduce the need for overhang or compensatory mandates (cf. Section 3.3). Incidentally, in deciding to leave the actual election results unchanged, we also specified that district candidates who scored more than 50% in the actual election would remain Condorcet winners in the simulation.

In the next step of the simulation, the order-lists were then applied to each voting district's actual data, which made it possible to calculate a preference profile for each constituency.

In order to consider regional differences in the simulated preferences, we furthermore computed the following regional groups (derived from the official statistical abbreviations of the German federal states, notably: BR denotes Brandenburg, HB denotes Bremen):

#	state or cluster	#	state or cluster
1	BY	2	BW
3	NW	4	HE
5	RP, SL	6	NI, HB
7	SH, HH	8	BR, MV
9	ST, SN, TH	10	BE

Table 1: States and state groups (clusters) considered for regionally-specific orderings.

Some states we had to group due to missing information in the data set concerning some (small) parties. There are very few Green-party voters in SL, for instance, or even too few LINKE voters in RP.

Armed with the regionally-sorted preference orderings, we used the R-package `vote` (Ševčíková et al., 2021), specifically, the function `Condorcet` by Barbaro and Ševčíková to determine the Condorcet winner in each electoral district.

4.2. Simulation Results

Looking at the preference profiles that were created for each type of voter per region (see Subsection 4.1), there are some obvious patterns that recur, but also some variations that emerge - especially when comparing western to eastern regions. These regional differences can be demonstrated as such:

Figure 1 does not depict all regional preferences for each type of voter but rather the most striking preference orders in terms of regional differences across voter groups. Most

[9] There is no precise data on the reasons for vote splitting in the last federal election, only on the share of votes that were split.

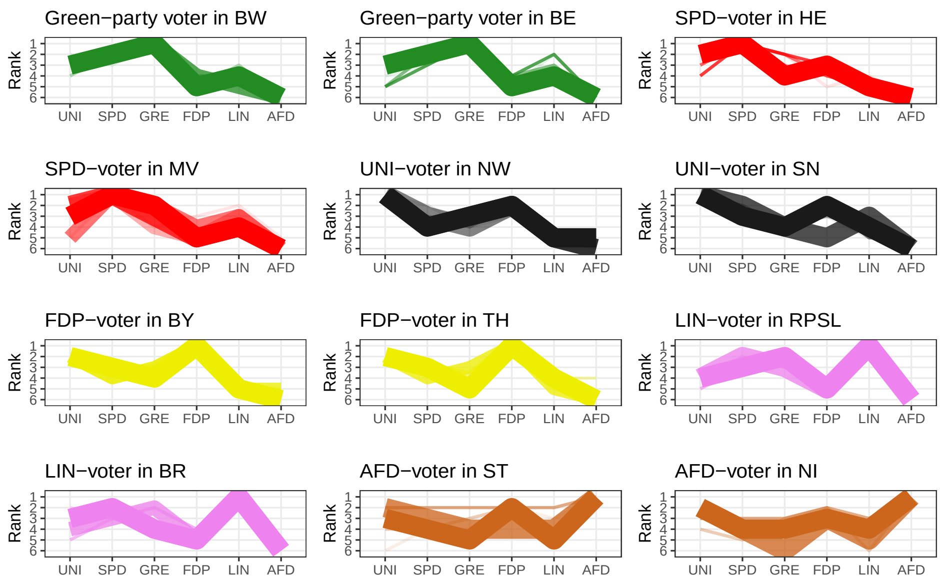

Figure 1: Regional preferences of different voters

abbreviations, which we used to describe different voters, are mostly equal to the ones mentioned in Section 3.1. Only the names for the GRÜNE (here *Green-party*) and the LINKE (here *LIN*) vary. The terminology used herein for the federal states corresponds with the official abbreviations. The abscissa shows the six parties represented in the Bundestag, and the ordinate shows the rankings of each voter group. The colours' saturation also indicates the frequency of the respective preference order - the more saturated the colour, the more frequently the ordering occurs. As illustrated in Figure 1, one of the most striking variations across regions can be observed among SPD voters, for instance. While the majority of SPD voters in Hesse (HE) places the FDP as their third preference and the *Green-party* as their fourth preference, this is not the case in Mecklenburg-Hither Pomerania (MV). In fact, not only does the majority of SPD voters there prefer the GRÜNE over the FDP. Furthermore, SPD-voters in Mecklenburg-Hither Pomerania prefer the LINKE over the FDP, which means that the FDP falls a full two ranks short compared to SPD-voters' preferences in Hesse. Instead, the LINKE moves up one rank and is thus ranked as the fourth preference by most of this voter group in MV. Green voters in BW rank the UNION far better than Green-party voters in Berlin. The list could be extended to several further examples.

It is, therefore, evident that there are regional differences, even within voter groups. In this context, however, it must be mentioned that we also conducted a simulation in which the regional differences were not taken into account. Instead, we draw a nation-wide average over the preferences for each voter group (nation-wide approach). Although the uniform simulation partly led to different constituency results compared to the regionally-differentiated situation, the resulting size of the Bundestag (see below) did not differ from the first simulation. This suggests that our simulation results are robust. (The corresponding result of this nation-wide approach are shown in this paper's Appendix A).

As mentioned in Section 4.1, after generating the regionally-sorted preference orders, the simple-majority rule was run on all 299 districts. As previously mentioned, we do not find any example of indeterminacy, meaning that we were able to find a Condorcet winner in each electoral district. Ultimately, all effects anticipated in Section 3 are compatible with the following results: As shown in Figure 2, applying the simple-majority rule leads to a differing electoral outcome compared to running the plurality rule.

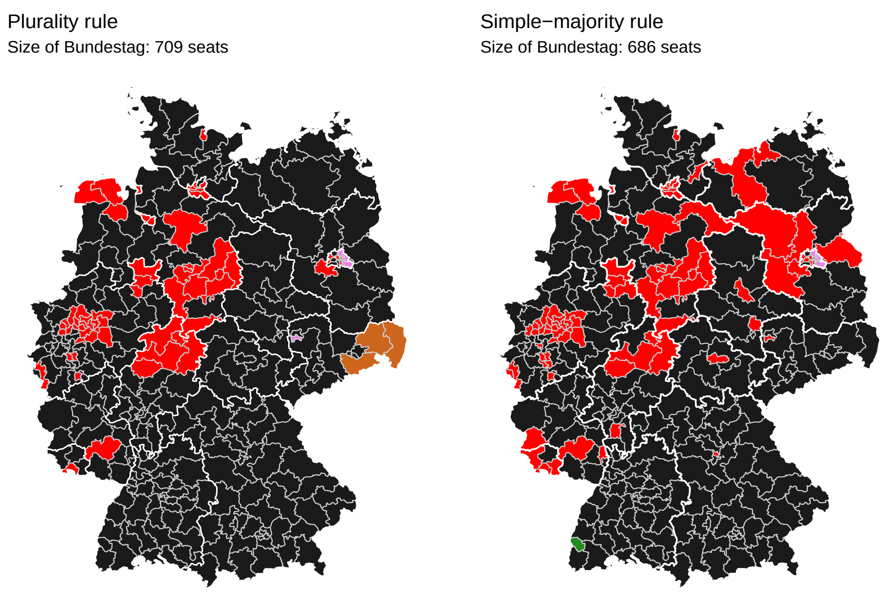

Figure 2: Districts actually won vs. Condorcet winners.

The left-hand side map depicts the actual electoral winners at the 2017 Bundestag election, while the map on the right-hand side illustrates the Condorcet winner simulated for the federal election in 2017. The colours are in alignment with the popular political colours[10].

Several districts flipped due to the change in voting method, i.e. the party that won this constituency under simple-majority rule differs from the one that won in the election under plurality rule. In order to obtain a better overview of those flipped districts, we create the following Figure 3 additionally:

The colours on the left map indicate the losing parties of our simulation, the colours on the right map the winning parties. Our simulation shows where the actual electoral outcome is driven by a violation of the condition **I**, as thoroughly described in Section 3.

Therefore, it is not surprising to observe that most districts whose winners change after applying the simple-majority rule switched from the Union to the SPD. This result is

[10]Black stands for the Union (i.e. CDU/CSU), red for the SPD, a chocolate colour we use for the AfD, yellow for the FDP, violet for the LINKE and green for the GRÜNE.

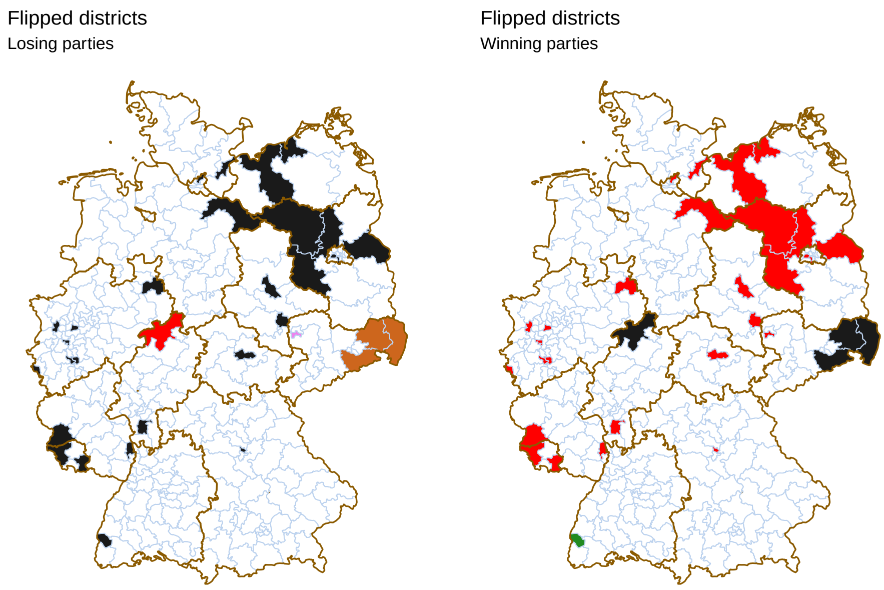

Figure 3: Map of flipped districts.

consistent with the claim of increased heterogeneity mentioned in Section 3.3. Although the Union does regain some districts originally won by the SPD, increased heterogeneity remains nevertheless due to the higher number of electoral districts switching from Union to SPD.

Using two flipped districts as examples, we will demonstrate how such different outcomes can occur purely as a result of a change in the democratic election process:

The left side of Table 2 shows the voter groups of the electoral district Freiburg (BW) for the SPD, LINKE (LIN), FDP and AfD. In addition to this, the data on how many of the individual voter groups in BW prefer the CDU/CSU to the GRÜNE (fav.uni) and vice versa (fav.gre) are each stated in per cent.

voters	state	fav.uni	fav.gre	voters	state	fav.uni	fav.spd
SPD	BW	0.343	0.657	GRE	HE	0.309	0.691
LIN	BW	0.250	0.750	LIN	HE	0.216	0.784
FDP	BW	0.826	0.174	FDP	HE	0.799	0.201
AFD	BW	0.517	0.483	AFD	HE	0.610	0.390

Table 2: Pair-wise comparison for Freiburg (UNI versus GRE) and Darmstadt (UNI versus SPD)

According to the official election data, roughly 179k valid first votes in the constituency of Freiburg were cast. Under plurality rule, 50,256 went to the CDU, 40,647 to the SPD, 13,172 to the LINKE, 46,115 to the GRÜNE, 9,546 to the FDP and 12,984 to the AfD. The rest of the votes were given to smaller parties that did not manage to enter the

Bundestag or to independent candidates. Thus, the Union won the electoral district of Freiburg, while the GRÜNE became the second-strongest force (lost by $\sim$ 4k votes).

Therefore, it is essential to consider what has been explained thoroughly in Section 3.1: *Votes* under plurality rule consists of first preferences only. However, if we now include the entire preference order in our simulation, i.e. selecting the direct mandates using the simple-majority rule in Freiburg, the GRÜNE emerges as the Condorcet winner instead[11].

The right side of Table 2 shows the voter groups of the electoral district Darmstadt (HE) for the GRÜNE (GRE), LINKE (LIN), FDP and AFD (thus, the match-up here is between UNION and SPD). In addition to this, the data on how many of the individual voter groups in Hesse prefer the CDU/CSU to the SPD (fav.uni) and vice versa (fav.spd) are each stated in per cent. There were $\sim$190k valid first votes in this district. 58,216 went to the CDU, 56,442 to the SPD. Thus, the Union won the electoral district of Darmstadt in the last federal election by less than 2k votes, while the SPD became the second-strongest force. If we now include the entire preference order in our simulation, just like we did above for Freiburg, the SPD emerges as the Condorcet winner instead. This is due to the fact that a vast majority of (many) Green-party voters in Darmstadt prefer SPD over the CDU. In our calculations we of course run any thinkable head-to-head comparison. We select only one of them here for the sake of simplicity and in order to demonstrate why districts flipped.

Other districts that have flipped in our simulation are located in Saxony, for example, where one went from the LINKE to the SPD. Furthermore, the AFD loses all of their three directly-won seats in Saxony to the UNION. This particular outcome is not very surprising either, as data analyses prior to the simulations have shown that the greatest and most unsystematic fluctuations occur in AfD voters' preferences. In contrast, most voters of all other parties mostly rank this party in the last place indicating that the AfD is highly polarizing. Therefore, the AfD has to be a clear Condorcet loser in all electoral districts, while it can nevertheless—contrary to all voters' full preference orders—very well be the election winner under plurality rule.

As a result, applying the simple-majority rule to the election of the directly-elected members of the Bundestag would have reduced the size of the Bundestag to 686 members. The flipped districts are responsible for most of the size reduction. It is, however,

[11] This can also be calculated manually using the data listed herein: Union-voters (50,256) prefer the CDU/CSU over the GRÜNE by 100%, and simultaneously, 100% of GRÜNE-voters (46,115) prefer the GRÜNE over the Union - so predictable it was not included in the table for that reason. Of SPD-voters (40,647), a majority of 65,7% prefer the GRÜNE to the CDU/CSU and thus, the opposite is true for 34,3% of this voter group. Likewise, a majority (75%) of LINKE-voters (13,172) prefers the GRÜNE over the Union, and 25% rank the CDU/CSU before the GRÜNE. The pattern is quite different for FDP-voters (9,546), who prefer the Union to the GRÜNE by 82,6%, while a minority of 17,4% prefers the reverse preference order. Of AfD-voters (12,984), 51,7% rank the CDU/CSU before the GRÜNE, while 48,3% of this type of voters prefer the GRÜNE over the Union. Multiplying the respective percentages with the corresponding numbers of first votes of the various voter groups, we come to the (rounded) result that 82,089 voters (of all parties in the Bundestag) prefer the UNION over the GRÜNE, while 90,631 voters rank the GRÜNE over the UNION. Consequently, under the simple majority rule, the GRÜNE wins as Condorcet winner in Freiburg.

essential to note that the AfD's loss of all of their three direct mandates to the Union temporarily resulted in an increase of the size of the Bundestag from 709 to 720 seats in total (all other changes omitted) since this step decreased heterogeneity.

Generally, most flipped districts are located in larger cities or cities with many young people, such as Freiburg, Darmstadt, Cologne, etc. Interestingly, the calculated size of 686 does not alter by ignoring a significant fraction of the flipped districts. Even assuming that no district in North Rhine-Westphalia (the German federal state with the largest population) would flip, the Bundestag would resize to the afore-mentioned 686 nevertheless. And even with national preference orderings that do not differ regionally, we end up with the same size result, as mentioned above, and depicted in the Appendix. This is a clear indication that our simulated size of the Bundestag is a robust result since it applies to different outcomes concerning flipped districts and differences in preference orders. The exciting thing about the Bundestag's resulting size of 686 members is that this is precisely the number which the legislator targeted with the amended electoral law explained in Section 2.2.

5. Conclusion and Outlook

Concluding this paper's findings, it was shown in Section 3 that the simple-majority rule, unlike the plurality rule, does not violate Condition **I**, and is therefore considered (axiomatically) superior to the plurality rule. Moreover, implementing the simple-majority rule for Bundestag elections does not contradict any constitutional norms or other laws, while the BVerfG is currently reviewing the 2020 reform as it is deemed unconstitutional. Apart from these scientific and legal advantages, the simulation results reveal that the Bundestag's size would have been at 686 seats by considering the simple-majority rule for selecting the directly-elected members of parliament in the 2017 election.

The remarkable aspect about this quantity is that it corresponds to the size produced as a result when applying the newly-adopted electoral reform to the actual 2017 election outcome. Therefore, the same effect, which the legislator targets with scientifically and legally questioned amendments of the voting law, would have been raised by substituting the plurality rule with the simple-majority rule.

Additionally, as explained in Section 4.1, our results represent a somewhat underestimated effect on the Bundestag size. Thus, in the case of actual implementation, and even more considerable size reduction is expected. Moreover, it is not out of the question to combine the simple-majority rule with other measures, such as reducing the number of constituencies to achieve even more effective downsizing.

Based on all results that have been found in the course of this work, it can thus be assumed that implementing the simple-majority rule for selecting the directly-elected members of parliament is to be deemed more advantageous than electing the direct mandates with the help of the 2020 electoral reform under plurality rule—not only scientifically and legally, but also size-wise.

As an outlook, it can be added that the first two aspects, i.e. the economic and legal superiority of the simple-majority rule, have been known for a long time. However, these have so far received little attention within parliament. Since the simple-majority rule may now even provide a solution to practical political problems, i.e. the excessive size of the Bundestag, this superior electoral law may and should be included in the political discourse—especially as part of the search for a solution to the parliament's size-problem.

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A. The nation-wide approach

In this Appendix, the nation-wide approach (preference orderings are computed without regional differentiation) is depicted. We restrain on the map indicating the flipped districts. Some other district flip here compared to Figure 3 (and concerning some district, both approaches yield the same output). However, the Bundestag size resulting from running the simple-majority rule remains unaffected compared to the regional differentiation.

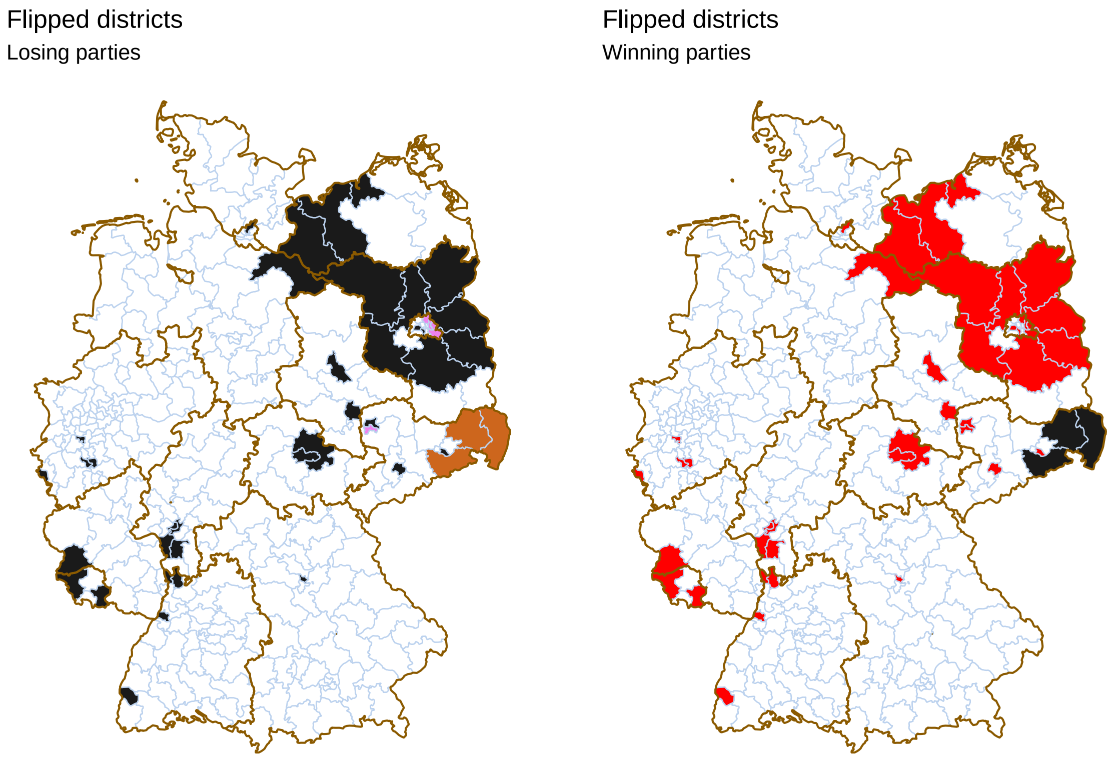

Figure 4: Flipped districts by running the nation-wide approach.