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# A social-choice perspective on authoritarianism and political polarization 

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# A social-choice perspective on authoritarianism and political polarization 

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Facing a spreading of polarization and authoritarianism, research from various disciplines attempted to explore the sources of this threatening development in many societies. Socio-economic factors, as well as the diffusion of social media, were identified as explaining factors. We emphasize another source for the success of polarizing politicians: The collective-decision rules. We show that several frequently used voting schemes greatly support polarizing candidates' success while other voting rules that are often scientifically proposed but rarely in use are much more appropriate to avoid polarizing candidates' success. The simple-majority rule and the Borda count are more suitable for preventing society from a polarizing candidate's landslide. By comparing the two last-mentioned voting rules, we find that the Borda count has some advantage over the simple-majority rule.

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## 1 Introduction

The world does face today a pandemic of authoritarianism, as well as a pandemic of disease, which debilitates human life in distinct but interrelated ways. Amartya Sen (2020a)

In a powerful speech and with vivid words, Amartya Sen recently warned against authoritarian governments' further propagation. There is a reason for alarm in the repressive tendencies in many countries that significantly limit the freedom of citizens and the liberal consensus of modern societies. In his acceptance speech on the occasion of the Peace Prize of the German Book Trade with which he was awarded, Sen listed exemplarily the governments in India, the USA, Poland, Hungary and Brazil. The governments in Belarus and Turkey can undoubtedly be added to this list. Authoritarianism is, unfortunately, not an issue of the past. It is a threat to liberty and social cohesion in our times.

Who would have thought it possible ten years ago that a mob would storm the Capitol in Washington DC with the tacit support of the running POTUS? Countries in Eastern Europe, which three decades ago started to free themselves from the yoke of dictatorship, now suffer from significant restrictions of civil rights and the freedom of expression as well as the massive persecution of homosexuals, including the establishment of particular regions that are to be kept as $L G B T$-free zones (Sen, 2020a, p. 74). The sad fact is that some political leaders seek to deepen social division rather than overcoming them. The 45 th US president was often accused of using inflammatory rhetoric or deeply divisive politics to distract from crises and his own mistakes. In Brazil, the president came to office by campaigning his "promise to save the country from such conservative nightmares as same-sex marriage, homosexuality, affirmative action, and so on" Sen (2020a).

Worrying examples of unprecedented restrictions on universities and courts' independence are currently seen in Hungary and Turkey. Despite international outcry, in 2018, the Hungarian parliament created a parallel-court system that cements executive control over the judiciary. Shortly after coming in power, the new "Law and Justice"-driven government in Poland enacted a new law to punish judges whose verdicts contradicted the views of the running government (Applebaum, 2020). Further, several attempts were made to take control over scientific institutions in Hungary and Turkey. Many further examples can be added to the list of attacks on democratic institutions.

Part of the overall picture of authoritarianism is the strong opposition from enlightened and liberal society groups. The predominantly female protesters in Belarus, who under the threat of massive repression, openly take to the streets, week after week, in their thousands against the regime in Minsk and risk arrest impressively exemplify this.

Like the President in Belarus, many authoritarian movements pursue misogynous rhetoric and policy, and often women give their face to the protest. Another recent example is the protests by women in Turkish cities after their president signed a decree annulling Turkey's ratification of the Istanbul Convention on violence against women.

The goal of authoritarianism is usually to encourage polarization: Seeking to find
faithful supporters and approvingly accepting an opposition that rejects them. An essential feature of authoritarianism is that it does not target to pursue consensual policy measures supported by a broad majority of the population. It is sufficient to achieve the required majority for their policy-howsoever it is measured. Authoritarianism and polarization are two sides of the same medal.

The political and economic consequences have widely been analysed. For instance, political scientists have shown that political polarization is often associated with more legislative gridlocks (Binder, 1999; Jones et al., 1995). Such legislative gridlocks can harm the economy in different ways. A gridlock often leads to a limited ability to take appropriate governmental measures to address societal changes and challenges (like the pandemic), and it tends to harm political innovations by yardstick competition. On the other hand, such gridlock-driven political failures themselves can lead to polarization (Dixit and Weibull, 2007).

### 1.1 Media's impact on polarization and voter turnout

A reasonable distinction is made in research between two kinds of political polarization: elite polarization and mass polarization. The first-mentioned focuses on the polarization of the political parties, and the latter means a polarization of the electorate (Hetherington, 2009). Accordingly, socio-economic developments or even changes in the media landscape can explain polarization both between elected officials and within citizens.

Several papers deal with the interaction between mass and elite polarization. For instance, in addressing the relationship between income dispersion and (ideological) polarization, Dettrey and Campbell (2013) find that despite the increases in income inequality and polarization, income inequality does not appear to have been a significant cause of growing polarization. Instead, the increase in ideological polarization in the US electorate was a consequence of the increased polarization of the political parties ${ }^{1}$.

Recent research pays particular attention to the media's role or, rather, the changes in the media landscape. Bernhardt et al. (2008) deal with the media bias. Such a media bias describes the media makers' decisions to selectively omit relevant information that conflicts with the beliefs and preferences of their readers, listeners, and viewers. Bernhardt et al. conclude that shifts in voting behaviour are more driven by media-bias effects than by changes in political views and preferences.

In their equally noteworthy paper, Campante and Hojman (2013) show the effect of two thoroughgoing developments in the media landscape on mass polarization: the introduction of the radio in the 20s of the last century and the diffusion of broadcast TV roughly twenty years later. Both milestones of media development lead to a reduction in

[^1]the polarization of the electorate. Nevertheless, Campante and Hojman show in a model (which we will discuss in more detail in Section 2) that a diminishing mass polarization can induce a higher degree of elite polarization. This can be the case when declining mass polarization is accompanied by low voter turnout. The authors report such a turnout drop due to the introduction of televisions, thus confirming a similar finding by Gentzkow (2006). On the other hand, the radio seems to have positively affected voter participation (a result also found by Strömberg (2004)).

Current work deals with the impact of social media usage. Social media is often blamed for exacerbating polarization by creating echo chambers (ideological bubbles) that prevent citizens from being exposed to information that contradicts their pre-existing beliefs (Levendusky, 2013). Levy and Razin (2020) argue that voters tend to move into echo chambers and that social media fosters this tendency. Assuming that citizens in echo chambers tend to be more vulnerable to polarization, social media increases the risk of polarization. This risk increases further by politicians' behaviour who deliberately and willingly use social media to manipulate voters' opinion. However, the causality between polarization and echo chambers is still under consideration in current research: Are echo chambers causing polarization? Or is it the other way around: Do echo chambers exist because people are polarized?

Regarding elite polarization, however, Bail et al. (2018) indicate in an exciting experimental setting that echo chambers affect the polarization of elected officials in the US. The two most recent works in this field report similar results concerning mass polarization. Allcott et al. (2020) show that a four-week Facebook "detox" (de-activation) makes people less polarized by at least some measures. In another field experiment, Levy (2021) shows that social media exaggerates polarization.

### 1.2 Collective-decision making and political polarization

All these attempts in explaining and analysing the diffusion of polarization are valuable and fruitful. Nevertheless, research pays little attention to the questions of which institutions ${ }^{2}$ enable the aforementioned social developments and contribute to the success of polarizing candidates ${ }^{3}$. This paper sheds light on the role of the collective-decision rules in explaining the strengthening of polarization. We argue that the frequently-used voting schemes give an incentive to pursue a polarizing policy.

The social-choice theory repeatedly emphasizes that often-used voting rules violate the principle of majority decision (Sen, 2017, Ch. 10*). Further, it has become part of the conventional insights that different voting rules can bring forth very different outcomes,

[^2]i. e., electoral winners (Saari, 1992; Sen, 1995). Consider the following profile of twenty (non-polarized) voters voting on a set of six candidates on the ballot (Table 1).

| Voter | A | B | C | D | E | F | Voter | A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 6 | 1 | 5 | 3 | 4 | 2 | 11 | 6 | 4 | 5 | 2 | 1 | 3 |
| 2 | 1 | 6 | 5 | 4 | 3 | 2 | 12 | 2 | 3 | 5 | 6 | 1 | 4 |
| 3 | 2 | 4 | 5 | 3 | 1 | 6 | 13 | 2 | 4 | 5 | 1 | 3 | 6 |
| 4 | 1 | 5 | 3 | 4 | 2 | 6 | 14 | 1 | 3 | 6 | 2 | 4 | 5 |
| 5 | 6 | 5 | 2 | 1 | 3 | 4 | 15 | 3 | 2 | 1 | 6 | 5 | 4 |
| 6 | 4 | 2 | 1 | 6 | 5 | 3 | 16 | 2 | 3 | 1 | 4 | 5 | 6 |
| 7 | 2 | 1 | 4 | 5 | 6 | 3 | 17 | 1 | 2 | 5 | 4 | 3 | 6 |
| 8 | 2 | 5 | 1 | 6 | 3 | 4 | 18 | 4 | 2 | 5 | 6 | 3 | 1 |
| 9 | 5 | 4 | 6 | 2 | 1 | 3 | 19 | 3 | 2 | 6 | 5 | 4 | 1 |
| 10 | 3 | 1 | 2 | 4 | 6 | 5 | 20 | 5 | 2 | 1 | 6 | 3 | 4 |

Table 1: Example of a profile which brings forth different electoral winners
The electoral outcome can easily be demonstrated depending on the voting scheme used to bring a candidate into office. Applying the most often used voting rule, the plurality rule, candidate $C$ will fill the office. By using the strict-majority rule cum run-off (Sen, 2017, Def. $10.5^{*}$ ), candidate $E$ is possibly the winner ${ }^{4}$. By using the most often proposed voting schemes in social-choice literature (cf. Dasgupta and Maskin, 2004; Saari, 2006), the winner is either $B$ by applying the simple-majority rule or $A$ when the Borda count selects the winner.

Now, consider the case that the electorate is polarized to a certain degree (we describe how we can measure such a degree of polarization in Section 2). The question now arises as to whether particular voting schemes are more likely to bring a polarizing candidate to power. We will show that all voting schemes, which focus on the voters' first (or even their second) preference only (their "votes"), give polarizing candidates an incentive. In line with Sen (2009), we refer to these voting rules as transcendental voting schemes, $T V S$. The plurality rule, also called the first-past-the-post, is an element in the TVS set. Here, the candidate with the most votes wins the election, regardless of how the electorate as a whole views him or her. By contrast, we label collective-decision rules that take into account the entire preference orderings of the voter, evaluative voting schemes, EVS, still in line with Sen (2009). The simple-majority rule, as well as the Borda count, are well-known EVS examples.

The following example illustrates the issue at stake. Consider a ballot, again consisting of six candidates. A (male) candidate on this ballot, say, $A$, can either pursue a polarizing campaign or an integrative, consensual one. By playing the polarizing strategy,

[^3]the candidate seeks to divide the electorate by stirring up xenophobic and misogynous feelings of resentment to catch more votes. In doing so, the candidate antagonizes over $50 \%$ of voters who rank him last among all the candidates on the ballot. However, $35 \%$ of the voters will find his campaign appealing and will rank him first.

A second, perhaps female candidate may pursue a consensual strategy. As a result, none of the voters would rank her fourth to sixth, but $32 \%$ would rank her first.

By pursuing the polarization strategy, $A$ will become the Condorcet loser as he will be defeated in any head-to-head match-up. He will be elected nevertheless under plurality rule (no matter how many votes the remaining four candidates will gain). Under the simple-majority rule, however, a Condorcet loser can not be successful. In this case, i. e. under the simple-majority rule, a consensual strategy is much more promising since the candidate ranked first by almost a third of the electorate and ranked no worse than third-place by any voter is likely to become the Condorcet winner.

The present paper is organized as follows: In Section 2 we will present some theoretical and econometric approaches to measure (mass) polarization. Section 3 describes the simulations we run in order to evaluate the success of a polarizing candidate under different voting schemes and with different degrees of polarization within the electorate. Our results are suitable to rekindle an interdisciplinary discussion on the simple-majority rule's appropriateness versus that of the Borda count (Section 4). We will conclude in Section 5 with some summarizing remarks.

## 2 Concepts and measurements

Facing the spread of polarization and authoritarianism, it is all but surprising that several studies deal with these phenomena. The research comes from different areas of social sciences. Economics-related papers ${ }^{5}$ often deal with the modelling and the measurement of polarization. The latter is far from being a new topic. Instead, the question of how to measure the heterogeneity of profiles was addressed several decades ago by Kendall and Smith (1939). They developed what is today known and widely used as the Kendall measure of concordance. The idea is appealingly simple: Let a table represent a profile with the $m>0$ alternatives (candidates) in the columns and the $n$ voters in the rows. If all the voters have the identical orderings (perfect resemblance), then the columns' sum would be $n, 2 \cdot n, 3 \cdot n, \ldots, m \cdot n$ in some order, and the corresponding squares-ofdeviations sum would be as large as possible, i. e., $S=\frac{n^{2}\left(m^{3}-m\right)}{12}$. The measure is based on the sum of squares of deviations around their mean value and is given by

$$
\begin{equation*}
W=\frac{12 S}{n^{2}\left(m^{3}-m\right)} \subset(0,1) \in \mathbb{R} \tag{1}
\end{equation*}
$$

Note that $W$ is a measure of resemblance, and therefore a higher $W$ indicates a higher concordance. So, in case of a perfect match (resemblance), $W=1$.

Some attempts have been made to analyse similarities between $W$ (and Spearman's

[^4]$\rho$ ) and the dispersion measures (e.g. Borroni and Zenga, 2006). However, Esteban and Ray (1994) have persuasively demonstrated that a mass polarization can not and shall not be expressed by using measures of income or wealth dispersion (like the Gini or the Atkinson measure). Instead, what accentuates polarization is the co-occurrence of intra-group homogeneity (cohesion (cf. Alcalde-Unzu and Vorsatz, 2012), "ingroup responsiveness" (cf. Diermeier and Li, 2019)) and inter-group heterogeneity (see also Baldiga and Green, 2011). The latter is represented by an alienation function, the first mentioned by an identification function, whose slope depends on a parameter $\alpha$, which Esteban and Ray label the degree of polarizing sensitivity. A measure for polarization, which has the homotheticity property and satisfies a small set of axioms (in particular: various forms of pooling leads to higher polarization), depends on $\alpha$ only. If $\alpha=0$, then the polarizing measure coincides with the Gini coefficient. Thus, the more citizens with an identical or similar variable (like income) feel integrated to each other, the higher polarization, and the higher the distinction between polarization and dispersion.

Based inter alia on this work, Can et al. (2015) present a polarization measure for strict preference orderings. Let us denote in line with standard notations in socialchoice theory the number of voters who prefer alternative $a$ over $b$ by $N(a \succ b)$. Then $d_{a b} \equiv|N(a \succ b)-N(b \succ a)|$ can be used to describe the heterogeneity within the (of $n$ voters consisting) electorate with respect to their opinions over any tuple ( $a, b$ ). Can et al. (2015) show that only the measure

$$
\begin{equation*}
\psi\left(\succ_{1}, \ldots, \succ_{n}\right)=\sum_{\{a, b\} \in \overline{\mathscr{B}}}^{n} \frac{n-d_{a b}}{n \cdot\binom{m}{2}} \tag{2}
\end{equation*}
$$

can satisfy a set of reasonable axioms. Here, $m$ denotes the number of alternatives and $\bar{B}$ the set of all subsets from the set of all alternatives, $\mathscr{B}$, with cardinality 2.

In their already-mentioned paper (see Subsection 1.1), Campante and Hojman present an intuitive model of polarization. They consider that two parameters can describe the electorate. The first is the share of moderate voters, the second the portion of strongly motivated voters. The latter are supposed to participate in elections for sure, while only part of the moderates cast a vote. Campante and Hojman show that in some cases, if political parties diverge to extreme positions, moderates with weak motivation do not vote while extreme voters do. Thus, the model sheds light on the connection between the success of extreme or polarizing candidates and voters' turnout.

Let us assume that we have a candidate, $A$, judged quite differently within the electorate. A group of supporters rank $A$ first, the second group of despisers rank him last, and a third group is moderate in the sense that they rank $A$ randomly. The two first-mentioned groups raise a particular polarization measure while the third group is likely to deepen it. The more moderate citizens decide not to go to the polls, the higher the probability of a polarizing candidate's victory under some voting schemes. We will adopt the basic features of the Campante and Hojman model for our simulations which we are going to present in the next section.

## 3 The success of polarizing candidates under various voting schemes

Several efforts were made to calculate the probability of a particular electoral outcome. One may think of the likelihood that there is no Condorcet winner (cf. Sen, 2017, Ch. 10.2). The calculations are still limited as the mathematical expressions take hardly-interpretable daunting shapes, especially when we try to go beyond existing literature (see Gehrlein et al. (2015); Maassen and Bezembinder (2002)). We, therefore, use the possibility of high-performing computing to simulate electoral outcomes in a large electorate with different numbers of alternatives (candidates) and different degrees of polarization. The frequently-used voting schemes and the often-proposed ones are applied to the profiles gained by our numerical simulations. We make use of the statistical package R (R Core Team, 2020; Ševčíková et al., 2021).

This is not the first attempt to raise electoral results by using extensive simulations (see, e.g. Jones et al., 1995; Dougherty and Heckelman, 2020). However, the growing computing capacities of so-called supercomputers (high-performing computers) allow us to go into deeper detail, in particular, to make cross-comparisons. To our best knowledge, the present paper is the first study to use simulations to analyse and explain the success of polarizing candidates.

### 3.1 Profiles

We simulated a large number of elections, each with an electorate consisting of 20k voters with strict preferences over a set of candidates. The number of candidates on the ballot, $m$, ranges from $m_{\min }=3$ to $m_{\max }=8$. One of them (we call the candidate $A$ ) is a (male) polarizing candidate (PC). $A$ is a PC because a share $\gamma$ ranks him first, and the same proportion of voters despise him and accordingly rank him last. Again, there is a third group of moderate voters in the sense that they rank $A$ randomly. Except for $A$, all the other candidates are ranked randomly in order to emphasize the effect of a single polarizing candidate.

We consider different levels of the polarization factor ranging from $10 \%$ to $40 \%$ in $5 \%$ steps; Thus: $\gamma \in\{0.10,0.15,0.20, \ldots, 0.40\}$. The following example in Table 2 shows an electorate of 20 voters with a polarization factor equal to $40 \%$.

Here, $A$ polarizes the voters since 16 out of 20 voters $(2 \gamma)$ rank him either first or last. Thus, a $\gamma$-value of 0.4 means that $80 \%$ of the voters have strong opinions about this particular candidate. A rising $\gamma$-value can either indicate that mass polarization increases or that-in line with Campante and Hojman (2013)—given a particular degree of polarization, the voter turnout drops.

Since $\gamma$ can take seven different values and we consider six distinct numbers of candidates on a ballot $(3, \ldots, 8)$, we compute $25 k \times 7 \times 6=1,050,000$ profiles each consisting of 20 k voters.

We show that our polarization factor is closely related to existing measures presented and discussed in Section 2. Figure 1 depicts this. We calculated regression lines over the around

| Voter | A | B | C | D | Voter | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 4 | 2 | 3 | 11 | 4 | 1 | 3 | 2 |
| 2 | 1 | 2 | 4 | 3 | 12 | 4 | 2 | 3 | 1 |
| 3 | 1 | 2 | 4 | 3 | 13 | 4 | 3 | 1 | 2 |
| 4 | 1 | 4 | 2 | 3 | 14 | 4 | 2 | 1 | 3 |
| 5 | 1 | 3 | 2 | 4 | 15 | 4 | 2 | 1 | 3 |
| 6 | 1 | 4 | 3 | 2 | 16 | 4 | 3 | 1 | 2 |
| 7 | 1 | 3 | 2 | 4 | 17 | 2 | 4 | 3 | 1 |
| 8 | 1 | 4 | 2 | 3 | 18 | 3 | 2 | 1 | 4 |
| 9 | 4 | 3 | 2 | 1 | 19 | 3 | 1 | 4 | 2 |
| 10 | 4 | 2 | 3 | 1 | 20 | 3 | 1 | 4 | 2 |

Table 2: 20-voter profile with a polarization factor of 0.4
one million profiles with respect to $1-W$ ( 1 minus Kendall's measure of concordance) and the polarization measure by Can et al. (2015). Though the levels differ, it can be seen that both measures correlate with the polarization factor used in this paper.

### 3.2 Collective-choice rules

In total, six voting schemes we applied to the roughly one million profiles. Thus our results are based on a total of more than six million electoral outcomes. From the wide range of existing collective-choice rules, we select the frequently used ones and the two voting schemes which are most often proposed in literature ${ }^{6}$.

1. Plurality rule, PV. Probably the most widely used voting method in most countries is the plurality rule with victory for the candidate that most voters rank first (Sen, 2017, Ch. A4*). First-past-the-post, as plurality voting is also called (particularly in Britain), is used to elect senators and representatives in the US and MP, e.g. in Britain and Germany.
2. Strict-majority rule cum run-off, AV. The strict-majority (also called absolute majority rule, two-round system, ballotage ...) requires a vote total of more than $50 \%$ (Sen, 2017, Ch. 10*3). In cases where no candidate receives the required share of votes, a run-off takes place between the candidates with the most and the secondmost votes. This voting rule applies, e.g., to the French presidential elections and for the elections of some US senators, such as in Georgia. It is also often used to fill the offices of local authorities.

[^5]

Figure 1: Measures by Kendall and Smith (1939) and Can et al. (2015) and how they correlate with the polarizing factor

Both PV and AV can be categorized as transcendental voting schemes in line with Sen (2009) since they focus on the voters' first and, where applicable, second preferences only rather than considering voters' whole preference orderings. It is a well-known property that even the Condorcet loser (the candidate who is defeated by any of her or his rivals) can be selected under plurality rule (Saari and Newenhizen, 1988). The Condorcet loser can also be a run-off participant under the strict-majority rule. However, seeing that a run-off is a pair-wise match-up, the Condorcet loser will be defeated in any run-off by definition.
3. Supplementary voting, SV. Supplementary voting is known as the voting scheme used for directly-elected mayors in England and Wales. The system was originally designed and (unsuccessfully) proposed by the Plant committee to be used in British parliamentary elections and to replace plurality rule. SV is a strict-majority voting scheme that allows the voters to cast their first and second preference. It, thus, belongs to the set of TVS. If no candidate receives more than $50 \%$ of the votes, then all but the two leading candidates are eliminated, and a second count (run-off) is held. In the run-off, the votes of those who supported eliminated candidates are distributed among the two remaining candidates so that one candidate eventually achieves an absolute majority. SV's properties (like monotonicity and consistency; both lacked by SV) are well elucidated by van der Kolk (2008).
4. Single-transferable vote, STV. Single-transferable vote, sometimes also called the Hare system after its first prominent supporter Thomas Hare, is an old voting rule dating back to 1819 (cf. Tideman, 1995) and is still used in elections today. It was envisioned originally and is still mainly applied as a voting system to select multiple winners in an election. However, it can be modified to work in single-winner elections as well. STV is still widely used in countries with Anglo-Saxon influence, such as Australia, Malta, and Ireland. Due to its wide use, there exist numerous variations of STV in varying degrees of complexity (cf. Levin and Nalebuff, 1995). For the sake of clarity and brevity, we will use the variation implemented in the statistical package $R$ and described in Raftery et al. (2021).

Aside from the voting schemes we observe in practice, there are two voting rules most often proposed in literature which date back to the times of the French revolution when these were discussed by the two French noblemen, Marquis de Condorcet and Charles de Borda.
5. Simple-majority rule, SMR. A frequently and presumably most often proposed method by social-choice theorists is the simple-majority rule, SMR (see, for instance, Dasgupta and Maskin (2004); Maskin and Sen (2016)). Here, the winner is the candidate who defeats each of his rivals in head-to-head comparisons (the Condorcet winner). Maskin (1995); Dasgupta and Maskin (2008) demonstrated the robustness of simple-majority rule (we will further discuss this robustness result in Chapter 3). The simple-majority rule ensures that if there is a Condorcet winner, she or he will be selected.
6. Borda rule, BR. It takes note of the position of a candidate vis-à-vis others in a ranking, weighting each candidate by her or his rank-order position. It was often used in pubs in the days of Arrow (see Arrow, 1963, p. 27) and is in use today, e.g. to elect the representatives of two minority groups in Slovenian parliament and used by several academic institutions. The Borda rule yields transitive social orderings; however, it violates Arrow 's condition I (independence of irrelevant alternatives) ${ }^{7}$. The Borda rule has received more and more attention in recent years due to the valuable contributions by Saari (2000a,b, 2010).

Let us define the set of all considered voting rules by $\mathscr{E}=\{\mathrm{PV}, \mathrm{AV}, \mathrm{SV}, \mathrm{STV}, \mathrm{SMR}$, BR \}. Further, we substantiate the dichotomy mentioned above between transcendental voting schemes (TVS) and evaluative voting schemes (EVS). The latter take into account the voters' entire orderings while the TVS considers only parts of the orderings (usually the first preference or even the first and the second preference.). Therefore, we have TVS $\equiv\{\mathrm{PV}$, $\mathrm{AV}, \mathrm{SV}\} \subset \mathscr{E}$ and $\mathrm{EVS} \equiv\{\mathrm{STV}, \mathrm{SMR}, \mathrm{BR}\} \subset \mathscr{E}$.

The celebrated Arrow impossibility theorem states that no collective-choice rule can satisfy specific elementary and mild axiomatic demands. In particular, a voting rule violates the

[^6]principle of majority decision, or it lacks decisiveness. Another celebrated result, provided by May (1952) states that conditions $\mathbf{U}, \mathbf{A}, \mathbf{N},{ }^{8}$ and positive responsiveness (S) are together necessary and sufficient for a decisive collective choice rule to be the method of majority decision ${ }^{9}$. Since earlier attempts to find "possibility theorems" relaxed $\mathbf{U}$ (such as "value restriction" of which single-peakedness is a special case (Sen, 1966)), more recent research varies the axiomatic demands in order to compare different voting schemes. Dasgupta and Maskin (2008), for instance, established their robustness-result ${ }^{10}$ based on six axioms and expand their work by considering strategy-proofness (Dasgupta and Maskin, 2020). Even I is currently under revision, as mentioned above (see Maskin (2020a,b).

For our purpose, we state the following

## Condition (Condition PPC)

A collective-choice rule shall not bring forth a polarizing candidate as an electoral winner.
It is evident that PPC conflicts with other reasonable axioms, particularly with the axiom that the Condorcet winner should be selected to the extent that such a winner exists ${ }^{11}$. However, this just-mentioned conflict is part of current research contributions questioning whether the Condorcet winner is an invariably satisfactory social choice (see Sen, 2020b) ${ }^{12}$.

We aim to show which voting rules can satisfy PPC for the broadest possible domain. Further, we demonstrate two robustness results similar to the robustness result by Dasgupta and Maskin (2008); Maskin (1995) in Section 3.3.

### 3.3 The polarizing candidate's success under different voting rules

After explaining our computational simulations and briefly introducing the collective-choice rules considered, we now provide results on a polarizing candidate's success under different voting rules. We denote by $\phi \in(0,1) \subset \mathbb{R}$ the probability for an electoral victory of the polarizing candidate (PC).

[^7]We introduce a second measure to describe the difference between the probability success of a PC compared to a non-polarizing candidate. For that, we compare the PC's success probability with the success probability of a candidate in a homogenous situation without any polarization and ideological predetermination. In such a homogenous situation, where the choice between two arbitrary candidates is the choice between Tweedledee and Tweedledum, the probability for a candidate to win an election is given by $1 / m$. We denote this second measure by

$$
\begin{equation*}
\Phi(\phi(\gamma, m))=m \cdot \phi-1 \tag{3}
\end{equation*}
$$

For instance, let the probability of a PC' success be $\phi=60 \%$ and consider $m=5$ candidates on the ballot. The probability of a PC's success is three times higher than in the homogenous situation such that the probability increases by $\Phi=200 \%$.


Figure 2: The success of the polarizing candidate under different voting rules - differentiated by the number of candidates.

Let us start by focussing on the $\phi$-value. Figure 2 depicts the probability for an electoral victory of the polarizing candidate in four spider-web plots. The plots consider four values of polarization, and the different lines indicate the $\phi$-value depending on the number of candidates.

For the sake of clarity, we will enumerate some of our observations in Results.
Our starting point is the plurality rule. It can easily be seen that any non-negative polarization factor increases the success probability of the PC. Since any of the PC's rivals can
expect a number of votes equal to $(1-2 \cdot \gamma) \frac{1}{m}+\gamma \frac{1}{m-1}$ and the PC itself has an expected number of votes equal to $\gamma+(1-2 \cdot \gamma) \frac{1}{m}$, it can readily be seen that the PC's expected amount of votes in his favour exceeds that of any of his rivals whenever $1>(m-1)^{-1}$. This condition holds for all $m>2$. In each of the roundabout one million simulated elections, the PC is the plurality winner.

## Result 3.1

For any pair $(\gamma, m)$, the polarizing candidate (PC) is always the plurality winner.
Empirical evidence for the success of polarizing candidates under the plurality rule is provided by Maskin and Sen (2016) with regard to the GOP primaries prior to the 2016 elections and by Barbaro and Specht (2021) with regard to the Bundestag election 2017 in the German state Saxony. In three out of 16 districts, polarizing Condorcet losers have been successful. All of them were seen best by roughly a third of voters, but roughly two-third regarded them as the worst candidates.

We obtain the same result for the supplementary voting (SV) by considering any ballot with more than three candidates. With two rivals only, the PC thrives in a range between $38.7 \%$ and $44 \%$ (increasing in $\gamma$ ). This is due to the fact that the voters can express twothirds of their orderings. Thus, the PC is often the only candidate who receives neither a vote nor a supplementary vote (the second preference). However, this effect vanishes already with four candidates, and SV violates the condition PPC for any domain with $m>3$.

## Result 3.2

For any pair $(\gamma, m>3)$ the polarizing candidate (PC) is thriving under the supplementary voting.

Next, we focus on the simple-majority rule (SMR), the strict-majority rule (AV) and the single-transferable vote (STV). They have in common what we note as:

## Result 3.3

The PC's probability of being successful under SMR, STV, and AV depends positively on $\gamma$ and negatively on $m$.

Thus, a higher polarization increases the PC's success expectation, and more opposing candidates reduce it in absolute terms ( $\phi$-value). By asking how a larger ballot size affects the relative probability, $\Phi$, we derive a reverse result: the higher the number of candidates, the higher $\Phi$. This is depicted in Figure 3 (where we show the $\Phi$-values also for the other voting rules).

It is important to note that despite Result 3.3, the probability of success ( $\phi$ ) differs remarkably among the three voting rules, as depicted in Figure 4. PC's probability for a successful electoral campaign is significantly smaller when pair-wise comparisons determine the electoral outcome.

Thus, it is worth focusing more detailed on the simple-majority rule. Since it brings forth a Condorcet winner, the PC's success under SMR indicates how often the PC is the Condorcet winner. Depending on the polarization factor and the number of candidates on the ballot, the respective value ranges from $12.7 \%$ to $35.6 \%$. The highest value occurs with three





SuppV


$$
\text { \# candidates }-3-5-7
$$

Figure 3: $\Phi$-values for different voting rules and distinct pairs of $(\gamma, m)$
candidates and $\gamma=0.4$, the lowest with eight candidates and a polarizing factor equal to $0.1^{13}$.

In this respect, SMR violates PPC because-as mentioned above-it conflicts with the objective of selecting a Condorcet winner. Therefore, a genuine issue to raise is how its performance compares with that of other voting rules. Our simulations yield a result similar to the already-mentioned one by Dasgupta and Maskin (2008).

## Result 3.4

Let $F \in \mathscr{E} \backslash\{B R\}$ be a collective-choice rule. For any $\gamma \in(0.1,0.4)$ and for any ballot with cardinality $m>3$ we can state: whenever SMR flaws (i.e., violates PPC), $F$ violates PPC, too. Conversely, there are domains in which F flaws, but SMR does not.

Another insight concerns the ballot's size. Regarding the EVS and AV (the other TVS can be set aside as their $\phi$-value does not alter with respect to $m$ ), the more candidates are on

[^8]

Figure 4: The distribution of $\phi$ and of $\Phi$ for the simple-majority rule, the singletransferable voting and the strict-majority rule.
the ballot, the more it is likely that the polarizing candidate will not be chosen. However, this connexion is most significant under the Borda rule. With eight candidates on the ballot, the PC is occasionally successful under SMR and even more rarely successful under the Borda rule. By the latter, with eight candidates total, the probability of the PC's success ranges from $2.5 \%$ to $10.8 \%$, depending on the degree of polarization. This insight is also depicted in Figure 2 by the spread of the lines, which is significantly higher on the Borda-rule ray than on the other rules' rays.

The reason is straightforward. Polarization leads voters to rank the PC lowest as often as voters rank him first. The more candidates run in the election, the higher the weight of those ranking in the last place. On the other hand, his supporters' weight (those who rank him first) does not depend on the ballot's size. What makes the Borda rule working well (in particular with many candidates) is exactly what is regarded as its primary weakness: the violation of Arrow 's condition $\mathbf{I}$, the independence of irrelevant alternatives.

Another interesting observation is depicted in Figure 5.

## Result 3.5

For all $\mathscr{E} \backslash\{B R\}: \frac{\partial \phi}{\partial \gamma} \geq 0$. Conversely, for $B R$ holds that $\frac{\partial \phi}{\partial \gamma}<0$.
A higher polarization factor leads to a higher probability of PC's success under any voting scheme except the Borda count. Here, the opposite is true. A candidate's strategy to


Figure 5: The effect of the polarizing factor on the PC's success.
polarize diminishes its prospects significantly. This observation even holds for the smallersized ballots; we depict the case of eight candidates in Fig. 5 for the sake of exposition. In explaining Result 3.5 we can rely on the reason given for the effect of $m$ on $\phi$ : A higher polarization does not only increase the number of supporters but also the number of despisers. The weight of the latter increases in $m$. In comparing this effect to the simple-majority rule, we should bear in mind that the PC loses any pair-wise comparison in the despiser-group. However, the sensitivity of preference intensities is not considered under SMR (we will look at this issue in the next chapter).

With these observations in mind, it is all but surprising that what holds true for the Condorcet winner in Result 3.4 even holds true for the Borda winner.

## Result 3.6

Let $F \in \mathscr{E} \backslash\{S M R\}$ be a collective-choice rule. For any $\gamma \in(0.1,0.4)$ and for any ballot with cardinality $m>3$ we have: whenever $B R$ flaws (i.e., violates PPC), $F$ violates PPC, too. Conversely, there are domains in which $F$ flaws, but $B R$ does not.

Summing up, our simulations show to which extent the EVS compared to TVS are more appropriate to prevent a PC's landslide. In particular, SMR and the Borda count are best suited to meet the requirement to prevent the PC. By focusing on the two best-working collective-choice rules, we found remarkable differences worth examining further.

## 4 Can the Chevalier de Borda beat the Marquis de Condorcet?

Modern exploration of the formal procedures of collective-decision making was pioneered in the late eighteenth century $(1784,1785)$ by the Marquis de Condorcet (1995) and JeanCharles de Borda (1995). While the Marquis de Condorcet found what social-choice theory labels a lack of decisiveness due to cycling patterns, Jean-Charles de Borda emphasized flaws still under consideration up to the 21st century. He marked that on positional rules, the voting outcome may depend on an irrelevant alternative, as referred to by Arrow (1963) more than 150 years later. Thus, while Condorcet showed that pair-wise comparisons might bring out a candidate (or social state) favoured over all alternatives by a majority of voters, Borda demonstrated that a voting rule based on rank-order counting most certainly will put forth a winner, even though the winner may out-rival the Condorcet winner.

Following the challenges experienced by the two French noblemen, a vast effort was made to find a stainless election scheme, free from the flaws mentioned above. It took until the 1950s for the effort to finally be declared over. Arrow set up a bleak conclusion with ubiquitous dimensions known as Arrow impossibility theorem. Considering four mild conditions that can be described as rational and democratic, there is no collective-choice rule that does not violate (at least) one of these requirements.

### 4.1 The Saari-Risse debate

An often-used claim is that if there is a Condorcet winner, then this alternative should be selected. This assertion, however, is fundamentally scrutinized by Saari (2000a), who puts forth vigorous objections to simple-majority voting and claims that a Condorcet winner would certainly not reflect the appropriate election outcome. His approach is not axiomatic. Rather he splits a preference profile into subspaces. One such subspace consists of a full majority cycle, which Saari labels "neutral sets", as they play a role similar to that of eigenspaces of a matrix. These neutral sets have "no effect on election outcomes; they just confuse and complicate the analysis" (Saari, 2000a, p. 3f). Its complement, which is referred to as effective parts of the profile, is seen as truly decisive. Saari shows that with a preference profile that yields a Condorcet winner, he or she would not win the election if the neutral subspace were removed from the profile and a majority decision were undertaken within the effective subspace.

Undoubtedly, Saari's contributions are the most elaborate ones attacking the claim favouring the Condorcet winner. Risse (2005) argued that Saari's reasoning shows fallacies in the philosophical reasoning. In particular, Saari's arguments fail because they reflect an inadequate view of what counts as an appealing argument about collective-decision rules, (cf. Risse, 2005, p. 97). ${ }^{14}$ Basically, Risse raises the question of the role of elections in a democracy. Elections are seen as an intermediate point in a long process of deliberation, which ensures political legitimation. In this context, the concept of the deliberative democracy is worth mentioning.

[^9]
### 4.2 Deliberative democracy

Despite their disputes (see Habermas, 1995), John Rawls and Jürgen Habermas shaped the notion of "deliberative democracy". Habermas argues against an empiristic and reductionist concept of democracy that separates the role of democratic legitimacy from political power. In such a "Hobbesian view", as Habermas (2009, p. 355) calls it, democracy means that a subset of the population rules over all others for a set period of time ${ }^{15}$.

In contrast, the concept of deliberative democracy is characterized by its procedural structure. When citizens deliberate, they exchange views and debate on the res publica. As (Rawls, 1997, p. 772) vividly emphasized, such public debate is crucial for the formation of citizen's opinion on institutional principles and matters of fundamental justice. Moreover, a deliberative democracy derives its legitimating force from the discursive structure. It can only fulfil its social function because citizens expect its result to have a reasonable quality (Habermas, 2009, p. 369). Rawls (1997) refers to "reasonable pluralism" as the basic feature of democracy. Given a plurality of conflicting religious, philosophical, or moral convictions, citizens realize that "they cannot reach agreement or even approach mutual understanding on the basis of their irreconcilable comprehensive doctrines", Rawls (1997, p. 766). This quote should be emphasized as it highlights that deliberative democracy is not the counting of citizens' "first preferences" or "best alternatives", even if most collective-choice rules in practice indeed focus on precisely this. The procedural nature of democracy unveils that social-will formation does not take place at the election days only. Rather, if any voter contributes to the process of will formation, then it can be questioned whether the split of the electorate into a neutral and an effective set at a particular moment (when it comes to tallying) is compatible with contemporary concepts of democracy.

On the other hand, doubts other than those of Saari were currently sown about the Condorcet winner's superiority. Since a Condorcet winner can be the candidate with no or very few first preferences (votes) only, Sen (2020b) reflects that not all Condorcet choices may be particularly compelling.

As mentioned above, Maskin (2020a,b) argue that the Arrovian condition I is unnecessarily stringent and that it makes a voting rule insensitive in respect to voters' preference intensities. By considering a modified form of $\mathbf{I}$, called MIIA, he shows that a voting rule satisfies MIIA, Arrow 's other conditions, and May (1952) 's axioms for majority rule if and only if it is the Borda rule.

Thus, we observe a slight tendency to greater support of Borda's rule over Condorcet's in

[^10]recent contributions. Our simulation results go in the same direction. As mentioned in the last chapter, both SMR and BR most often (i. e., for the broadest set of domains) fulfil condition PPC. However, the Borda rule is seen to be much more robust against higher degrees of polarization. The PC's success increases with increasing degrees of polarizing under SMR but decreases when the Borda count determines the electoral winner. The reason for this is what Maskin (2020b) called the sensitivity to preference intensities. The Borda rule weights the group of despised voters much higher than the SMR. Thus, the incumbent's strategy to deepen polarization can bring him out of his office with the Borda rule when times comes for re-election. However, the opposite holds for any other voting rule being an element of $\mathscr{E}$.

When the polarizing candidate is the Condorcet winner...
she or he is also successfull under the Borda-rule

| 0.4 | 0.681 | 0.51 | 0.405 | 0.318 | 0.281 | 0.235 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.74 | 0.611 | 0.528 | 0.458 | 0.408 | 0.366 |
|  | 0.792 | 0.677 | 0.611 | 0.548 | 0.513 | 0.471 |
|  | 0.823 | 0.736 | 0.681 | 0.619 | 0.611 | 0.578 |
|  | 0.846 | 0.777 | 0.725 | 0.69 | 0.651 | 0.644 |
|  | 0.869 | 0.807 | 0.783 | 0.735 | 0.726 | 0.707 |
| 0.1 | 0.879 | 0.827 | 0.799 | 0.768 | 0.765 | 0.757 |
|  |  | 4 |  | ${ }^{6}$ |  | 8 |

Figure 6: How often is the Condorcet winner also the Borda winner?
This insight is also depicted in the heat-maps (Figures 6 and 7). By considering the set of domains in which the PC becomes the Condorcet winner, how often will he also win under the Borda rule? Let us recall Result 3.4. We found out that the PC is also successful in all such domains under any other voting rule, despite the Borda count. Figure 6 depicts that with few candidates only and a low polarizing factor, the electoral outcomes coincide more often. However, with a larger ballot size and an increased polarization, the PC as the Condorcet winner is less and less a Borda winner as well. For both factors, the number of candidates $m$ and the polarizing factor $\gamma$, we can conclude that the higher their respective value, the smaller the PC's success probability under the Borda rule. In this, the Borda rule is unique among the considered voting schemes.

Let us now take a closer look at the other case: The one where the PC is a Borda

When the polarizing candidate is the Borda winner...
she or he is also successfull under simple-majority rule


Figure 7: How often is the Borda winner also the winner under the simple-majority rule?
winner. According to Result 3.6, in these cases, the PC is successful in any other voting rule with the exception of the SMR. Figure 7 depicts how often the PC as Borda winner is also a Condorcet winner. The figure yields remarkable differences compared to the results depicted in the first heatmap (Fig. 6). In approximately eight out of ten cases, the Borda winner and the Condorcet winner coincide and are the polarizing candidate when the ballot is small $(m=3)$. The concordance value decreases in the number of candidates and notably diminishes with an increasing factor of polarization. The lowest coincidence applies to the case of eight candidates and a polarizing factor of $10 \%$ only.

## 5 Conclusion

Individuals' liberty rights and peaceful coexistences are seriously threatened by polarization. In explaining its origin and diffusion, research has found several factors. Socio-economic factors and the dissemination of (dis-)information through social media channels play a role. Aside from the numerous explanations for the increase of polarization and authoritarianism, the role of collective decision-making in explaining the success of polarizing candidates has not yet received attention in the literature. Our paper shows that distinct voting rules can have quite different contributions to the success and the spreading of polarization. We show that polarizing candidates benefit greatly from voting rules that consider voters' first or even
the second preference only. By contrast, those voting rules which evaluate the entire order of voter preferences are less likely to select the polarizing candidate as the winner. Especially the simple-majority rule as well as the Borda count are best suited in this sense.

By comparing these two voting rules, we are contributing to an ongoing and presumably accelerating discussion on whether the Condorcet or the Borda winner is the best social choice. In attempting to prevent a polarizing candidate's selection, we found a strong indication that the Borda rule works best. Two effects, both inherent in the Borda rule, explain this: The first and most important one is that the Borda rule also considers the opinion of those who reject a polarizing candidate. Their worst-ranking behaviour is evaluated when it comes to tallying. The Borda rule has this feature in common with other evaluative voting schemes (EVS). The second effect, which makes the Borda rule the most appropriate electoral procedure for avoiding the success of polarization, is what Maskin (2020a) recently labelled the sensitivity to preference intensities. Precisely the property for which the Borda rule was always criticized, namely the violation of $\mathbf{I}$, now leads to the fact that the Borda rule works better than the simple-majority rule. Jointly with the insight that the Borda rule does not violate a sound weakening of $\mathbf{I}$, this paper contributes to a long-lasting discussion the roots of which date back to the troubling time of the two French noblemen, who gave their names to the most appropriate voting schemes.

Nevertheless, our paper's essential and most crucial finding remains that the voting schemes frequently used favour polarization and thus social divide. Since the advocates of evaluative voting schemes from the late $18^{\text {th }}$ century up to the present have mostly underlined their reasonable concern with axiomatic considerations, we now argue in favour of considering the voters' whole preferences in view of the threat of polarization. We raise the politically important issue that evaluative voting schemes are axiomatically superior and can contribute to more consensus-based policies.

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The related R -script is available on the author's gitlab site.

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[^1]:    ${ }^{1}$ This result, however, is disputed. Using data on the US-state level Voorheis et al. (2015) find a significant link between income inequality and polarization. Further, they argue that there is a mutual reinforcement: income inequality leads to political polarization, and the gridlock induced by polarization reduces incumbents' ability to alleviate rising dispersion. Conversely, Bosancianu (2017) argues that the causation does not come from income inequality but educational inequality instead. At least, Bosancianu does not find a significant connection between income inequality and polarization when controlling for inequality in educational skills, which are causing mass polarization.

[^2]:    ${ }^{2}$ We regard institutions in line with North (1992) as the "rules of the game in a society" and mean the collective-decision rules in particular.
    ${ }^{3}$ In a recent contribution, Diermeier and Li (2019) show that political parties bias their programmes toward their partisans if voters respond more strongly to their own party's policy deviations than to that by the other parties. Diermeier and Li model two candidates only, an incumbent and a challenger. Since voting rules bring forth quite different outcomes if more than two alternatives are on the ballot, we will extend our assessment to the multiple-candidate case. However, Diermeier and $\mathrm{Li}(2019)$ is the first vigorous and valuable contribution to shed light on the incentive to polarize.

[^3]:    ${ }^{4}$ The run-off takes place between the candidates with the most votes. As it is enshrined in most voting laws, the coin toss decides who will participate in a run-off if two or more candidates receive the same amount of votes. In this example, the coin brings $E$ into the run-off as $E$ ties with $A$.

[^4]:    ${ }^{5}$ An overview of the state in the political sciences is provided, e.g. by Hetherington (2009).

[^5]:    ${ }^{6}$ One voting scheme which is often proposed and sometimes also used is the approval voting (AV) (see Brams and Fishburn, 2005). We omit it from our list as AV's outcome depends on the cut-off level, which has to be set arbitrarily. Even with an arbitrary cut-off level, it is a strong assumption that this level applies to all individuals. Further, with a cut-off level between the first and the second-ranked candidate, AV coincides with the plurality rule.

[^6]:    ${ }^{7}$ In two recent contributions, Maskin (2020a,b) argue that I is unnecessarily strong. He demonstrates that the Borda rule does not violate a weaker form of condition I, called MIIA. See Ch. 4.

[^7]:    ${ }^{8}$ Unrestricted domain, anonymity and neutrality, for further explanations, see, e.g. (Sen, 2017, Ch. 5*)
    ${ }^{9}$ The method of majority decision holds if and only if $\forall x, y \in \mathscr{B}: x R y \Leftrightarrow[N(x P y) \geq N(y P x)]$ and belongs ta a class of I-respecting collective-choice rules.
    ${ }^{10}$ Dasgupta and Maskin (2008) consider six conditions: the well-known A, N, I, P, decisiveness (D), and the sound condition called ordinality $\mathbf{O}$. The latter states that the winner should depend only on citizens' ordinal rankings and not on cardinal information like preference intensities. Since simple-majority rule (SMR) satisfies all conditions except $\mathbf{D}$, and no rule can satisfy all conditions simultaneously (as a consequence of Arrow's impossibility theorem), Dasgupta and Maskin show that the simple-majority rule satisfies the six conditions for the broadest class of (restricted) domain. Moreover, if some voting rule $F$ satisfies the six axioms on a given domain, then the simple-majority rule must also satisfy the axioms on that same domain. Unless $F$ is itself the SMR, there is another domain on which SMR satisfies all six axioms, and the original voting rule does not.
    ${ }^{11}$ To provide a simple example: Consider 100 voters and 10 candidates. A PC is ranked first by 51 voters and ranked last by 49. A second candidate, $B$, is ranked second by every voter. The PC in this case is the Condorcet winner and the outcome violates PPC. A voting rule which brings forth not the PC must, thus, violate $\mathbf{I}$.
    ${ }^{12}$ Condorcet consistency, as this requirement is also named, implies that votes (first preferences) are mostly irrelevant (Barbaro, 2021). Thus, a Condorcet winner can be the candidate with none or only a few votes. Among others, Sen (2020b) emphasizes that not all Condorcet choices may be particularly compelling.

[^8]:    ${ }^{13}$ The PC is even a strong Condorcet winner (a strong Condorcet winner is a candidate who receives more than half of the votes total; see Barberà et al. (2020)) if enough voters from the not-polarized group rank him first. With $\gamma=0.4$, twenty per cent of the electorate rank the candidates randomly. With three candidates on the ballot only, the PC can expect to get $40 \%+20 / 3 \% \sim 46.67 \%$ of the votes total. However, the PC is likely to become a Condorcet winner without reaching an absolute majority of the votes. In particular, if a large share of voters in the not-polarized group rank him second.

[^9]:    ${ }^{14}$ See Saari (2006) for a comprehensive reply.

[^10]:    ${ }^{15}$ Habermas argue that even the liberal view, at least in economics, is not interested in the inputs of a rational will-formation but is orientated towards the performance of governmental activities. In this line, for instance, Roubini and Sachs (1989) argue that higher public debt in countries with many political parties in a coalition (where small parties use a veto power leading to inefficient governmental spending) is by far more likely. Thus, these voting systems may be seen as preferable which favour a two-party system like plurality voting. The liberal model hinges on the constitutional framework of an economic society rather than on the democratic self-determination of deliberative citizens. Seen from a liberal viewpoint, the only and exclusive function of democratic decision making is legitimating the exercise of political power (see Habermas, 2009, p. 361-363).

