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Majority principle and indeterminacy in German elections

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Abstract

Out of the many possible voting schemes, the notoriously-used plurality rule is far from being the best. Previous research from France and the US reveals how plurality winners fall short of majority support. Therefore, eminent scholars advocate the simple-majority rule. The latter, however, faces the threat of indeterminacy due to cycling patterns. To contribute to the scarce evidence on the empirical occurrence of these phenomena, we used survey data from the 2017 German election to simulate preference orderings on district candidates. We find that violations of the majority principle are frequent. Conversely, we do not uncover any indeterminacy.

Keywords: *Elections, Plurality voting, simple-majority rule, indeterminacy*
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1. Introduction

A striking result in social-choice theory is that—letting voters' preferences remain fixed—different election outcomes emerge with changes of the voting method (Saari, 1992, 2000, 2010; Sen, 1995). Each candidate can be selected or even bottom-ranked depending on the voting scheme. The electoral outcome, therefore, may more accurately reflect the choice of a voting method than voters' preferences (Saari, 2000, p. 3).

The most often used voting rule around the world, the plurality rule, can bring forth the Condorcet loser (the candidate being defeated in any head-to-head match-up) into office. More generally, the plurality rule and most other voting rules select a candidate even if short of majority. In such a case, the electoral outcome violates the principle of majority decision.[1] Pushed by some puzzling electoral results in 2016 and before in the US, a debate on electoral reform is ongoing. Based on their decade-long research involvement and considerable achievement (see Dasgupta and Maskin (2020) for a recent survey), eminent social-choice theorists repeatedly advocated simple-majority rule[2] for elections (Maskin (2014b); Maskin and Sen (2016, 2017a,b). Further cf. Dasgupta and Maskin (2004) and (Sen, 2017, A4*)).

The simple-majority rule avoids violations of the majority principle. However, the simple-majority rule may lack decisiveness. The latter occurs when voting schemes based on pair-wise comparisons face a cycling pattern (Condorcet paradox). It could be that after the vote tally, no winner exists—a nightmarish thought for many lawyers, policy-makers and others.

Ever since Arrow's impossibility theorem, every anonymous[3] (thus, non-dictatorial) and Paretian voting scheme must either violate condition **I** (see next section), or risks to lack determinacy. The consequence for policy-makers and election designers is that they cannot continue to insist on each. Even worse, violations of the majority principle are more likely to emerge with increasing candidacies (ballot fragmentation). Yet, such fragmentation may also increase the fear of majority cycles in the first place.

Given this inescapable dilemma, a natural question to raise is: how often do various voting rules violate either condition **I** or exhibit indeterminacy (or both)? Because the answer ultimately depends on the distribution of preference orderings, this question

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[1]Formally, the principle of majority decision holds if and only if $\forall x, y \in \mathscr{B} : x\,P\,y \Leftrightarrow [N(x \succ y) > N(y \succ x)]$ (Sen, 2017, Ch.5*), where $\mathscr{B}$ denotes the set of candidates (ballot). $N(x \succ y)$ denotes the number of voters for whom $a \succ b$. Finally, $a\,P\,b$ denotes that a is socially preferred to b. All notations in this article are identical to those in Sen (2017).

[2]By simple-majority rule, it is labelled the rule originally introduced in 1785 by the Marquis de Condorcet (1995) demanding that an alternative should be selected if and only if it beats every other alternative on the ballot in pair-wise contests. See also May (1952); Sen (1966, 1969); Inada (1969).

[3]The social-choice theory not only consists of a remarkable number of somehow disappointing impossibility theorems, but it also confuses readers with non-uniform uses of terms. May (1952) used a slightly different characterisation of neutrality than Arrow. The definition by (Sen, 2017, Def. 5*2), which implies independence from irrelevant alternatives (IIA), is closely related to Arrow's characterisation. The latter term, IIA, however, is also used differently. Nash's IIA is a requirement on rationality, also known as Sen's property α (Sen, 2017, Ch. 1*6). From the latter, Arrow's IIA, henceforth **I**, is completely independent (Ray, 1973; Sen, 1969, Fn. 1).

needs to be addressed through empirical means. Yet, likely because data on voters' preference orderings are rarely available, studies that address either the violation of the majority principle through plurality voting or indeterminacy of the simple-majority rule—let alone both in one study—are few (see below). To enhance our knowledge on the occurrence of these phenomena, we turn to German federal elections. In the mixed-member electoral system used in German federal elections, single-member plurality voting is used to determine district winners. This case is characterised by the use of plurality voting in a context of relatively high levels of party fragmentation—such that in the 2017 election only 4% of plurality winners received an absolute majority of votes in their district.

To simulate sets of preference orderings for candidates in all 299 electoral districts, we combine data on official election results with a large data set from repeated wave polls before and after the 2017 Bundestag elections. Given that we cannot observe candidate preference orderings directly, we use rankings and ratings of political parties to infer preference orderings. We present two main findings. Firstly, Condorcet winners exist in every single electoral district. In other words, indeterminacy is not an issue. Secondly, violations of the majority principle are frequent. According to our simulation, 33 (11%) of the plurality winners were not Condorcet winners. Even Condorcet losers were selected in Germany. Notably, we find that the three plurality district winners from the radical-right AfD would have lost against generic candidates from all of the other main parties in head-to-head contests. Our findings have implications for debates over reforms of electoral systems for German federal elections and beyond.

The present paper is organised as follows: We compare the widely-used plurality rule axiomatically with the scientifically proposed simple-majority rule in section 2 and present the seminal robustness result in favour of the latter. In section 3, we discuss our application, the candidate vote in German federal elections. We highlight that, with increasing party fragmentation in Germany, plurality rule may (more) often violate the majority principle, and we discuss reasons for why, or why not, Condorcet cycles may exist when it comes to candidate preferences of German voters. In section 5 we present our data simulation and its findings. A summarising section 6 concludes our contribution.

2. Comparing plurality voting and the simple-majority rule: the robustness result

2.1. Violations of the majority principle and lack of decisiveness

The public may see violations of the Arrovian condition **I** as a laboratory problem, far from relevant in real-live elections. However, violating condition **I** is more than a formal anomaly. It concerns a fundamental democratic principle, the principle of majority decision.

The latter states that if a candidate y is regarded worse compared to her or his competitor x by a majority of voters, then y shall not be selected (see footnote 1).

This view's appeal is obvious: there exists a superior alternative to y from the majority of voters' point of view. Now, consider a third candidate on the ballot, say z. Let a majority of voters prefer z to y, too. Then there is no argument added for y, the Condorcet loser, to win the election. The Arrovian condition **I** states that a voting rule shall not violate the principle of majority decision. Specifically, the voting rule shall not allow the twice-defeated candidate y to win the election. Instead, the majority principle implies the reasonable argument favouring the Condorcet winner (the candidate beating all the competitors in pair-wise comparison) as the ideal electoral outcome (Sen, 1995).

There are some prominently reported condition-**I** violations in real-world elections. The 2015 primaries of the Republican party took place under plurality rule. Maskin and Sen (2016); Kurrild-Klitgaard (2018); Dasgupta and Maskin (2020) showed that the then-POTUS was a defeated candidate in the primaries and could have been stopped under voting schemes that are not vulnerable to condition-**I** violations. Aggravating, he benefited twice since he was short of a majority even in the 2016 presidential elections (Potthoff and Munger, 2021). Put differently, the $45th$ president benefited from violations of the majority principle enshrined in the voting procedure. The 2002 French presidential election provided another case in point. From a nine-candidate ballot, the far-right winger Le Pen participated in the run-off against Chirac after none of the candidates received an absolute majority in the first round. Dasgupta and Maskin (2004) vividly argued that most available evidence suggests that in a head-to-head contest, the socialist Jospin would have easily won against Le Pen. Dasgupta and Maskin (2004) argued further that it is even plausible that Jospin could have defeated Chirac had he made it to the run-off. Other examples concern the 1992 and the 2000 US presidential elections (Tabarrok, 2001; Maskin, 2014a). In 2000, the outcome in Florida was pivotal, with an extraordinarily slim margin favouring George W. Bush. Maskin argued that Al Gore would have won quite handily if the third candidate, Ralph Nader, had not run.[4]

Aside from elections to select incumbents, several examples for condition-**I** violations were reported in decisions by law-makers within parliaments. The German federal parliament, Bundestag, provides some examples. One concerns the post-reunification location of the German capital, as discussed by Pappi (1992) and Leininger (1993) in this journal. Another example is the 1992 decision about abortion regulation (Pappenberger and Wahl, 1995; Kleiner and Moldovanu, 2022). For an overview, see von Oertzen (2003); Kleiner and Moldovanu (2022).

This paper will argue that similar anomalies also occur in German elections to select federal lawmakers. For that (and for a state election) some experimental evidence exists. Alós-Ferrer and Granić (2012) conducted two field experiments on Approval Voting in Germany. Both took place in medium-sized cities in 2008 (state elections in Hesse) and 2009 (federal elections). They found that electoral outcomes differ by comparing the plurality rule with Approval Voting.

Avoiding condition-**I** violations are possible only at the expense of running at risk of indeterminacy. Applying the so-called *impartial culture*, the assumption that every

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[4]Further examples are provided by Sen (2017). In this context, see the elaborations over the myth of Hitler as a majority winner by (Sen, 2017, Ch. A4).

logically possible profile arises with equiprobability, with three candidates on the ballot, the probability of cyclical majorities was calculated to be 8.77% (Niemi and Weisberg, 1968). However, this number is susceptible to the ballot size. With four candidates, the number is 17.55%, and with ten candidates on the ballot, almost every second election lacks decisiveness (Jones et al., 1995; Deemen, 1999; Huang and Chua, 2000; Gehrlein, 2002; Maassen and Bezembinder, 2002; Dougherty and Heckelman, 2020). However, voters' preferences are not randomly distributed but ideologically driven instead. The impartial culture approach is the worst-case scenario such that any deviation reduces the probability of majority cycles (Tsetlin et al., 2003).

Perhaps because the evidence on voters' preference orderings are rarely available (Miller, 2019), empirical studies on the occurrence of indecisiveness are rare. Chamberlin et al. (1984) used data from five presidential elections of the American Psychological Association (APA). Voters had the opportunity to rank the candidates (however, less than a majority rank-order all candidates, a quarter specified their top-choice only). Focusing on those who submitted their orderings, Chamberlin et al. find no cycling majorities in the APA elections. Feld and Grofman (1992) analysed 36 elections held by private organisations in Great Britain (the elections took place under the single-transferable voting system). They, too, find no cycling pattern in any one election. By analysing seven elections held by academic institutions, Regenwetter and Grofman (1998) concluded "that transitive majority orderings may be expected in real-world settings more often then the formal social choice literature suggests". Finally, Tideman (2006) reports from 87 laboratory elections employed by the Electoral Reform Society of Great Britain and Ireland. He reported "extremely small" cases of majorities involved in cyclic anomalies. Evidence for the existence of a cycling pattern in a large electorate (based on Danish polling data) was provided by Kurrild-Klitgaard (2001). While most of these contributions indicate that cycling patterns are not a big issue, the evidence is limited and partly comes from rather unconventional settings.

2.2. Arrow and beyond

Arrow's disenchanting impossibility theorem is a result of ubiquitous reach and rightfully one of the most important insights of the past century. However, it does not state that electoral systems will work all of the time badly, but that all schemes can work badly at times. Maskin (1995); Dasgupta and Maskin (2008, 2020) raised the follow-up question: Which voting rule satisfies the Arrovian conditions[5] for the broadest class of restricted preference domains? We will focus on the relationship between plurality rule and the simple-majority rule for our purpose.

Concerning plurality rule, it works well (i.e., complies with **I**) if voters preferences fulfil *limited favouritism* (LF) (Dasgupta and Maskin, 2008). It requires that for any

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[5]Specifically, Dasgupta and Maskin (2008) consider six conditions: the well-known anonymity, neutrality, **I**, the Pareto principle, *decisiveness* (**D**), and *ordinality* **O**. They use May (1952)'s definition of neutrality, thus it does not imply **I** as the neutrality definition by Sen (2017) do. Furthermore, they use the Nash-IIA but claim in their follow-up paper (Dasgupta and Maskin, 2020) that their results hold for Arrow's **I**, too.

triple of candidates, all agree that one candidate is never top-ranked. LF ensures the electoral success of a Condorcet winner under plurality rule.

Indeterminacy, the flaw of the simple-majority rule, does not occur if voters' preferences fulfil value restriction (VR). It requires that all voters agree that some alternative is not best, or all agree that it is not medium-ranked, or that all agree that it is not worse in anyone's ordering (Sen, 1966). Single peakedness is a special kind of VR (a candidate in a triple is never medium-ranked).

Next, it can readily be seen that LF $\implies$ VR. Thus, if voters' preferences are such that LF holds, the same preferences do not yield a cycling outcome. The intuition is straightforward. Plurality rule violates condition **I** due to vote splitting.[6] However, if a candidate in a triple is never the favourite such that no voter casts a vote in her or his favour, then vote splitting is not an issue.

Thus, LF $\implies$ VR constitute (a part of) the *robustness result* by Dasgupta and Maskin (2008): If plurality rule works well (i.e., does not violate a reasonable set of axioms) on a profile R, the simple-majority rule does too. Conversely, there is a domain on which the simple-majority rule works well, but the plurality rule does not. The answer to the question raised above on the voting scheme that satisfies the Arrovian conditions for the widest class is sharp: the simple-majority rule.[7] The importance of the robustness result can hardly be underestimated. It states that election designers do not face a trade-off.

The second question remains: how serious are the problems of both, **I**-violating and indeterminacy? Though we know that the first-mentioned issue occurs at least as often as the latter, it is unclear how often elections are in danger of failing decisiveness. However, there are indications that, in developed democracies, both issues can apply more often today than in previous decades. One reason is the increase in party fragmentation, another an increasing multi-dimensionality of political competition in conjunction with the emergence of anti-establishment voters. The following sections deal with both phenomena. Then, drawing on German polling data, we empirically assess actual occurrences of both.

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[6]The term *vote splitting* is used to describe the situation when candidate x would beat y in a one-on-one contest but loses to y when z runs too because z splits off some of the votes that otherwise would go to x (see e.g., Maskin, 2020a,b; Sen, 2020). The term shall not be confused with split ticket voting. The latter denotes voting for a different party for different contests in the same election (Barnes et al., 2017).

[7]It should be noted that the robustness result is much more general than we briefly described in this chapter. It also holds for rank-order voting rules. The Borda count is the most famous method among the rank-order voting rules. Rank-order voting does not violate **I** if a candidate (in each triple) is always ranked top, or ranked medium, or ranked bottom. This condition is dubbed quasi agreement (QA). As can readily be seen, QA also implies VR.

3. Vote splitting and the role of anti-establishment voters in German federal elections

We will provide a brief overview of the voting system used in German federal elections, motivate our choice to study this case against the background of increasing party fragmentation and the spread of anti-establishment voters in Germany. Then, we will discuss why or why not on actual domains (a) the majority principle may be violated and (b) Condorcet winners may not exist but cycling majorities instead.

The voting system used for German federal elections is a mixed-member proportional (MMP) system (Shugart and Wattenberg, 2003; Hainmueller and Kern, 2008; Alós-Ferrer and Granić, 2012). It combines features of plurality voting in 299 single-member districts with proportional representation at the national level. Voters are eligible to cast two ballots: A candidate vote ("Erststimme") for one of the candidates standing in their district and a party vote ("Zweitstimme") for a party list. The district winners are selected by plurality rule. The mechanics of seat allocation in the proportional tier are more complex and need not be discussed in detail here. Suffice to say that the party seats are allocated in such a way as to make the overall seat distribution proportional to the party-vote shares obtained.[8] 299 seats are regularly reserved for the proportional tier, leading to a regular Bundestag size of 598 lawmakers.[9]

Like in many Western democracies, the German party system has become increasingly fragmented over the last decades. The Bundestag elected from 1961 to 1980 consisted of three parliamentary groups, with the Social Democrats and the Christian Democrats together consistently accounting for over 80% of the party votes. Since 2017, the parliament hosts six parliamentary groups: The Social Democrats (SPD), the two conservative parties CDU and CSU, henceforth referred to as UNION, the Green party (henceforth: GREEN), the liberal party (FDP), the leftist party (henceforth: LEFT), and the radical-right AfD. The combined vote share of the SPD and UNION has fallen to 53.4% in the 2017 election. In 2021, it missed the 50%-mark (with 49.8%) for the first time in the Federal Republic's history.

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[8]However, it should be noted that the proportional system does not patch any of the flaws concerned with the plurality rule. The universality of Arrow's impossibility theorem implies that the problems of single-winner systems even hold for multiple-winner schemes. All extant proportional systems provide representation based solely or primarily on voters' first preferences, which makes the violation of the majority principle obvious. Without going into deeper detail, a simple intuition can be provided: since the ideal victorious party, the Condorcet winner, can be the party without any vote, it could be that the most-favoured party receives no seat in parliament. It is, thus, evident that the considerations on the flaws of plurality voting also apply to multiple-winner systems for proportional-representation schemes. It remains the pertinent questions raised by Monroe (1995): Proportional to what? And the representation of what?

[9]However, parties may win more constituency mandates in a state than are awarded to the statewide party list based on the proportional allocation. These so-called "Übergangmandate" (surplus mandates) may, in turn, cause additional mandates for the other parties (so-called "Ausgleichsmandate", compensation mandates) to achieve overall proportionality. In 2017, this led to a Bundestag with 709 seats. In 2021, the Bundestag's size further increased to 736. The increasing size of the German Bundestag has been the primary impetus for ongoing discussions on electoral reform.

Figure 1: *Party system fragmentation and share of absolute-majority winners in electoral districts in German federal elections 1949—2017. Data are taken from Michael Gallagher and the Constituency-Level Elections Archive.*
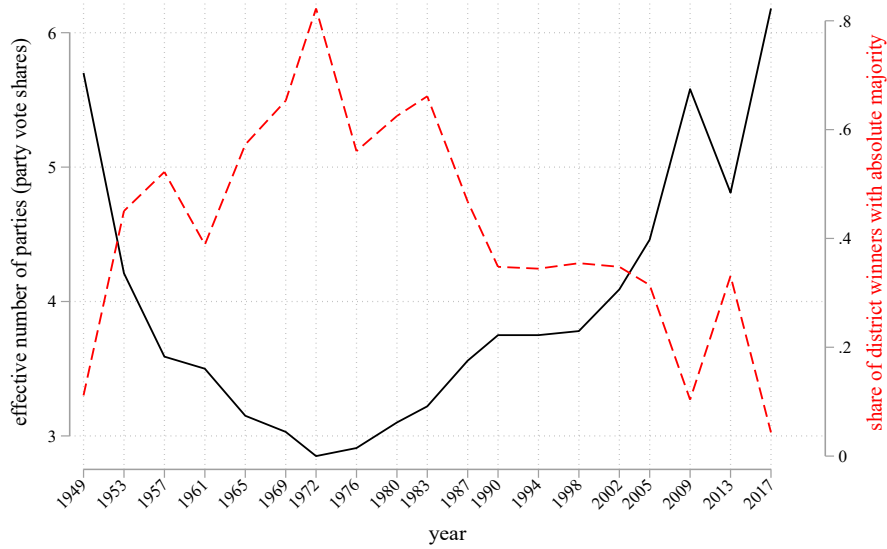
Party system fragmentation is usually measured by the *effective number of parties*[10] (Laakso and Taagepera, 1979). We depict this measure through the black/solid line in figure 1 for all federal elections from 1949 up to the 2017 election. As can be seen, the effective number of parties has never been higher than after the 2017 election, with fragmentation being about twice as high as in the 1960s and 1970s.

What does this increasing fragmentation imply for the electoral outcome? Does it mean that the share of plurality winners who are not Condorcet winners has increased? We will estimate this quantity for the 2017 election below, but we cannot calculate it for all elections due to missing data. However, we can calculate for each election how often the district winner received more than half of the vote totals. Since an absolute-majority winner is also a Condorcet winner (Sen, 2017, Lemma 10*d), the number indicates how many Condorcet winners existed at minimum. This number has also fallen markedly—as shown by the red/dashed line in figure 1.

In 1972, 82% of the plurality district winners received more than half of the votes in their district. In 2017, only 4% —13 candidates in total—managed to do so. That the fragmentation in terms of party vote shares is closely mirrored in fewer and fewer district winners receiving an absolute majority is no surprise given that most voters align their candidate with their party votes.[11]

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[10]The *effective number of parties*, $\mathcal{N}$, is defined as $\mathcal{N} \equiv \sum_{i=1}^{n} v_i^{-2}$ with v_i being the vote share of party $i \in \{1, \ldots, n\}$.

[11]For the 2017 Bundestag election, data on reported vote choices from the post-election cross-section of the German election study (GLES, 2019) indicate that 70% of voters voted for the same party with their candidate and party vote. If we exclude those whose party vote was for one of the smaller

To illustrate how results of the candidate vote may look like in the increasingly frag-mented environment, we show the results in the district of Görlitz in the 2017 election in figure 2.

Figure 2: *Candidate vote shares in electoral district of Görlitz in the 2017 German federal election. Data are taken from the Federal Election Commissioner (Bundeswahlleiter)*

Note: Candidate with vote shares below 2.5 not shown.
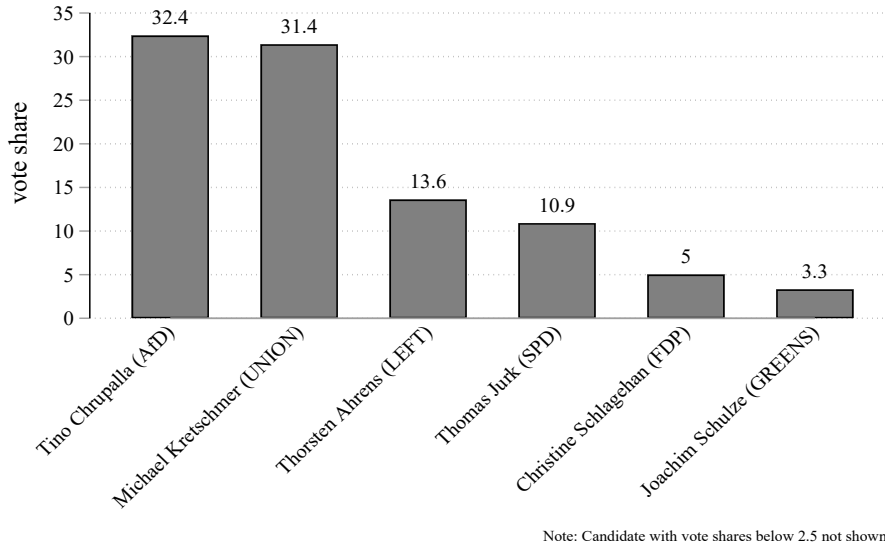
Here, the far-right AfD candidate was able to narrowly defeat his rival from the UNION with a vote share of below a third. There are doubts whether a majority of voters actually preferred the right-winger over his UNION adversary. The answer depends on whether voters of the other candidates would have preferred the right-winger over the conservative in a head-to-head comparison. This question cannot be answered with certainty in the absence of immediate information on voters' preference orderings. Yet, intuitively one might guess that those who voted for candidates from the mainstream parties SPD, FPD and GREEN would overwhelmingly prefer a UNION candidate over a candidate from the radical-right—and perhaps the same might be true of those who voted for a candidate of the leftist party (LEFT). Our data simulation below builds on

parties, who do not even field a candidate in many districts, this number rises to 72%. A big chunk of *ticket-splitting* (see Fn. 6) is by either FDP party voters voting for a UNION candidate (4.5%), or by GREEN party (3.5%) or LEFT party (2%) voters voting for an SPD candidate—presumably due to strategic voting. None of the other, arguable more idiosyncratic, combinations occurred in more than 2% of cases. We observe similar patterns by using the Politbarometer data set (see below). We depict related results in the supplementary material to this paper. At the aggregate level of electoral districts, candidate and party vote shares correlate with 0.97 for the 2017 election. In 264 of the 299 electoral districts (88.3%) the same party won a plurality of candidate and party votes (based on calculations with data from the Bundeswahlleiter). See also Herrmann and Pappi (2008).

this intuition of inferring candidate preference orderings from party preferences. We will thereby assess the prevalence of plurality winners not being Condorcet winners.

The second consequence of increasing party fragmentation—and most plurality winners not receiving an absolute majority—could be that, in many districts, Condorcet winners do not even exist. In the example above, perhaps the UNION candidate was preferred over the AfD candidate by most voters, but a majority preferred the LEFT candidate over the UNION candidate, and a majority preferred the AfD candidate over the candidate from the LEFT.

Under which conditions are such Condorcet cycles likely to occur? As discussed above, this is a matter of the shape of voter's preference orderings. If we assume the preference orderings of (most) voters to follow a one-dimensional left-right logic, indeterminacy is unlikely to occur (value restriction). However, there is ground for concern that voters preferences are more and more unlikely to exhibit value restriction. Alós-Ferrer and Granić (2015), by using the data from their field experiments in Germany (see section 1), argue that political landscapes, as perceived by the voters, are inherently multi-dimensional and cannot be reduced to a single left-right dimension. Political science scholarship agrees that political competition in established democracies today must be conceived by, at least, two dimensions. In some accounts, an anti-establishment dimension now adds to an ideological left-right dimension (Uscinski et al., 2021). Others put forth a two-dimensional model distinguishing between an economic and a cultural axis of political conflict (Hillen and Steiner, 2020; Lefkofridi et al., 2014; Kriesi et al., 2008; Oesch and Rennwald, 2018). Both types of multidimensionality may lead to Condorcet cycles.

Consider first the spread of *anti-establishment voters*. An anti-establishment orientation may lead voters to dislike candidates they associate with the established parties. They prefer parties who voice discontent with the political status quo, possibly in a populist fashion (Tolvanen et al., 2021). In line with this logic, studies show that voters of extreme left-wing parties and parties of the radical-right share certain attitudes, like populist sentiment (Akkerman et al., 2014, 2017; van Hauwaert and van Kessel, 2017) or low political trust (Akkerman et al., 2017).

To see how this may cause Condorcet cycles, consider three equally-sized voter groups: left-wing voters (ℓ), center-right voters (c), and *anti-establishment* voters (a). They have preferences over the triple {UNION (U), LEFT (L), AfD (A)} such that they yield the following perfect-cycle profile:

$$L \succ_\ell U \succ_\ell A$$
$$U \succ_c A \succ_c L$$
$$A \succ_a L \succ_a U$$

Alternatively, preference profiles such as $\{\succ_a\}$ could arise from policy preferences on multiple *policy* dimensions. For example, in a study of the 2017 German federal election, Steiner and Hillen (2021) analyze the voting behaviour of citizens who are left-wing on economic issues (favouring higher social spending) and culturally authoritarian

(preferring a restrictive immigration policy). These "left-authoritarian" voters over-proportionally support both the AfD and LEFT. From a spatial voting perspective, it may be perfectly rational for a left-authoritarian to hold a preference ordering of parties like $\{\succ_a\}$. The first preference for the AfD may reflect its proximity on the cultural dimension. The LEFT might be attractive due to its economic position, whereas the UNION does not offer an appealing position on either policy dimension. Despite the AfD and the LEFT being usually located at opposite ends of the left-right spectrum, for a left-authoritarian, these parties might well be closer to his/her ideal point in a two-dimensional policy space than the UNION. Thus, it may well be possible for Condorcet cycles to exist regarding candidate and party preferences of German voters.

However, there are essential counterarguments. Despite such commonalities between voters of the extreme left and the extreme right, both camps also hold distinct political outlooks and identities that may lead to intense antagonisms. In their recent paper on sympathy towards out-group parties in Germany, Wagner and Harteveld (2021) show that mean ratings of LEFT are about as low among AfD supporters as among supporters of the UNION and the FDP, and are much more favourable among voters of the centre-left SPD and GREEN. The AfD is rated poorly by supporters of all the other major parties and even worse by LEFT supporters compared to UNION and FDP supporters. More generally, distance on the left-right scale is a strong predictor of out-party dislike (Wagner and Harteveld, 2021). This probably reflects the identity component of left-right orientations (Mason, 2018) in combination with elite cueing: Voters who identify as left-wing perceive right-wing parties as part of the out-group, and this tendency is amplified by the messages they receive from political elites they feel close to about which parties to dislike. In this argument, value restrictions on profiles arise from political identities and are thus due to affective mechanisms. Value restrictions may also arise from rational, information-based mechanisms. For example, the democratic process of deliberation may lead to more enlightened preference orderings that then satisfy single-peakedness (List et al., 2013; Rad and Roy, 2021).

Given these competing considerations and the possibly complex mixture of different preference profiles we observe in different parts of Germany, it is an open question whether indeterminacy is a relevant issue. This question needs to be addressed by empirical means. Our data simulation below will do so—alongside testing the expectation that plurality rule failed to comply with the Arrovian condition **I** in a significant number of districts in the 2017 Bundestag election.

4. Simulation approach and data description

We first discuss the building blocks of our approach and then discuss in more detail how we implemented it. Given that information on voters' preference orderings of the individual candidates standing in the 299 electoral districts in 2017 are, of course, unavailable, we need to infer sets of preference orderings through a simulation exercise. Our approach starts from the known information on the vote shares received by the party's candidates in all 299 electoral districts, which were obtained from the Federal

Election Commissioner (Bundeswahlleiter). Building on the intuition from above (see section 3), we then ask: what are the sets of preference orderings of those who voted for, say, a Green-party candidate in Bavaria, and what about of those who voted for a SPD-candidate in Saxony, and so on. To infer this information, we utilise a large survey data set containing information on how 36,689 individuals across Germany ranked and rated the major parties: The cumulative data set of all cross-sectional "Politbarometer" surveys carried out by the polling firm Forschungsgruppe Wahlen (2018) in 2017. We then assign these sets of preference profiles observed per most preferred party (and region) to the electoral districts based on the observed candidate vote shares and determine the Condorcet winner.

This approach entails a number of—we think: reasonable—assumptions. First, as this approach abstracts from individual candidates, it invokes the assumption that preference orderings of candidates are similar to preference orderings of parties. This assumption finds justification in the observation that candidate and party votes correlate strongly, at the individual level and even more so at the aggregate level (see footnote 11). Still, it is advisable to interpret our results in terms of generic candidates.[12] Second, because we assign the preference profile of those whose most preferred party is i to those who voted for a candidate from party i, we further abstract from strategic voting regarding the candidate votes. For example, some of those whose most preferred *party* is the FDP might have voted for a more competitive UNION *candidate* for strategic reasons. However, a strength of this approach is that it recovers the actual plurality winners. Importantly, we can thus be confident that divergences between actual plurality winners and Condorcet winners, as determined by our simulation, result solely from the voting rule.[13] Third, we assume one set of preference profiles for voters whose most preferred party is i per region (mostly states, see below). We thereby take into account that the sets of preference profiles for, say, a voter whose first preference is the LEFT might not be the same in all parts of Germany, but may for example differ between Bavaria and Saxony. We acknowledge that this is still a (necessary) simplification, as the set of preference profiles of voters of a given party may also differ within states, for example, across rural and urban places.

We compiled preference profiles as well as the corresponding relative probabilities of the different orderings with the help of the statistical package R (R Core Team, 2020). In order to keep the analysis tractable, our simulation is limited to the six parties represented in the 2017 Bundestag (AfD, UNION, FDP, SPD, GREENS, LEFT).[14] The

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[12]For example, rather than, for example, stating that Michael Kretschmer was the Condorcet winner in Görlitz (see section 3), it would be more accurate to say that a generic candidate from the UNION would have been a likely Condorcet winner against generic candidates from the other parties.

[13]Alternatively, we could have used the party vote shares to assign preference profiles to districts. But then the plurality winner in our simulation would differ from the actual plurality in those 35 districts for which the party winning a plurality of party votes is not the same as the party winning a plurality of candidate votes.

[14]According to the Politbarometer data set, more than 96% cast a vote in favour of one of these parties. Also note that independent candidates are a negligible phenomenon in German federal elections: Of the candidates not running on a party ticket none received a vote share of more than 10 %, and only three of more than 2%.

Politbarometer data set involved the variables named V61, V62, and V63, for which the survey participants disclosed what party they rank first, second, and third. We utilised this information to generate the individual orderings of the ranks one to three. For the ranking of the fourth to sixth places, we used the variables V49 to V55. Here, participants gave each party a rating ranging from −5 to +5, where −5 is the worst rating they could give and +5 the best.[15]

In the the next step, we calculated how often each particular ordering (e.g., UNION first, GREEN second, SPD third, . . ., AfD last) occurred per party voter in a particular region (e.g. GREEN voter in Bavaria). To identify voters (of different parties), we used another variable in which survey participants disclosed the party they (were intending to) vote for in an upcoming Bundestag election (the so-called "Sonntagsfrage").[16]

In order to consider regional differences in preference orderings, we distinguished the following regional groups (table 1, using the official statistical abbreviations of the German federal states, notably: BR denotes Brandenburg, HB represents Bremen). Note that we had to group some (neighbouring) states due to sparse information in the data set concerning some (smaller) parties. For instance, there are very few GREEN-party voters in SL, or even too few LEFT voters in RP.

Table 1: *States and state groups (clusters) considered for regionally-specific orderings.*

#	state or cluster	#	state or cluster
1	BY	2	BW
3	NW	4	HE
5	RP, SL	6	NI, HB
7	SH, HH	8	BR, MV
9	ST, SN, TH	10	BE

Based on this classification, we generated the percentage distribution of the different types of voters to create subsets of particular voters in particular states or state groups. For instance, we calculated a list for the state Bavaria consisting of six data frames (UNION-voters, SPD-voters, . . ., AfD-voters). We then calculate any ordering's frequency for each type of voter. It calculated, for instance, how often Bavarian GREEN-voters have an ordering such as Union 4 - SPD 2 - LEFT 3 - GREENS 1 - FDP 5 - AfD 6 and so forth. Summarising this information, it means that GREEN-party voters in Bavaria can be described as the group of voters within the scope of our simulation,

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[15]The subdivision into different variables was necessary to obtain accurate preference orderings. The rankings (variables V61, V62, and V63) correspond to the votes that would be cast in an election according to the survey participants' response to the "Sunday question". However, they partially contradict the ratings collected through the variables V49 to V55, presumably due to a primacy effect. Therefore, the more accurate rankings were used to generate the individual orderings from the first to the third preference since they are given only up to that number. We then used the ratings for computing the fourth rank and above.

[16]Note that this a different variable than V61, i.e., the vote intention was recorded in a different variable than the party ranked first. In order to identify voters, we used the perhaps slightly more accurate actual vote intention. However, we find an almost perfect coincidence between both variables.

which is entitled to cast a vote in Bavaria and prefers the green party over all other parties. Now, if, for instance, 15% of the Green-voters in Bavaria have an ordering such as

$$\text{GREEN} \succ \text{CSU} \sim \text{SPD} \succ \text{FDP} \succ \text{LEFT} \succ \text{AfD},$$

then, we assigned such an ordering to 15% of green-party voters observed in Bavaria's actual Bundestag election.

In the next step of the simulation, the order lists were then assigned to each voting district's actual candidate vote data, making it possible to calculate a set of preference profiles for each district. Armed with the regionally-sorted preference orderings, we used the R-package `vote` (Ševčíková et al., 2021) to determine the Condorcet winner in each electoral district.

5. Empirical results

5.1. Preference orderings

Looking at the preference profiles that were created for each type of voter per region, there are some obvious patterns that recur, but also some variations that emerge—especially when comparing western to eastern regions. In Figure 3, we show the most striking preference orders in terms of regional differences across voter groups. The abscissa shows the six parties represented in the Bundestag, and the ordinate shows the rankings of each voter group. The colours' saturation also indicates the frequency of the respective preference order—the more saturated the colour, the more frequently the ordering occurs.

Figure 3: *Regional preferences of different voters.*

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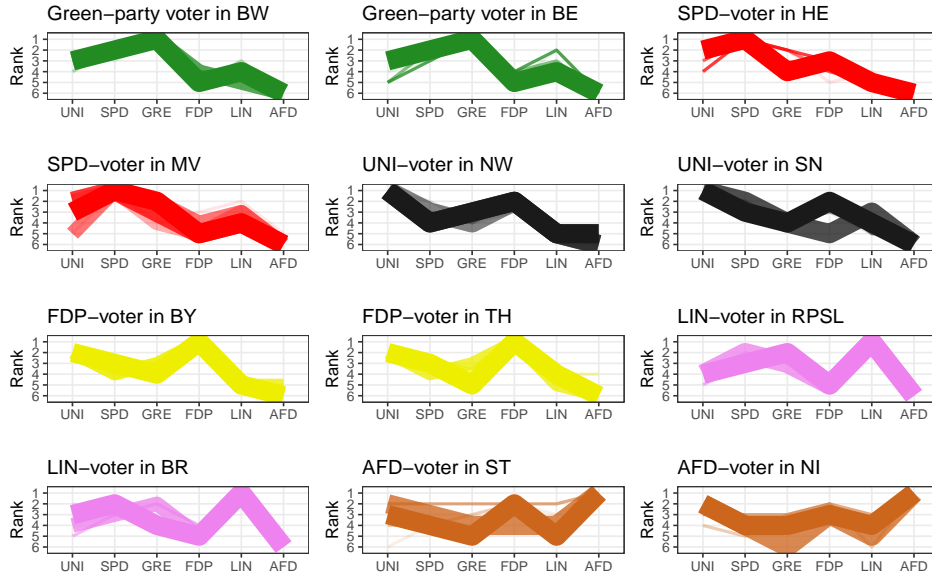
As illustrated in figure 3, one of the most striking variations across regions can be observed among SPD voters, for instance. While the majority of SPD voters in Hesse (HE) places the FDP as their third preference and the *GREEN-party* as their fourth preference, this is not the case in Mecklenburg-Hither Pomerania (MV). In fact, most SPD voters there prefer the GREENS over the FDP.

Furthermore, SPD-voters in Mecklenburg-Hither Pomerania prefer the LEFT over the FDP, which means that the FDP falls a total of two ranks short compared to SPD-voters' preferences in Hesse. Instead, the LEFT moves up one rank and is thus ranked as the fourth preference by most of this voter group in MV. Green voters in BW rank the UNION far better than Green-party voters in Berlin. The list could be extended to several further examples. Beyond these differences, most orderings fulfil value restriction, thus do not lead to cycling majorities. We will demonstrate that in the next but one subsection.

5.2. Anti-establishment voters

As outlined in section 3, the existence and spread of so-called *anti establishment* voters can contribute to cycling majorities. With the help of the above-mentioned rating variables (V49 to V55) we assessed empirically how LEFT-party voters judge the far-right AfD compared to the judgement of all voters (except for the AfD-voters). Analogously, we look at how all of their non-voters judge the LEFT and compare this grading to the AfD-voters rating.

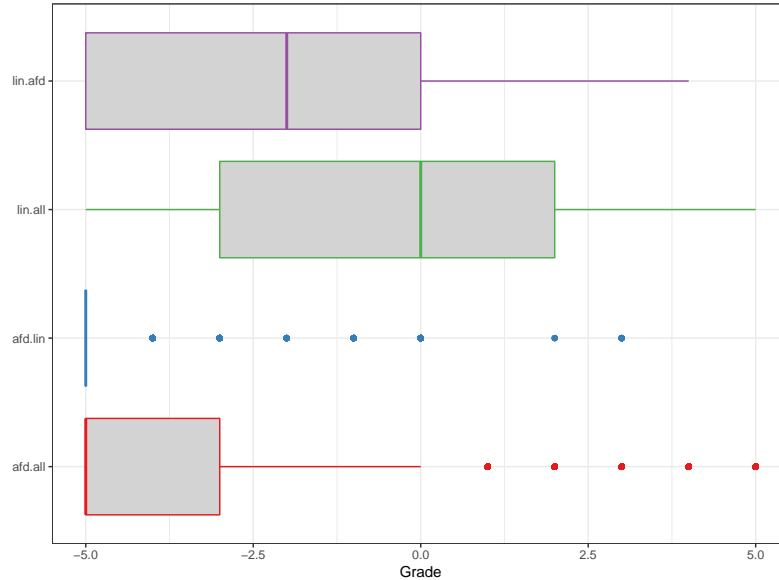**Figure 4:** *Grading of the extreme parties.*

Figure 4 depicts the results and provides support for our expectation that voters of the extreme parties do not exhibit what is sometimes called a horseshoe shape. The AfD was

judged worse by all non-AfD voters (the boxplot dubbed "afd.all"). However, it turns out that the LEFT voters graded the AfD (boxplot "afd.lin") even worse. A mirror image appears regarding ratings of the LEFT: The LEFT is rated worse by AfD voters (boxplot "lin.afd") than by voters of the other non-LEFT parties (boxplot "lin.all").

Although both groups, the AfD voters and the LEFT voters, might share some common views, it seems that fundamental differences between both camps outweigh existing commonalities.

5.3. The existence of Condorcet winners

Having generated the regionally-sorted preference orders, we run the simple-majority rule on all 299 districts. As a result, we do not find any evidence for indeterminacy. Rather, we found in each electoral district a Condorcet winner. Our data analysis brings forth similar results as in previous studies (Chamberlin et al., 1984; Feld and Grofman, 1992; Regenwetter and Grofman, 1998; Tideman, 2006). However, it was not a priori clear that German polling data bring forth the same result as data from a lab experiment in GB or even data from private-organisation elections. It appears that, at least in the German context, preference orderings of ordinary voters regarding political parties are structured in such a way as to prevent the emergence of majority cycles. This results was not preordained, as majority cycles are quite likely under the assumption of unstructured, random preference orders (see section 2). Even if structured, the (increasing) multidimensionality of political competition could well bring about majority cycles (see section 3). However, it appears that preference orders exhibit overall sufficient value restriction to prevent indeterminacy. We attribute this results to the power of ideological identities that lead to preference orders that mostly are in line with a left-right alignment of political parties—despite the tendencies toward multidimensionality (i.e. the emergence of an anti-establishment dimension or of multiple policy dimensions).

5.4. Plurality winners vs. Condorcet winners

Concerning the violations of the Arrovian condition **I**, we calculated in which districts the existing Condorcet winner was defeated by the plurality winner. As shown in figure 5, applying the simple-majority rule leads to a differing electoral outcome compared to running the plurality rule. The left-side map depicts the actual electoral winners at the 2017 Bundestag election, while the map on the right-hand side illustrates the Condorcet winner simulated for the federal election in 2017. The colours are in alignment with the popular political colours.[17]

Overall, 33 districts—11% of all districts—flipped due to the change in voting method, i.e. the party that won this constituency under simple-majority rule differs from the one that won in the election under plurality rule.[18] In order to obtain a better overview of

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[17]Black stands for the Union, red for the SPD, a chocolate colour we use for the AfD, violet for the LEFT and green for the GREENS.

[18]Using two flipped districts (Darmstadt and Freiburg) as examples, we demonstrate how such different outcomes can occur purely as a result of a change in the election process step by step in the

Figure 5: *Districts actually won vs. Condorcet winners.*

Plurality rule   Simple–majority rule
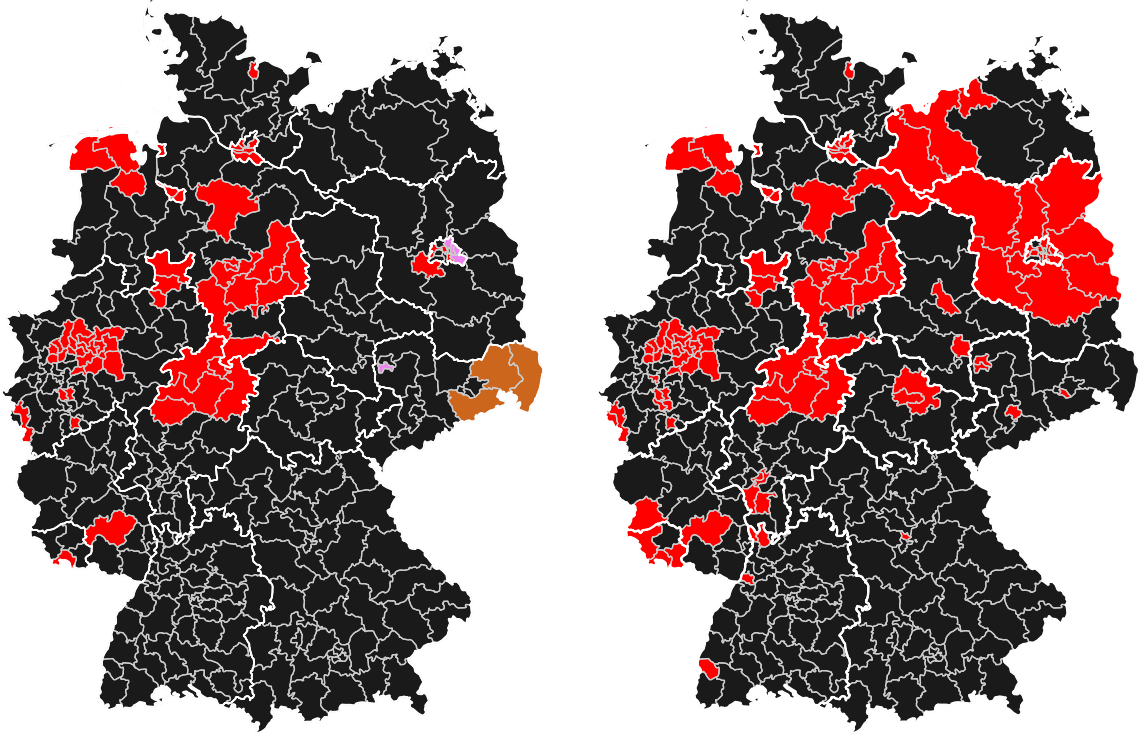
those flipped districts, we created the same map as in figure 5, but coloured the flipped districts only. The figure is presented in appendix A (fig. 6). We also list all flipped districts in appendix B (table 2).

Most districts whose winners change after applying the simple-majority rule switched from the UNION to the SPD (n=28). In these districts, there is a substantial amount of vote splitting on the left-side of the political spectrum with candidates from the GREENS and/or the LEFT gaining substantial vote shares. As most of these voters prefer the SPD over the UNION, we see these districts flipping once we move from plurality voting to the simple-majority rule. Most of these flipped districts are located in larger cities or cities with many young people, such as Berlin, Cologne, Darmstadt, Hamburg, etc. For example, in Berlin-Tempelhof-Schöneberg the UNION candidate received a plurality of 28.9% of votes, but with strong vote splitting among the left-wing parties (SPD: 22.0%, GREENS: 18.9%, Left: 10.8%) our simulation indicates the SPD candidate as the Condorcet winner.[19] A similar mechanic takes place in a bunch of

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supplementary material.

[19]In one district—the city of Freiburg, won by the UNION candidate—it is the GREEN candidate who

more rural districts in Eastern Germany won by the UNION but with substantial vote shares especially for the LEFT candidate. This is, for example, in the district Märkisch-Oderland - Barnim II, northeast from Berlin: Here, the LEFT candidate received more than a fifth of the votes (22.5%) and the SPD candidate, albeit receiving a low share under the plurality system (15.8%), emerges as the Condorcet winner.[20]

The most fatal violations of the condition **I** we find in Saxony. Here, our simulation indicates that the plurality winners in three districts were Condorcet losers, i.e. they would have very likely been defeated in pairwise comparison against candidates from all five other major parties. After applying the simple-majority rule, the far-right winger AfD loses all of their three directly-won seats in Saxony to the UNION, including the aforementioned district "Görlitz" (figure 2).

Our data indicate that the AfD is highly polarising (which is indicated already in figure 4). For far more than 90%, the AfD is either the top choice or the worst alternative. We rarely observe that voters rank the party on the second to the fifth rank. Even in Saxony, the overwhelming majority of voters regard the AfD in no way as a feasible option. Only a minority (roughly a third) sees their favourable party in the AfD. However, the majority's opinion is not considered under the plurality rule but could be so under the simple-majority rule (or even the Borda count).

6. Conclusion

For a long time, the appropriate choice of the voting scheme was like holding the wolf on the ears. Either one risks the elections not bringing forth a winner or filling the office with an incumbent short of a majority. The contributions by Dasgupta and Maskin (2008, 2020) provide an essential step to resolve the dilemma. The simple-majority rule has proved superior to the often used plurality rule. Although the claim to rethink voting schemes in the US is apparently pushed by some electoral outcomes in 2016, there is reason to re-evaluate the voting methods also outside the US. We argued that even in Germany, the violation of the Arrovian condition **I** is increasingly an issue—given the increase in party fragmentation.

On the other hand, we find little evidence for the substantial threat of cycling outcomes. We, therefore, took a step to revitalising the discussion in Germany by providing some theoretical insights and empirical evidence. This paper aims at contributing to-

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emerges as the Condorcet winner from such a dynamic.

[20]This example illustrates how the Condorcet winner can be a candidate who received a low share of votes under the plurality system. We look at this more systematically in the supplementary material where we plot the distribution of candidate vote shares received by the different types of candidates. All but one of the plurality winners with a voter share larger than 40% is also a Condorcet winner. Thus, high plurality shares that approach an absolute majority essentially ensure that plurality winners are also Condorcet winners—at least for these plurality winners from the mainstream parties. However, in districts without such a clear front-runner the Condorcet winning candidate may have received a (very) low share of plurality votes. The most extreme example is the SPD Condorcet winner from the district Leipzig II who obtained a vote share of 13.7% in a heavily fragmented electoral environment.

wards a scientifically-based discussion.

There is a burgeoning discussion in German judiciary. An attempt to substitute the run-off elections in North Rhine-Westphalia by plurality rule failed in court as it did not meet the constitution's requirements, according to the ruling of the state court in 2019 (VerfGH 35/19, see Barbaro (2021)). Because of the plurality rule's flaws regarding the majority principle, its compatibility to constitutional law is currently questioned in jurisprudence literature (Barbaro and Wieland, 2021).

Although reliable data for the 2021 Bundestag elections are not available yet, it can be reasonably assumed that the far-right AfD increasingly profited from condition-**I** violation in the 2021 elections. Though disdainful of democratic norms (Arzheimer and Berning, 2019), AfD candidates prevailed in sixteen districts. The reason is vote-splitting behaviour favoured by the weakening of the UNION party and the strengthening of the social democrats, particularly in the eastern states. In a sound electoral system, a shift within the democratic spectrum exclusively affects the pair-wise judgement between UNION and SPD. However, it appears that a (the) "irrelevant" alternative was among the profiteers.

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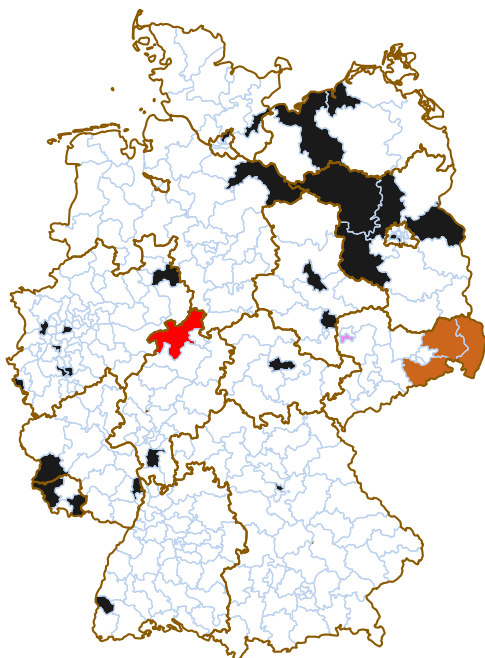
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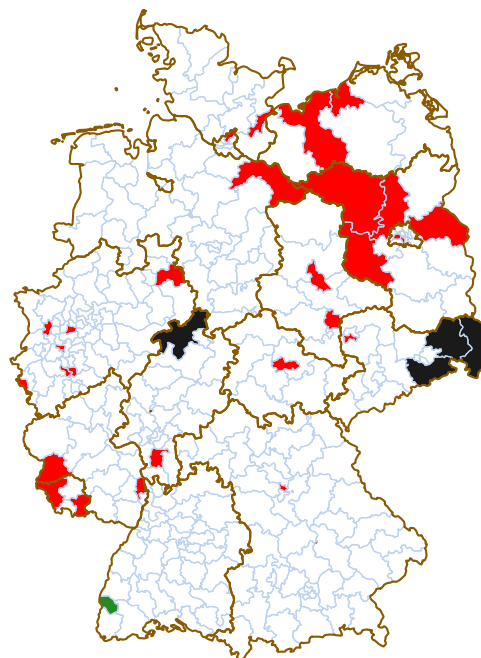
A. Map of flipped electoral districts

The colours on the left map indicate the losing parties of our simulation, the colours on the right-hand side map the winning parties. Our simulation shows where the actual electoral outcome is driven by a violation of the condition **I**.

Figure 6: *Map of flipped districts.*

B. List of flipped districts

Table 2: *PW indicates the plurality winner, CW the Condorcet winner (simple-majority winner).*

#Distr	District Name	State	PW	CW
11	Lübeck	SH	UNI	SPD
13	Ludwigslust-Parchim II – Nordwestmecklenburg II – LK Rostock I	MV	UNI	SPD
14	Rostock – Landkreis Rostock II	MV	UNI	SPD
21	Hamburg-Nord	HH	UNI	SPD
37	Lüchow-Dannenberg – Lüneburg	NI	UNI	SPD
56	Prignitz – Ostprignitz-Ruppin – Havelland I	BR	UNI	SPD
58	Oberhavel – Havelland II	BR	UNI	SPD
59	Märkisch-Oderland – Barnim II	BR	UNI	SPD
60	Brandenburg a.d.H. – Potsdam-M. I – Havelland III – Teltow-Fl. I	BR	UNI	SPD
69	Magdeburg	ST	UNI	SPD
72	Halle	ST	UNI	SPD
80	Berlin-Charlottenburg-Wilmersdorf	BE	UNI	SPD
81	Berlin-Tempelhof-Schöneberg	BE	UNI	SPD
87	Aachen I	NW	UNI	SPD
93	Köln I	NW	UNI	SPD
94	Köln II	NW	UNI	SPD
107	Düsseldorf II	NW	UNI	SPD
114	Krefeld II – Wesel II	NW	UNI	SPD
120	Essen III	NW	UNI	SPD
135	Lippe I	NW	UNI	SPD
153	Leipzig II	SN	LIN	SPD
156	Bautzen I	SN	AFD	UNI
157	Görlitz	SN	AFD	UNI
158	Sächsische Schweiz-Osterzgebirge	SN	AFD	UNI
167	Waldeck	HE	SPD	UNI
186	Darmstadt	HE	UNI	SPD
193	Erfurt – Weimar – Weimarer Land II	TH	UNI	SPD
203	Trier	RP	UNI	SPD
207	Ludwigshafen/Frankenthal	RP	UNI	SPD
244	Nürnberg-Nord	BY	UNI	SPD
281	Freiburg	BW	UNI	GRE
297	Saarlouis	SL	UNI	SPD
299	Homburg	SL	UNI	SPD