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Highlights

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- Plurality-based electoral systems foster polarization.
- Considering preference intensities, the Borda rule deters polarization.
- This study demonstrates that parties on the political fringes benefit most from pluralitybased systems such as the proportional representation system.
- Radical right parties are particularly successful in gaining more parliamentary representation than public backing, benefiting from their polarizing behavior.

On the Polarization Premium for radical parties in PR electoral systems

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Abstract

Western democracies are grappling with escalating political polarization. While scholars have explored various societal and economic factors contributing to this phenomenon, the influence of the electoral system has received limited attention. In this paper, we argue that the use of the proportional representation system (PR), a common electoral approach, contributes to the rise in polarization. PR systems prioritize voters' top preferences, allowing candidates to increase their electoral support even as the proportion of citizens strongly opposed to them grows.

We formally demonstrate that PR systems incentivize candidates and parties to adopt extreme positions, a trend discouraged under the Borda count system. Using the Borda count as a benchmark, our empirical analyses with data from the Comparative Study of Electoral Systems (CSES) confirm that polarizing candidates, often positioned at ideological extremes, benefit under PR voting rules. Our study reveals an additional asymmetric effect of PR systems, which disproportionately favors right-leaning polarizing candidates. This phenomenon is attributed to the more pronounced disagreement that far-right ideology provokes among moderate and left-leaning citizens compared to radical left ideologies.

Keywords: Elections, Voting Schemes, Political Polarization, Proportional Representation. *JEL:* D71

1. Introduction

Political polarization poses a significant challenge to democratic norms and the functionality of institutions. It creates barriers to government formation and stability and undermines social cohesion overall. Literature has noted the potential for polarization to intensify political gridlocks, perpetuating a cycle of escalating polarization (Dixit and Weibull, 2007).

Increasingly, scholars are investigating the origins of these troubling trends. They highlight the transformation of partianship into a social identity that promotes social sorting

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and engenders animosity between supporters of different political parties (Mason, 2015; Mason and Wronski, 2018; Iyengar et al., 2019; West and Iyengar, 2022). The role of social media has been identified as a significant factor in this transformation (Bail et al., 2018; Allcott et al., 2020; Levy, 2021; Azzimonti and Fernandes, 2023; Törnberg, 2022). Furthermore, current research on representation also focuses on the rising polarization, identifying political parties as key drivers of this trend. These parties increasingly craft their strategies by making representative claims to specific social groups (Saward, 2006; Disch, 2021).

This paper contributes to the discourse by demonstrating that the electoral system, which precedes party strategies causally, may create incentives for polarizing strategies. We build on recent findings by Lachat and Laslier (2024), who identified a link between political polarization and the mechanical effects of electoral rules. Their results show that in contexts of polarized and fragmented party systems, plurality voting in single-member districts significantly underperforms other electoral methods in selecting the Condorcet winner. This supports claims from the social choice literature that plurality voting frequently flaws such that the legitimacy of any election outcome that involves the plurality rule has been questioned (Saari, 2008, p. 190f). While PR systems are seldom criticized in this manner, they also adhere to the plurality logic by requiring voters to vote for a single candidate or party only. We argue that plurality-based proportional electoral systems incentivize polarizing strategies by political leaders and parties, whereas systems that rely on the Borda count to distribute seats proportional to vote shares discourage such behavior. Divisive strategies by political leaders may thus not only be a response to social developments but can also partly result from the incentives induced by PR electoral systems.

This study aims to empirically evaluate the extent to which PR systems incentivize political parties to adopt extremist platforms. We will demonstrate that a policy shift to the extremes consistently results in a substantial decline in popular support for the respective political party. However, polarizing political factions may willingly embrace this broad alienation, as it does not adversely impact their chances of electoral success. On the contrary, employing a divisive campaign among the electorate can potentially serve as a viable electoral tactic, despite the risk of alienating a larger number of voters compared to those it manages to attract. We empirically show that this behavior is more frequently observed among candidates from the ideological right.

In Section 2, we will provide a succinct review of the related literature. Section 3 will formally illustrate how proportional representation (PR) systems encourage political candidates and parties to adopt radical stances. We will then demonstrate the reduction of these incentives under the Borda rule (henceforth: BR), which shows near immunity to polarizing strategies. The Borda rule serves as a primary benchmark for assessing the biases inherent in plurality-based PR systems. Based on our theoretical framework, we formulate several empirically testable hypotheses in Section 4. In Section 5, our empirical analysis, which utilizes voter preference data from the Comparative Study of Electoral Systems (CSES), validates the empirical significance of the formal results.

Our findings substantiate two key assertions. First, candidates who espouse polarizing views are more frequently positioned at the extreme ends of the ideological spectrum. Second, within political systems employing plurality-based voting rules, these candidates tend

to garner electoral advantages. Furthermore, our analysis reveals that the impact of the PR system disproportionately favors politicians aligned with right-wing orientations. We interpret this outcome by considering that far-right ideology tends to provoke a greater degree of intense disagreement among voters identifying as moderate or left-leaning, compared to the level of disagreement elicited by radical left ideologies. Consequently, proportional representation systems contribute to political polarization, with right-wing political forces emerging as the primary beneficiaries.

2. Related Literature

Long predating the current discourse on polarization, political scientists have extensively examined the impact of electoral systems on this phenomenon. The groundbreaking work by Cox (1990) established a significant connection between the two areas. By considering four characteristics¹ of electoral systems, Cox assessed their incentive to centrifugal or centripetal outcomes. The latter term refers to the phenomenon wherein parties are motivated to promote viewpoints that foster consensus, commonly known as "centrist policies".

Calvo and Hellwig (2011) empirically supported Cox's findings. Aligning with this, Myerson and Weber (1993, Theorems 3 and 4) compared plurality rule with Approval Voting and demonstrated that the latter is less susceptible to polarization. However, to derive this result, several strong assumptions were made, particularly concerning a uni-dimensional policy space. Bol et al. (2019) demonstrate that such a squeeze-out of centrist parties is associated with higher electoral-rule disproportionality. The study at hand integrates into the existing body of political science literature and expands the research scope within the subject.

In many respects, our contribution is closely related to the recent work by Lachat and Laslier (2024), published in this journal. They explore how often different electoral systems produce different election winners, using data from the Comparative Study of Electoral Systems (CSES) project. They generate voter preference orderings from so-called thermometer variables, namely the ratings of parties or candidates on a Likert scale. We utilize exactly the same dataset and the same thermometer data as Lachat and Laslier (2024). While their focus is primarily on comparing Condorcet and plurality winners, our study concentrates on seat distribution in parliaments. The core of Lachat and Laslier's work is the election results under the plurality rule in constituencies (First-Past-The-Post). In contrast, our focus lies on PR systems. Although both systems share a plurality-based character, they function in significantly different ways, particularly concerning the issue of wasted votes. A third distinction is the reference point. In Lachat and Laslier (2024), the reference is the Condorcet winner, while our reference point is the Borda rule because—as we demonstrate in our theoretical section—it exhibits greater immunity to political polarization.

¹(i) The electoral system, (ii) the ballot structure (how many votes can be cast, abstention possibility, votes cumulation), (iii) magnitude (the number of seats to be filled), and (iv) the ballot size (number of candidates).

3. Theoretical Background

Distinct electoral systems can yield markedly different outcomes, even when voters' preferences remain the same Saari (2000a,b); Brandt et al. (2022). Additionally, various voting systems generate unique motivations for the strategic positioning of political platforms. Therefore, when assessing the impact of voting rules, it is beneficial to categorize election results into the "arbitrariness" component and the "incentive" component. This categorization facilitates ongoing assessment. The example in Table 1 serves to illustrate these considerations.

Election at t_1				Election at t_2				
Group 1	Group 2	oup 2 Group 3		Group 1	Group 2	Group 3		
34%	33.5%	32.5%		34%	33.5%	32.5%		
a	b	С		d	b	с		
d	d	d		a	с	b		
с	С	b		с	a	a		
b	a	a		b	d	d		

Table 1: Voters' preferences and electoral outcomes

The electorate² consists of three distinct voter groupings, with all voters within a group sharing the same strict preferences over the candidates in set $\mathscr{B} \equiv \{a, b, c, d\}$. The first election occurs at time t_1 . Under plurality voting, candidate *a* emerges as the victor, garnering the highest number of votes. However, *a*'s victory overlooks the strong alienation felt by a two-thirds majority, as all voters not belonging to the first group consistently rank each other candidate higher than *a*. This scenario exemplifies the *Borda paradox* (Gehrlein and Lepelley, 2009), which could have been averted using either the Condorcet method or the Borda rule (BR), both of which would result in the election of candidate *d*. This example highlights the concept of *arbitrariness* in electoral outcomes.

The section on *incentives* primarily focuses on the implications of electoral outcomes on candidates' campaign strategies in preparation for the election at t_2 . In response to the unfavorable election outcomes in t_1 , candidate d (who received no single vote in t_1) revises his political agenda to align with populist right-wing ideologies, thereby appealing to misogynous and xenophobic sentiments. This shift to the populist right is favorably received by the voters in the first group, as indicated by the observed rise of d in their orderings. However, the assertive approach adopted by the candidate results in the alienation of a substantial portion of the electorate. Consequently, while support for candidate d grows within group 1, there is a notable decline in rankings among all other voters. Despite initially

²We assume a continuum of voters, indexed by points on the unit interval [0, 1]. The continuum is defined over the Borel set such that the preference proportions do not fall into some finite exceptional set (because the Lebesgue measure is not defined for all subsets of the continuum).

being recognized as both a Condorcet and Borda winner, candidate d is now identified as the Condorcet loser. Nevertheless, this combative strategy culminates in electoral success. This example serves to illustrate how the plurality rule, and by extension, plurality-based systems such as PR systems, can incentivize polarization. In contrast, such a divisive campaign is not endorsed if the winner is determined by applying Borda's rule or the Condorcet method.

The incentive to polarize under plurality-based voting schemes stems from the fact that candidates or parties are not obligated to consider the perceptions of individuals who do not cast votes for them. Voting rules that take into account voters' entire preference orderings address this issue to some extent. However, the effectiveness of rank-order voting³ and the simple-majority rule (aka the Condorcet method) in preventing the electoral successes of polarizing political entities can vary significantly.

Table 2. I follies of a polarized electorate									
	Pi	ofile	(\mathbf{I})	$\operatorname{Profile}\left(\overline{\mathrm{II}}\right)$					
Rank	S	\mathcal{D}	\mathcal{M}	S	\mathcal{D}	\mathcal{M}			
Rank	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3			
1	A	В	A C B	A	В	С			
2	В	С	\mathbf{C}	C	С	А			
3	C	А	В	B	А	В			

Table 2: Profiles of a polarized electorate

To illustrate the specific differences, we utilize a well-established social choice framework relevant to a divided electorate (Gehrlein, 2005; Campante and Hojman, 2013). A polarizing candidate is defined as one who garners fervent support from a substantial section of voters while being vehemently despised by another large fraction (see Table 2). The electorate is divided into three voter groups: the supporters (S), the despisers (D), and the moderates (\mathcal{M}). The moderates are those who rank a divisive candidate in either position, whereas the other (larger) groups rank the controversial candidate (here: candidate A) either at the top (the supporters) or at the bottom (the despisers). The proportion of each group is denoted as γ_i , with $\sum_{i=1}^{3} \gamma_i = 1$. To depict a polarized electorate, we assume the group of moderates to be smaller than the respective bipartisan groups. Additionally, to avoid trivialities, we ensure that no single voter group constitutes an absolute majority, i.e., $1/2 > \gamma_1 > \gamma_3$; 1/2 > $\gamma_2 > \gamma_3$. A more in-depth formal representation of our model is provided in Appendix A. In this section and in the Appendix, we employ the Borda rule by assigning a score of one to a candidate ranked first, two to a candidate ranked second, and three to those ranked last by voters. The winner is the candidate with the lowest total score.

In our first example, labeled Profile (I), Candidate A also secures the support of the moderates, which makes her the Condorcet winner despite being a polarizing figure. Thus, A will occupy the office under both the plurality rule and the Condorcet method for any admissible configurations of γ_i . According to Borda's rule, the divisive candidate will not win if the condition $\gamma_1 > 2 - 4 \cdot \gamma_2$ is satisfied.

³By rank-order voting we mean the set of scoring rules without plurality rule.

For example, if $\gamma_1 = 38\%$ and $\gamma_2 = 40\%$, then the polarizing candidate A will emerge as the Borda winner. However, if the number of her fervent supporters reaches a level close to the absolute majority threshold (e.g., $\gamma_1 = 48\%$, $\gamma_2 = 40\%$), the polarizing candidate A will be defeated by B. Under the plurality rule, a surge in unwavering support for a candidate consistently facilitates or solidifies their electoral success. Conversely, under the Borda rule (BR), such an increase may result in electoral defeat. At first glance, this outcome may seem counter-intuitive. The cause can be attributed to the role played by the *irrelevant alternative*⁴ C. If C had not run, A would have emerged victorious regardless of the voting system employed. The inclusion of candidate C does not affect the outcome of the plurality vote, as no voter selects C as their top choice, a property termed Limited Agreement (Sen, 2017, Def. 10*4) or Limited Favoritism (Dasgupta and Maskin, 2008). Moreover, the candidacy of C has no impact on the outcome according to Condorcet's method due to this specific scheme's adherence to Arrow's IIA axiom.

How does this seemingly paradoxical outcome arise when applying the BR? The answer lies in the fact that the increase in support for γ_1 comes at the expense of γ_3 . Since both groups place candidate A in the first position, this switch has no bearing on the plurality-rule result. However, the two groups differ in their perception of candidate C. The moderate group positions C between A and B, indicating a higher preference intensity. In contrast, in the supporters group, C is not ranked between the other two candidates, suggesting a lower intensity of preferences. In other words, the purpose of variable C is to signify the variations in the levels of preference intensity among different voter groupings (Maskin, 2024). Note that candidate B defeats the Condorcet winner. It is precisely the (disputed) axiomatic weakness of the BR—its vulnerability to the IIA requirement—that explains its immunity to polarization.

The immunity of Borda's rule against polarization, through the consideration of preference intensities, is further demonstrated in Profile (II) in Table 2. In this scenario, both candidates A and B emerge as polarizing figures. Depending on the sizes of the groups γ_1 and γ_2 , one of these two candidates would be selected under the plurality rule. However, when applying the Borda rule (BR), neither of the polarizing candidates can win the election. Instead, the consistent outcome across the entire admissible domain of group sizes is the victory of the only non-polarizing candidate, C, who, incidentally, receives the fewest votes under plurality.

Political polarization signifies an increase in preference intensities. A voting method that exacerbates these intensities is thus disadvantageous for divisive figures. In this light, the BR presents the a significant obstacle for a polarizing candidate. It discourages divisive strategies, contrasting sharply with the incentives induced by the plurality rule.

In this context, it is necessary to carefully qualify our statement regarding the particular

⁴In Arrow's seminal theorem (Arrow, 1963), the IIA (independence from irrelevant alternatives) is implied by the neutrality requirement. The latter means that all alternatives shall be treated equally. If a majority favors *a* over *b*, then the electoral rule (the social welfare function) should not alter this binary decision solely because *c* is running, too. The Arrovian IIA requirement is more general than against a third alternative. IIA is violated if in the collective choice involving two alternatives, *anything* other than the individual orderings over the tuple gets a place, including the *preference intensities* (see Sen, 2017, Ch. 7).

immunity of the BR to polarizing strategies. This immunity arises because candidates ranked from second to k - 1 are weighted, a feature absent in the plurality rule. The more the middle ranks are considered, the more significantly a placement in the last position is negatively weighted. Therefore, it can be shown that the Anti-plurality rule (APR, Baharad and Nitzan (2005)) represents the voting method that particularly—and even more so than the BR—impedes the electoral success of polarizing candidates or parties. Under the APR, the candidate who is ranked last the least often emerges as the winner. Since polarizing candidates are, by definition, frequently placed in the last position, they tend to fare poorly when the APR is applied.

The reason we still consider the BR as a benchmark relates to its multitude of desirable characteristics (Maskin, 2024), unlike the APR. The latter fails to account for which parties are particularly preferred by voters. For instance, if a party x is consistently seen in the top two positions, and a party y is viewed less favorably by all voters (but never as the worst option), then the Anti-Plurality Rule would generate a tie between x and y. This outcome does not align with the principle of majority decision-making.

Our Appendix provides analytical proof that the observations made in the specific cases (I) and (II) are applicable to all conceivable polarized profiles. Specifically, it establishes that if a polarizing candidate is unable to win an election under the plurality rule, they will also be unable to secure a victory under the Borda rule (BR) across the same domain. Conversely, there are scenarios where the Borda winner outperforms a divisive candidate, who may nonetheless become the plurality winner within the same domain. We detail the robustness of the BR against polarizing behavior in Theorem 1 in the Appendix.

4. Polarizing candidates and ideology

The remainder of the paper addresses the empirical relevance of the above stated theoretical insight for a broad range of party systems in parliamentary democracies that are largely shaped by ideology. This is done by first analyzing the connection between polarizing candidates and political ideology, and second by analyzing potential asymmetries in this relation.

Following the idea that citizens' preferences for political candidates in parliamentary democracies are mostly single-peaked on an underlying ideological dimension, we naturally expect candidates at the ideological poles to be more likely to polarize the electorate, as compared to ideologically moderate candidates. Assuming single peakedness, citizens from the right will prefer moderate to left parties, and citizens from the left will prefer moderates to right parties. Thus, moderates are likely to be consensus candidates. We should not take that for granted, however. Besides ideology, personal characteristics of political leaders also contain the potential to polarize the electorate, see Donald Trump as an example, who is deeply admired by his supporters and deeply despised by his political opponents. While acknowledging the possibility that also moderate political leaders might polarize, we still expect political ideology to be a strong structuring force for individual preferences. This intuitively results in a positive relation between ideological extremeness and the potential to polarize the electorate. Thus we formulate the following hypothesis. **Hypothesis** H_1 . Ideologically extreme parties are more likely to be polarizing candidates than moderate parties.

Furthermore, we postulate that radical right parties polarize more than their left counterparts because right-wing ideology provokes more intense disagreement among moderate voters than vice versa (Harteveld et al., 2022). Consequently, we expect them to perform disproportionately well under proportional representation (PR) voting rules compared to Borda's rule. The reason for this asymmetry in the rejection of extreme left and right ideologies lies in the exclusionary and anti-system character of radical right ideologies. These ideologies are marked by a rejection of democratic, pluralist, and liberal values (Rydgren, 2007), and are characterized by their endorsement of exclusionary nationalism and authoritarianism (Carter, 2018). While radical left ideology also contains anti-system elements, particularly when combined with populism—a 'thin' ideology (Mudde and Kaltwasser, 2013)—these elements primarily target the political and economic system, not third-party groups. Consequently, radical left populism is described as dyadic, targeting the elite, while radical right populism is triadic, attacking both the elites and out-groups (March, 2017). Thus, leftwing ideology is often described as inclusionary, focusing on socio-economic egalitarianism, whereas right-wing ideology is exclusionary, centering on nativism (Mudde and Kaltwasser, 2013; March, 2017).

These exclusionary elements make radical right ideology significantly more consequential. While radical left-wing parties aim to fulfill their election pledges through redistribution and expropriation, the promises of radical right parties typically entail depriving members of ethnic groups and other minorities of their rights, their homeland, and their citizenship. These consequences are difficult to reconcile with the basic democratic norms endorsed by moderate voters. Hence, we hypothesize that the exclusionary elements of radical right ideology provoke a much stronger rejection among ideologically moderate and left-leaning citizens, more so than radical left ideology, irrespective of their association with populism. Consequently, radical right parties should also be more likely to polarize than ideologically left parties.

Hypothesis H_2 . Ideologically right parties are more likely to be polarizing candidates than ideologically left parties.

By radicalizing their ideological stances, right parties can appeal to segments of the electorate that are dissatisfied with the current government, endorse nativism, and oppose liberal and democratic norms. By taking an ideologically extreme standpoint, radical right parties can thus gain supporters who are unlikely to support any other established party. By so extending their supporter base, they simultaneously increase the share of voters who despise them. Thus, they gain electoral support under PR voting while increasing polarization in their preference profile and provoking more rejection.

What remains less understood is the extent to which this tendency is facilitated by the incentives of PR systems. Based on the theoretical insights articulated above, we expect plurality-based voting rules to set such incentives for polarization. Conversely, the Borda

rule (BR) is relatively immune to such behavior and instead encourages consensus-seeking behavior. Therefore, we expect, first, that polarizing candidates perform better under PR than under Borda voting, and second, that rightist platforms achieve better results under PR than under Borda voting rules. These insights lead to the formulation of the following hypotheses:

Hypothesis H_3 . Polarizing candidates will get more votes under PR voting as compared to Borda voting.

Hypothesis H_4 . Ideologically right parties will get more votes under PR voting as compared to Borda voting.

To quantify this effect, we will compare the vote/seat⁵ bonus of ideologically extreme parties under PR voting rules with their seat share under the Borda rule (BR). Conversely, we aim to demonstrate how the BR might impede the electoral success of divisive candidates at the ideological extremes. This analysis will enable us to draw conclusions about the extent to which PR voting rules incentivize polarization.

5. Empirical Results

The following section first describes the data and methods used to empirically test Hypotheses $H_1 - H_4$, and subsequently presents the results of the statistical analyses.

5.1. Data and Methods

The dependent variable in H_1 and H_3 is a dummy variable indicating polarizing candidates. The tests of the other hypotheses use party vote (or seat) shares based on empirical preference profiles, first under PR voting rule, and second under the Borda voting rule as dependent variables. We therefore seek empirical indicators of parties' preference profiles among national electorates. We get these from a large data base covering multiple countries and points in time, by drawing on data from the Comparative Study of Electoral Systems (CSES), waves 1-5 (CSES, 2020, 2023). Restricting the data set to parliamentary elections in democracies⁶ that employ a PR voting rule leads to 89,110 respondents with valid responses on the required survey items for which we can construct individual preference rankings. These respondents are nested in 27 countries covering the time span from 1996 until 2020, resulting in data on 549 non-unique party observations at 79 national elections. The set of countries consists mainly of European countries, covering various regions like North (e.g. Denmark), West (e.g. Austria), South (e.g. Portugal), East (e.g. the Czech Republic) and the Balkan region (Albania). It also contains democracies from other parts of the world like New Zealand, South Africa and Israel.

⁵For the sake of simplicity and without loss of generality, we use votes and seats interchangeably in the following analysis, assuming that seats are allocated purely proportional to votes.

⁶We choose to only include countries with a Polity IV rating of at least nine.

We treat respondents' stated vote choice as their top preference, using the reported thermometer feeling scores to rank preferences on the remaining parties accordingly. Thermometer feeling variables ask respondents to rate parties on an 11-point like-dislike scale, providing a nuanced spectrum of voter sentiment. Page and Jones (1979) demonstrate that these ratings are reliable indicators of voters' sincere preferences over parties, commonly used in constructing party rankings (Aldrich et al., 2011; Abramson et al., 2013; Barbaro and Specht, 2022; Lachat and Laslier, 2024). We include only respondents who participated (post-election studies) or intended to participate (pre-election studies) in the election and who rated all parties. Respondents may assign the same rating to multiple parties, creating ties in their rankings. We account for ties at all ranks except the first, where the vote choice variable serves as a definitive indicator of their primary preference. This method helps mitigate biases in party preference profiles that could arise from individual turnout decisions. Our dataset for analysis comprises 549 party-level observations nested within countries and time, including data on 228 unique parties. To accommodate the clustered data structure, we estimate mixed regression models with random intercepts at the party or country level.⁷

5.1.1. Polarizing Candidates

Based on the individual preference rankings available in the CSES, we construct preference profiles for all parties at each national election by calculating the share of supporters and despisers within the national subset. Supporters are defined as respondents who rank the party in the first place, while despisers are those who rank it last. All other respondents are categorized as moderates for the respective party. Using these aggregate-level party preference profiles, we construct a dummy variable *Polarizing candidate*, indicating whether a party exhibits a polarizing preference profile. To align with empirical realities in multi-party settings, we slightly modify the theoretical definition of a polarizing candidate. We consider parties as polarizing if their share of supporters exceeds 10 percent of the electorate—representing a substantive segment—and their share of despisers surpasses that of moderates. This criterion classifies 38 parties as polarizing candidates, approximately 17 percent of the parties in our sample. Notable examples include right-leaning parties such as the nationalist Freedom Party of Austria (FPO) in 2008, 2013, and 2017, and the Spanish conservative People's Party (PP) in 1996, 2004, and 2008⁸; left-leaning examples include the Democratic Left Alliance (SLD) in Poland in 1997 and 2005, and the Communist Party of Bohemia and Moravia (KSCM) in the Czech Republic in 1996, 2002, and 2013.

5.1.2. Ideological party positions

We estimate parties' ideological positions based on perceptual data as included in the CSES survey. Concretely, each respondent is asked to place each party on a 11-point left-right scale. We consider those parties for which the survey also provides feeling thermometer

⁷All models were estimated with different specifications, including random intercepts at the country and year levels. The results proved robust against these variations. Given that there is often only one observation per party, opting for a party fixed-effects model instead of a mixed regression model with random intercepts at the party level would lead to a significant loss in efficiency.

⁸According to our definition, the PP is not classified as a polarizing candidate in the 2000 election.

scores, which usually corresponds to all relevant and viable parties at a national election. The reported party placements are further rescaled to be free from bias by differential item functioning and idiosyncratic scale usage Aldrich and McKelvey (1977).⁹ Thus, neither projection nor persuasion effects bias our party estimates. This rescaling is performed for each national election separately, resulting in party configurations with mean at zero and standard deviation of one. To get a meaningful interpretation across countries and over time, we center the ideological scale to the mean voter's position, and further standardize on the 20th and 80th percentile of the voter distribution. Based on these ideological party positions, we construct a variable *Distance from mean voter* capturing the ideological distance of the party from the mean voter at the respective election. We define all parties to the left of the 20th percentile of the voter distribution as *ideologically left parties*, and all parties located to the right of the 80th percentile of the voter distribution as *ideologically right parties*.

As a robustness test, we further rely on a classification of left and right parties based on the party family classification included in the CSES. We classify Ecology, Communist, Socialist, and Left Liberal parties as *left party families*. We classify Right Liberal, National, and Ethnic parties as *right party families*.

5.1.3. Parliamentary representation under PR and Borda

We do not use official election results as a measure of PR vote shares, but instead calculate vote shares based on the individual-level party rankings directly. This approach offers two advantages: firstly, it avoids biases introduced by institutional settings such as multiple election tiers and district magnitude; secondly, by using a consistent set of preference profiles, the comparison between Borda and PR votes accurately reflects the impact of the aggregation rule. The PR system essentially extends the plurality rule to a seat allocation mechanism, where each party list receives parliamentary mandates proportional to its share of first-place preferences. Despite criticisms of its axiomatic foundation, which we have discussed previously, the application of the PR system remains straightforward. Accordingly, we define the party-level variable PR votes as the proportion of respondents who rank the party first.

A methodological challenge of our approach lies in adapting the results of a Borda election for seat allocation in parliamentary elections. This raises the same questions begged by Monroe (1995, p.925) associated with the PR system: 'Proportional to what and representation of what?' To the best of our knowledge, there is no well-established method for applying the Borda count to closed-list elections, where the number of seats typically exceeds the number of party lists. Traditionally, the Borda count is used to select one candidate from many, although there are adaptations like the k-Borda rule for selecting multiple candidates, as described by Elkind et al. (2017). Monroe (1995) developed a similar method, termed 'Pure Fully Proportional Representation' (PFPR). However, these approaches are not directly applicable to our study for two reasons. Firstly, Monroe, along with related literature Chamberlin and Courant (1983); Elkind et al. (2017), primarily focuses on committee formation where a subset of candidates is selected such that the number of winners

⁹We use the aldmck function provided by the basicspace R package (Poole et al., 2016).

k is less than the number of party lists, J. In contrast, in parliamentary elections, the scenario typically involves k > J, where a party (list) receives multiple seats, making it a distinct context. Secondly, the PFPR system is fundamentally different from our approach as it aims for utilitarian efficiency by minimizing coercion in selection. In cases where the committee consists of only one person, PFPR aligns with the Borda voting method. However, with two or more committee members, PFPR uses the principles of the Borda rule but leads to different outcomes¹⁰. This is demonstrated in the empirical application case provided by (Monroe, 1995, p. 928f), where the second committee member is the fourthplaced in a conventional Borda ranking. Thus, the individual second-placed in the Borda count does not get selected, which differs from the committee comprising two alternatives. Comparing a PFPR system with the PR system would therefore be inappropriate. Instead, in our study, we proportionally map the Borda scores of parties to seats. The party with the highest Borda score receives the most seats, followed by the next highest scores respectively. The relative differences in seat allocation (i.e., the number of additional seats the top party receives compared to the second-placed) are derived from the ratio of their respective Borda scores.

In particular, we utilize individual party rankings over the J parties considered in the respective national election. The top-ranked party receives J - 1 points, the second-ranked party J - 2 points, and so forth, down to the least ranked party, which receives zero points. Denote these numbers for a party $k \in 1, ..., J$ by r(k). Then, $B(k) = \int_{i=0}^{1} r_i(k) \cdot d, \gamma(i)$ represents party k's Borda score. We transform the Borda scores for party $j \in 1, ..., k, ..., J$ into a *Borda-based proportional representation*¹¹ (B-PR) by considering the shares of the Borda scores relative to the total number of scores, normalized to unity. For instance, party k is assigned a seat share (s_k) in parliament by dividing the party's Borda score by the sum of the scores of all parties, (i.e., $s_k = B(k) / \sum_j B(j)$). To illustrate the divergent outcomes arising from employing B-PR compared to (plurality-based) PR systems, consider our illustrative election (t_1) in Table 1. The Condorcet-winner party d would not be represented under PR, while the Condorcet loser party occupies 34% of the total seats. In contrast, under B-PR, party d secures 30.5% of the seats, emerging as the dominant faction in parliament. Party a, with only 20% of the seats, constitutes the smallest parliamentary group.

One final caveat must be mentioned. Since we infer the Borda vote directly from party preferences, we exclude the possibility of casting strategic votes under Borda. However, the primary focus of this paper is not to explore voting behavior under B-PR but to use the Borda vote as a benchmark to evaluate potential biases in PR voting. Therefore, we argue

¹⁰Exactly the same pattern arises by applying the Borda-related procedures proposed by Chamberlin and Courant (1983).

¹¹It is worth noting that the B-PR, as employed in this paper, has a weak axiomatic foundation. We maintain that the Borda results from candidate selection should be proportionally translated into a seat distribution. Without additional axioms, B-PR incentivizes strategic entry, potentially resulting in a party obtaining seats even if largely rejected by voters. To apply B-PR in practice, further elaborations, such as introducing a minimum entry requirement (analogous to the frequently applied minimum-vote threshold in PR systems), would be necessary. However, our approach complies with the modified IIA (MIIA) requirement imposed by Maskin (2024), because every Borda-based ordering prevails in B-PR.

that the assumption of sincere voting under B-PR is justified for our purposes (see also Lachat and Laslier (2024)).

Finally, we generate the variable *Diff PR-Borda*, which represents the difference between the two vote shares. In the illustrative example, *Diff PR-Borda* for party d on profile t_1 is -0.305. Positive values indicate that a party secures a larger vote share under PR than under Borda, given the preference profile at the current election. Conversely, negative values suggest a larger vote share under Borda.

5.2. Test of Hypotheses

We begin by presenting empirical evidence supporting Hypothesis H_1 , which posits that polarizing candidates are more frequently found at the ideological extremes than at the center. To test this claim, we regress the variable *Polarizing Candidate* on the distance from the mean voter, with the results displayed in Table 3, Model 1. Additionally, we explore potential asymmetries in this effect by introducing dummy variables for ideologically right and left parties (Model 2), as well as right and left party families (Model 3), as proposed in Hypothesis H_2 . The model estimates provide evidence that the likelihood of being a polarizing candidate increases with greater distance from the mean voter (Model 1). Moreover, this effect is more pronounced for right-leaning parties than for left-leaning ones (Model 2 and Model 3). While the difference in effect size between right and left parties is modest based on our operationalization that relies on perceived ideological positions, it becomes more distinct when using party family classifications. Parties associated with leftist party families do not show a greater likelihood of polarizing the electorate compared to ideologically moderate party families. Conversely, the probability is significantly higher for parties affiliated with rightist party families.

Because of Hypothesis H_2 , we also expect that radical right parties provoke more rejection than both moderate and radical left parties. We test this by regressing the share of despisers on the same set of variables as previously discussed. The results are presented in Table 3, Models 4 to 6. The data confirms our hypothesis: the share of despisers generally increases with the distance from the mean voter's position, and notably, the share of despisers is higher for ideologically right parties than left parties (Model 5), as well as for right party families compared to left party families (Model 6). Furthermore, the difference between left and right parties is more pronounced when classifications are based on party family rather than on perceived ideological placement relative to the voter distribution.

Next, we evaluate Hypotheses H_3 and H_4 by regressing the vote difference between the PR and Borda systems on a set of explanatory variables, with the results presented in Table 4. H_3 suggests that polarizing candidates gain larger seat shares under PR than under Borda voting. We test this by regressing the vote difference on a dummy variable indicating polarizing parties. The results from the first model show a statistically significant positive effect of 0.06 on the vote difference between PR and Borda voting, supporting H_3 . This finding indicates that polarizing candidates achieve approximately a 6 percentage point vote premium under PR compared to their vote share in B-PR.

The second model includes a covariate for the ideological left-right position, which also shows a statistically significant positive effect, thus supporting H_4 . Models 3 and 4 introduce

	Table 5.	ricourto or mi	litilevel regres	sion models				
	Pola	rizing candi	date	Share of despisers				
	0	neralized lin nixed-effect		linear mixed-effects				
	(1)	(2)	(3)	(4)	(5)	(6)		
Dist. mean voter	1.44^{***} (0.30)			0.16^{***} (0.01)				
Ideol. right		2.35^{***} (0.50)			0.17^{***} (0.02)			
Ideol. left		1.90^{***} (0.51)			0.13^{***} (0.02)			
Right family		()	1.48^{***} (0.41)		()	0.13^{***} (0.02)		
Left family			(0.11) 0.55 (0.46)			(0.02) 0.05^{***} (0.02)		
Constant	-4.35^{***} (0.50)	-4.02^{***} (0.45)	(0.10) -3.19^{***} (0.32)	0.17^{***} (0.02)	0.25^{***} (0.02)	(0.02) 0.27^{***} (0.02)		
Ν	` 539´	539	535	539	539	535		
Log Likelihood	-122.87	-120.98	-129.79	251.32	215.37	211.58		
AIC	251.74	249.97	267.57	-494.64	-420.74	-413.16		
BIC	264.61	267.12	284.70	-477.48	-399.29	-391.75		

Table 3: Results of multilevel regression models

*p < .1; **p < .05; ***p < .01

alternative operationalizations for the ideological position, estimating effects for dummy variables indicating far-left and right parties, as well as classifications based on left and right party families. These results show that, when controlling for the effect of polarizing parties, the effect reported in Model 2 is rather due to left parties doing disproportionally poor as compared to rightist parties, than rightist parties enjoying a vote premium under PR over Borda. Still, the vote premium for polarizing candidates is robust at around 6 percentage points in all models.

		diff_plu	r_borda	
	(1)	(2)	(3)	(4)
Polarizing cand.	0.06***	0.06***	0.06***	0.06***
	(0.01)	(0.01)	(0.01)	(0.01)
Ideological position		0.01^{**}		
		(0.005)		
Ideol. right			0.001	
			(0.01)	
Ideol. left			-0.04^{***}	
			(0.01)	
Right family				-0.01
				(0.02)
Left family				-0.02
				(0.01)
Constant	-0.01	-0.005	0.003	0.003
	(0.01)	(0.01)	(0.01)	(0.01)
Ν	549	539	539	535
Log Likelihood	592.82	577.47	577.93	569.21
AIC	$-1,\!177.64$	-1,144.94	-1,143.86	-1,126.42
BIC	-1,160.41	-1,123.50	-1,118.13	-1,100.73
$^{*}p < .1; ^{**}p < .05;$	$^{***}p < .01$			

Table 4: Results of multilevel regression models of seat difference PR vs. Borda

These findings support the notion that PR voting systems incentivize parties to adopt more extreme stances, even if it involves pursuing a divisive strategy, because the degree of rejection does not impact the electoral outcome.

To elucidate the effects of ideology on vote shares under both voting systems, we estimate additional models using the PR vote share and the Borda vote share as dependent variables, respectively. The results are displayed in Table 5. Initially, we regress the vote shares under both rules on the ideological left-right position of the party and a dummy variable indicating polarizing candidates. Models 1 and 3 confirm the expected pattern: both variables positively influence the vote share under PR, with polarizing candidates gaining an additional 5 percent of the votes compared to non-polarizing candidates, after controlling for their ideological position. While the effect of ideological position is not statistically significant at conventional levels, it trends positively, suggesting that right-leaning parties gain slightly more votes than left-leaning parties.

Conversely, under Borda voting, these effects are not evident. Polarizing candidates do not outperform non-polarizing parties, and the ideological position of the party does not significantly influence electoral outcomes.

Table 5: Results of multilev	Table 5: Results of multilevel regression models of PR and Borda votes							
	PR v	votes	Borda	votes				
	(1)	(2)	(3)	(4)				
Ideological position	0.01		-0.002					
	(0.01)		(0.003)					
Polarizing candidate	0.05**		-0.01					
	(0.02)		(0.01)					
Dist. from mean voter		-0.02		-0.02^{*}				
		(0.03)		(0.01)				
(Dist. from mean voter) ²		-0.02^{*}		-0.01^{**}				
		(0.01)		(0.005)				
Right side		-0.02		-0.03^{***}				
		(0.02)		(0.01)				
Dist. from mean voter * right side		0.04^{**}		0.02**				
-		(0.02)		(0.01)				
Constant	0.14^{***}	0.18***	0.14^{***}	0.18***				
	(0.01)	(0.02)	(0.004)	(0.01)				
Ν	539	539	539	539				
Log Likelihood	378.04	379.68	779.35	801.68				
AIC	-746.07	-745.35	-1,548.69	-1,589.36				
BIC	-724.62	-715.33	-1,527.25	-1,559.33				

p < .1; p < .05; p < .01

The purpose behind Models 2 and 4 is to more intricately depict the relationship between ideology and electoral performance. To achieve this, we estimate an interaction effect between the distance from the mean voter and an indicator variable for the ideological side, and we include a squared term to enhance model fit. Fig. 1 visually represents the results. This graph highlights a more pronounced relationship between the ideological position and the vote share under PR voting compared to Borda voting, indicating that PR voting is more sensitive to the ideological stance of the party.

The asymmetry in the effect is also evident: the vote share decreases more steeply for parties situated to the left of the mean voter compared to those on the right. Specifically, a party located two scale points to the right of the mean voter is predicted to gain an additional 12 percentage points under PR, whereas it would only secure 5 percent of the votes if positioned an equal distance from the mean voter but on the left side. This represents a seven-percentage-point disadvantage for the left party. Under Borda voting, the discrep-

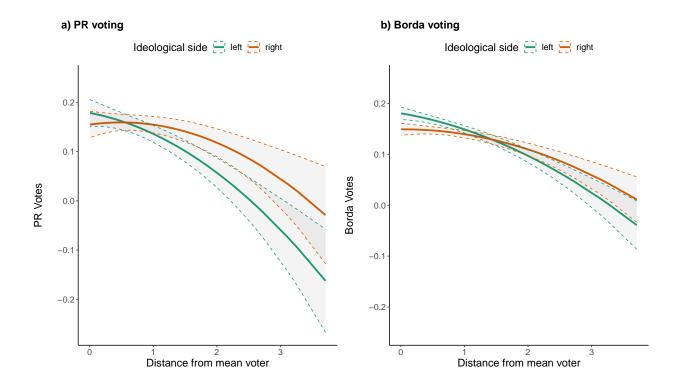


Figure 1: Marginal effect plot of distance from mean voter on PR and Borda vote share by ideological side

ancy in vote shares between a left and right party located two scale points from the mean voter is merely one percentage point (10 percent for the left vs. 11 percent for the right party). It is important to note that these models are derived from the exact same preference profiles.

6. Conclusion

The rise of political polarization in Western democracies has spurred increased scholarly attention towards understanding its underlying causes. This study shifts focus towards an additional, yet previously marginalized, explanatory factor: the amalgamation of voter preferences facilitated by the electoral procedure. We present both formal and empirical evidence elucidating how plurality-based voting systems, such as the widely-used PR system, create incentives for candidates to adopt divisive strategies. Specifically, we identify the Borda count as a voting method with greater efficacy in mitigating the electoral success of polarizing candidates, employing it as a benchmark to assess the inherent bias in PR voting.

Our comparison between the Borda rule (BR) and the PR system is not arbitrary; it is grounded in a coherent theoretical framework.

We demonstrate the empirical relevance of our formal findings by utilizing data from the Comparative Study of Electoral Systems, which spans a broad array of countries and time periods. Our empirical results reveal a robust association between the propensity for political polarization and ideological orientation. Consequently, we show that polarizing candidates achieve enhanced electoral performance under PR compared to the Borda-based seat allocation. Notably, this effect manifests asymmetrically: specifically, radical right candidates and parties perform disproportionately better under PR voting than their equally extreme counterparts from the ideological left, an advantage that is absent when employing the seat allocation methodology dictated by the B-PR.

One conclusion we draw from these results is that PR systems are not well-equipped to mitigate escalating polarization, especially when instigated by political actors aligned with the right end of the ideological spectrum. Although the Borda count is not perfectly immune to divisive strategies, our empirical analysis demonstrates its relative lack of significant ideological bias compared to PR voting. Consequently, the Borda-based systems offers an alternative to PR that encourages consensus-seeking behavior rather than exacerbating affective polarization. This shift has the potential to strengthen social cohesion and prevent divisive political tendencies.

A politically relevant conclusion from our analysis is that the support for right-wing populist and radical right parties is systematically overestimated when their share of parliamentary seats is used as a measure. Conversely, a high seat share may suggest that these right-wing parties have substantial backing in the population, which could attract new voters to them. Therefore, the danger posed by autocratic, misogynistic, and xenophobic forces can be exacerbated by our electoral system granting them seats that are not justified by actual support in the population.

Appendix A. On the comparative immunity of Borda's rule against the electoral victory of a polarizing candidate

We apply the standard methods and notations used in social choice theory. We assume a continuum of voters, indexed by points on the unit interval [0, 1], although they are sorted within three groups (see below). The Borel-set continuum ensures generic decisiveness.¹² We make use of the framework by Campante and Hojman (2013) to model a polarized electorate, where voters have quite different opinions over (at least) one candidate. This approach was first launched by Gehrlein (2005). In particular, a fraction γ_1 of the voters (supporters) ranks A first. A second group (fraction γ_2), the despisers, ranks A least. Hence, we assume a polarizing alternative, A, to be judged quite differently within the citizenry. Specifically, a polarizing candidate (PC) is either ranked first-place by voter group γ_1 and ranked least by γ_2 or ranked top-place by voter group γ_2 and ranked least by γ_1 .

A third, moderate, group rank the candidates randomly. The latter group is sized $\gamma_3 = 1 - \gamma_1 - \gamma_2$. The domain of polarized profiles is denoted by \mathscr{R} . Note that assessing three groups, as it is usually done in research, does not limit the generality of our evaluation. Rather, the framework with three groups and three alternatives is the worst case for a PC under plurality rule. The PC party can win though short of majority because of vote-splitting behavior. The more parties are running, the more likely is that vote splitting distorts majority will.

Imposing three groups is senseless if they can have equal preference orderings. Thus, we assume that all groups are distinct (i.e., have distinct preference orderings from each other). The model allows two candidates to comply with the definition of a polarizing candidate. Specifically, a candidate B becomes a PC if (and only if) s/he is bottom-ranked by the supporters and top-ranked by the group of despisers.

We assume that both the group of supporters and the group of despisers are large. This presumption guarantees that the profiles we evaluate accurately reflect a polarizing electorate.

Assumption 1. $1/2 > \gamma_1 > \gamma_3; \quad 1/2 > \gamma_2 > \gamma_3.$

The larger γ_1 and γ_2 , the higher the polarization. Assumption 1 admits a wide range of configurations of γ -values. We denote this admissible set by Ω .

We will focus on the comparison between Borda's rule and plurality rule. More generally, between the family of rank-order voting with the class of plurality-based electoral methods (like PR systems).

Formal representations of the considered voting schemes are frequently provided in the literature (Riker, 1982; Dasgupta and Maskin, 2008), so that we abstain from introducing the rules in detail. Our main interest is to explore the domain on which the Borda rule is more suitable to prevent the electoral victory of a polarizing candidate. Formally,

¹²According to (Dasgupta and Maskin, 2008, Section 3), generic decisiveness requires that a voting scheme be decisive for regular profiles, where the preference proportions do not fall into some finite exceptional set. This holds if voters are aligned on a Borel set interval.

Property 1 (Property PPC). Property **PPC** holds if, for a given profile, a voting rule does not bring forth a polarizing candidate as the electoral winner.

Given the structure of a polarized electorate, eight different patterns of profiles can result. These are summarized in Table A.6

			14	ble A		014112	cu pi	onnes.	•			
		(1)			2			3			4	
Rank	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3
1	A	В	В	А	В	А	A	В	В	A	В	А
2	В	С	А	В	С	\mathbf{C}	C	\mathbf{C}	А	C	\mathbf{C}	В
3	C	А	С	С	А	В	B	А	С	В	А	С
		5			6			\bigcirc			8	
Rank	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3
1	A	В	\mathbf{C}	Α	В	С	A	В	С	A	В	С
-												
2	В	С	А	В	\mathbf{C}	В	C	С	А	C	\mathbf{C}	В

Table A.6: Polarized profiles.

We call (1) to (8) patterns of the respective profiles because eight further profiles where B and C are substituted with each other are isomorphic to those profiles encompassed in Tab. A.6. Profiles (3), (4), (7), (8) comprise two polarizing candidate. We evaluated profiles (2) and (7) (as (1) and (11)) in depth above in the theoretical background section.

For the sake of notational simplicity, we denote by $a P^p b$ that a defeats b under the plurality rule. Applying the Borda rule, we use the notion P^b for the respective binary relation. In line with standard notation in social choice theory, we denote a choice set by $\mathcal{C}(R, r, x)$. It reads that x wins the election by applying the electoral rule r on profile R. For instance, $\mathcal{C}(\odot, p, A)$ denotes that candidate A is the winner under plurality rule on domain \odot . Similarly, $\mathcal{C}(\odot, b, \neg C)$ denotes that candidate C cannot emerge as the winner under Borda's rule on domain \odot .

Applying the plurality rule and the Borda rule, respectively, to each profile yields the following results:

- (1) C((1), (b, p), B): the (unique) polarizing candidate cannot succeed under plurality rule or Borda rule on the entire domain of admitted profiles Ω .
- (2) $\mathcal{C}(2, (p, b), \neg C)$ and $\mathcal{C}(2, p, A)$. By applying the Borda rule, there is a domain on which *B* emerges as the Borda winner. We have indicated in the theoreticalbackground section that this domain is the set of all γ -values $\in \Omega$, for which $y_1 > 2 - 4 \cdot \gamma_2$ holds. This can easily proved as follows:

$$BP^{b}A \Leftrightarrow -\gamma_{1} - 2\gamma_{2} + 3 < 2\gamma_{2} + 1 \Leftrightarrow \gamma_{1} > 2 - 4\gamma_{2}.$$
(A.1)

- (3) There are two polarizing candidates, A and B. C cannot win in any case. Thus, on Ω , the electoral result is the victory of a polarizing candidate. This can be proved as follows: we show that the domain on which $BP^b C$ holds belongs to Ω : $BP^b C \Leftrightarrow \gamma_1 < \gamma_2 + 2 \cdot \gamma_3 \implies \gamma_1 \in \Omega$.
- (4) Same as in Profile (3).
- (5) Candidate A (the PC) becomes plurality winner if and only if $\gamma_1 > \gamma_2$. We have to show that $\gamma_1 > \gamma_2$ is a necessary, but not a sufficient condition for A to emerge as Borda winner. We proceed in two steps:

First, we show that $\mathcal{C}(5, b, \neg C)$. Secondly, we show that the domain on which A wins implies $\gamma_1 > \gamma_2$, but not vice versa.

- (a) $C((5), b, \neg C)$. We prove that C is never an element in the Borda choice set by showing that C is defeated by B on the entire domain Ω : $BP^bC \Leftrightarrow 2 \cdot \gamma_1 + \gamma_2 + 3 \cdot \gamma_3 < 3 \cdot \gamma_1 + 2 \cdot \gamma_2 + \gamma_3 \Leftrightarrow \gamma_3 < \frac{\gamma_1 + \gamma_2}{2}$. By the definition of a polarized profile, this condition consistently holds since otherwise, γ_3 must exceed either γ_1 or γ_2 , whereby violating Assumption 1.
- (b) For the PC to become the Borda winner, A has to defeat B, too. This is possible for the following domain: $AP^bB \Leftrightarrow \gamma_1 + 3 \cdot \gamma_2 + 2 \cdot \gamma_3 < 2 \cdot \gamma_1 + \gamma_2 + 3 \cdot \gamma_3 \Leftrightarrow \gamma_2 < 1/3$. The last expression denotes the sufficient condition for A's victory. Note that whenever this condition holds, we must have $\gamma_1 > \gamma_2$, because if $\gamma_2 < 1/3 < \gamma_3$, then the group γ_1 must garner more than one-third of the electorate among its members.

Hence, whenever A becomes a Borda winner, s/he will also emerge as plurality winner, but not vice versa.

- (6) First, C cannot beat B, whatever rule is used for election: $BP^bC \Leftrightarrow \gamma_3 < \gamma_1 + \gamma_2$. Further: $AP^pB \Leftrightarrow \gamma_1 > \gamma_2$, but $\forall \gamma_i \in \Omega : BP^bA$. It is well-known that a Condorcet loser cannot become a Borda winner Saari (2000a,b). Because of A emerging as exactly that, a Condorcet loser, we proved the general victory of B over A under Borda's rule. Formally: $BP^bA \Leftrightarrow 2 \cdot \gamma_1 + \gamma_2 + 2 \cdot \gamma_3 < \gamma_1 + 3 \cdot (\gamma_2 + \gamma_3) \Leftrightarrow \gamma_1 < \frac{2}{3} - \frac{\gamma_3}{3}$. The opposite case, thus, is only feasible for $\gamma_3 > 0.5$.
- (7) Both, A and B are polarizing candidates. Because of the numerical inferiority of the moderates, the plurality rule brings forth one of the polarizing candidates for the entire domain of Ω . Under rank-order voting, however, the result is totally different: $\forall \gamma_i \in \Omega : CP^bA \& CP^bB$. To prove this, consider that $CP^bA \Leftrightarrow 2(\gamma_1 + \gamma_2) + \gamma_3 < \gamma_1 + 3 \cdot \gamma_2 + 2 \cdot \gamma_3 \Leftrightarrow \gamma_1 < \gamma_2 + \gamma_3 \implies \gamma_1 < \frac{1}{2}$, which holds $\forall \gamma_1 \in \Omega$. Further, $CP^bB \Leftrightarrow 2(\gamma_1 + \gamma_2) + \gamma_3 < 3 \cdot (\gamma_1 + \gamma_3) + \gamma_2 \Leftrightarrow \gamma_2 < \gamma_1 + 2 \cdot \gamma_3$. The right-hand side of the latter expression must exceed 1/2, indicating that every $\gamma_2 \in \Omega$ fulfills the condition.

(8) Similar to (7), with A and B interchanged.

Taking together the results so far in this Appendix, we can state a robustness result regarding the Borda rule.

Theorem 1 (Robustness results in polarized profiles)

Let $\mathscr{R} \in \mathscr{R}$ denote the domain on which plurality rule prevents a victory of a PC. Then, the Borda rule prevents the PC victory on the same domain, too. Conversely, there exists another domain $\ddot{\mathscr{R}} \in \{\mathscr{R} \setminus \dot{\mathscr{R}}\}$ on which the Borda rule prevents the PC-success, and plurality rule does not.

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