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Electoral Methods and Political Polarization

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Highlights

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- Voting schemes exert significant influence on candidates' incentives, potentially fostering either moderation or polarization.
- The commonly-used plurality rule tends to encourage the proliferation of wedge issues.
- Scientifically-proposed voting methods such as the simple-majority rule and the Borda count serve to deter polarization.
- In a broad framework, the anti-plurality rule emerges as a formidable deterrent against polarizing candidates or parties. However, its efficacy is somewhat diminished when embedded within sequential scoring methods.
- Little support is provided for the optimistic view on Approval Voting's capacity to hinder polarisation.

Electoral Methods and Political Polarization

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Abstract

Research from various disciplines has addressed the relationships between electoral systems and political polarization. The results are inconclusive. This paper systematically examines how different electoral systems either promote political polarization or render it unattractive for candidates to distinguish themselves through polarization. We assume a polarized electorate and investigate Condorcet-consistent voting procedures as well as scoring rules, both single and two-staged.

Keywords: Elections, Voting Schemes, Political Polarization, Scoring Rules
JEL: D71

1. Introduction

The escalation of political polarization endangers democratic principles and peaceful coexistence in modern societies (Sen, 2020a).

In this paper, we investigate how voting systems rewards polarising strategies. This research is motivated by the fact that the often-used plurality rule (PR) has a substantial incentive to polarize. Other voting rules, such as the simple-majority rule (Condorcet method) and the Borda count (rank-order voting), take into account more accurately the views of those who shun combative politicians. However, both voting systems accomplish this in distinct ways.

2. The model

We utilize the standard methods and notations employed in social choice theory. We assume a continuum of voters, indexed by points on the unit interval $[0, 1]$, although they are sorted within three groups (see below). The continuum is defined over the Borel set such that the preference proportions do not fall into some finite exceptional set (because the Lebesgue measure is not defined for all subsets of the continuum).

For our assessment, we adopt most of the framework developed by Dasgupta and Maskin (2008, 2020). In particular,

1. we consider a triple of alternatives on a ballot $\mathscr{B}$ from which to choose,

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2. voters' preferences are anti-symmetric (strict), the voters are perfectly informed about candidates' policy choice.

3. we compare the three voting systems used in the aforementioned paper: the simple-majority rule (Condorcet method), plurality rule, Borda count. In Section 4, we will generalize the results by considering further electoral schemes.

The set of all logically possible voters' preference orderings over the finite set of alternatives, $\mathscr{B}$, is denoted by $\mathfrak{R}$. We neglect *menu dependency* (Sen, 1993, 1994, 1995), such that voters' top (best) preference is always identical with their *votes* under plurality rule (PR). As in Dasgupta and Maskin (2008) and most of the other related papers, we neglect strategic considerations of voters.

2.1. Polarized Electorates

Our focus lies on the success of polarizing candidates (PC) in elections. The candidates are labeled in alphabetical order A, B, C. One among possibly two polarizing contestants is candidate A.

We adopt an already established model to evaluate electoral outcomes in a polarized citizenry. In particular, we use the framework of Campante and Hojman (2013) to model a polarized electorate as a restricted domain on $\mathfrak{R}$, where voters have quite different opinions over (at least) one candidate. A similar approach was first launched by Gehrlein (2005). In particular, a fraction γ_1 of the voters (the 'supporters') ranks A first. A second group (fraction γ_2), the 'despisers', ranks A least. Hence, we assume a polarizing alternative, A, to be judged quite differently within the electorate (see Definition 1 below). A third, 'moderate', group rank the candidates randomly. The latter group is sized $\gamma_3 = 1 - \gamma_1 - \gamma_2$. Note that the assessing three groups, as it is usually done in research, does not limit the generality of our evaluation.

Imposing three groups is senseless if they can have equal preference orderings. Thus, we impose our first assumption as follows:

Assumption 1. All groups are distinct (i.e., have distinct preference orderings from each other).

The restricted domain of polarized profiles is denoted by $\mathscr{R} \subset \mathfrak{R}$. Table 1 illustrates the domain of polarized profiles.

We allow for a second polarizing alternative, B, if s/he is ranked bottom by the supporters and top-ranked by the despisers.

We assume that both the group of supporters and the group of despisers are large. This presumption guarantees that the profiles we evaluate accurately reflect a polarizing electorate. In particular, we will assume that together, supporters and despisers constitute an absolute majority.

Assumption 2. ${}^1\!/_2 > \gamma_1 > \gamma_3; \quad {}^1\!/_2 > \gamma_2 > \gamma_3.$

Table 1: The restricted domain of polarized preferences, $\mathscr{R} \subset \mathfrak{R}$.

Rank	Supporters γ_1	Despisers γ_2	Moderates γ_3
1	A	$\cdot$	$\cdot$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	$\cdot$	A	$\cdot$

The set of γ_i-values that are admissible with respect to Assumption 2 will be denoted $\mathfrak{S}$. It is helpful to introduce and define the following four notions for the ongoing.

Definition 1 (Polarizing candidates). A polarizing candidate (PC) is either

1. ranked best by voter group γ_1 and ranked least by γ_2 or
2. ranked best by voter group γ_2 and ranked least by γ_1

Definition 2 (Consensus candidates). A consensus candidate (CC) is a candidate never ranked bottom.

This definition corresponds to the 'centrist candidate' by Miller (2013). Gehrlein (2005) used the term 'unifying candidate' to describe a similar pattern.

Definition 3 (Daunou-Condorcet winner). A candidate is a Daunou-Condorcet winner (DCW) if and only if s/he is top-ranked by at least the absolute majority of voters.

Analogously, we define the Daunou-Condorcet loser by

Definition 4 (Daunou-Condorcet loser). A candidate is a Daunou-Condorcet loser (DCL) if and only if s/he is ranked least by an absolute majority of voters.

2.2. Voting schemes

A voting rule is a collective choice rule that maps preference profiles into an electoral outcome. Definitions of the various electoral rules can be found in Moulin (1991, Ch. 9) and other standard references in voting theory.

Our main interest is which voting scheme is most suitable to prevent the electoral victory of a polarizing candidate. For this purpose, we state the following property.

Property 1 (Property PPC). Property **PPC** holds if, for a given profile, a voting rule brings not forth a polarizing candidate as the electoral winner.

A polarizing candidate can be the Condorcet winner. Therefore, it is evident that **PPC** conflicts with the claim that the Condorcet winner should be selected to the extent that such a winner exists (Condorcet consistency).

3. Polarized Electorates

Given the structure of polarized profiles and bearing in mind that all groups are distinct from each other (Assumption 1), for any triple sixteen possible profiles can appear. Eight of them are isomorphic to the other eight profiles since just both B and C are substituted by each other. Thus, we can focus on eight profiles, depicted in Table 2.

Table 2: Polarised profiles $\mathscr{R}$.

	①			②			③			④		
Rank	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3
1	A	B	B	A	B	A	A	B	B	A	B	A
2	B	C	A	B	C	C	C	C	A	C	C	B
3	C	A	C	C	A	B	B	A	C	B	A	C
	⑤			⑥			⑦			⑧		
Rank	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3	γ_1	γ_2	γ_3
1	A	B	C	A	B	C	A	B	C	A	B	C
2	B	C	A	B	C	B	C	C	A	C	C	B
3	C	A	B	C	A	A	B	A	B	B	A	A

Remark 1 (Characterization of the profiles). The following characterization of the profiles are useful for the ongoing analysis.

1. Profiles ① to ④ fulfill limited favoritism (LF). Thus, applying PR on profiles ① to ④ comply with the Arrovian IIA requirement such that the plurality winner coincides with the Condorcet winner. We will refer to profiles ① to ④ as $\mathscr{R}_{LF}$ or simply LF-profiles.

2. No profile $R \in \mathscr{R}$ fulfills quasi-agreement (QA). We prove this in Appendix A. Hence, the Borda winner can potentially defeat the Condorcet winner and even the Daunou-Condorcet winner.

3. There can be one or even two polarizing candidates. If there are two PCs in an LF-profile, there cannot exist a consensus candidate (CC) in the same profile. Otherwise, the profile would fail to comply with Assumption 1. To show this, consider that limited favoritism requires that C cannot be top-ranked by the moderates. If the latter group ranks C medium, then their orderings are similar to that of one of the other voter groups.

4. Profile ⑤ represents a cyclical pattern (Condorcet paradox).

5. A Daunou-Condorcet loser exists in all profiles ⑥ to ⑧. A Condorcet loser cannot be a Borda winner (Fishburn and Gehrlein, 1976; Moulin, 1991, Ch. 9).

6. A consensus candidate (CC) exits in profiles ⑥ to ⑧. A CC cannot exist in LF-profiles.

It is useful to state the following Lemma regarding consensus candidates in polarized profiles.

Lemma 1 (Consensus candidate in polarized profiles)
If an alternative is a CC, then s/he is a Condorcet winner. A CC cannot be defeated under Borda's rule.

Proof of Lemma (1). By the definition of polarized profiles, the bottom-ranked candidates of the supporters and the despisers must differ. Thus, only candidate C can become a consensus candidate. By the nature of profiles $R \notin \mathscr{R}_{LF}$, C must be top-ranked by the moderates . Therefore, a CC beats each candidate twice in a head-to-head match up. This part of the proof is an application of Lepelley et al. (2000, Proof of Prop. 1) to the domain of polarized profiles.

Any voter who rank another candidate above the CC, does rank her or him single-spaced (the CC is never ranked bottom, so the distance to any CC-defeating candidate is one rank at maximum). On the other hand, a CC beats any of her or his rivals twice (see the previous paragraph of this proof). Thus, the Borda count requires that a Condorcet winner must be defeated with a larger rank distance. Since this is impossible, the CC can not be defeated under the Borda rule.

4. Electoral rules and electoral success of polarizing figures

4.1. Condorcet-Consistent Rules

A Condorcet-consistent voting rule is defined by its ability to elect a Condorcet winner whenever such a candidate exists (Felsenthal and Tideman, 2014; Moulin, 1991, Def. 9.2). This category encompasses a range of voting schemes all of which are unified by their adherence to the Condorcet principle (for an extensive review, refer to Felsenthal and Tideman (2014); Moulin (1991, Ch. 9.2)). The simple-majority rule, often cited as the most straightforward example of Condorcet-consistency, aptly represents this class by ensuring the selection of the Condorcet winner when one is identifiable.

It can easily be seen that A is the Condorcet winner in ② and ④. In ①, ③, and ⑥ B emerges as the Condorcet winner. In the remaining profiles (except ⑤, which exhibits a cyclical pattern), C will be elected under a Condorcet-consistent voting rule. A PC is only successful if s/he is a DCW.

However, this congruence in outcome assurance under the umbrella of Condorcet-consistent voting rules does not extend to scoring methods. Specifically, the plurality rule and the Borda count, despite both being categorized as scoring rules, diverge significantly in the results they produce.

4.2. Single-Stage Scoring Rules

Single-stage scoring rules (SSSR, henceforth we simply use the abbreviated term scoring rules) constitute a class of electoral systems where points are allocated to candidates based on the order of preference expressed by voters, with the winner being the candidate accumulating the highest total score. In a contest involving k candidates and assuming anti-symmetric preferences, a scoring vector is represented as a k-tuple $w = (s_1, s_2, \ldots, s_{k-1}, s_k)$, comprising real numbers that satisfy the conditions $s_0 \leq s_1 \leq \ldots \leq s_{k-1}$, and $s_0 < s_{k-1}$. For elections featuring three candidates, the diversity of scoring rules can be encapsulated by a normalized vector $(1, \lambda, 0)$, illustrating the spectrum of methodologies within scoring rules.

For instance, the plurality rule corresponds to the scenario where $\lambda = 0$. Conversely, assigning $\lambda = 1/2$ mirrors Borda's count (Lepelley et al., 2000). The anti-plurality rule (Baharad and Nitzan, 2005; Bossert and Suzumura, 2016) aligns with $\lambda = 1$. Furthermore, the Dowdall System, utilized in Nauru, operates on a descending fractional allocation, awarding 1 point to the first choice, $1/2$ to the second, $1/3$ to the third, and so on, effectively showcasing a λ-value of $1/4$ for three-candidate elections after an affine transformation (Diss et al., 2023).

As the parameter λ increases within the scoring vector framework, the allocation of points shifts to favor medium-ranked candidates more prominently. This phenomenon underscores a critical feature of scoring rules: higher λ values inherently penalize candidates who find themselves positioned at the lower end of voter preferences. Consequently, this structural aspect of scoring systems serves as a strategic disincentive for polarizing candidates, who are likely to be ranked unfavorably by a broader segment of the electorate. This dynamic illustrates the nuanced manner in which scoring rules, through the choice of λ, can influence electoral outcomes and potentially moderate or foster political polarization.

In the following, we investigate the λ-values necessary to create a tie between a PC and a non-PC. We denote by $x\, I^{\lambda}\, y$ the scenario where candidates x and y receive the same score, given λ. The λ-value required to achieve this tie is termed the bliss point, denoted as $\tilde{\lambda}^{\textcircled{p}}_{x,y}$. It is defined as follows: Setting this λ-value in profile $p \in \left\{\textcircled{1}, \ldots, \textcircled{8}\right\}$ results in an equal score for candidates x and y. Candidate x (the left-mentioned in the binary relation) prevails over y for given (γ_1, γ_2) if the electoral rule assigns a λ-value below $\tilde{\lambda}^{\textcircled{p}}_{x,y}$. For example, in Table 3 we find that $\tilde{\lambda}^{\textcircled{1}}_{A,B} < 0$. It suggests that candidate A cannot surpass candidate B under a scoring rule. Theoretically, A's victory would necessitate a negative λ-value, indicating that a voter's second-ranked candidate is penalized compared to their last choice. Similarly, $\tilde{\lambda}^{\textcircled{p}}_{x,y} > 1$ implies that x consistently prevails over y across all feasible λ-values, as y would require a λ-value beyond the viable range to secure victory against x.

A well-established method for visualizing electoral outcomes under scoring rules is through Saari triangles (Saari, 1995, 1999, 2011), which we will employ for illustrative purposes below (see Fig. 2, and Appendix B). In this depiction, each candidate is represented by a vertex of an equilateral triangle. Points on the red vertical lines signify a tie between A and B, thus a bliss point $\tilde{\lambda}^{p}_{A,B}$ lies on the vertical line. Similarly, a bliss point between A and C resides on the green line, while the bliss point between B and C is positioned on the red line. The electoral outcomes are delineated by the dashed orange-colored 'procedure lines' (Saari,

Table 3: Bliss Points

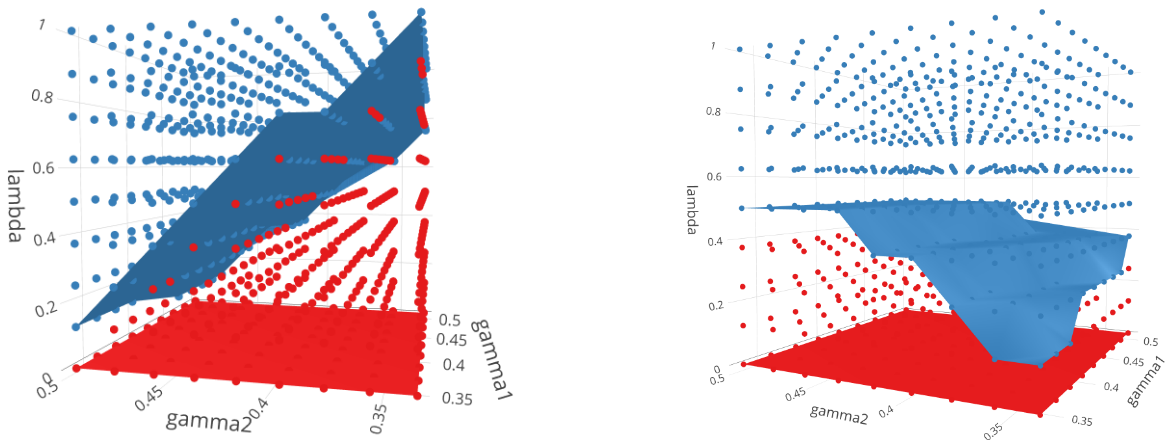
$\mathscr{R}$	$A\, I^\lambda\, B$	$A\, I^\lambda\, C$	$B\, I^\lambda\, C$	PPC
①	$\tilde{\lambda}^{①}_{A,B} = \frac{2\cdot\gamma_1-1}{\gamma_1-\gamma_3} < 0$	$\tilde{\lambda}^{①}_{A,C} = \frac{1-\gamma_2-\gamma_3}{\gamma_2-\gamma_3} > 1$	$\tilde{\lambda}^{①}_{B,C} = \frac{\gamma_1-1}{\gamma_1-\gamma_2} \notin (0,1)$	$\forall\lambda \in (0,1)$
②	$\tilde{\lambda}^{②}_{A,B} = \frac{1-2\cdot\gamma_2}{\gamma_1} \in (0,1)$	$\tilde{\lambda}^{②}_{A,C} = \frac{\gamma_2-1}{\gamma_1-1} > 0$	$\tilde{\lambda}^{②}_{B,C} = -\frac{\gamma_2}{2\cdot\gamma_1-1} > 1$	$\forall\lambda > \tilde{\lambda}^{②}_{A,B}$
③	$\tilde{\lambda}^{③}_{B,A} = \frac{1-2\cdot\gamma_1}{\gamma_3} > 0$	$\tilde{\lambda}^{③}_{A,C} = \frac{\gamma_1}{\gamma_1+\gamma_2-\gamma_3} \in (0,1)$	$\tilde{\lambda}^{③}_{B,C} = \frac{\gamma_2+\gamma_3}{\gamma_2+\gamma_1} \in (0,1)$	$\forall\lambda > \max\left[\tilde{\lambda}^{③}_{A,C}, \tilde{\lambda}^{③}_{B,C}\right]$
④	$\tilde{\lambda}^{④}_{A,B} = \frac{\gamma_1-\gamma_2}{\gamma_3} + 1 > 0$	$\tilde{\lambda}^{④}_{A,C} = \frac{\gamma_1+\gamma_3}{\gamma_1+\gamma_2} \in (0,1)$	$\tilde{\lambda}^{④}_{B,C} = \frac{\gamma_2}{\gamma_2+\gamma_1-\gamma_3} \in (0,1)$	$\forall\lambda > \max\left[\tilde{\lambda}^{④}_{A,C}, \tilde{\lambda}^{④}_{B,C}\right]$
⑤	$\tilde{\lambda}^{⑤}_{A,B} = \frac{\gamma_1-\gamma_2}{\gamma_1-\gamma_3} < 1$	$\tilde{\lambda}^{⑤}_{A,C} = \frac{\gamma_1-\gamma_3}{\gamma_2-\gamma_3} > 0$	$\tilde{\lambda}^{⑤}_{B,C} = \left| \frac{\gamma_2-\gamma_3}{\gamma_2-\gamma_1} \right| > 1$	$\forall\lambda > \tilde{\lambda}^{⑤}_{A,B}$
⑥	$\tilde{\lambda}^{⑥}_{A,B} = \frac{\gamma_1-\gamma_2}{\gamma_1+\gamma_3} < \frac{1}{3}$	$\tilde{\lambda}^{⑥}_{A,C} = \frac{\gamma_1-\gamma_3}{\gamma_2} \in (0,1)$	$\tilde{\lambda}^{⑥}_{B,C} = \left| \frac{\gamma_2-\gamma_3}{\gamma_2-\gamma_1} \right| > 1$	$\forall\lambda > \tilde{\lambda}^{⑥}_{A,B}$
⑦	$\tilde{\lambda}^{⑦}_{B,A} = \frac{\gamma_2-\gamma_1}{\gamma_3} < 1$	$\tilde{\lambda}^{⑦}_{A,C} = \frac{\gamma_1-\gamma_3}{\gamma_1-\gamma_3+\gamma_2} < \frac{1}{2}$	$\tilde{\lambda}^{⑦}_{B,C} = \frac{\gamma_2-\gamma_3}{\gamma_1+\gamma_2} < \frac{1}{2}$	$\forall\lambda > \max\left[\tilde{\lambda}^{⑦}_{A,C}, \tilde{\lambda}^{⑦}_{B,C}\right]$
⑧	$\tilde{\lambda}^{⑧}_{A,B} = \frac{\gamma_1-\gamma_2}{\gamma_3} < 1$	$\tilde{\lambda}^{⑧}_{A,C} = \frac{\gamma_1-\gamma_3}{\gamma_1+\gamma_2} < \frac{1}{2}$	$\tilde{\lambda}^{⑧}_{B,C} = \frac{\gamma_2-\gamma_3}{\gamma_2-\gamma_3+\gamma_1} < \frac{1}{2}$	$\forall\lambda > \max\left[\tilde{\lambda}^{⑧}_{A,C}, \tilde{\lambda}^{⑧}_{B,C}\right]$

1995, Ch. 4.2.3), which indicate the electoral results for fixed (γ_1, γ_2)-values but varying λ-values. Note that the position and slope of the respective procedure line depend on the distribution of γ_i-values. In Appendix B, we outline a Saari triangle for each of the eight profile pattern, utilizing identical γ_1-values in each case.

Table 3 presents all bliss points. For completeness, we also include those that result in a tie between two polarizing candidates. These values are crucial for evaluating which of the polarizing candidates (if two are in contention) poses the greatest challenge for a non-PC. The final column specifies the range of λ-values required to satisfy the **PPC**.

Regarding profile ①, $\tilde{\lambda}^{①}_{A,B} < 0$ indicates that the unique PC is consistently defeated by B. Consequently, the property PPC holds for all λ-values within the $(0,1)-$space. Regarding profile ②, the PC can be surpassed by B, or C, or both. However, whenever C prevails over A, B also triumphs over A, but not vice versa. This is indicated by $\tilde{\lambda}^{②}_{B,C} > 1$. Hence, the pertinent bliss point is $\tilde{\lambda}^{②}_{A,B}$. By setting $\lambda = 1/2$, we precisely obtain the outcomes discussed earlier: higher γ_1-values can hinder the electoral success of the PC. Figure 1a depicts by the blue points the combinations of γ_i and λ-values that yield a defeat of the PC. Along to $\lambda = 1/2$, the effect that higher γ_1-values can hinder the electoral success of the PC is depicted. The blue plane illustrates the shape of the bliss points.

In profile ③, $\tilde{\lambda}^{③}_{A,B}$ characterizes the competition among the two PC. Regarding the possibility to avoid the victory of one of them, the other two bliss points are of interest. The only not polarizing candidate, C, can win, if s/he gets more scores than A and B, respectively. Hence $\lambda > \max[\tilde{\lambda}^{③}_{A,C}, \tilde{\lambda}^{③}_{B,C}]$. Figure 2 (left panel) illustrates the electoral outcomes in a Saari triangle. As an numerical example, we set $\gamma_1 = 0.48, \gamma_2 = 0.32, \gamma_3 =$



(a) $BP^{\lambda}A$ vs. $AP^{\lambda}B$ in Profile ②.

(b) $BP^{\lambda}C$ vs. $CP^{\lambda}B$ in Profile ⑧.

Figure 1: **PPC** in Profiles ② and ⑧

0.20. These numbers yield bliss points $\tilde{\lambda}^{\textcircled{3}}_{A,B} = 0.2$, $\tilde{\lambda}^{\textcircled{3}}_{A,C} = 0.8$, and $\tilde{\lambda}^{\textcircled{3}}_{B,C} = 0.6875$. Candidate B emerges victorious under plurality rule ($\lambda = 0$). The other PC, A, get more scores than B for $\lambda > 0.2$, such that A becomes the Borda winner, as indicated by the orange-colored circle at $\lambda = 1/2$. The Borda rule ranks B also above C. With increasing λ, C eventually gets more scores than B ($\lambda > 0.6875$). For C to emerge as electoral winner, the λ-value has to exceed not only $\tilde{\lambda}^{\textcircled{3}}_{B,C}$, but also $\tilde{\lambda}^{\textcircled{3}}_{A,C}$. Hence, **PPC** is fulfilled for $\lambda > 0.8$, indicating that the anti-plurality rule prevents the victory of a PC within this domain and with this particular voter allocation. A similar pattern recurs in the case of profile ④.

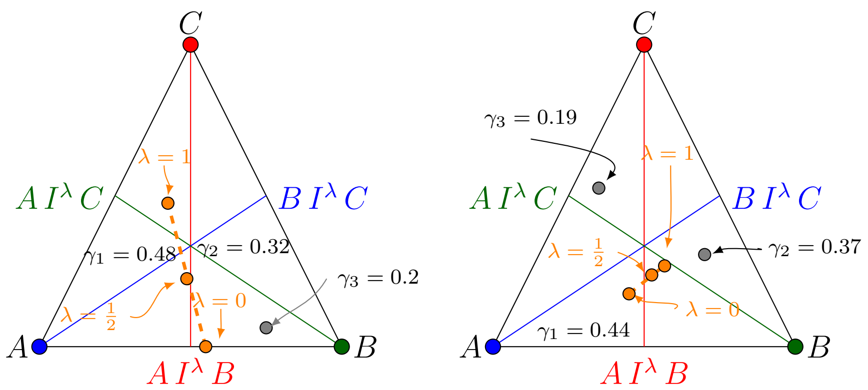


Figure 2: Illustration of electoral outcomes in profiles ③ (left panel) and ⑤ in Saari-triangles.

In profile ⑤, characterized by a cyclical pattern, the sole PC, A, cannot emerge victorious if $\gamma_1 < \gamma_2$. However, with increasing λ-values, A gets defeated even when $\gamma_1 > \gamma_2$. The right panel in Figure 2 illustrates this scenario.[1]

As demonstrated in the previous section, the sole PC in ⑥, A, is consistently defeated by B under the Borda rule. The bliss point $\tilde{\lambda}^{⑥}_{A,B}$ indicates that A can obtain a higher score than B for $\lambda < 1/3$ (encompassing Downdall, and PR). Since B prevails over C across the entire admissible domain, the competition between A and B is pivotal for determining whether **PPC** holds or not.

The findings for profiles ⑦ and ⑧ align with the analysis of election results conducted using the Borda voting method in Section 3. The consensus candidate, C, consistently emerges victorious when $\lambda > \frac{1}{2}$. However, for λ values below the bisection point, a polarizing candidate can secure victory. This suggests, for instance, that according to the Dowdall method, a polarizing candidate might succeed while failing under the Borda method.

Figure 1b illustrates for profile ⑧, using blue points, all combinations of γ_i and λ values indicating when the CC prevails. Conversely, the red points indicate when one of the two PCs succeeds.

The following theorem summarizes our main insight from assessing single-stage scoring rules.

Theorem 1 (PPC under single-staged scoring rules)
*Consider a polarized profile characterized by a particular distribution of $\gamma_i \in \mathfrak{S}$. If a scoring rule with $\lambda = \dot{\lambda} \in [0, 1]$ applied on this profile satisfies **PPC**, when every scoring rule with $\lambda > \dot{\lambda}$ satisfies **PPC**, too.*

In words, our analysis in the present subsection has demonstrated that scoring rules can either promote or hinder political polarization to varying degrees. A higher λ-value corresponds to greater difficulty in fostering polarization, whereas a lower λ-value increases the likelihood of a polarizing candidate prevailing. Thus, we have represented the two extreme cases with the plurality rule and the anti-plurality rule. Both rules share the characteristic of considering only a minimal portion of voter preferences. While the anti-plurality rule may seem preferable for mitigating political polarization, it does not appear universally advisable. At the very least, one may doubt the feasibility of a electoral system garnering public acceptance if it prevents voters from expressing their preferences for the candidates or parties they genuinely favor.

4.3. Sequential Scoring Rules

Against this backdrop, sequential scoring rules offer a potential solution. Sequential scoring rules involve the application of a scoring rule initially to generate a social ranking, with the least preferred option being eliminated. In cases involving more than three alternatives, this process is iterated until only two alternatives remain for selection. A majority vote is then conducted to choose between these remaining options (Lepelley and Valognes, 2003). This approach allows for greater consideration of information derived from voters' preference rankings.

[1]Notice that C cannot defeat B, because this would require $\lambda \cdot \gamma_2 + \gamma_3 > \lambda \cdot \gamma_1 + \gamma_2 \Leftrightarrow (\lambda - 1)\gamma_2 - \lambda\gamma_1 > \gamma_3$. The λ-values that allow C to get a higher score than B lie above 1 or must be negative.

In a three-candidate scenario, we can once again apply the $(1, \lambda, 0)$ sequence, resulting in the exclusion of the candidate with the lowest number of votes for $\lambda = 0$ (*plurality elimination rule*). For $\lambda = 1$, we implement the anti-plurality rule in the first stage (*anti-plurality elimination rule*), thereby eliminating the candidate ranked last by the fewest voters. $\lambda = 1/2$ describes the Baldwin rule (Borda-elimination rule) (Diss et al., 2023; Kamwa, 2022, see).

In the subsequent analyses, we must consider a distinction and formulate restrictive assumptions. Firstly, we differentiate between immediate and temporally sequential stages of the electoral process. Immediate procedures involve directly connecting the second round of voting to the first, utilizing the preference rankings conveyed by voters in the initial stage. In contrast, temporally sequential procedures feature a time gap of two or more weeks between rounds. During this period, voters may reconsider whether and how they will vote in the second round. This possibility clearly allows room for strategic considerations. Since our model neither accounts for decisions to abstain from voting nor strategic deliberations, we assume that all voters participate in all rounds and vote according to their original preference rankings.

4.3.1. Plurality-Elimination Rules

Examples of plurality-elimination rules include the Hare system (aka instant runoff aka Single-Transferable Vote), the plurality-runoff method, and supplementary voting. There are two reasons why we focus on plurality-elimination rules: firstly, they are widely employed, and secondly, empirical research suggests that their application reinforces moderate forces and weakens political polarization.

The Hare system is prevalent in countries with Anglo-Saxon legal and political traditions. The plurality-runoff (henceforth: p-r) is notably utilized in the French presidential elections and the election of US senators in Georgia. Supplementary voting stands as the designated electoral scheme for the selection of directly-elected mayors within England, Wales, and Norway (van der Kolk, 2008; Christensen and Aars, 2010). These schemes may differ, particularly with more than three candidates. In the three candidate environment, all have in common that two best vote-getters face each other in the runoff, where the head-to-head majority determines the winner. Note that the elimination of one candidate results in all remaining candidates moving up in the preference rankings of the voters, given that they were ranked below the removed candidate.

An interesting application of the p-r system is observed in the Italian municipal elections. Specifically, cities with more than 15,000 inhabitants employ the p-r, while smaller communities revert to the (single-staged) PR. Bordignon et al. (2016) empirically demonstrate a diminished influence of extremist voters in larger cities, attributing this effect to the runoff system's structure. Similar features were attributed to the Hare system (Grofman and Feld, 2004; Tomlinson et al., 2024).

In our framework examining polarized electoral scenarios, plurality-elimination systems often lead to the exclusion of consensus candidates from the final round of voting. This tendency is particularly notable for candidate C, who, despite not being a polarizing figure, is consistently eliminated in the initial stage. Consequently, this implies a recurring victory for polarizing candidates. There are two exceptions, found in profiles ① and ⑥. Profile ⑥ stands out as the sole case where plurality elimination rules perform better (with regard to **PPC**) than the plurality rule. However, the opposite holds true for Profile ⑤, where plurality-elimination rules remove C from contention, leaving the only candidate capable of defeating the polarizing candidate in a head-to-head comparison. This outcome persists even in domains where the plurality rule adheres to **PPC** criteria (viz., $\gamma_1 < \gamma_2$). Overall, plurality-elimination rules fail to rectify the recurring deficiencies inherent in the PR.

Although this result was generated within a restricted model framework, it nevertheless points to a fundamental weakness of plurality-elimination rules. In a polarized electorate, consensus-oriented and moderate candidates face difficulty in garnering a sufficiently high number of votes. They may still be Condorcet winners, but this characteristic only becomes pivotal in the final stage (a Condorcet winner obviously prevails in every two-candidate runoff). The primary obstacle for consensus-oriented candidates lies in the first stage. Thus, plurality-elimination rules are highly advantageous for polarizing candidates, as they often eliminate those candidates whom they cannot defeat in a runoff. Detailed results for each profile are reported below in Subsection 4.5.

4.3.2. Anti-plurality elimination rules

While plurality-elimination rules are commonly encountered in practice, the sequential consideration of the anti-plurality rule is more firmly rooted in theory than in practice. In theoretical contexts, it is known as the Coombs method. According to Chamberlin et al. (1984, p. 489) it involves eliminating the candidate accruing the highest number of last-place votes in the initial round (see also Lepelley and Valognes, 2003, Sec. 4.2).[2] Subsequent iterations of this elimination process lead to a head-to-head contest, ultimately determining the winner by a simple majority. For instance, consider Profile ①: Candidate C, most-often ranked at the bottom, is removed from contention. The ensuing head-to-head comparison results in a victory for candidate B.

The determination of the eliminated candidate in Profiles ② through ⑤ hinges on whether γ_1 surpasses γ_2 or not. For instance, in Profile ②, the PC is the deleted candidate if the faction supporting her or him is outnumbered by her or his detractors ($\gamma_1 < \gamma_2$). Analogous to the scenario discussed earlier under Borda's rule, more fervent supporters can diminish the electoral prospects of the PC; in this case, C is ousted, leading to A prevailing over B. Notably, Coombs rule fails to satisfy the PPC condition in Profiles ③ and ④. Moreover, both Coombs rule and the PR thwart the PC's victory in Profile ⑤ when $\gamma_2 > \gamma_1$; otherwise, candidate A themselves faces elimination. Finally, Coombs method adheres to **PPC** in Profiles ① and ⑥ through ⑧.

An interesting observation is that the Coombs Rule is by no means better suited than the Borda Rule for mitigating polarization. On the contrary, the application of the Coombs

[2]The Coombs Rule shares a close connection with the concept of *exhaustive voting* by Duncan Black (1958[1988], p. 86ff) . One notable distinction between these electoral methodologies lies in the temporal dynamics: Coombs operates instantaneously, whereas Black's model explicitly allows for temporally sequential stages.

Rule in profiles ③ and ④ results in the victory of a PC who would have had no chance of winning under the Borda Rule. This outcome is even more pronounced when compared to the anti-plurality rule. While the latter consistently satisfies the PPC condition, this is not the case under the Coombs Rule. Table 4 in Subsection 4.5 provides a synoptic comparison, illustrating the results described here.

4.4. Approval Voting

Hardly any electoral procedure has garnered as much attention in recent decades as Approval Voting (AV). It entails each voter being able to either approve (some) individual candidates. The candidate with the most approvals is elected. Assuming voters exhibit dichotomous preferences, AV possesses a variety of desirable properties (Brams and Fishburn, 1978, 2005). Particularly in comparison to the PR, AV demonstrates superiority in many respects. For instance, Myerson and Weber (1993, Theorems 3 & 4) demonstrated that AV is less susceptible to polarization. Gehrlein et al. (2016) showed that the electoral success of a Condorcet loser is less frequent under AV than under PR.

Results from field studies suggest that voters handle AV effectively, seemingly without being overwhelmed by the task of categorizing candidates into two camps. Pioneering work was conducted by (Laslier and Van der Straeten, 2008) during the first round of the 2002 French presidential election. In an in-situ field experiment, voters were briefed on the functioning of AV one week before the election and subsequently asked to approve (or not to approve) of candidates. Indeed, the far-right candidate fared significantly worse under AV than under the plurality rule. This finding, along with the broader observation of remarkable disparities between AV and plurality vote outcomes, was reaffirmed in a subsequent in-situ experiments Baujard et al. (2014); Alós-Ferrer and Granić (2012).

In light of these investigations, the analysis within the theoretical framework discussed here does not reveal a clear moderating effect of AV. Conceptually, the challenge lies in determining voters' cut-off levels. Initially assuming that all voters approve the same number of candidates, AV can either significantly favor or significantly hinder polarization. The election results under AV are identical to those under plurality voting when all voters approve only one candidate. In this case, AV would frequently produce polarizing candidates as winners. However, if voters always approve two candidates, the result corresponds entirely to that of the election under anti-plurality rule, leading to a consistent defeat of polarizing candidates.

Even when assuming that voters have different cut-off levels, as discussed by Gehrlein et al. (2016, Sec. 2.2), no uniform picture emerges. AV can even more strongly promote the electoral success of a polarizing candidate than the PR. This effect occurs when the two major groups (supporters and despisers) only approve one candidate, while the moderates approve two. For example, in Profile ①, if $\gamma_1 > \gamma_2$. In this case, candidate B wins under PR, but the polarizing candidate A wins under AV. To see this, consider that A is approved by $\gamma_1 + \gamma_3$, whereas B by $\gamma_2 + \gamma_3$. However, it must be noted here with reservations that the moderates harm themselves by approving two candidates, as their voting behavior leads to the electoral victory of candidate A, although candidate B is more preferred by them. This situation underscores the notion that the assumption of honest voting may not necessarily be compelling.

4.5. Section summary

We conclude this section with a synoptic summary of the results. Table 4 indicates when the condition PPC is met. An upward arrow (↑) signifies that **PPC** is satisfied for the entire permissible range of γ_i values, thereby indicating that a polarizing candidate cannot succeed. If it is not possible to prevent the electoral success of a polarizing candidate, this is indicated by a downward arrow (↓). If the fulfillment of **PPC** hinges on the specific distribution of group sizes, this is expressed by a rightward arrow (→).

Table 4: Synoptical table

Profile	Condorcet Consistent Rules	Single-Stage Scoring Rules				Sequential Rules	
		PR ($\lambda = 0$)	Dowdall ($\lambda = 1/4$)	Borda ($\lambda = 1/2$)	Anti-PR ($\lambda = 1$)	Plurality-elim. rule	Anti-Plur. elim. rule
①	↑	↑	↑	↑	↑	↑	↑
②	↓	↓	→	→	↑	↓	→
③	↓	↓	→	→	↑	↓	↓
④	↓	↓	→	→	↑	↓	↓
⑤	→	→	→	→	↑	↓	→
⑥	↑	→	→	↑	↑	↑	↑
⑦	↑	↓	→	↑	↑	↓	↑
⑧	↑	↓	→	↑	↑	↓	↑

5. Conclusion

We find that polarizing candidates benefit considerably from the plurality rule. In contrast, the simple-majority rule offers minimal incentive for polarization, while the Borda count strongly discourages polarizing campaigns and promotes centripetal outcomes.

The superiority of two electoral procedures originating from the French Revolution era over the plurality rule is grounded in their incorporation of voters' comprehensive preference orderings. Within a framework involving three candidates, those positioned at the bottom are at a relative disadvantage. This occurs as candidates in intermediate positions receive more attention during the tallying process. Aside from the three well-known electoral systems, we demonstrate that the anti-plurality rule, which significantly emphasizes the middle position, is particularly effective in mitigating strategies designed to factionalize the electorate.

While this study primarily focuses on political polarization, it is essential to recognize the axiomatic properties that electoral procedures either meet or fail to meet. As a scoring rule, the anti-plurality rule adheres to properties formulated by Young (1975) and further highlighted in subsequent literature (Baharad and Nitzan, 2005; Bossert and Suzumura, 2016; Kurihara, 2018). Despite subjective perceptions, the electorate is likely to reject a voting system that does not accommodate preferences for preferred candidates or parties. Notably, Sen (2020b) recently emphasized the significant role of first preferences among voters. Given this context, sequential voting procedures offer a potential solution by applying the anti-plurality rule only in the initial stage, followed by a decisive runoff between candidates. This approach is exemplified by the Coombs Rule. Nevertheless, our findings indicate that it is less effective than the Borda Rule in curtailing the success of polarizing candidates.

On the other hand, the distinctiveness of the Borda rule among scoring rules has been recognized since the seminal work by Young (1975, Theorem 3), which highlighted its appealing characteristics.[3] Taking this analysis further, Maskin (2024) demonstrates that the Borda rule is the unique aggregation method fulfilling three out of four classical Arrovian conditions, May's axioms, and a weakened version of Young's consistency criterion. Although no scoring rule, including the Borda rule, can satisfy Arrow's **I**-condition, the Borda rule adheres to a relaxed version of **I**, dubbed Modified IIA (MIIA). This adaptation permits the consideration of preference intensities inferred from ordinal data.

The consideration of preference intensities,[4] which often attracts scrutiny from proponents of Condorcet-consistent methods towards the Borda rule, actually supports its superior resilience against polarization compared to Condorcet methods. When contrasted with the plurality rule, the Condorcet method exhibits a significantly higher immunity to polarization and fulfills key axiomatic criteria (Horan et al., 2019).

Within the constraints of our model, we acknowledge that it does not capture two empirical phenomena. First, the moderating effect observed with the plurality run-off, and second, a similar effect associated with Approval Voting. Approval Voting has received substantial support in scholarly research. However, in our three-candidate model where voters operate with identical cutoff levels, AV behaves similarly to either the PR or the anti-plurality rule. Relaxing the assumption of uniform cutoff levels reveals that AV could, in certain instances, intensify support for polarizing candidates more than the frequently criticized plurality rule. Nevertheless, this aspect should not be exaggerated, as it largely depends on the specific assumption mentioned earlier.

[3] A pivotal feature of this rule is its *consistency*—termed *reinforcement* by Moulin (1991).

[4] Arrow's IIA is violated if, in the collective choice involving two alternatives, *anything* other than the individual orderings over the tuple is considered, including the *preference intensities* (see Sen, 2017, Ch. 7).

Appendix A. Proof of Remark 1, Second item

Proof (Proof of Remark 1, second item.). We have to show that no candidate is top-ranked, or medium-ranked, or worse-ranked in all distinct voter groups for any profile $R \in \mathscr{R}$.

1. Top-rank: Candidate A is top-ranked by the supporters but bottom-ranked by the despisers. Thus, no candidate is top-ranked by all.
2. Bottom-rank: The same as for the top-rank applies to the bottom-rank. Therefore, no candidate is bottom-ranked by all groups.
3. Medium-rank: If LF holds, a candidate, say C, is never top-ranked. If C is always medium-ranked, the groups are not distinct (cf. Assumption 1). The moderate group then has the same preference ordering as either the supporters or the despisers. If LF does not hold, all candidates are ranked top once, and no one can be ranked medium or worse by all voters.

Appendix B. Numerical Examples for Scoring-Rule Outcomes

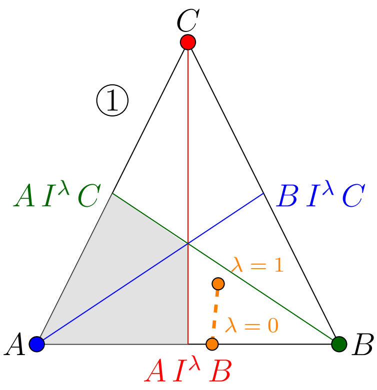
In this appendix section, we provide detailed illustrations of Saari triangles corresponding to profiles ① through ⑧, utilizing the parameters $\gamma_1 = 0.42$ and $\gamma_2 = 0.35$ consistently across all instances. These parameters are chosen to reflect the distribution of support that positions the PC A as the plurality winner in profiles ② and ④ through ⑧. The gray shaded areas specifically denote the areas where polarizing candidate(s) secure a victory.

The λ-values essential to satisfy the PPC condition exhibit significant variation across different electoral profiles. Notably, in certain cases, such as profiles ③ and ④, the required λ-values are so elevated that, in practical terms, only the application of the anti-plurality rule is effective in precluding the electoral triumph of a PC. This underscores the unique capability of this rule to counteract the polarization dynamics inherent in specific voting scenarios.

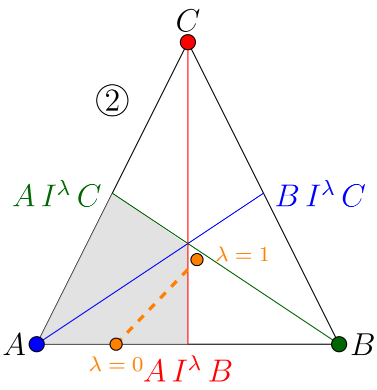
Conversely, for other profiles, such as ⑦, the Borda method demonstrates adequacy in meeting the **PPC**. However, it's noteworthy that the Dowdall method does not achieve the same level of effectiveness in this context, indicating a nuanced distinction between scoring rules and their applicability in curbing the success of polarizing figures.

Moreover, the PR predominantly facilitates the electoral victory of a PC, with the exception of profile ①. On the other side of the spectrum, the application of the anti-plurality rule consistently lead to a defeat of the PC(s), indicated by the procedure lines achieving the unshaded area in each profile at $\lambda = 1$.

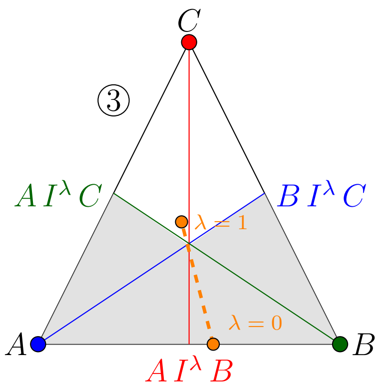
Figure B.3: Saari-triangles and respective procedure lines for $\gamma_1 = 0.42, \gamma_2 = 0.35, \gamma_3 = 0.23$



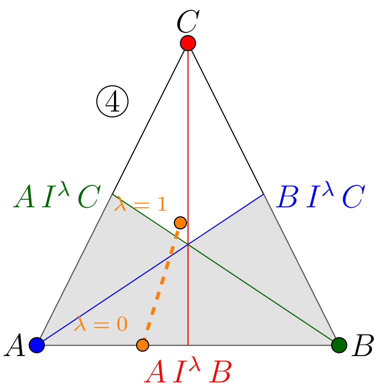
①: **PPC** if $\lambda > 0$.



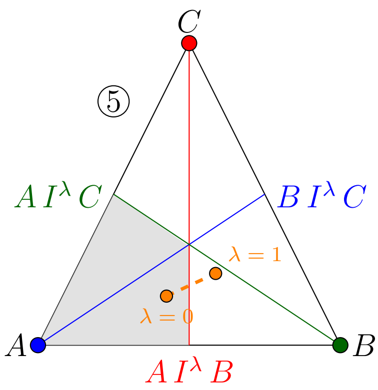
②: **PPC** if $\lambda > 0.741$.



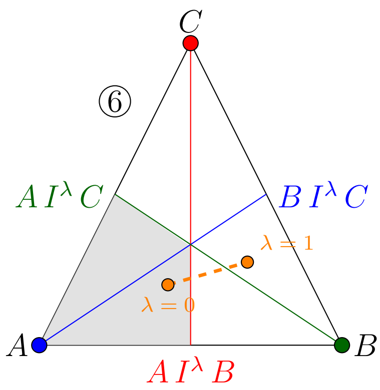
③: **PPC** if $\lambda > 0.\overline{77}$.



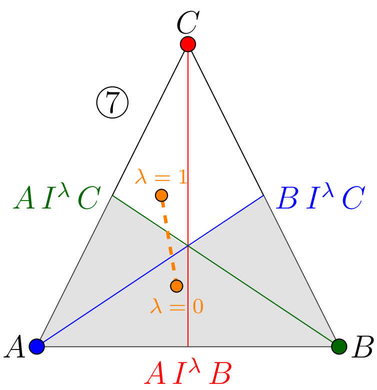
④: **PPC** if $\lambda > 0.844$.



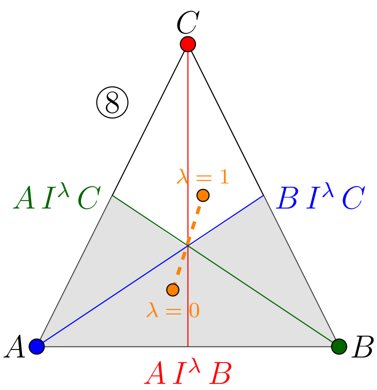
⑤: **PPC** if $\lambda > 0.368$.



⑥: **PPC** if $\lambda > 0.108$.



⑦: **PPC** if $\lambda > 0.352$.



⑧: **PPC** if $\lambda > 0.247$.

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