Advanced Macroeconomics

Chapter 5: Government: Expenditures and public finances

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Overview

1. The government budget constraint
   - The nominal government budget constraint
   - The real government budget constraint
2. Financing government expenditures: Tax finance
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The nominal government budget constraint

- The nominal government budget constraint is given by:

$$P_t g_t + P_t h_t + (1 + R_t) B_t = B_{t+1} + \Delta M_{t+1} + P_t T_t. \qquad (1)$$

- Notation:
  - P_t: Price level.
  - g_t: Real government expenditure.
  - h_t: Real transfers to households.
  - B_t: Government "revenue" from issuing bonds in period $t-1$.
  - R_t: Interest rate on government bonds issued in period $t-1$.
  - M_t^B: Monetary base supplied at the start of period t ($\Delta M_{t+1} = M_{t+1} - M_t$).
  - T_t^B: Total real taxes.

The real government budget constraint

- Dividing the nominal government budget constraint by the price level P_t yields:

$$\begin{aligned} g_t + h_t + \frac{(1+R_t)\,B_t}{P_t} &= \frac{B_{t+1}}{P_t} + \frac{\Delta M_{t+1}}{P_t} + T_t \Longleftrightarrow \qquad (2) \\ g_t + h_t + (1+R_t)\,b_t &= \frac{P_{t+1}}{P_t}\frac{B_{t+1}}{P_{t+1}} + \frac{P_{t+1}}{P_t}\frac{M_{t+1}}{P_{t+1}} - \frac{M_t}{P_t} + T_t \Longleftrightarrow \\ g_t + h_t + (1+R_t)\,b_t &= \frac{(1+\pi_{t+1})\,P_t}{P_t}\left(b_{t+1} + m_{t+1}\right) - m_t + T_t \Longleftrightarrow \\ g_t + h_t + (1+R_t)\,b_t &= (1+\pi_{t+1})\left(b_{t+1} + m_{t+1}\right) - m_t + T_t. \end{aligned}$$

Financing government expenditures: Tax finance

- Assumption: Consider a **permanent** increase in government spending in period t by Δg_t which is financed by an increase in lump-sum taxes, T_t.
- Since the increase is permanent we have:

$$\Delta g_{t+s} = \Delta g_t = \Delta g, \ \forall s \geq 0. \tag{3}$$

- Moreover, we have:

$$\Delta T_{t+s} = \Delta T_t = \Delta T = \Delta g, \ \forall s \geq 0. \tag{4}$$

- Since the additional government expenditures are financed by tax increases no additional deficit is generated, i.e. $\Delta b_{t+s} = 0, \ \forall s \geq 0$.

Financing government expenditures: Tax finance

- The permanent tax increase will reduce the households' available income, $x - T$, from period t on.
- The households' permanent income without the tax increase (denoted by W_t^o) is given by:

$$W_t^o = (1+R)\, a_t + \sum_{s=0}^{\infty} \left(\frac{1}{1+R} \right)^s \left[x_{t+s} - T_{t+s} \right]. \tag{5}$$

where we assumed that $\pi = 0$ and therefore $r = R$.

- The households' permanent income with the tax increase (denoted by W_t^n) is given by:

$$W_t^n = (1+R)\, a_t + \sum_{s=0}^{\infty} \left(\frac{1}{1+R} \right)^s \left[x_{t+s} - T_{t+s} - \Delta T \right] \tag{6}$$

Financing government expenditures: Tax finance

- The households' permanent income with the tax increase can be rewritten as follows:

$$\begin{aligned} W_t^n &= (1+R)\,a_t + \sum_{s=0}^{\infty}\left(\frac{1}{1+R}\right)^s [x_{t+s} - T_{t+s} - \Delta T] = \\ &= (1+R)\,a_t + \sum_{s=0}^{\infty}\left(\frac{1}{1+R}\right)^s [x_{t+s} - T_{t+s}] - \sum_{s=0}^{\infty}\left(\frac{1}{1+R}\right)^s \Delta T = \\ &= (1+R)\,a_t + \sum_{s=0}^{\infty}\left(\frac{1}{1+R}\right)^s [x_{t+s} - T_{t+s}] - \frac{1}{1-\frac{1}{1+R}}\Delta T = \\ &= (1+R)\,a_t + \sum_{s=0}^{\infty}\left(\frac{1}{1+R}\right)^s [x_{t+s} - T_{t+s}] - \frac{1+R}{R}\Delta T. \end{aligned} \quad (7$$

- The change in households' permanent income, $\Delta W_t = W_t^o - W_t^n$, is therefore given by:

$$\Delta W_t = W_t^n - W_t^o = -\frac{1+R}{R}\Delta T. \quad (8)$$

Financing government expenditures: Tax finance

- Given the reduction in households' permanent income their consumption will also fall.
- Denoting by c_t^o the households' consumption without tax increases, by c_t^n the households' consumption with tax increases and by Δc_t the change in consumption then we have (given the assumptions of chapter 4.2):

$$\begin{aligned}\Delta c_t &= c_t^n - c_t^o = \left(\frac{R}{1+R}\right) W_t^n - \left(\frac{R}{1+R}\right) W_t^o = \\ &= \left(\frac{R}{1+R}\right)(W_t^n - W_t^o) = \left(\frac{R}{1+R}\right)\left(-\left(\frac{1+R}{R}\right)\Delta T\right) \\ &= -\Delta T.\end{aligned} \tag{9}$$

$\Longrightarrow$ The tax-financed increase in permanent government spending leads to an equally-sized decrease in private consumption.

Financing government expenditures: Tax finance

- The overall effect of the permanent increase in government spending on the economy (denoted by Δy_t) is given by:

$$\Delta y_t = \Delta c_t + \Delta g_t = -\Delta T + \Delta T = 0. \tag{10}$$

$\Longrightarrow$ Permanent increase in government spending has no effect on the economy.

The sustainability of the fiscal stance

- Starting point: Nominal government budget constraint

$$P_t g_t + P_t h_t + (1 + R_t) B_t = B_{t+1} + \Delta M_{t+1} + P_t T_t. \qquad (11)$$

- Dividing both sides of this equation by nominal GDP, $P_t y_t$, yields:

$$\frac{P_t g_t}{P_t y_t} + \frac{P_t h_t}{P_t y_t} + \frac{(1 + R_t) B_t}{P_t y_t} = \frac{B_{t+1}}{P_t y_t} + \frac{M_{t+1}}{P_t y_t} - \frac{M_t}{P_t y_t} + \frac{P_t T_t}{P_t y_t}. \qquad (12)$$

- Rearranging and simplifying yields:

$$\begin{aligned} \frac{g_t}{y_t} + \frac{h_t}{y_t} + (1 + R_t) \frac{b_t}{y_t} \quad = \quad & \frac{T_t}{y_t} + \frac{B_{t+1}}{\frac{1}{(1+\pi_{t+1})} P_{t+1} \frac{1}{(1+\gamma_{t+1})} y_{t+1}} + \\ & + \frac{M_{t+1}}{\frac{1}{(1+\pi_{t+1})} P_{t+1} \frac{1}{(1+\gamma_{t+1})} y_{t+1}} - \frac{m_t}{y_t} \end{aligned}$$

The sustainability of the fiscal stance

- The just derived equation can be simplified as follows:

$$\frac{g_t}{y_t} + \frac{h_t}{y_t} + (1 + R_t)\frac{b_t}{y_t} = \frac{T_t}{y_t} + (1 + \pi_{t+1})(1 + \gamma_{t+1})\left(\frac{b_{t+1}}{y_{t+1}} + \frac{m_{t+1}}{y_{t+1}}\right) - \frac{m_t}{y_t}$$

  where γ_{t+1} denotes the growth rate of real GDP between periods t and $t+1$.

- The question we want to analyze in this section is whether a given government deficit, $P_t D_t$, is sustainable or not.

- The government deficit in a given period t is given by:

$$P_t D_t = P_t g_t + P_t h_t + R_t B_t - P_t T_t - (M_{t+1} - M_t). \quad (13)$$

The sustainability of the fiscal stance

- Dividing the just derived equation by nominal GDP, $P_t y_t$, yields:

$$\frac{D_t}{y_t} = \frac{g_t}{y_t} + \frac{h_t}{y_t} + R_t \frac{b_t}{y_t} - \frac{T_t}{y_t} - \left(1 + \pi_{t+1}\right)\left(1 + \gamma_{t+1}\right)\frac{m_{t+1}}{y_{t+1}} - \frac{m_t}{y_t}. \tag{14}$$

- From the budget constraint we see that the right-hand side of this equation is given by:

$$\frac{g_t}{y_t} + \frac{h_t}{y_t} + R_t \frac{b_t}{y_t} - \frac{T_t}{y_t} - \left(1 + \pi_{t+1}\right)\left(1 + \gamma_{t+1}\right)\frac{m_{t+1}}{y_{t+1}} - \frac{m_t}{y_t} = \tag{15}$$

$$= \left(1 + \pi_{t+1}\right)\left(1 + \gamma_{t+1}\right)\frac{b_{t+1}}{y_{t+1}} - \frac{b_t}{y_t}.$$

- That is, we have:

$$\frac{D_t}{y_t} = \left(1 + \pi_{t+1}\right)\left(1 + \gamma_{t+1}\right)\frac{b_{t+1}}{y_{t+1}} - \frac{b_t}{y_t}. \tag{16}$$

The sustainability of the fiscal stance

- Note: The difference between the actual deficit, D_t and the interest payments on accumulated debt, $R_t B_t$, is called (nominal) primary deficit, $P_t d_t$, and is given by:

$$P_t d_t = P_t D_t - R_t B_t. \tag{17}$$

- Assuming that inflation, π and output growth, γ are constant, the expression for $\frac{D_t}{y_t}$ becomes:

$$\frac{D_t}{y_t} = (1+\pi)(1+\gamma)\frac{b_{t+1}}{y_{t+1}} - \frac{b_t}{y_t} \Longleftrightarrow \tag{18}$$

$$\frac{b_{t+1}}{y_{t+1}} = \frac{1}{(1+\pi)(1+\gamma)}\frac{D_t}{y_t} + \frac{1}{(1+\pi)(1+\gamma)}\frac{b_t}{y_t} \tag{19}$$

$\Longrightarrow$Interpretation?

The stability and growth pact

- The stability and growth pact requires that

$$\frac{D_t}{y_t} \leq 0.03 \tag{20}$$

and

$$\frac{b_t}{y_t} \leq 0.60 \tag{21}$$

hold.

- Assuming that the deficit-GDP ratio remains constant over time the equation for the dynamics of the debt-GDP ratio (equation (19)) represents a stationary difference equation.
- Thus, the debt-GDP ratio will converge to a (constant) steady-state level and we will have in equilibrium:

$$\frac{b_{t+1}}{y_{t+1}} = \frac{b_t}{y_t} = \frac{b_t}{y_t} \tag{22}$$

The stability and growth pact

- Using the equation for the dynamics of the debt-GDP ratio we then obtain:

$$\begin{aligned}
\frac{b_{t+1}}{y_{t+1}} &= \frac{1}{(1+\pi)(1+\gamma)}\frac{D_t}{y_t} + \frac{1}{(1+\pi)(1+\gamma)}\frac{b_t}{y_t} \\
\frac{b}{y} &= \frac{1}{(1+\pi)(1+\gamma)}\frac{D}{y} + \frac{1}{(1+\pi)(1+\gamma)}\frac{b}{y} \\
\frac{(1+\pi)(1+\gamma)-1}{(1+\pi)(1+\gamma)}\frac{b}{y} &= \frac{1}{(1+\pi)(1+\gamma)}\frac{D}{y} \\
\frac{b}{y} &= \frac{1}{(1+\pi)(1+\gamma)-1}\frac{D}{y}.
\end{aligned}$$

- Since $(1+\pi)(1+\gamma) \approx 1+\pi+\gamma$ we obtain:

$$\frac{b}{y} = \frac{1}{\pi+\gamma}\frac{D}{y}. \tag{23}$$

The stability and growth pact

- Implications of the analysis:Assume that the current debt-GDP ratio is given by $\frac{\bar{b}}{\bar{y}}$.
- Then:
  - If the debt-output ratio is kept constant over time and we have:
    $$\frac{1}{\pi+\gamma}\frac{D}{y} < \frac{\bar{b}}{\bar{y}} \tag{24}$$
    then the debt-GDP ratio will fall.
  - If the debt-output ratio is kept constant over time and we have:
    $$\frac{1}{\pi+\gamma}\frac{D}{y} = \frac{\bar{b}}{\bar{y}} \tag{25}$$
    then the debt-GDP ratio will remain constant.
  - If the debt-output ratio is kept constant over time and we have:
    $$\frac{1}{\pi+\gamma}\frac{D}{y} > \frac{\bar{b}}{\bar{y}} \tag{26}$$
    then the debt-GDP ratio will increase.