The Informational Content of Ratings: Ordinal Regressors in Econometrics*

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Abstract. Globalization has increased the demand for information about creditworthiness of firms. Such information has been provided by rating agencies for decades already in the United States. Since ratings are one-dimensional, they have the advantage of compact information. On the other hand they are derived mainly from subjective considerations which raises the question of the informational content of ratings. To answer this question, we analyse an econometric model which has already been considered by Liu and Thakor (1984) and Kao and Wu (1990/1994). We show that the use of ratings as explanatory variables leads to special methodological problems of "ordinal regressors". In particular we employ a new estimation method first proposed by Gourieroux, Monfort and Renault (1993) which is more suitable than the method of conditional expectations first suggested by Terza (1987). Empirical results are given for a data set already used in the literature.

1 Introduction

Globalization of markets forces firms to adapt international standards of accountancy and publicity. This is especially important if the firm acts on capital markets. To provide investors with information about creditworthiness, ratings such as those published by Standard & Poor's or Moody's[1] have been used in the US already for decades. Such ratings have the advantage of providing information in a compact (mostly unidimensional) form. However, it is not clear how these ratings are derived, and, obviously, many subjective elements are involved. It is, therefore, not surprising that their informational content has been questioned by practitioners and academics as well.

We consider this question from an econometric perspective as it has been done by Liu and Thakor (1984) and, in greater detail, by Kao and Wu (1990, 1994). We will show that the use of (ordinal) ratings as regressors leads to methodological problems, as was noted by Terza (1987) who proposes an estimation method which, however, is reasonable only when there is merely *one* "ordinal regressor". Breslaw and McIntosh (1998) give a detailed exposition of an adequate estimation procedure based on simulation. We will use an alternative approach of Kukuk (1999) which is based on the "indirect

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[1] See Moody's (1999) and Standard & Poor's (1996).

inference"-approach of Gourieroux, Monfort and Renault (1993). We illustrate our method by a data set that was also used by Kao and Wu (1990).[2]

The paper has the following structure: In two subsections we review the economic and statistical aspects of ratings and describe the meaning of the "marginal effect" of ratings. Section 2 contains methodological considerations for microeconometric models with ordinal regressors. Empirical results are given in section 3. Some summarizing remarks are collected in section 4 which also gives an outlook on future work to be done.

1.1 Economic aspects of ratings

As already noted in the introduction, ratings have become increasingly important in evaluating creditworthiness. In Europe this has become a hot topic since the Bank for International Settlements at Basle made ratings for all firms obligatory.[3] In Germany there is currently an intensive debate whether "internal" ratings by banks are as good as those published by rating agencies.[4]

1.2 Statistical aspects of ratings

Von Tessin (1999) has analysed a data set containing information on 545 industrial bonds evaluated by Standard & Poor's in March 1996. We use this data set to briefly describe their main characteristics. Letter "A" denotes a high creditworthiness (or low default risk) whereas letter "C" signals a high risk of losing the investment. Ratings are given both for firms and for single bonds as can be seen from columns 1 and 2 of table 1 which shows that ratings may differ between a specific bond and those of the issuing firm. Column 3 displays the average yield to maturity in the year 1996. Table 1 shows that a lower rating corresponds to a higher (average) yield indicating a higher risk.

It is evident that the rating variable is ordinal. Its informational content may be investigated by econometric models in two ways:

- In a probit or logit regression the rating may be specified as the dependent variable which is influenced by a set of explanatory variables. See subsection 1.3 for an example.[5]
- Contrary to this approach one may ask what influence the rating variable has on some performance measure (such as yield to maturity). Then we have the situation of an ordinal regressor which is discussed formally in

[2] We thank Shihwa Kao for providing the data.
[3] See in particular BIS (1999). We are grateful to Hans Hirth, Universität Tübingen, for informing us about this.
[4] A research project by Jan-Pieter Krahnen and Martin Weber has analysed this topic. See in particular Weber, Krahnen and Voßmann (1999).
[5] Some authors have used discriminant analysis which, however, disregards the ordinal nature. See Kao and Wu (1994).

section 2. Since the rating will be specified together with other regressors we call its effect on the dependent variable "marginal". This will be discussed in more detail in the following subsection.

Table 1. Rating-Data

firm rating	bond rating	mean yield	number of bonds
AAA	AAA	7,145	32
AA+	AA+	7,427	2
AA	AA	7,303	37
AA-	AA-	7,086	48
A+	A+	7,068	18
A	A	7,497	64
A	A-	7,354	5
A-	A-	7,751	48
A-	BBB+	8,333	2
BBB+	BBB+	7,806	18
BBB+	BBB	8,419	5
BBB	BBB	7,726	30
BBB	BBB-	8,889	6
BBB-	BBB-	8,222	59
BBB-	BB+	8,501	13
BB+	BB+	7,928	18
BB+	BB-	8,943	18
BB	BB	8,900	14
BB	BB-	9,270	1
BB	B+	10,048	11
BB-	BB-	10,181	10
BB-	B+	9,957	4
BB-	B	10,213	16
B+	B+	8,838	9
B+	B	10,063	5
B+	B-	11,007	8
B	BBB-	12,233	3
B	B	13,550	2
B	CCC+	10,391	3
B-	B-	18,177	1
B-	CCC	14,947	2
CCC	CCC	11,317	1

1.3 Marginal effect of rating on yield?

The additional or "marginal" effect of ratings on the performance of a firm and on single bonds has been examined first by Liu and Thakor (1984). However, their estimation method was not adequate as pointed out by Kao and

Wu (1990, 1994). They employed Terza's (1987) method which uses conditional means as regressors (see subsection 2.3). The model of Liu and Thakor is displayed graphically in figure 1. There it is assumed that the rating is not only an influencing variable but is itself influenced by other variables which act, however, also directly on the performance measure. Consequently, only the marginal or residual effect of the rating is estimated in this model. Quite

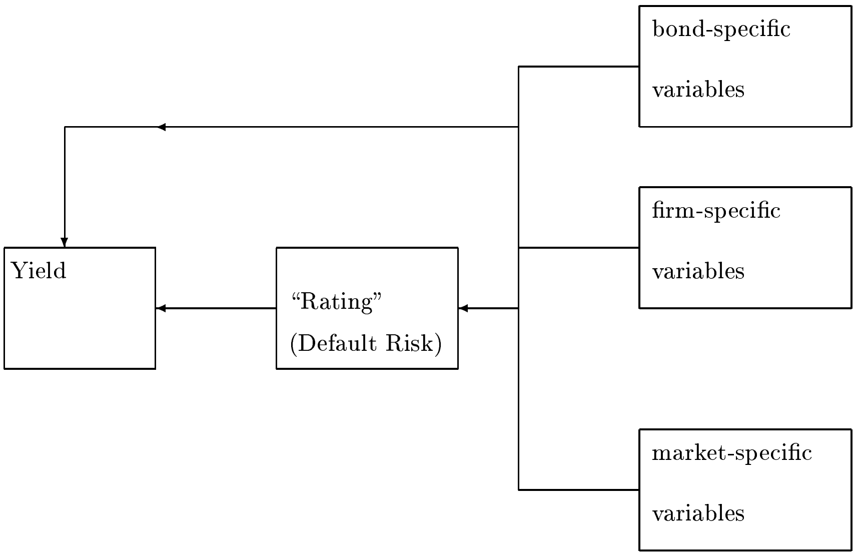

Fig. 1. The model used by Liu/Thakor and Kao/Wu

interestingly, the results regarding the marginal effect of ratings are different for the two data sets examined.

- In Kao and Wu (1990) rating of corporate bonds is investigated. The dependent variable is the corporate bond yield. The authors find a significant positive marginal effect.
- In Kao and Wu (1994) rating of municipal bonds is considered. The dependent variable is municipal bond yield. In this study, however, the marginal effect of rating is not significant.

This leads to the following remarks and questions:

- Does rating have a different quality for municipal bonds compared to private corporate bonds? As far as we know this important question has not yet been analyzed.
- The marginal effect always depends, of course, on the set of other explanatory variables included. However, the low explanatory power in case of

municipal bonds suggests that the addition of other variables would not change the result.

The empirical results from the two studies are summarized in table 2 for Kao and Wu (1990) and in table 3 for Kao and Wu (1994). Both tables show the probit regression results as well as the results from the equation in which the rating acts as a regressor. Note that in both studies the rating scheme of Moody's is employed (1 indicating best rating, k the worst rating). We will use the data set of table 2 in our own analysis in section 3.

Table 2. Estimation Results (Kao and Wu, 1990, p. 322)

P	**R**	variable	description
	1	Yield	Corporate Bond Yield
	$\oplus$	T-bond rate (TB)	The average annualized interest rate of a treasury bond with same maturity as the corporate bond at the issuance date
1	$\oplus$	Agency bond ratings (RISK)	1 = Aaa; 2 = Aa; 3 = A; 4 = Baa; 5 = Ba; 6 = B
+	+	Interest coverage (INTCOV)	(Net income + interest)/interest
+	+	Size of issue (SIZE)	The size of the bond issue in millions of dollars
$\oplus$	$\oplus$	Debt ratio(DR)	Book value of total debts/(market value of equity + book value of total debts)
$\oplus$		Returns on assets (ROA)	Net income/total assets
+	+	Dividend payout ratio (DPR)	Annual dividends/net income
$\oplus$	+	Subordination(SUB)	1 = subordination; 0 = no subordination
+	+	Asset growth rate(GR)	The five-year average of total-assets growth rate
$\oplus$	+	Total firm assets(TA)	Current assets + long-term assets
+	+	Call	1 = call provision; 0 = no call provision
$\oplus$	$\oplus$	amortization rate (AMORT)	Total promised principal payments before maturity/total principal payments(sinking-fund amortization rate)
$\oplus$	$\oplus$	DAMORT	Amortization rate * rating dummy RD (RD = 1 for investment-grade bonds and 0 otherwise)
$\oplus$	+	Financing restriction (FR)	1 = restriction on additional issue of debts with the same or higher priority, 0 = otherwise
$\oplus$	+	Industry type (IND)	1 = industry bonds; 0 = utility bonds
	+	Time to maturity (T)	The number of years to maturity
Notes:		number of observations: 231 **P** = probit regression using rating as dependent variable **R** = linear regression using yield as dependent variable + = variable included $\oplus$ = marginal effect significant 1 = dependent variable	

2 Ordinal variables in microeconometrics

2.1 Ordinal regressors

When an ordinal variable is specified as an explanatory variable in a linear regression or a probit regression, we call it an ordinal regressor. A scatter plot of the rating together with some performance measure shows the typical

Table 3. Estimation Results (Kao and Wu, 1994, p. 67)

P	R	variable	description
	1	y	municipal bond yield
⊕	⊕	X_1	the total net direct debt burden of the state in billions of dollars
		X_2	the ratio of net direct debt to estimated full valuation (on real and personal property)
+		X_3	the ratio of net direct debt to total personal income
⊕	⊕	X_4	the per capita debt in thousands of dollars
⊕	⊕	X_5	unemployment rate of the state as of 1975
+		X_6	total state population in 1975
		X_7	the percentage population change between 1970 and 1975
⊕	+	X_8	median home value in thousands of dollars
		X_9	per capita personal income in thousands of dollars
1	+	$\hat{RISK}$	(estimated) risk (Terza's method) (1 = Aaa, 2 = Aa, 3 = A1, 4 = A, 5 = Baa)
Notes:			number of observations: 29 **P** = probit regression using rating as dependent variable **R** = linear regression using yield as dependent variable + = variable included ⊕ = marginal effect significant 1 = dependent variable

pattern of data sets when there exists a monotonic relation. In figure 2 we show the scatter plot taken from Machauer and Weber (1998, p. 1365) which displays a positive relation between the two variables. Note that Machauer and Weber use the interest rate premium instead of the yield.

2.2 (In-)Adequate econometric treatment

It seems natural to employ the threshold model which is well known from ordered probit models also in the case when the ordinal variable is used as an explanatory variable. Denoting the observed ordinal variable by y and the underlying (continuous) "latent" variable by η, the linear regression model with an ordinal regressor may be briefly described as follows: Let the latent structural model

$$\zeta = \alpha + \beta\,\eta + \varepsilon$$

be given with $\zeta = z$ directly observable. For the regressor variable we have then from the threshold model

$$y = \begin{cases} 1 & \eta \le \mu_1 \\ 2 & \mu_1 < \eta \le \mu_2 \\ \vdots & \vdots \\ r & \eta > \mu_{r-1} \end{cases},$$

where we assumed that the ordinal variable has r categories.

Fig. 2. Source: Machauer and Weber (1998) , p. 1365.

In many studies (see, e.g., Machauer and Weber, 1998, and Elsas and Krahnen, 1998), however, the rating variable is considered as a nominal variable leading to the well-known dummy variable approach. This leads to the following problems[6]

- Since a greater number of parameters has to be estimated, there is an efficiency loss.
- There is no possibility to relate the estimated effects to the "marginal" effect in the underlying structural model.
- The advantage of a more flexible specification using dummy variables is at least partly compensated by the separation from the latent model.
- Most important: The dummy variable approach introduces a severe errors-in-variables problem[7]. Inconsistency of the estimator occurs if the rating variable is correlated with other regressors which will typically be the case.

2.3 Conditional means as regressors

Terza (1987) considered the problems arising from ordinal regressors. He proposed the following approach: First, estimate the conditional mean of η given the event $y = k$ (that is category k has been observed), and, second,

[6] Breslaw and McIntosh, 1998, p. 26.

[7] See the results in Breslaw and McIntosh (1998), p. 27, and appendix D.

use this expression as a regressor. Kao and Wu generalized this approach in deriving the conditional means with additional regressors which leads to[8]

$$E\,[\eta \,|\, y = k] = -\,\frac{\phi(\mu_k - \mathbf{x}'\,\boldsymbol{\beta}) - \phi(\mu_{k-1} - \mathbf{x}'\,\boldsymbol{\beta})}{\Phi(\mu_k - \mathbf{x}'\,\boldsymbol{\beta}) - \Phi(\mu_{k-1} - \mathbf{x}'\,\boldsymbol{\beta})}.$$

Parameter estimates are obtained from a probit regression. However, the standard errors have to recognize the additional noise introduced by the "generated" regressors. The corresponding formulae have been derived by Kao and Wu (1990). However, this method may only be recommended if a single ordinal regressor is specified whereas for more than one ordinal regressor the correlation of the underlying latent variables is neglected. Kao and Wu consider just one ordinal regressor and, therefore, their estimation results are sound.[9]

2.4 Efficient estimation with regard to ordinal regressors

Usually it is sufficient to consider the *conditional* density of the dependent variable for given values of the regressor variables. This is the standard approach in linear regression theory. This approach is adequate since in general for this model the regressor variables are exogenous in the sense of Engle, Hendry and Richard (1983). Ronning and Kukuk (1996) have shown that this is no longer true if the observed regressor variable is ordinal and the threshold model obtains which implies that the thresholds μ_k $(k = 1, \ldots, r-1)$ enter as additional parameters to be estimated. For this reason the usual factorization of the joint density is no longer possible. As pointed out by Ronning and Kukuk (1996) the estimation based on the conditional distribution[10]

$$p(z \,|\, rating, \mathbf{x})$$

rather than on the joint distribution

$$p(z\,,\, rating \mid \mathbf{x})$$

implies an efficiency loss due to the nonexogeneity of the ordinal regressor that may be substantial.[11] This result suggests that ordinal variables should always be treated as jointly endogenous in a statistical sense. Conditional expectations are derived from this joint distribution. This, by the way, is the strategy taken by standard computer software like LISREL.

[8] See, for example, Ronning (1991), p. 28.

[9] Results of Breslaw and McIntosh (1999) suggest that the correlation with other regressors should also be taken into account. Probit regression results in tables 2 and 3 show that these correlations may be strong.

[10] $\mathbf{x}$ is the set of all regressor variables including the ordinal rating.

[11] A more general result regarding efficiency loss due to the non-exogeneity of ordinal regressors is given in Ronning (2001).

Maximum likelihood estimation is feasible for this model only if the rating is the only regressor variable.[12] However, if more than one ordinal regressor or additional nominal and continuous regressor variables are taken into account, numerical problems become too complex when employing the ML method. Kukuk (1999) discusses this in detail and proposes a new estimation procedure via "indirect inference" for microeconometric models with ordinal regressors.[13]

We briefly describe the estimation procedure for the case of one continuous regressor($x_1 = x_1^*$), two ordinal regressors (x_2 and x_3) as well as a nominal explanatory variable (D).[14] In addition to the "true" model

$$y^* = \beta_0 + x_1^*\beta_1 + x_2^*\beta_2 + x_3^*\beta_3 + D\beta_4 + \varepsilon$$

we consider some "auxiliary" model

$$y^* = \theta_0 + x_1^*\theta_1 + x_2\theta_2 + x_3\theta_3 + D\theta_4 + \eta$$

which specifies ordinal regressors from the outset and treats them as discrete variables taking on the values $1, 2, 3, \ldots, r$. The set of parameters from the true model is denoted by $\boldsymbol{\beta}$ and the corresponding set for the auxiliary model by $\boldsymbol{\theta}$.

We start by simulating the true model for a given parameter vector $\boldsymbol{\beta}_0$ (where joint normality is assumed for all x's). The simulated latent regressors are transformed into ordinal variables via the threshold model which is estimated from the observed data set.[15] The resulting data set is used to estimate the parameter vector $\boldsymbol{\theta}$ of the auxiliary model (for a given parameter vector $\boldsymbol{\beta}_0$). We call this estimate $\tilde{\boldsymbol{\theta}}(\boldsymbol{\beta}_0)$. On the other hand, we directly estimate the auxiliary model on the basis of the observed sample and obtain an estimate $\hat{\boldsymbol{\theta}}$. Since the two estimates would be close to each other if the parameter vector $\boldsymbol{\beta}_0$ in the simulation is near to the true parameter vector, we will iterate our procedure to minimize the distance between the two estimates. Certain regularity conditions have to be fulfilled in order to obtain a minimum.[16] The resulting estimates may be shown to be consistent.

3 Empirical analysis

We use the data set from Kao and Wu (1990) to illustrate our method. Estimation results are given in table 4. First we present results for Terza's method

[12] See, for example, Ronning and von Tessin (1998) and von Tessin (1999). See also Lee and Poon (1986), and Lee, Poon and Bentler (1990) for a more extensive discussion.

[13] See Gourieroux, Monfort and Renault (1993). This method had been proposed in another context also by Gallant and Tauchen (1996).

[14] Kukuk (1999), p. 21 figure 3.

[15] Alternatively, the thresholds could be included into the set $\boldsymbol{\beta}$.

[16] For details see Kukuk (1999).

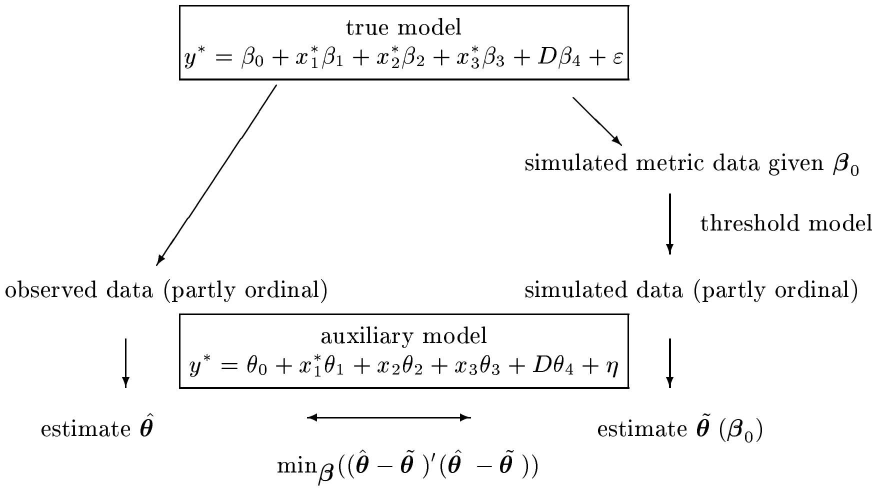

Fig. 3. Estimation via indirect inference

as used by Kao and Wu. In addition we present estimation results from our own estimation procedure via indirect inference. Note that we consider two alternatives: (a) financing restrictions (FR) are treated as ordinal, (b) financing restrictions (FR) are treated as nominal. In the first case we have two ordinal regressors. In all three estimation procedures the standard errors of estimates are obtained by bootstrapping from which the t-values were computed. For a description of variables see table 2. The rating variable (RISK)

Table 4. Estimation Results (Data from Kao and Wu, 1990)

	Terza's Method		**Indirect Method**			
			FR nominal		FR ordinal	
variable	estimate	t-value	estimate	t-value	estimate	t-value
C	-28.70	-22.82	1.20	4.91	1.40	3.11
TB	48.02	19.10	45.74	19.93	45.15	13.62
RISK	0.35	13.24	0.82	16.34	0.80	11.06
INTCOV	0.02	4.24	0.01	2.25	0.02	2.64
SIZE	0.00	0.75	0.00	1.35	0.00	1.27
TA	0.00	1.94	0.00	1.50	0.00	1.03
FR	0.12	1.24	0.11	0.82	0.08	0.47
DATE	0.01	2.89	0.01	3.82	0.01	0.84
IND	0.42	4.19	0.37	4.22	0.43	4.16
All t-values computed from bootstrapped variances.						

has the expected strong positive effect on the yield. Moreover, the treasury bond rate (TB) is significant in all three procedures which shows the strong link between bond yields and the "market rate". Unfortunately, the financing restrictions are not significant at all, and, accordingly, we cannot tell whether the ordinal or the nominal structure is the more adequate description. In the case of ordinality an underlying latent variable such as the (good) relationship between a firm and its bank would be assumed. For example, Weber, Krahnen and Voßmann (1999) find a strong influence of a close relationship with a particular bank ("Hausbank").

4 Concluding remarks

Our paper shows that models involving ordinal regressors may be estimated quite easily via indirect inference. Contrary to Terza's method an arbitrary number of ordinal regressors may be specified together with both continuous and nominal regressors. We hope to study the marginal effect of ratings on the basis of a data set taken from German banks in a further study which also contains partial ratings for some characteristics of firms.[17]

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