

From LDA++ to X+DMFT:

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Strategies for interfacing electronic structure and
many-body theory

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Collaborators and Funding

Ceria

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Sr₂RhO₄

C. Martins, M. Aichhorn, L. Vaugier, S. Biermann

BaFe₂As₂

P. Werner, M. Casula, T. Miyake, F. Aryasetiawan, A. Millis
ARPES: Veronique Brouet

BaCo₂As₂

A. Van Roekeghem, J. Tomczak, M. Casula, T. Ayrar,
H. Jiang, F. Ferrero, O. Parcollet
ARPES: Hong Ding's group

SrVO₃/SrTiO₃

A. Van Roekeghem

SrVO₃ within GW+DMFT

J. Tomczak, M. Casula, T. Miyake

Sn/Si(111)

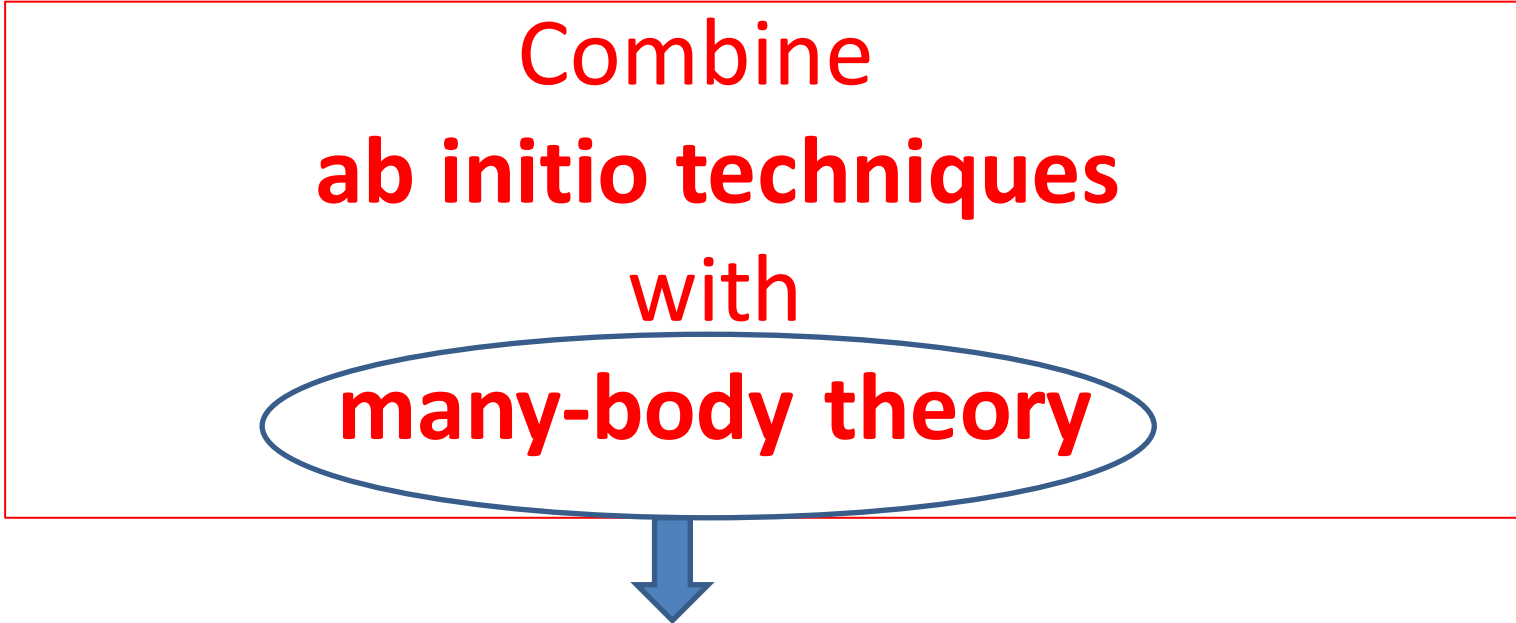
P. Hansmann, T. Ayrar



Spectral properties of materials ...

... beyond the band picture ?

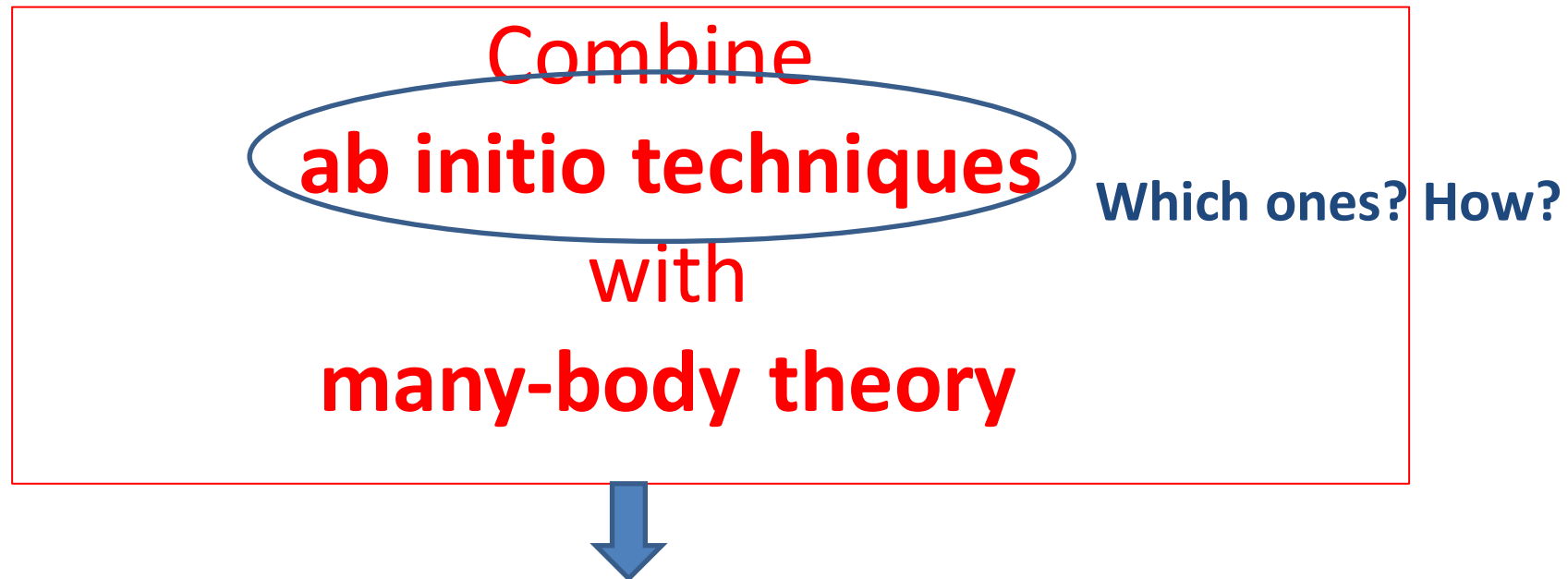
Combine
ab initio techniques
with
many-body theory



Here: **Dynamical Mean Field Theory**

Spectral properties of materials ...

... beyond the band picture ?



***Ab initio* calculations of quasiparticle band structure in correlated systems: LDA++ approach**

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Institute of Metal Physics, Ekaterinburg 620219, Russia

(Received 11 July 1997)

We discuss a general approach to a realistic theory of the electronic structure in materials containing correlated *d* or *f* electrons. The main feature of this approach is the taking into account of the energy dependence of the electron self-energy with the momentum dependence being neglected (local approximation).

Ab initio calculations of quasiparticle band structure in correlated systems: LDA++ approach

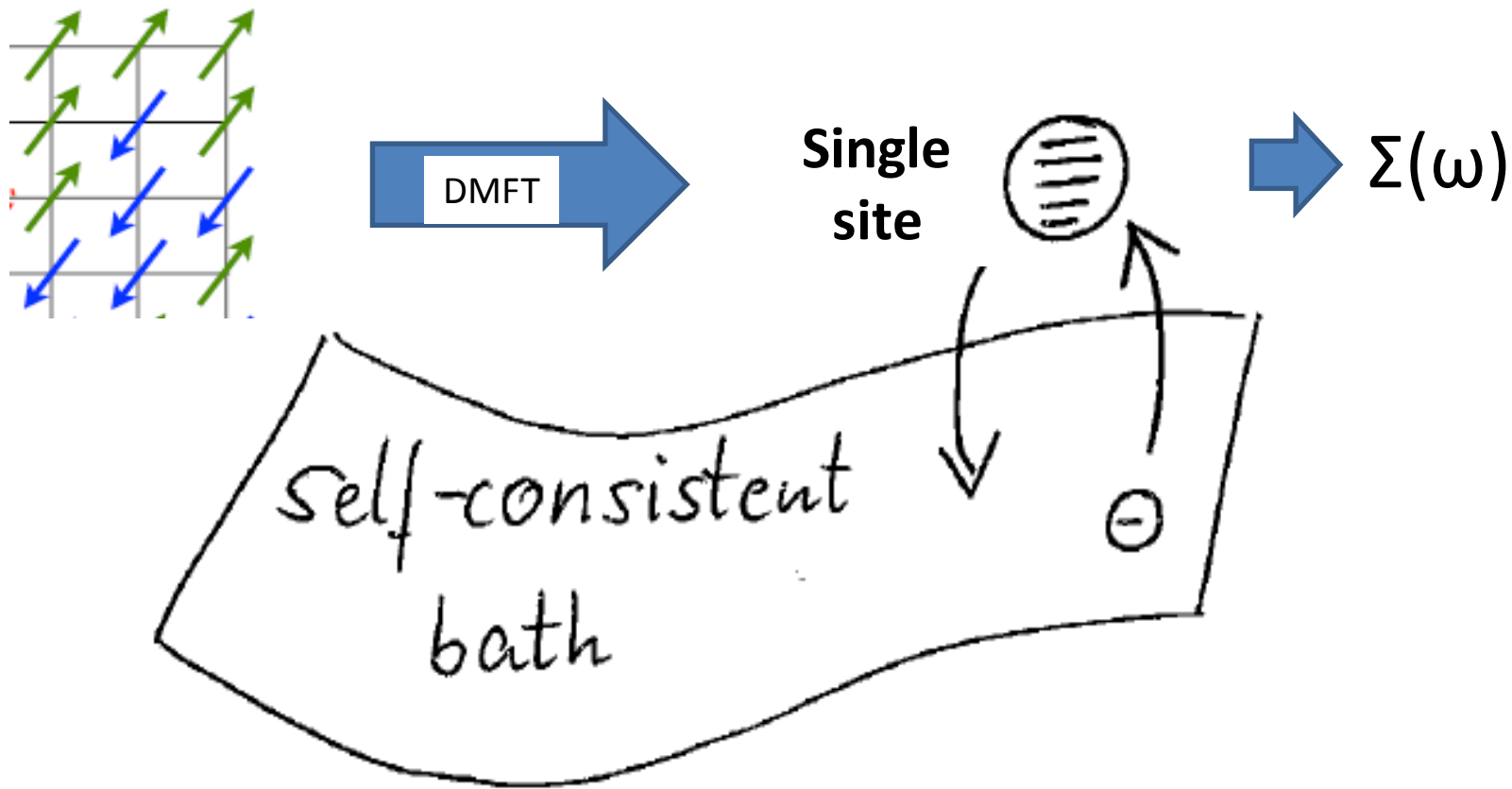
$$\begin{aligned}
H = & \sum_{ij\sigma\{m\}} t_{m_1 m_2}^{ij} c_{im_1\sigma}^+ c_{jm_2\sigma} \\
& + \frac{1}{2} \sum_{i\{\sigma m\}} U_{m_1 m_2 m'_1 m'_2}^i c_{im_1\sigma}^+ c_{im_2\sigma'}^+ c_{im'_2\sigma'} c_{im'_1\sigma}, \quad (1)
\end{aligned}$$

where (i,j) represents different crystal sites, $\{m\}$ labels different orbitals, and the $\{\sigma\}$ are spin indices. Coulomb matrix elements are defined in the usual way:

$$\begin{aligned}
U_{m_1 m_2 m'_1 m'_2} = & \int \int d\mathbf{r} d\mathbf{r}' \psi_{m_1}^*(\mathbf{r}) \psi_{m_2}^*(\mathbf{r}') \\
& \times V_{ee}(\mathbf{r}-\mathbf{r}') \psi_{m'_1}(\mathbf{r}) \psi_{m'_2}(\mathbf{r}'); \quad (2)
\end{aligned}$$

here $V_{ee}(\mathbf{r}-\mathbf{r}')$ is the screened Coulomb interactions and $\psi_m(\mathbf{r})$ are localized on-site basis functions (the site index being suppressed).

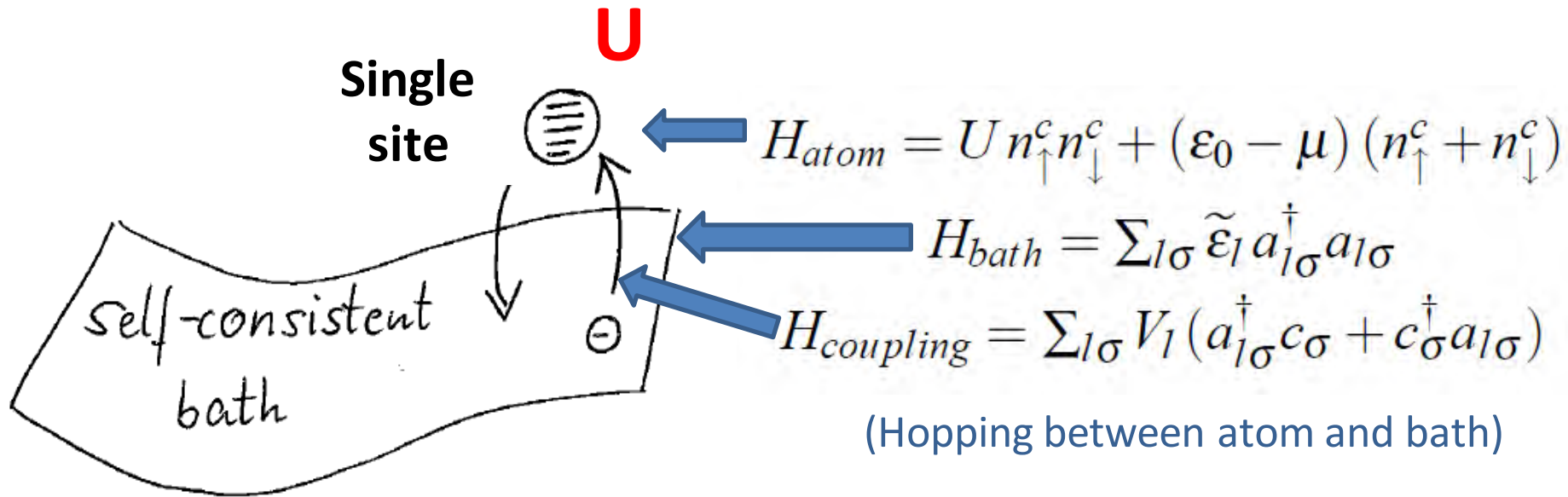
Dynamical mean field theory



Georges, Kotliar, Krauth, Rozenberg, Rev. Mod. Phys. 1996

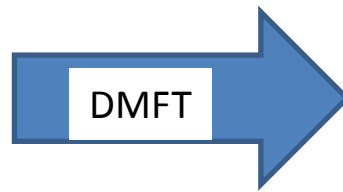
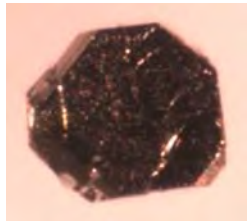
DMFT

Calculate G_{loc} from an impurity problem (that is, a single site with Hubbard interaction U coupled to a bath):

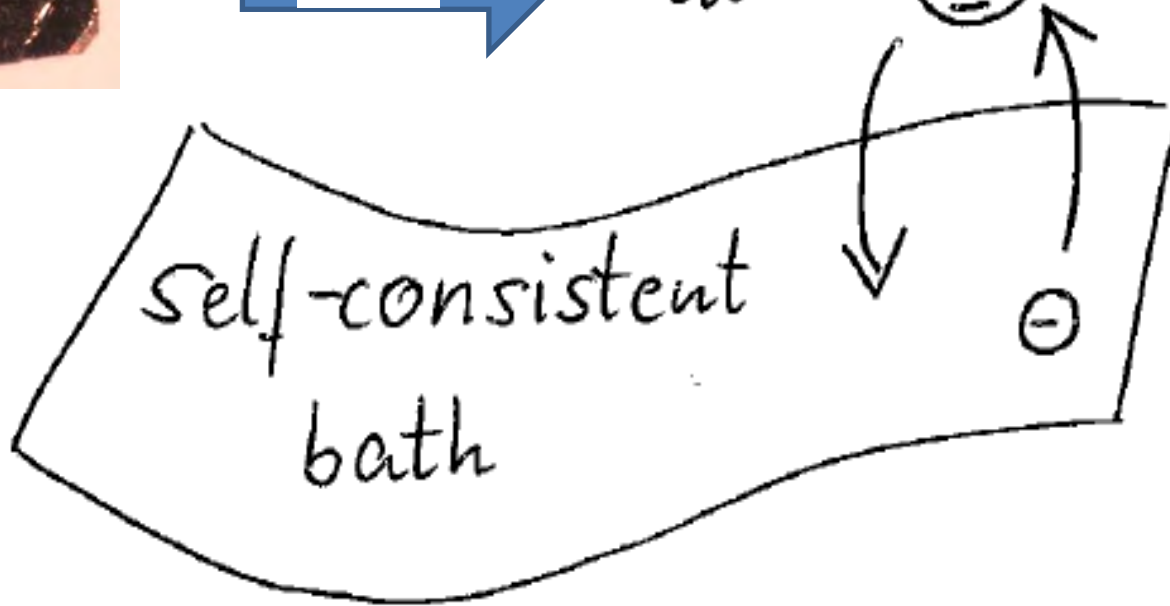
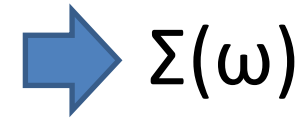


Determine bath (the dynamical mean field) self-consistently

Dynamical mean field theory within realistic electronic structure calculations: “DFT+DMFT”



atom



Lichtenstein, Katsnelson, 1998

Anisimov, ... Kotliar, 1997

Ab initio calculations of quasiparticle band structure in correlated systems: **LDA++ approach**

A. I. Lichtenstein

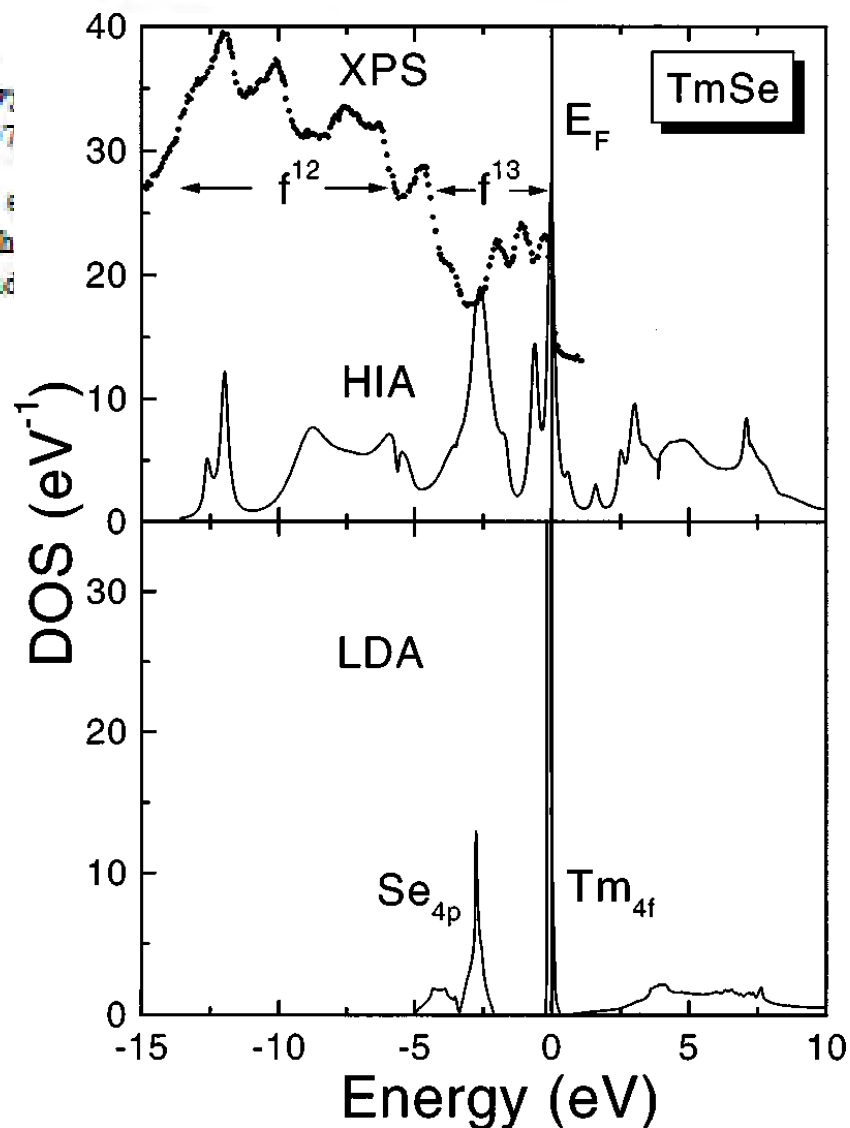
Forschungszentrum Jülich, D-52428 Jülich, Germany

M. I. Katsnelson

Institute of Metal Physics, Ekaterinburg

(Received 11 July 1997)

We discuss a general approach to a realistic theory of the s -correlated d or f electrons. The main feature of this approach is the dependence of the electron self-energy with the momentum dependence



Outline

- Density functional theory + dynamical mean field theory (“DFT+DMFT”)
 - Examples: CeSF, Sr₂RhO₄, BaCo₂As₂
- DFT+DMFT revisited
 - How to get rid off U?
 - How to get rid off DFT?
- Screened Exchange Dynamical Mean Field Theory
 - Concepts and relation to GW+DMFT
 - Examples: BaCo₂As₂, SrVO₃
 - Relation to DFT? – Example: SrTiO₃
- GW+DMFT
 - Examples: Adatoms on surfaces, SrVO₃
- Summary/perspectives

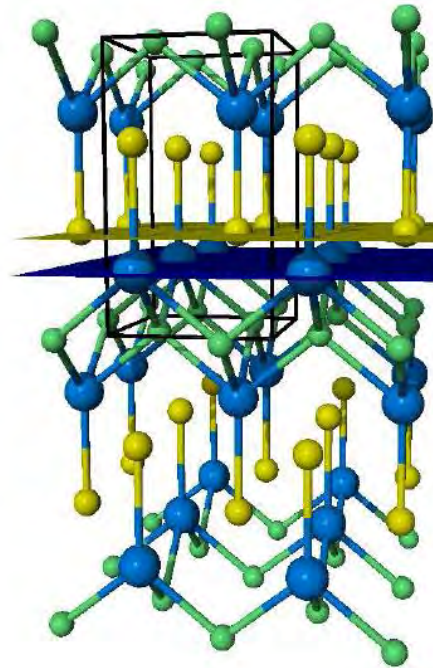
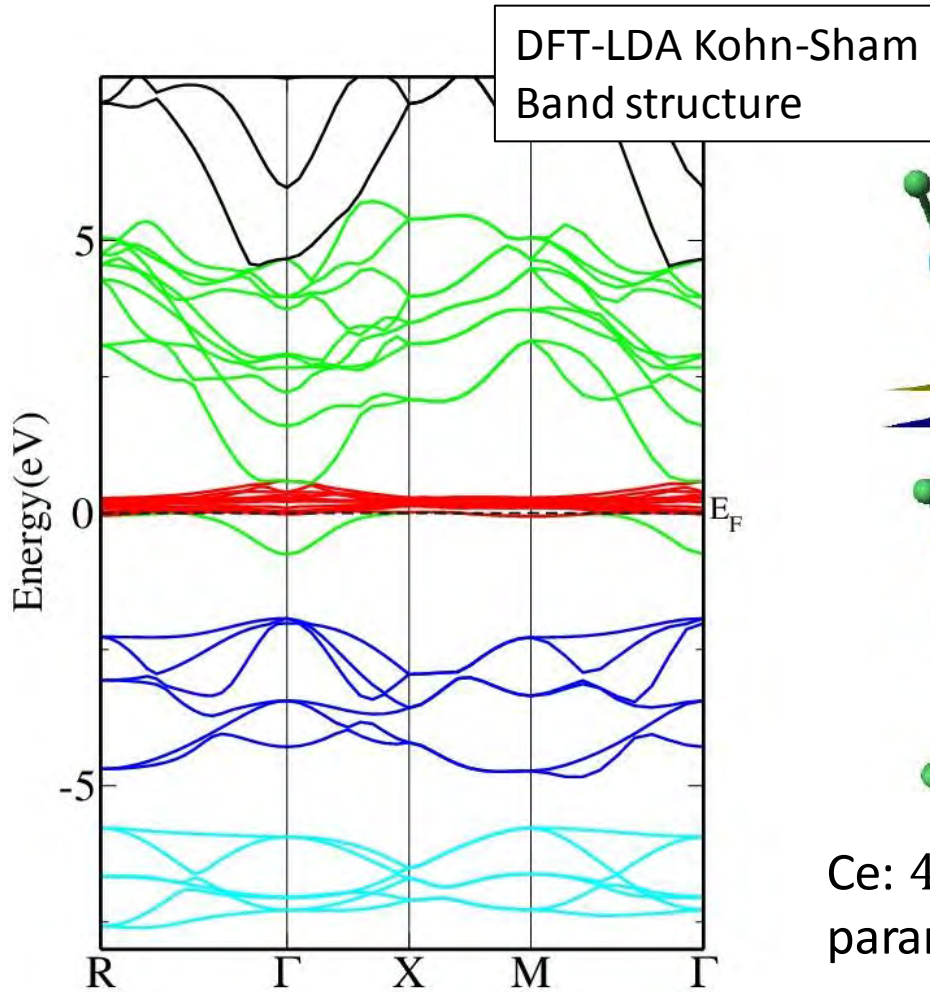
Appetizer: it works!

But it shouldn't

Why it works ... !
(in which situations...)

How to go beyond ...

Example 1: CeSF – an f-electron pigment (cf. Rhodia's Neolor series)

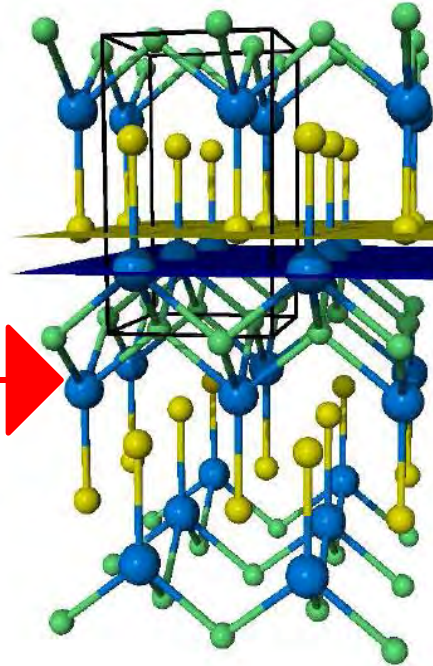
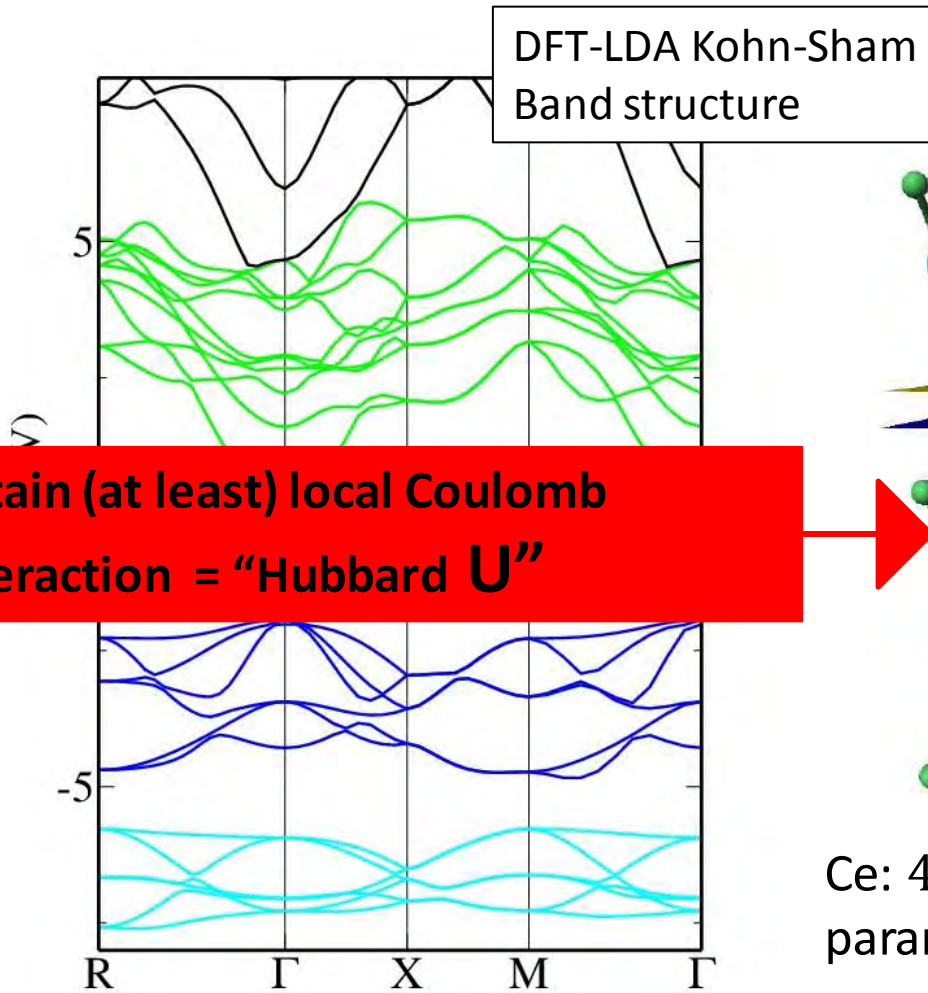


Ce: $4f^1$ configuration,
paramagnetic

Example 1:

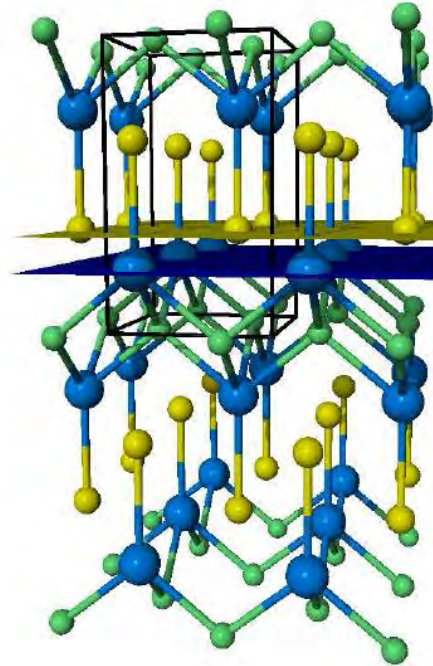
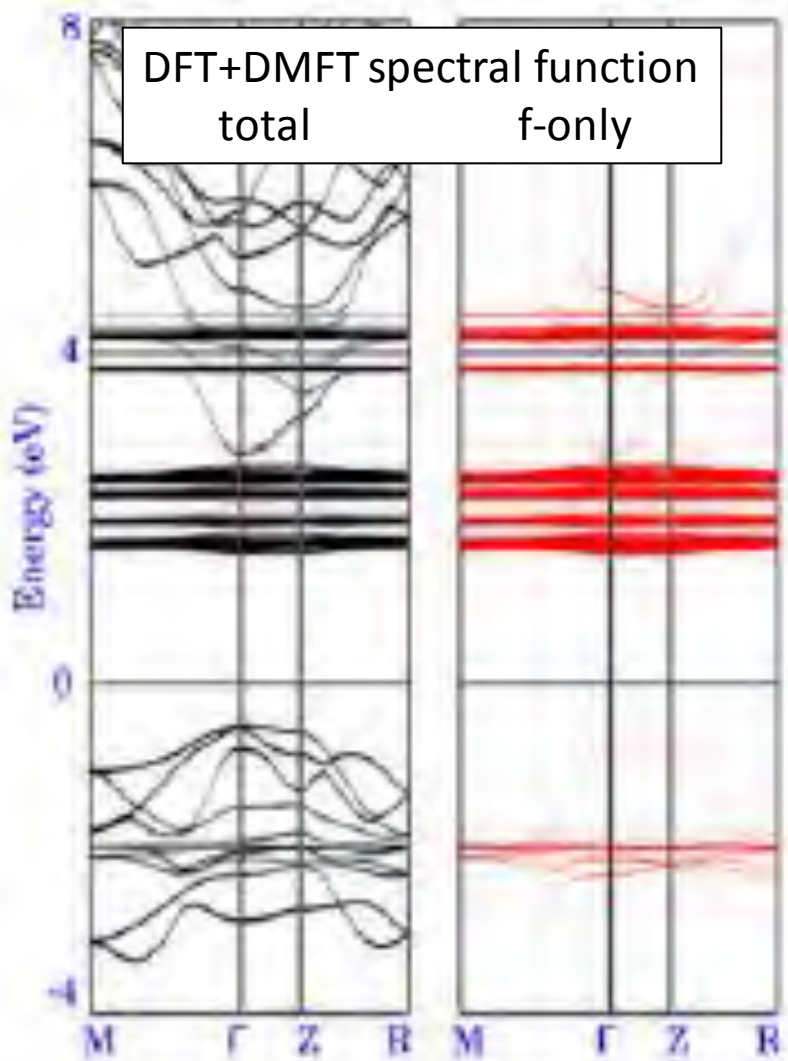
CeSF – an f-electron pigment

(cf. Rhodia's Neolor series)



Ce: $4f^1$ configuration, paramagnetic

Example 1: CeSF – an f-electron pigment (cf. Rhodia's Neolor series)

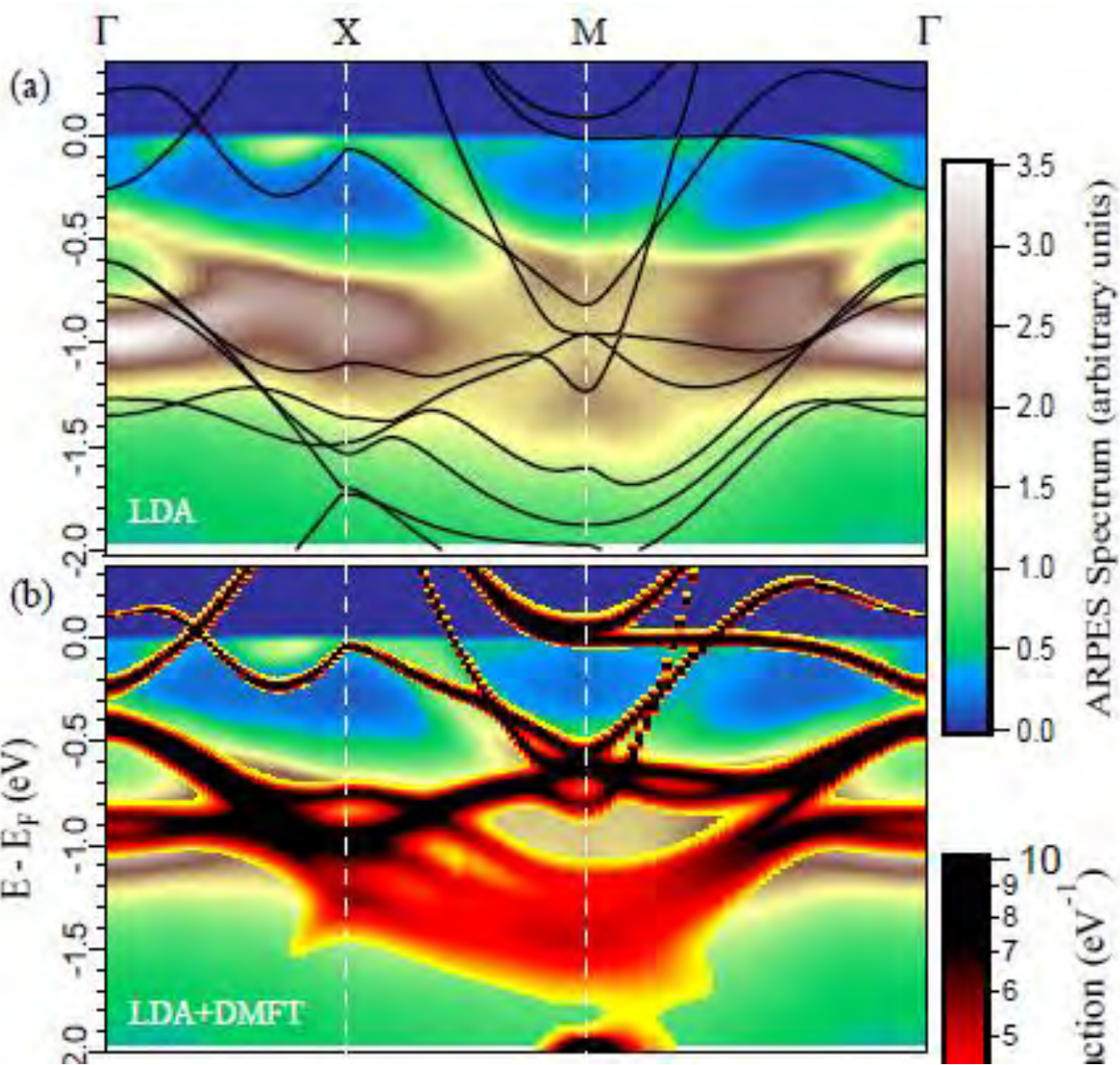


Ce: $4f^1$ configuration,
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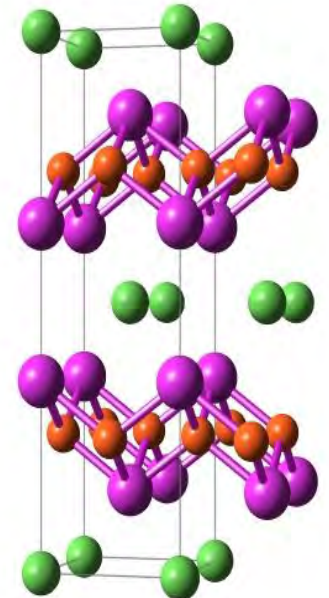
Calculated colour of CeSF:



Example 2: BaCo2As2

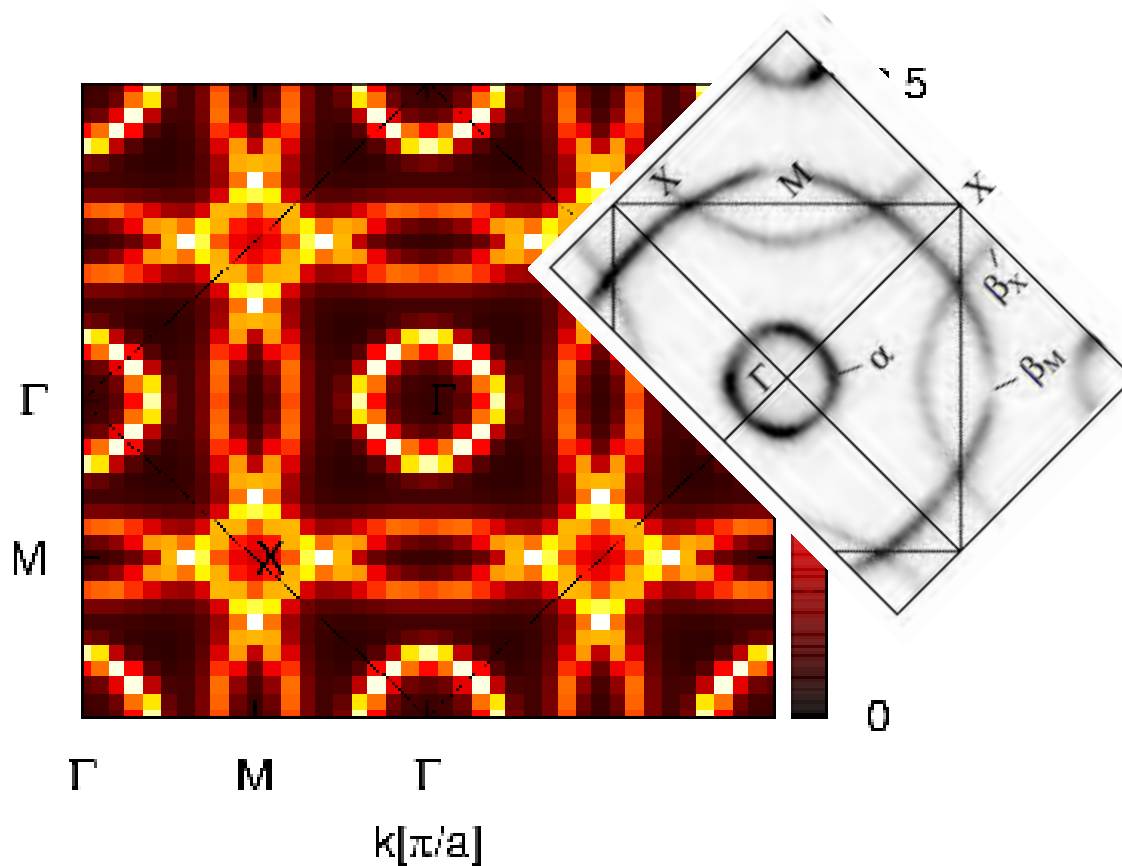


A. Van Roekeghem



Xu et al., PRX (2013) & A. van Roekeghem et al., PRL 2014

Example 3: Fermi surface of Sr₂RhO₄



Martins et al, Phys. Rev. Lett. (2011) and in preparation
Cf. Liu et al, PRL 2008.

DFT+DMFT ... a story of success!

Applications to

- 3d transition metal oxides, sulphides
[SrVO₃, CaVO₃, LaTiO₃, YTiO₃, VO₂, V₂O₃, BaVS₃, ...]
- Transition metals [Mn, Ni, ...]
- f-electron elements and compounds [Ce, CeSF, RE₂O₃]
- Iron Pnictides [LaFeAsO, FeSe, BaFe₂As₂, BaCo₂As₂, Ba₂Ti₂Fe₂As₄O ...]
- Spin-orbit materials [Sr₂IrO₄, Sr₂RhO₄]
- Low-dimensional systems (organics)
-

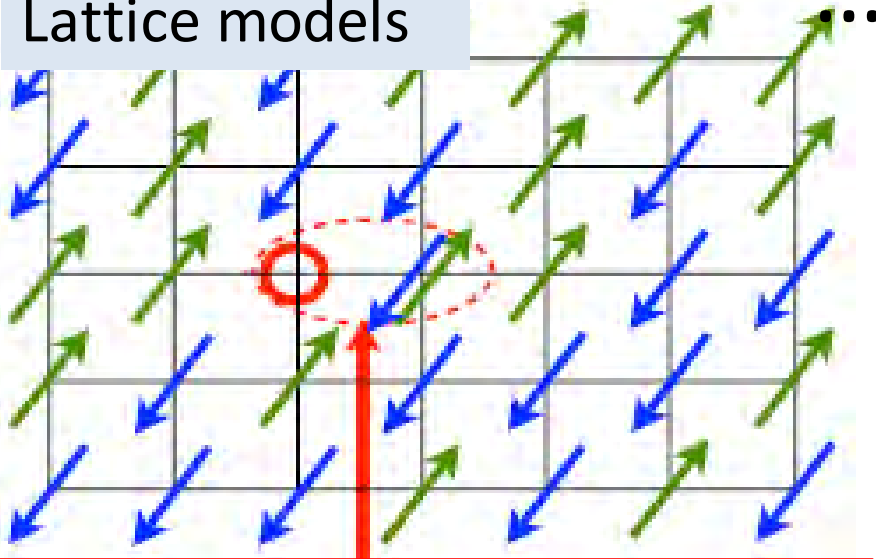
However:

- What's U in the solid?
- Why start from Kohn-Sham Hamiltonian?
- How to avoid double counting of interactions? Of screening?

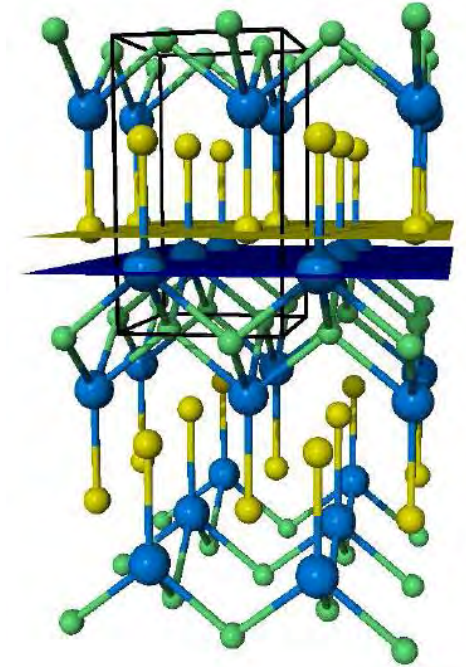
**Central question: How to “downfold”
to an effective reduced Hilbert space?**

Wanted: Low-energy effective Hamiltonian

Lattice models



**Local Coulomb interaction
= "Hubbard U"**



Microscopic link between model and material ?

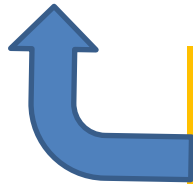
In practice: “DFT+DMFT” calculates hoppings from density functional theory

One-particle part of Hamiltonian from DFT-LDA



$$H = \sum_{ij\sigma\{m\}} t_{m_1 m_2}^{ij} c_{i m_1 \sigma}^+ c_{j m_2 \sigma}$$

$$+ \frac{1}{2} \sum_{i\{\sigma m\}} U_{m_1 m_2 m_1' m_2'}^i c_{i m_1 \sigma}^+ c_{i m_2 \sigma}^+ c_{i m_2' \sigma'} c_{i m_1' \sigma'}$$

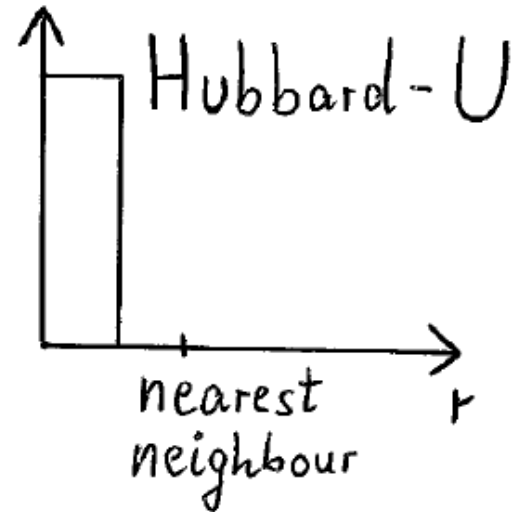
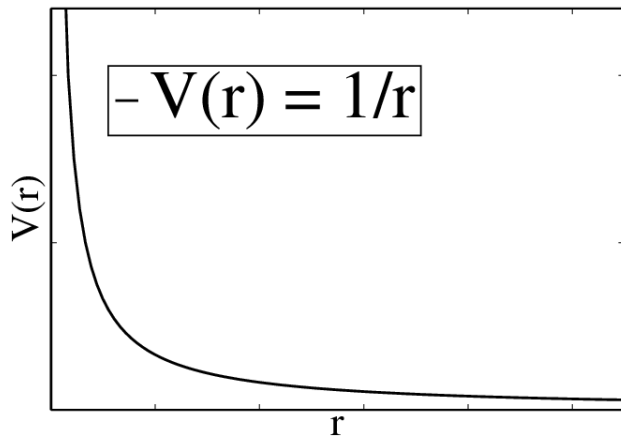


Hubbard interactions (and possibly Hund's coupling) for “correlated shell”

But what about U?

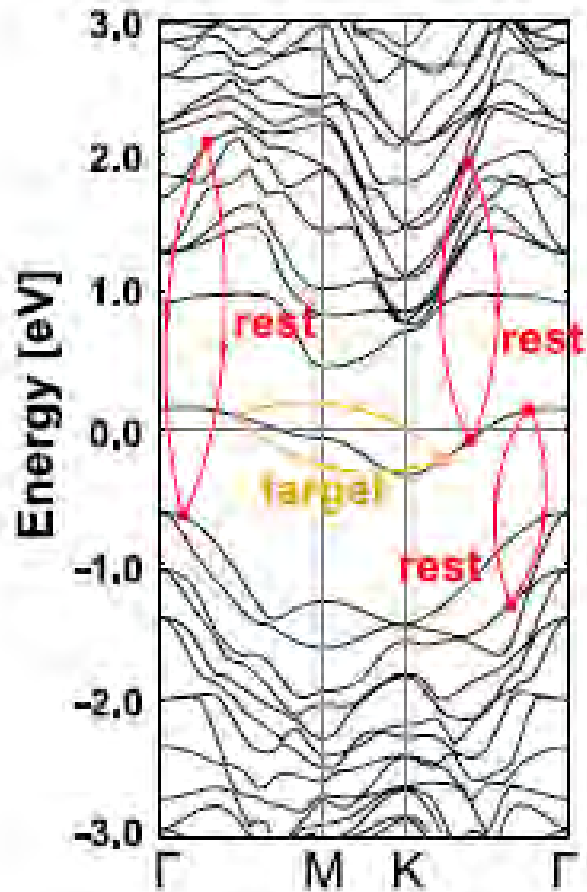
How to bridge between ...

1/r Coulomb interaction & its description as “+U” ?



Description of screening?

Hubbard U ...

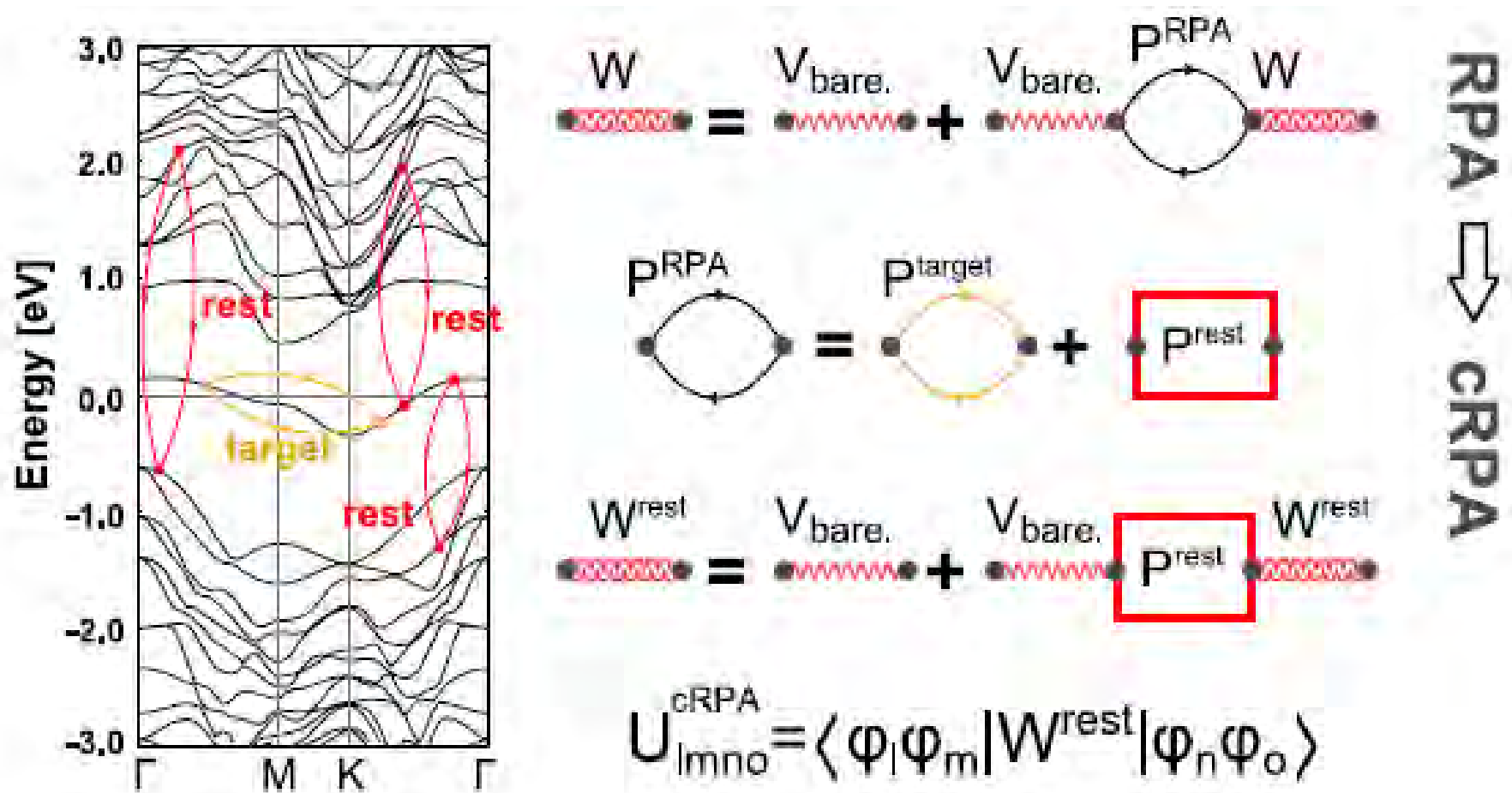


- the bare interaction within a low-energy subspace
- a partially screened interaction in the full space
- How can we calculate partial screening by higher-energy degrees of freedom?

Aryasetiawan, Imada, Georges, Kotliar, Biermann, Lichtenstein, PRB 2004.

[Figure from Hansmann et al., JPCM 2013]

Constrained Random Phase Approximation



Aryasetiawan, Imada, Georges, Kotliar, Biermann, Lichtenstein, PRB 2004.

[Figure from Hansmann et al., JPCM 2013]

Hubbard U – take home messages:

- Can be calculated *ab initio* within cRPA
- Matrix element of a *partially screened interaction*
[NB. Interatomic interactions also calculable!]
- Two consequences:
 - U depends on subspace and orbitals
 - U is *frequency-dependent* (“dynamically screened”)

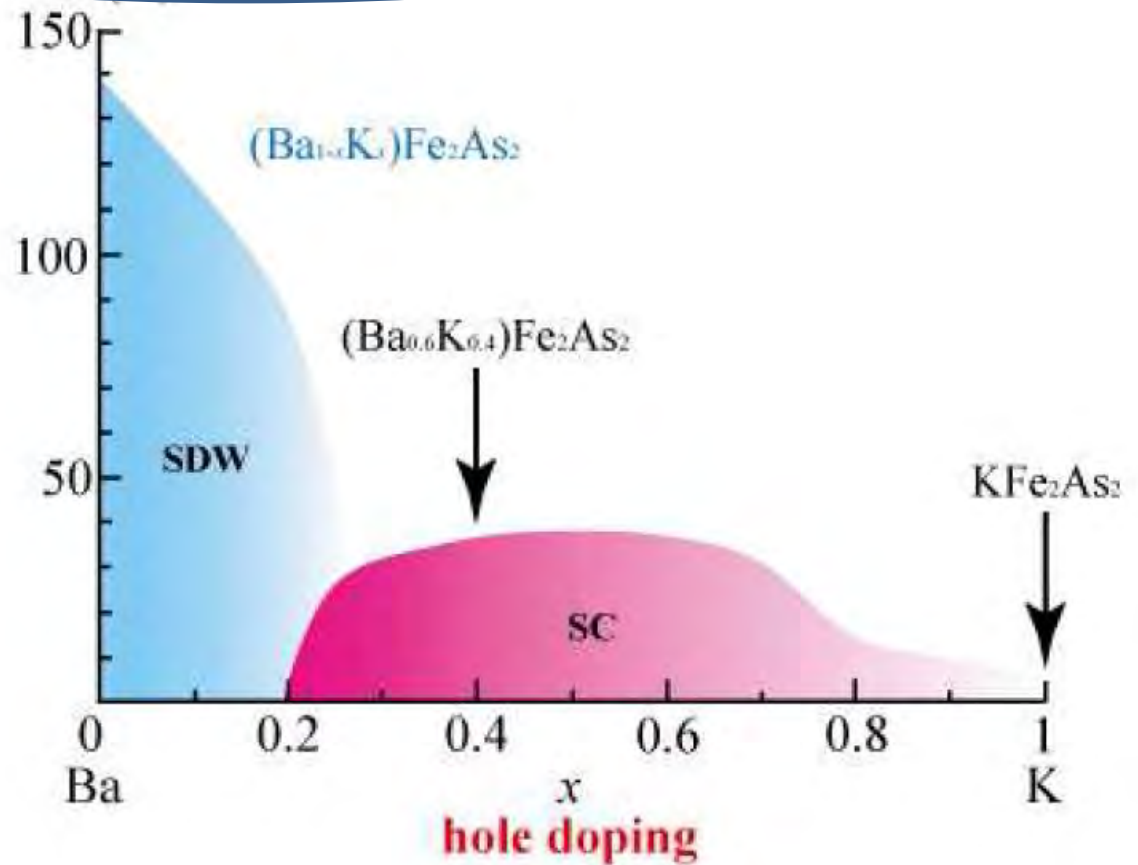
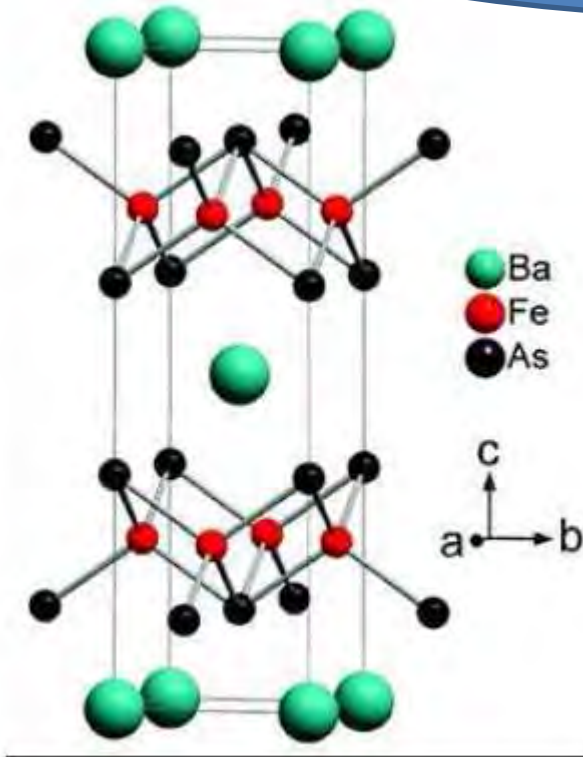
Aryasetiawan, Imada, Georges, Kotliar, Biermann, Lichtenstein, PRB 2004.

[Implementation into Wien2k: Vaugier, Jiang, Biermann, PRB 2012]

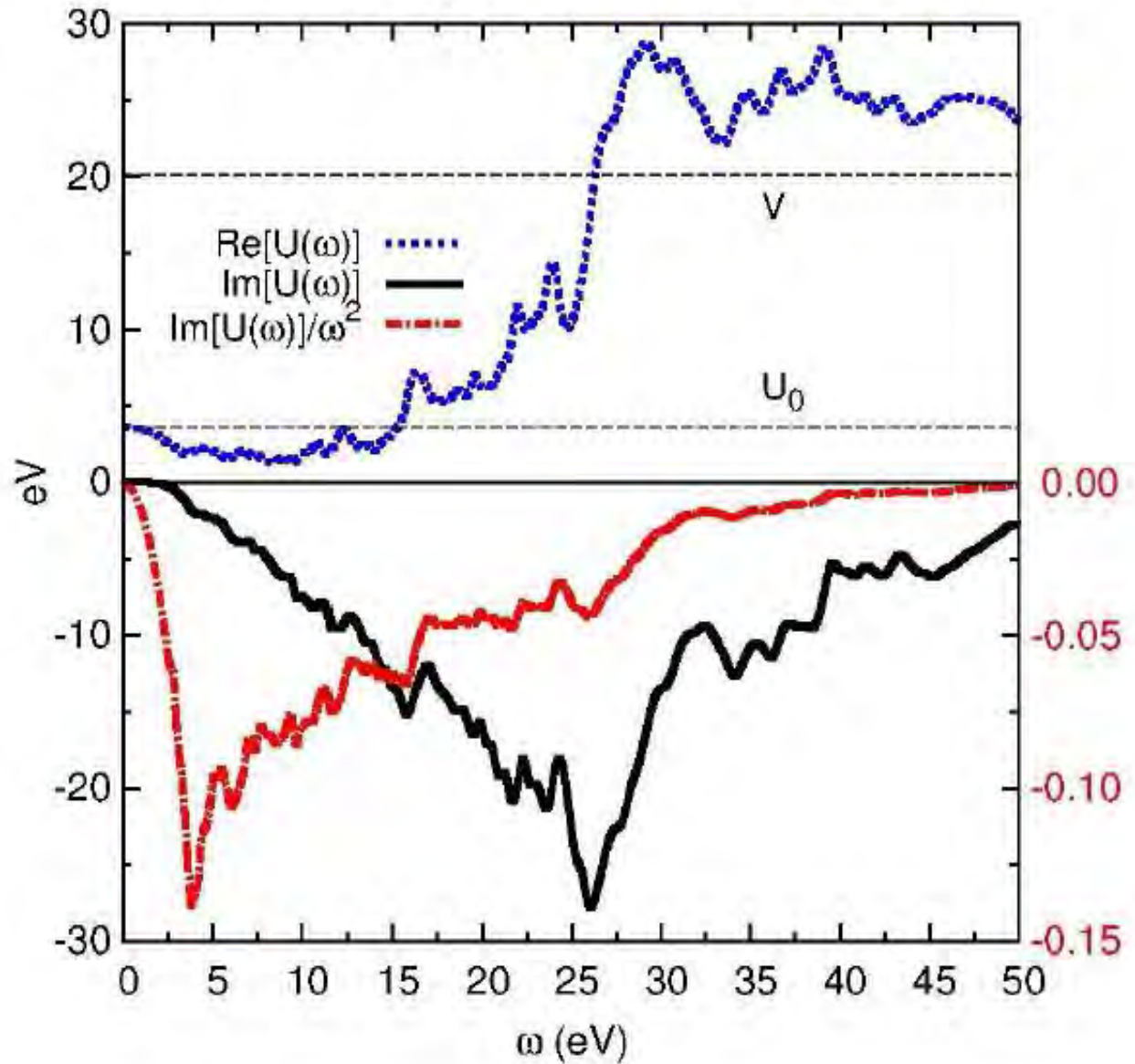
See also work by: Imada and Solovyev, Miyake, Nakamura ...

BaFe₂As₂

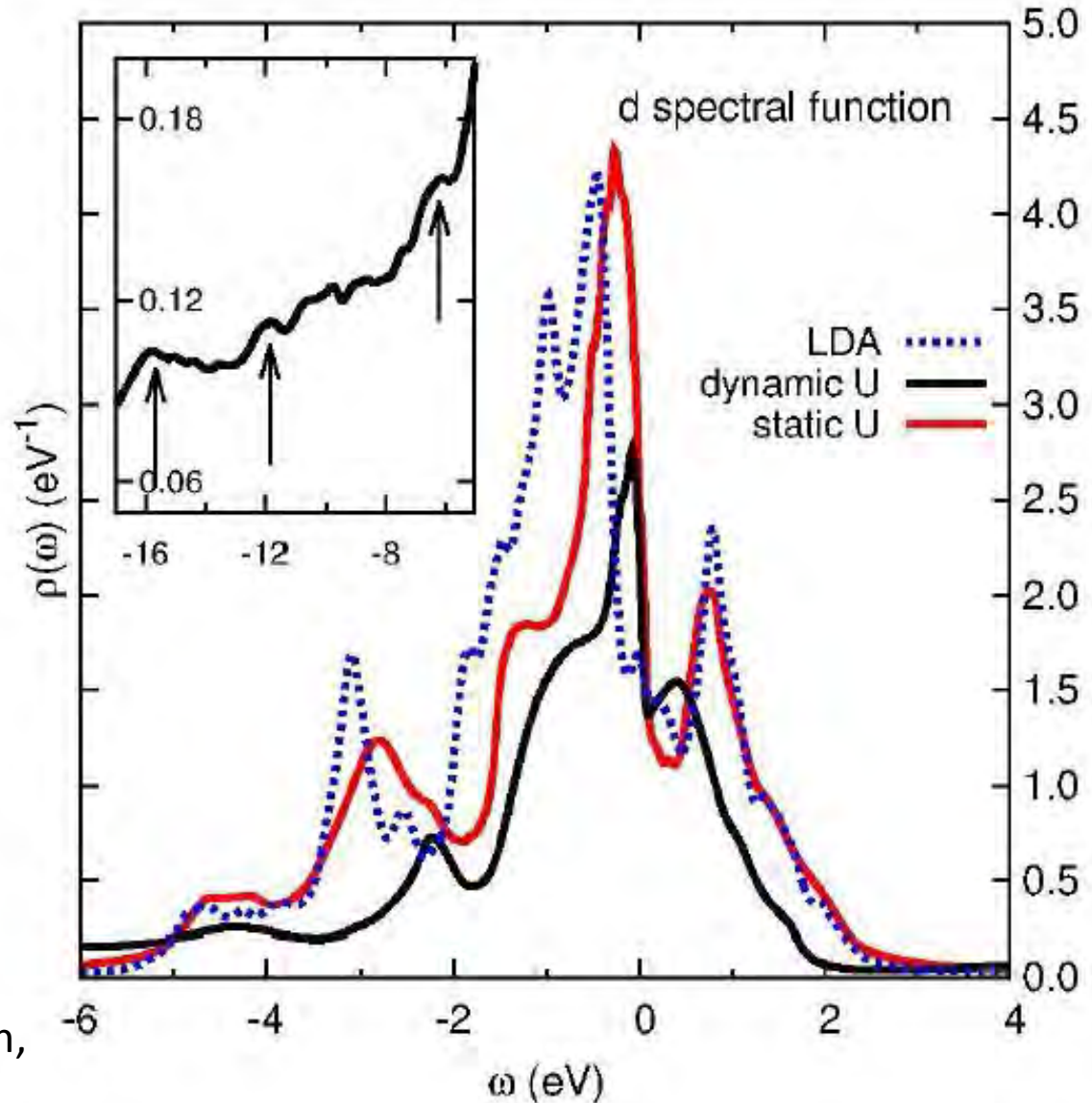
Our focus ...



$U(\omega)$ for BaFe_2As_2



BaFe₂As₂



Werner, Casula, Miyake, Aryasetiawan,
Millis, SB, Nature Physics 2012

See also: “electronic polaron effect” Casula, Werner, Miyake, Aryasetiawan, Millis, SB, PRL 2012

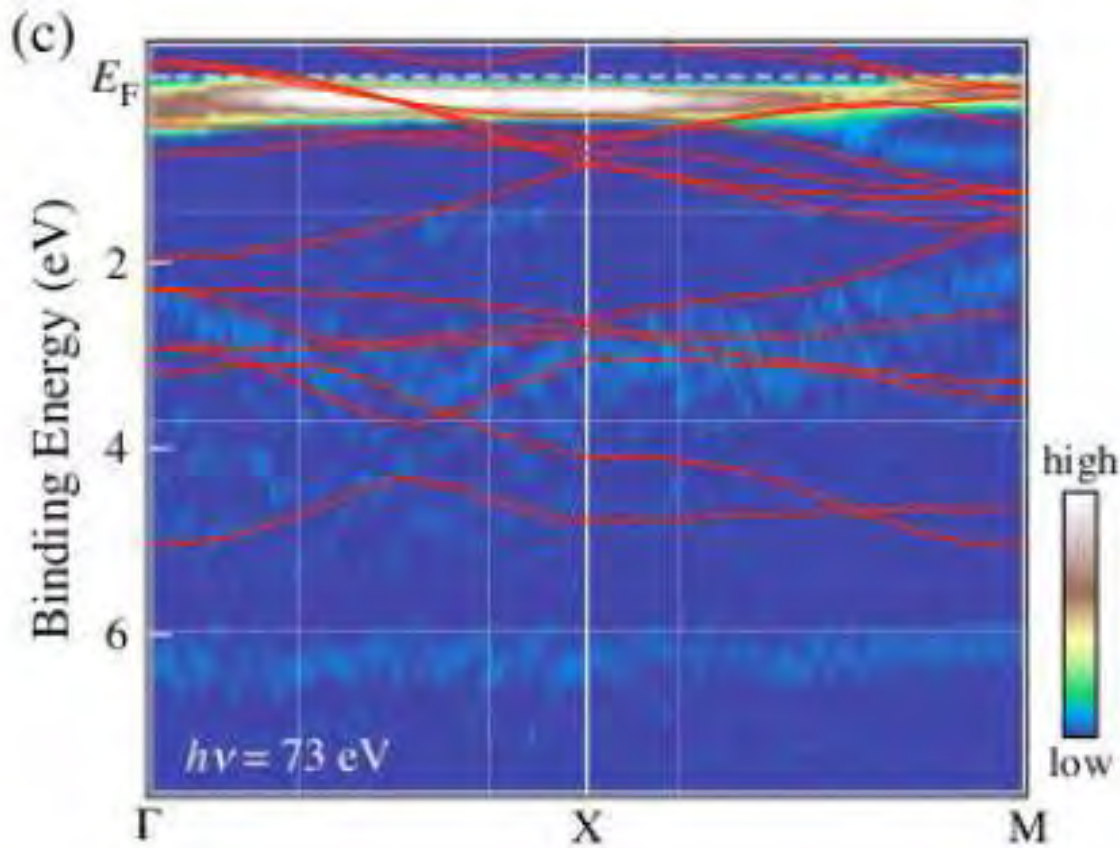
Origin of spectral weight loss?

- Problem with dynamical interactions \Leftrightarrow static problem with *screening bosons*
- Diagonalisation of fermion-boson problem leads to “*electronic polarons*”
- Spectral function contains *plasmon replicae*
- **Weight** of main peak reduced to

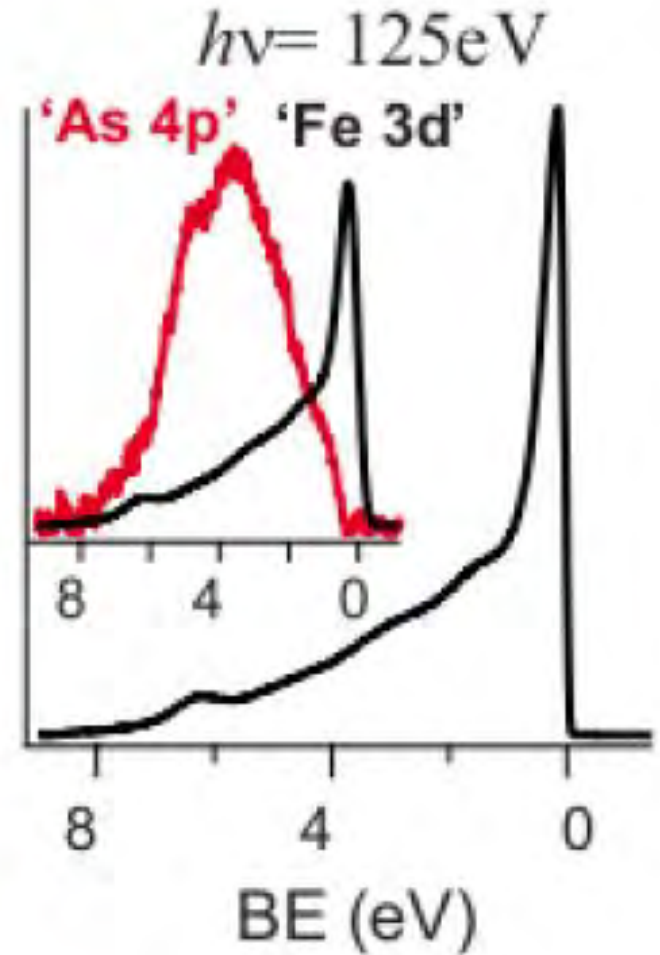
$$Z_B = e^{-\frac{1}{\pi} \int_0^\infty d\omega \frac{\text{Im} U(\omega)}{\omega^2}}$$

[Casula, Werner, Miyake, Aryasetiawan, Millis, SB, PRL 2012]

BaFe₂As₂ Photoemission



Ding et al





Yi et al.

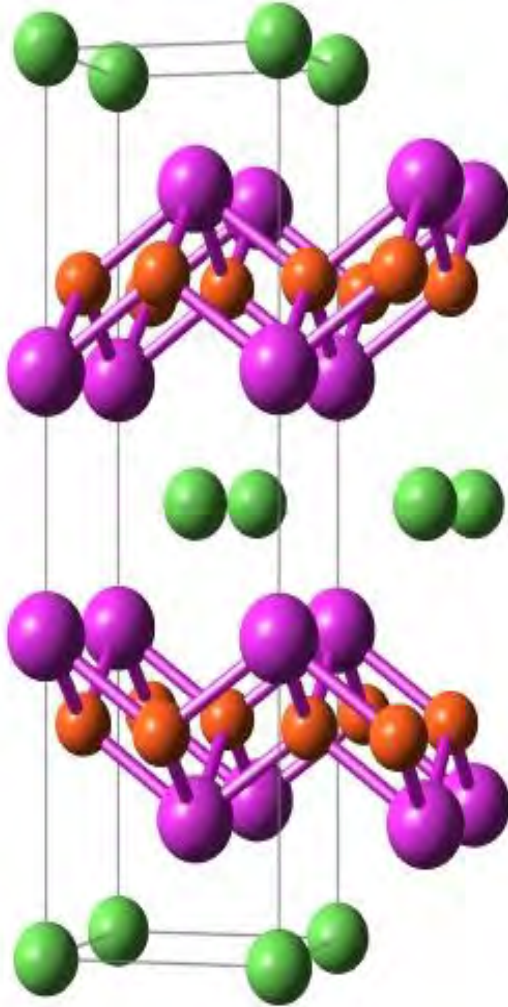
Side remark

- DMFT with dynamical $U(\omega)$ closely related to cumulant approach: approximate DMFT solver for dynamical impurity problem (Casula et al., PRB 2012) can be understood as a generalized cumulant ansatz
- See also: “slave rotor” method for dynamical impurity problems: Krivenko & SB, PRB 2015
- Cf. description of plasmons with GW+cumulant approaches (cf. work by L. Hedin, F. Aryasetiawan, S. Louie, L. Reining, J. Rehr ...)

Effects of dynamical interactions:

- Replicaes and spectral weight transfers 
- Additional renormalisations 
 - (with respect to what starting electronic structure?)

Cobalt Pnictide: BaCo₂As₂

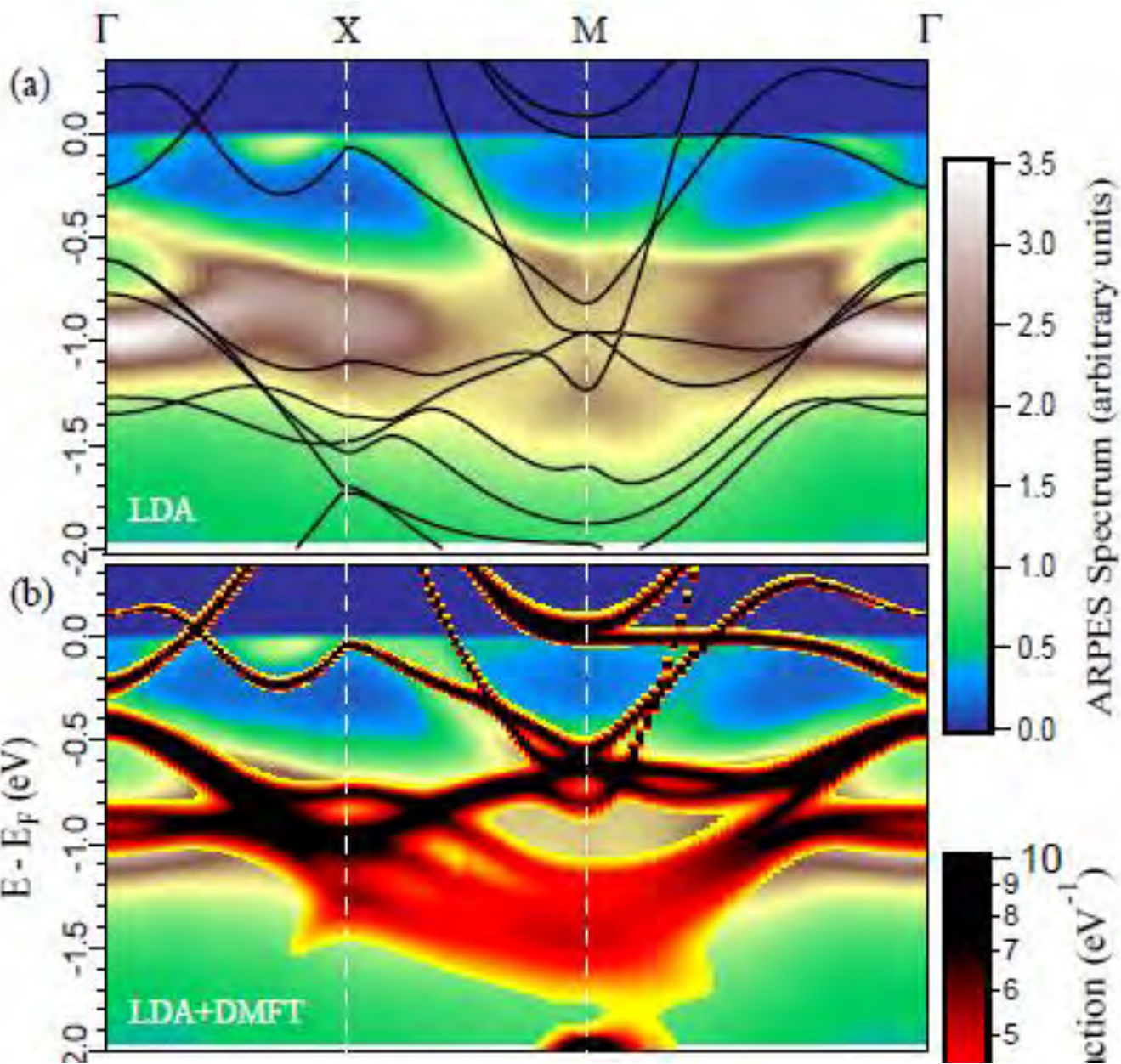


Fe-d7 configuration
=> weakly correlated

Cobalt Pnictide: BaCo₂As₂



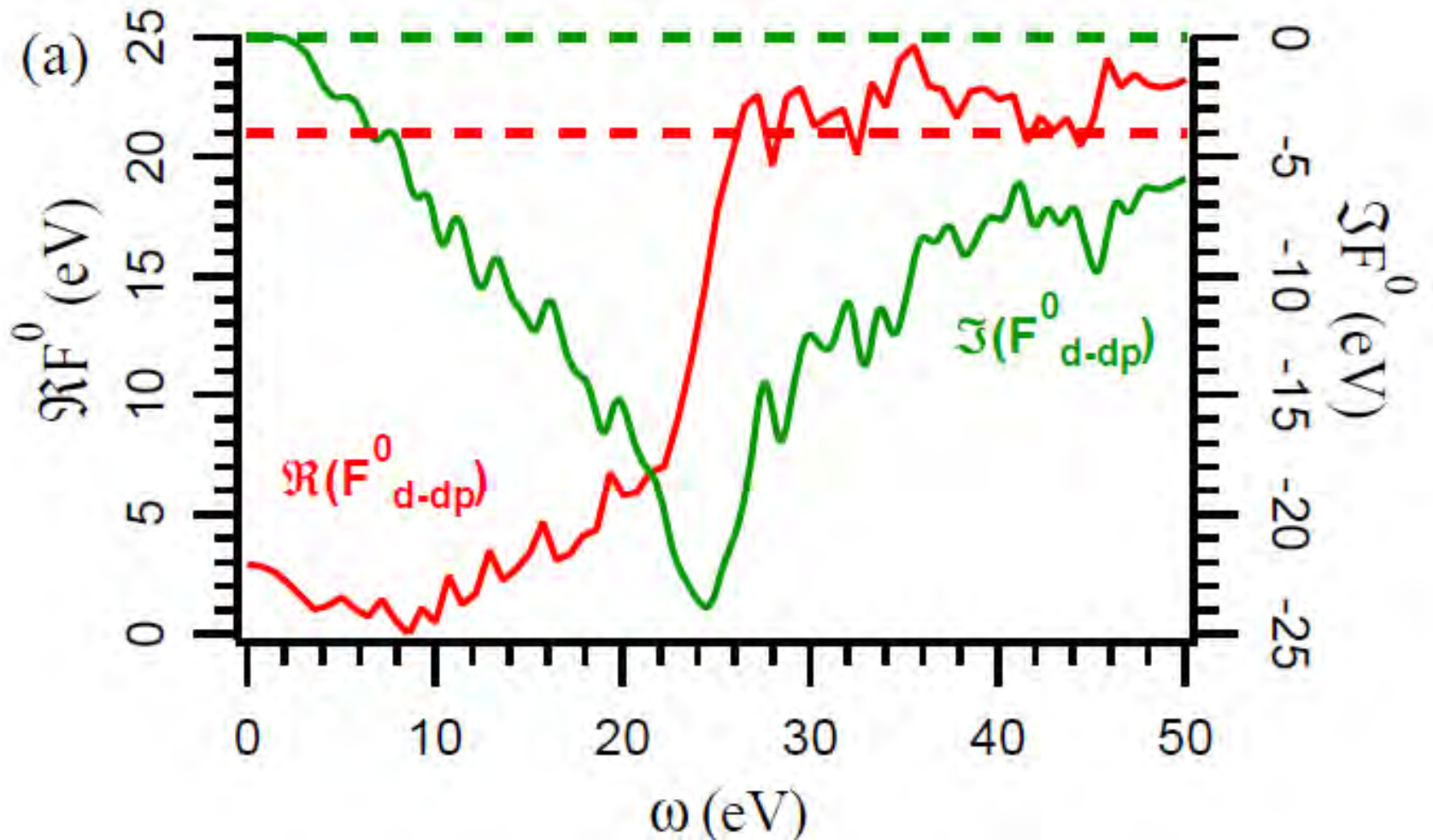
A. Van Roekeghem
IOP-CAS & Ecole
Polytechnique



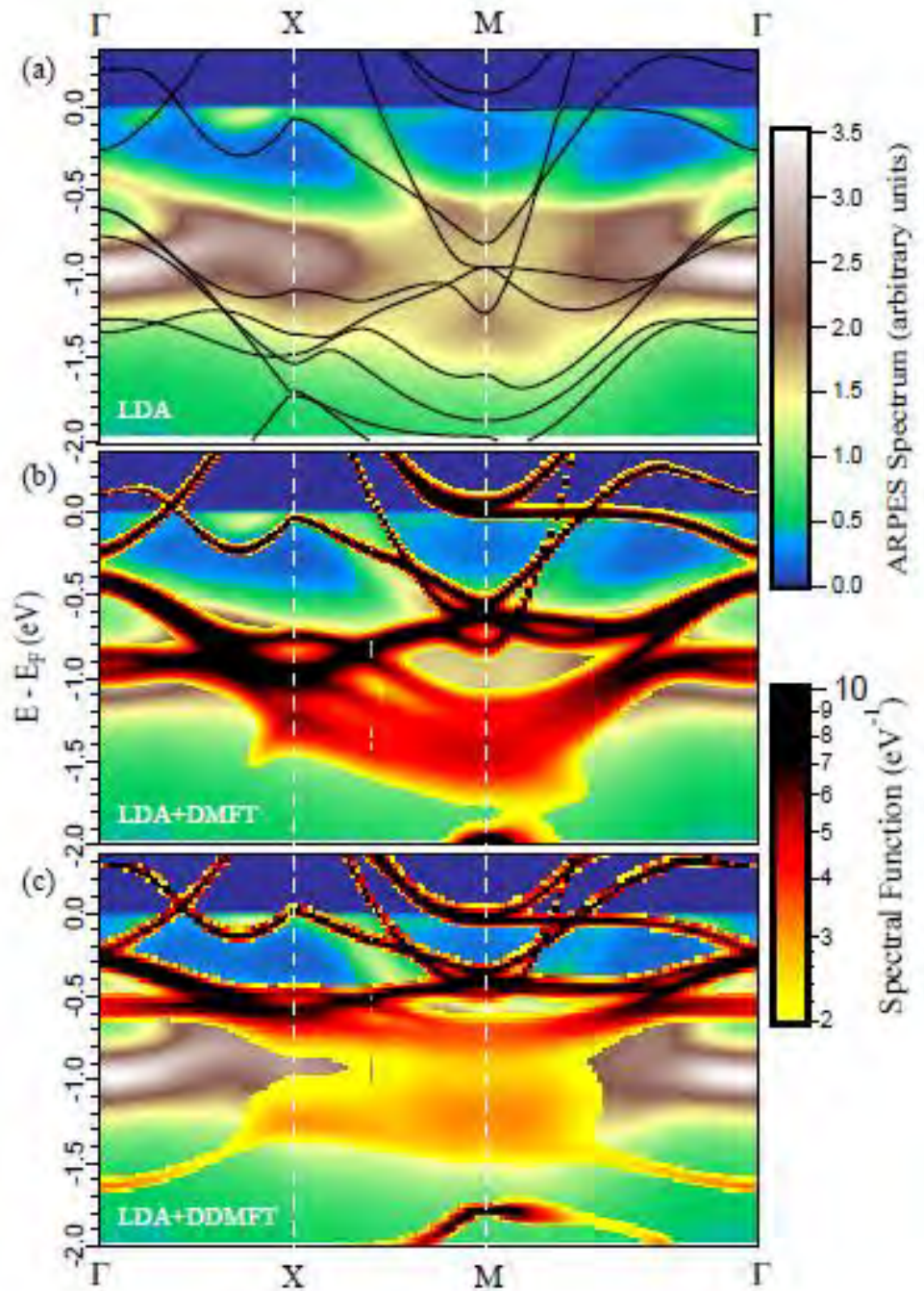
Xu et al., PRX (2013) &
A. van Roekeghem et
al., PRL 2014

See also:
PES by Dhaka et al.

Dynamical interaction for BaCo₂As₂ (from cRPA)



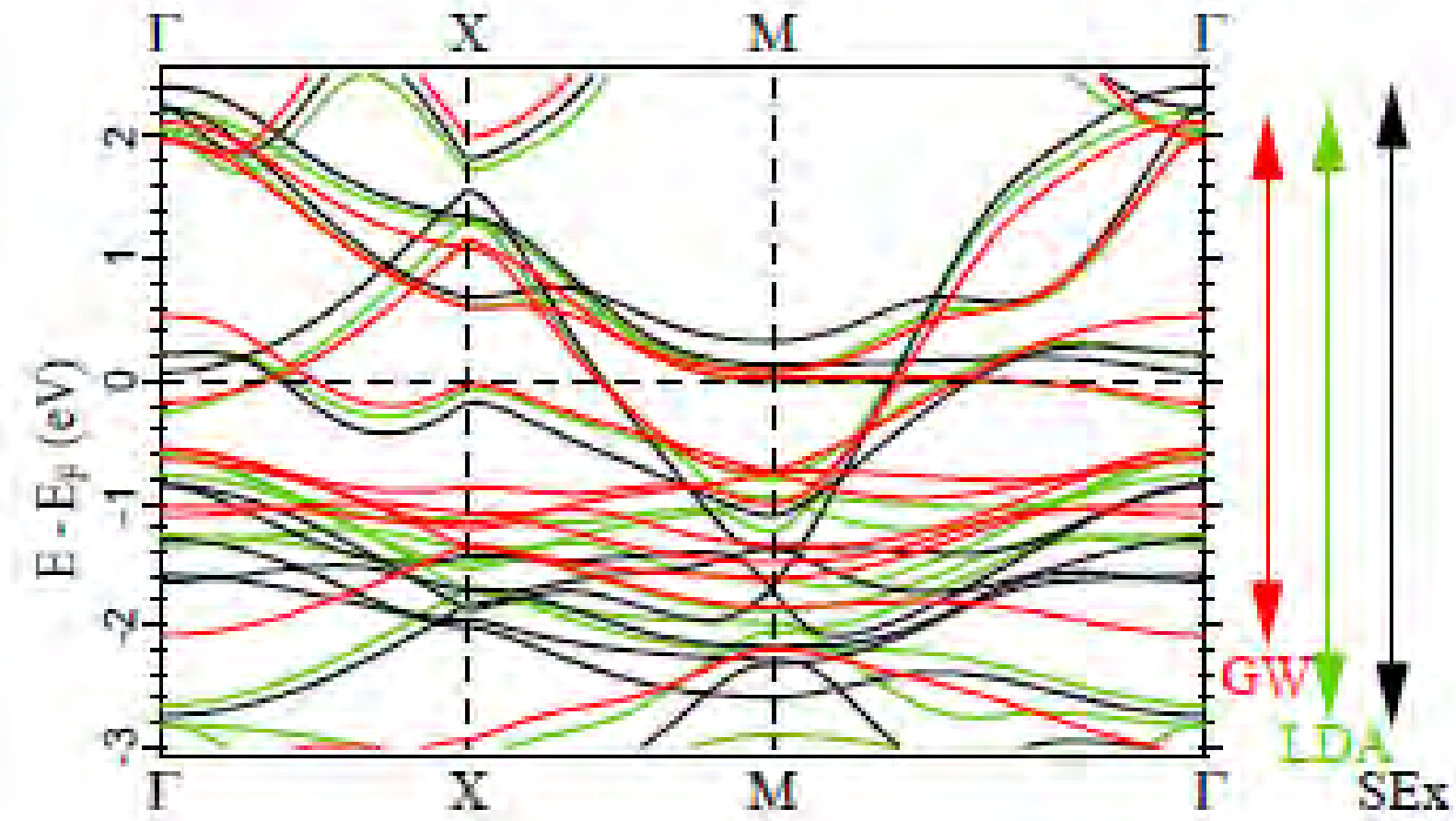
With
dynamical
effects



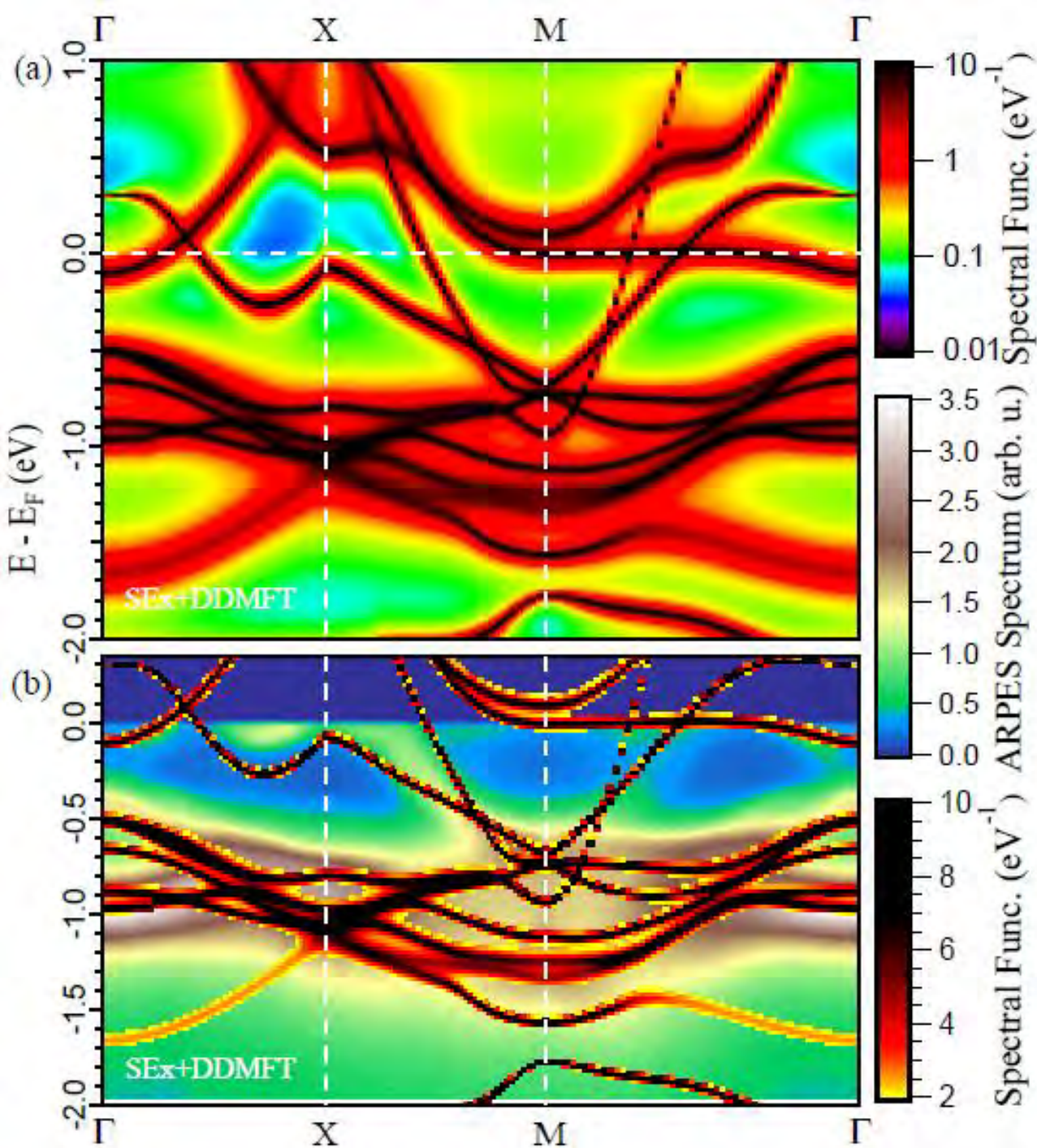
Have we worked too much?

No! Not enough!

Nonlocal corrections: Screened exchange vs LDA



Screened Exchange + DMFT

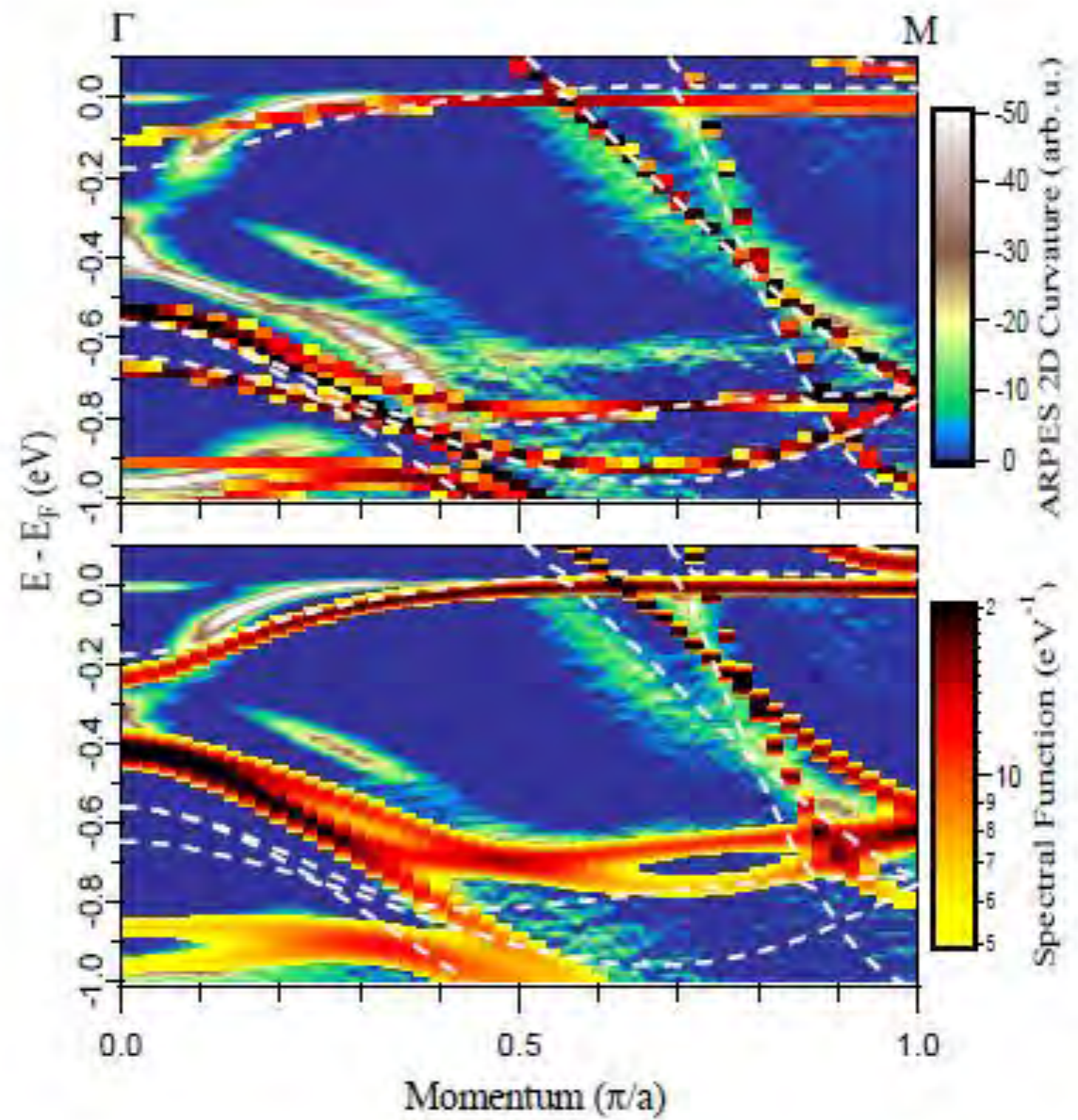


A. van Roekeghem et al.,
Phys. Rev. Lett. 2014

“Screened
exchange+DMFT”

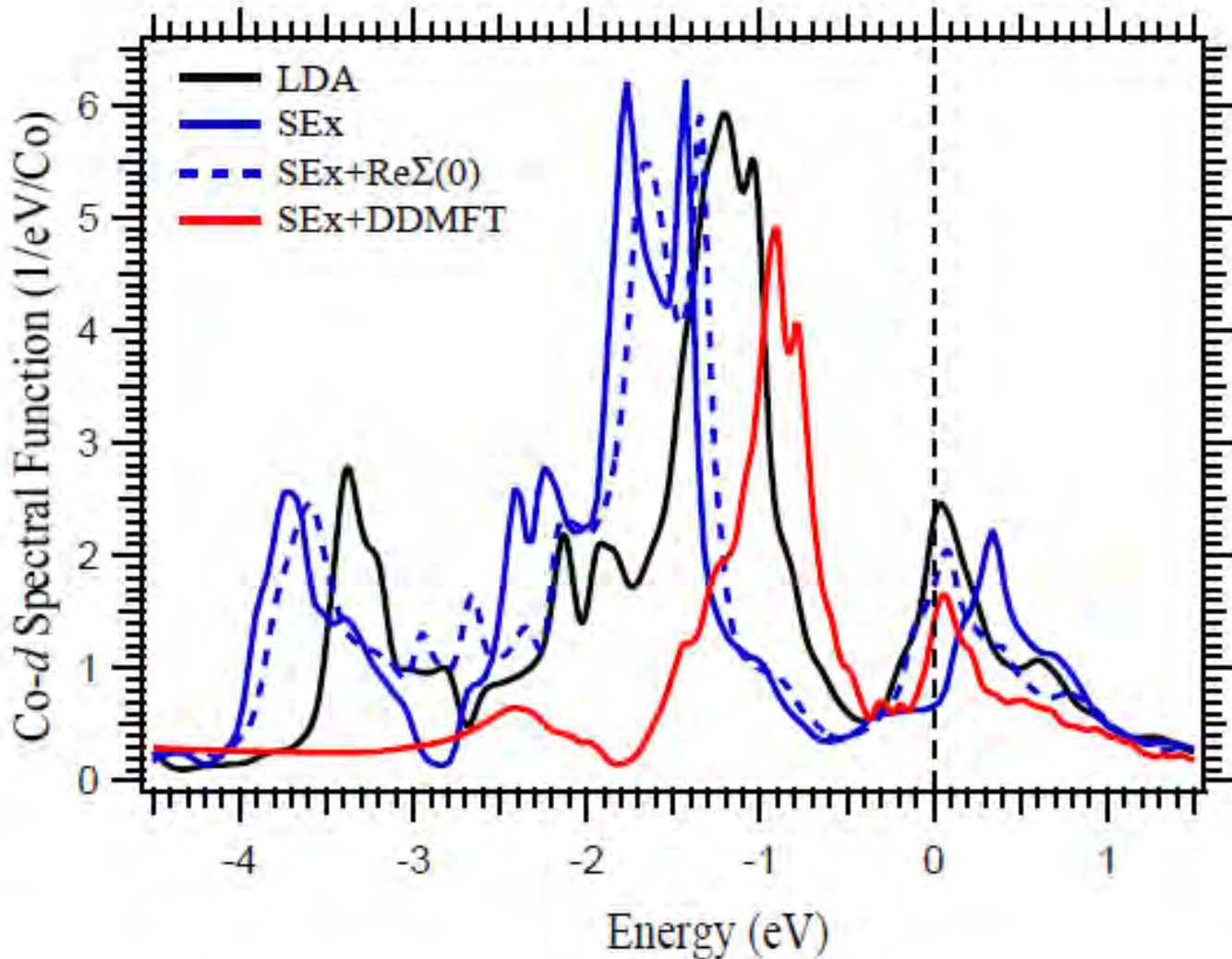
VS.

DFT+DMFT



Description of Fermi surface corrected by Screened Exchange+DMFT !

Momentum-integrated Co-3d spectral function

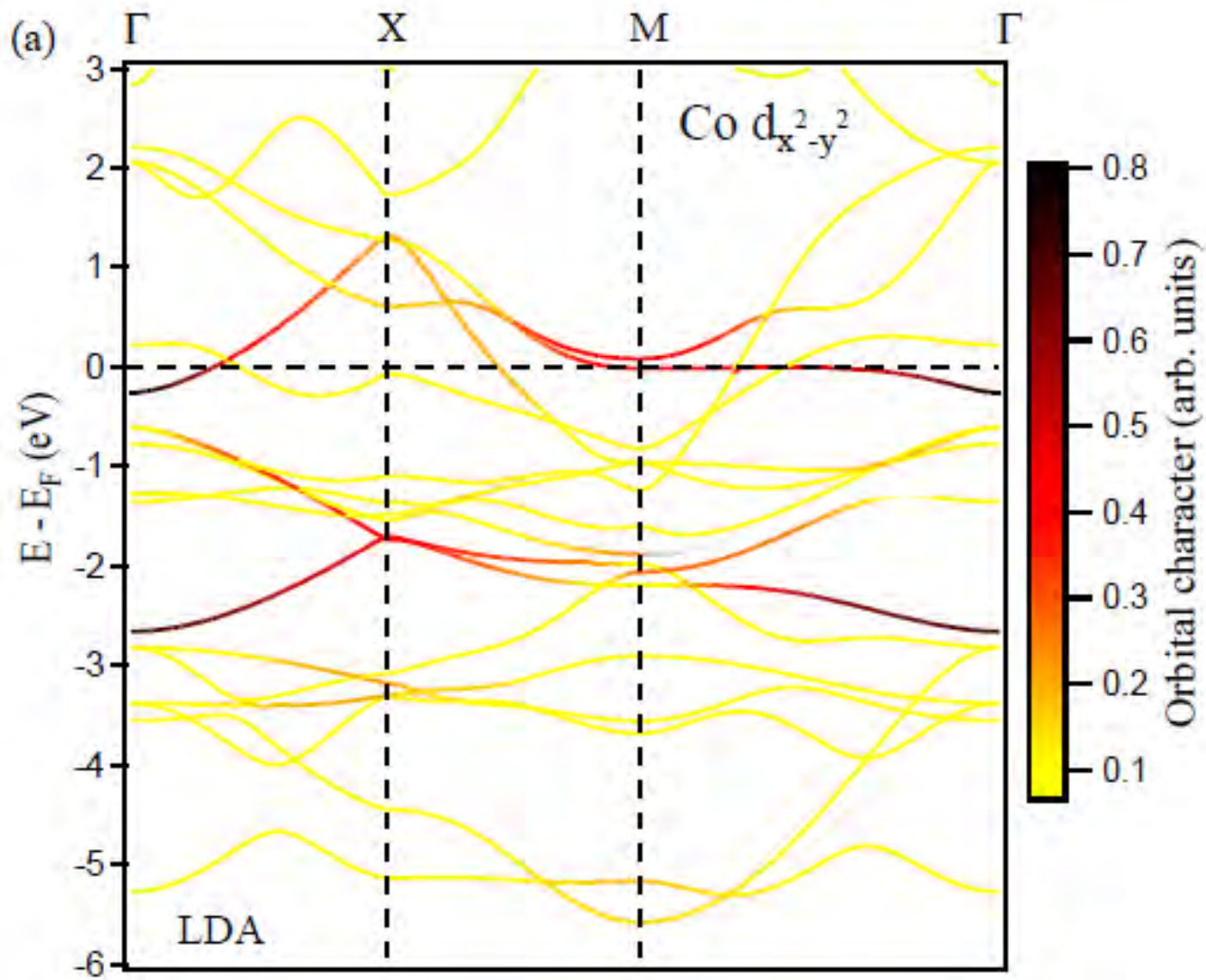


Solves yet another puzzle:

LDA DOS would suggest Stoner ferromagnetism (Sefat et al., PRB 2009),
however: in nature no magnetic order.

[Note: CaCo_2As_2 is ferromagnetic]

Dx2-y2 – a very sensitive proxy ...



Screened Exchange Dynamical Mean Field Theory ...

- ... a simplified version of GW+DMFT

Biermann, Aryasetiawan, Georges, PRL 2003

Sun and Kotliar, PRL 2004

(see also related “GD+SOPT+DMFT” scheme by Sun & Kotliar, PRL 2002)

Application to real materials:

SrVO₃: Tomczak et al. , EPL 2012 and PRB 2014, for GW+DMFT

Fully self-consistent implementation for Sn/Si(111) : Hansmann et al., PRL 2013]

“GW+DMFT”

- A combination of Hedin’s GW approximation and DMFT

The $\Psi[G,W]$ functional

The free energy of a solid can be expressed as a functional $\Gamma[G,W]$ of

- 1) the Green's function G and
- 2) the screened Coulomb interaction W .

[Almbladh et al., Int J. Qu. Chem. 1999, Chitra et al. 2000]

$$\Gamma[G,W] = \text{Hartree part} + \Psi[G,W]$$

The GW+DMFT functional

- $\Gamma[G,W] = \text{Hartree part} + \Psi[G,W]$

- $\Psi \approx \Psi^{\text{EDMFT}}[G_{ii}, W_{ii}] + \Psi_{\text{nonloc}}^{\text{GW}}[G_{ij}, W_{ij}],$

where $\Psi_{\text{nonloc}}^{\text{GW}} = \Psi^{\text{GW}} - \Psi_{\text{loc}}^{\text{GW}}.$

Biermann, Aryasetiawan, Georges, PRL 2003, and arxiv 2004

Sun & Kotliar, PRL 2004

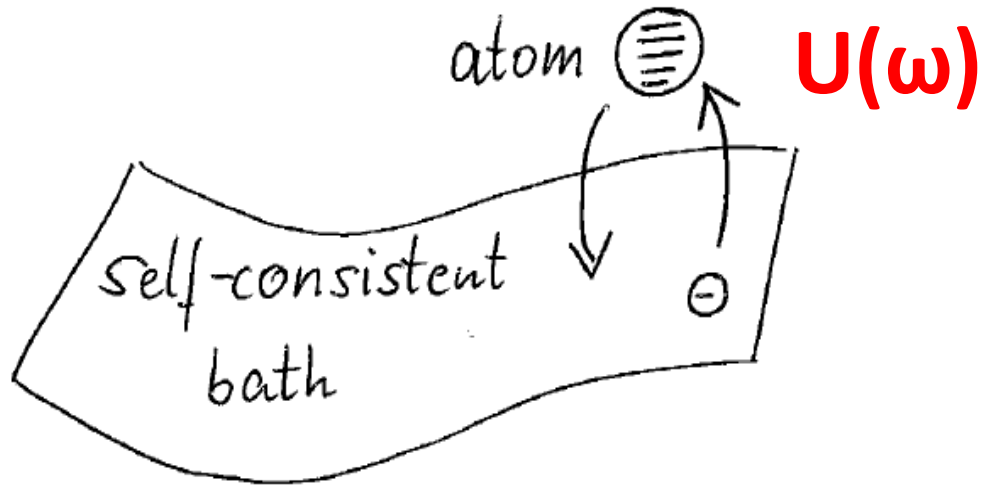
Ayral, Werner, Biermann, PRL 2012, PRB 2013

Hansmann, Ayral, Vaugier, Werner, Biermann, PRL 2013

Review: Biermann, J. Phys. Cond. Matt. (2014)

EDMFT

Calculate G_{loc} and W_{loc} from a dynamical impurity model (that is, an impurity model with bath and dynamical Hubbard interactions).



Determine bath (the dynamical mean field) and dynamical Hubbard $U(\omega)$ self-consistently

GW+DMFT Eqs.

Impurity model :

$$\mathcal{G}(\tau), \mathcal{U}(\tau)$$

$$G_{imp} \equiv -\langle T_{\tau} c c^{\dagger} \rangle_S \rightarrow \Sigma_{imp}^{xc} = \mathcal{G}^{-1} - G_{imp}^{-1}$$

$$W_{imp} = \mathcal{U} - \mathcal{U} \chi \mathcal{U} \quad P_{imp} = \mathcal{U}^{-1} - W_{imp}^{-1}$$

↑

↓

Update

Combine :

$$G^{-1} = G_{loc}^{-1} + \Sigma_{imp} \quad \Sigma = \Sigma_{imp} + \Sigma_{GW}^{nonlocal}$$

$$U^{-1} = W_{loc}^{-1} + P_{imp} \quad P = P_{imp} + P_{GW}^{nonlocal}$$

↑

↓

Self - consistency

$$G_{loc} = \sum_{\mathbf{k}} [G_H^{-1} - \Sigma^{xc}]^{-1}$$

$$W_{loc} = \sum_{\mathbf{q}} [V_{\mathbf{q}}^{-1} - P]^{-1}$$

+ outer loop: self-consistency over GW calculation: update $P_{nonlocal}$ and $\Sigma_{nonlocal}$

Motivation:

- Separation, at the GW level, of self-energy into local dynamical and nonlocal static part:

$$\Sigma = \text{GW} = \Sigma_{\text{loc}}(\omega) + \Sigma_{\text{nonlocal}}(\mathbf{k})$$

(empirically found for pnictides in Tomczak, Schilfgaarde, Kotliar, PRL 2012;
for SrVO₃, see Miyake et al, PRB 2012, Tomczak et al., PRB 2014)

=> GW+DMFT equivalent to DMFT on top of effective single-particle Hamiltonian $H=H_0+\Sigma_{\text{nonlocal}}(\mathbf{k})$

- Now: identify nonlocal part with screened exchange GW(0)

Screened Exchange Dynamical Mean Field Theory ...

- ... a simplified version of GW+DMFT

[Biermann, Aryasetiawan, Georges, PRL 2003]

[See also Tomczak et al. , EPL 2012 and PRB 2014, for GW+DMFT for SrVO₃, and the recent self-consistent implementation for Sn/Si(111) : Hansmann et al., PRL 2013]

- ... a dynamical non-perturbative generalization of Hedin's Coulomb-Hole-Screened-Exchange (“COHSEX”) scheme

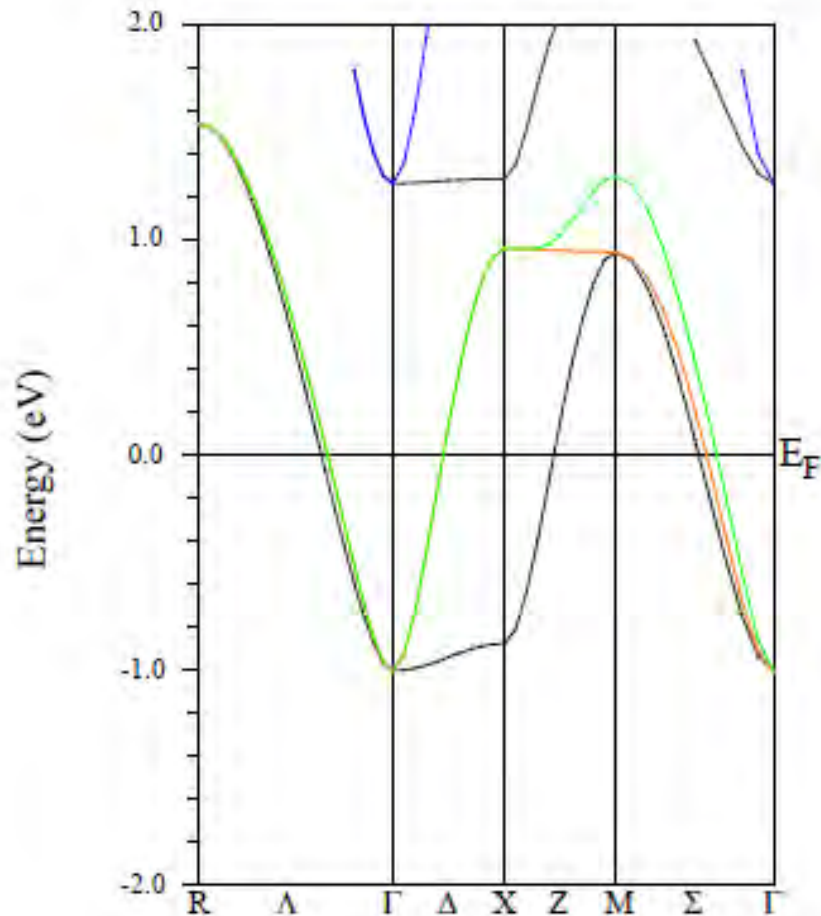
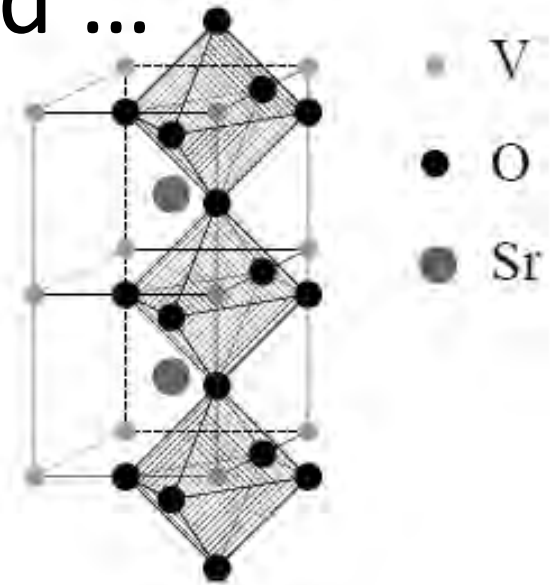
[Hedin, PRB (1965)]

- ... a combination of generalized Kohn-Sham schemes [Goerling and others] with “dynamical DMFT”(*)

(*) “Dynamical DMFT” = DMFT with frequency-dependent interactions

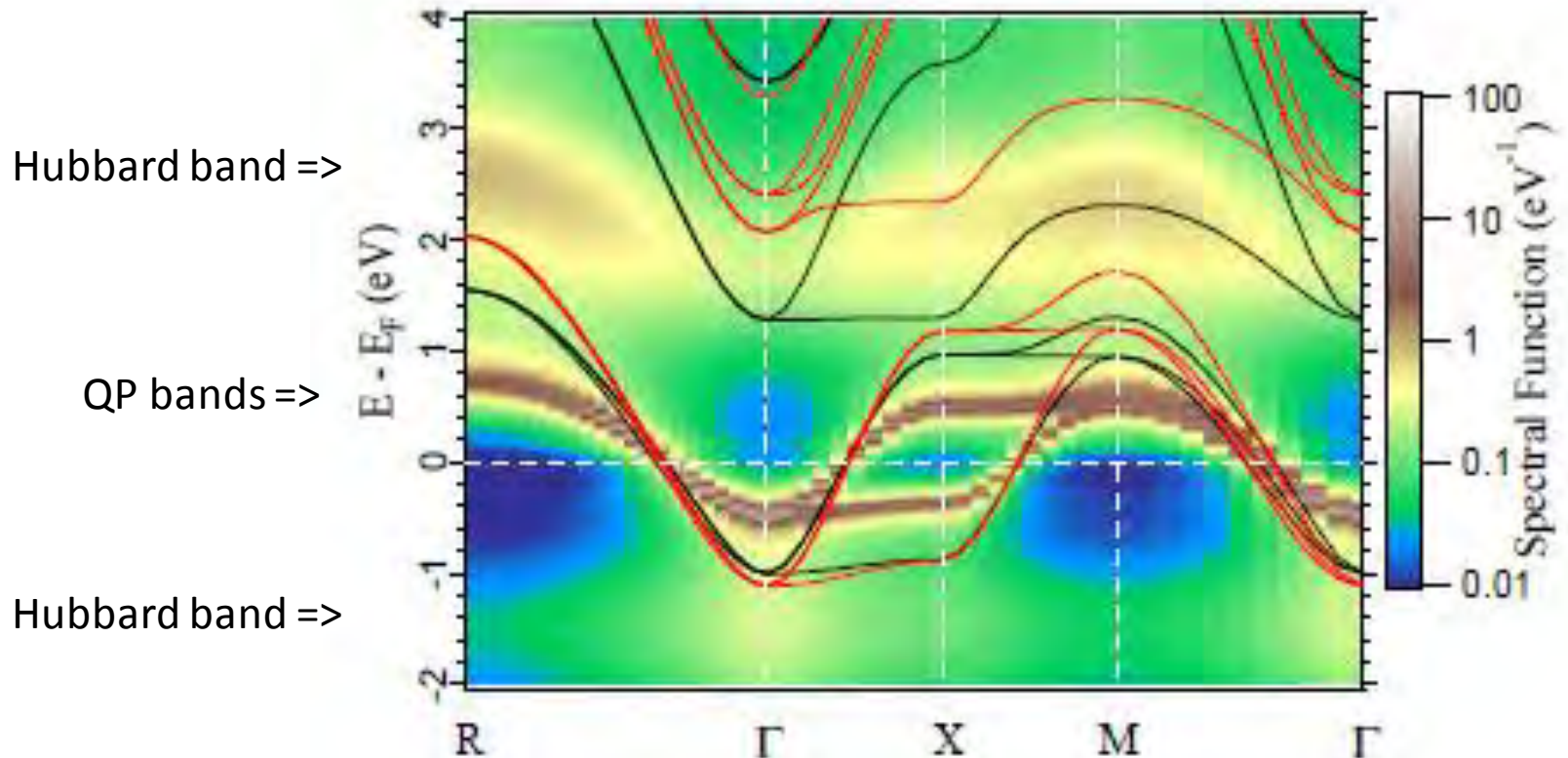
A simpler example: SrVO₃

SrVO₃: a drosophila compound ...



- Test compound for DFT+DMFT implementations (see e.g. Pavarini et al., PRL 2004 and various others ...)
- GW+DMFT: Tomczak et al., EPL 2012, PRB 2014
- LDA+U(w)+DMFT: Casula et al., PRB 2012
- Various GW- or DMFT-inspired schemes: Gatti&Guzzo, PRB 2013, Sakuma PRB 2013, Taranto PRB 2013.

A simpler example: SrVO₃



Black: LDA

Red: Screened exchange

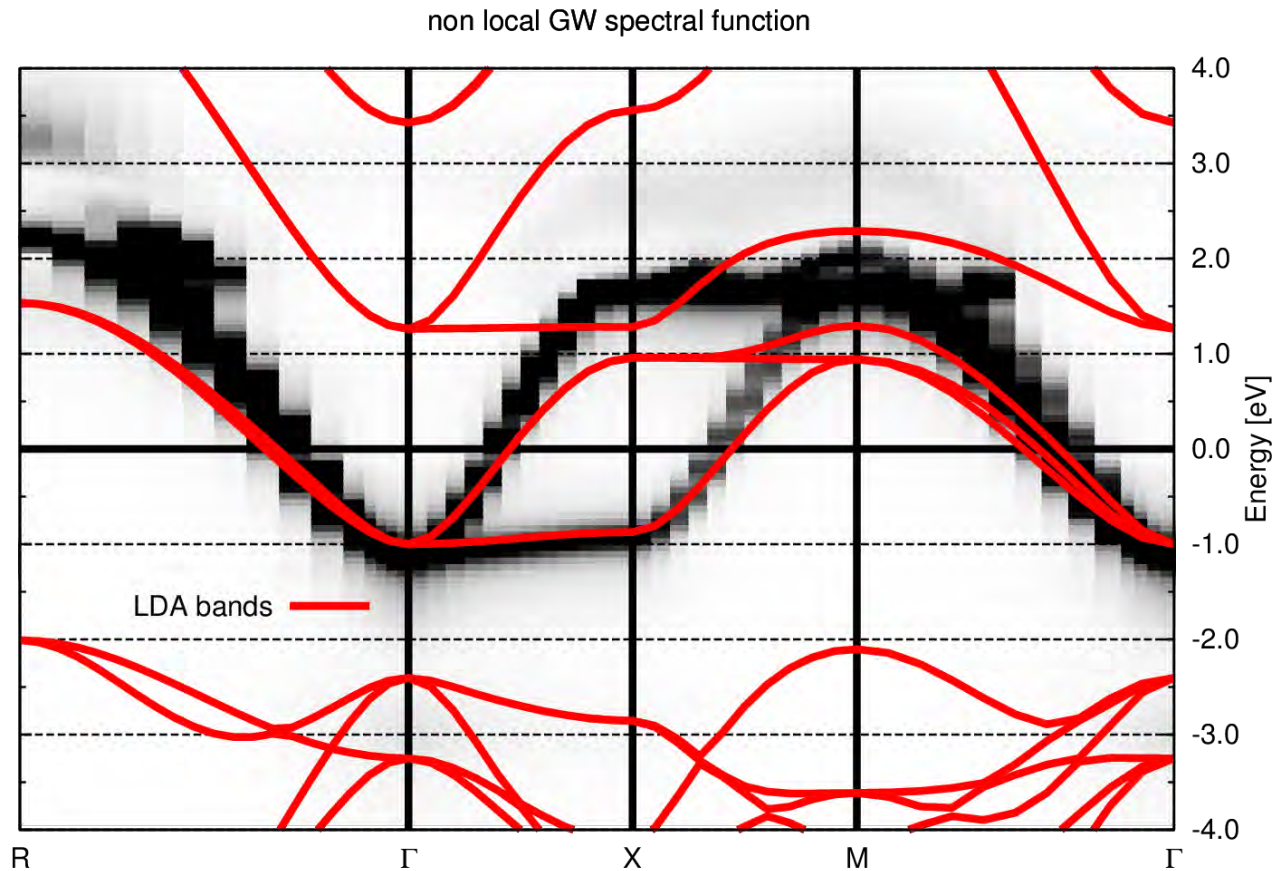
Color: Screened exchange dynamical mean field theory

Excellent agreement with ARPES!

See work by Fujimori's group

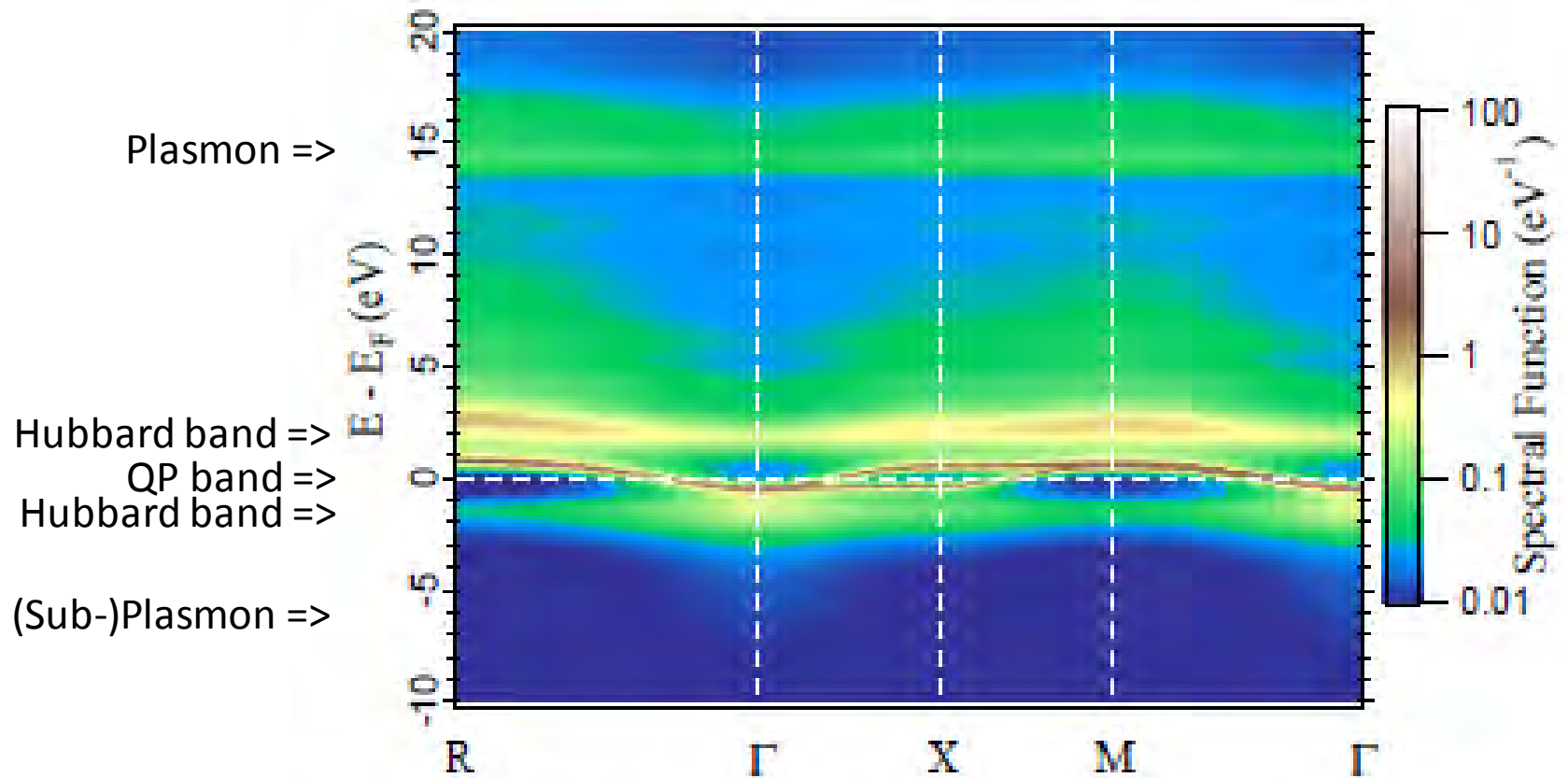
A. Van Roekeghem and S. Biermann, Europhysics Letters (2014)

Spectral function from “non-local GW”



From: Tomczak, Casula, Miyake, Biermann, PRB 2014

A simpler example: SrVO3

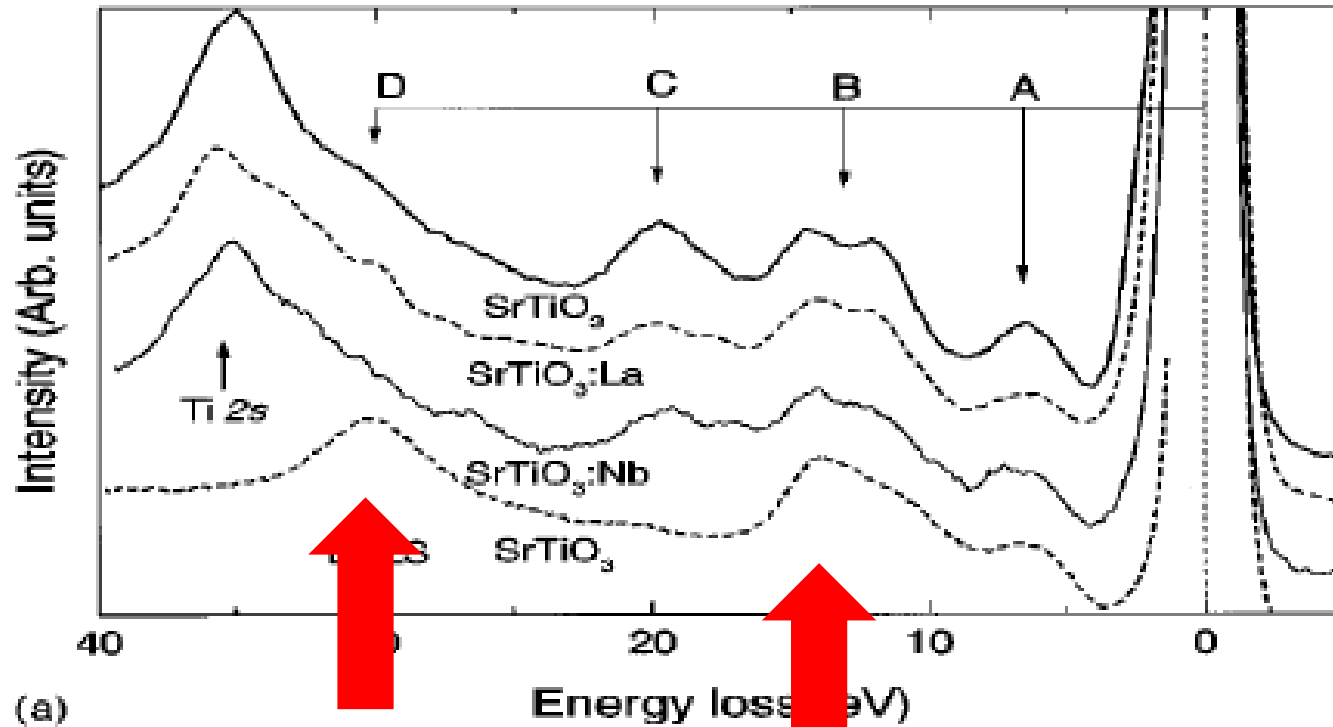


SrVO3 within Screened Exchange Dynamical Mean Field Theory

A. van Roekeghem and S. Biermann, Europhysics Letters (2014)

Consistent with full GW+DMFT calculations in Tomczak, Casula, Miyake, SB, PRB 2014

Plasmons seen in Electron Energy Loss Spectroscopy (EELS) !



(a)
Kohiki et al., PRB 2000

Side remark: why do DFT bands give a reasonable approximation to single-particle excitations of weakly correlated metals?

- Error cancellation between exchange and correlation ! (well-known for total energies. Here: for excitations ...)
- Example: n-doped SrTiO₃:

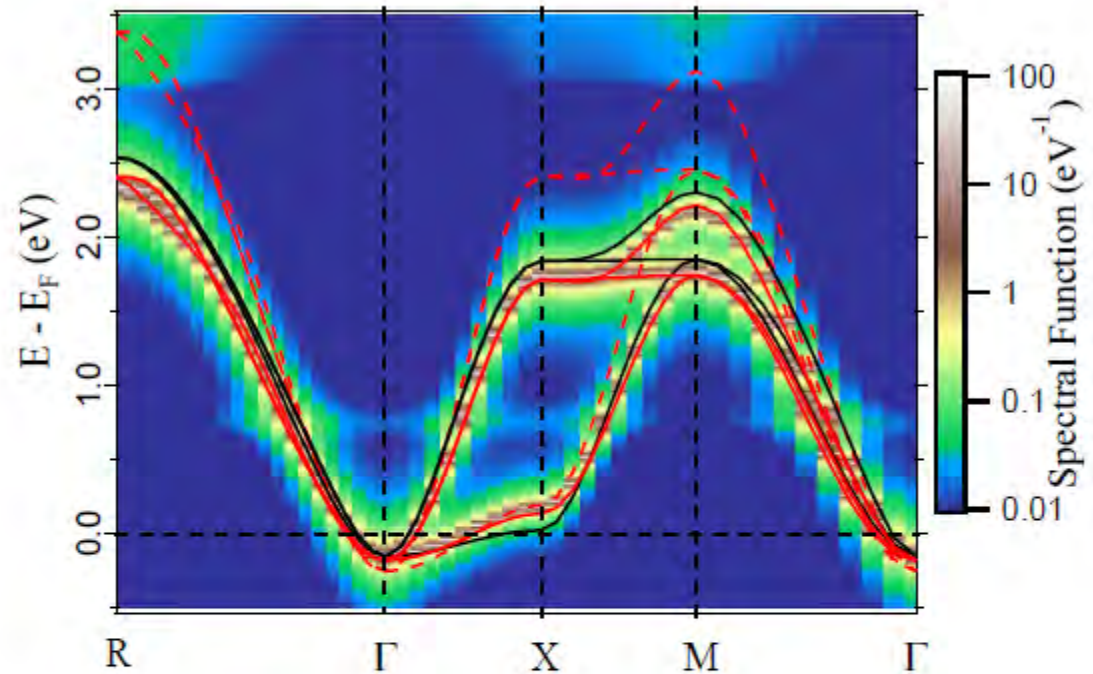


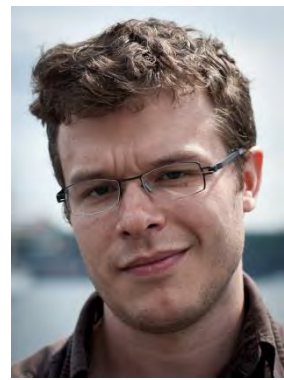
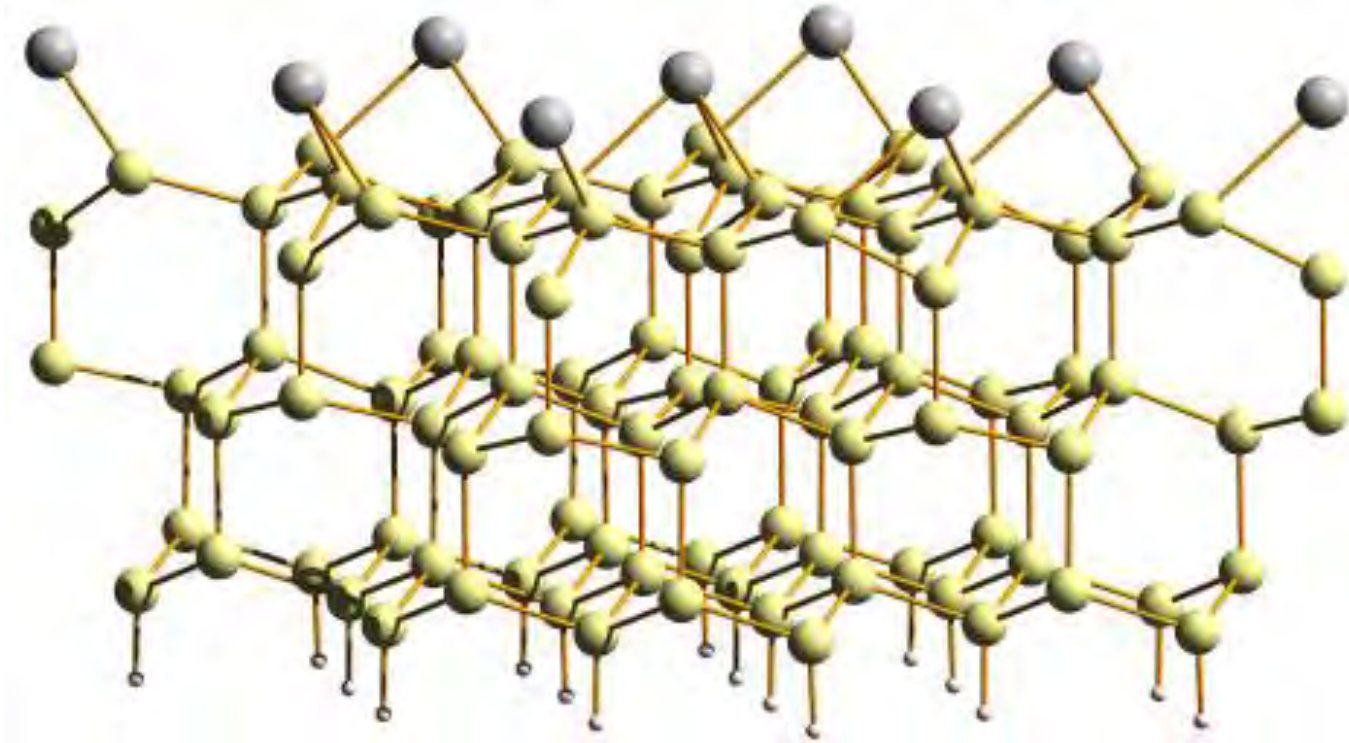
Fig. 4: Band structure of the t_{2g} states of SrTiO₃ within LDA (black lines), Screened Exchange (red dashes) and SEx renormalized by a plasmonic factor $Z_B = 0.7$ (red lines) superimposed on the SEx+DDMFT t_{2g} spectral function. The chemical potential corresponds to $n = 0.05$ electron doping per Ti atom, which gives a self-consistent Thomas-Fermi screening-length of $\lambda = 0.6 a_0^{-1}$ according to the SEx density of states.

GW+DMFT for “real” systems?

- Ferromagnetic Nickel: static U, “[LDA+DMFT]_{local}+GW_{nonloc}” for Σ , self-consistent in DMFT part (inner loop) for fixed GW calculation
(Biermann, Aryasetiawan, Georges, PRL 2003)
- SrVO₃: as above, but based on quasi-particlized-GW (\Rightarrow problem: GW_{loc} counted twice!)
(Taranto et al., PRB 2013)
- SrVO₃: dynamical U from cRPA, “[LDA+U(w)+DMFT]_{local}+GW_{nonloc}” for Σ as a one-shot combination without self-consistency
(Sakuma et al., arXiv2013)
- **SrVO₃: dynamical U from cRPA, true GW+DMFT, self-consistent in DMFT part (inner loop) for fixed GW calculation.**
(Tomczak et al., Europhys. Lett. 2012 and PRB 2014)
- **Systems of adatoms on surfaces: “Sn:Si(111)”**: full GW+DMFT (self-consistency at DMFT and GW level) within low-energy space
(Hansmann, Ayrál, Vaugier, Werner, Biermann, Phys. Rev. Lett. 2013)

See also calculations for extended Hubbard model: Sun&Kotliar, PRL 2004, Karlsson JPCM 2007, Ayrál, Biermann, Werner, PRL 2012 and PRB 2013

A realistic example: Sn/Si(111)



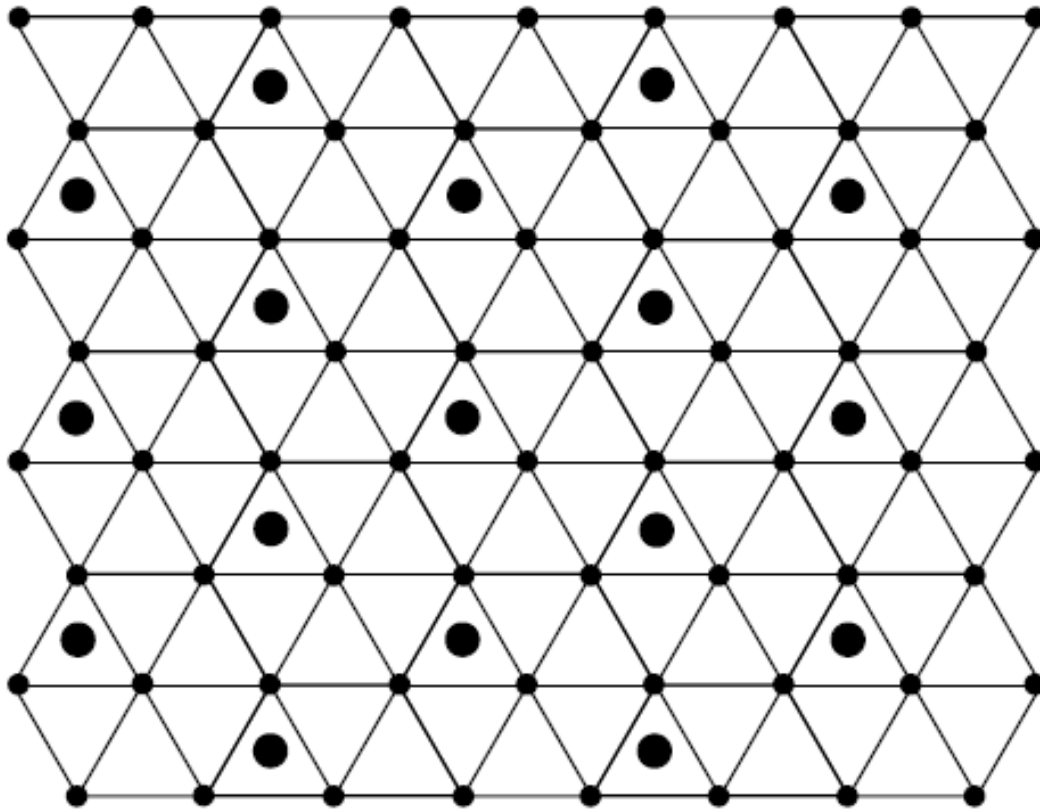
P. Hansmann



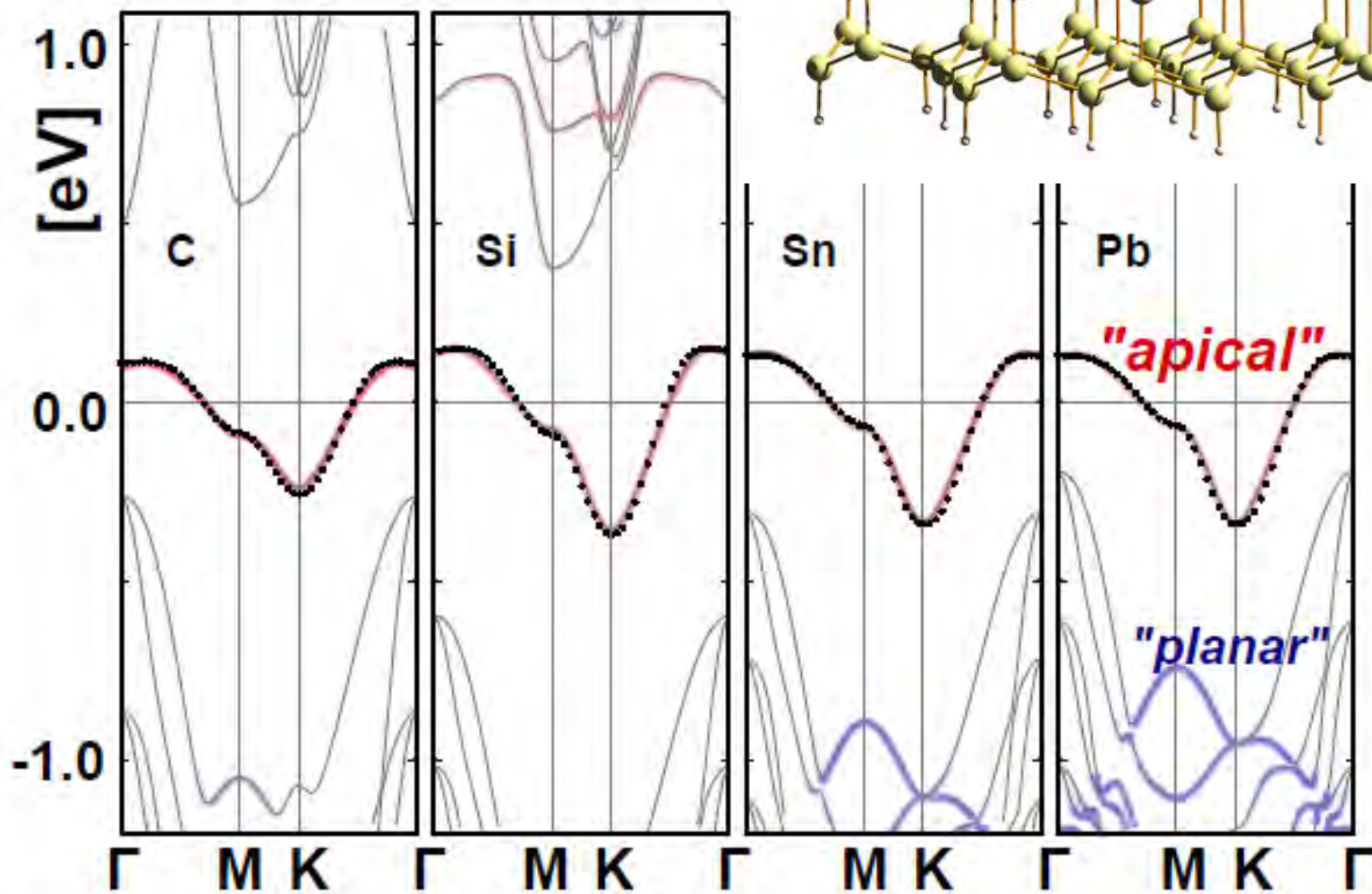
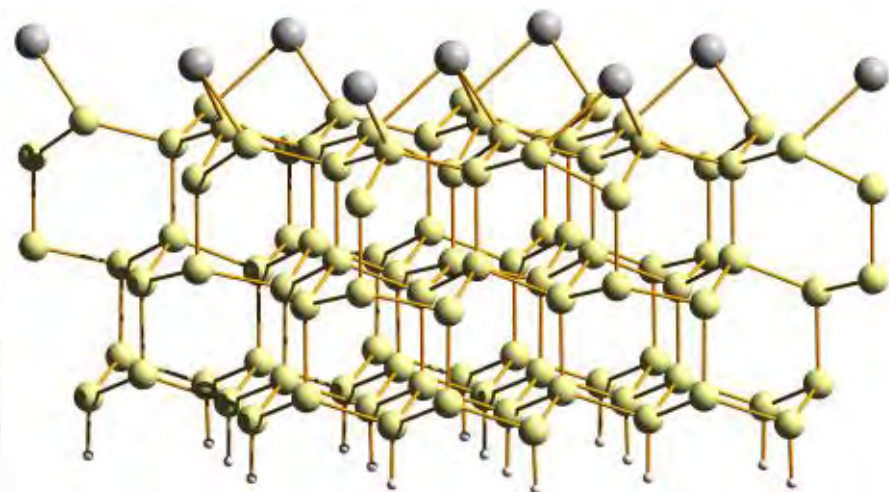
T. Ayrál

Monolayer of adatoms forms triangular lattice

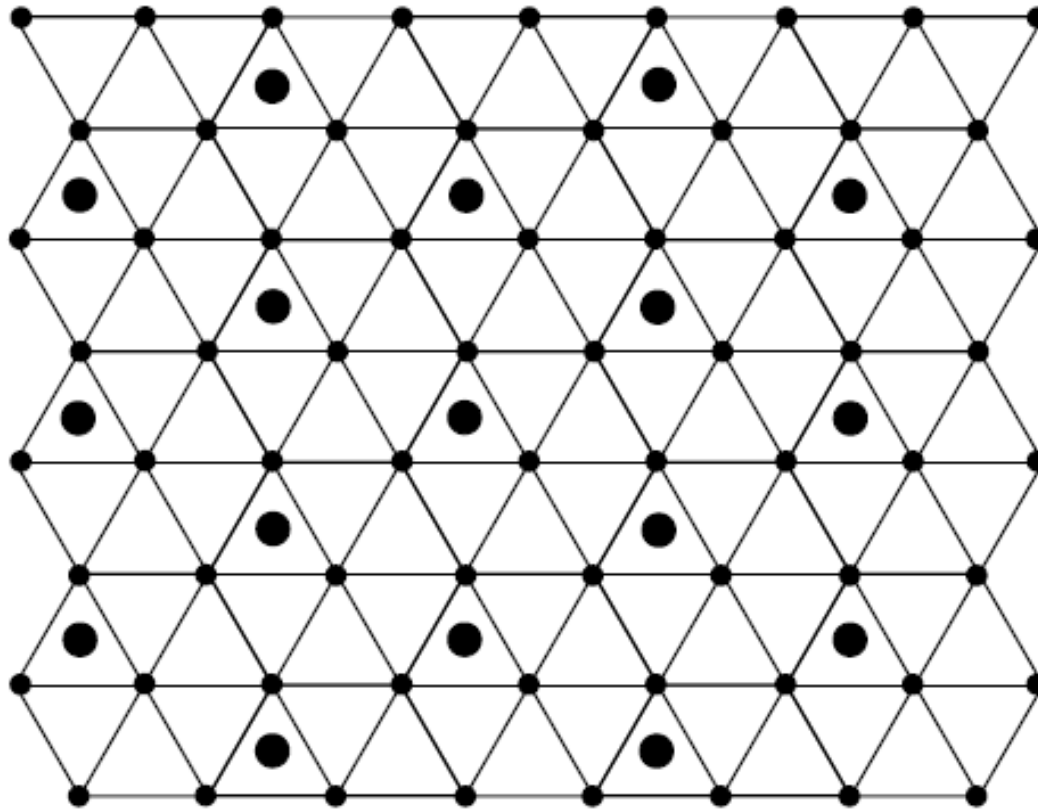
2d triangular lattice by adatoms



X/Si(111) with
X=C, Si, Sn, Pb



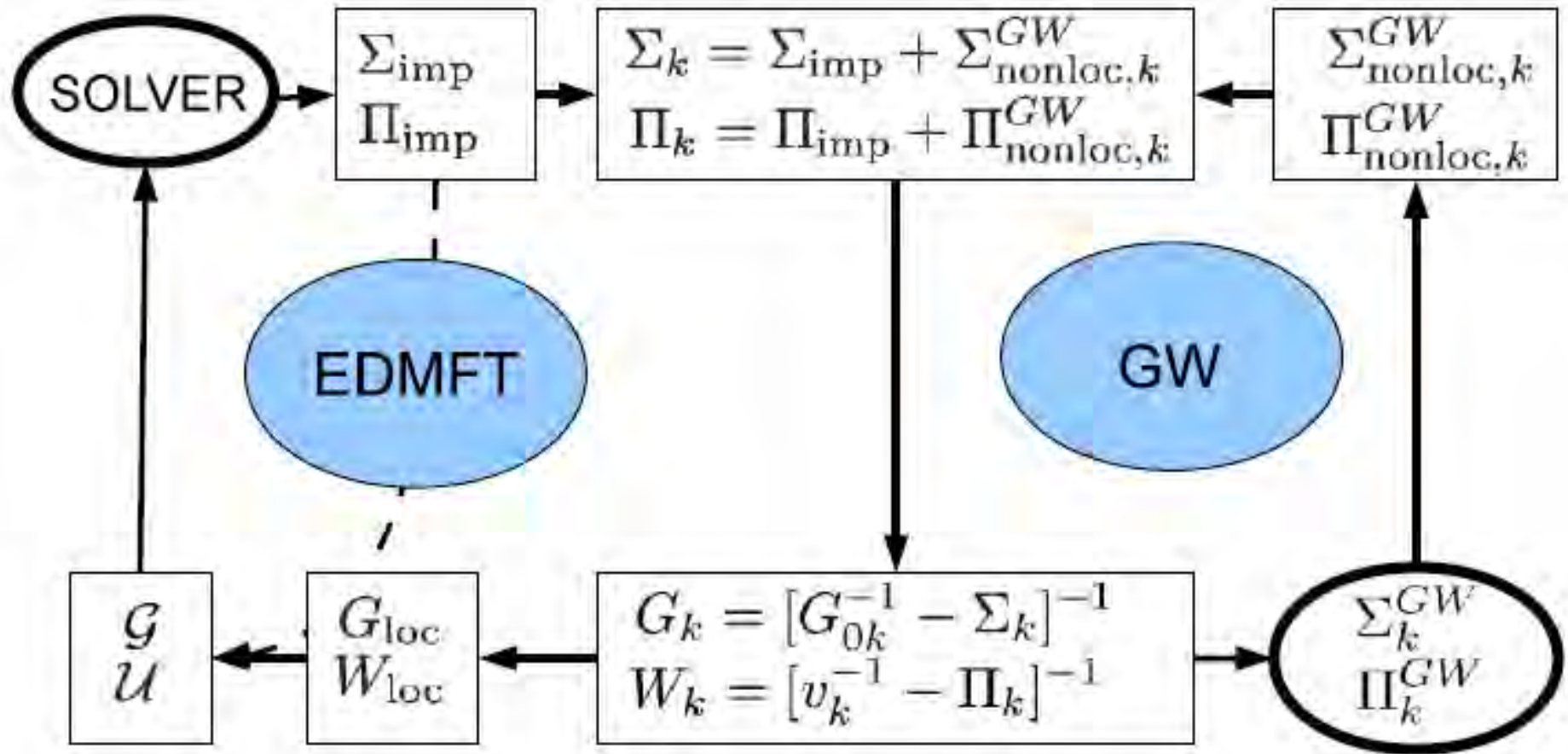
Hubbard model on 2d triangular lattice formed by adatoms



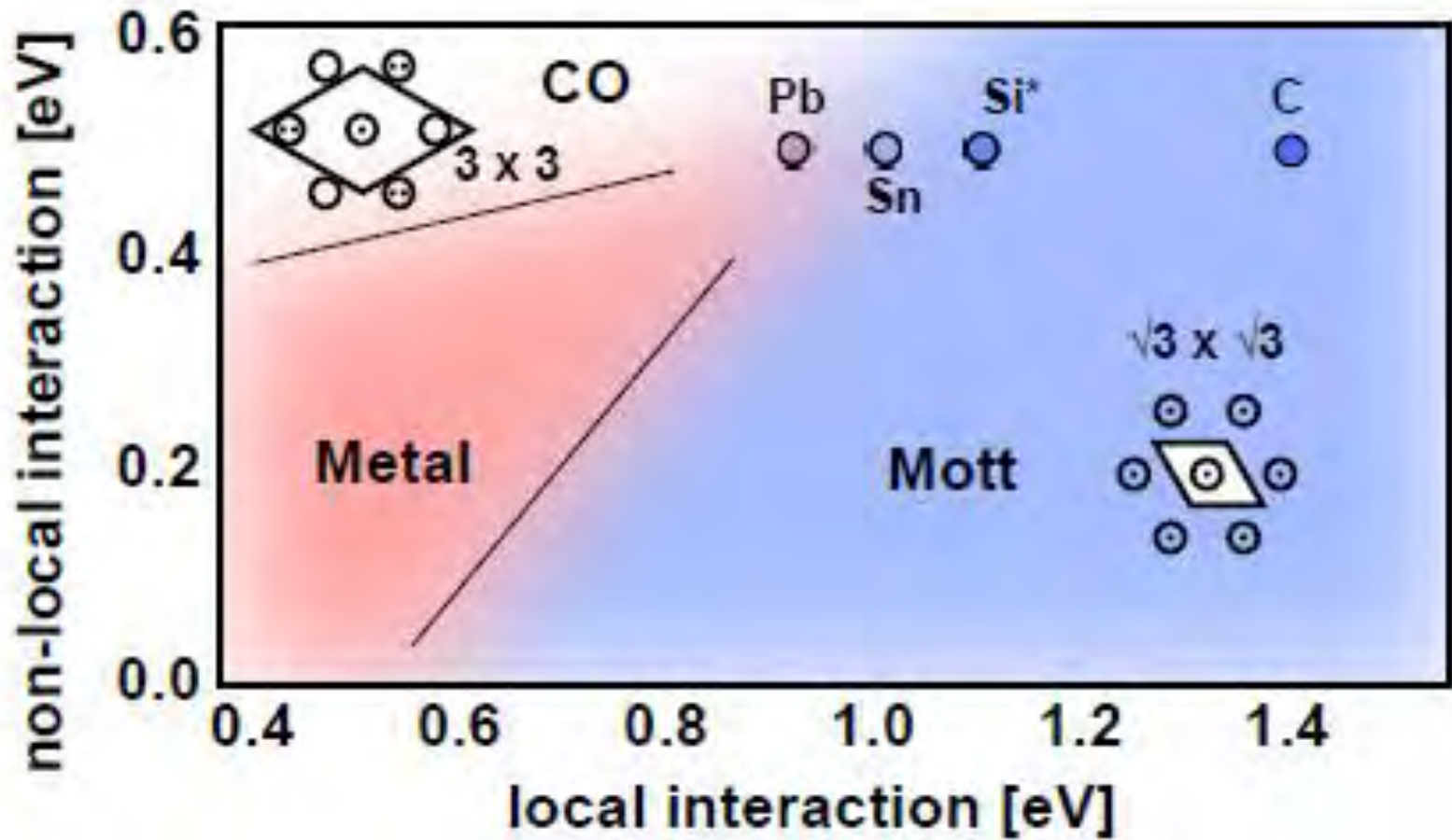
Extended Hubbard Model from LDA and cRPA ?

	C	Si	Sn	Pb	
t	38.0	50.0	42.0	42.0	[meV]
$-t'$	15.0	23.0	20.0	20.0	[meV]
t''	0.5	5.0	10.0	10.0	[meV]
U_0	1.4	1.1	1.0	0.9	[eV]
U_1	0.5	0.5	0.5	0.5	[eV]
U_n			U_1/r_a		
V_0	6.0	4.7	4.4	4.3	[eV]
V_1	2.8	2.8	2.7	2.8	[eV]

Fully self-consistent GW+DMFT:



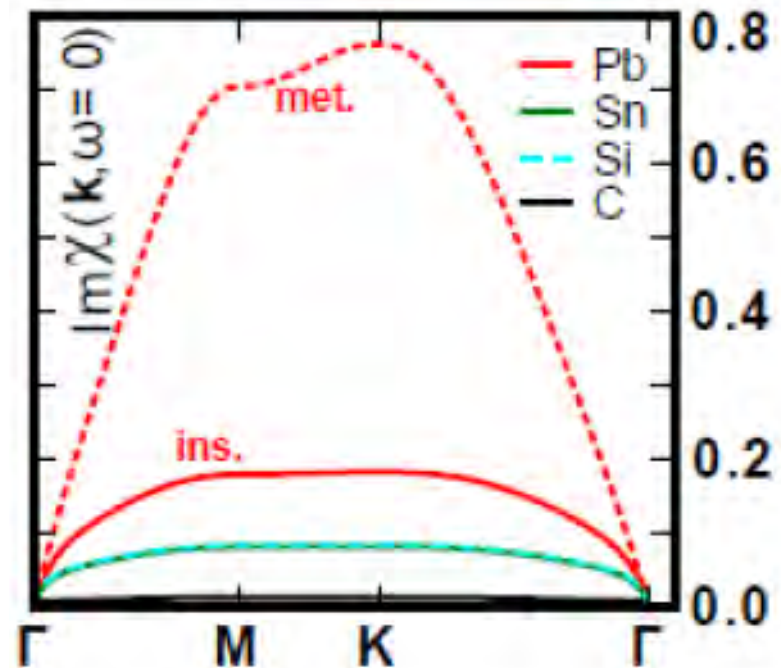
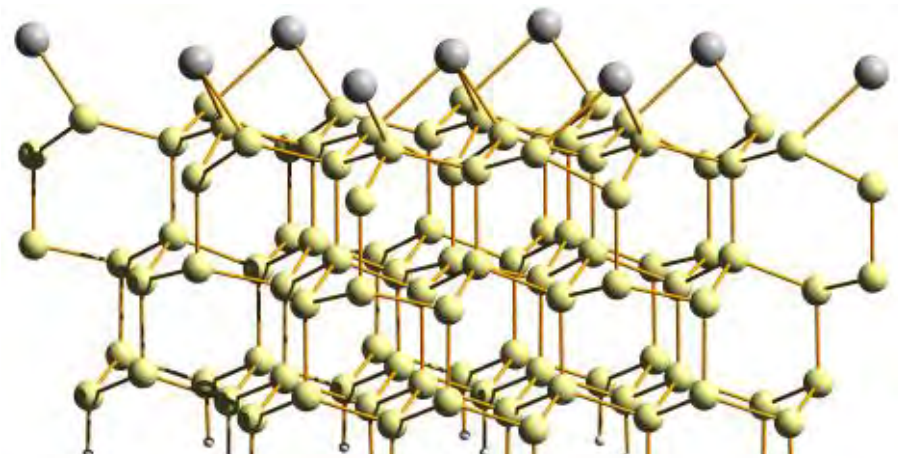
Phase diagram



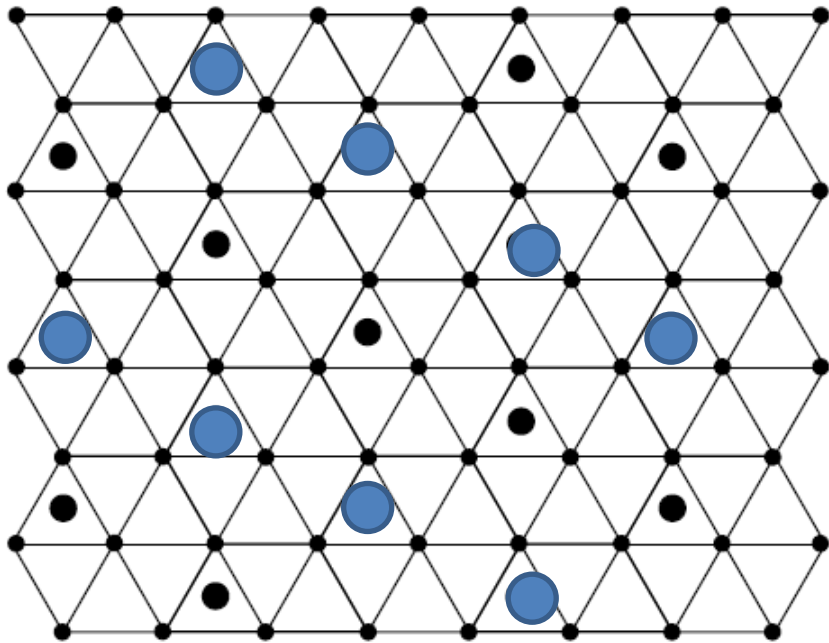
GW+DMFT for Sn/Si(111)

Charge-charge
correlation function:

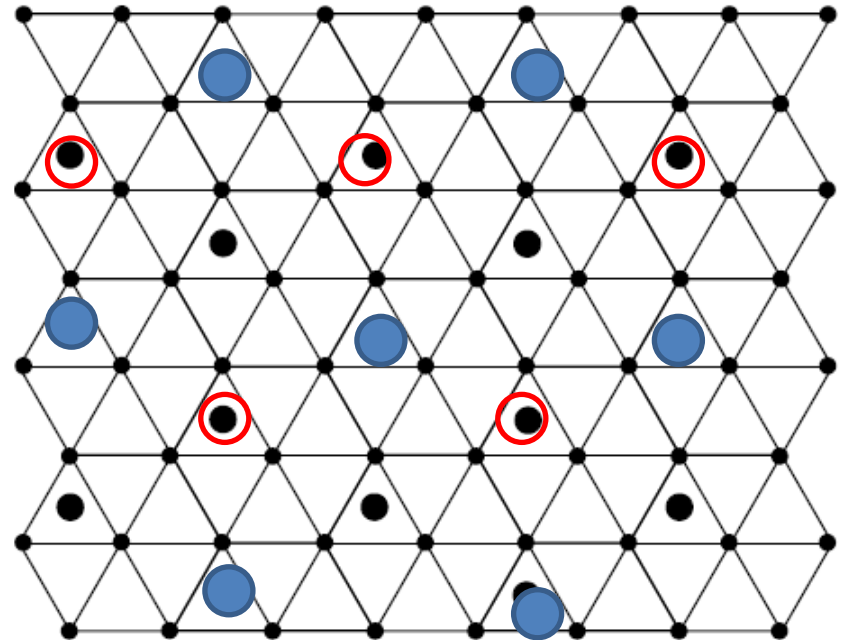
$$\text{Im}\chi(\mathbf{k}, \omega = 0)$$



2 charge order patterns



$k=M$

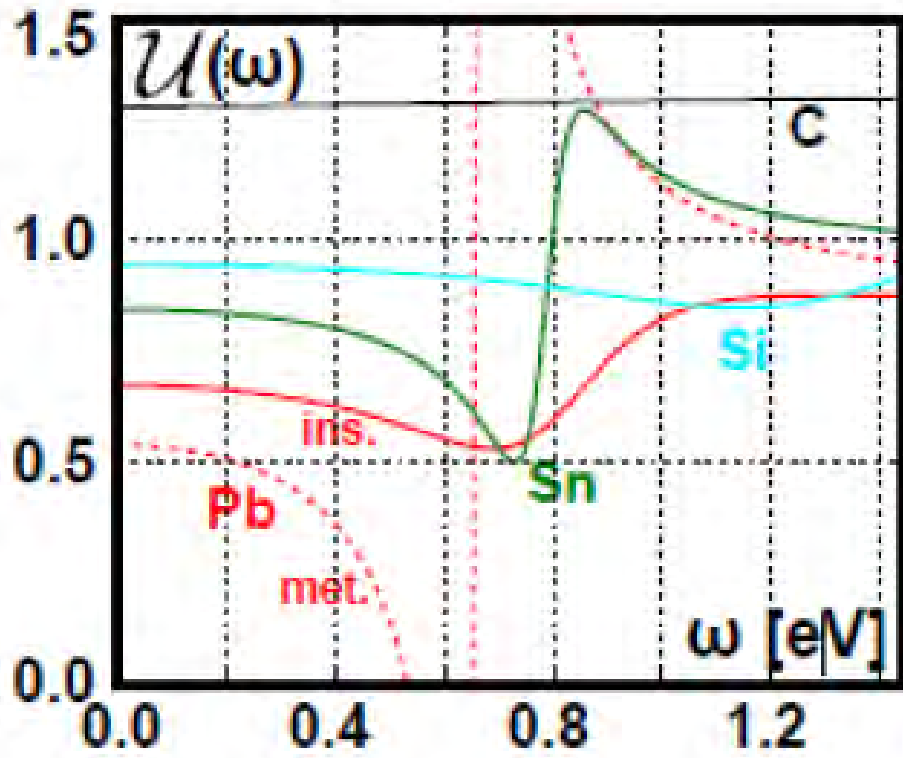
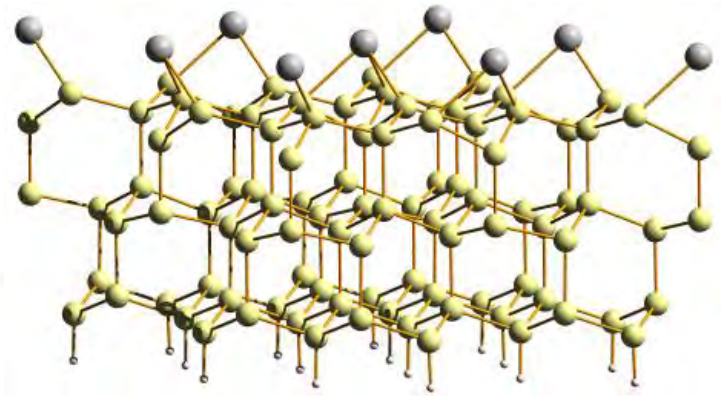


$k=K$

● Doubly occupied site

○ singly occupied site

$U(\omega)$ from GW+DMFT



P. Hansmann, T. Ayrar, L. Vaugier, P. Werner, SB, PRL 2013

Extended Hubbard Model from LDA and cRPA ?

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V_0	6.0	4.7	4.4	4.3	[eV]
V_1	2.8	2.8	2.7	2.8	[eV]
$V_1/\epsilon_{\text{Sisurf}}^{\text{stat.}}$	0.47	0.47	0.45	0.47	[eV]
$\mathcal{U}(\omega=0)$	1.3	0.94	0.84	0.67(ins.) 0.54(met.)	[eV] [eV]

Effective local interaction from GW+DMFT

Extended Hubbard Model from LDA and cRPA ?

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Effective local interaction from GW+DMFT

“U-V” is a lower bound !

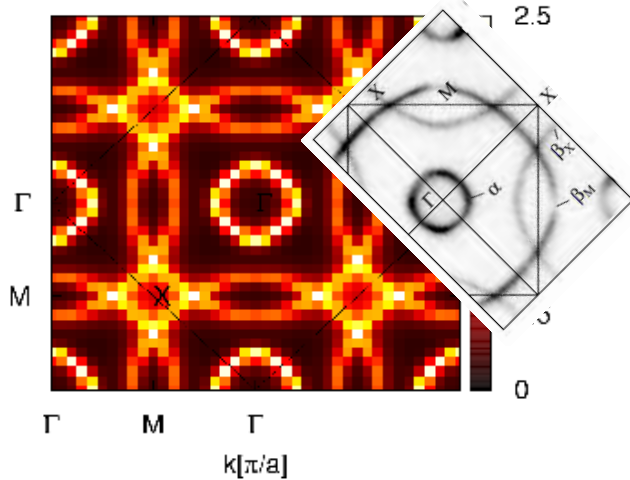
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“U-V” is a lower bound !

Summary ...

Sr₂RhO₄ Fermi surface



Martins et al, Phys. Rev. Lett. (2011)

f-electron pigments: ceria



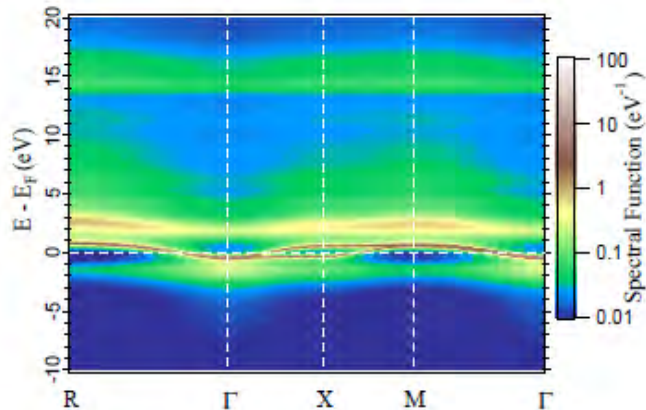
(cf. Rhodia's Neolor series)

Calculated colour of
CeSF:



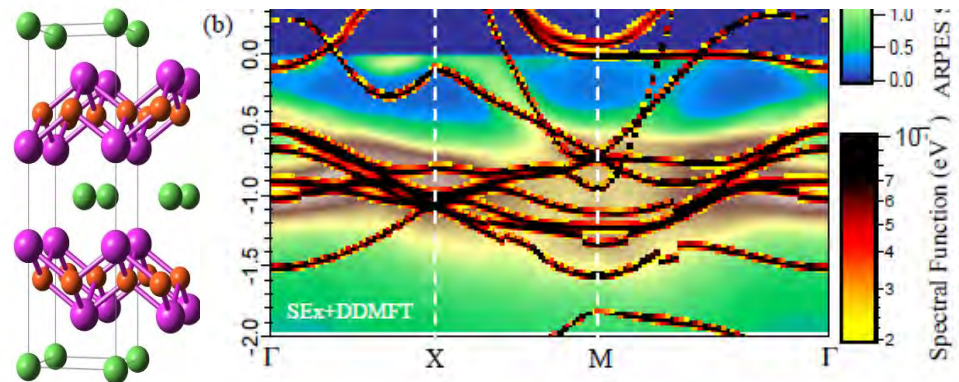
Tomczak et al., PNAS (2013)

SrVO₃ and SrTiO₃



Van Roekeghem, SB, Europhys. Lett. (2014)

BaCo₂As₂



Van Roekeghem et al., PRL 2014