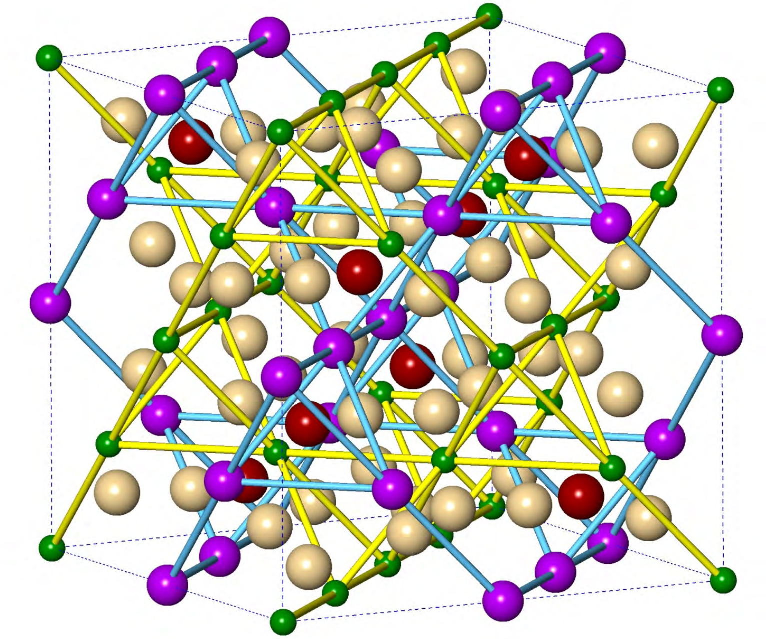
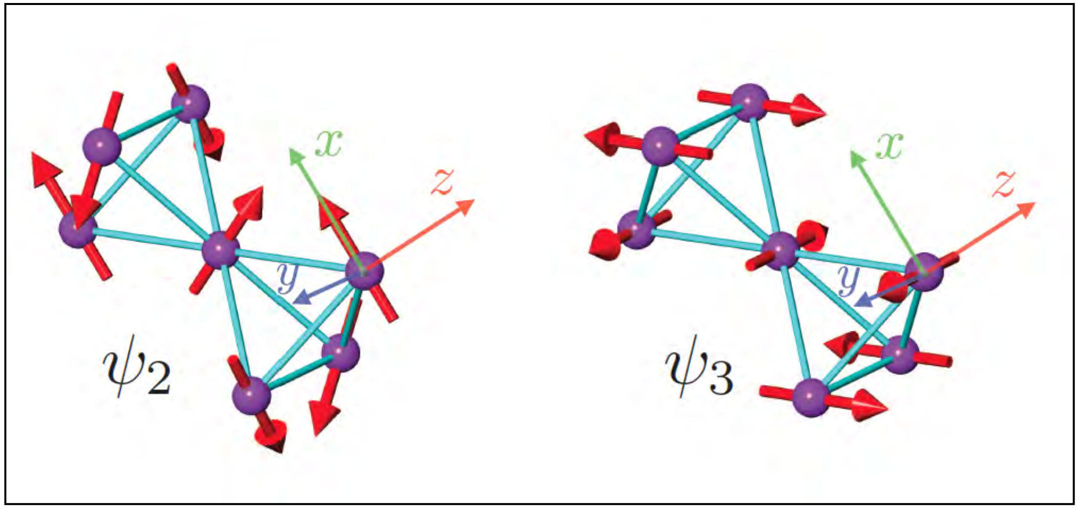
SPICE-Workshop on "Computational Quantum Magnetism", Mainz – May 2015

XY Rare-Earth Pyrochlore Oxides – an Order-by-Disorder Playground





Michel Gingras
Department of Physics & Astronomy, University of Waterloo,
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Collaborators

$Er_2Ti_2O_7$:

- **Zhihao Hao (UW)**
- Paul McClarty
- Pawel Stasiak
- **Behnam Javanparast (UW)**
- Alex Day
- Jaan Oitmaa
- Rajiv Singh
- Behnaz Bagheri
- Anson Wong

$Yb_2Ti_2O_7$:

- **Ludovic Jaubert (OIST)**
- **Zhihao Hao (UW)**
- **Jeff Rau (UW)**
- Owen Benton
- Nic Shannon
- Rajiv Singh
- Jaan Oitmaa
- Robert Hayre
- Ryan Applegate
- Taoran Lin
- Anson Wong
- Alex Day

Outline

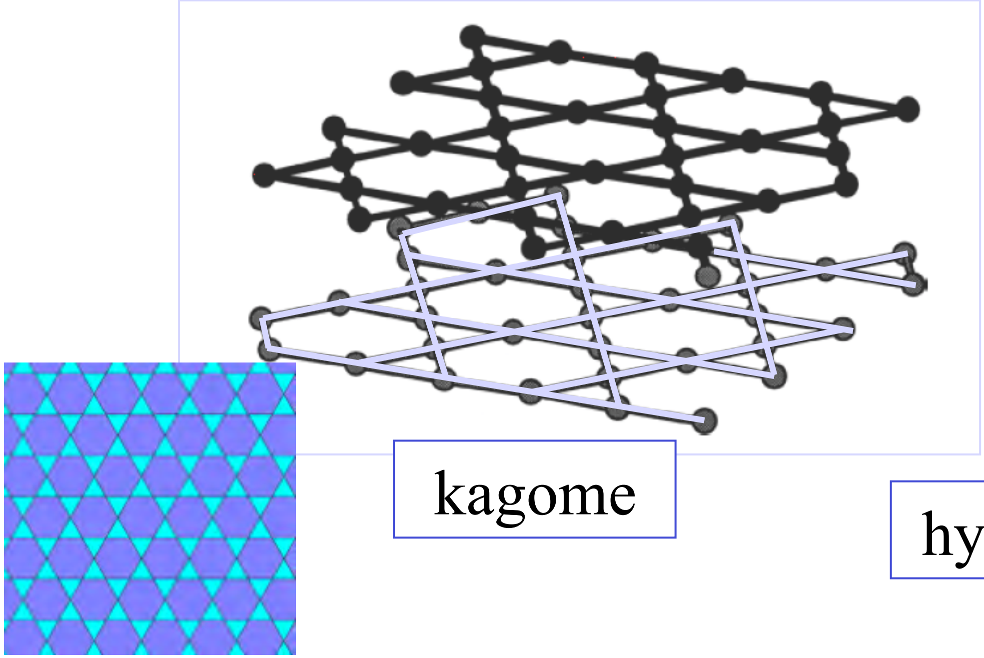
1. Rare-earth pyrochlore oxides
2. Order-by-disorder (ObD) in $Er_2Ti_2O_7$
3. Why should one care about ObD?
4. $Yb_2Ti_2O_7$: what is the hidden order?
5. Conclusion

Outline

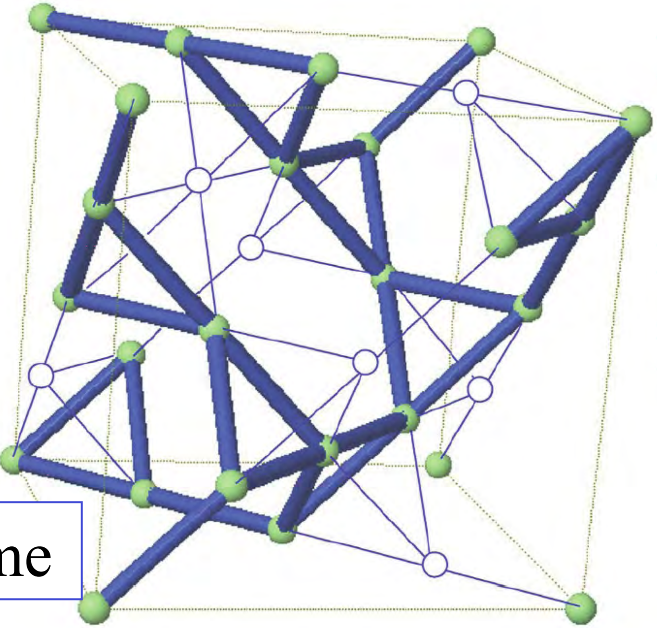
1. Rare-earth pyrochlore oxides
2. Order-by-disorder (ObD) in $Er_2Ti_2O_7$
3. ~~Why should one care about ObD?~~
4. $Yb_2Ti_2O_7$: what is the hidden order?
5. Conclusion

Message of Talk

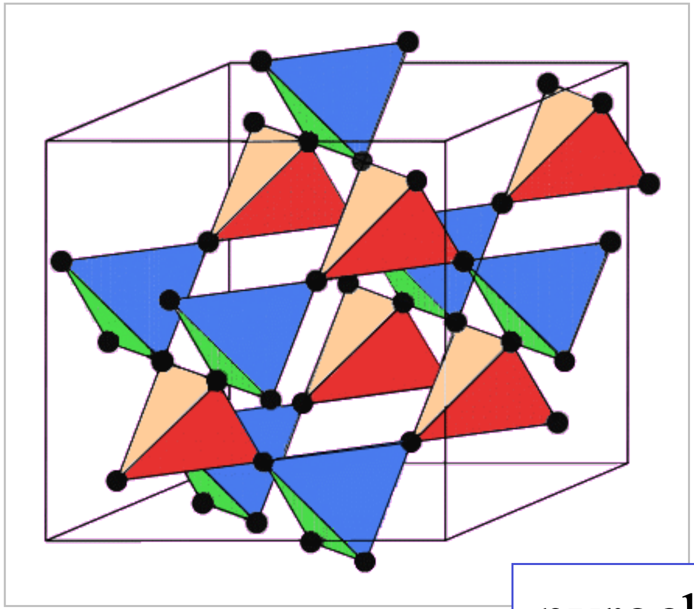
1. Order-by-disorder (ObD) appears quite likely in $Er_2Ti_2O_7$
2. $Yb_2Ti_2O_7$ is puzzling:
   - *Some form of hidden order in $Yb_2Ti_2O_7$?*
   - *Are (i) competing orders and (ii) the consequential ObD the missing links in $Yb_2Ti_2O_7$?*



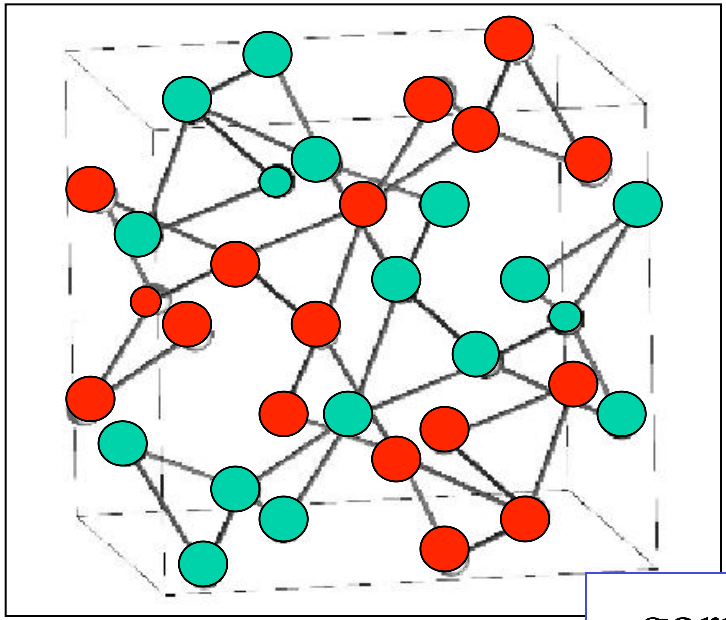
kagome



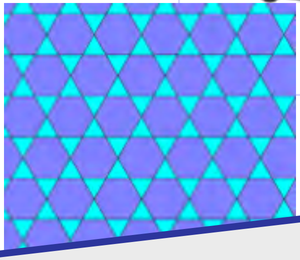
hyperkagome

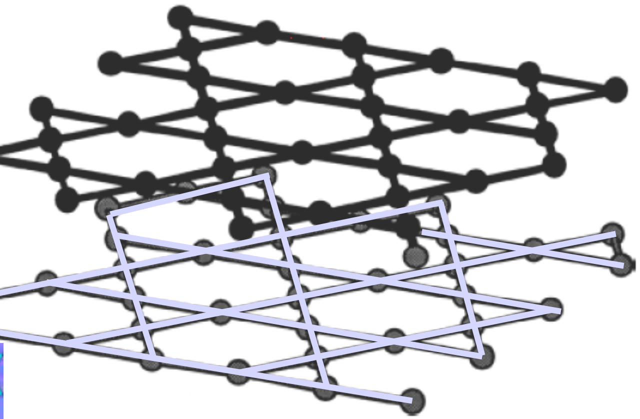


pyrochlore

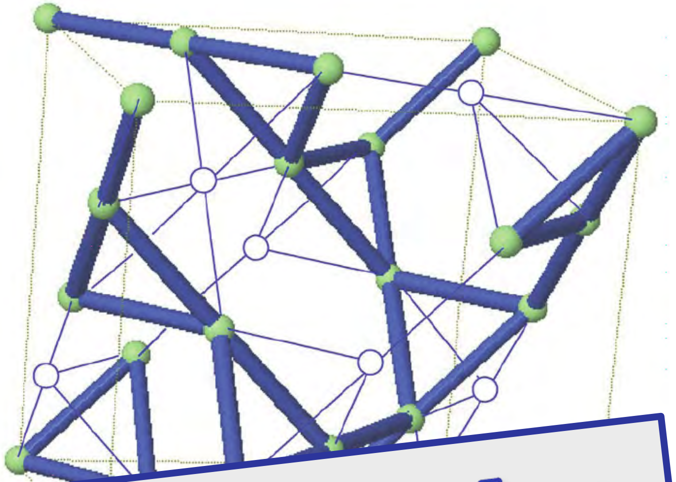


garnet





kagome

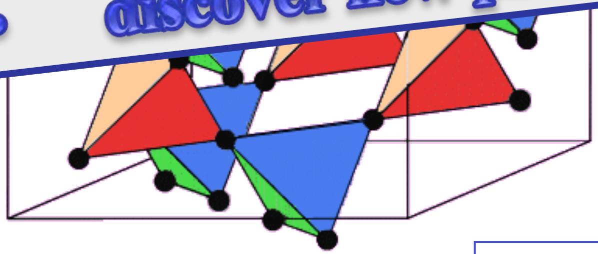


Lattices of corner-sharing triangles or tetrahedra:

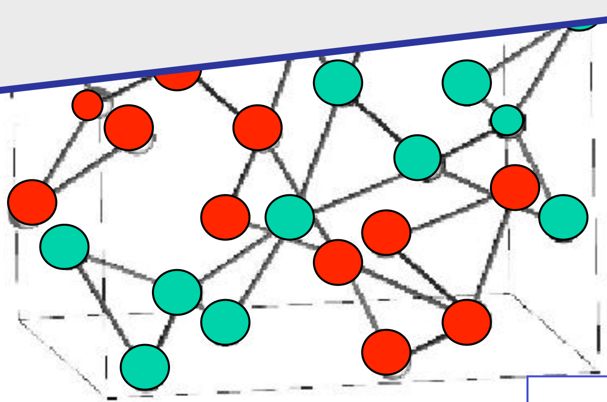
- attractive architectures to search for quantum spin liquids

&

- discover new phenomena



pyrochlore



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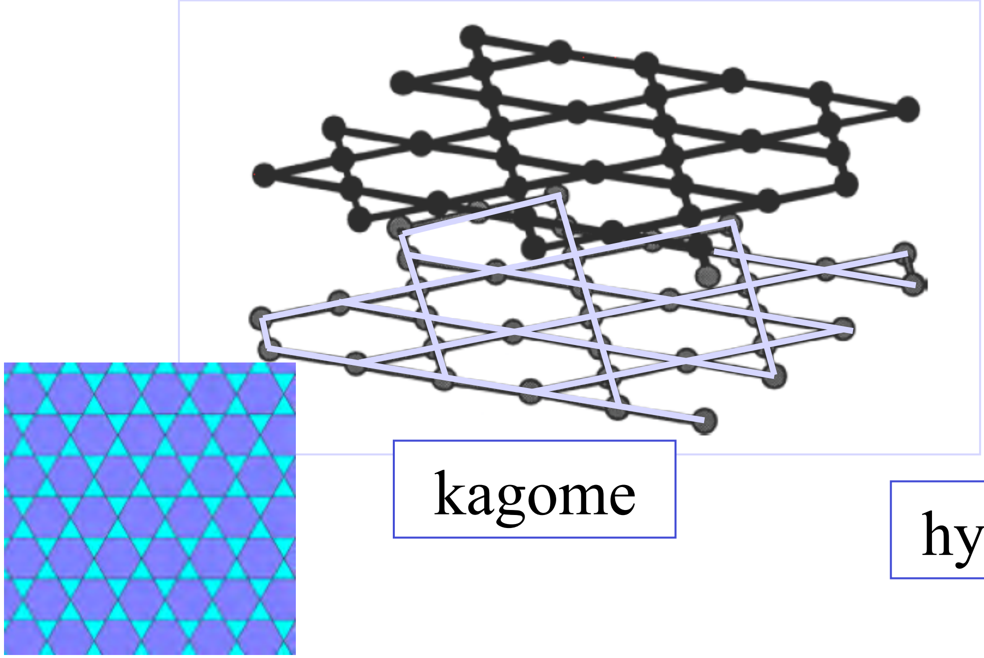

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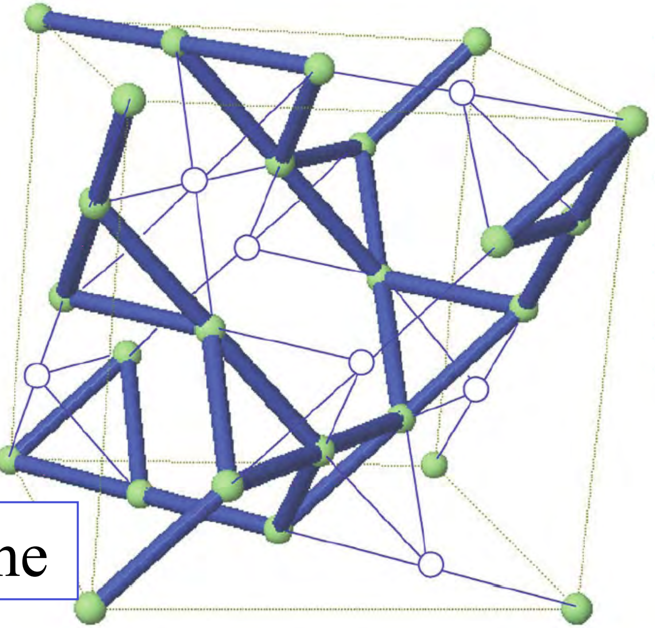
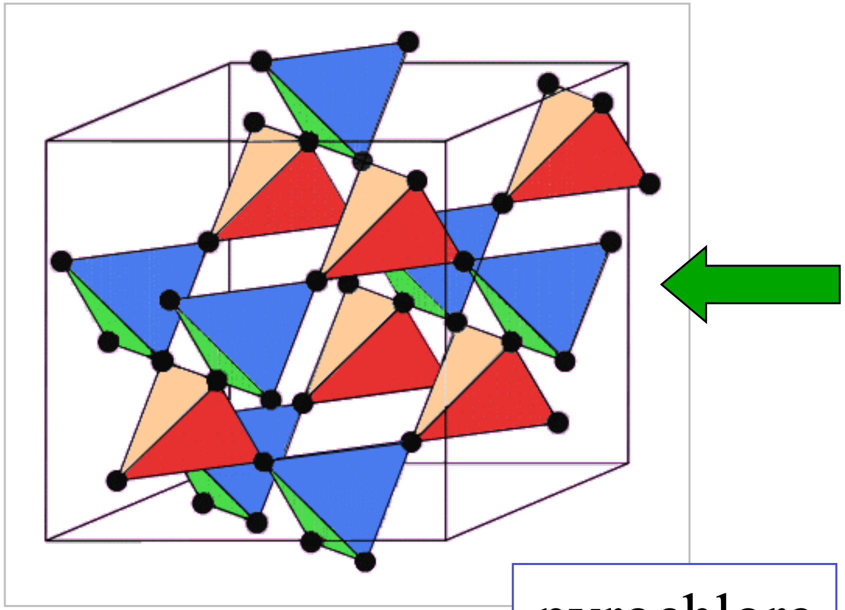
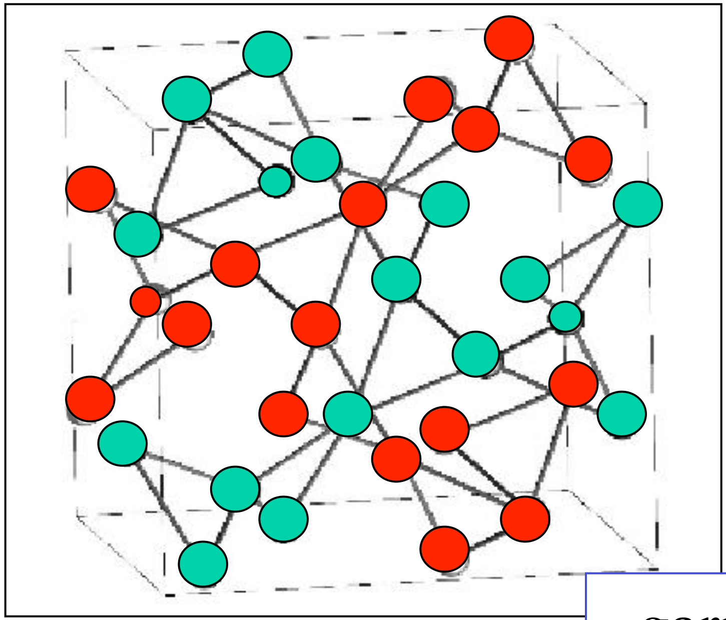


hyperkagome



pyrochlore

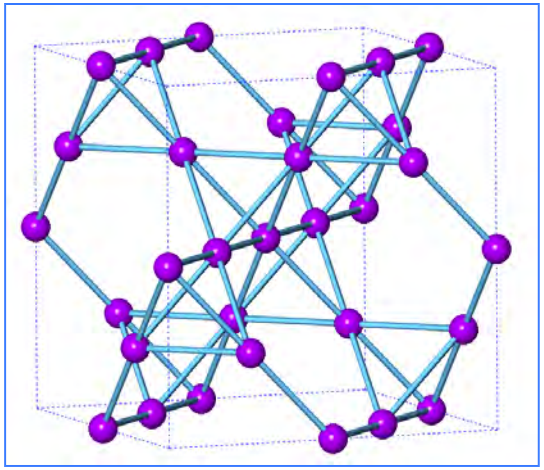


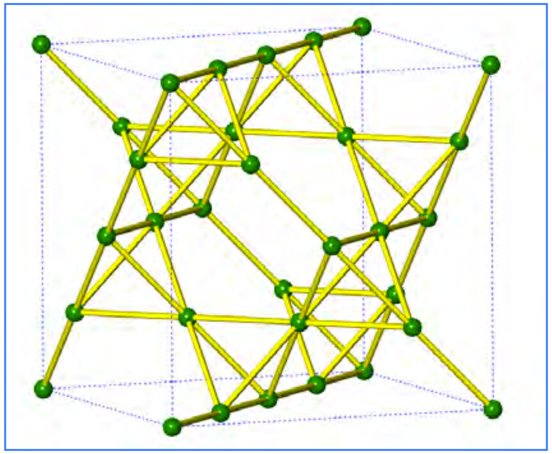
garnet

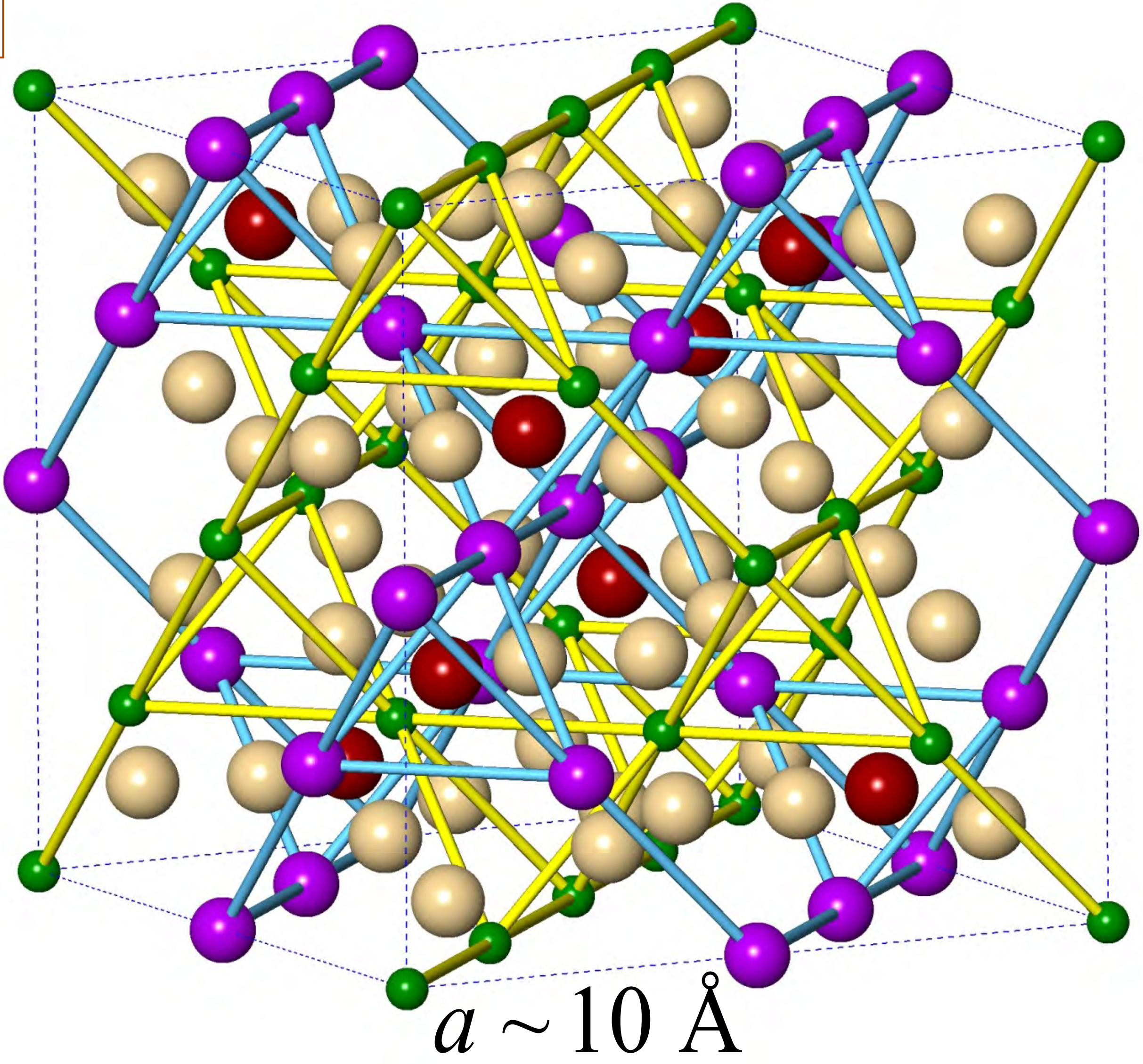
$A_2B_2O_7$

A=Pr,Gd,Tb,Dy,Ho,Er,Yb

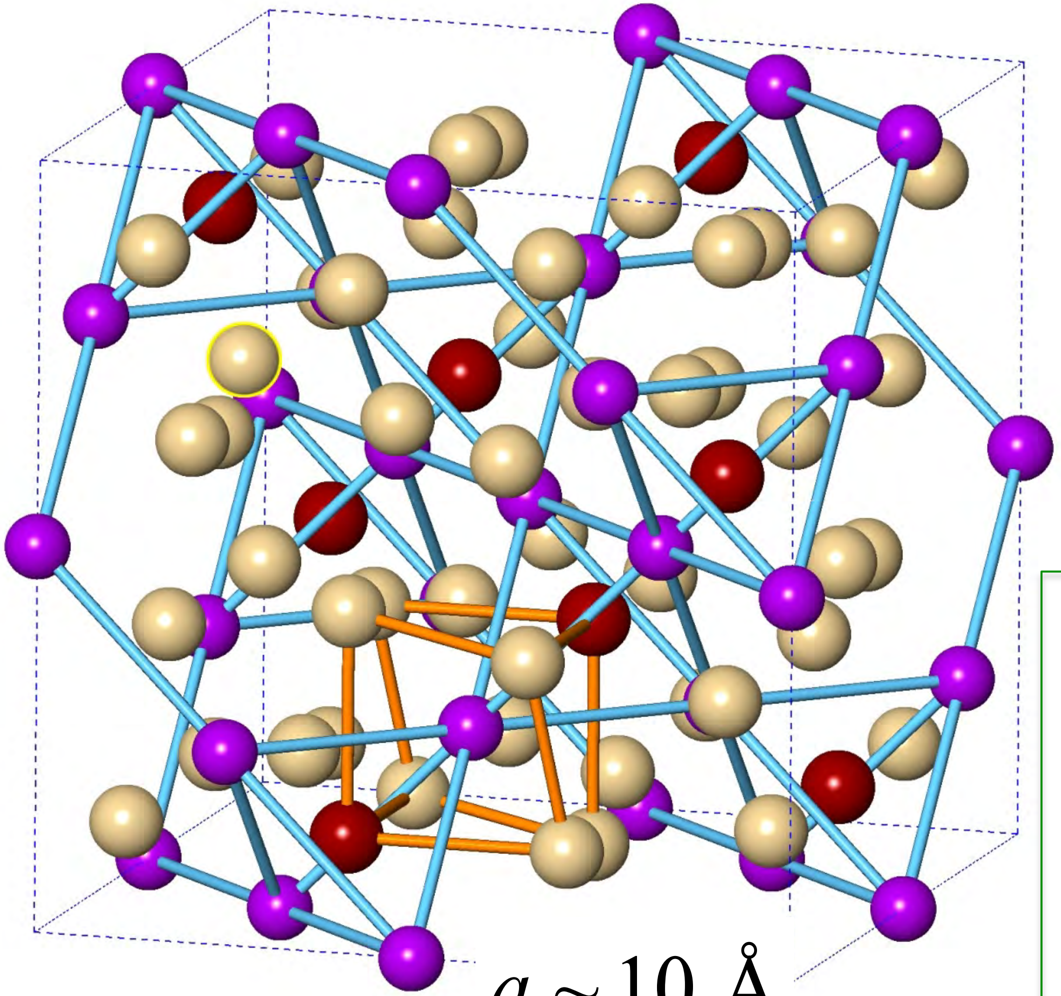
B=Ti,Sn, Zr,Sn







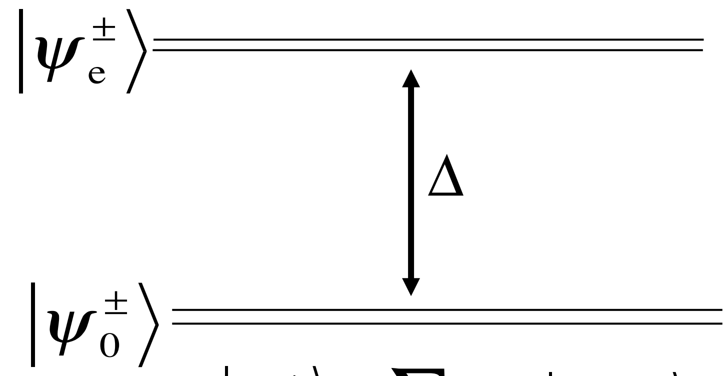
$R_2Ti_2O_7$ R=Gd, Tb, Dy, Ho, Er, Yb



For example:

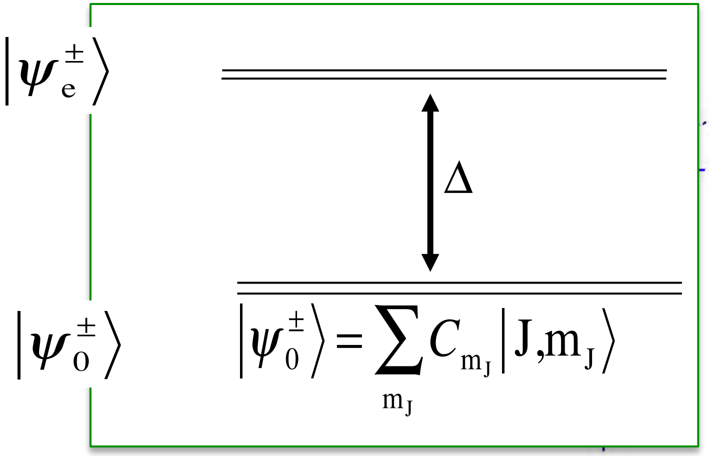
Ho^{3+}: J=L+S = 6+2 = 8

crystal electric field lifts
2J+1 degeneracy

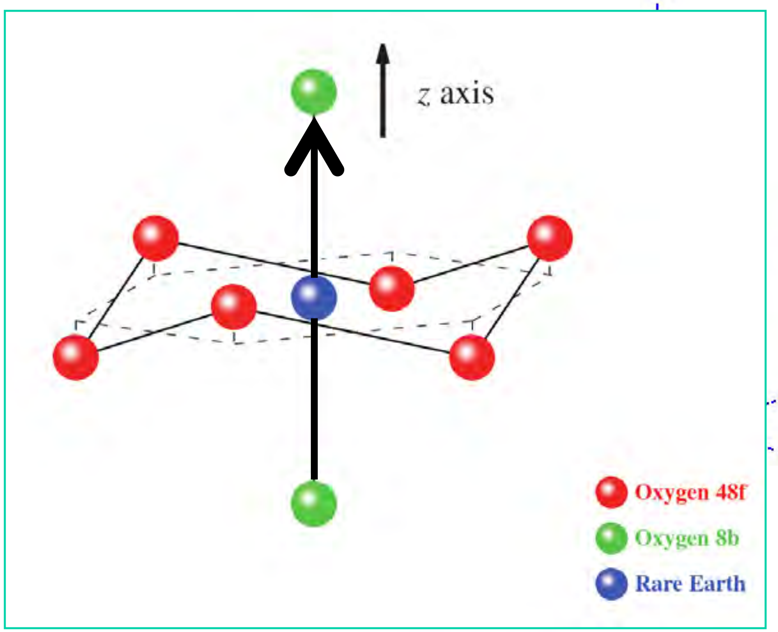


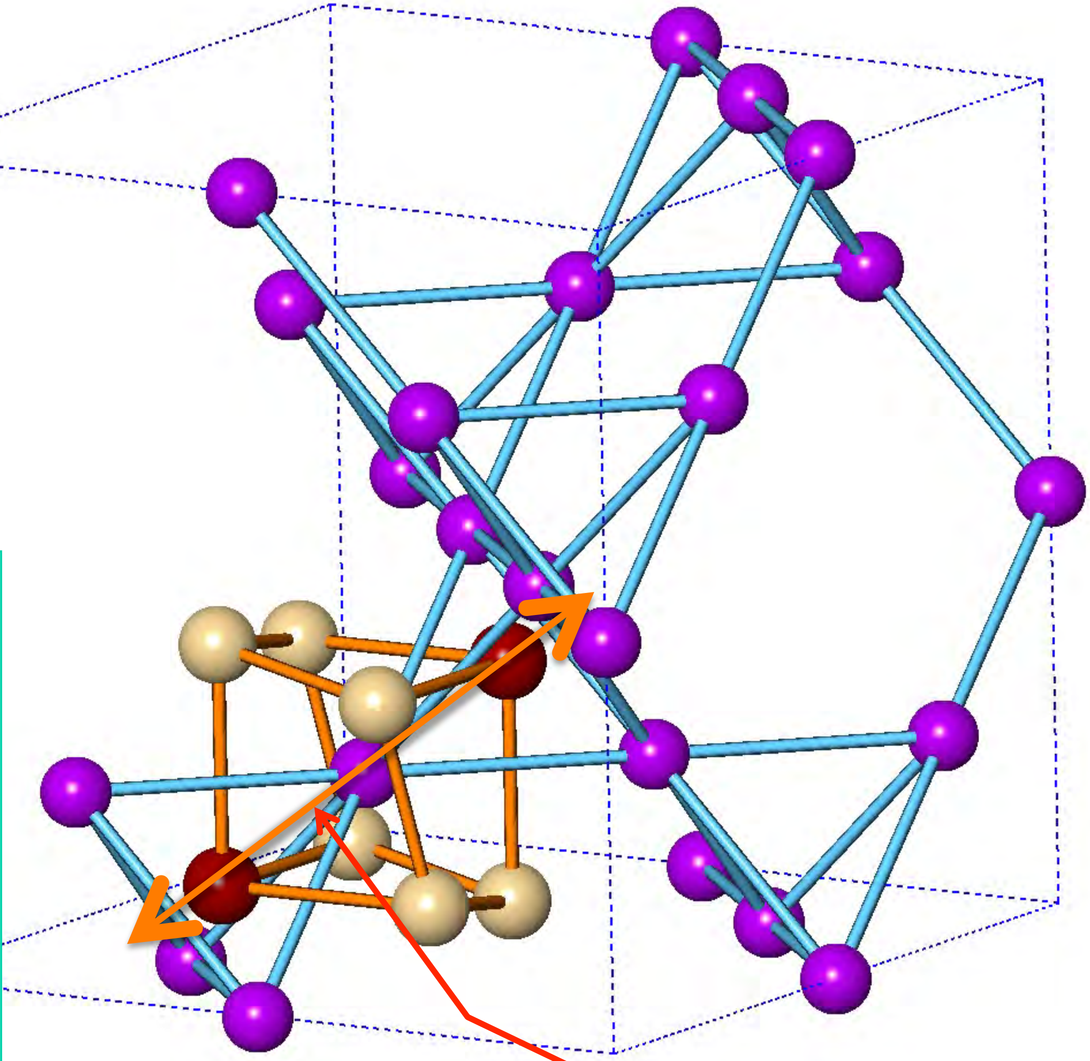
$$|\psi_0^\pm\rangle = \sum_{m_J} C_{m_J} |J,m_J\rangle$$

crystal electric field splits
2J+1 degeneracy



$$|\psi_0^\pm\rangle = \sum_{m_J} C_{m_J} |J,m_J\rangle$$

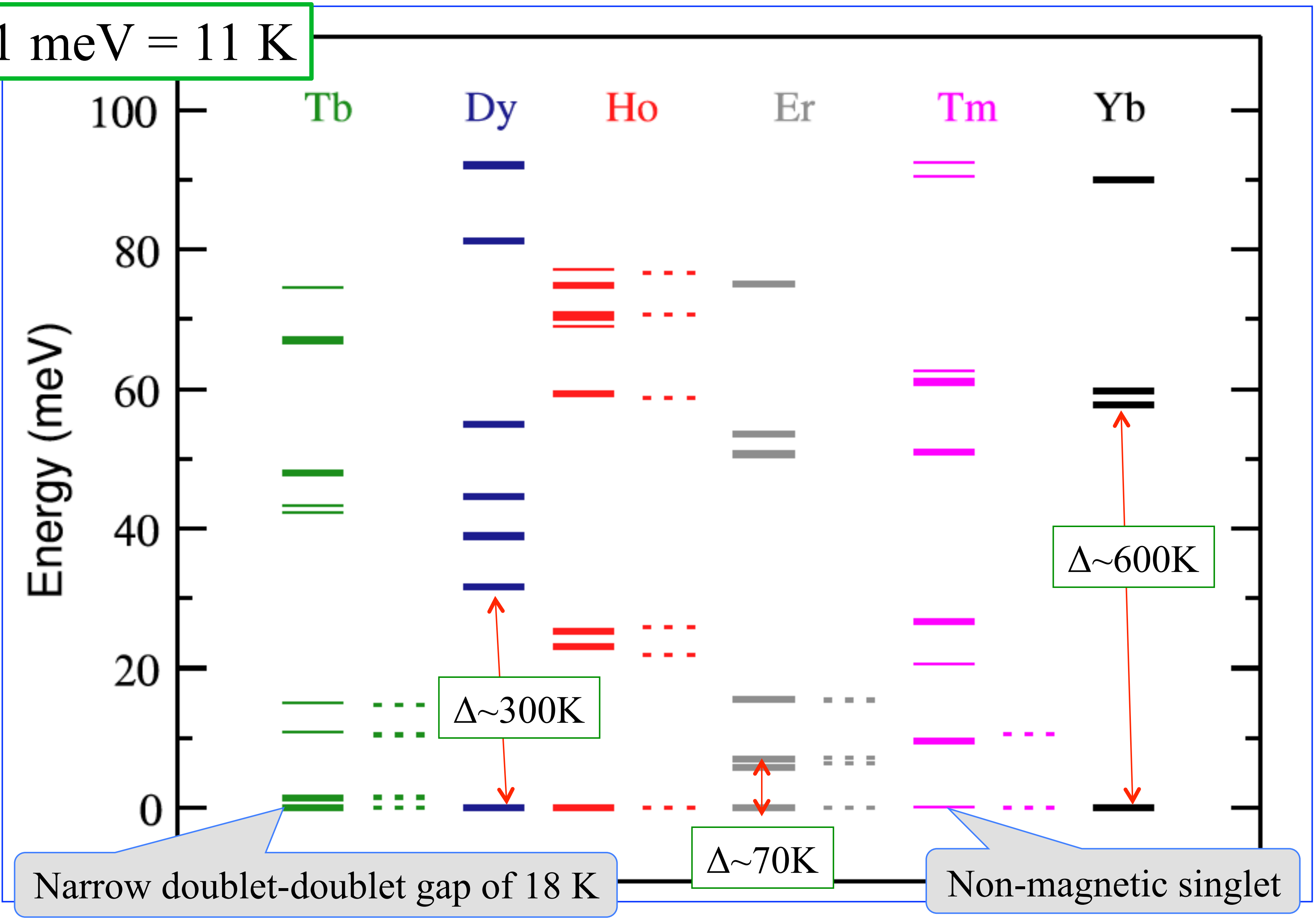




Parallel to cubic [111] direction

Crystal field energy levels in $R_2Ti_2O_7$

1 meV = 11 K



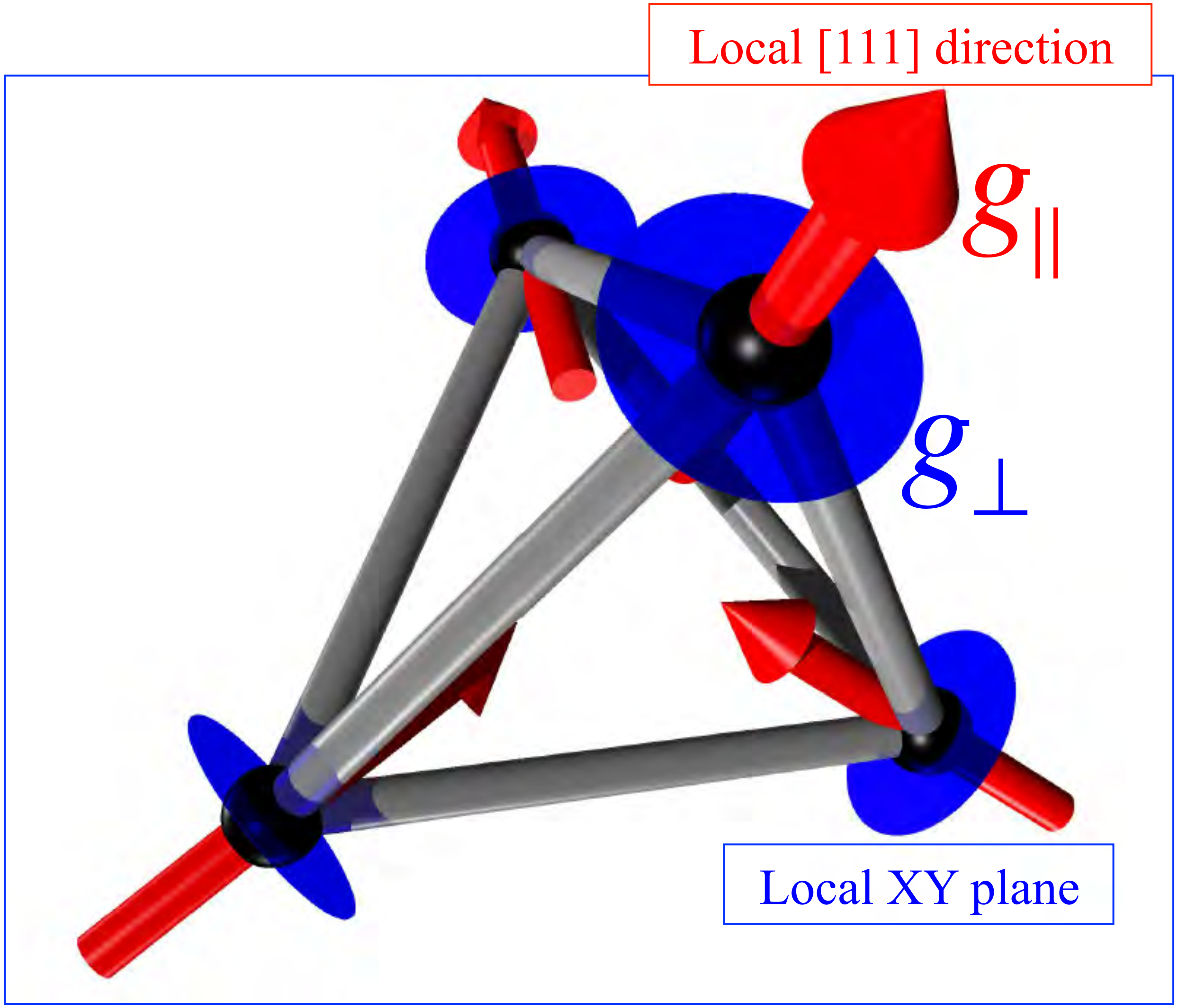
Δ~300K

Δ~600K

Δ~70K

Narrow doublet-doublet gap of 18 K

Non-magnetic singlet



Ising
- $Ho_2Ti_2O_7$
- $Dy_2Ti_2O_7$
- $Tb_2Ti_2O_7$

XY
- $Yb_2Ti_2O_7$
- $Er_2Ti_2O_7$

Heis.
- $Gd_2Sn_2O_7$
- $Gd_2Ti_2O_7$

We know that symmetry plays an important role in the statistical mechanics of "conventional" (i.e. non-frustrated) magnets – what happens in frustrated ones?

Effective Hamiltonian Method

$$H = H_{\text{cef}} + H_{\text{int}}$$

$$H_{\text{int}} = \sum_{i>j}\sum_{K,K'}\sum_{Q,Q'} I_{KK'}^{QQ'} O_K^{Q}(\mathrm{J}_i) O_{K'}^{Q'}(\mathrm{J}_j)$$

$$H_{\text{eff}} = P H_{\text{int}} P + P H_{\text{int}} \mathrm{Q} H_{\text{int}} P + \cdots$$

$$\mathrm{P} = \sum_{\alpha\in \mathrm{P}} |\alpha\rangle\langle\alpha|$$

Large (CEF gap Δ) denominator compared to the energy scale of H_{int}

$$\mathrm{Q} = \sum_{\beta\notin P} \frac{|\beta\rangle\langle\beta|}{E_0^{\alpha} - E_0^{\beta}}$$

$$H_{\text{eff}} = P H_{\text{int}} P + P H_{\text{int}} \mathrm{Q} H_{\text{int}} P + \cdots$$

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Large (CEF gap Δ) denominator compared to the energy scale of H_{int}

$$\mathrm{Q} = \sum_{\beta \notin P} \frac{|\beta\rangle\langle\beta|}{E_0^\alpha - E_0^\beta}$$

Note: Fine for Dy, Ho, Er, Yb; not Tb

$$H_{\mathrm{eff}} = P H_{\mathrm{int}} P + \xcancel{P H_{\mathrm{int}} \mathrm{Q} H_{\mathrm{int}} P + \cdots}$$

$$H_{\text{eff}} = J_{zz}\sum_{\langle i,j\rangle} S_i^z S_j^z - J_{\pm}\sum_{\langle i,j\rangle}\left(S_i^+ S_j^- + S_i^- S_j^+\right)$$

For Δ/J_{zz}>>1, ground doublet projection is good enough

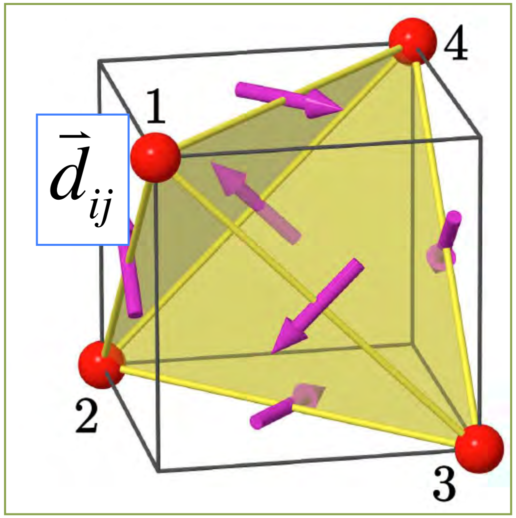
$$+J_{z\pm}\sum_{\langle i,j\rangle}\left[S_i^z\left(\zeta_{ij}S_j^+ + \zeta_{ij}^* S_j^-\right) + i \leftrightarrow j\right] + J_{\pm\pm}\sum_{\langle i,j\rangle}\left(\gamma_{ij}S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-\right)$$

$S = \frac{1}{2}$ operator

K.A. Ross, L. Savary, B.D. Gaulin, and L. Balents; PRX **1**, 021002 (2011)

$$H_{\text{eff}} = J_{\text{Ising}}\sum_{\langle i,j\rangle} S_i^{z_i} S_j^{z_j} + J_{\text{iso}}\sum_{\langle i,j\rangle}\vec{S}_i \bullet \vec{S}_j + J_{\text{DM}}\sum_{\langle i,j\rangle}\vec{d}_{ij}\bullet\vec{S}_i \times \vec{S}_j$$

$$+ J_{\text{pd}}\sum_{\langle i,j\rangle}\left(\vec{S}_i \bullet \vec{S}_j - 3\vec{S}_i \bullet \hat{r}_{ij}\hat{r}_{ij}\bullet\vec{S}_j\right)$$



P.A. McClarty, S.H. Curnoe, and M J.P. Gingras,
J. Phys.: Conf. Ser. **145**, 012032 (2009)

J.D. Thompson *et al.* Phys. Rev. Lett.**106**, 187202 (2011)

Frustrated Rare-earth Pyrochlore Oxides

Examples of phenomena

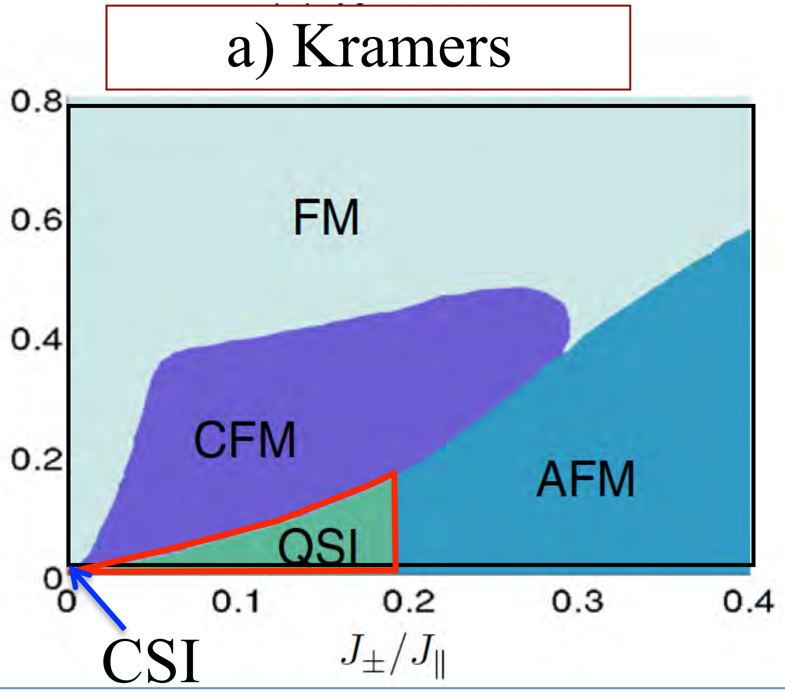
- $(Ho,Dy)_2(Ti,Sn,Ge)_2O_7$: spin ice
- $Yb_2Ti_2O_7$: quantum spin ice ?
- $Tb_2Ti_2O_7$: spin liquid / QSI?
- $Gd_2Ti_2O_7$: multiple transitions
- $Er_2Ti_2O_7$: order-by-disorder

J.S. Gardner, M.J.P. Gingras, and J.E. Greedan,
Magnetic Pyrochlore Oxides, Rev. Mod. Phys. **82**, 53 (2010)

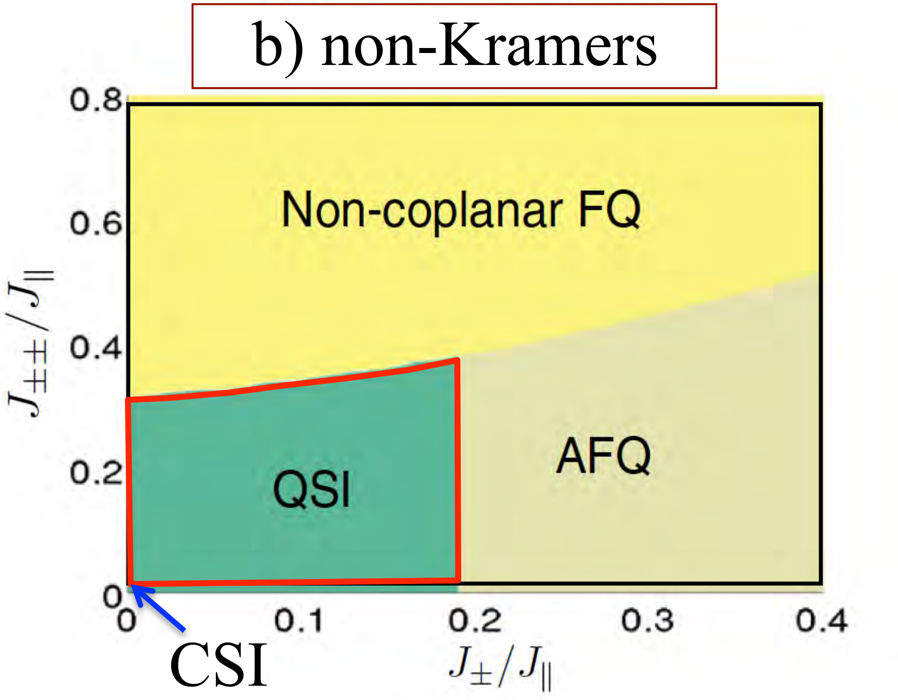
$$H = J_{\parallel}\sum_{\langle i,j\rangle} S_i^z S_j^z - J_{\pm}\sum_{\langle i,j\rangle}\left(S_i^+ S_j^- + S_i^- S_j^+\right)$$

Savary & Balents, PRL **108**, 037202 (2012).

$$+J_{z\pm}\sum_{\langle i,j\rangle}\left[S_i^z\left(\zeta_{ij}S_j^+ + \zeta_{ij}^* S_j^-\right) + i\leftrightarrow j\right] + J_{\pm\pm}\sum_{\langle i,j\rangle}\left(\gamma_{ij}S_i^+S_j^+ + \gamma_{ij}^* S_i^- S_j^-\right) \quad S=\tfrac{1}{2}\ \text{operator}$$



Savary & Balents, PRL **108**, 037202 (2012).

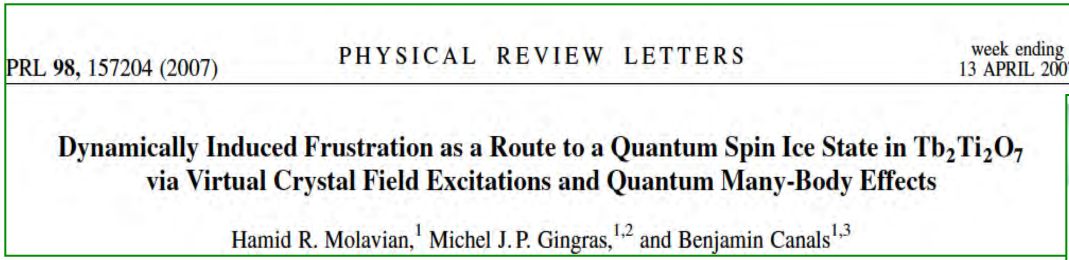


Lee, Onoda & Balents, PRB **86**, 104412 (2012).

QSI is expected to be a U(1) quantum spin liquid

Gingras & McClarty, Rep. Prog. Phys. **77**, 056501 (2014)

There is an ongoing search for quantum spin ice materials



PRL **98,** 157204 (2007) PHYSICAL REVIEW LETTERS week ending 13 APRIL 2007

Dynamically Induced Frustration as a Route to a Quantum Spin Ice State in $Tb_2Ti_2O_7$ via Virtual Crystal Field Excitations and Quantum Many-Body Effects

Hamid R. Molavian,[1] Michel J. P. Gingras,[1,2] and Benjamin Canals[1,3]

nature COMMUNICATIONS

ARTICLE

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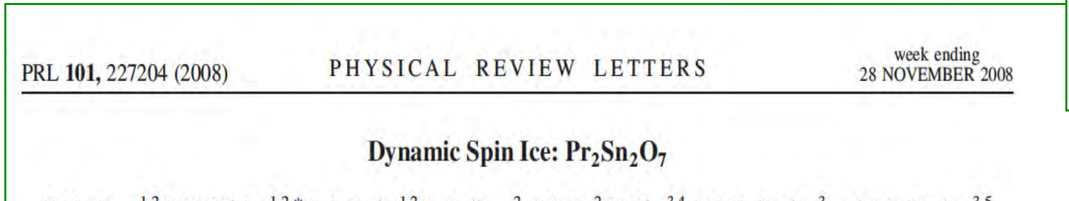
Quantum fluctuations in spin-ice-like $Pr_2Zr_2O_7$

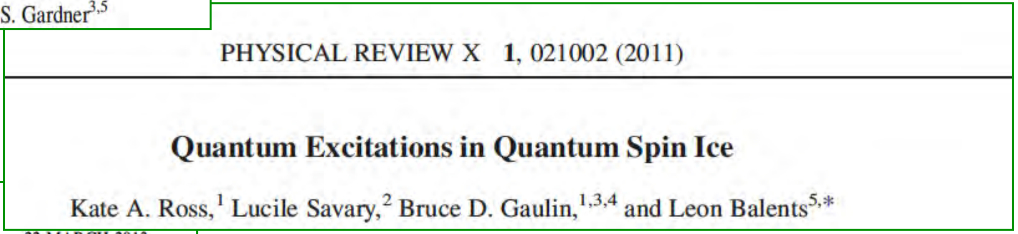
K. Kimura[1], S. Nakatsuji[1,2], J.-J. Wen[3], C. Broholm[3,4,5], M.B. Stone[5], E. Nishibori[6] & H. Sawa[6]



PRL **101,** 227204 (2008) PHYSICAL REVIEW LETTERS week ending 28 NOVEMBER 2008

Dynamic Spin Ice: $Pr_2Sn_2O_7$

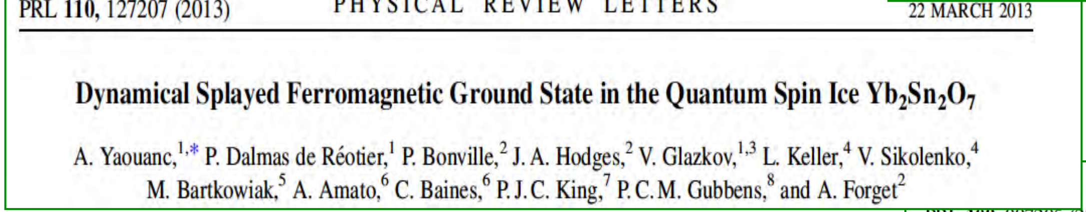
H. D. Zhou,[1,2] C. R. Wiebe,[1,2,*] J. A. Janik,[1,2] L. Balicas,[2] Y. J. Yo,[2] Y. Qiu,[3,4] J. R. D. Copley,[3] and J. S. Gardner[3,5]



PHYSICAL REVIEW X **1,** 021002 (2011)

Quantum Excitations in Quantum Spin Ice

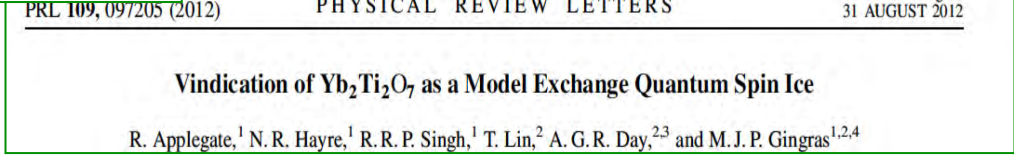
Kate A. Ross,[1] Lucile Savary,[2] Bruce D. Gaulin,[1,3,4] and Leon Balents[5,*]



PRL **110,** 127207 (2013) PHYSICAL REVIEW LETTERS 22 MARCH 2013

Dynamical Splayed Ferromagnetic Ground State in the Quantum Spin Ice $Yb_2Sn_2O_7$

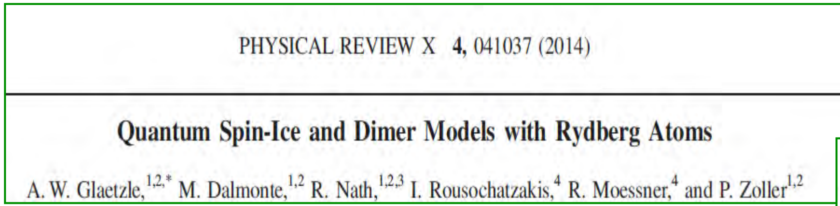
A. Yaouanc,[1,*] P. Dalmas de Réotier,[1] P. Bonville,[2] J. A. Hodges,[2] V. Glazkov,[1,3] L. Keller,[4] V. Sikolenko,[4]
M. Bartkowiak,[5] A. Amato,[6] C. Baines,[6] P. J. C. King,[7] P. C. M. Gubbens,[8] and A. Forget[2]



PRL **109,** 097205 (2012) PHYSICAL REVIEW LETTERS week ending 31 AUGUST 2012

Vindication of $Yb_2Ti_2O_7$ as a Model Exchange Quantum Spin Ice

R. Applegate,[1] N. R. Hayre,[1] R. R. P. Singh,[1] T. Lin,[2] A. G. R. Day,[2,3] and M. J. P. Gingras[1,2,4]



PHYSICAL REVIEW X **4,** 041037 (2014)

Quantum Spin-Ice and Dimer Models with Rydberg Atoms

A. W. Glaetzle,[1,2,*] M. Dalmonte,[1,2] R. Nath,[1,2,3] I. Rousochatzakis,[4] R. Moessner,[4] and P. Zoller[1,2]

PRL **105,** 047201 (2010) PHYSICAL REVIEW LETTERS week ending 23 JULY 2010

Quantum Melting of Spin Ice: Emergent Cooperative Quadrupole and Chirality

Shigeki Onoda and Yoichi Tanaka
Condensed Matter Theory Laboratory, RIKEN, 2-1, Hirosawa, Wako 351-0198, Saitama, Japan

Frustrated Rare-earth Pyrochlore Oxides

Examples of phenomena

- $(Ho,Dy)_2(Ti,Sn,Ge)_2O_7$: spin ice

→ **$Yb_2Ti_2O_7$ (XY) : quantum spin ice ?**

- $Tb_2Ti_2O_7$: spin liquid / QSI?
- $Gd_2Ti_2O_7$: multiple transitions

→ **$Er_2Ti_2O_7$ (XY) : order-by-disorder**

J.S. Gardner, M.J.P. Gingras, and J.E. Greedan,
Magnetic Pyrochlore Oxides, Rev. Mod. Phys. **82**, 53 (2010)

Outline

1. Rare- earth pyrochlore oxides
2. Order-by-disorder (ObD) in $Er_2Ti_2O_7$
3. ~~Why should one care about ObD?~~
4. $Yb_2Ti_2O_7$: what is the hidden order?
5. Conclusion

Order-by-disorder

ObD is a phenomenon via which thermal fluctuations and/or quantum fluctuations lift an accidental classical *T*=0 ground state degeneracy by

(i) selecting the subset of states of the degenerate around which the system **fluctuate the most**

which

(ii) then "allows" for a spontaneous symmetry breaking and long-range order

Order-by-Disorder

"Degeneracy parameter"

$$E = E_{\text{classical}}(\phi) = E_{\text{classical}} \quad \text{(independent of } \phi\text{)}$$

$$E(\phi) = E_{\text{classical}} + \sum_k \tfrac{1}{2}\hbar\omega_k(\phi)$$

Thermal contribution to ObD
- *either at T=0$^+$*
- *or at T~T_c*

Quantum contribution to ObD

$$F = E - TS(\phi) \qquad E = E_{\text{classical}} + \sum_k \tfrac{1}{2}\hbar\omega_k(\phi)$$

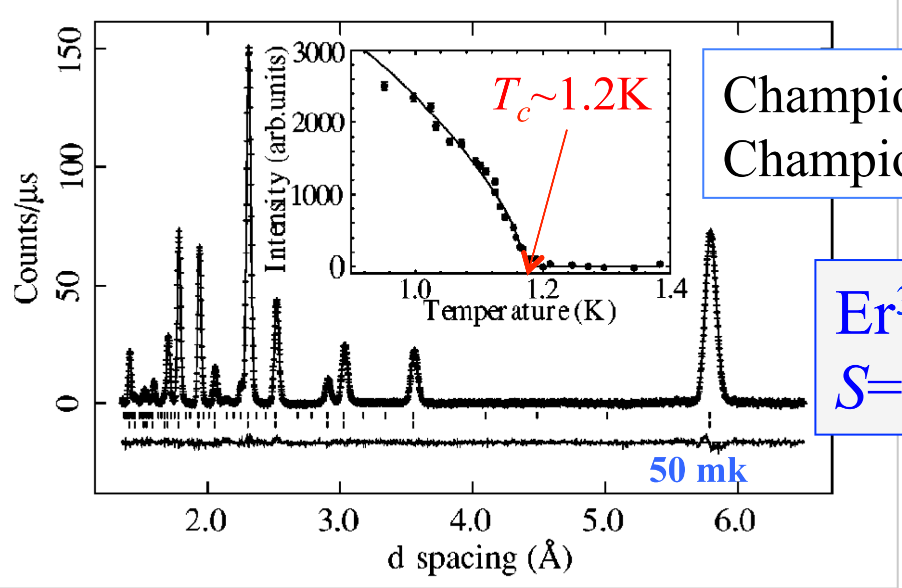
Order-by-disorder – does it “occur” ?

The concept of (thermal/quantum) *order-by-disorder* (ObD) is compelling (and presumably correct) for a number of theoretical models described by some minimal Hamiltonian.

➔But does it really occur in nature, in real materials?

➔Evidence for ObD is strong in $Er_2Ti_2O_7$

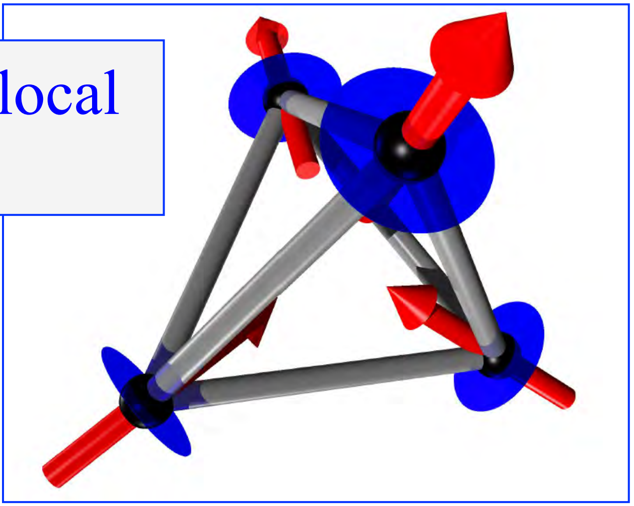
The problem of order in XY-like $Er_2Ti_2O_7$

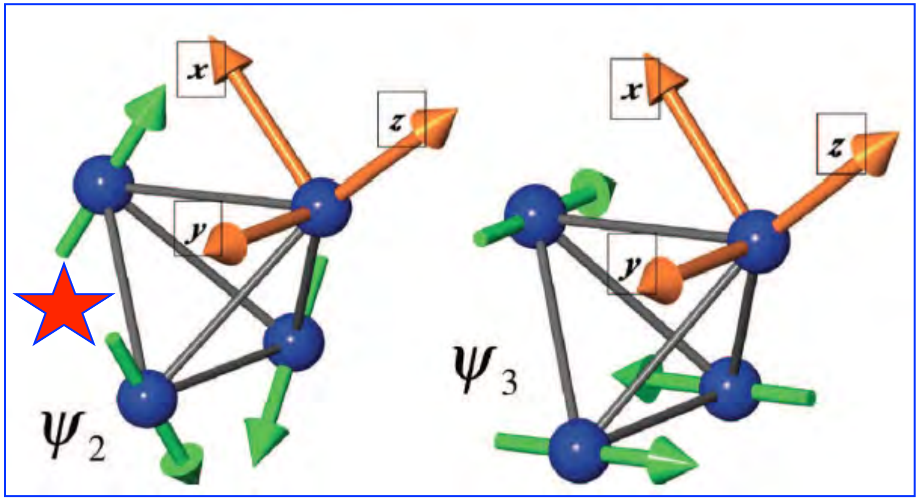


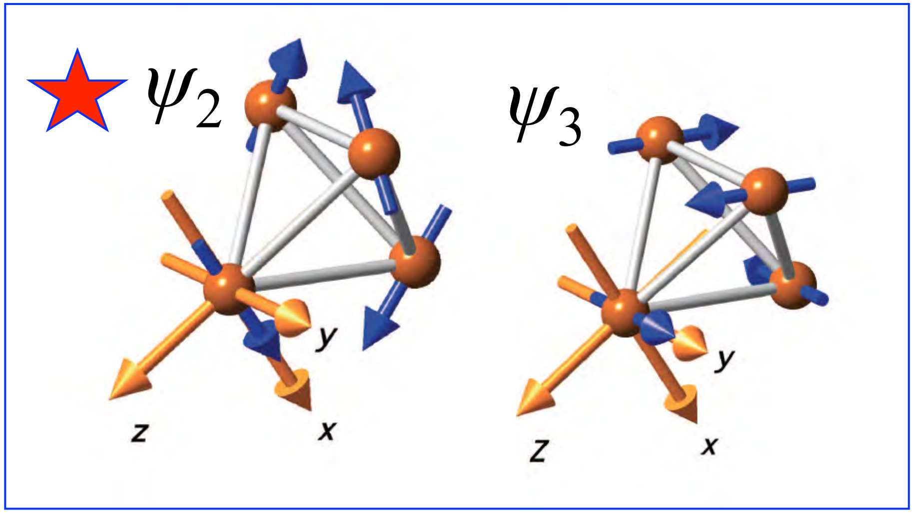
Champion *et al.*, PRB (RC) **68**, 020401 (2003)
Champion and Holdsworth, JPCM **16**, S665 (2004)

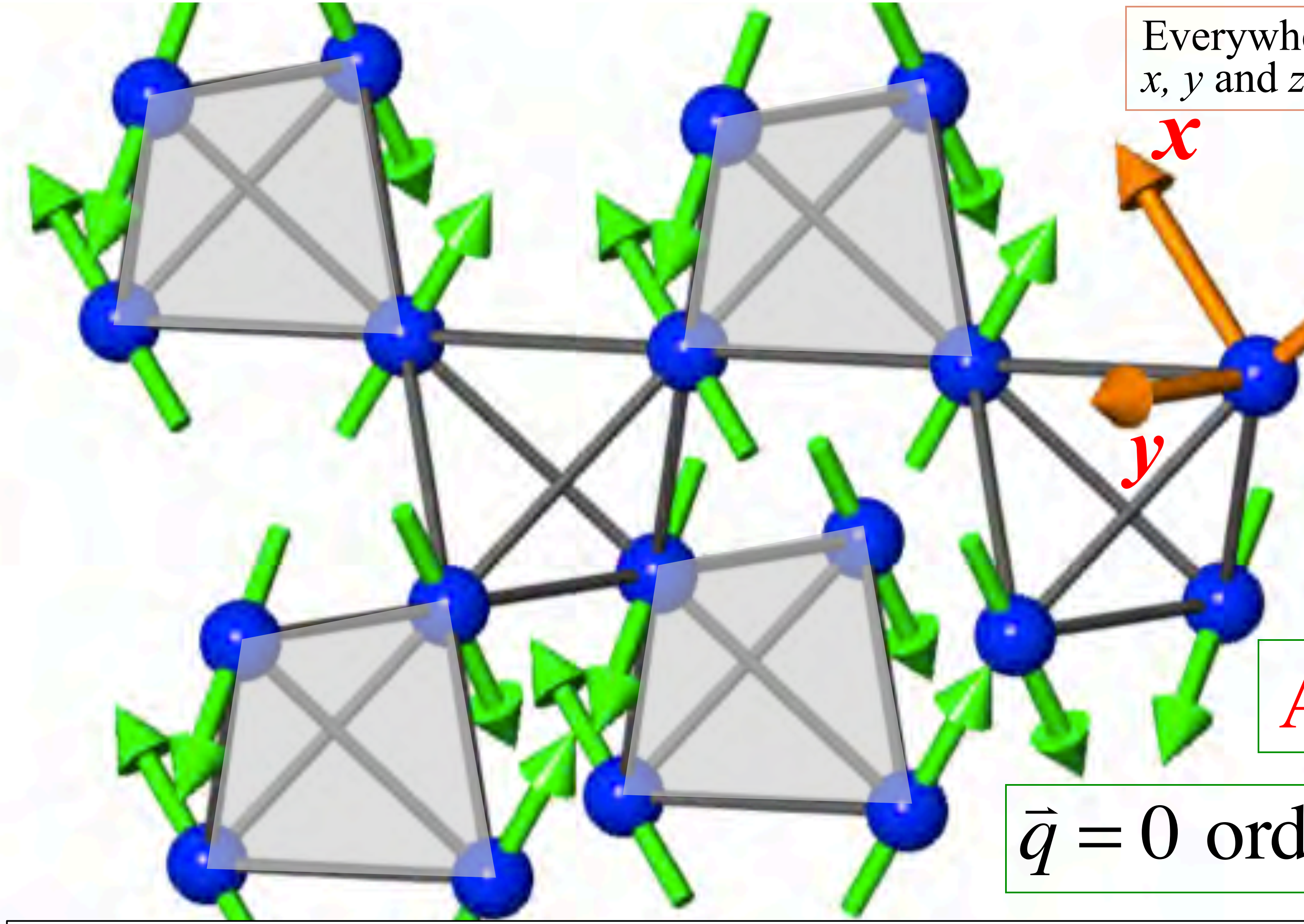
Poole *et al.*, JPCM **19**, 452201 (2007)

Er^{3+} behave as local
S=1/2 XY spin









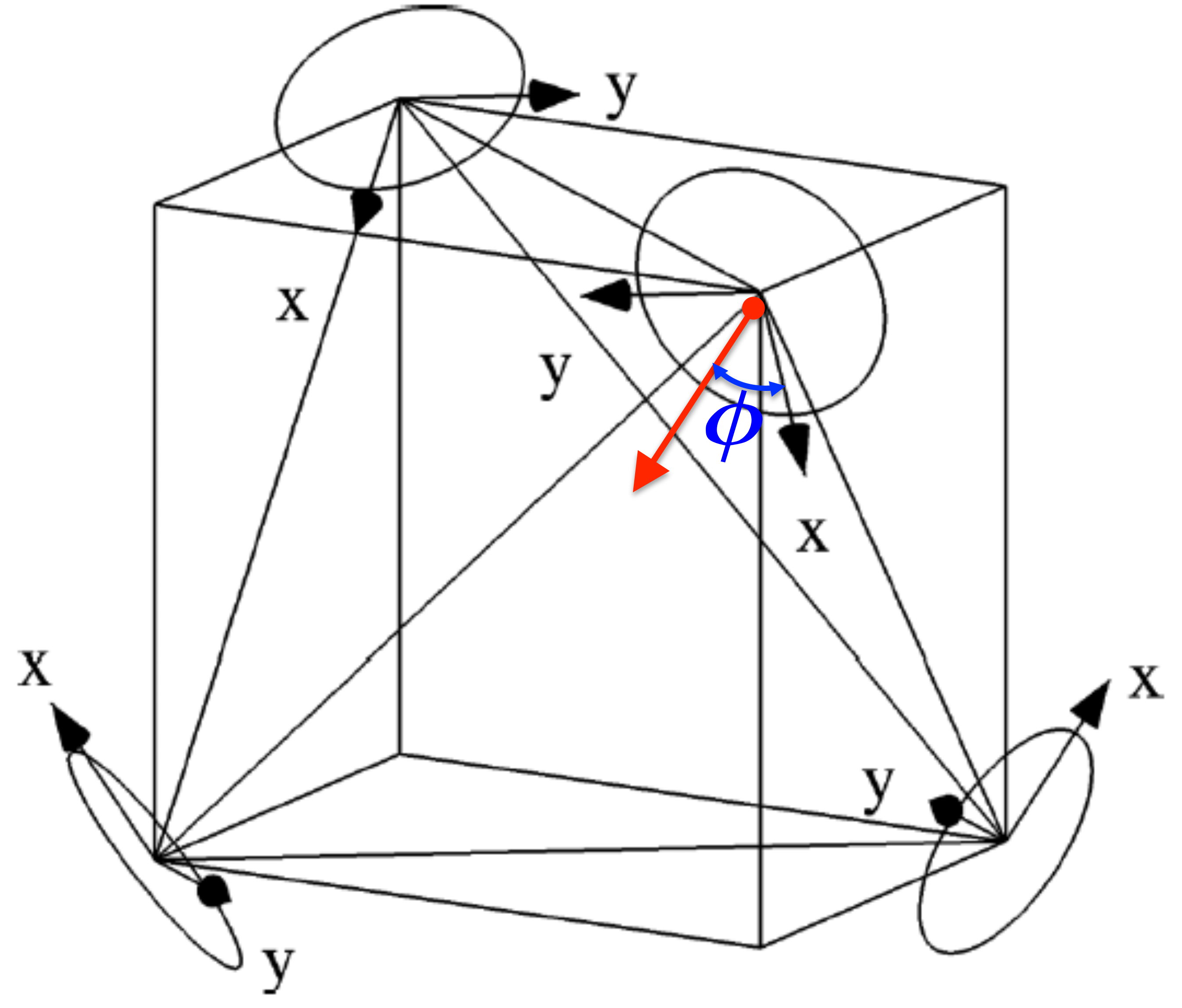
Everywhere in this talk, *x*, *y* and *z* are local axes

AF / ψ_2

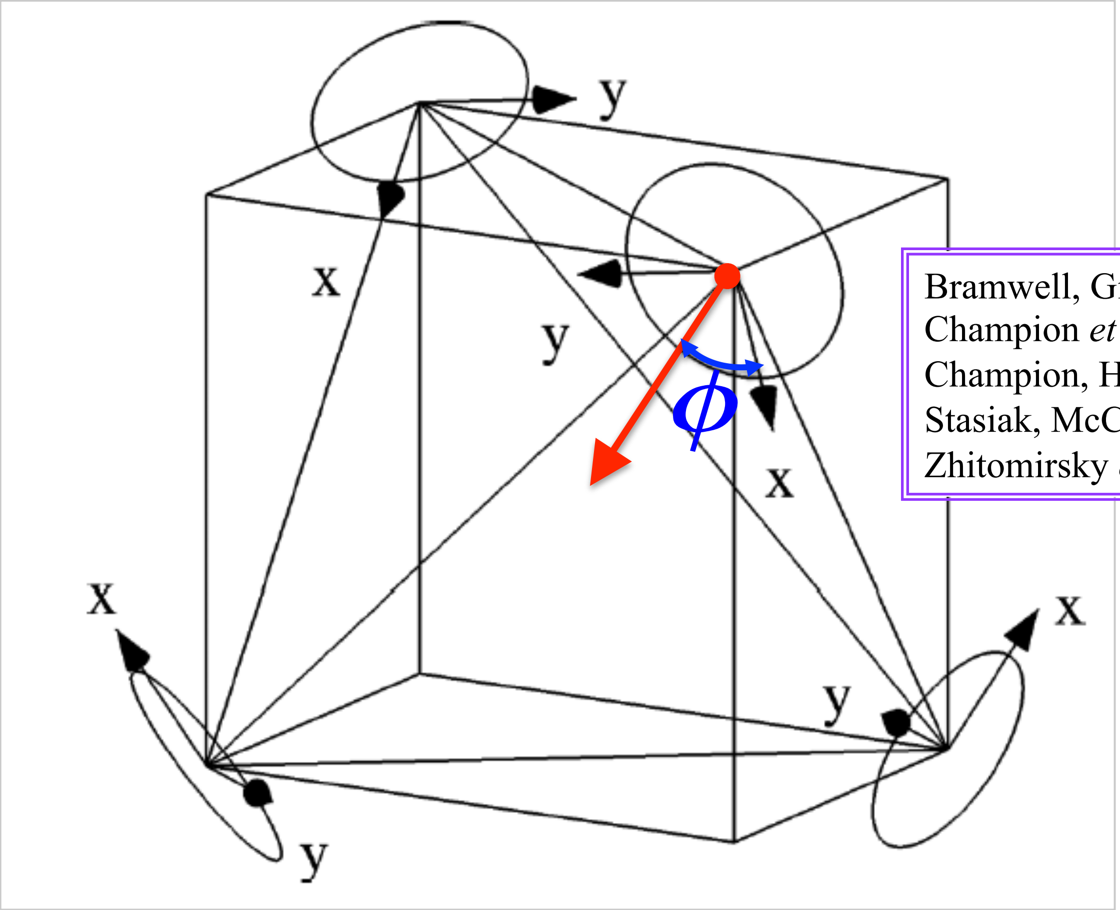
$\vec{q} = 0$ order

1. Champion *et al.* found in a classical pyrochlore *XY* antiferromagnet toy-model a selection of ψ_2 proceeding via <u>thermal</u> *order-by-disorder* (ObD)
2. They further speculated that quantum fluctuations could proceed similarly and that a ψ_2 ground state could be stabilized via <u>quantum</u> ObD

Degeneracy of Classical Ground State



Degeneracy of Classical Ground State



Bramwell, Gingras, Reimers, J, Appl. Phys. **75**, 5523 (1994)
Champion *et al.* PRB **68**, 020401R (2003)
Champion, Holdsworth, J. Phys. Cond Matt. **16**, S665 (2004)
Stasiak, McClarty, Gingras, Phys. Rev. B **89**, 024425 (2014)
Zhitomirsky *et al.*, Phys. Rev. Lett. **109**, 077204 (2012)

"No bilinear spin-spin interaction of arbitrary range and anisotropy can lift the classical degeneracy of the ψ_2/ψ_3 manifold"

Savary, Ross, Gaulin, Ruff, and Balents; Phys. Rev. Lett. **109**, 167201 (2012).

Proposed Solution to the $Er_2Ti_2O_7$ Problem

Order by Quantum Disorder in $Er_2Ti_2O_7$

L. Savary, K.A. Ross, B.D. Gaulin, J.P.C. Ruff, and L. Balents; Phys. Rev. Lett. **109**, 167201 (Published 15 October 2012)

$$H = J_{zz}\sum_{\langle i,j\rangle} S_i^z S_j^z - J_{\pm}\sum_{\langle i,j\rangle}\left(S_i^+ S_j^- + S_i^- S_j^+\right)$$

$$+J_{z\pm}\sum_{\langle i,j\rangle}\left[S_i^z\left(\zeta_{ij}S_j^+ + \zeta_{ij}^* S_j^-\right) + i \leftrightarrow j\right] + J_{\pm\pm}\sum_{\langle i,j\rangle}\left(\gamma_{ij}S_i^+S_j^+ + \gamma_{ij}^* S_i^- S_j^-\right) \quad S = \tfrac{1}{2} \text{ operator}$$

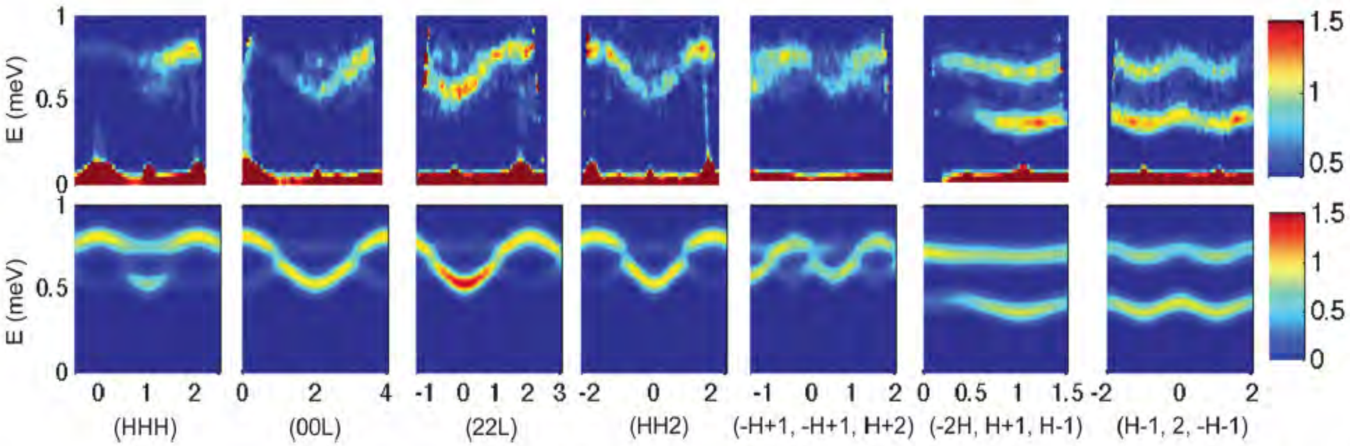
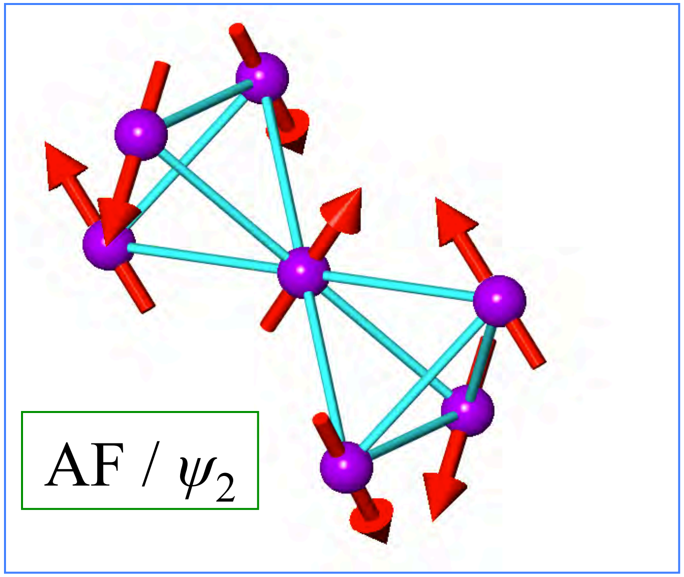
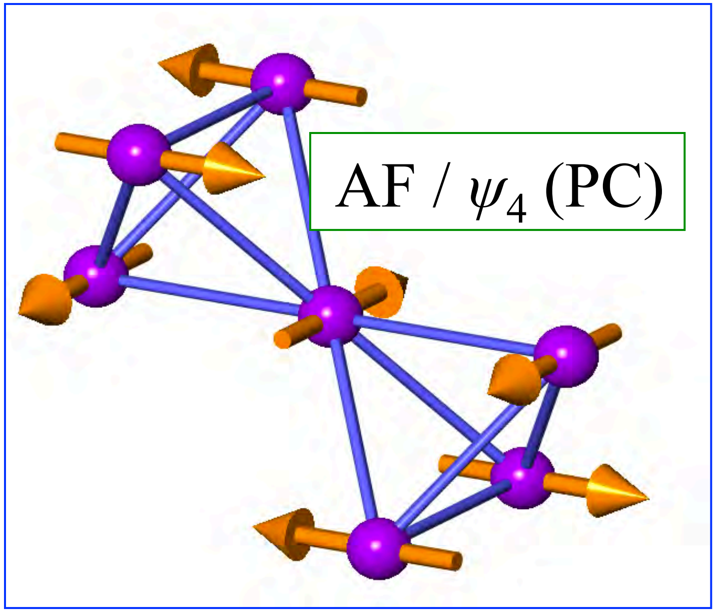
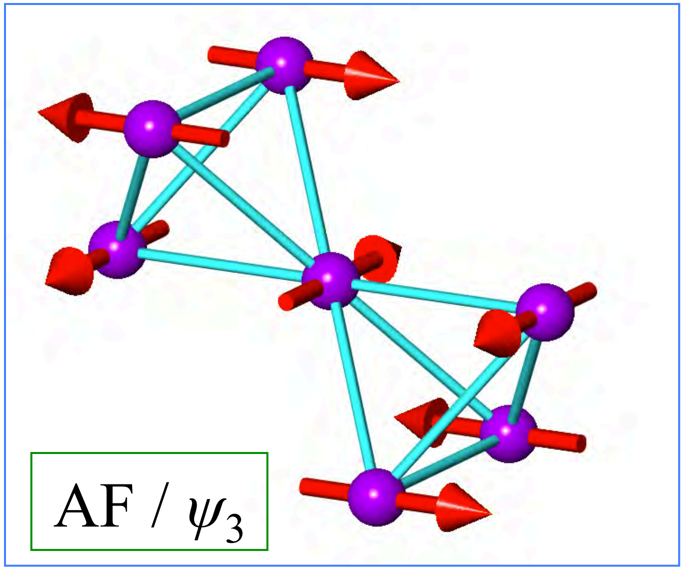


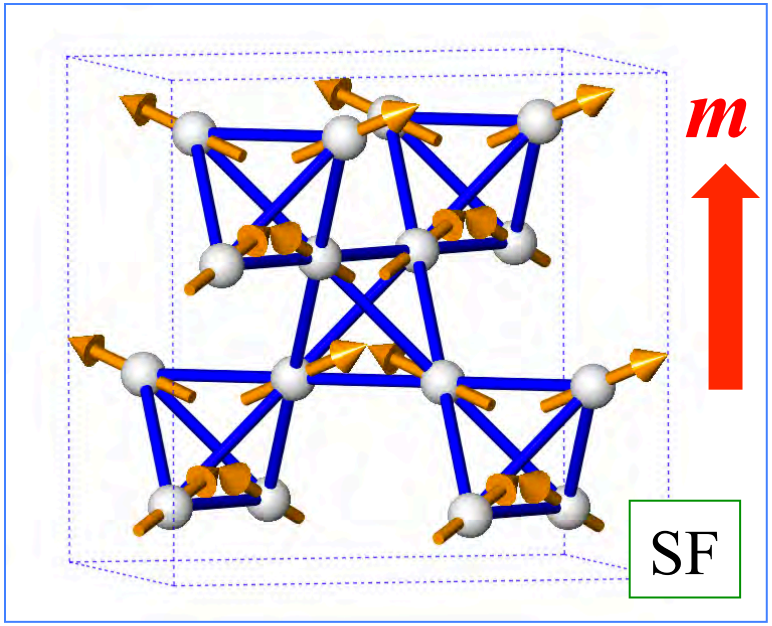
FIG. 1 (color). The measured $S(\mathbf{Q}, \omega)$ at $T = 30$ mK, $H = 3$ T sliced along several directions. The first five columns show $S(\mathbf{Q}, \omega)$ in the *HHL* plane, with the field applied along $[1\bar{1}0]$, while the last two columns show $S(\mathbf{Q}, \omega)$ for the field along [111]. Top row: measured $S(\mathbf{Q}, \omega)$. Bottom row: calculated $S(\mathbf{Q}, \omega)$, based on an anisotropic exchange model with six free parameters (see text) that were extracted by fitting to the measured dispersions.

What are (classical) long-range ordered phases?

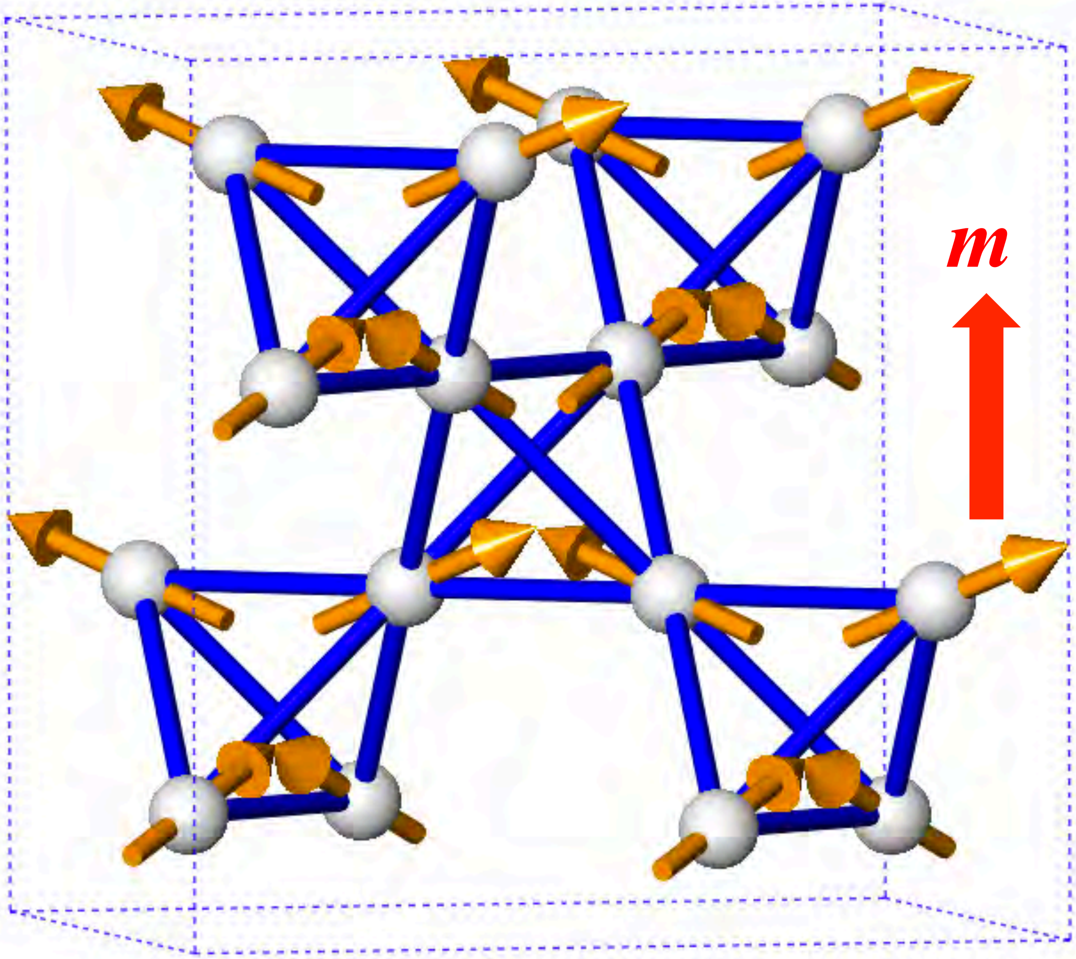






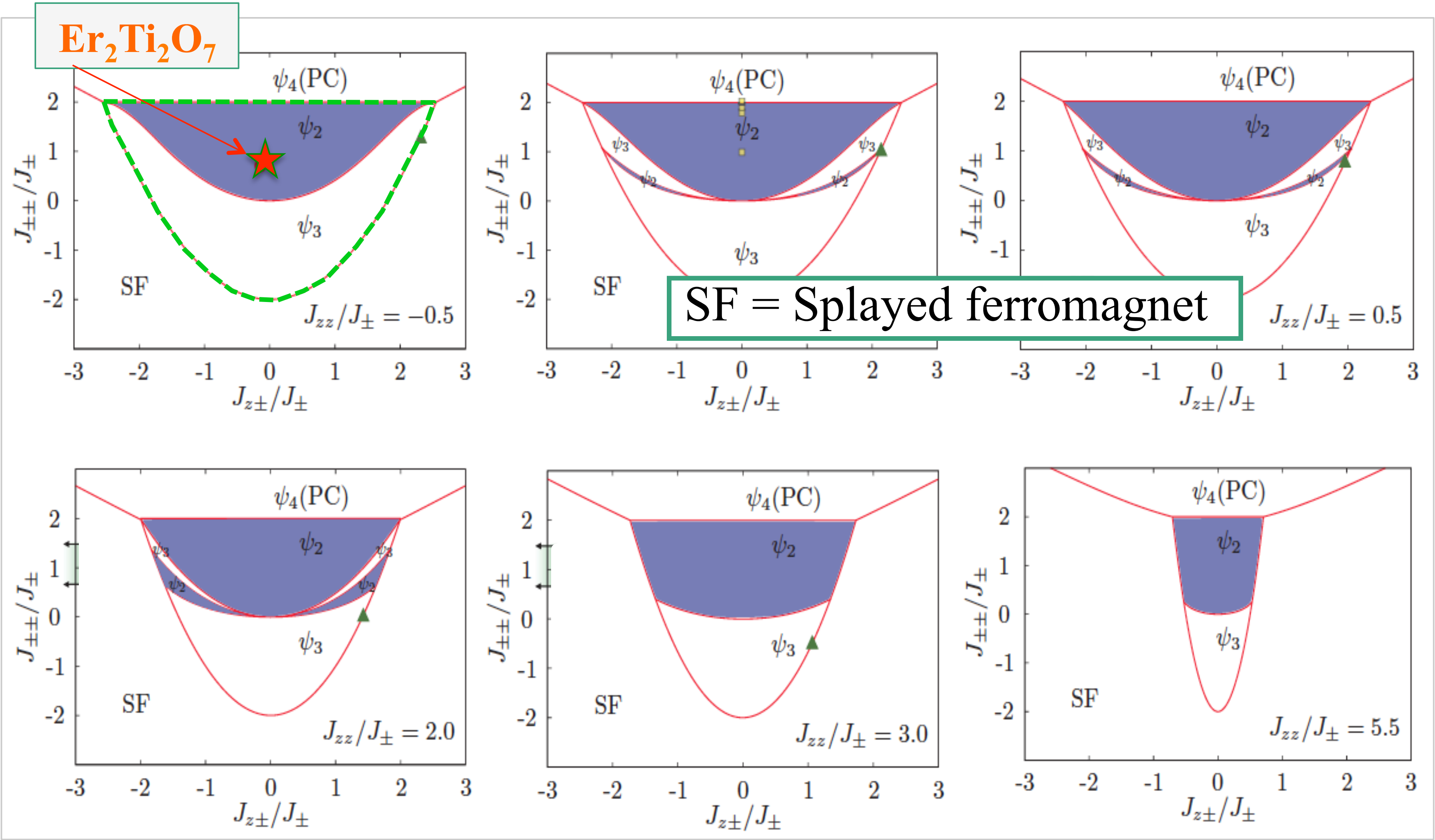


More about the splayed-ferromagnet phase



- Canted spin ice ("2-in/2-out")
- Splayed ferromagnet (SF)
- Net magnetization along <100>

Stability of ψ_2 for $Er_2Ti_2O_7$ against long-range dipolar interaction

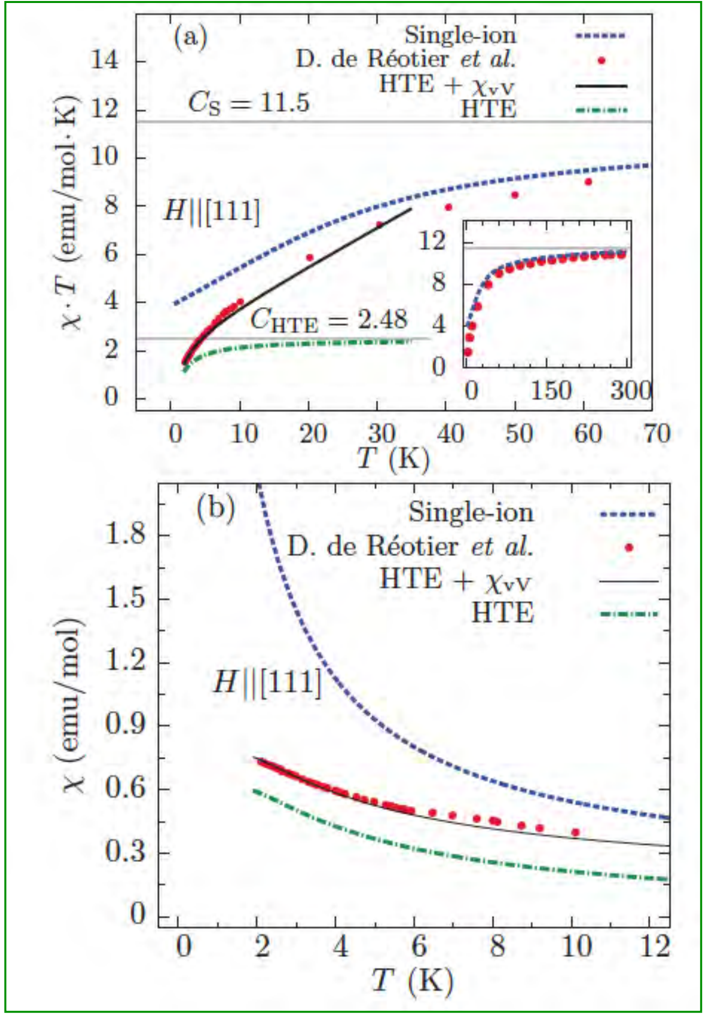


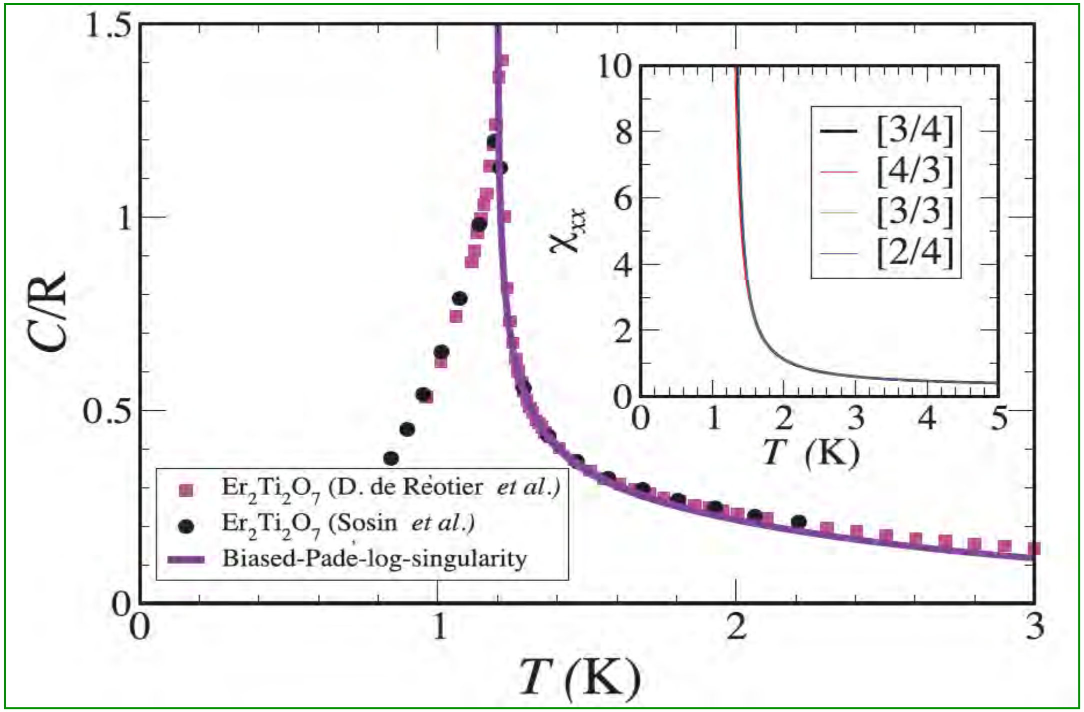
$Er_2Ti_2O_7$

SF = Splayed ferromagnet

Ground state phase diagram of generic XY pyrochlore magnets with quantum fluctuations;
Anson W.C. Wong, Zhihao Hao, and M.J.P. Gingras, Phys. Rev. B **88**, 144402 (2013)

"Phase Transition and Thermal Order-by-Disorder in the Pyrochlore Quantum Antiferromagnet $Er_2Ti_2O_7$: a High-Temperature Series Expansion Study";
J. Oitmaa, R.P. Singh, A.G.R. Day, B.V. Bagheri, M.J.P. Gingras; Phys. Rev. B (RC) **88**, 220404 (2013).



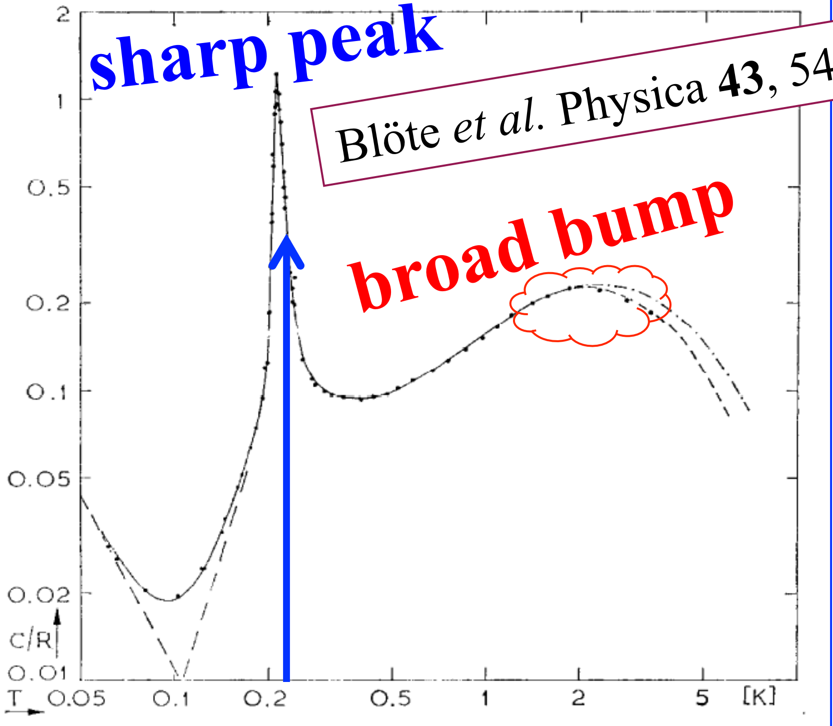


6th order "order-parameter" susceptibility shows that all terms in the $1/T$ expansion are larger <u>term by term</u>, starting at order $1/T^4$, for ψ_2 than for ψ_3

→ "demonstration" of thermal ObD operating at T_c in H_{Savary}

Outline

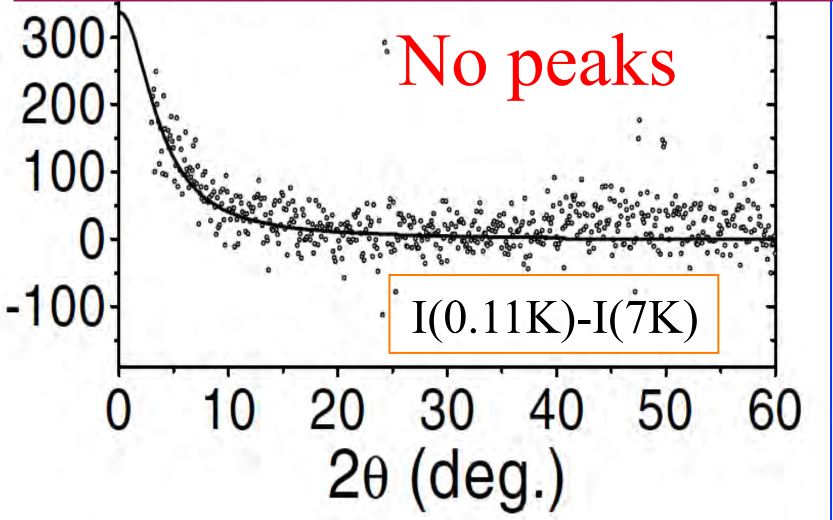
1. Rare- earth pyrochlore oxides
2. Order-by-disorder (ObD) in $Er_2Ti_2O_7$
3. ~~Why should one care about ObD?~~
4. $Yb_2Ti_2O_7$: what is the hidden order?
5. Conclusion



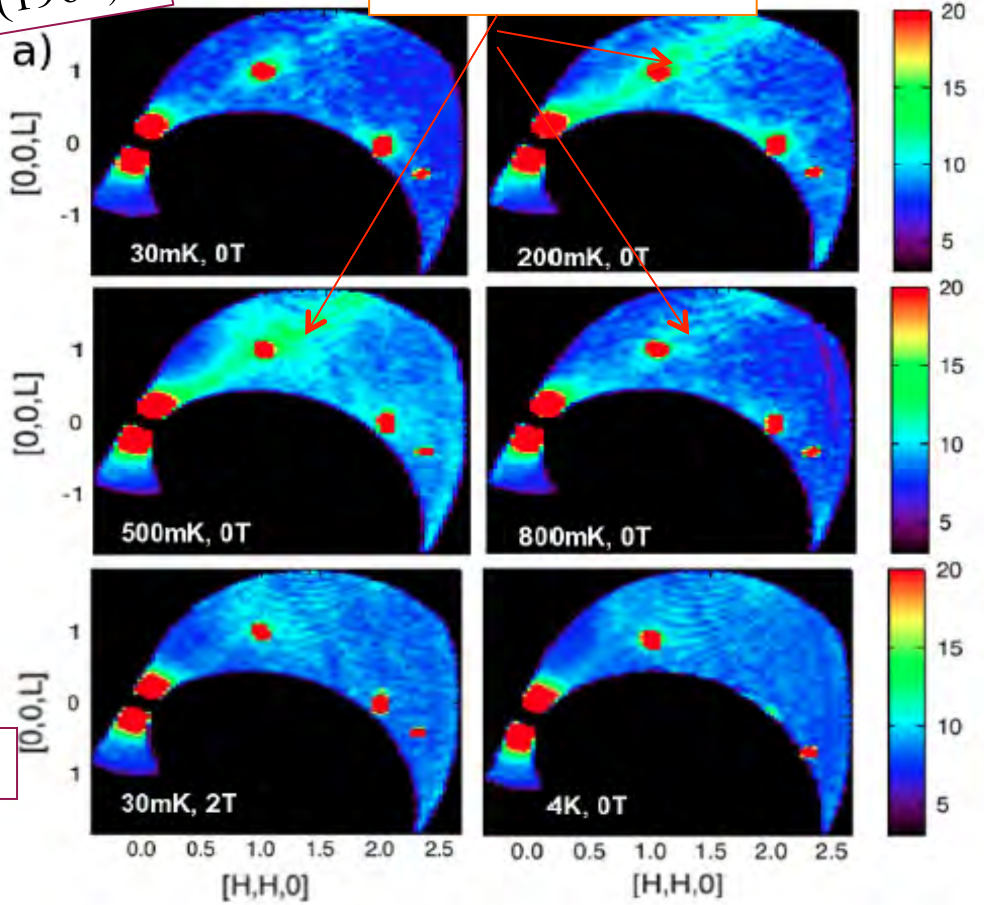
Blöte *et al.* Physica **43**, 549 (1969)

Fig. 12. Heat capacity of $Yb_2Ti_2O_7$.

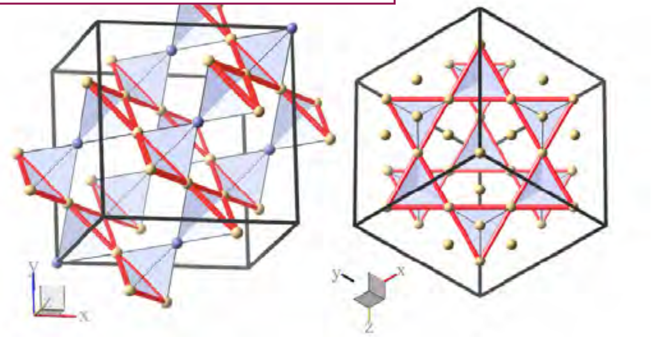
Hodges *et al.* Phys. Rev. Lett. **88**, 077204 (2002)



rods at $T<1$K



Ross *et al.* PRL **103**, 227202 (2009)

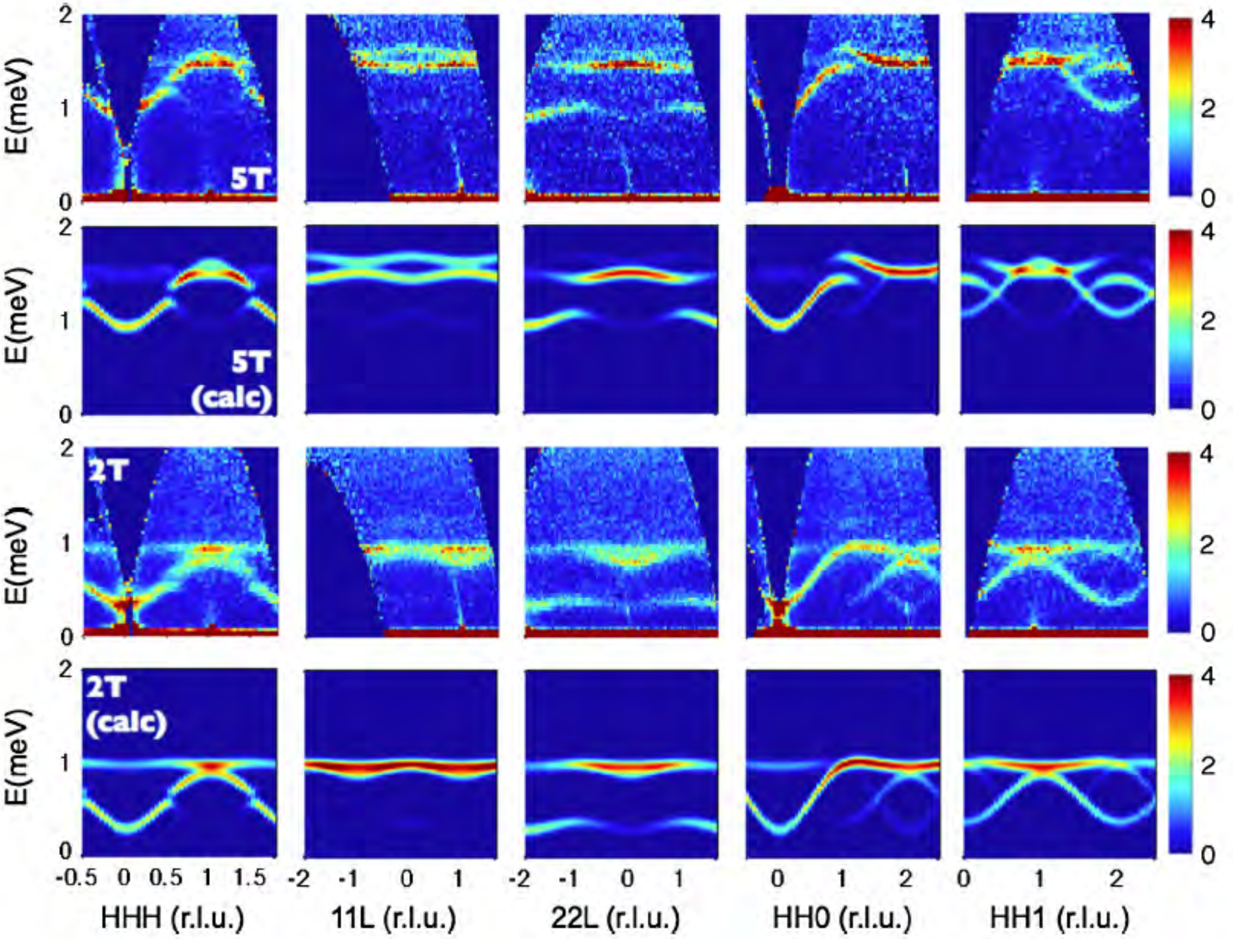


PHYSICAL REVIEW X 1, 021002 (2011)

Quantum Excitations in Quantum Spin Ice

Kate A. Ross,[1] Lucile Savary,[2] Bruce D. Gaulin,[1,3,4] and Leon Balents[5,*]

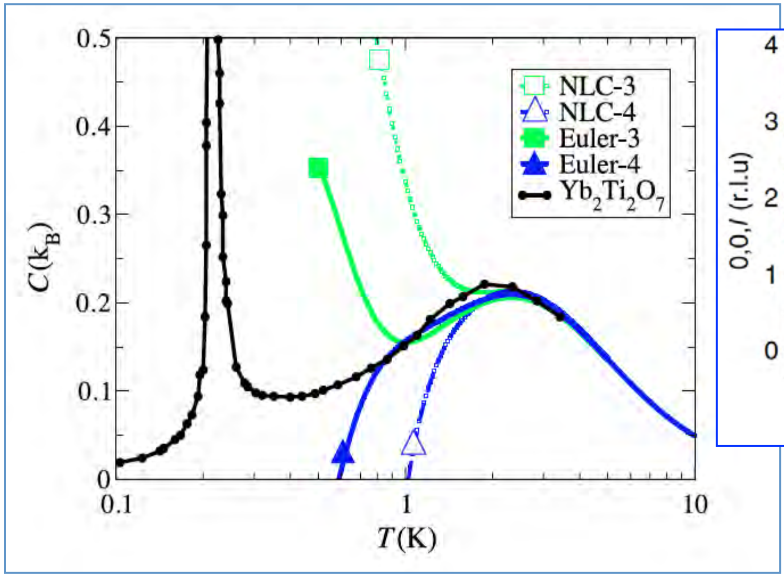
$$H = J_{zz}\sum_{\langle i,j\rangle} S_i^z S_j^z - J_{\pm}\sum_{\langle i,j\rangle}\left(S_i^+ S_j^- + S_i^- S_j^+\right) + J_{z\pm}\sum_{\langle i,j\rangle}\left[S_i^z\left(\zeta_{ij}S_j^+ + \zeta_{ij}^* S_j^-\right) + i \leftrightarrow j\right] + J_{\pm\pm}\sum_{\langle i,j\rangle}\left(\gamma_{ij}S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-\right)$$

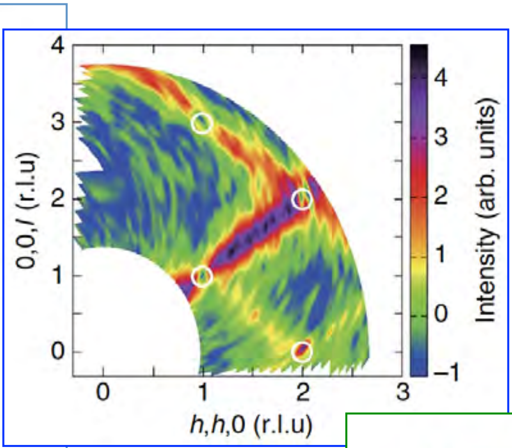


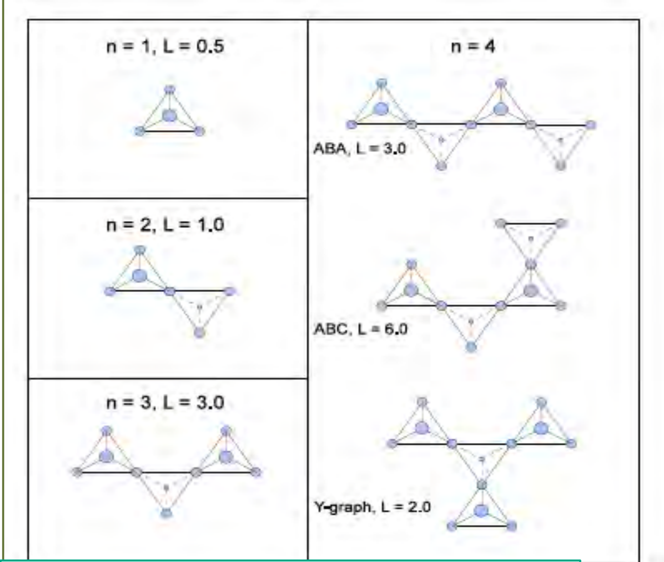
PRL **109,** 097205 (2012) PHYSICAL REVIEW LETTERS week ending 31 AUGUST 2012

Vindication of $Yb_2Ti_2O_7$ as a Model Exchange Quantum Spin Ice

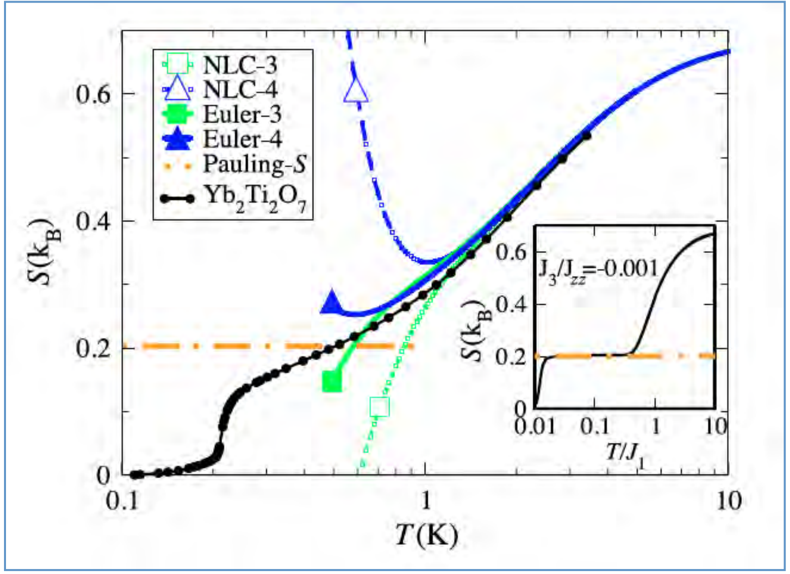
R. Applegate,[1] N. R. Hayre,[1] R. R. P. Singh,[1] T. Lin,[2] A. G. R. Day,[2,3] and M. J. P. Gingras[1,2,4]

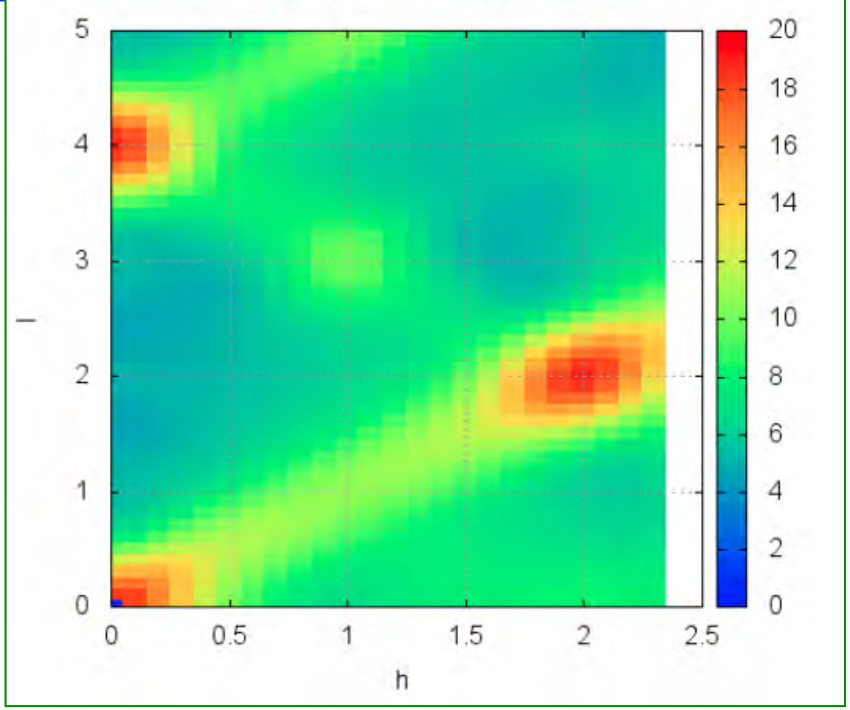




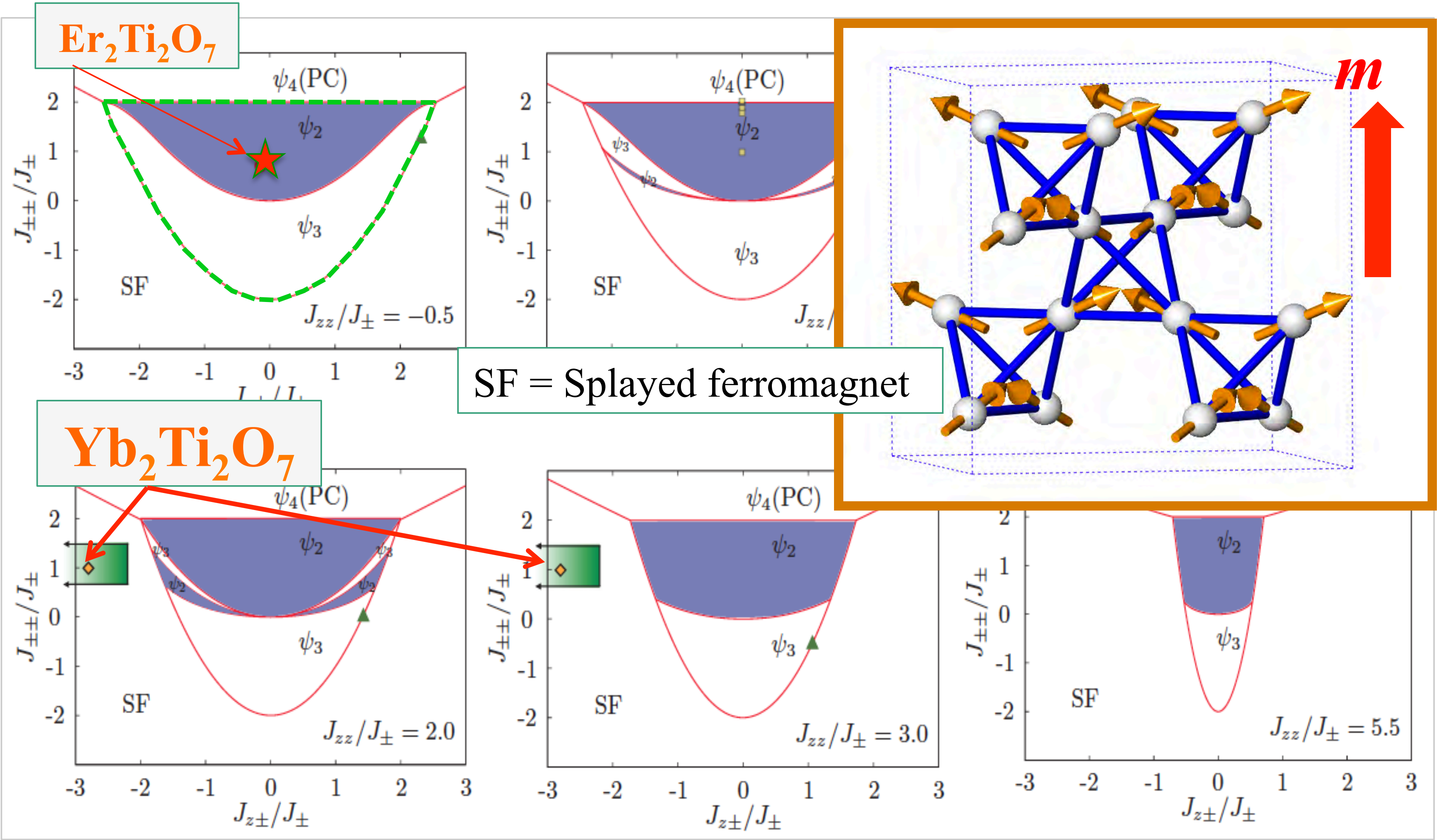


Numerical linked-cluster expansion (NLC)





Where is $Yb_2Ti_2O_7$?



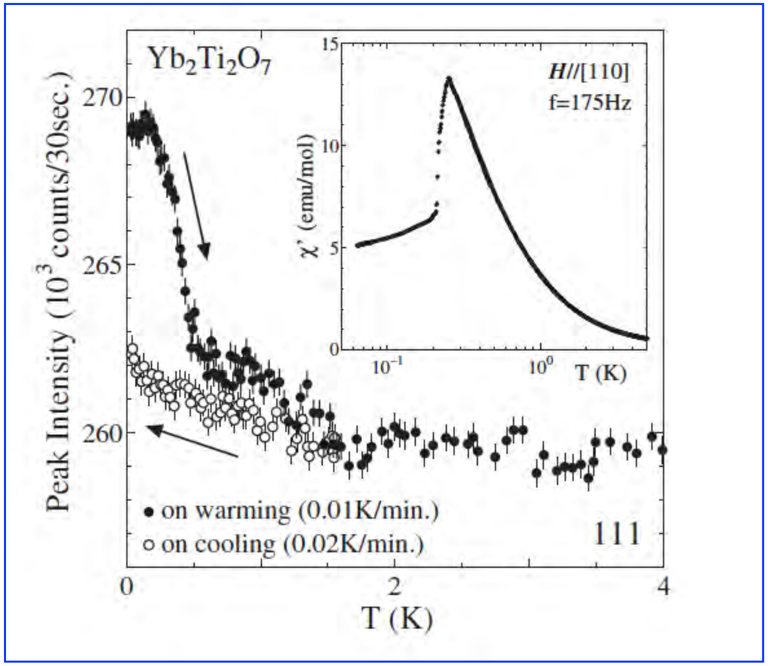
$Er_2Ti_2O_7$

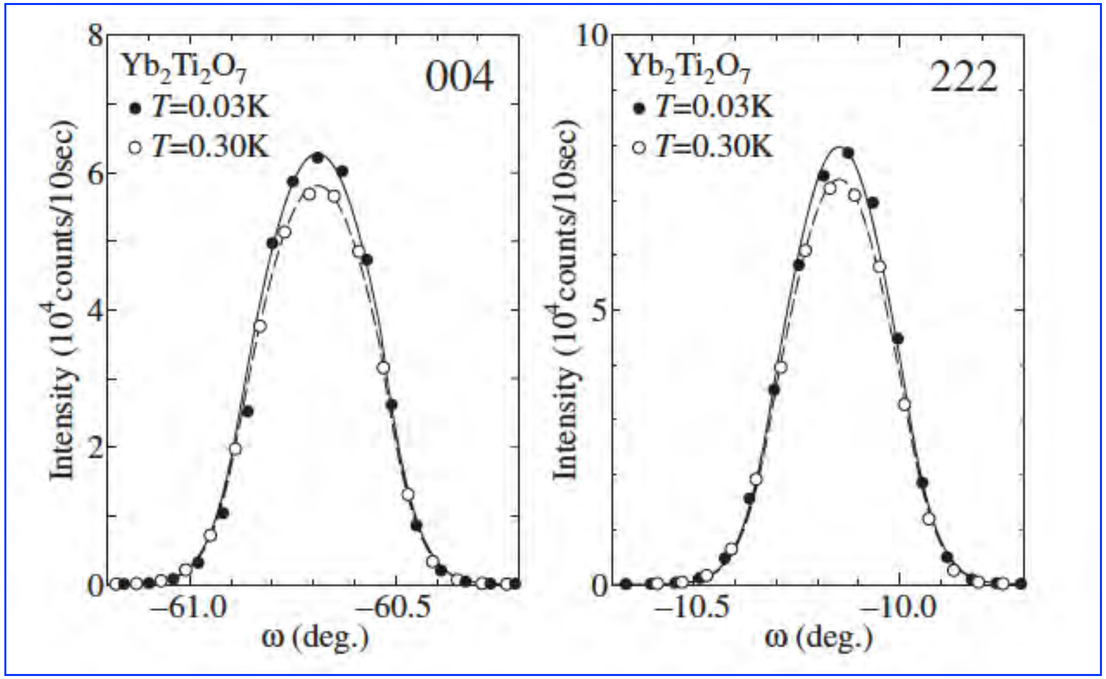
$Yb_2Ti_2O_7$

SF = Splayed ferromagnet

***Ground state phase diagram of generic XY pyrochlore magnets with quantum fluctuations*;**
Anson W.C. Wong, Zhihao Hao, and M.J.P. Gingras, Phys. Rev. B **88**, 144402 (2013)

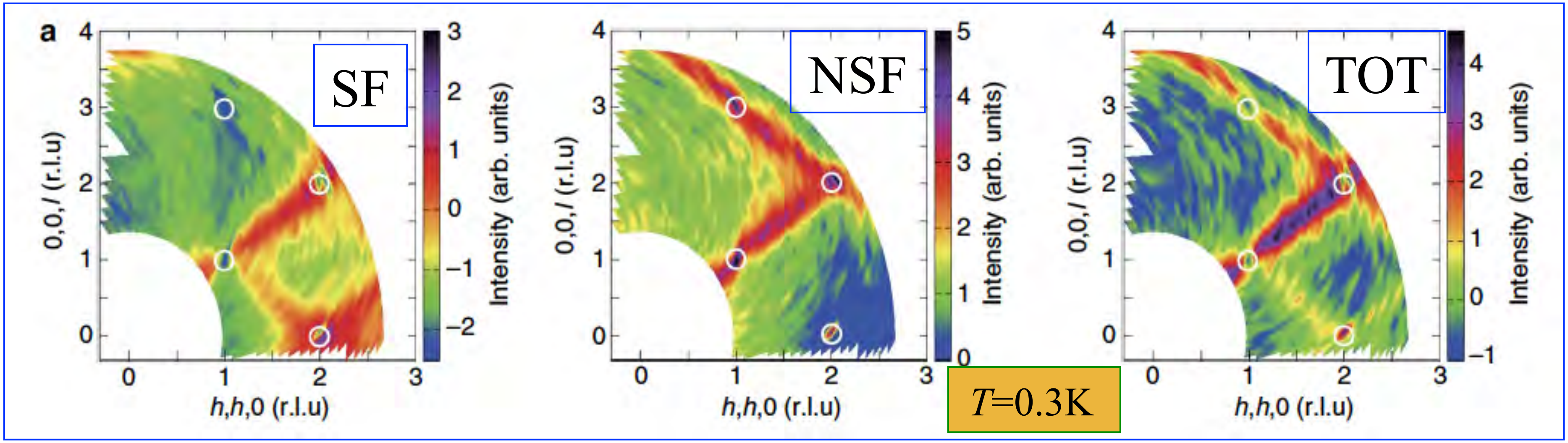
Yasui *et al.*, J.Phys. Soc. Jpn **72**, 3014 (2003)



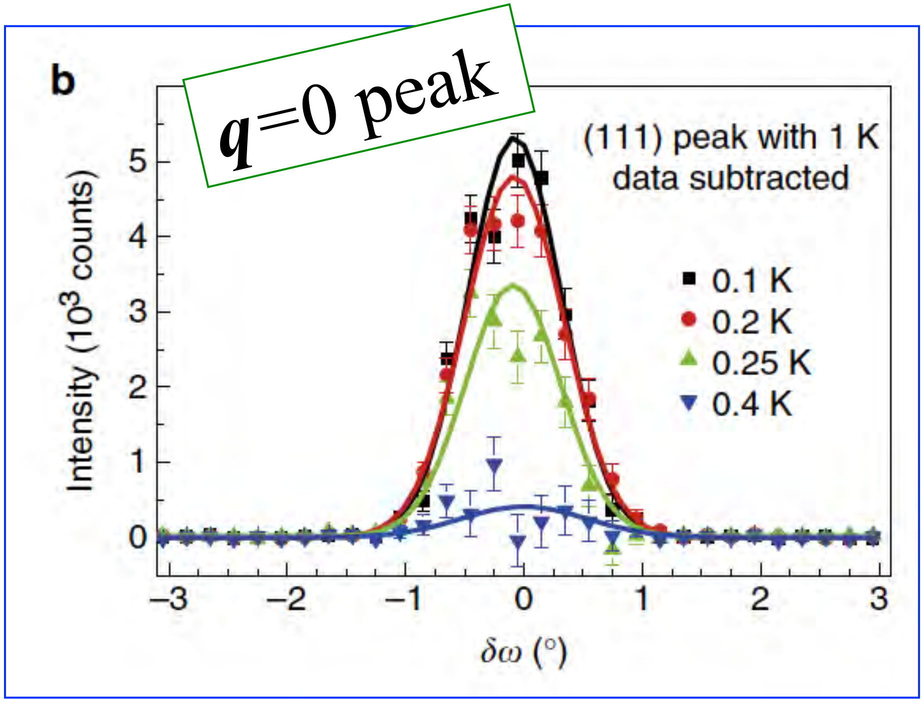


- ***q***=0 peaks + ferromag.-like bulk at 0.03K < T_c ~ 0.24K
- But not confirmed by neutron depolarization experiment [Gardner *et al.*, Phys. Rev. B **70**, 180404 (2004).

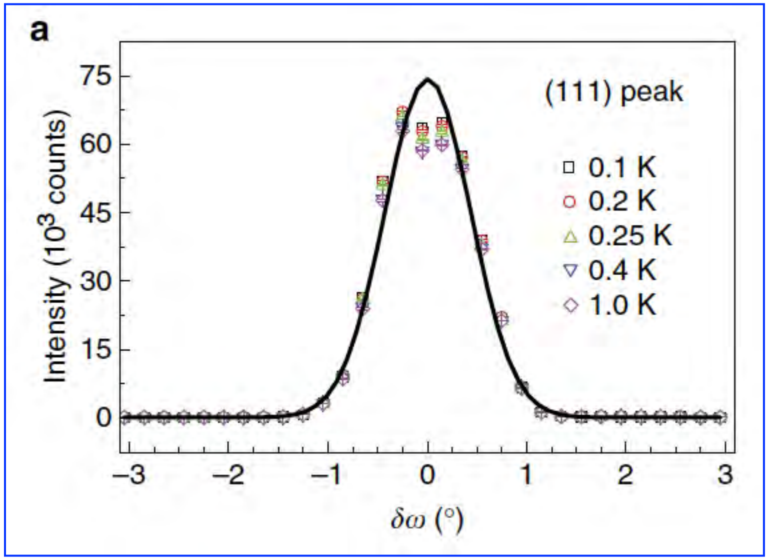
Chang *et al.*, Nature Communications | 3:992 | DOI: 10.1038/ncomms1989

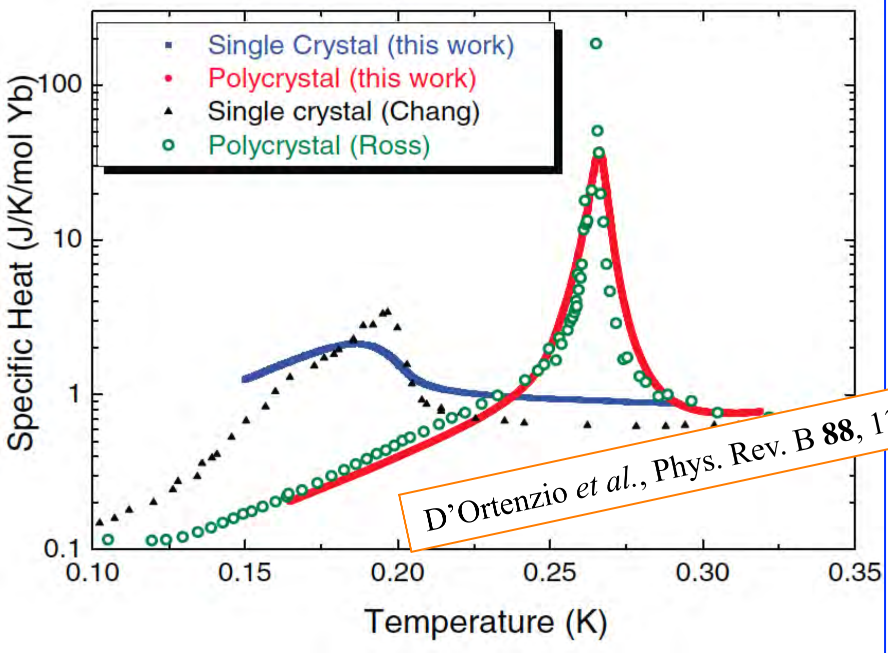


T=0.3K

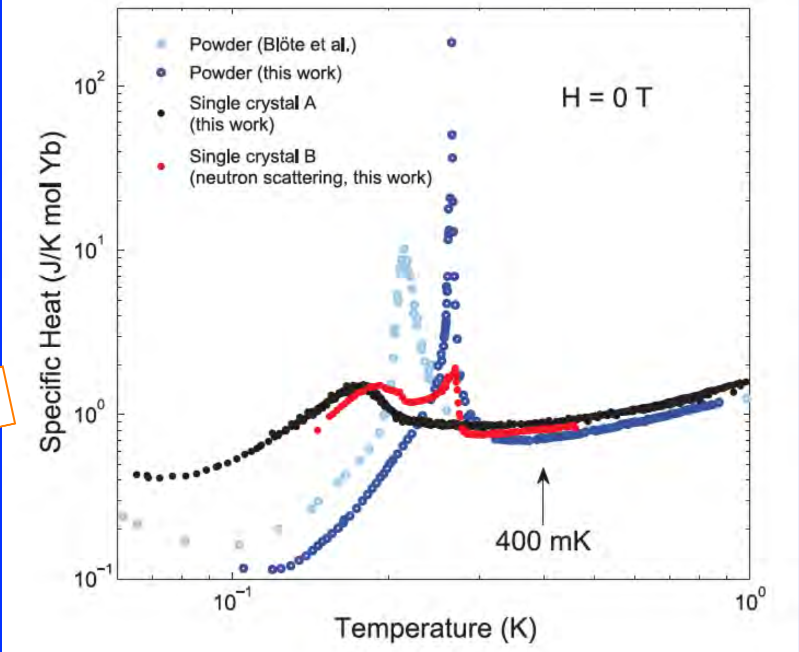


***q*=0 peak**

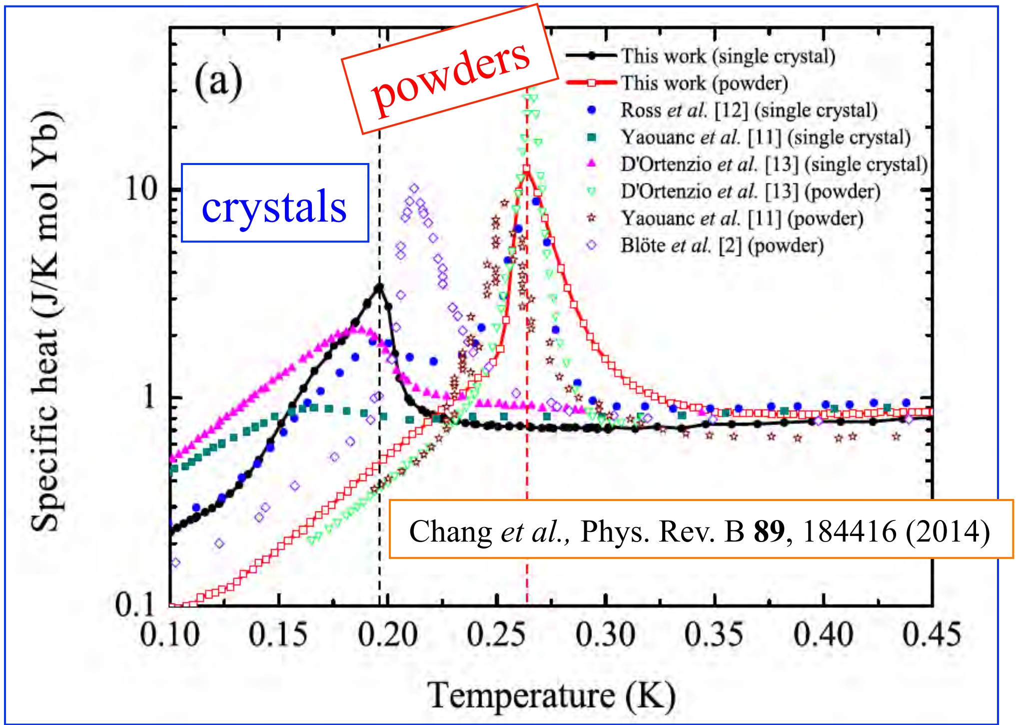




D'Ortenzio *et al.*, Phys. Rev. B **88**, 134428 (2013)



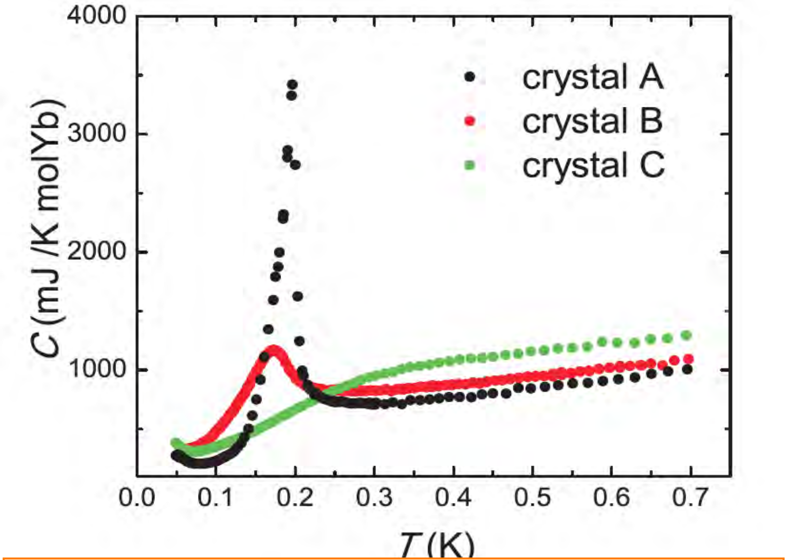
Ross *et al.*, Phys. Rev. B. **84**, 174442 (2011



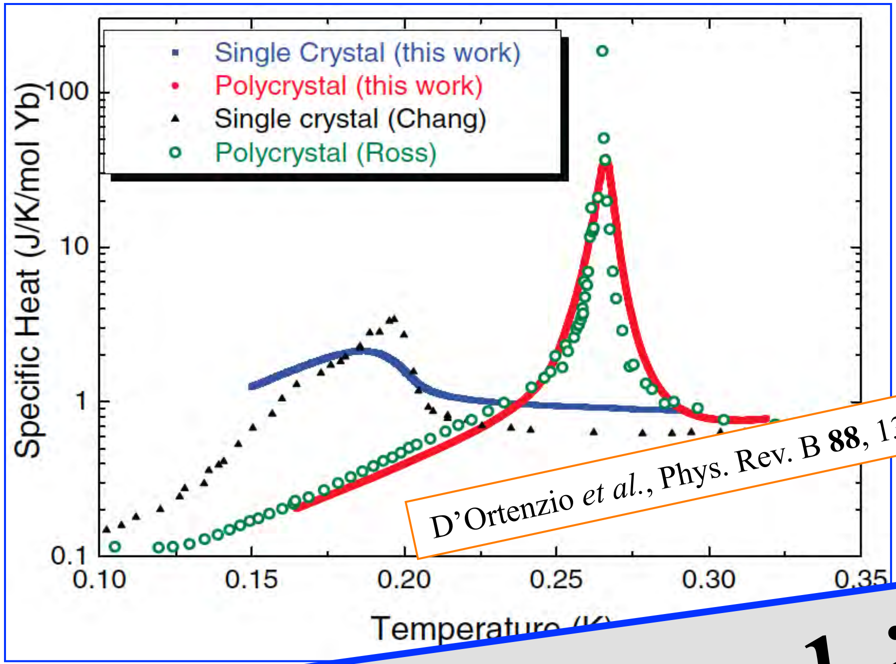
powders

crystals

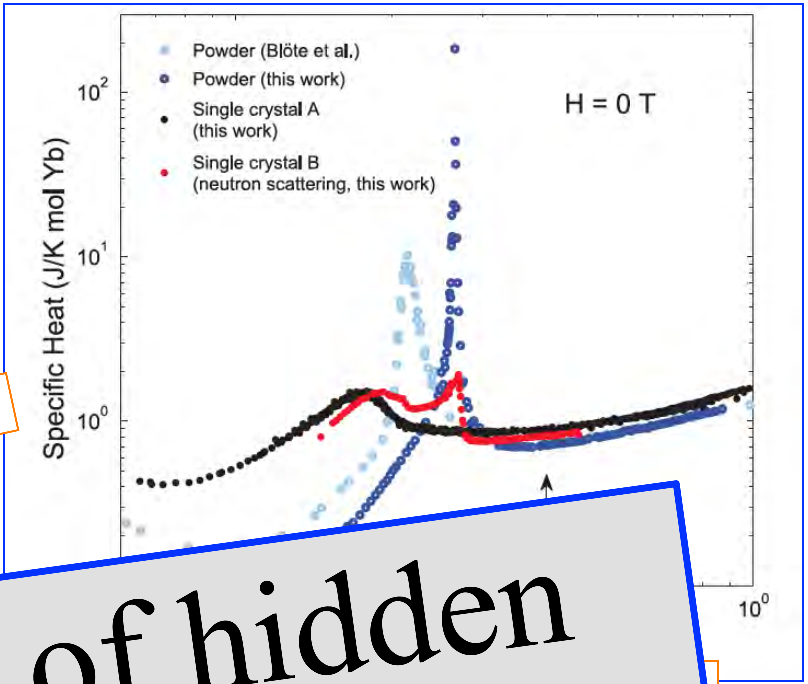
Chang *et al.*, Phys. Rev. B **89**, 184416 (2014)



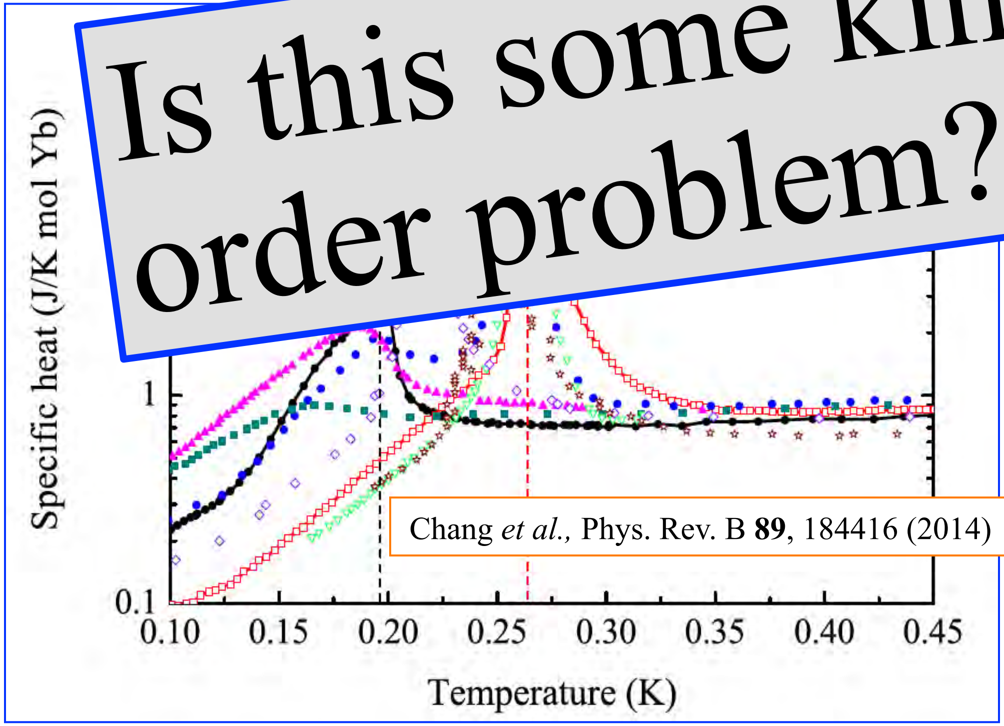
Chang *et al.*, Nat. Comm. | DOI: 10.1038/ncomms1989



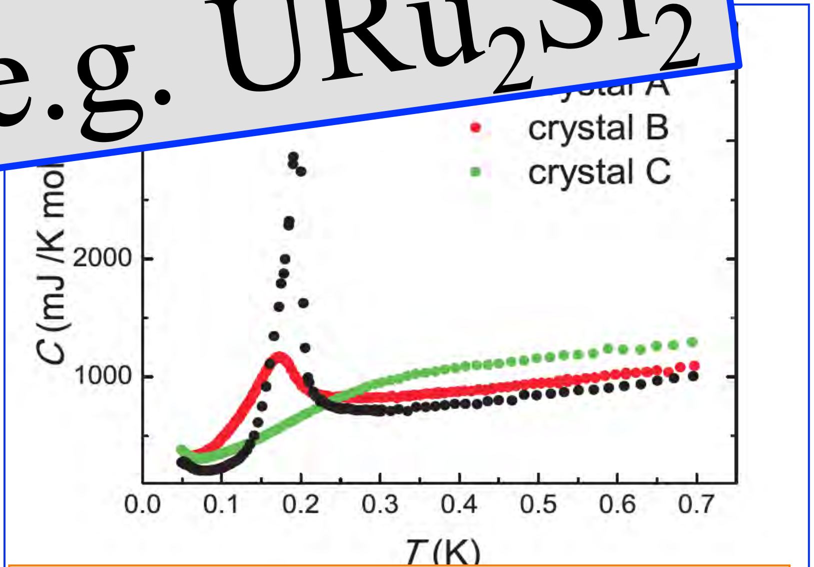
D'Ortenzio *et al.*, Phys. Rev. B **88**, 134428 (2013)



Is this some kind of hidden order problem? e.g. URu_2Si_2

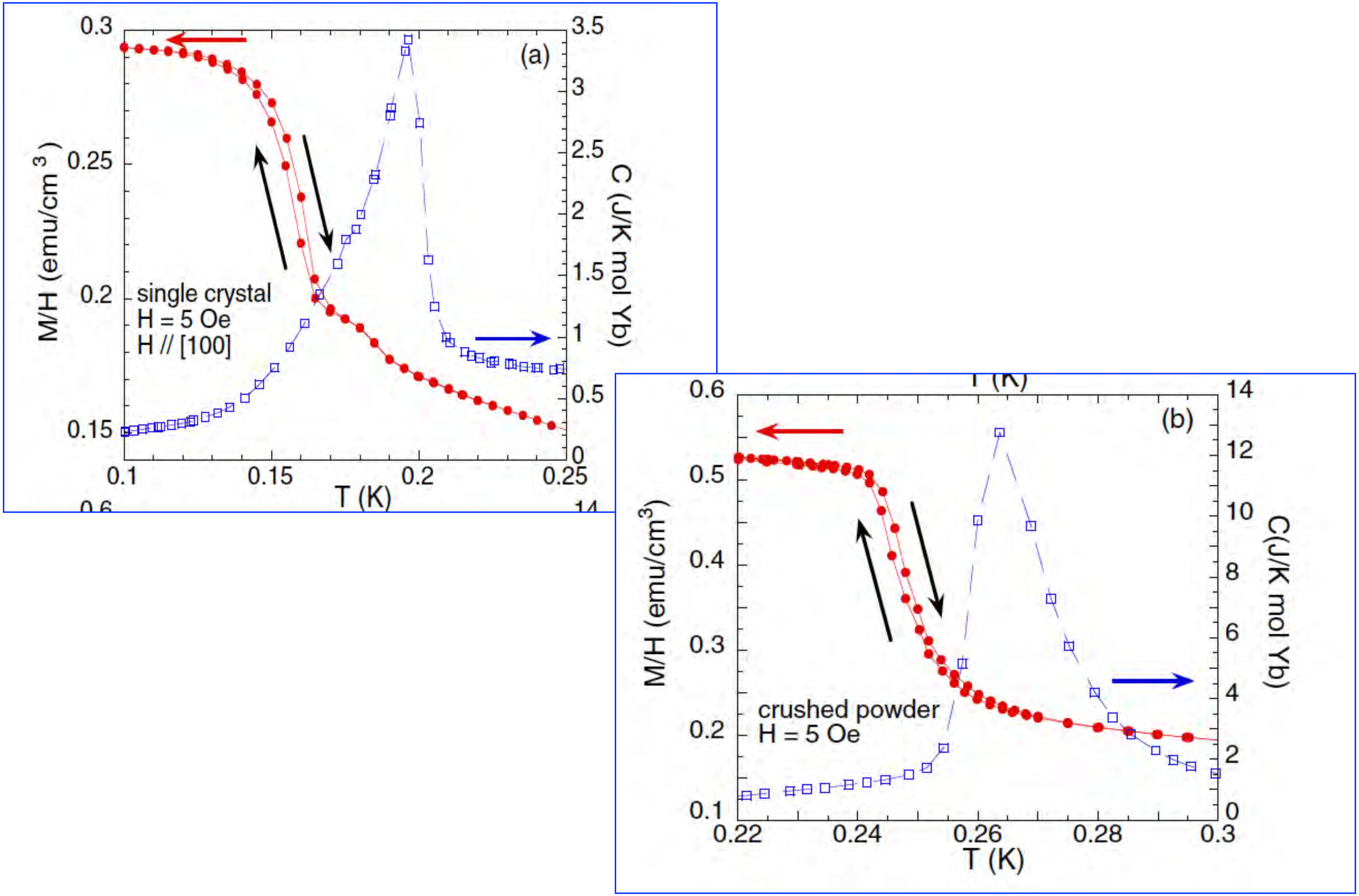


Chang *et al.,* Phys. Rev. B **89**, 184416 (2014)



Chang *et al.*, Nat. Comm. | DOI: 10.1038/ncomms1989

<u>Crucial hint</u>: Lhotel *et al.* Physical Review B **89**, 224419 (2014)



Summary of $Yb_2Ti_2O_7$ Experiments

- Large sensitivity to sample preparation (powders vs single crystals)
- Clear tendency towards ferromagnetic-like state
- A few (powder and single crystal) samples show a 5-25% difference in rise of magnetization vs highest-temperature magnetic specific heat peak

➔ Suggests that the material is in the vicinity of a ferromagnetic-like state and a competing order

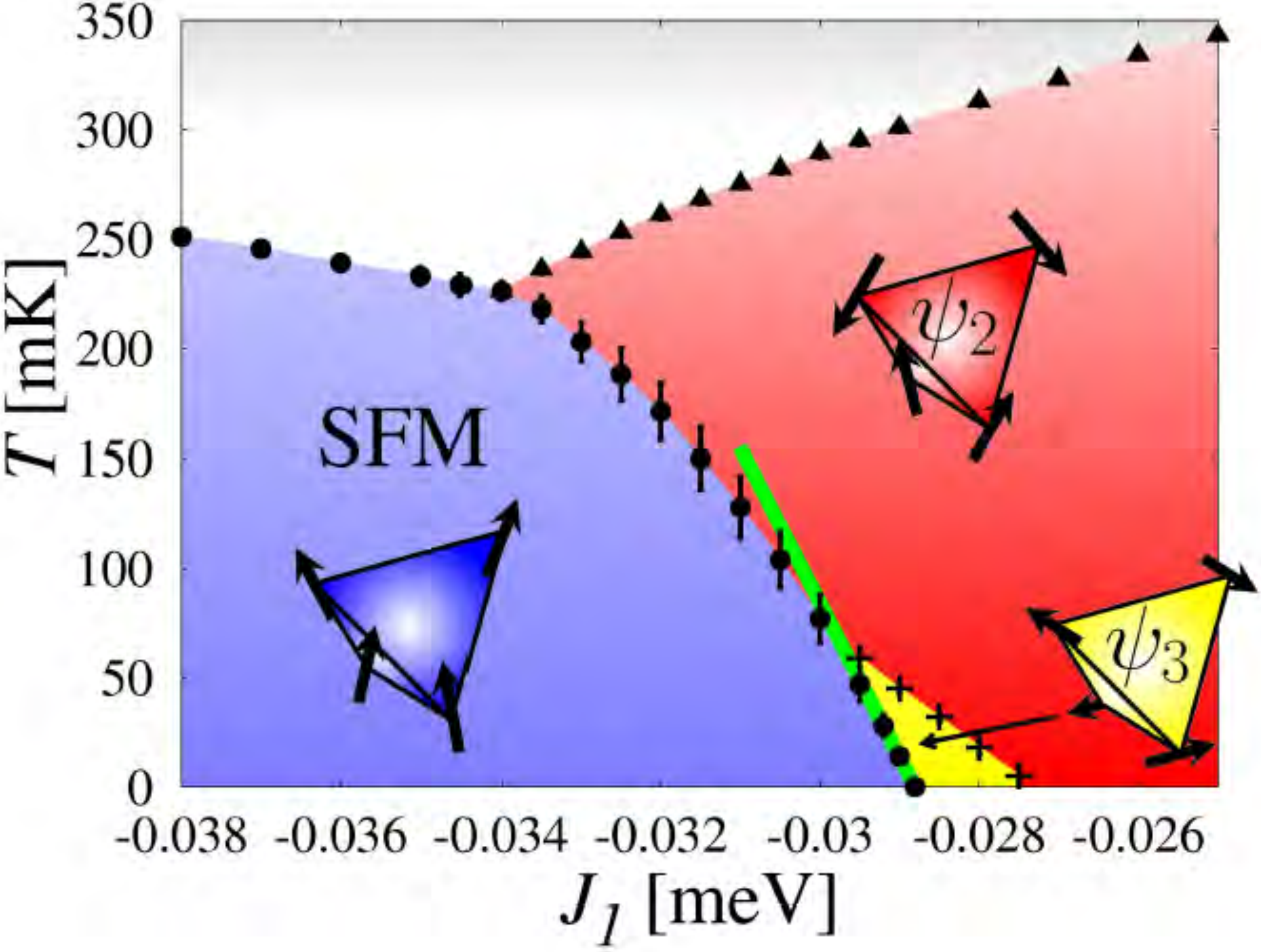
Jaubert, Benton, Rau, Oitmaa, Singh, Shannon, Gingras;
"Are multiphase competition & order-by-disorder the keys to understanding $Yb_2Ti_2O_7$*?"*; arXiv:1505.05499

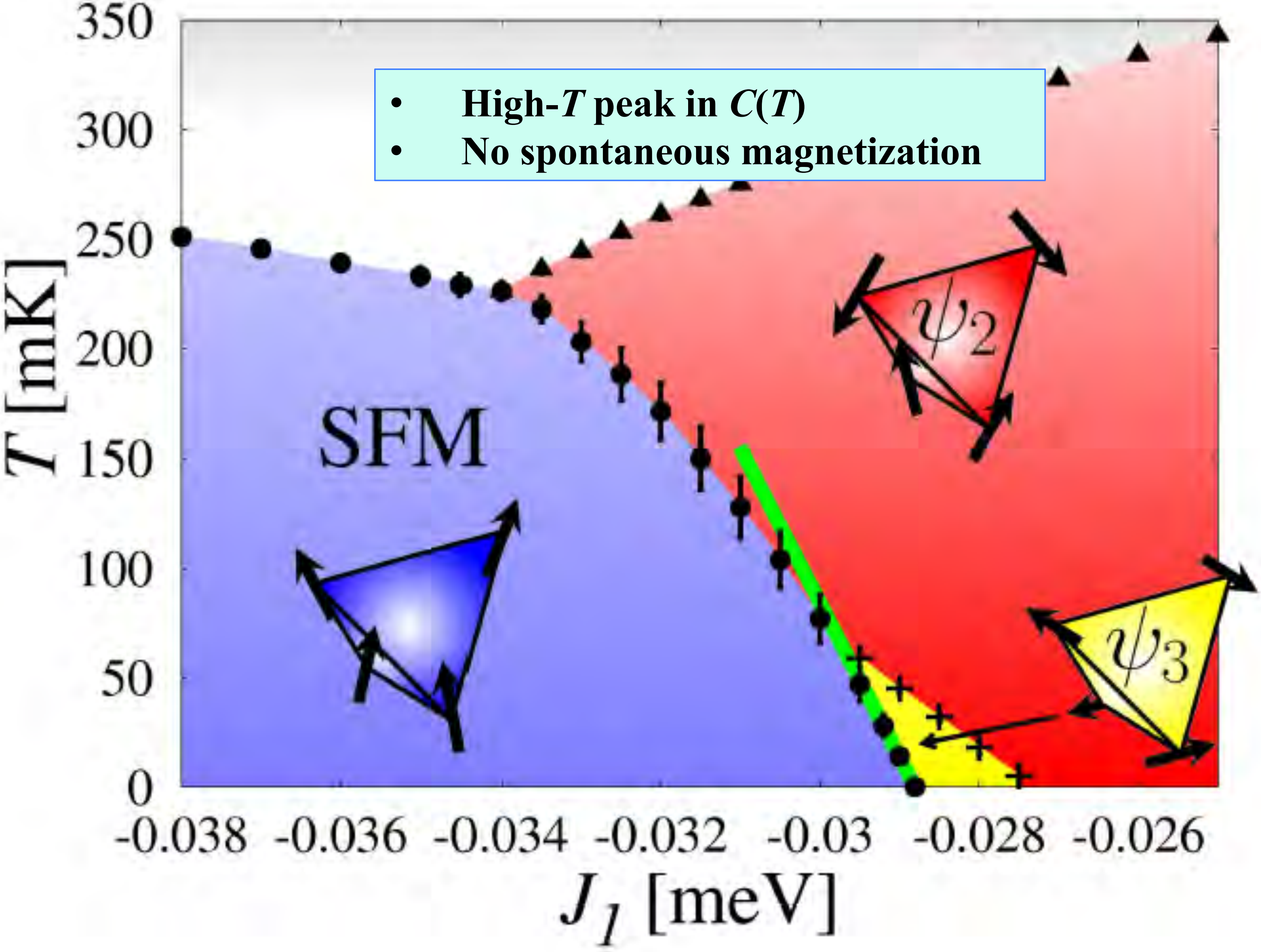
By combining:

- Classical Monte Carlo
- Classical and quantum spin wave calculations
- Exact diagonalization
- Cluster mean-field theory
- Numerical linked-cluster expansion
- High-temperature series expansion

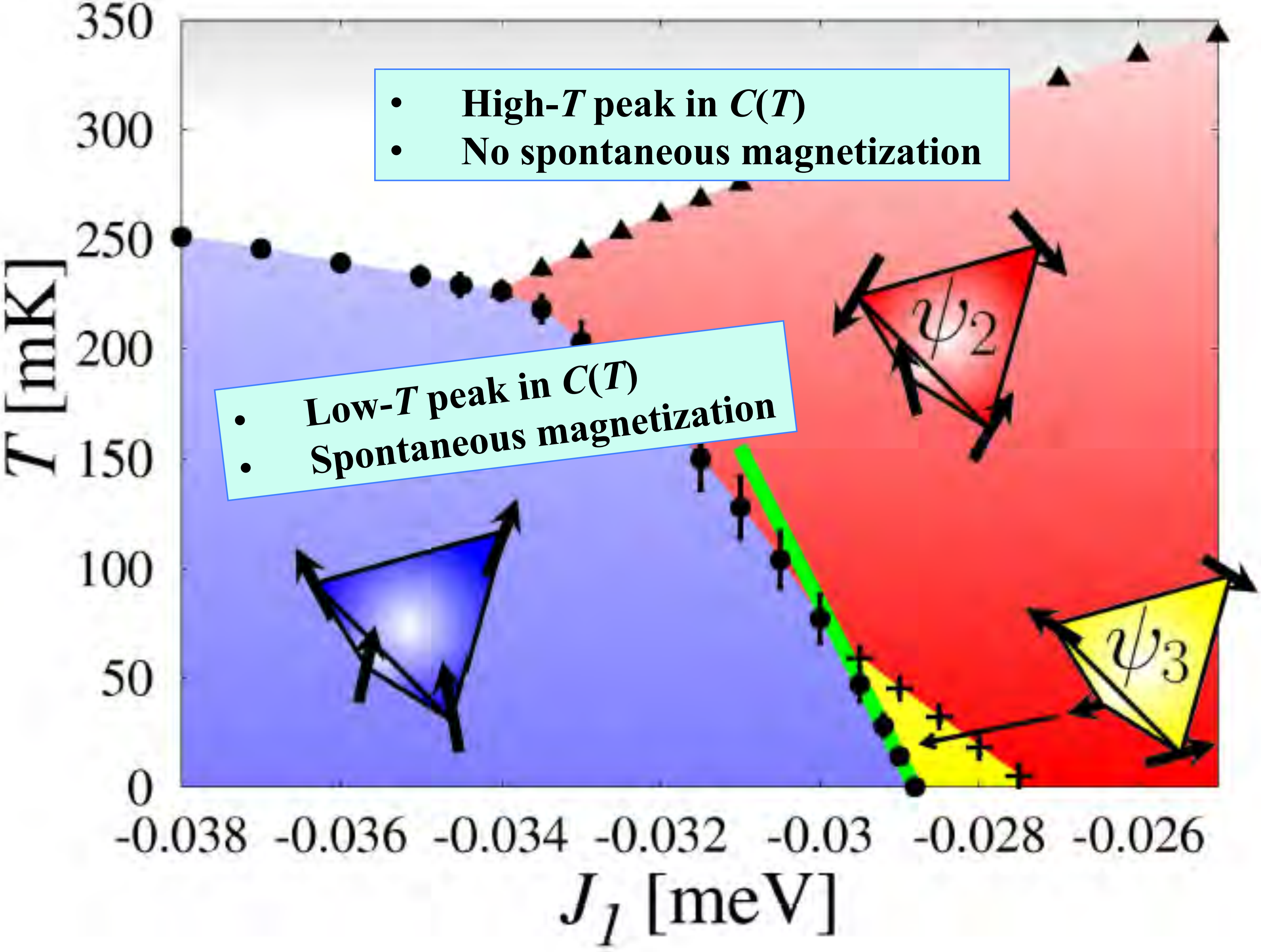
- Given the uncertainty on the exchange parameters determined by Ross, Savary, Gaulin and Balents; Phys. Rev. X **1**, 021002 (2011)
- and the computed quantum shift/renormalization of the SFM-AF boundary

→ We found that $Yb_2Ti_2O_7$ is very close to the SFM-AF boundary, and …


350
300
250
200
150
100
50
0
T [mK]
-0.038
-0.036
-0.034
-0.032
-0.03
-0.028
-0.026
J_1 [meV]
SFM
ψ_2
ψ_3

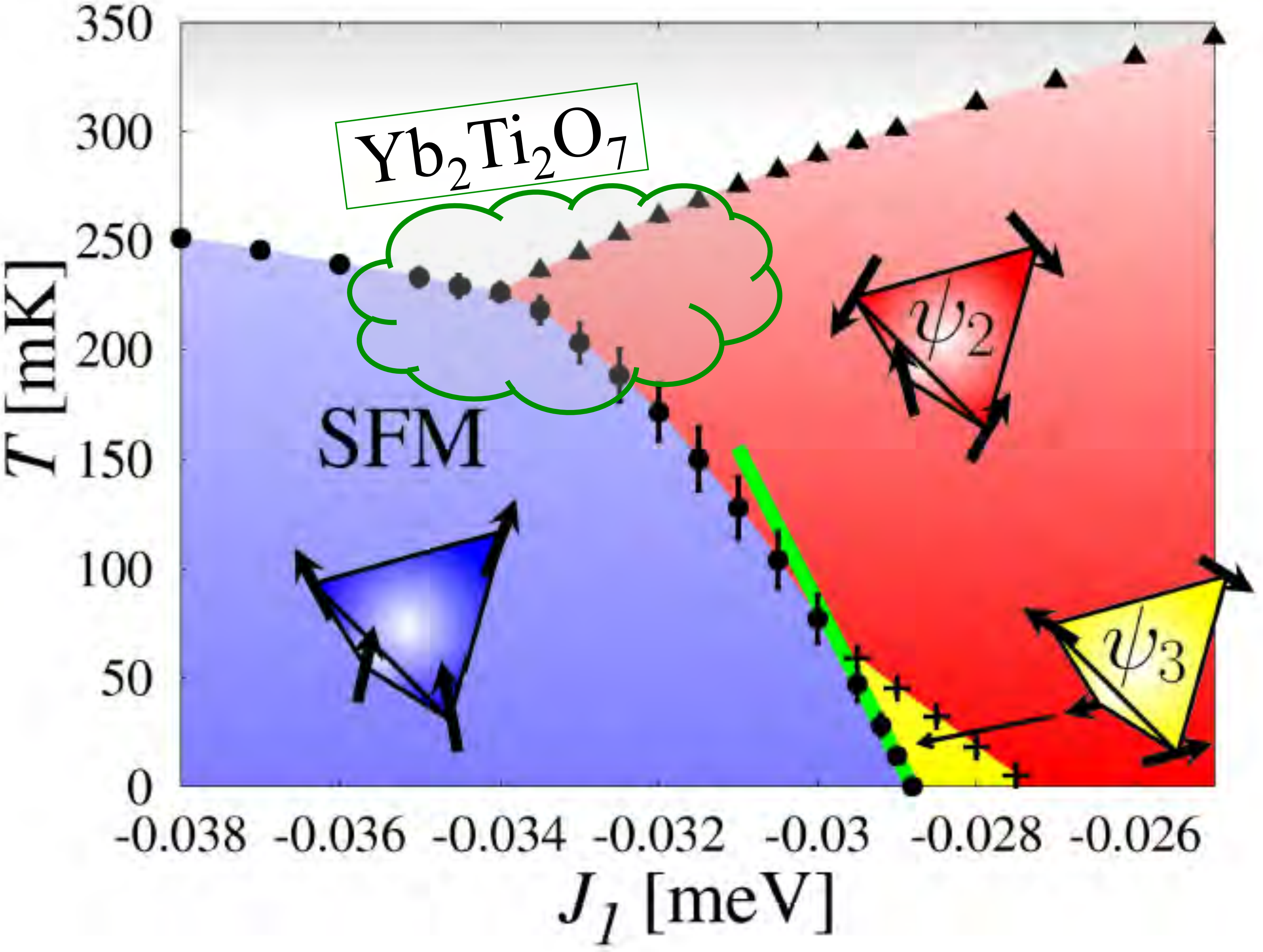


- **High-*T* peak in *C*(*T*)**
- **No spontaneous magnetization**



- **High-*T* peak in *C*(*T*)**
- **No spontaneous magnetization**

- **Low-*T* peak in *C*(*T*)**
- **Spontaneous magnetization**


350
300
250
200
150
100
50
0
T [mK]
-0.038
-0.036
-0.034
-0.032
-0.03
-0.028
-0.026
J_1 [meV]
$Yb_2Ti_2O_7$
SFM
ψ_2
ψ_3

Message of Talk

1. Order-by-disorder (ObD) appears quite likely in $Er_2Ti_2O_7$
2. $Yb_2Ti_2O_7$ is puzzling:

The competition of energetically versus entropically stabilized orders are likely (the) essential missing ingredients to understand $Yb_2Ti_2O_7$.

Alighiero Fabrizio Boetti,
1940-1994