

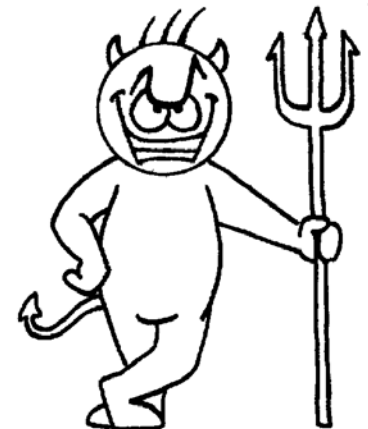
Bad metals, *an introduction*

Mischievous cuprates

(The possible role of incoherence in shaping the phase diagram of the copper-oxide high- T_c superconductors)

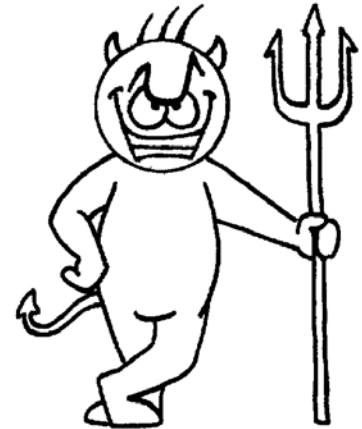
Nigel Hussey

*High Field Magnet Laboratory
Radboud University, Nijmegen*



Outline

- *Introduction*
- *Saturating metals*
 - *Bad metals*
 - *New ideas*
- *Mischievous cuprates*

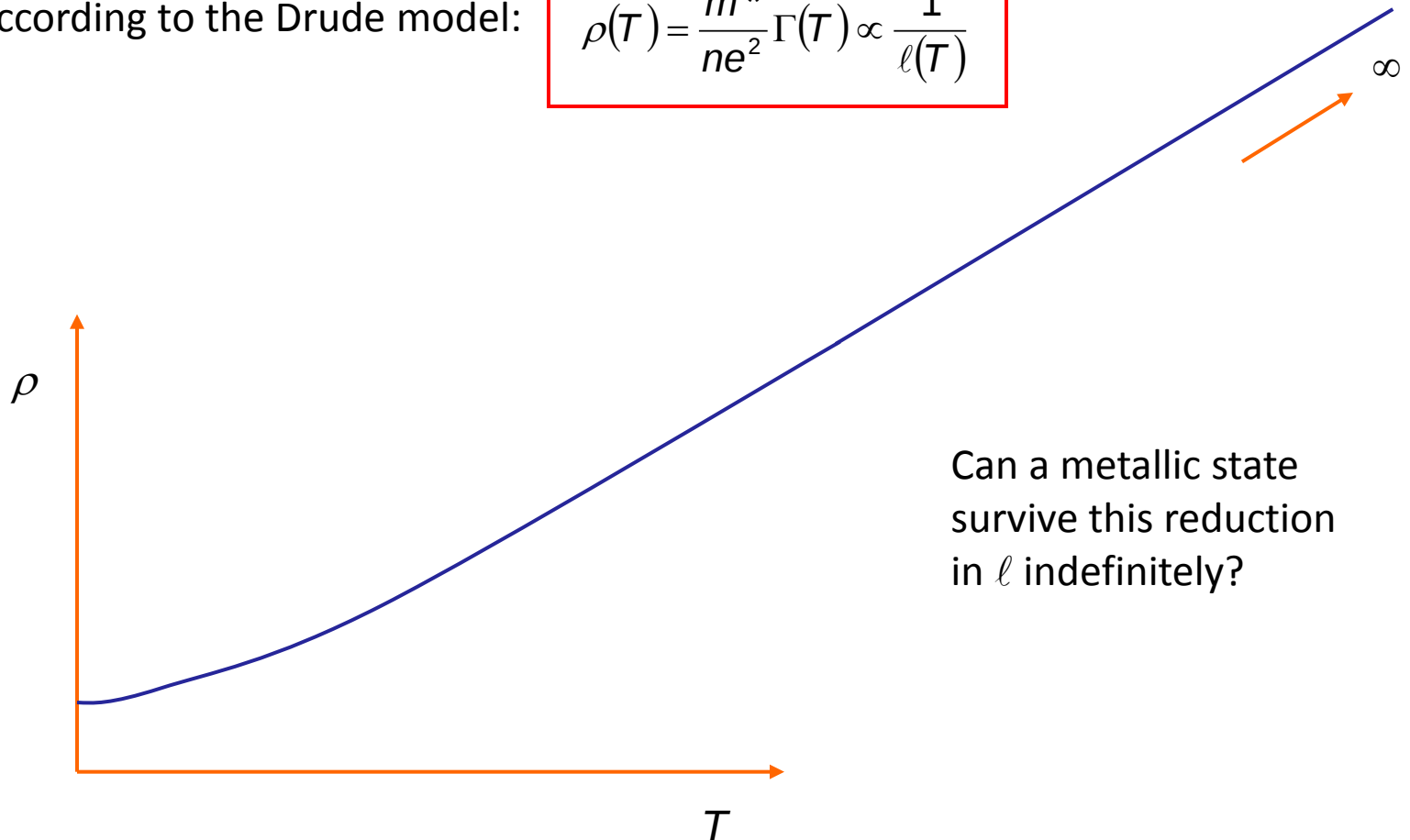


Qu. What constitutes metallic behaviour?

Basic definition: A material whose resistivity increases with temperature

According to the Drude model:

$$\rho(T) = \frac{m^*}{ne^2} \Gamma(T) \propto \frac{1}{\ell(T)}$$



Can a metallic state survive this reduction in ℓ indefinitely?

Mott-Ioffe-Regel limit

Basic definition:

$$\ell \geq a$$

Other definitions:

$$k_F \ell \geq 1$$

$$k_F \ell \geq 2\pi$$

$$\hbar \Gamma \leq \gamma \mathcal{W}$$

$$\left\{ \begin{array}{l} \varepsilon = 2t \cos(k_x a) \\ v_x = \frac{1}{\hbar} \frac{\partial \varepsilon}{\partial k_x} = \frac{2ta}{\hbar} [\sin(k_x a)] \\ \ell = \frac{2ta}{\hbar} \tau \geq a \Rightarrow \frac{\hbar}{\tau} \leq 2t \end{array} \right.$$

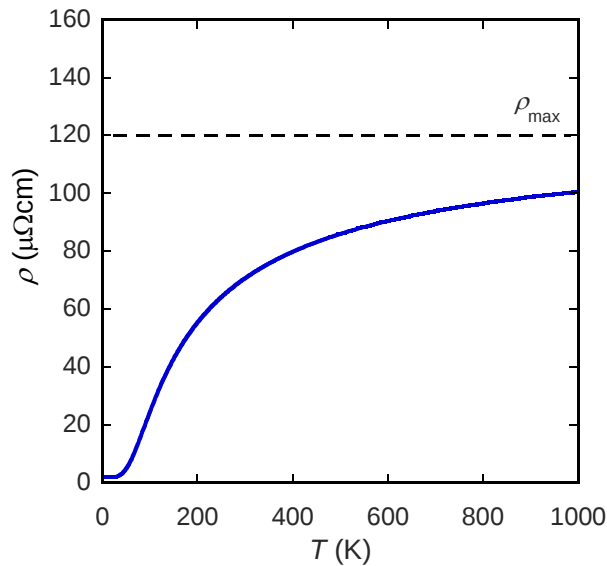
Question is... what lies beyond the MIR limit?

Two types of metals

Emery & Kivelson,
PRL **74** 3253 (95)

Saturating metals

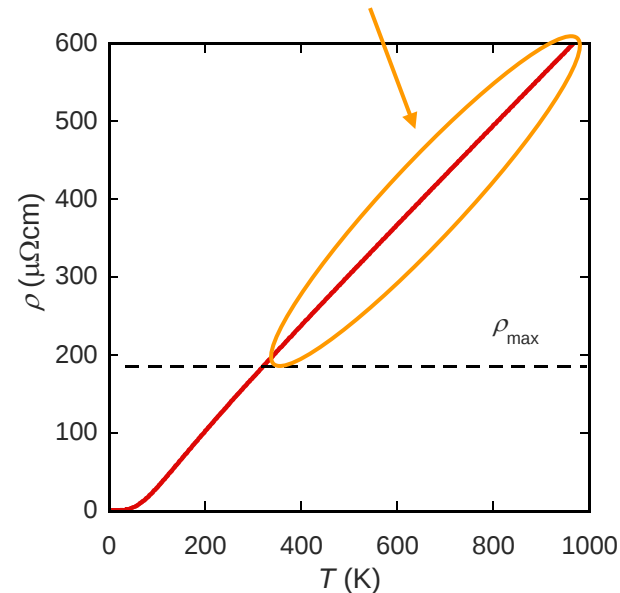
$$\ell \geq a$$



A-15 compounds
Alloys, Elements
Heavy Fermions

Bad metals

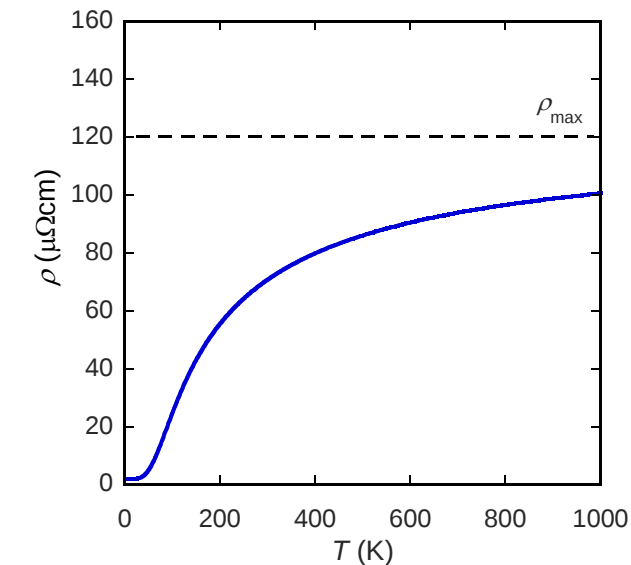
$$\ell \leq a?$$



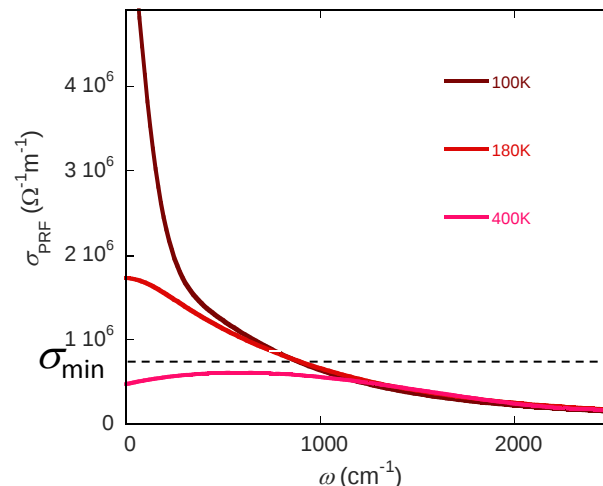
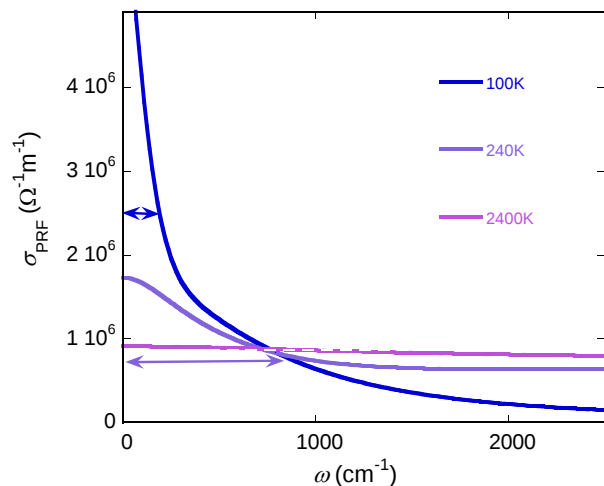
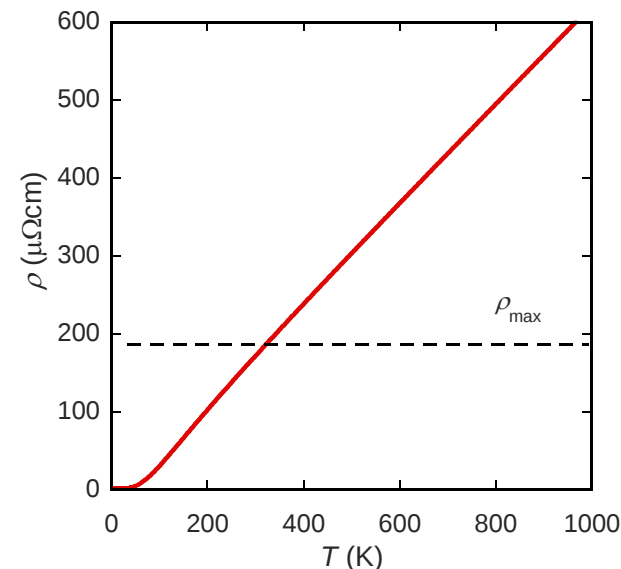
Cuprates, Nickelates
Manganites, Ruthenates
Vanadates, Pyrites, Organics

- Optical response found to be very different in the two cases
- The phrase “bad metal” is a misnomer – it’s not *metallic* at all

Saturating metals



Bad metals



Spectral weight preserved below $\omega \sim W$

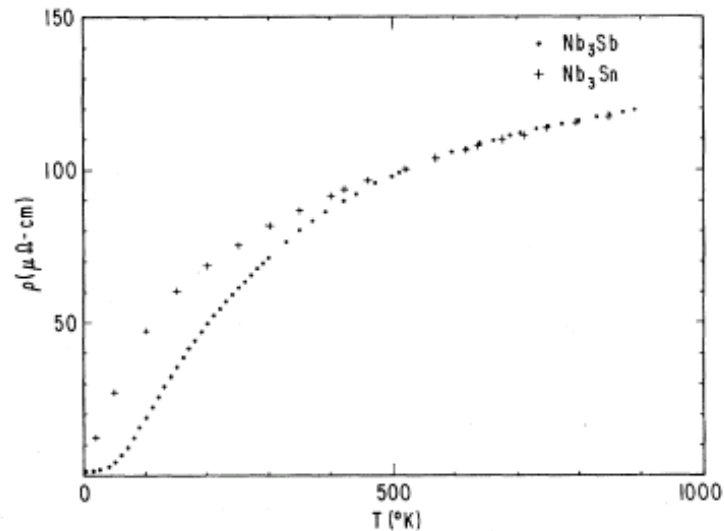
Low-freq. spectral weight displaced to $\omega > W$

Saturating metals

Nb_3Sb , Nb_3Sn

- At high enough T , $\rho(T)$ tends towards a constant value consistent with $\ell = a$

Fisk & Webb, *PRL* (76)



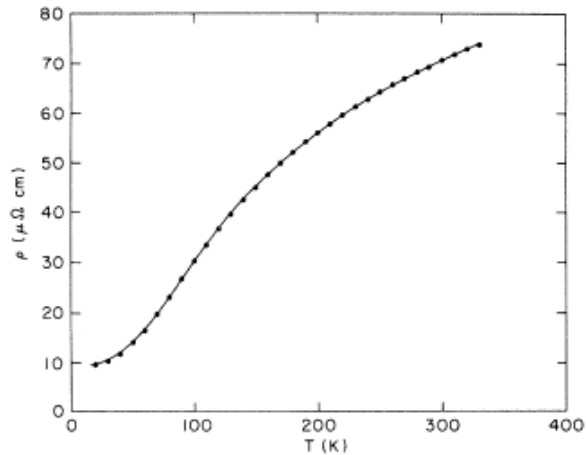
- In many cases, the approach to saturation is found to follow a simple “parallel-resistor” formula, where ρ_{max} corresponds to $\ell = a$

$$\frac{1}{\rho(T)} = \frac{1}{\rho_{\text{ideal}}(T)} + \frac{1}{\rho_{\text{max}}}$$

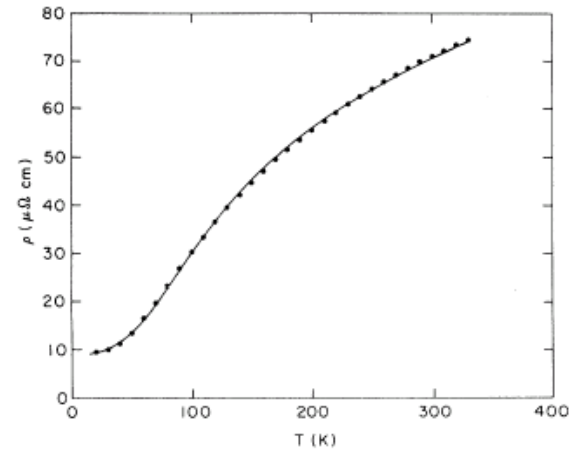
Wiesmann *et al.*, *PRL* (77)

A15 compounds – fits to PRF

V_3Si

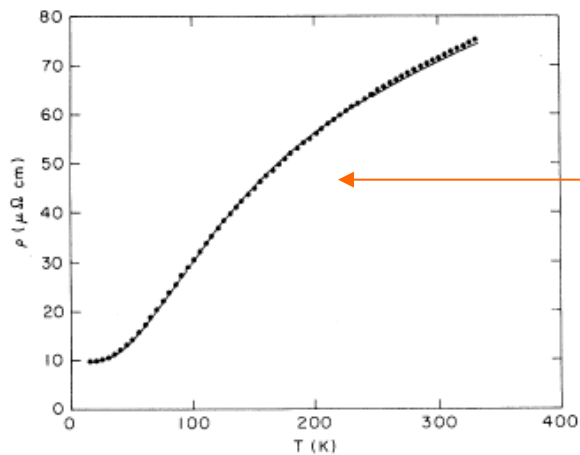


$$\rho_{ideal} = \rho_{sd} \text{ (Wilson's formula)}$$



$$\rho_{ideal} = \rho_{BG}$$

Caton & Viswanathan,
PRB (82)



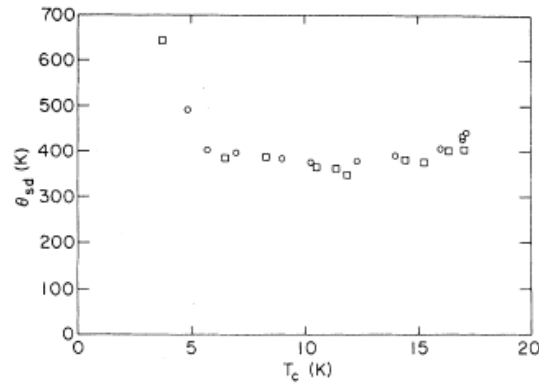
Other fits have been attempted, e.g.

$$\rho = \rho_0 + \rho_1 T + C \exp(T_0/T)$$

but Wilson *sd* model and PRF
found to fit data best.

A15 compounds – fits to PRF

V_3Si

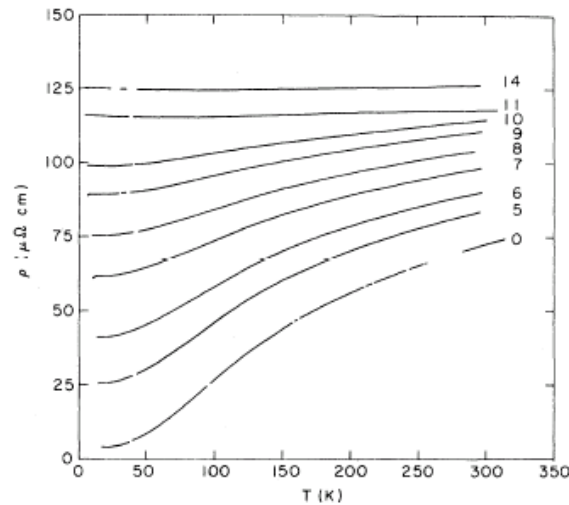


Values of both $\rho_{\max}$ and θ_{sd} do not change appreciably with irradiation, suggesting that indeed, the fitting is robust...

Caton & Viswanathan, *PRB* (82)

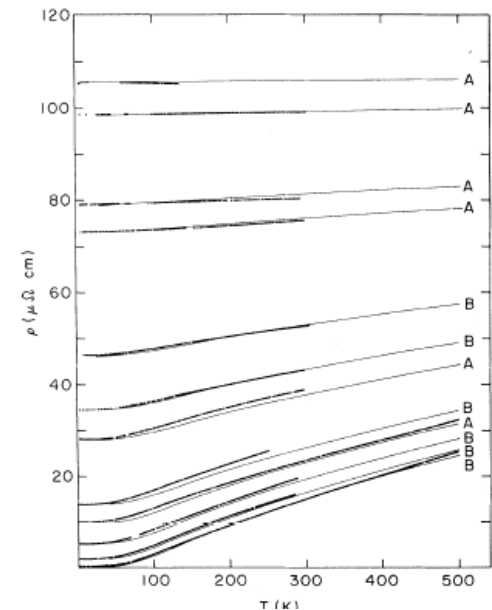
Similar trend observed by on Mo_3Ge

(Gurvitch *et al.*, *PRL* (78))



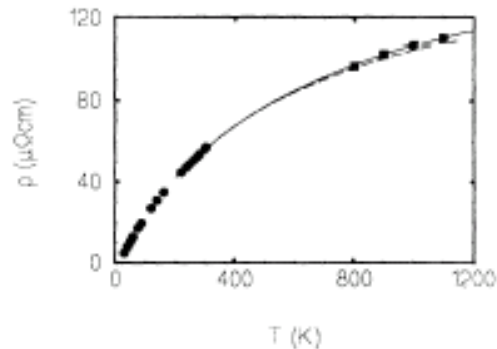
$$\frac{1}{\rho(T)} = \frac{1}{\rho_{\text{ideal}}(T)} + \frac{1}{\rho_{\max}}$$

... and PRF works all the way up to $\rho_0 \sim \rho_{\max}$!



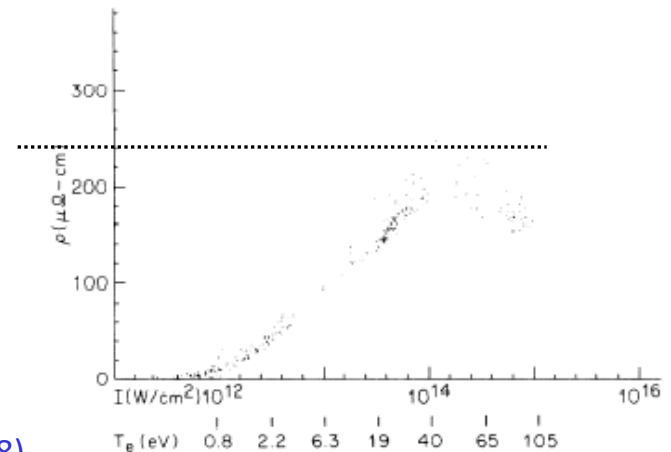
PRF works for elements too...

La



Sundqvist, *PRL* (92)

Al



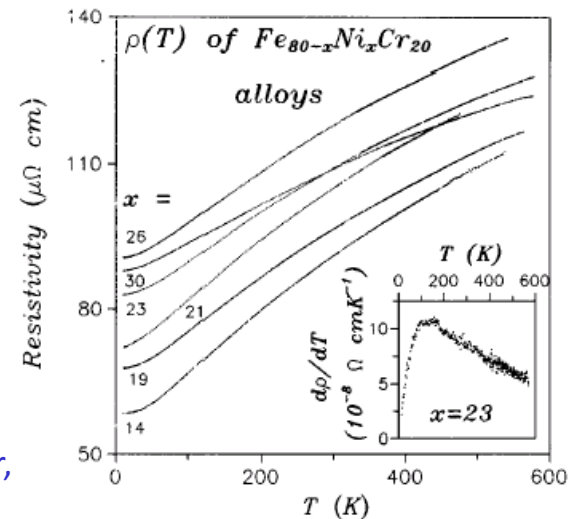
Milchberg *et al.*, *PRL* (88)

Similar trends seen in Sc, Y, Os (Sanborn *et al.*, *PRB* (89)) and Ti (Tzeng *et al.*, *JSPJ* (92))

... and alloys

$\gamma\text{-Fe}_{80-x}\text{Ni}_x\text{Cr}_{20}$

In all cases, saturation value consistent with the MIR criterion $\ell \sim a$



Nath & Majumdar,
PRB (96)

Proposed origin of PRF

Rather than having a unique ℓ for all qp's, we must consider a **distribution of scattering times** within a given metal.

By invoking the MIR limit, Gurvitch argued that there must be a **minimum time τ_0** before which no scattering can take place.

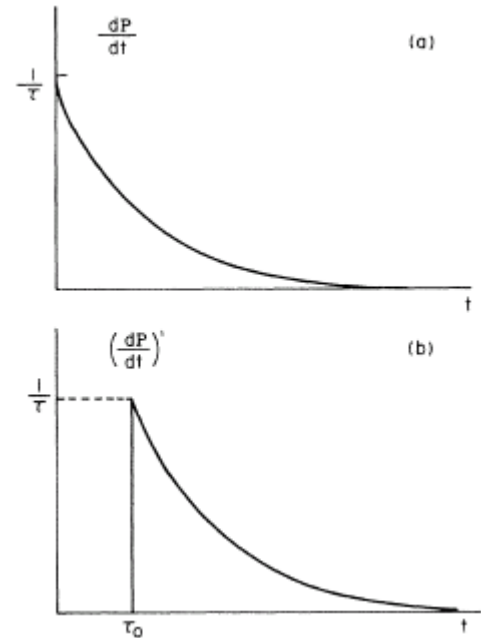
Gurvitch, *PRB* (81)

So instead of

$$V_D = \int V(t) dP = \frac{eE}{m\tau} \int_0^{\infty} t e^{-t/\tau} dt = \frac{eE}{m} \tau$$

we have

$$V'_D = \int V(t) dP' = \frac{eE}{m\tau} \int_0^{\infty} t e^{-(t-\tau_0)/\tau} dt = \frac{eE}{m} (\tau + \tau_0) \Rightarrow \sigma' = \frac{ne^2}{m^*} (\tau + \tau_0) = \sigma_{\text{ideal}} + \sigma_{\text{min}}$$



Note that this **does not imply** the existence of two channels...

Optical conductivity of metals

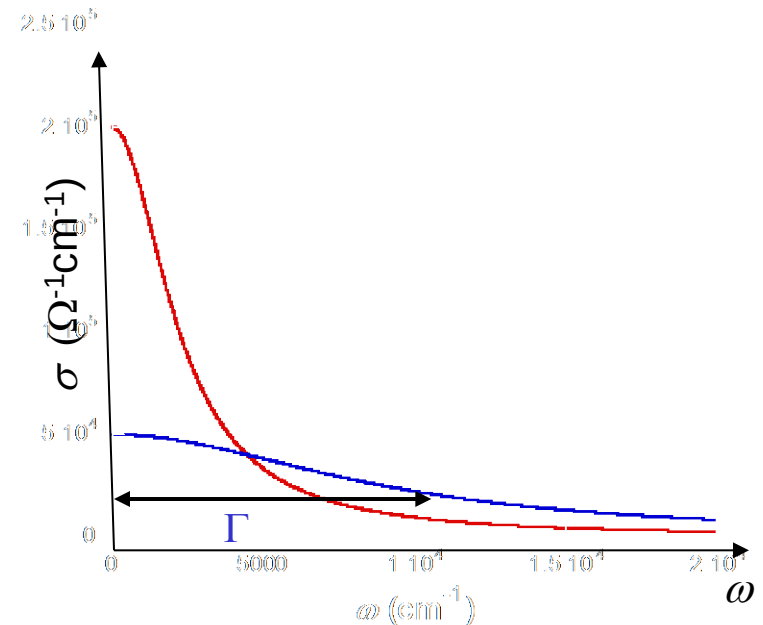
$$\sigma = \frac{ne^2}{m^*} \frac{1}{\Gamma - i\omega} = \frac{\Omega_p^2}{4\pi} \left(\frac{1}{\Gamma - i\omega} \right) \left(\frac{\Gamma + i\omega}{\Gamma + i\omega} \right)$$

$$\Omega_p^2 = \frac{4\pi ne^2}{m^*}$$

$$\sigma = \text{Re}(\sigma) + i\text{Im}(\sigma) = \frac{\Omega_p^2}{4\pi} \frac{\Gamma}{\Gamma^2 + \omega^2} + i \frac{\Omega_p^2}{4\pi} \frac{\omega}{\Gamma^2 + \omega^2}$$

Optical sum rule

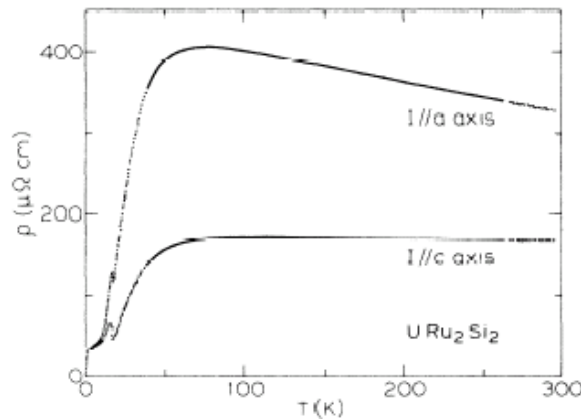
$$\int_0^{\omega} \sigma(\omega) d\omega = \frac{\Omega_p^2}{8}$$



Heavy fermions - the most extreme example of saturating metals

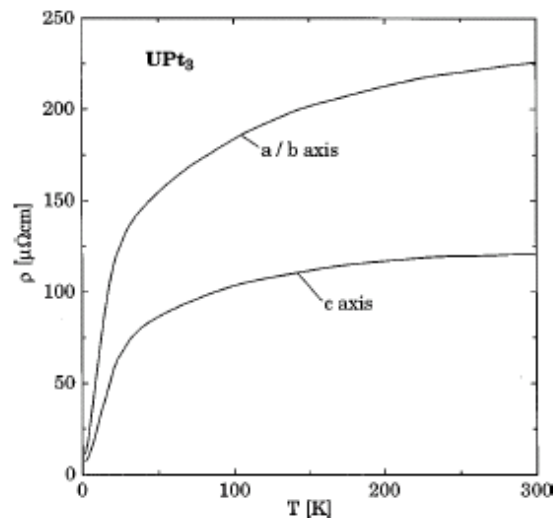
URu_2Si_2

Palstra *et al.*,
PRB (86)



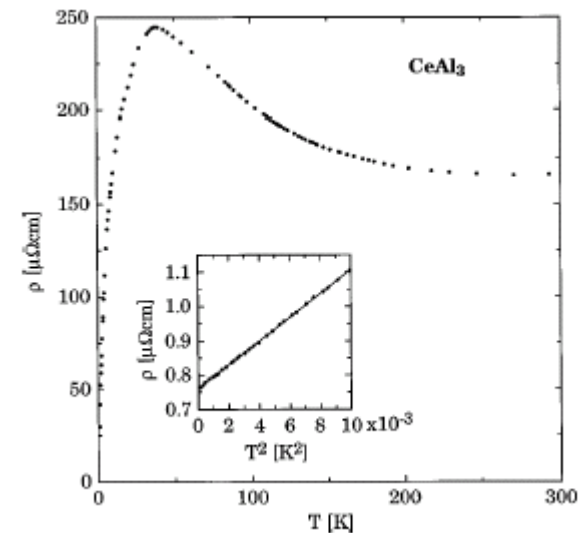
UPt_3

de Visser *et al.*,
JMMM (84)



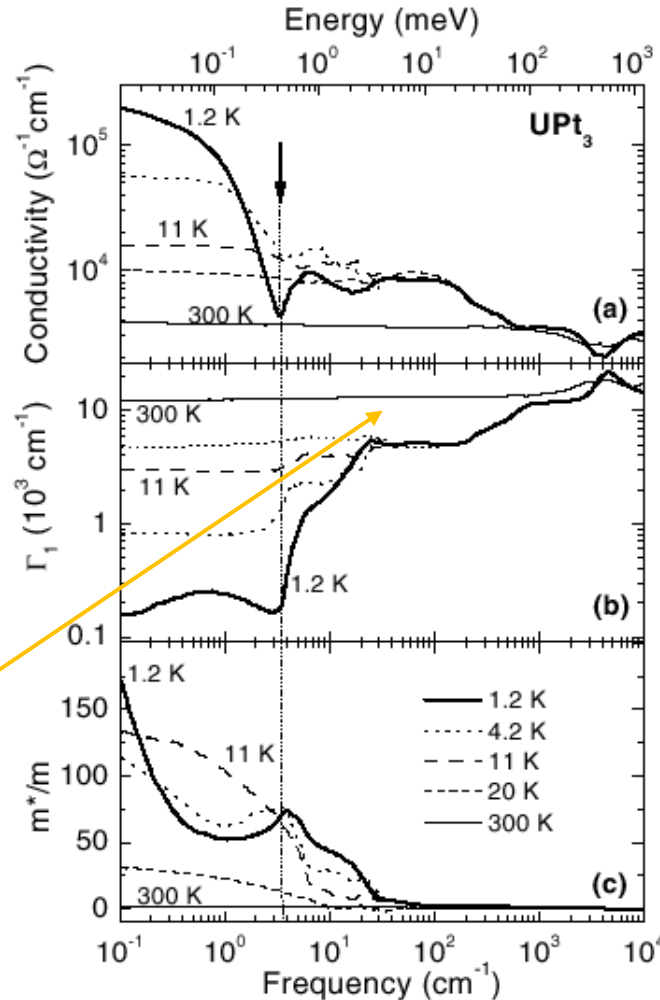
CeAl_3

Andres *et al.*, PRL (75)

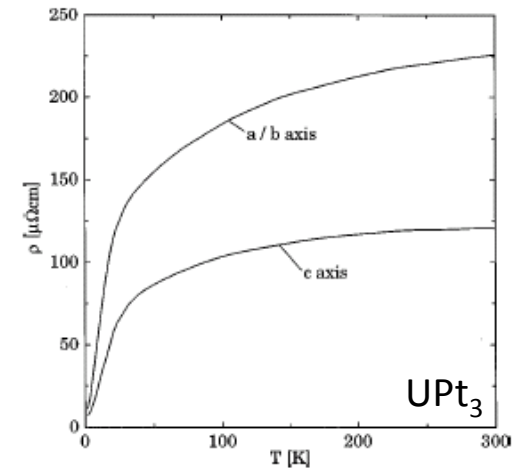


- Resistivity saturation sets in above 100K at a value $\sim 100 - 300 \mu\Omega \text{ cm}$, depending on current direction.
- Possibility to measure $\sigma(\omega)$ in saturation limit.

- At low T , sharp Drude peak appears due to strong renormalization of conduction electrons via interaction with localized moments.
- Renormalization reflected in strong suppression of Γ and strong enhancement of m^* .
- At high T (in the saturation regime), $\sigma(\omega)$ becomes flat and featureless at a value corresponding to $1/\rho_{\max}$
- Similarly, $1/\tau(\omega)$ saturates at MIR limit $\sim W/2$.



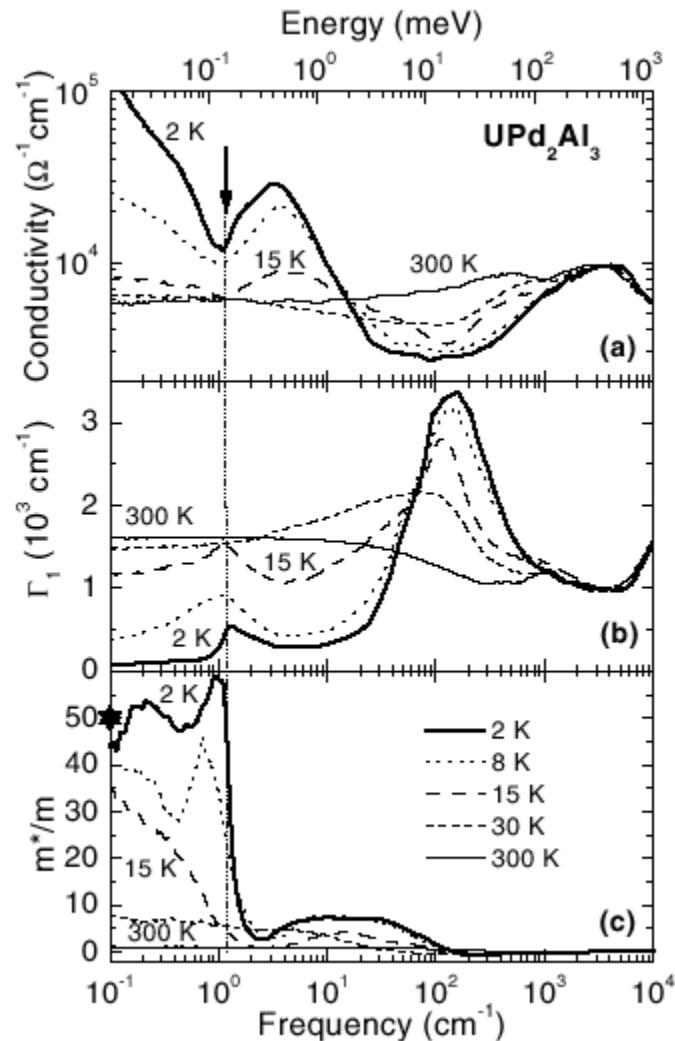
Dressel *et al.*,
PRL **88** 186404 (02)



- Resistivity saturation sets in above 100K at a value $\sim 100 - 300 \mu\Omega\text{cm}$, depending on current direction.
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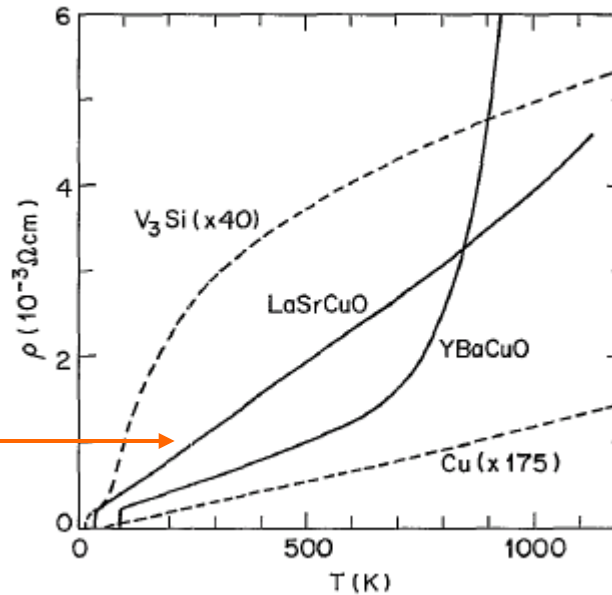
de Visser *et al.*,
JMMM (84)

- Again, same phenomenology observed in UPd_2Al_3
- Important point here is that in saturating metals, spectral weight redistributed within W
- Ultimate limit of redistribution is a conductivity spectrum resembling a rectangle of height $\sigma_{\min}$ and width W
- Seems more energetically favourable for system to remain metallic, **even with the shortest possible mean free path!**



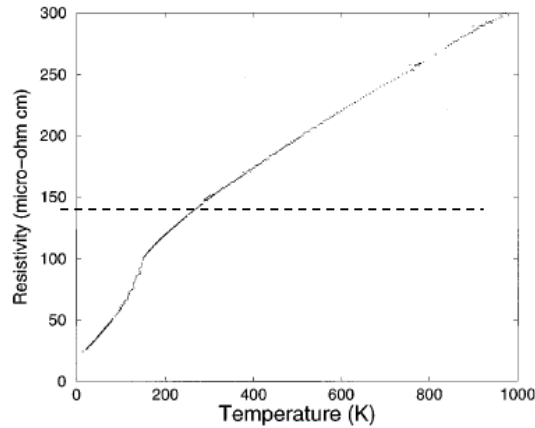
Non-saturating (bad) metals

- First observed in high- T_c cuprates.
- $\rho(T)$ increases approx. linearly with T beyond MIR limit

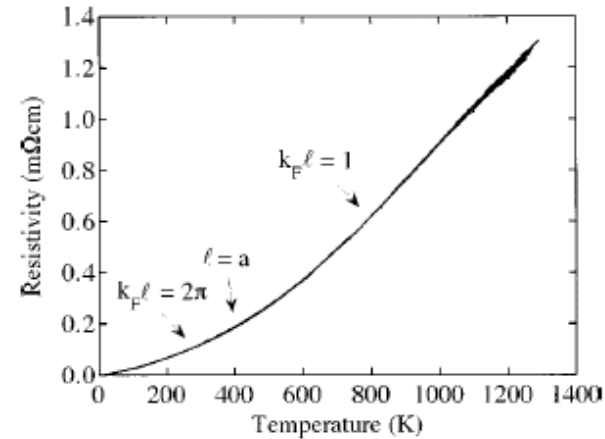
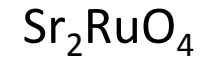


Gurvitch & Fiory, PRL (87)

- Absence of resistivity saturation in bad metals claimed to be a manifestation of their non-FL ground state at 0K. Emery & Kivelson, PRL (95)
- If carriers do not “sense” the approach to resistivity saturation at moderate to high T , then minimum scattering time should not exist, even at low T . *This is not necessarily the case however...*

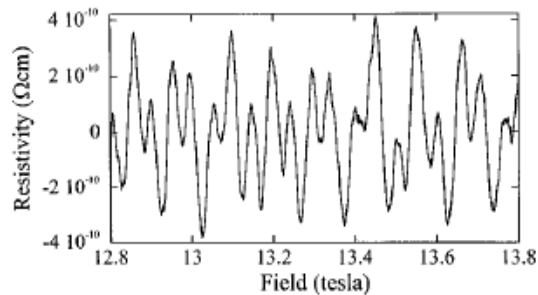


Allen *et al.*, PRB (96)

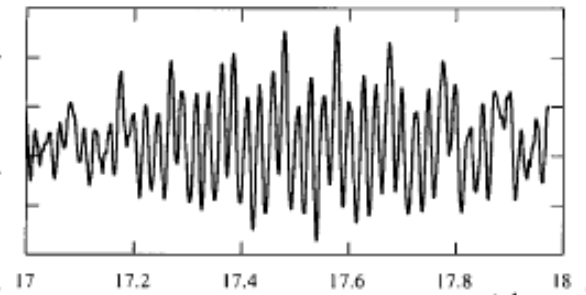


Tyler *et al.*, PRB (98)

In both cases, $\rho(T) \sim T^2$ at low T and dHvA oscillations, of FL form, are observed.



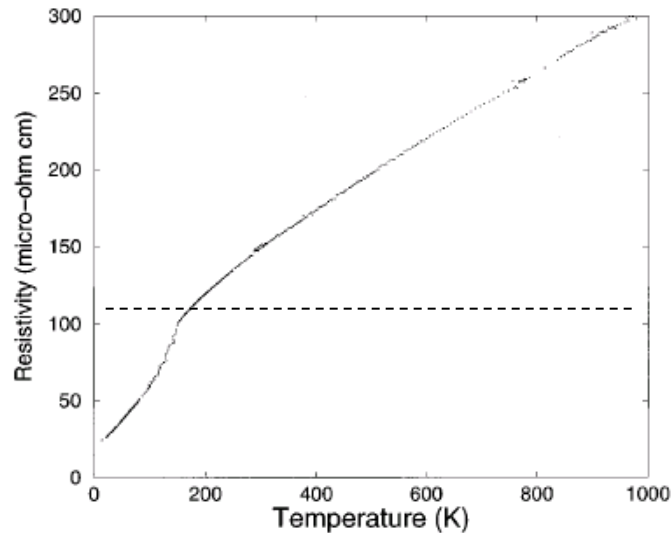
Mackenzie *et al.*, PRB (98)



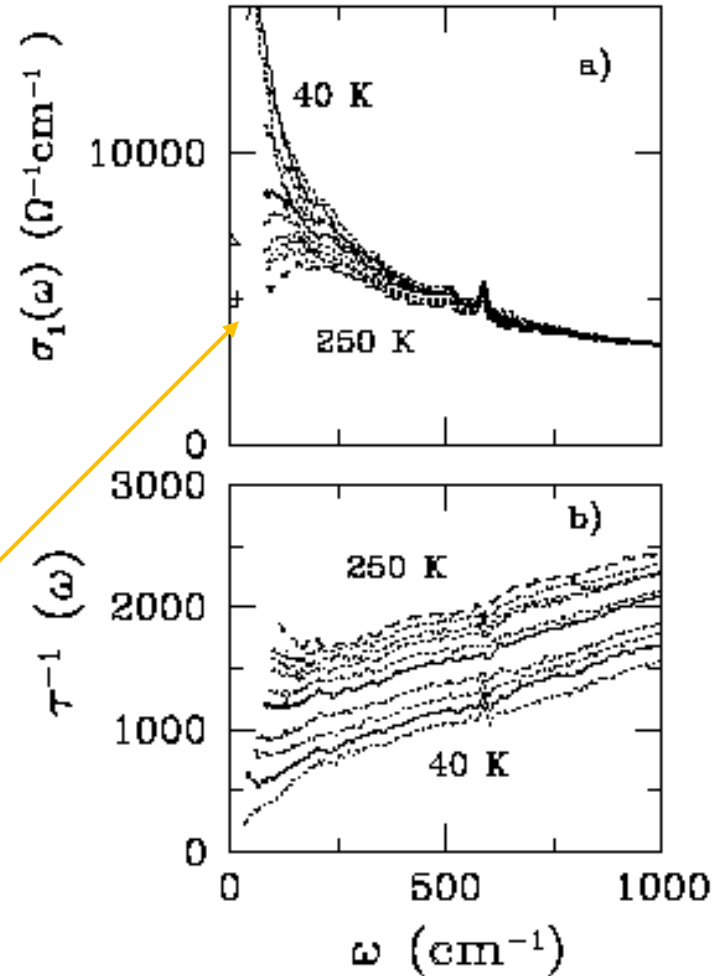
Mackenzie *et al.*, PRL (96)



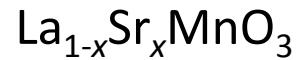
Allen *et al.*,
PRB **53** 4394 (96)



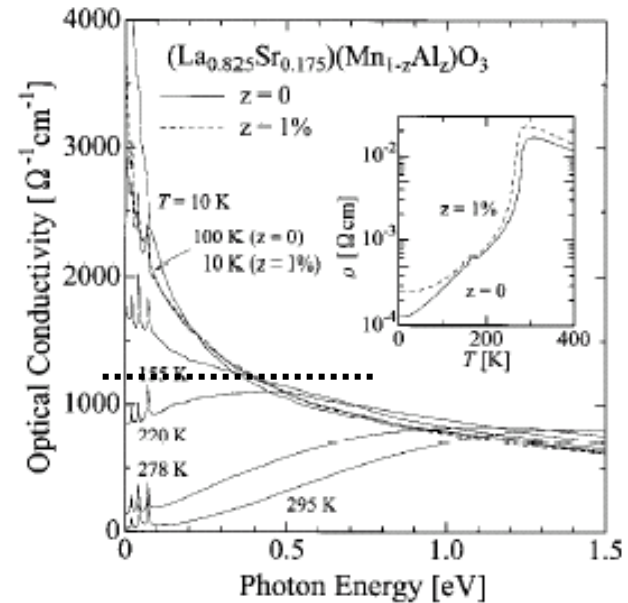
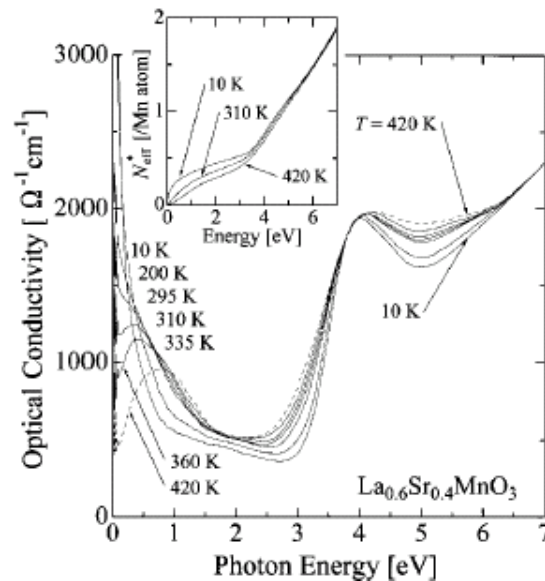
- Drude peak evolves into finite-frequency peak as T increases
- Coherent/incoherent crossover identified by low-freq. upturn in $1/\tau(\omega)$.



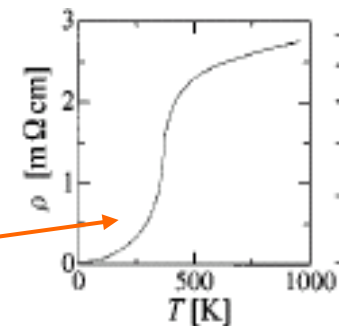
Kostic *et al.*,
PRL **81** 2498 (98)

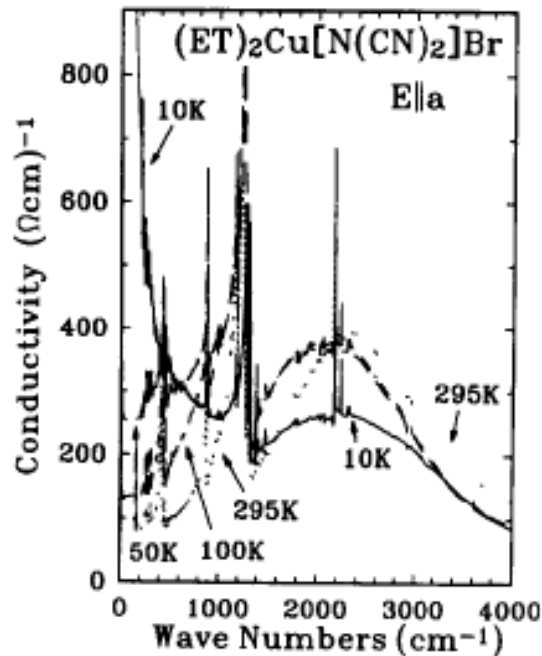


Takenaka *et al.*, PRB (02)

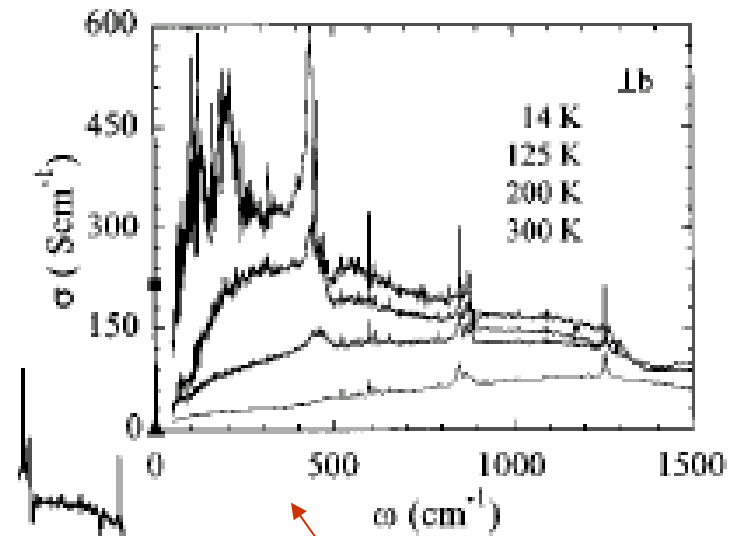


- Again, Drude peak disappears around $\sigma_0 \sim 1000 - 1400 \text{ W}^{-1}\text{cm}^{-1}$ even though Drude “tail” still survives.
- Redistribution of spectral weight beyond $\omega \sim W$
- $\rho(T)$ rapidly increases above 1 m Ωcm , saturating at a higher value above T_c



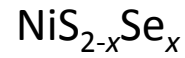
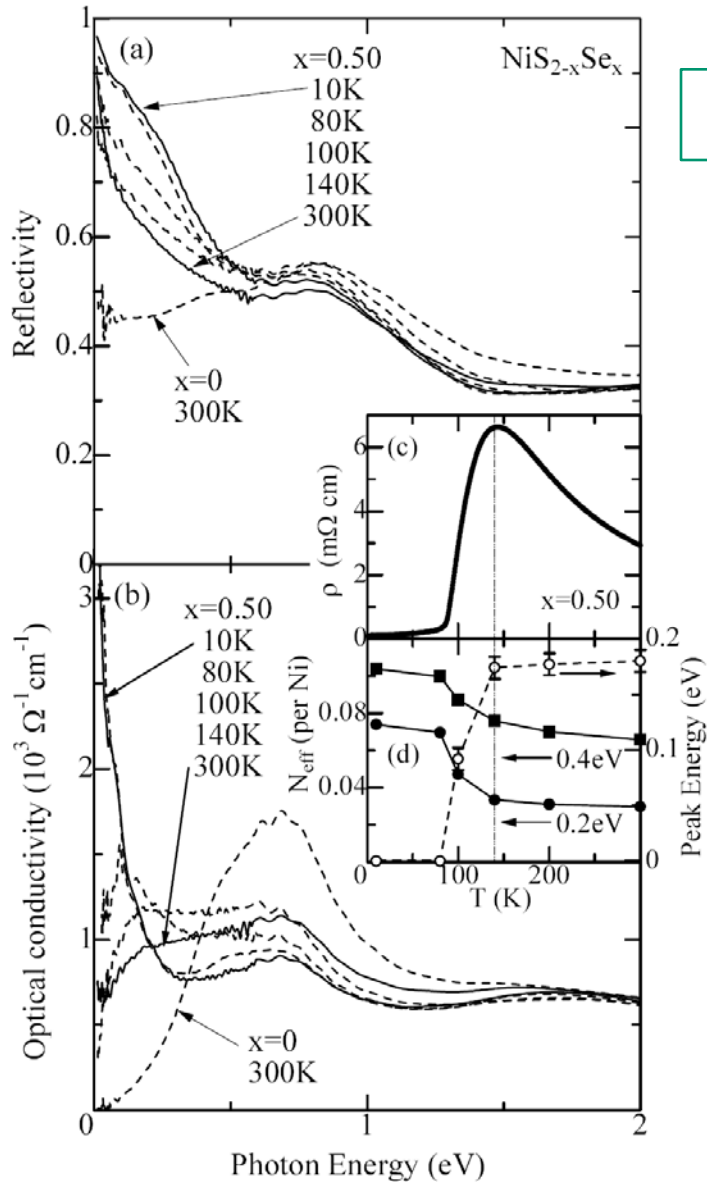
β -(ET) salts

Eldridge *et al.*, SSC (91)



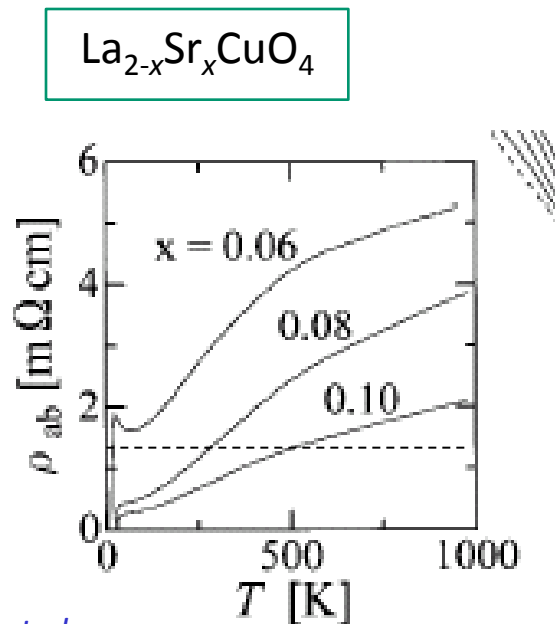
Dong *et al.*, PRB (99)

Same phenomenology but coherent /incoherent crossover occurs at much lower temperatures, sometimes even below the temperature range of the experiments!



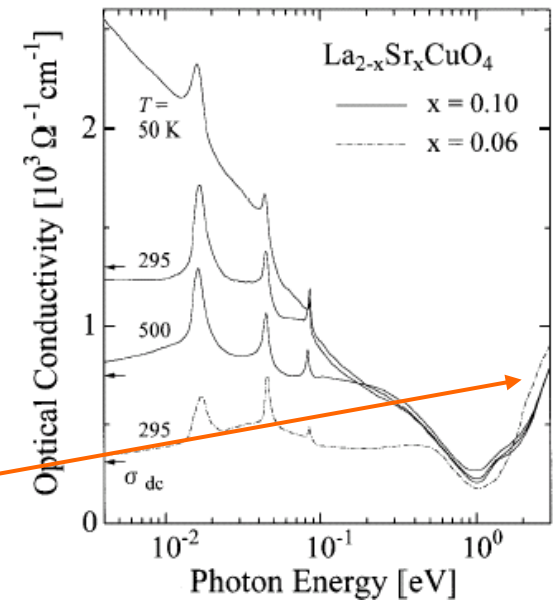
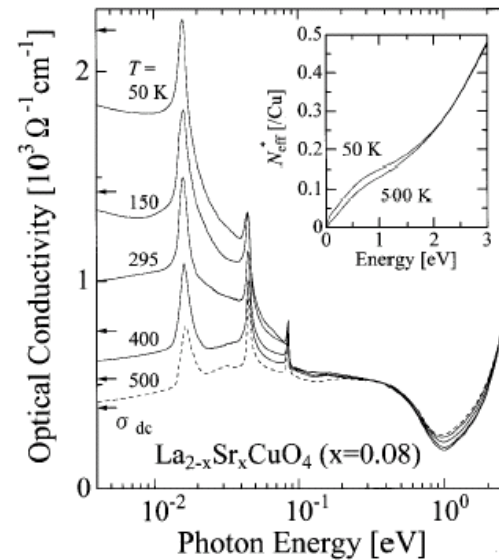
- LiV_2O_4
- Na_xCoO_2
- Cuprates
- (Pnictides)
-
-
-

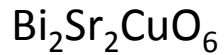
Miyasaka *et al.*,
unpublished



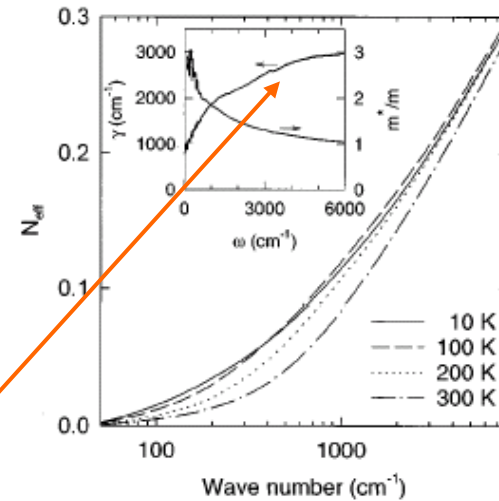
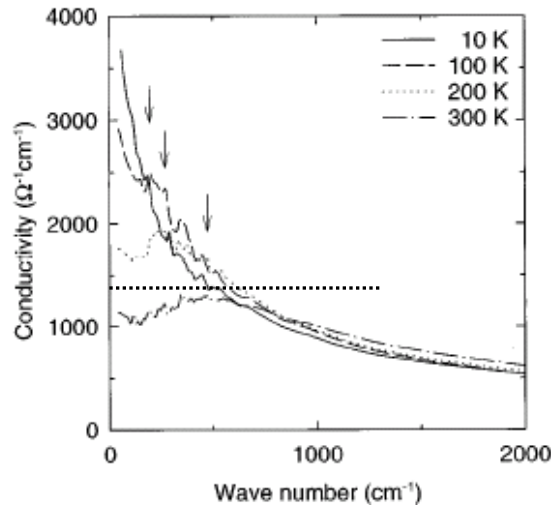
Takenaka *et al.*,
PRB 65 092405 (02)

- $\rho(T)$ continues to increase above 1 mΩcm, tending to saturate at a much higher value
- Drude peak disappears around $\sigma_0 \sim 700 - 1000 \Omega^{-1}\text{cm}^{-1}$ though Drude “tail” still survives
- Lost spectral weight recovered beyond $\omega = W$



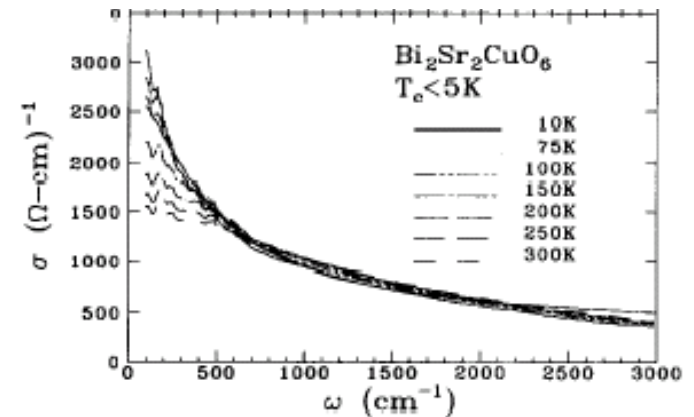


Tsvetkov *et al.*, PRB (97)



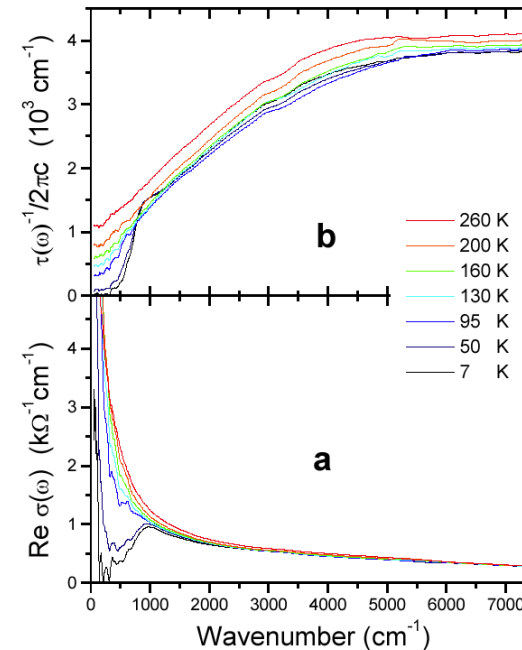
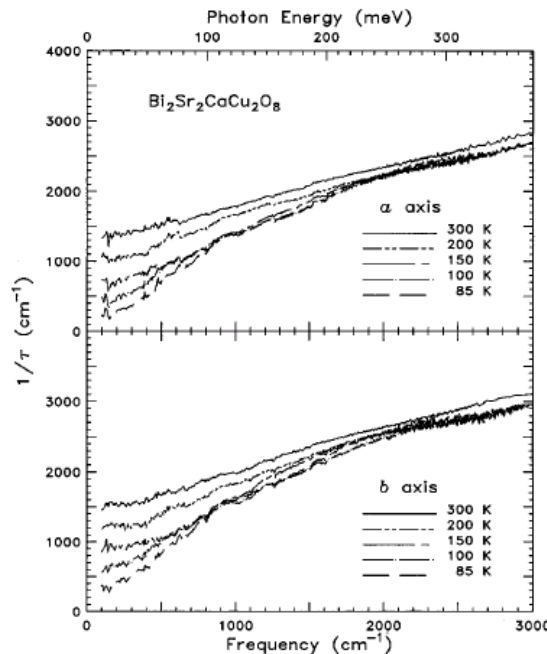
- Drude peak disappears around $\sigma_0 \sim 1200 \Omega^{-1}\text{cm}^{-1}$ though Drude “tail” still survives.
- $1/\tau(\omega)$ appears to saturate around 3000 cm^{-1} ($\sim 0.35 \text{ eV}$) at $T = 10\text{K}$

Romero *et al.*, SSC (92)



$\Gamma(\omega)$ seen to saturate in both Bi-based and Tl-based cuprates where phonons do not complicate spectrum. (Slight T -dependence in $\Gamma_{\max}$ may be due to $v_F(T)$)

Saturation value (3000 – 4000 cm^{-1}) compatible with $\Gamma_{\max} \sim W/2$



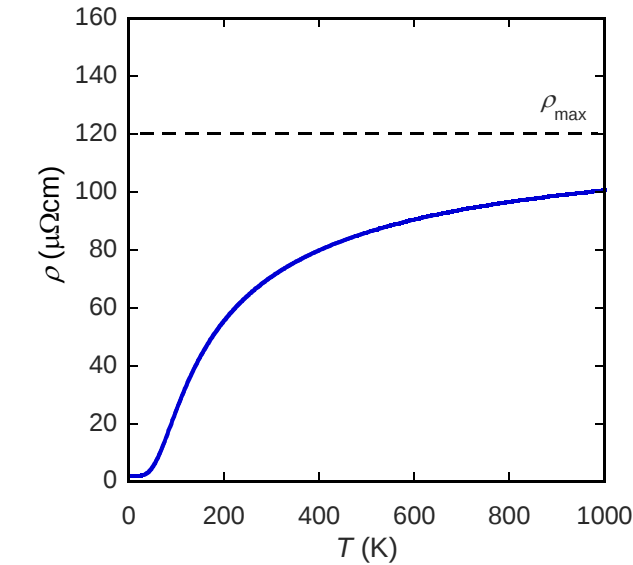
Quijada *et al.*,
PRB (99)

Bi-2212

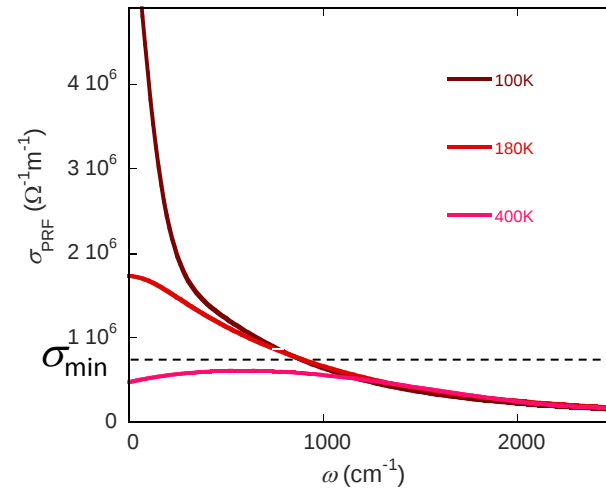
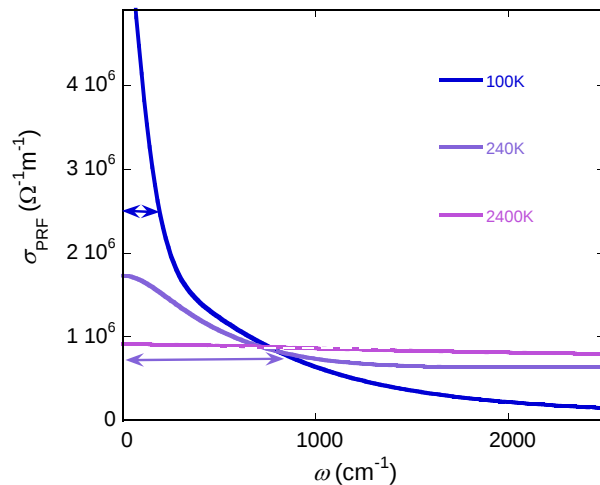
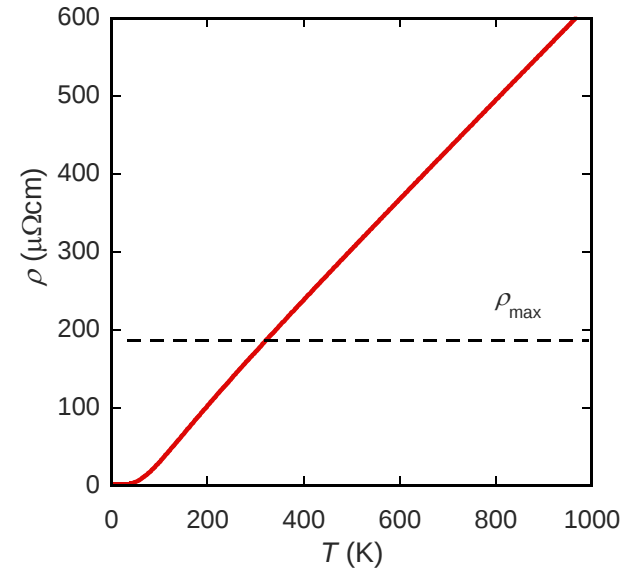
van der Marel *et al.*
Nature (03)

Y-Bi2212

Saturating metals



Bad metals



Spectral weight preserved below $\omega \sim W$

Low-freq. spectral weight displaced to $\omega > W$

What is the origin of bad metallic behavior?

So what is the cause of bad metallic behaviour?

What is NOT the cause

Thermal expansion



Lattice anharmonicity



Non-FL ground state



Two-dimensionality



Band degeneracy



OR

Interband scattering



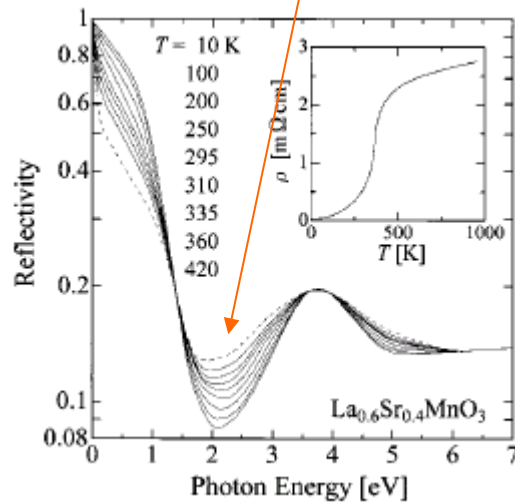
What is the origin of bad metallic behavior?

What about narrow bands?

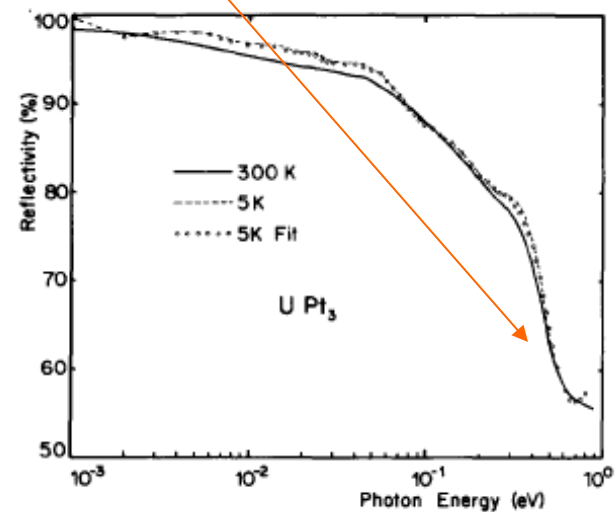


No!

c.f. Manganites and Heavy fermion metals



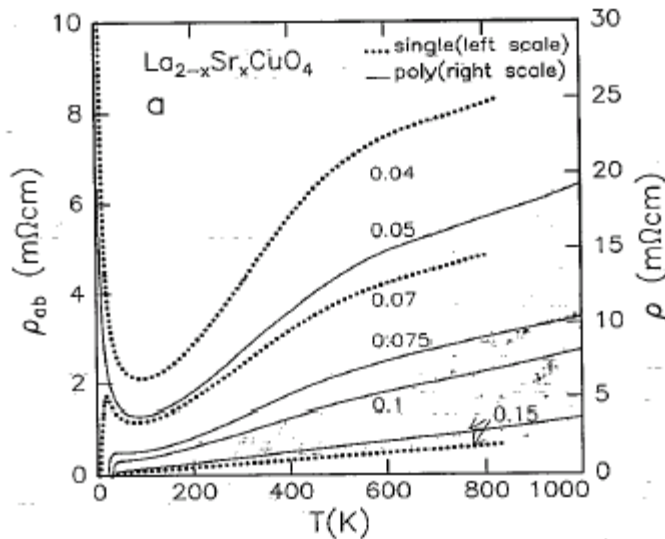
Takenaka *et al.*, PRB (02)



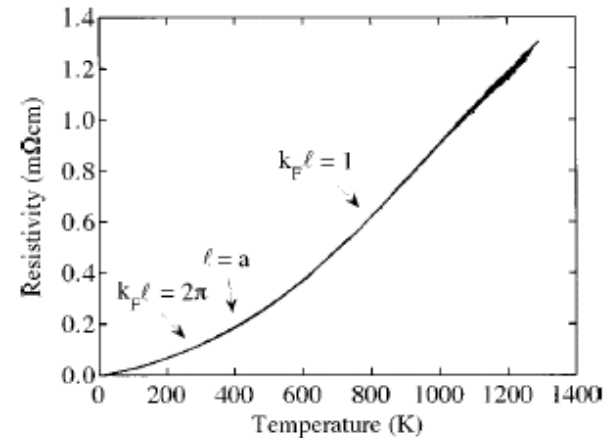
Marabelli *et al.*, SSC (86)

What is the origin of bad metallic behavior?

What about the size of the Fermi surface?



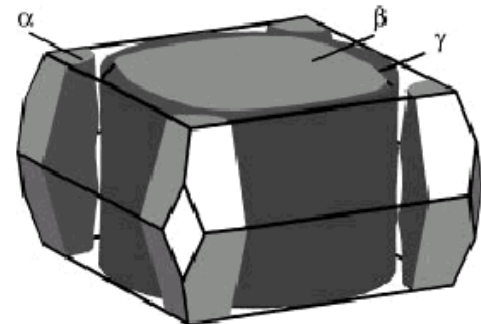
Takagi *et al.*, PRL (92)



Tyler *et al.*, PRB (98)

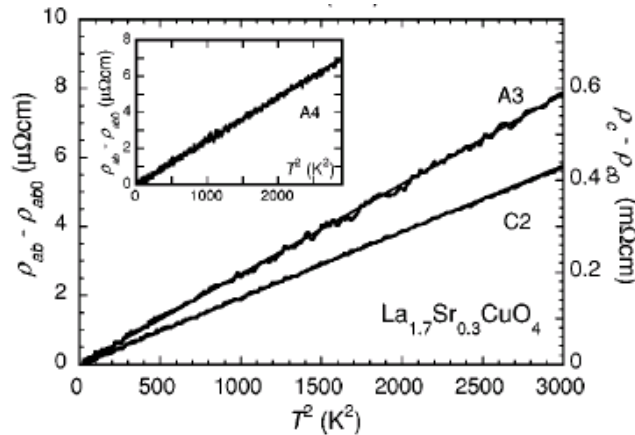
No!

c.f. Sr_2RuO_4



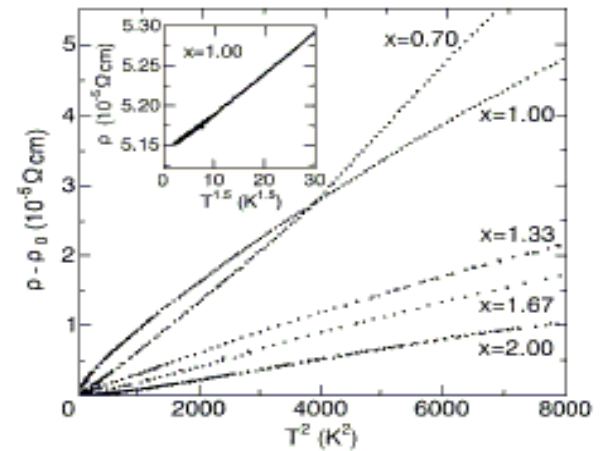
What are the defining characteristics of a bad metal?

2D cuprate - $\text{La}_{1.7}\text{Sr}_{0.3}\text{CuO}_4$ Nakamae *et al.*,
PRB (03)

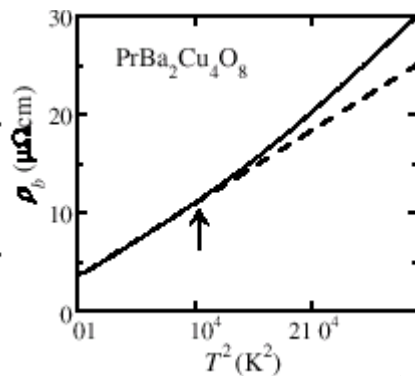


Pyrite - $\text{NiSe}_{1-x}\text{S}_x$

Miyasaka *et al.*,
JPSJ (00)



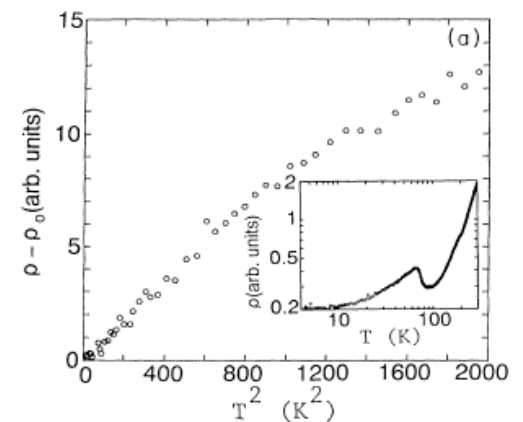
1D cuprate - $\text{PrBa}_2\text{Cu}_4\text{O}_8$



McBrien *et al.*, JSPJ (00)

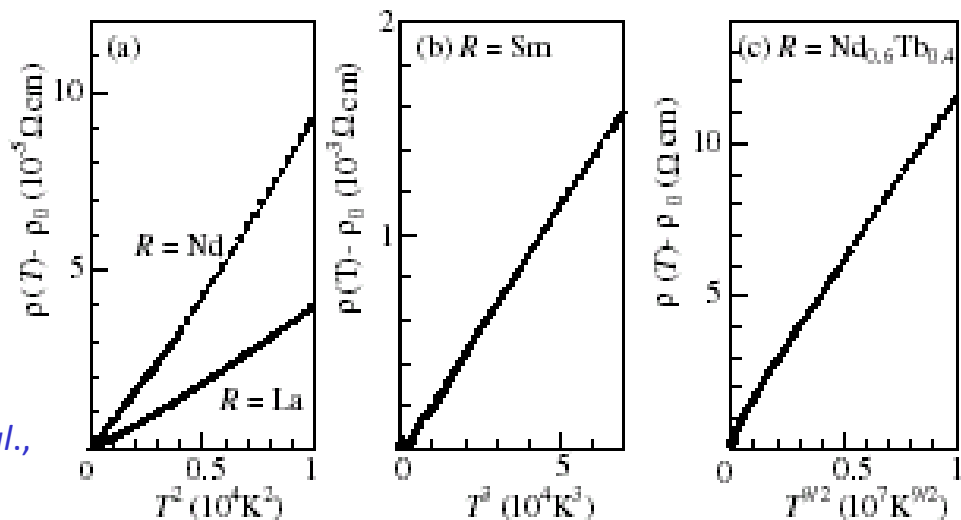
Organics
DCNQI-Cu

Kobayashi *et al.*,
PRB (93)



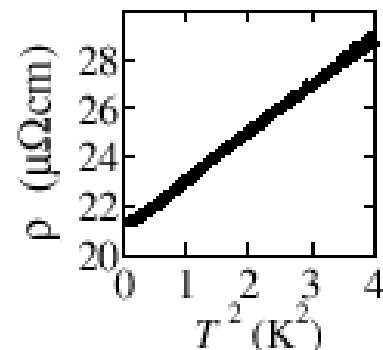
What are the defining characteristics of a bad metal?

Manganites



Akimoto *et al.*,
PRL (00)

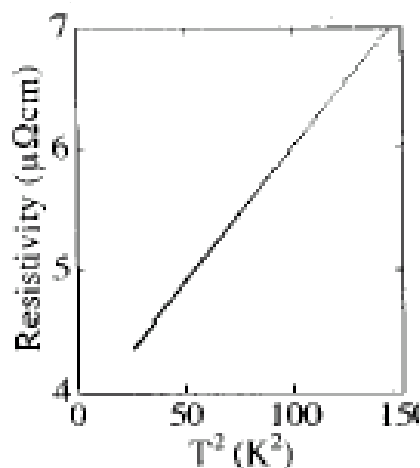
LiV_2O_4



Urano *et al.*, PRL (00)

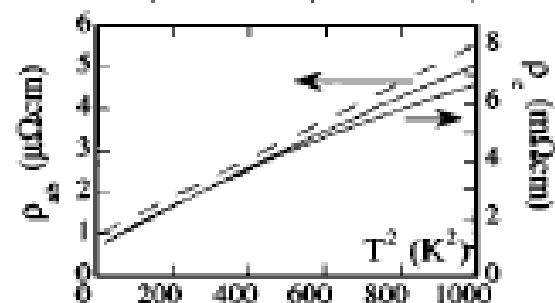
Ruthenates

SrRuO_3



Mackenzie *et al.*,
PRB (98)

Sr_2RuO_4



Hussey *et al.*, PRB (98)

What are the defining characteristics of a bad metal?

Close to Mott transition

T^2 resistivity at low- T
extending over wide temperature range

Strong electron correlations

$$U \geq W$$



Cuprates
Manganites
Ruthenates
Vanadates
Organics
Pyrites
...

Previous attempts to explain bad metallic behaviour

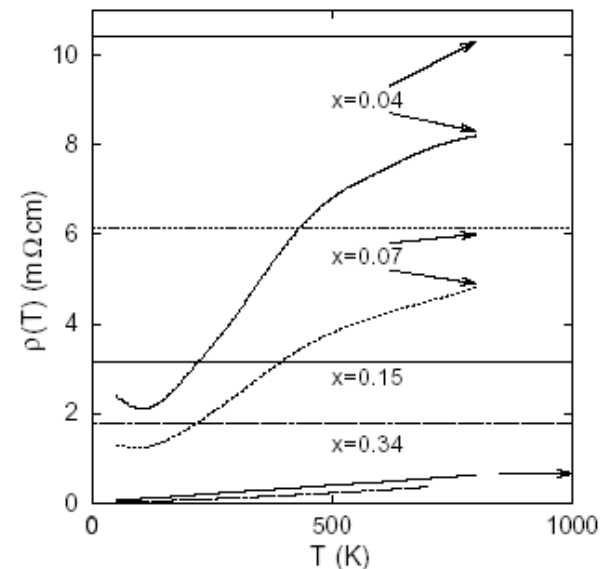
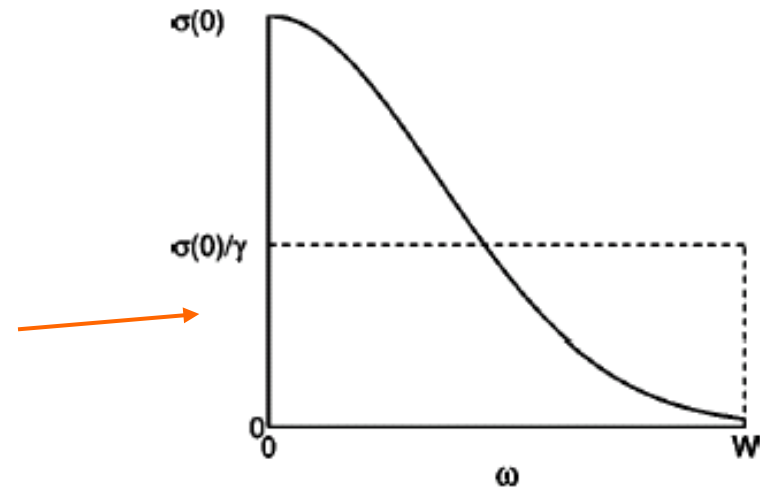
1) tJ model

Calandra & Gunnarsson,
EPL **61** 88 (03)

- Reduction of kinetic energy term by strong correlations leads to suppressed spectral weight in $\sigma(\omega)$ and correspondingly smaller $\sigma_{\min}$ (larger $\rho_{\max}$).

$$\frac{2}{\pi} \int_0^{\infty} \sigma(\omega) d\omega = -\frac{1}{2} \frac{d^2 e^2}{N \Omega \hbar^2} \langle T_K \rangle$$

- In C&G's model, the onset of “metallic” $\rho(T)$ below saturation limit coincides with development of Drude tail.
- This does not account for the loss of low frequency spectral weight with increasing T .

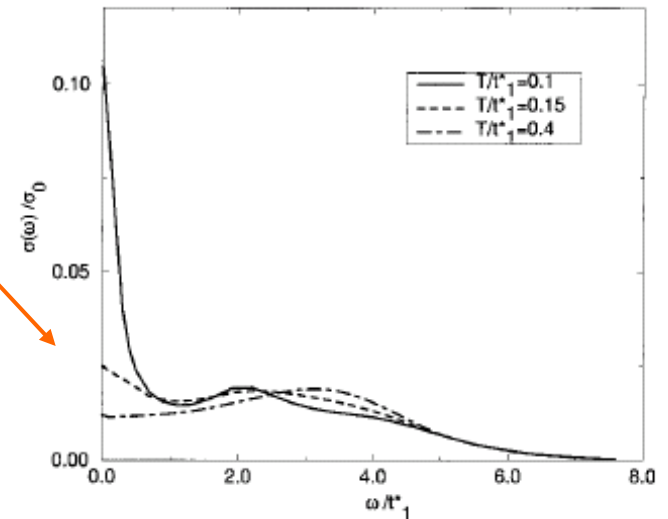
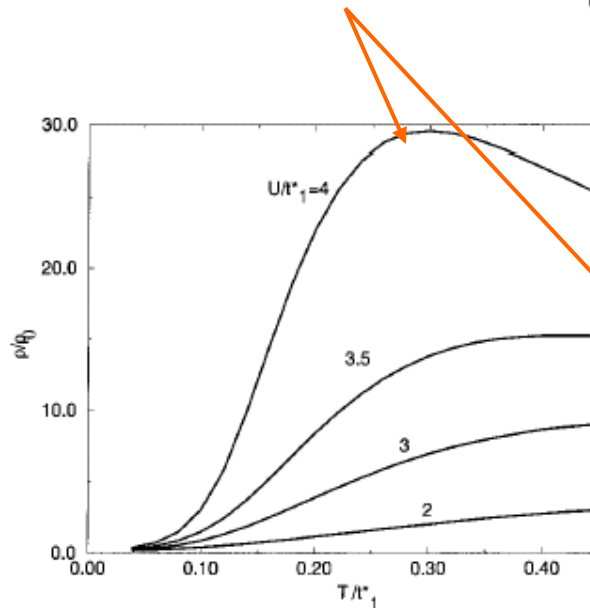
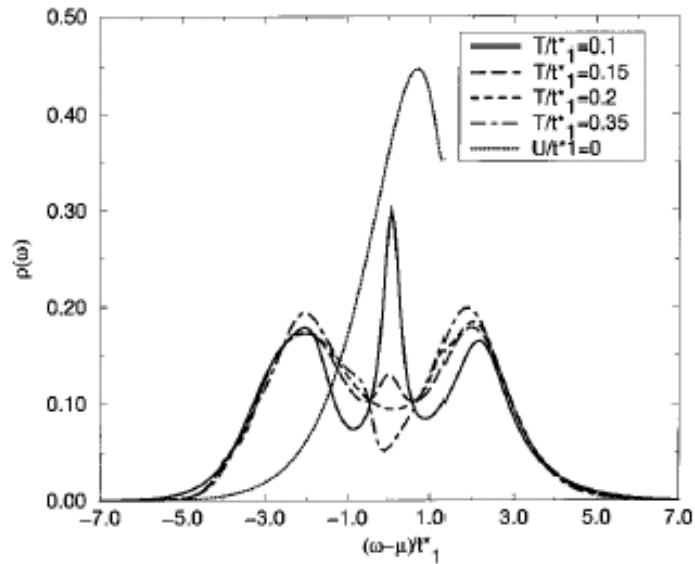


Previous attempts to explain bad metallic behaviour

2) DMFT

Rozenberg *et al.*,
PRL **75** 105 (95)
Merino & McKenzie,
PRB **62** 2416 (00)

- As T increases, qp weight vanishes.
- Inspection of $\rho(T)$ and $\sigma(\omega)$ however show that Drude tail and “metallic” resistivity vanish at similar temperature.



Previous attempts to explain bad metallic behaviour

3) Nanoscale inhomogeneity

Arnason *et al.*,
PRL **81** 3936 (98)

VOLUME 81, NUMBER 18

PHYSICAL REVIEW LETTERS

2 NOVEMBER 1998

Bad Metals Made with Good-Metal Components

S. B. Arnason, S. P. Herschfield, and A. F. Hebard

Department of Physics, University of Florida, Gainesville, Florida 32611

(Received 22 December 1997; revised manuscript received 12 June 1998)

We have grown thin stable films of a good metal, Ag, that have characteristics of bad metals: high resistivity, strong temperature dependence of resistivity, and lack of resistive saturation. For films of different thickness, the temperature-dependent resistance and the Hall effect resistance provide evidence that the apparent bad metallicity is a consequence of the microstructure of the film rather than the result of new physics. This microstructure, which we characterize with scanning probe techniques, occurs on length scales comparable to the mean free path, thereby changing the sign of the classical magnetoresistance from positive to negative. [S0031-9007(98)07453-5]

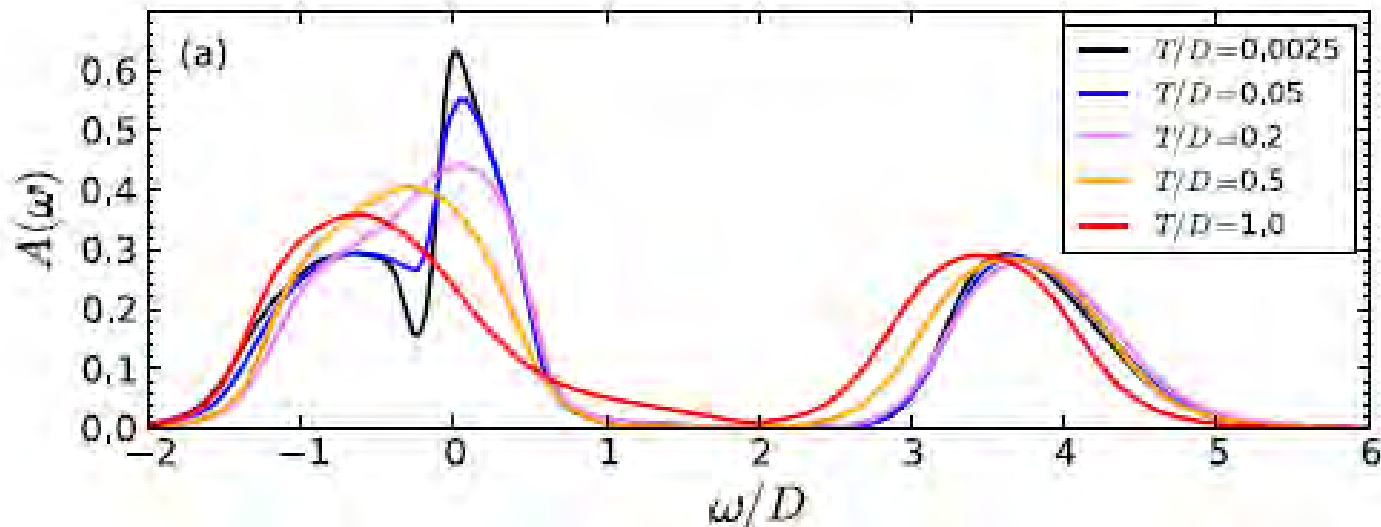
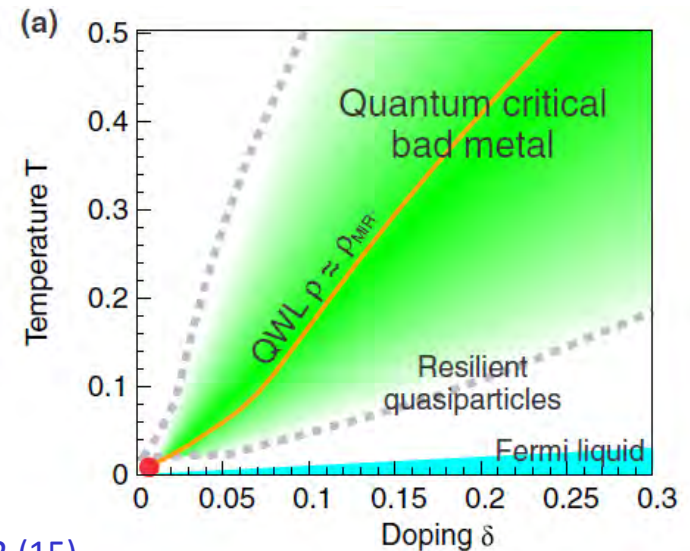
New approaches to understand MIR violation

Resilient quasiparticles

Revisits MIR violation using DMFT
- finds a large region in temperature
beyond the usual FL T^2 resistivity
regime where quasiparticles survive.

Deng *et al.*,
PRL **110** 086401 (13)

Vucicevic *et al.*,
PRL **114** 246402 (15)



New approaches to understand MIR violation

T -dependent Ω_p

$$\sigma = \text{Re}(\sigma) + i\text{Im}(\sigma) = \frac{\Omega_p^2}{4\pi} \frac{1}{-i\omega \frac{m^*(\omega)}{m_b} + 1/\tau(\omega)}$$

Analyses optical conductivity of VO₂

Deng *et al.*, *PRL* **113** 246404 (14)

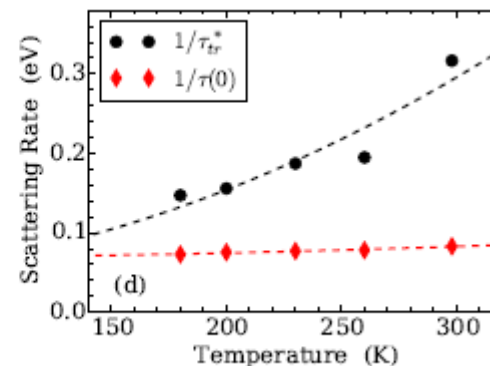
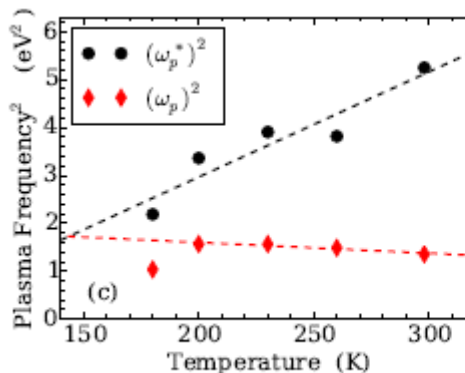
$$\int_0^W \sigma(\omega) d\omega = \frac{\Omega_p^2}{8}$$

Introduces new quantities

$$(\Omega_p^*)^2 = \frac{m_b}{m^*(0)} \Omega_p^2$$

$$\frac{1}{\tau_{tr}^*} = \frac{m_b}{m^*(0)} \frac{1}{\tau(0)}$$

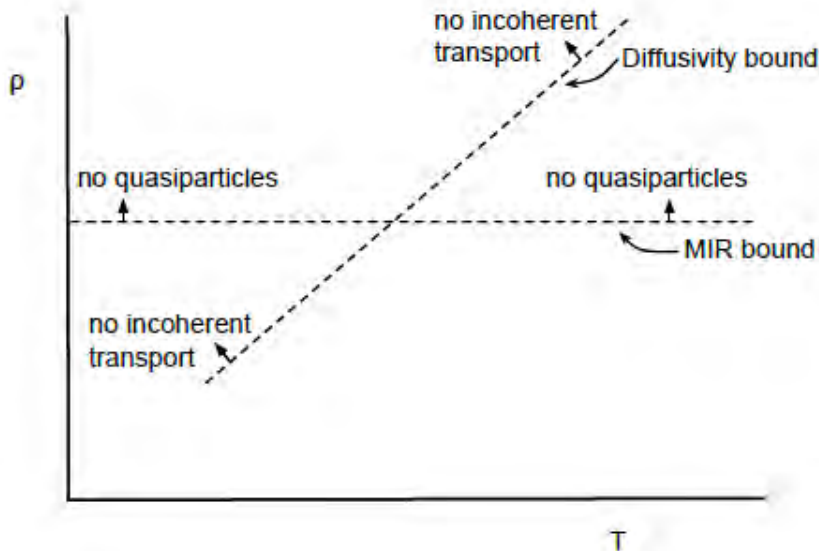
Finds strong T -dependence in both suggesting bad metallic transport arises due to strong correlation-driven T -dependence in the electronic structure.



New approaches to understand MIR violation

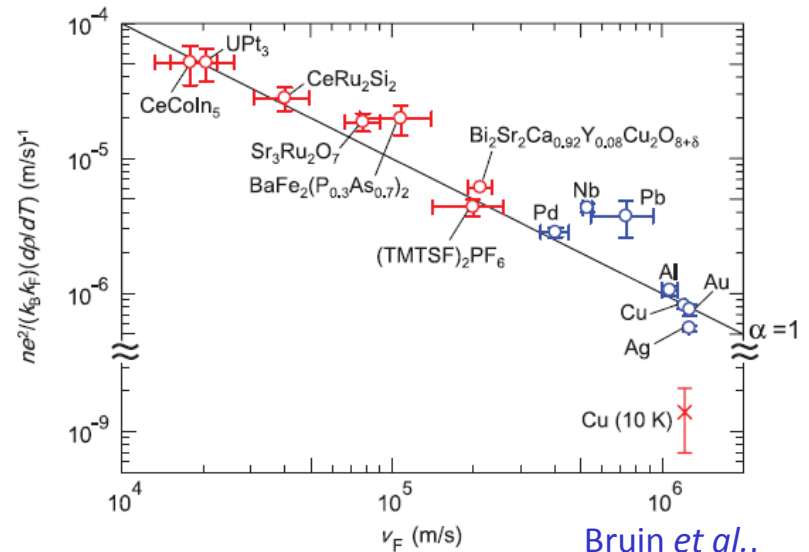
“Hartnoll+”

Motivated by empirical observation of a universal value in the slope of T -linear resistivities in many systems.



This bound, however, can be violated in the strongly correlated (large U) regime.

Pakhira & McKenzie, *PRB* **91** 075124 (15)



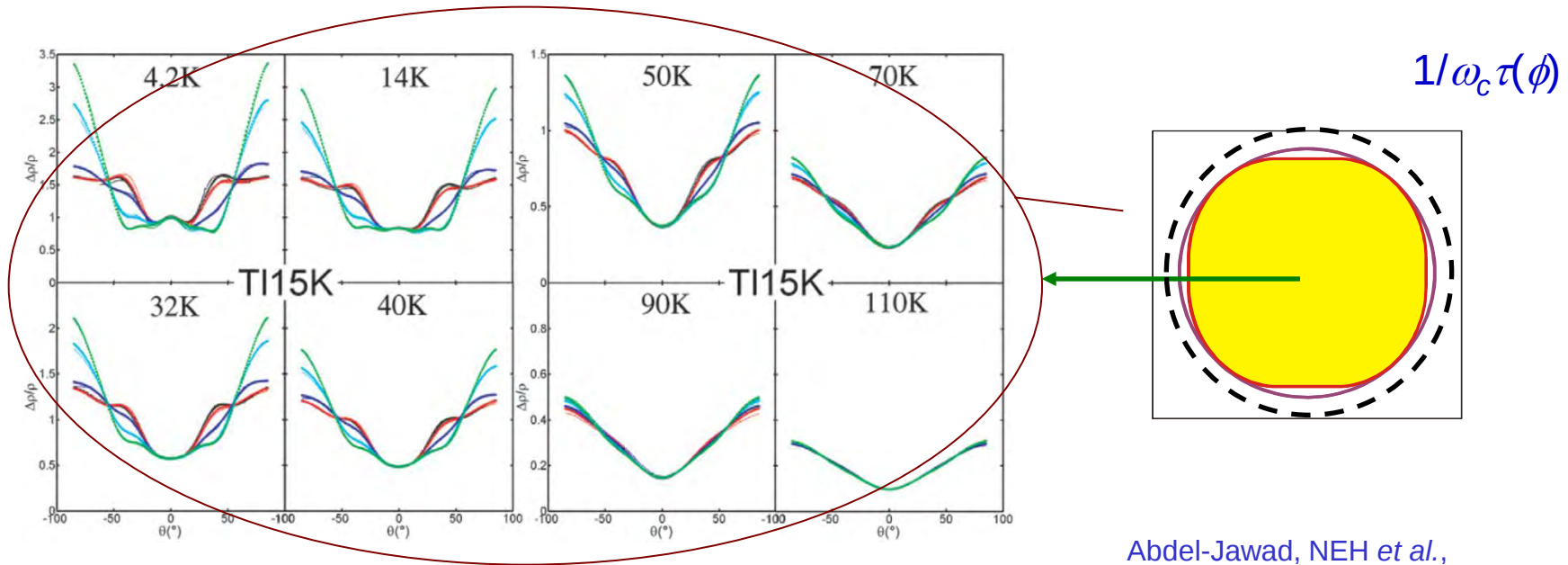
Bruin *et al.*,
Science **339** 804 (13)

Considers incoherent (non-quasiparticle) transport in terms of **diffusion**, rather than momentum relaxation, and offers new insight into the seemingly ubiquitous T -linear resistivity observed in bad metals at high temperatures.

Bad metals (SCES) are subject to a **diffusivity bound**

Hartnoll, *Nature Phys.* **11** 54 (15)

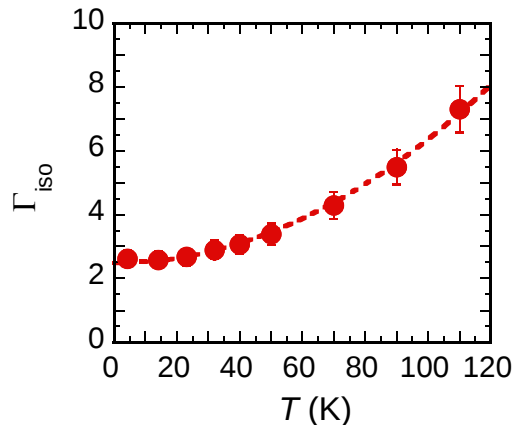
Anisotropic incoherence in cuprates – part I



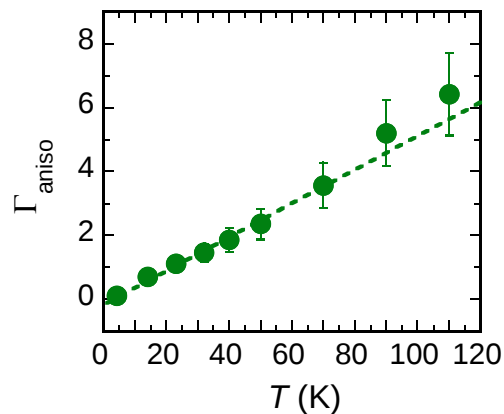
French, NEH *et al.*, *NJP* **11** 055057 (2009)

Abdel-Jawad, NEH *et al.*,
Nature Phys. **2** 821 (2006)

$\Gamma(T)$ at the nodes



Additional scattering
along $(\pi, 0)$

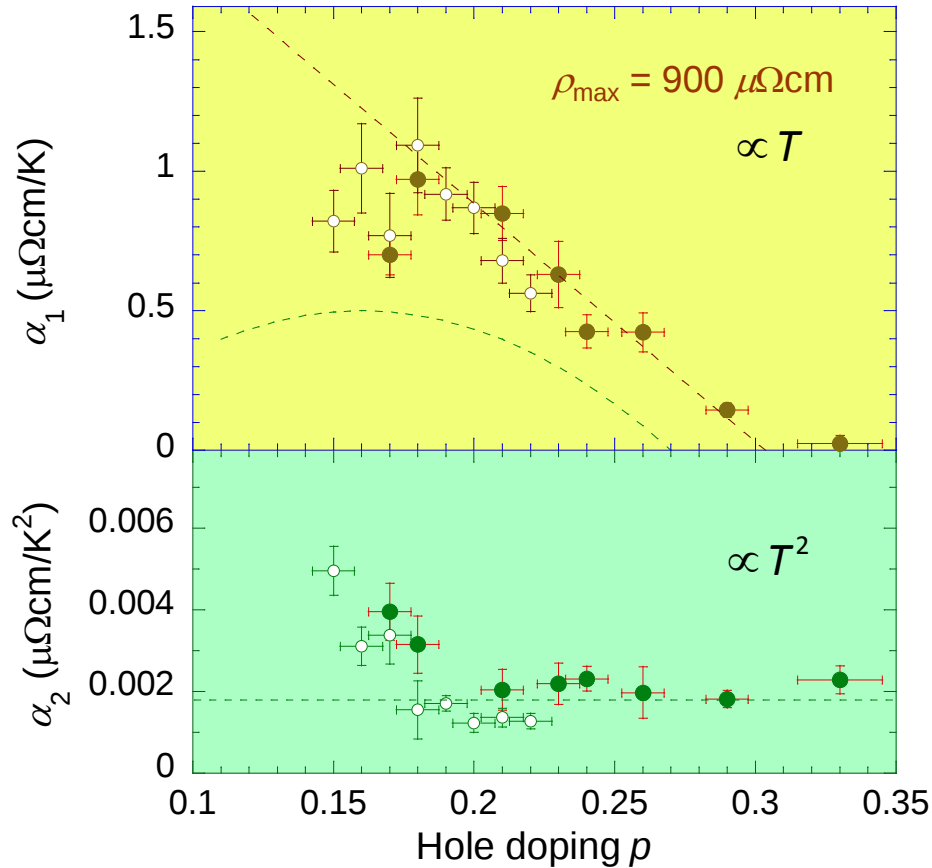


$$\begin{aligned}\Gamma(\phi, T) &= \Gamma_{\text{iso}} + \Gamma_{\text{aniso}} \\ &= \Gamma_0 + BT^2 + CT \cos^2 2\phi\end{aligned}$$

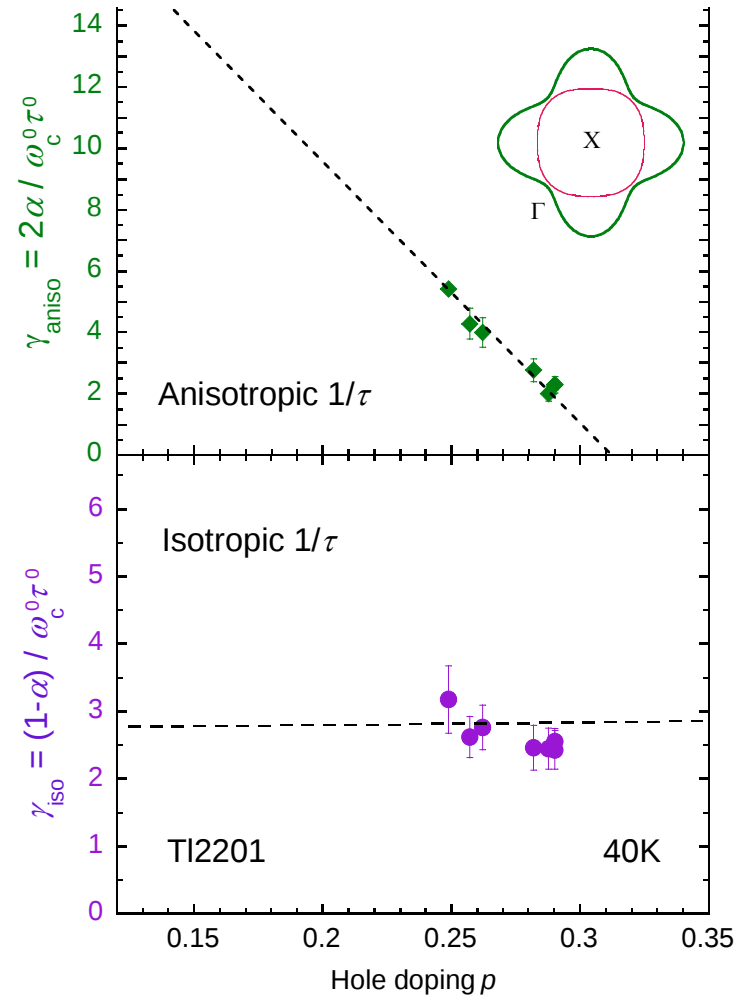
Two independent scattering processes everywhere on FS, one isotropic and T^2 , the other anisotropic and linear-in- T .

Doping dependence of α_1 and α_2

Cooper, NEH et al.,
Science **323** 603 (2009)

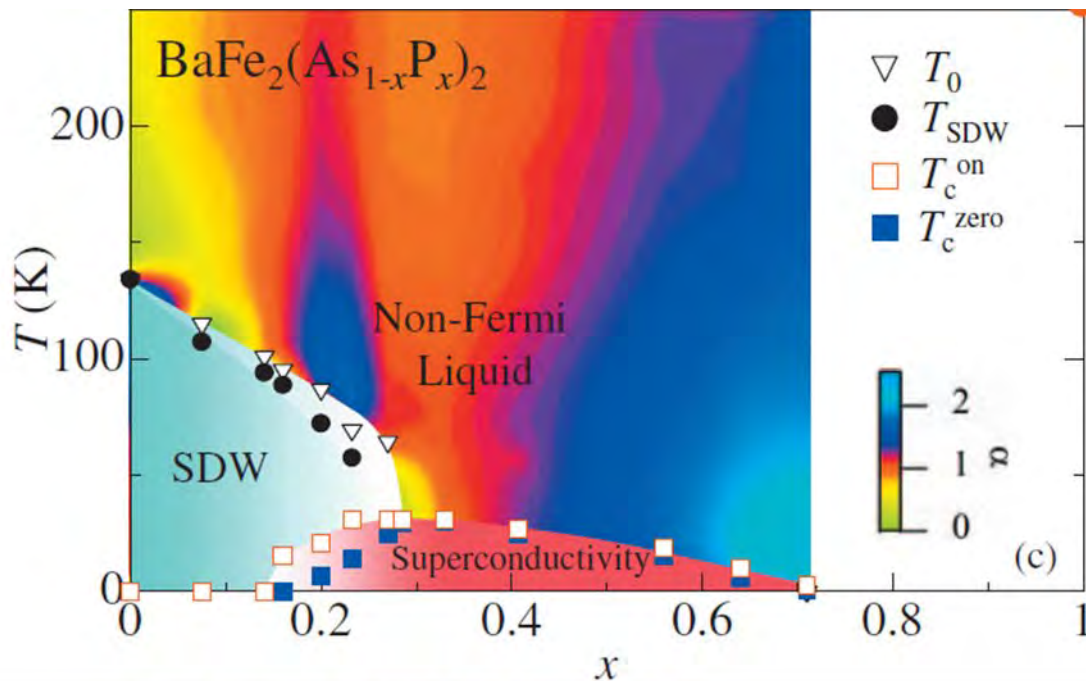


$$\frac{1}{\rho_{\text{eff}}} = \frac{1}{\alpha_0 + \alpha_1 T + \alpha_2 T^2} + \frac{1}{\rho_{\text{max}}}$$

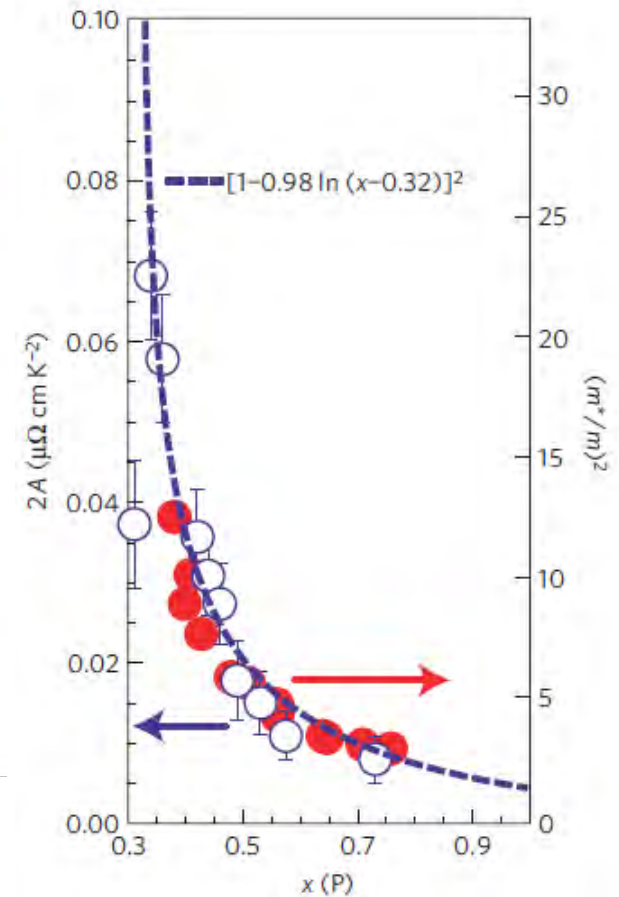


Abdel-Jawad et al., PRL **99** 107002 (2007)

Cuprates vs. the pnictides



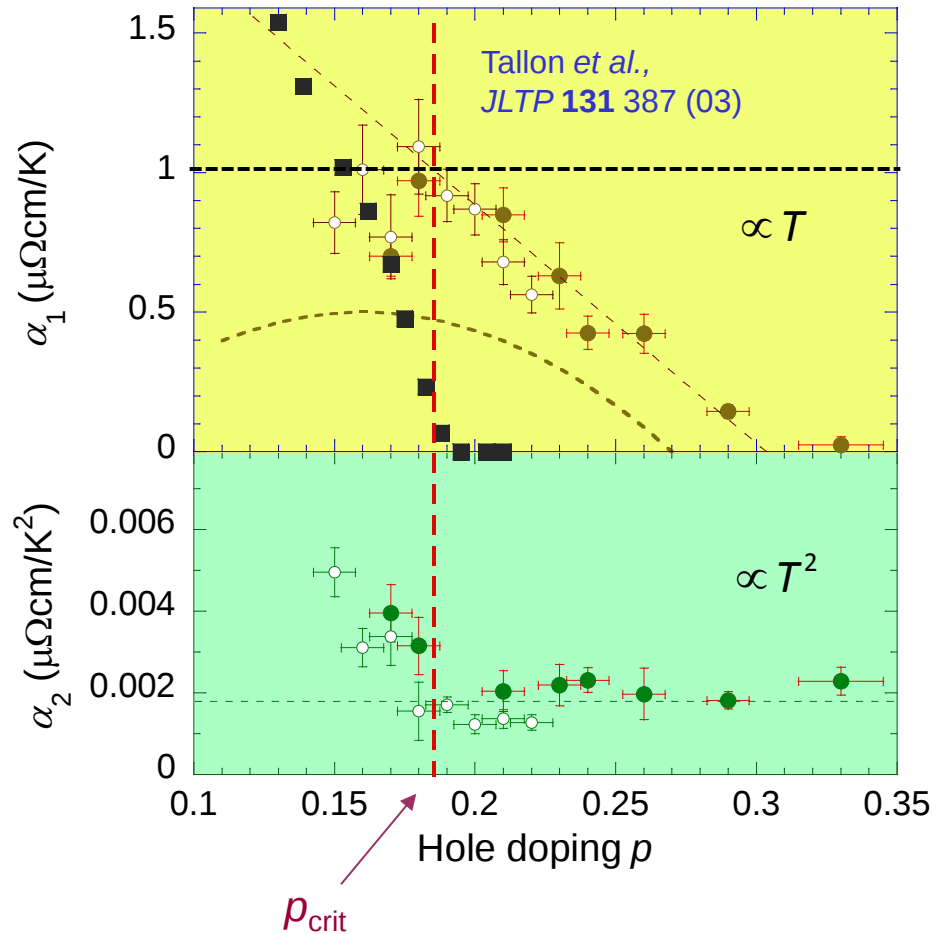
Kasahara et al., *PRB* **81** 184519 (2010)



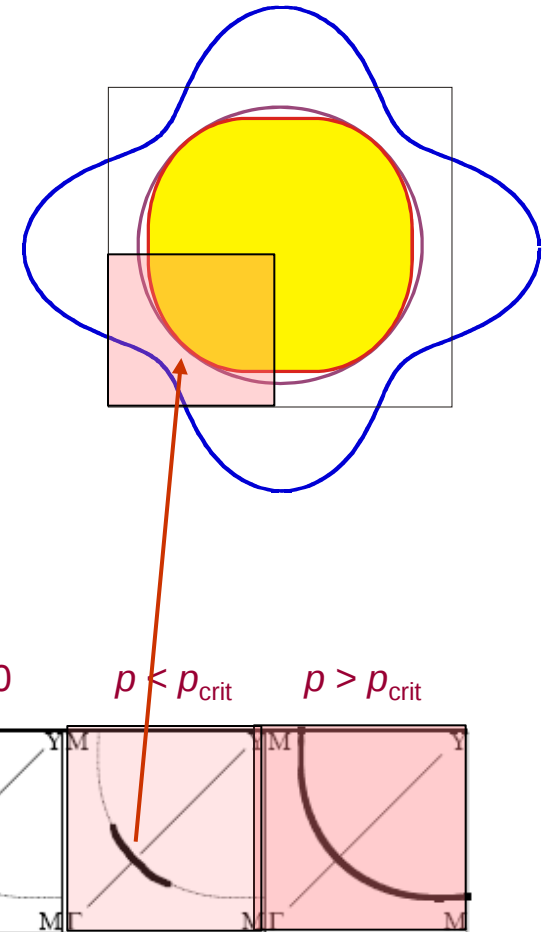
Analytis, NEH et al.,
Nature Phys. **10** 194 (14)

Doping dependence of α_1 and α_2

Cooper, NEH et al.,
Science **323** 603 (2009)



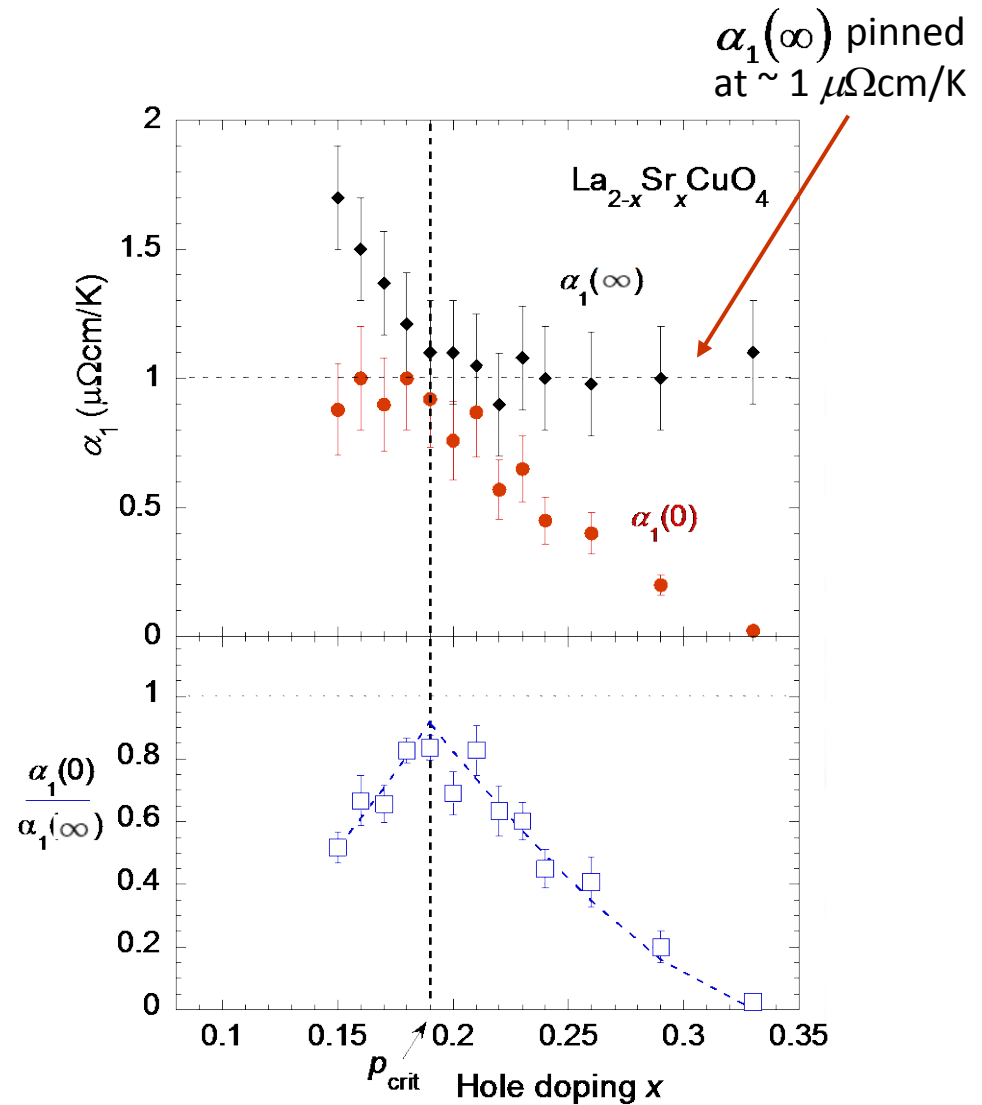
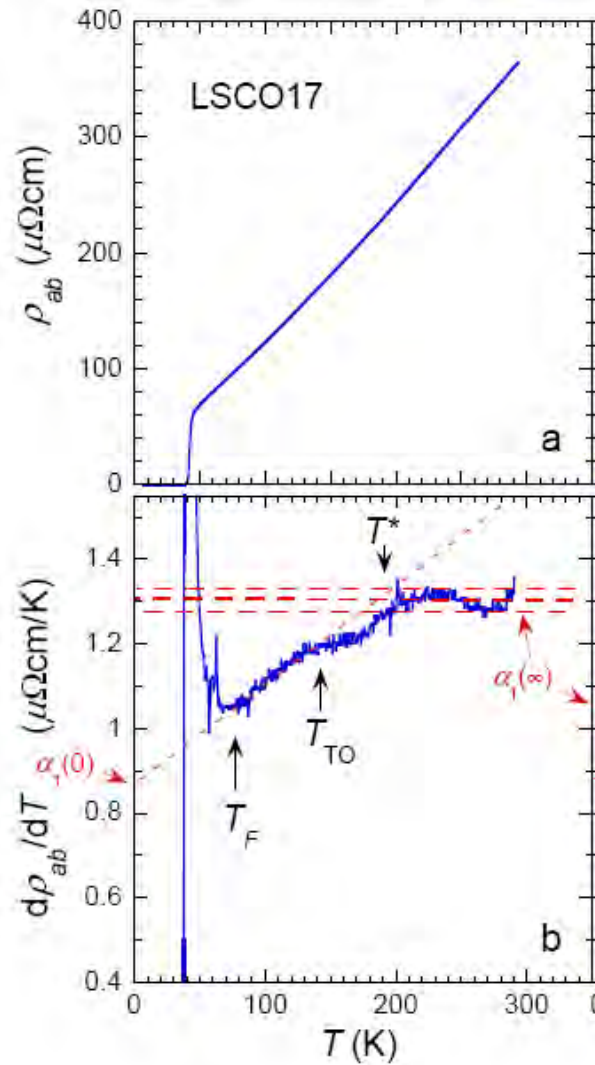
- Change in transport coefficients coincides with onset of pseudogap



- Evolution of transport coefficients across p_{crit} consistent with k-dependence of each component

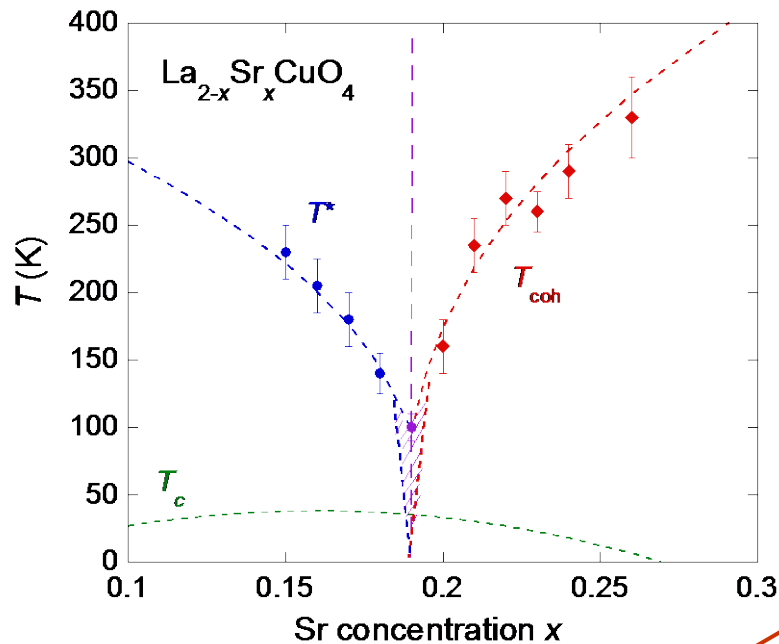
Onset of incoherence?

NEH et al.,
PTRSA **369** 1626 (2011)



Onset of incoherence?

NEH et al.,
PTRSA **369** 1626 (2011)

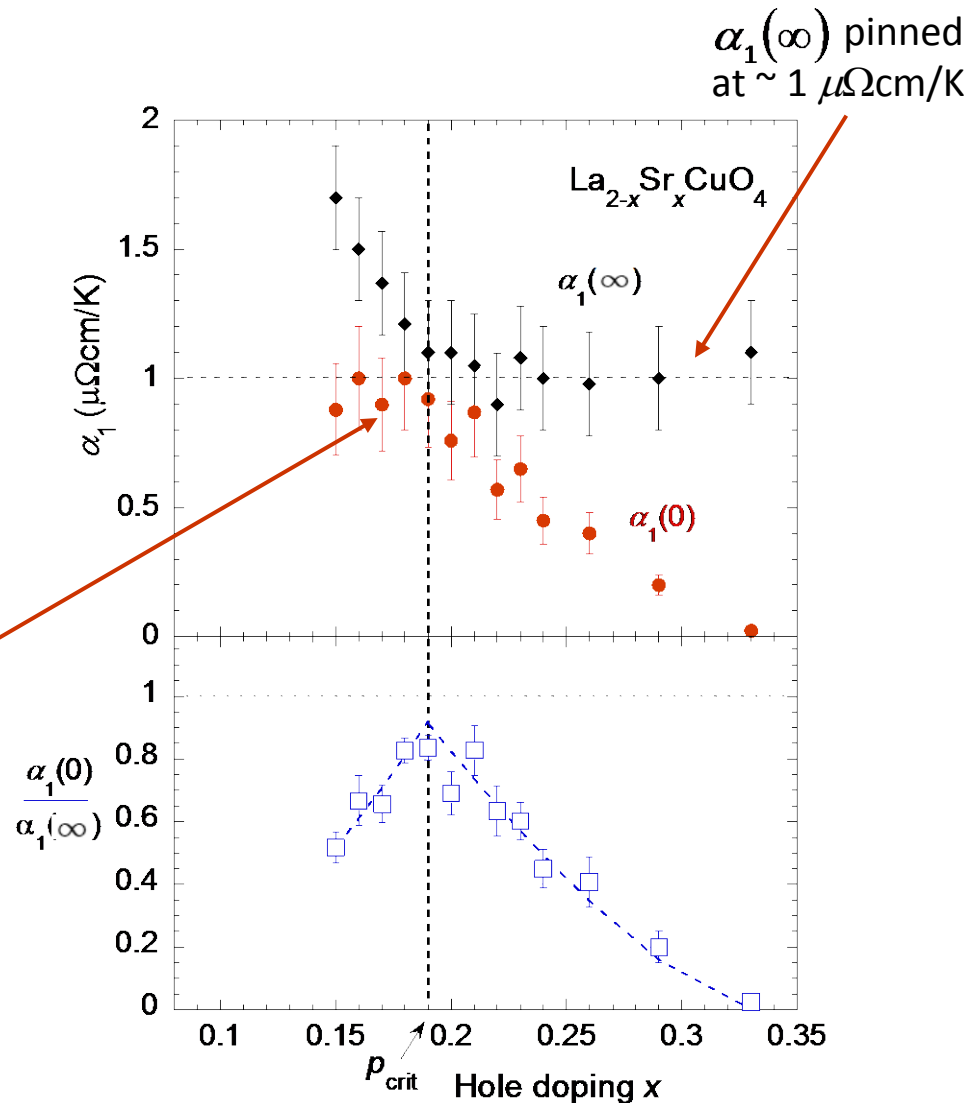


At p_{crit} , α_1 reaches $1 \mu\Omega\text{cm/K}$
– is this significant?

$1 \mu\Omega\text{cm/K}$ for OD LSCO corresponds to

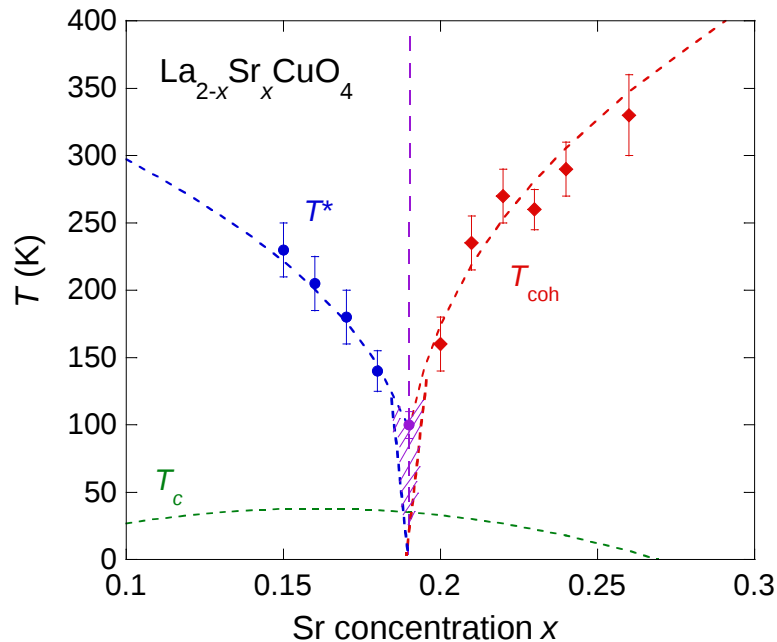
$$\hbar/\tau = 2\pi k_B T \text{ at } (\pi, 0)$$

“Planckian dissipation limit”
- onset of incoherent transport



Onset of incoherence?

NEH *et al.*,
PTRSA **369** 1626 (2011)

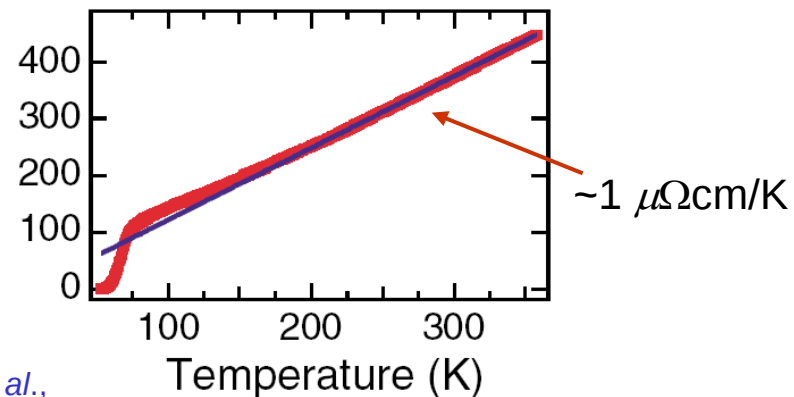
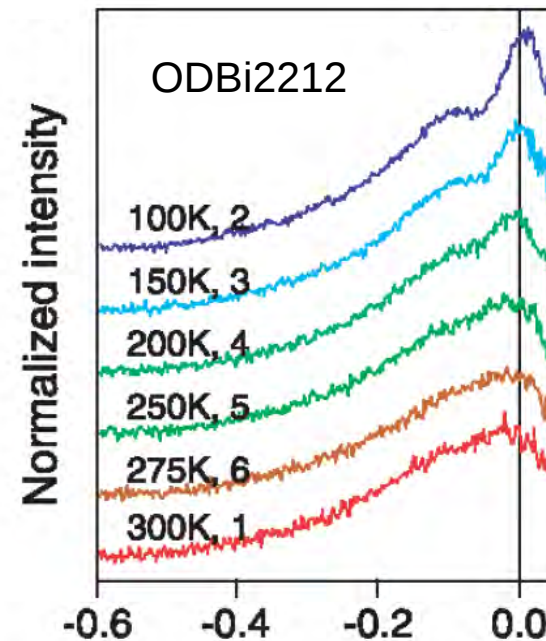


1 $\mu\Omega\text{cm/K}$ for OD LSCO corresponds to

$$\hbar/\tau = 2\pi k_B T \text{ at } (\pi, 0)$$

“Planckian dissipation limit”
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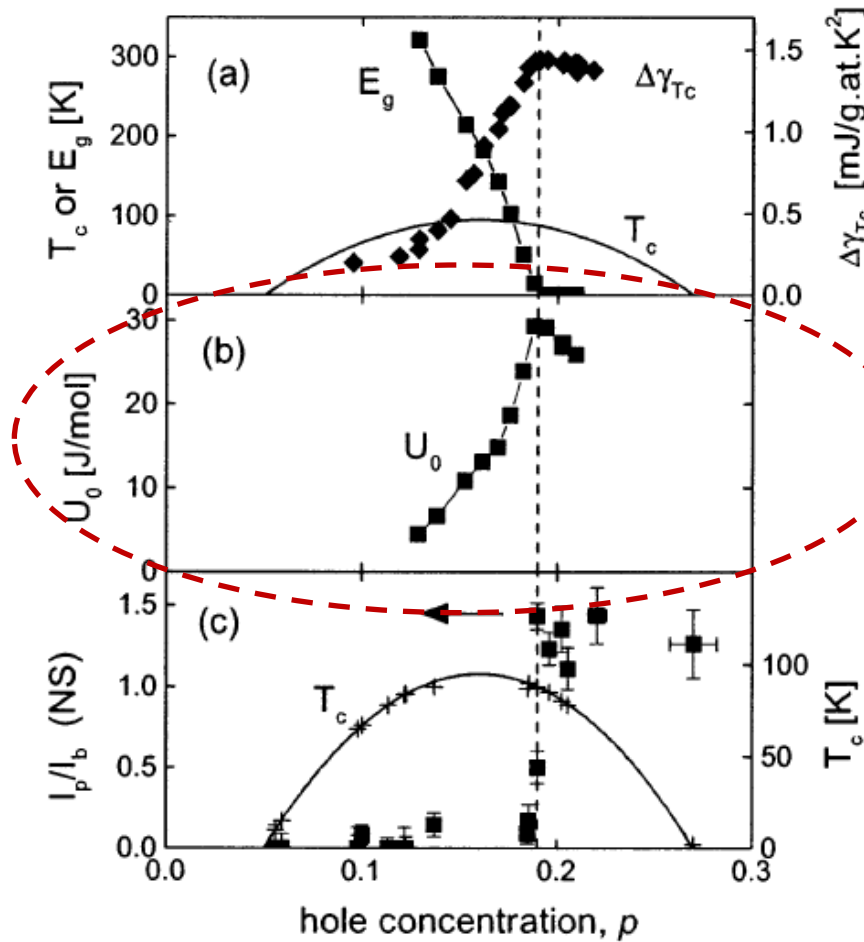
Zaanen, *Nature* **430** 512 (2004)



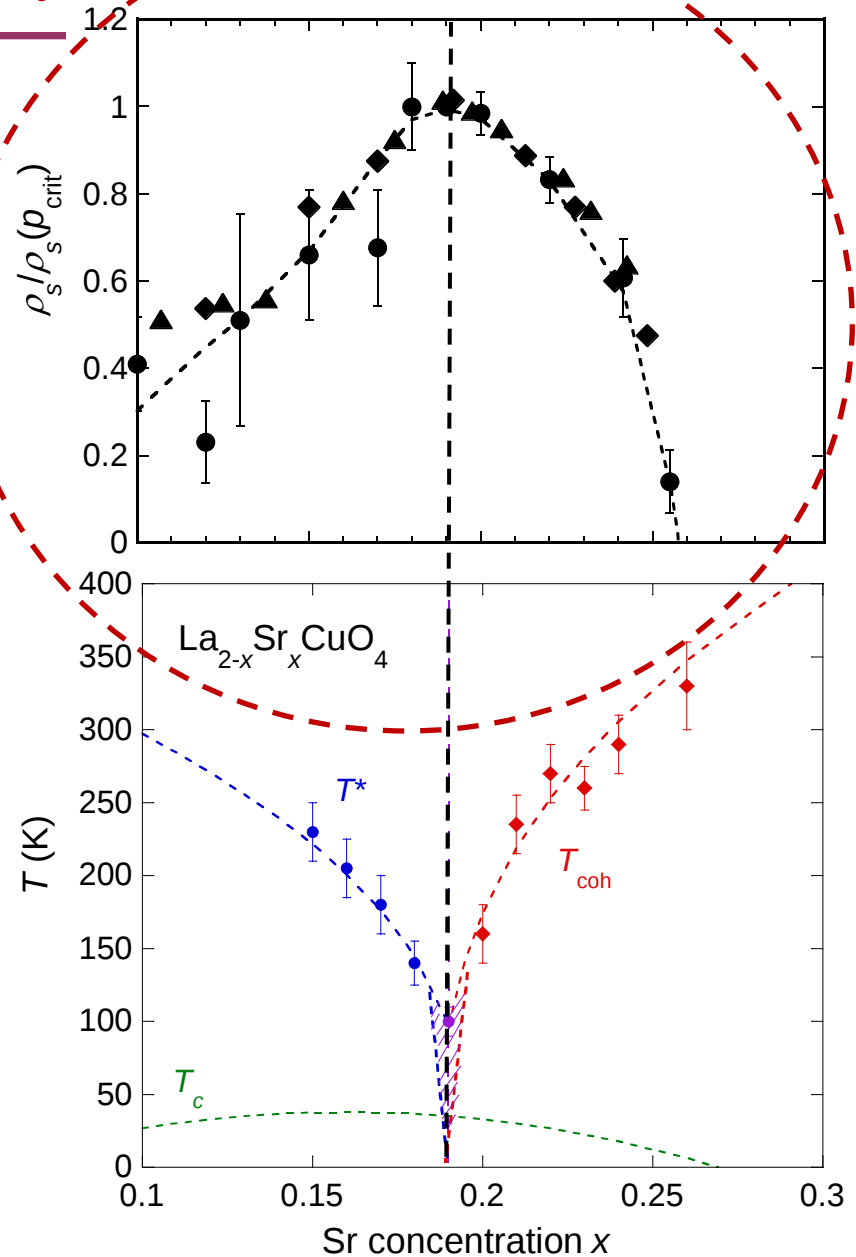
Kaminski *et al.*,
PRL **90** 207003 (2003)

Impact of T_{coh} on superconductivity

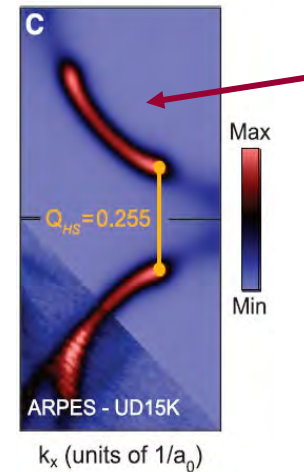
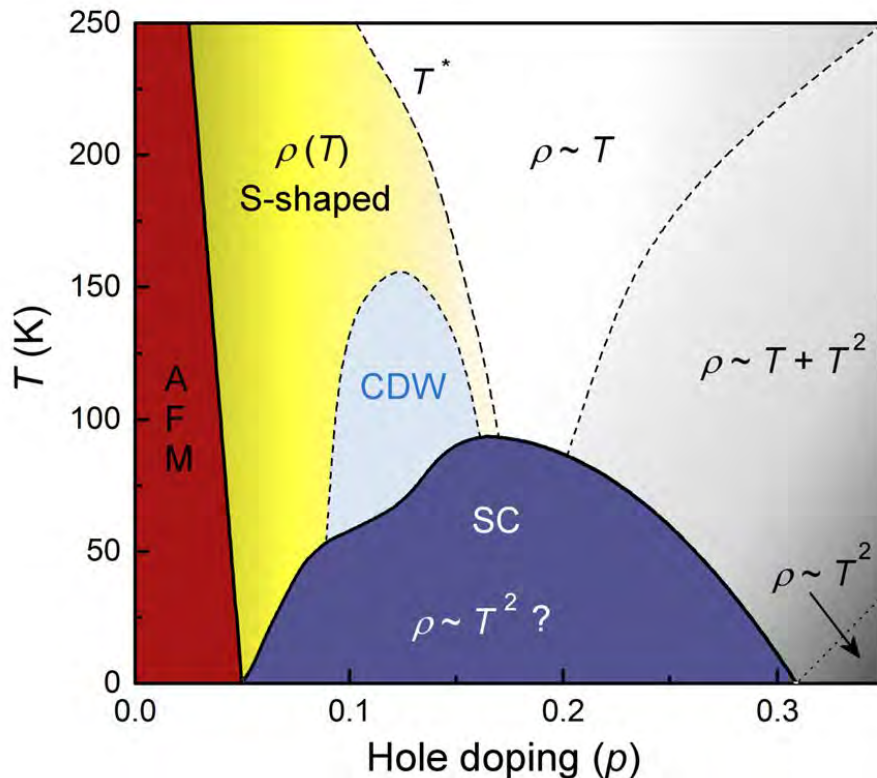
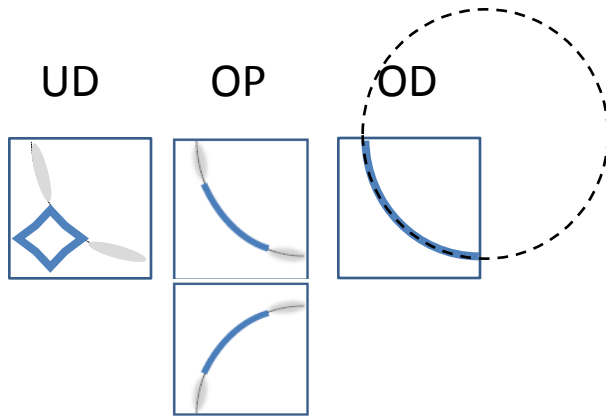
Rourke, NEH et al.,
Nature Phys. 7 445 (2011)



Tallon et al., JLTIP 131 387 (2003)



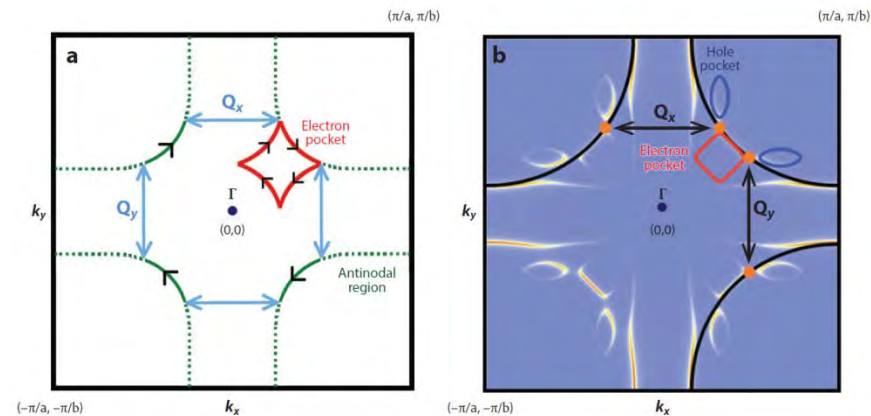
Anisotropic incoherence in cuprates – part II



Biaxial charge order causes Fermi surface reconstruction (FSR).

But is there single pocket only or extra hole pockets too?

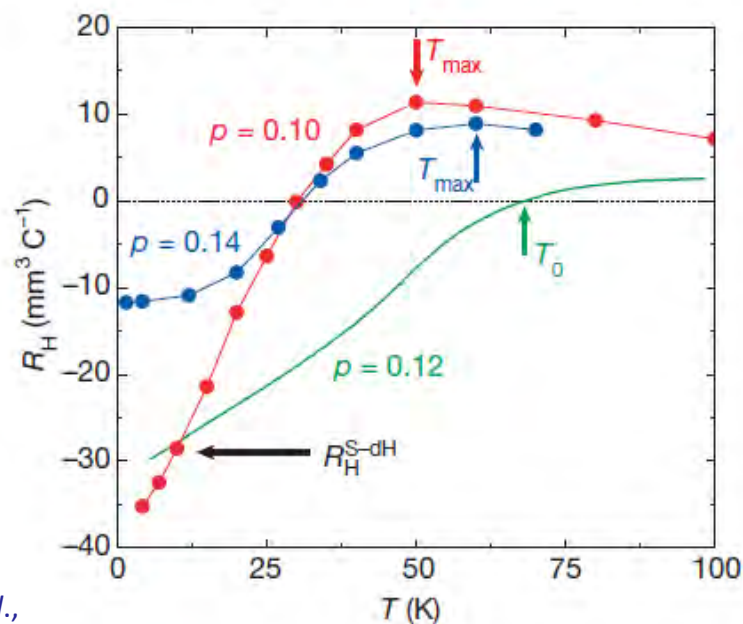
Comin et al.,
Science **343**, 390 (2014)



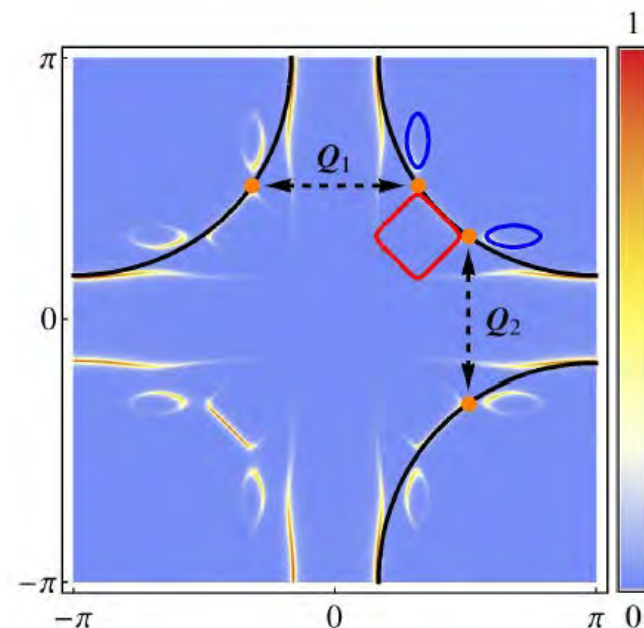
Sebastian & Proust,
ARCOMP **6**, 411 (2015)

Allais et al.,
Nat. Commun **5**, 5771 (2014)

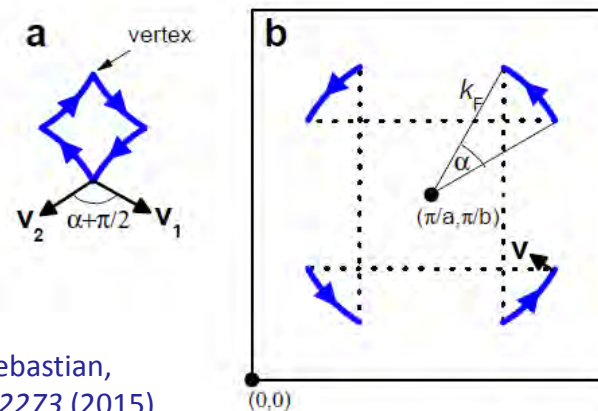
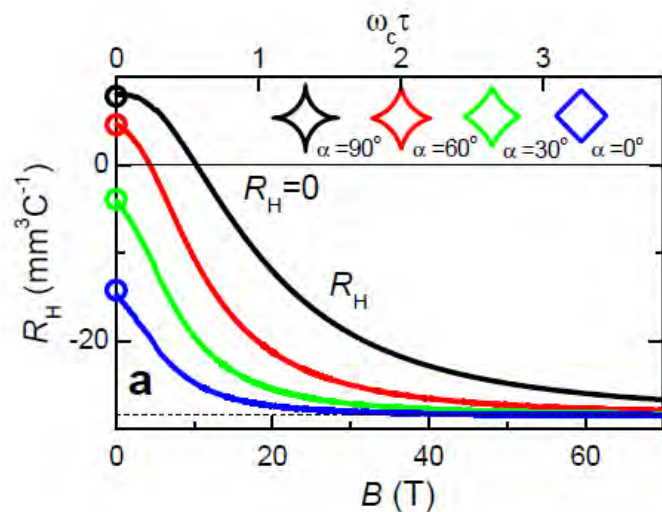
Hall coefficient in UD cuprates



LeBoeuf *et al.*,
Nature **450**, 533 (2007)



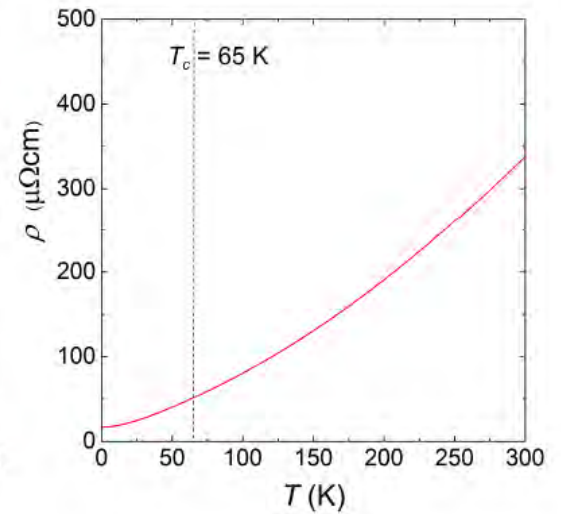
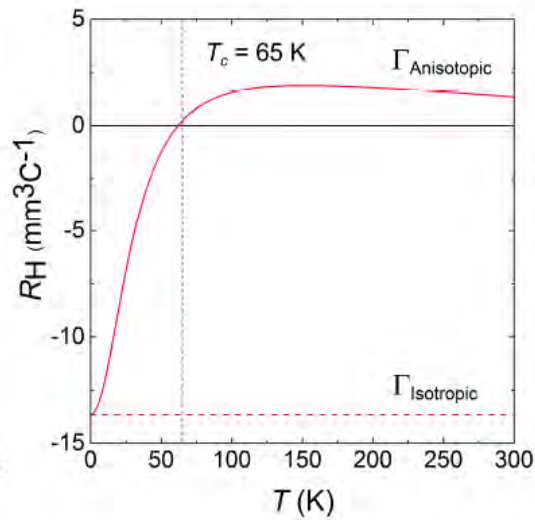
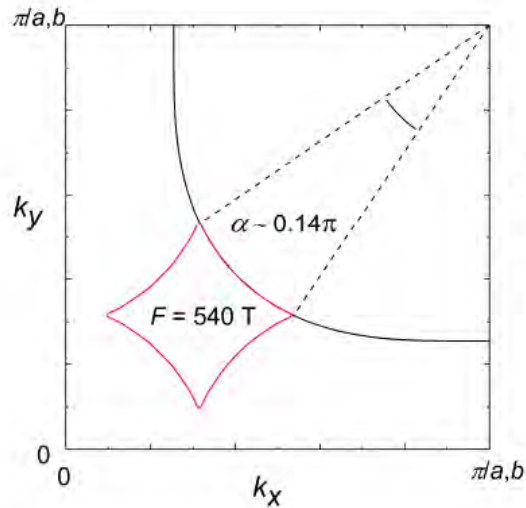
Allais *et al.*,
Nat. Commun **5**, 5771 (2014)



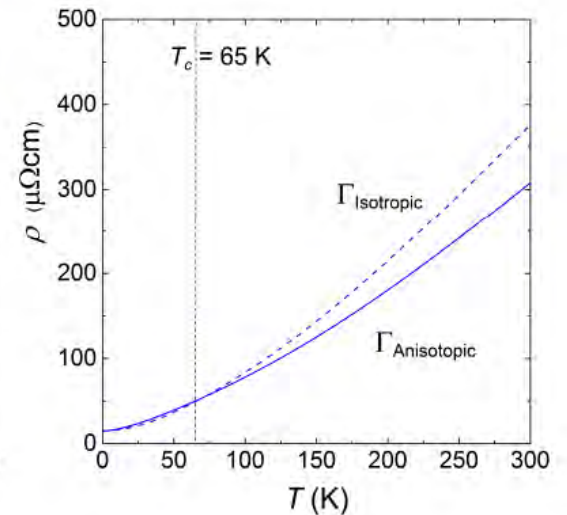
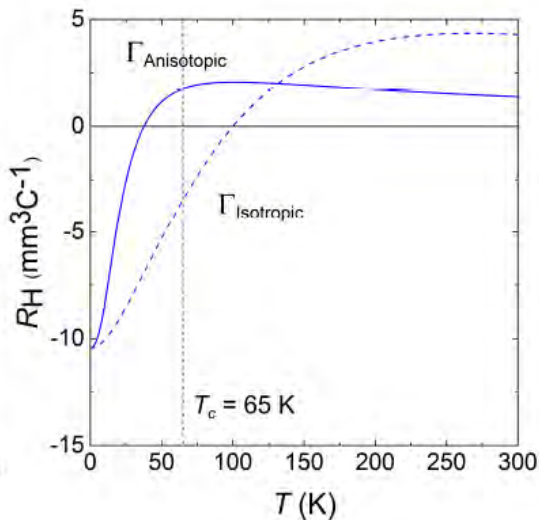
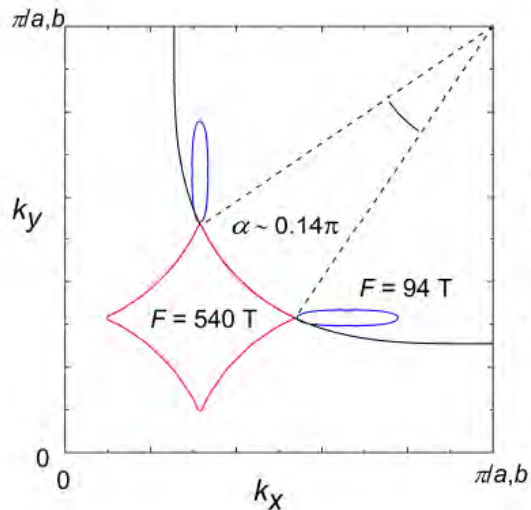
Harrison & Sebastian,
arXiv:1502.02273 (2015)

Inclusion of anisotropic scattering

Robinson & NEH,
unpublished (2015)

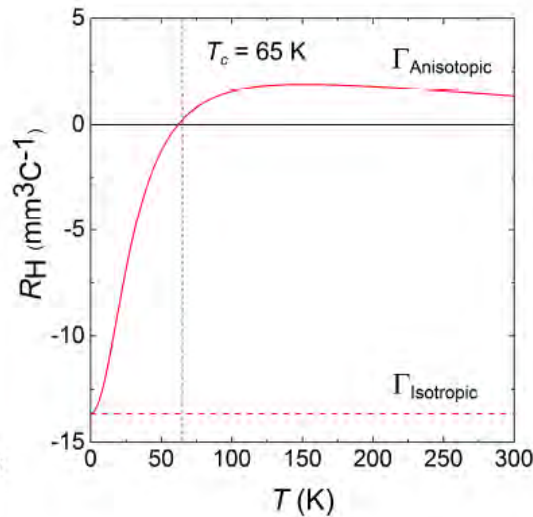
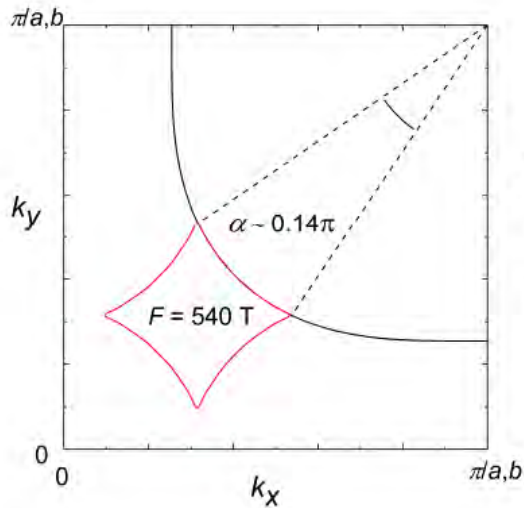


$$\Gamma(T) = \Gamma_0 + \Gamma_2 \cos^2 2\varphi T^2$$

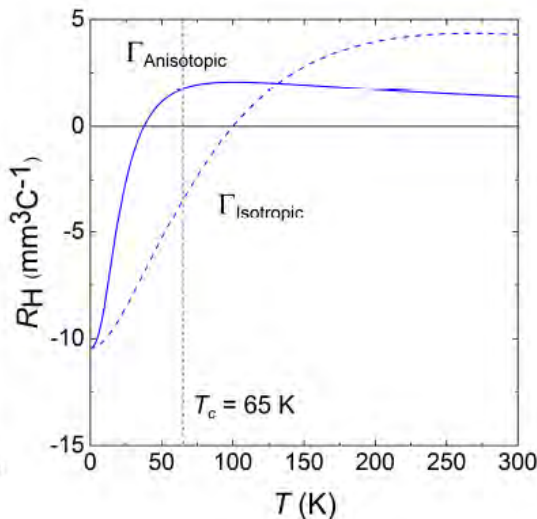
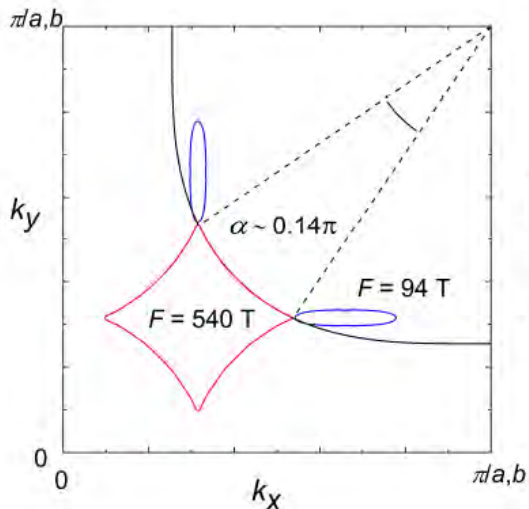


Inclusion of anisotropic scattering

Robinson & NEH,
unpublished (2015)



$$\Gamma(T) = \Gamma_0 + \Gamma_2 \cos^2 2\varphi T^2$$



In either scenario, the strength of the scattering rate is found to approach $\Gamma_{\max} = v_F/a$ at $T = T_{CO}$.

Thus, it may well be two incoherence bounds influencing the phase diagram of high- T_c cuprates, the MIR bound for the near-nodal quasiparticles inside the PG regime, and the diffusive bound beyond the PG regime for the qp states near the zone boundaries.

Summary

