# Disorder effects in strongly interacting metals, insulators and superconductors

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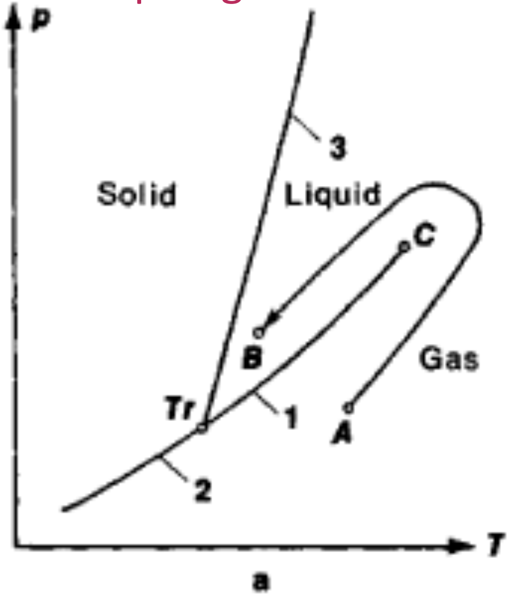
*Bad metal behavior in Mott systems*
Mainz, Germany, July 01, 2015

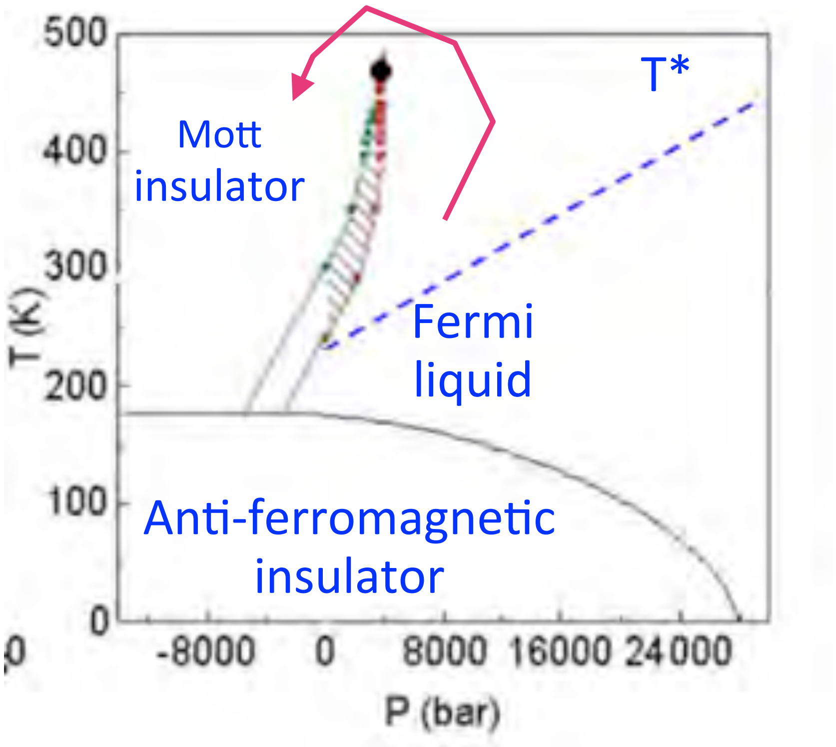
# The Mott transition as a liquid-gas transition

(Castellani et al. PRL (1979); Rozenberg et al. PRL (1999); Kotliar et al. PRL (2000))

P. Limelette et al., Science **302**, 89 (2003)

Liquid-gas transition





$(V_{0.989}Cr_{0.011})_2O_3$

- First-order line ending at a critical point
- Can go continuously from one state to the other

# The Mott transition as a liquid-gas transition

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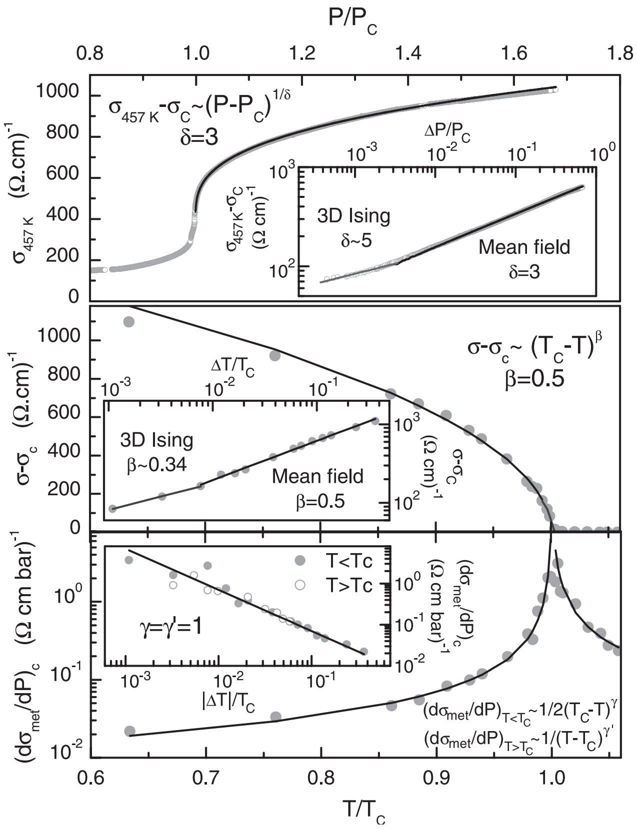


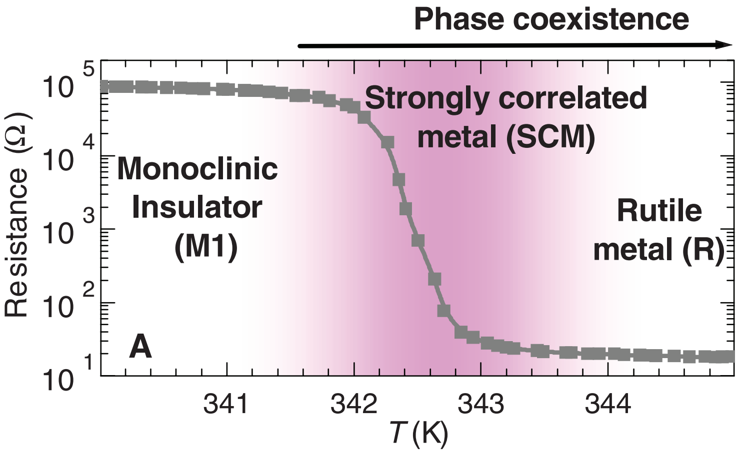
$(V_{0.989}Cr_{0.011})_2O_3$

- Can be characterized by the conductivity behavior
- Ising universality class: mean field behavior (but 3D Ising very close to the transition point) (Abdel-Jawad et al. PRL 2015)
- Scalar order parameter (like the density in the liquid-gas transition): consistent with the conductivity behavior (Papanikolau et al. PRL 2008; M. Abdel-Jawad et al, PRL 2015)

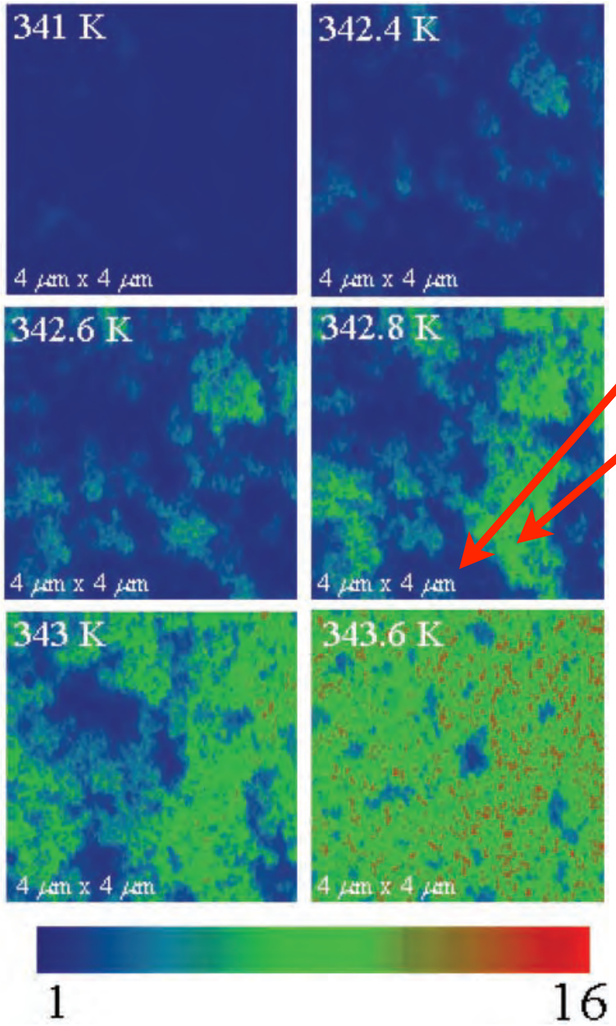
# Effects of inhomogeneities and finite temperature

Scanning Infrared Microscopy (M. M. Qazilbash et al., Science 38, 1750 (2007))

$VO_2$



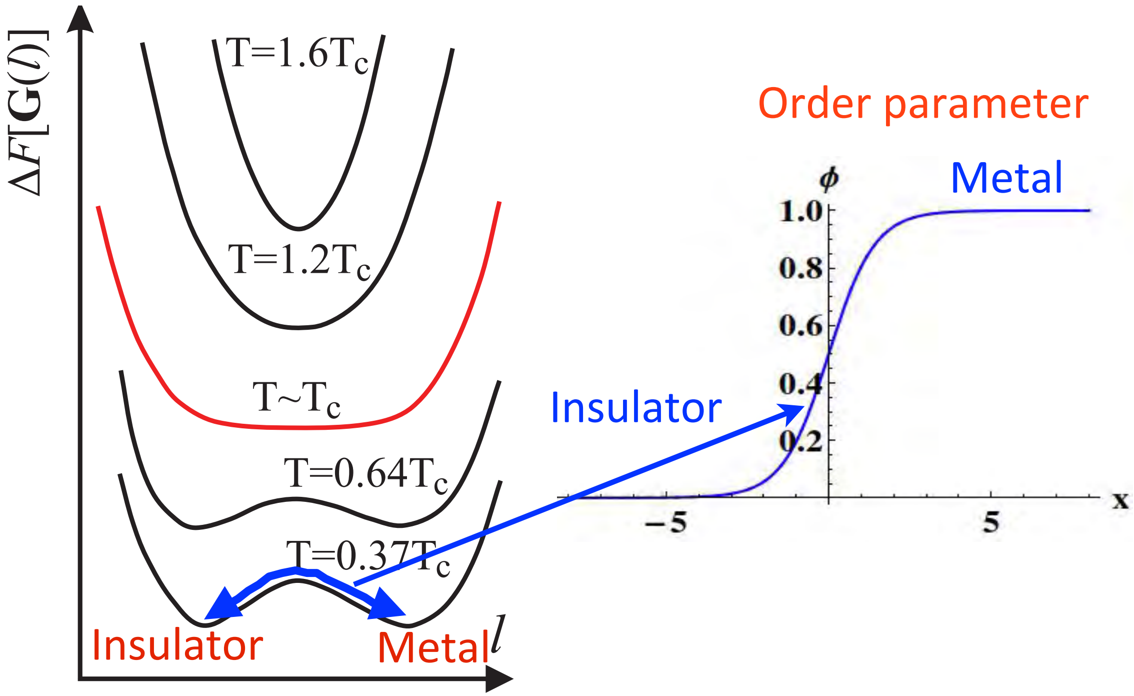
Ivan Schuller's talk



Inhomogeneous state: domains of Mott insulator + domains of correlated metal

# Mott transition: Landau functional description

(Rozenberg et al. PRL (1999); Kotliar et al. PRL (2000))



- The domain wall interpolates between metal and insulator
- At the center of the domain wall is the unstable solution
- The unstable solution is crucial!

# Taking into account inhomogeneities (disorder): statDMFT

$$H_{DH} = -t \sum_{\langle i,j\rangle,\sigma} \left( c^{\dagger}_{i\sigma} c_{j\sigma} + c^{\dagger}_{j\sigma} c_{i\sigma} \right) + \sum_{i,\sigma} \epsilon_i c^{\dagger}_{i\sigma} c_{i\sigma} + U \sum_{i} c^{\dagger}_{i\uparrow} c_{i\uparrow} c^{\dagger}_{i\downarrow} c_{i\downarrow}$$

$$P\left(\varepsilon_i\right) = \begin{cases} 1/W; & \left|\varepsilon_i\right| < W/2 \\ 0; & \text{otherwise} \end{cases}$$

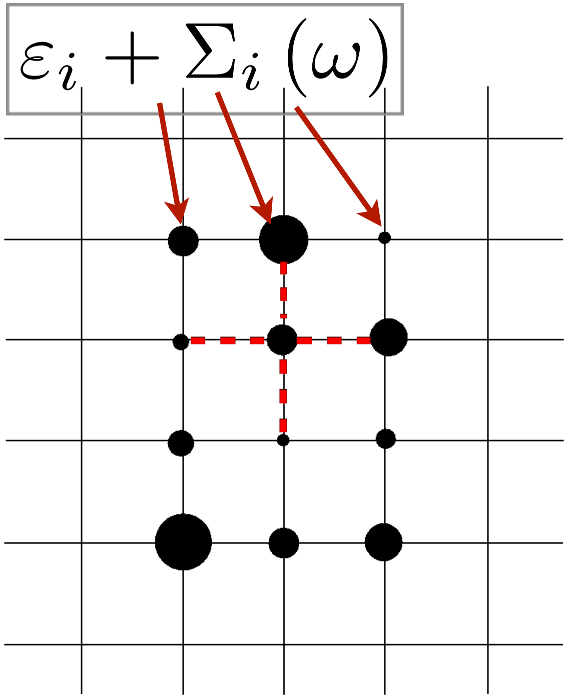
- Statistical Dynamical Mean Field Theory

(Kotliar and Dobrosavljević, PRL **78,** 3943 (1997)):

(i) assume local, though spatially fluctuating, self-energies;

(ii) self-consistency: each site "sees" the spatially fluctuating density of states of its neighbors.

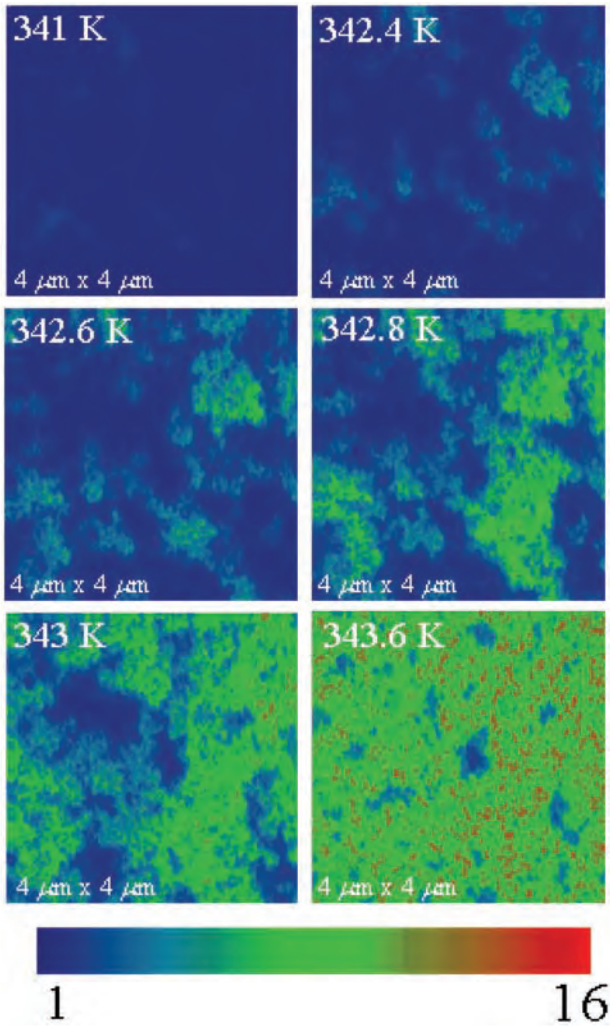
(iii) Note that this is exact at $U$=0 (including localization effects)



# Effects of inhomogeneities and finite temperature

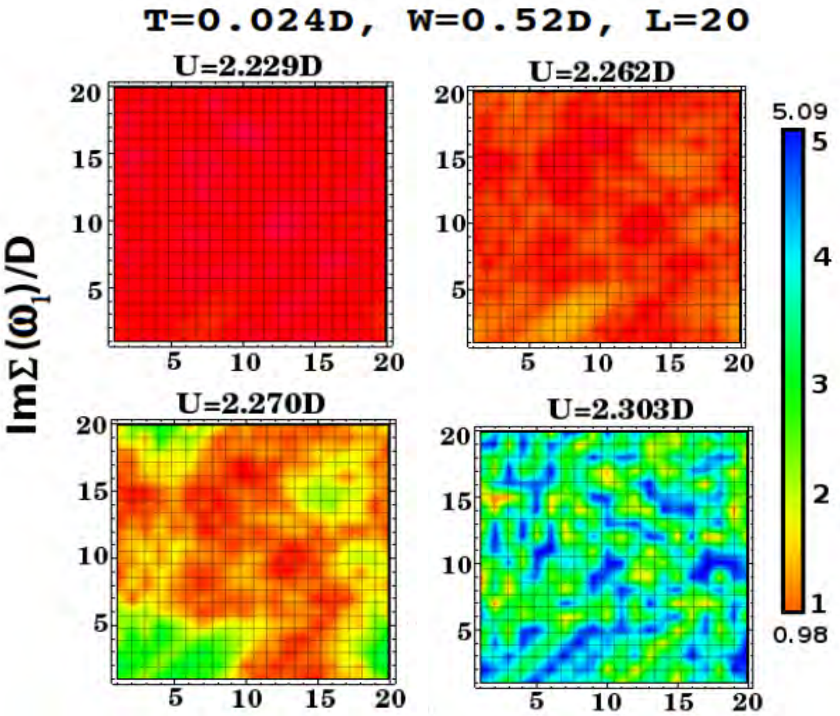
Scanning Infrared Microscopy (M. M. Qazilbash et al., Science 38, 1750 (2007))

$VO_2$



StatDMFT shows meso-scale space fluctuations:

- large domain walls between Mott insulator and metallic droplets

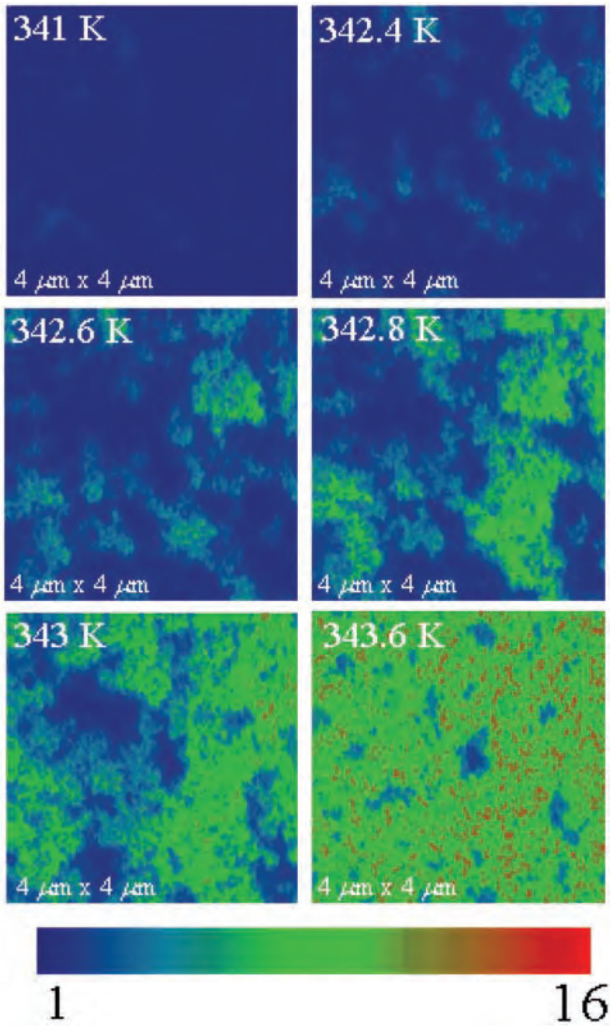


Martha Suarez Villagrán's PhD Thesis (2014)

# Effects of inhomogeneities and finite temperature

VO$_2$

Scanning Infrared Microscopy (M. M. Qazilbash et al., Science 38, 1750 (2007))



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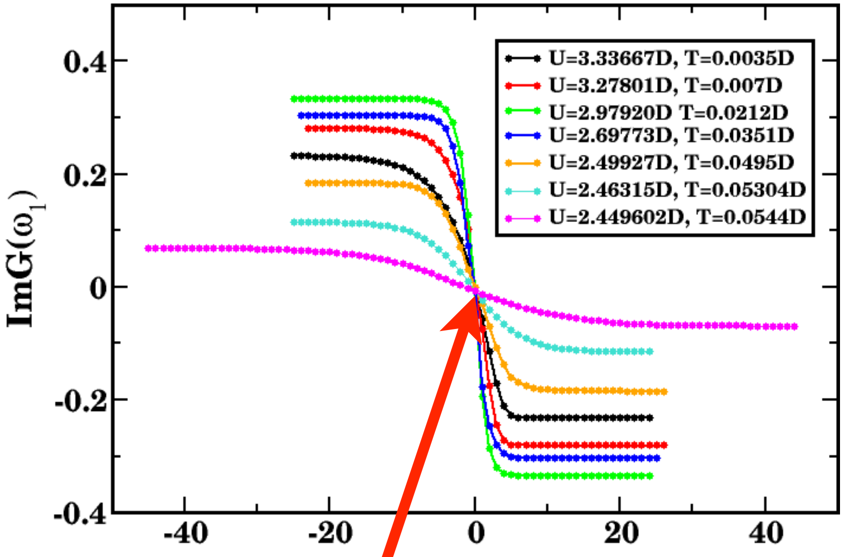
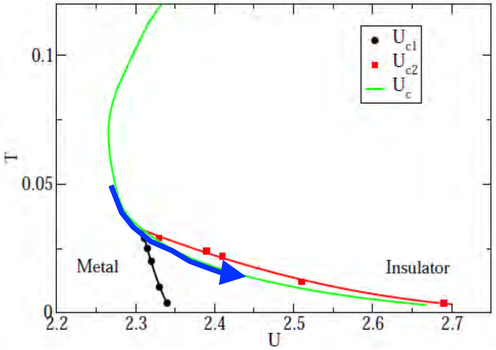


Let’s look at the behavior at the domain wall center: the unstable solution

# The unstable solution along the 1st order line

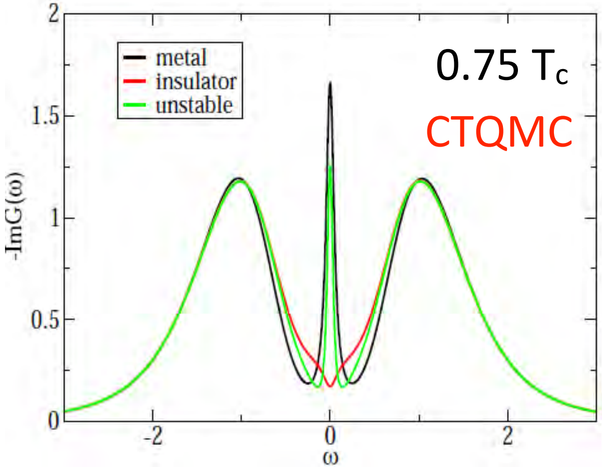
(Tsung-Han Lee, J. Vučičević, D. Tanasković,E. M., V. Dobrosavljević, in preparation)

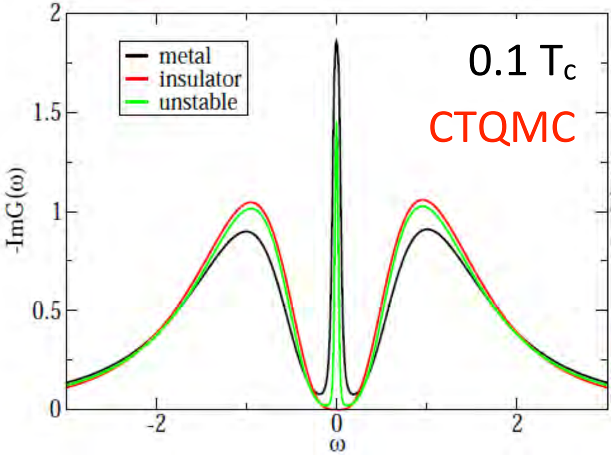


$$G_{loc}(\omega) = \frac{1}{\omega - \Delta(\omega) - \Sigma(\omega)}$$

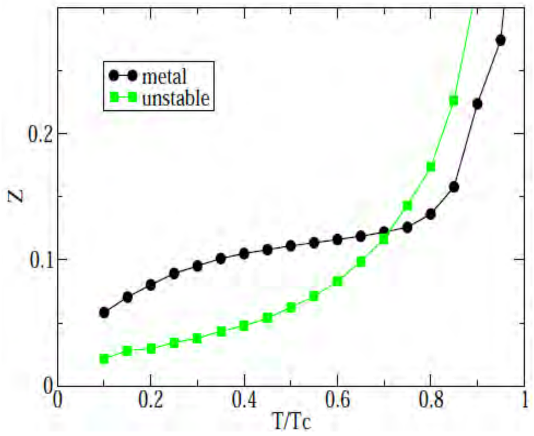
$$\Sigma(\omega) \approx \left(1 - Z^{-1}\right)\omega - i\Gamma$$

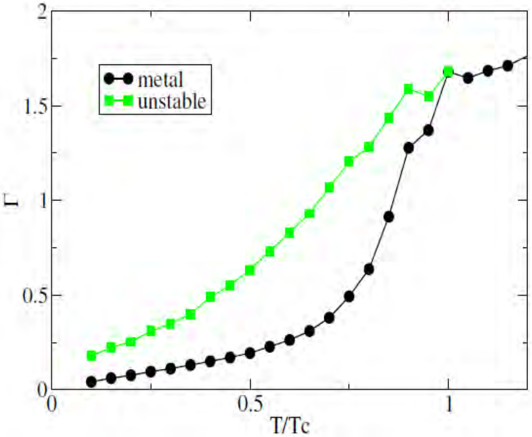
Local DOS: clear quasiparticle peak

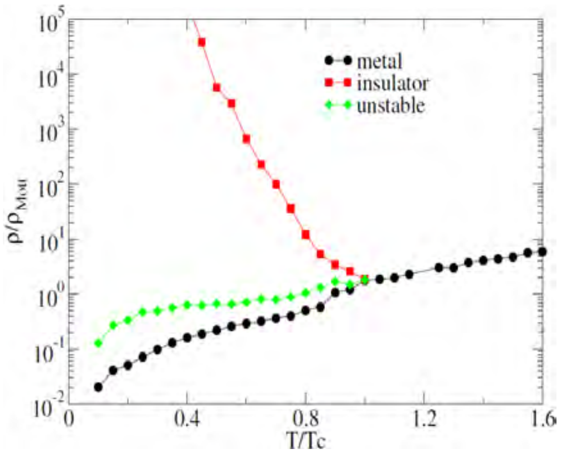




- Resilient quasiparticles (W. Xu *et al.*, PRL **111**, 03642 (2013))
- Incoherent metal



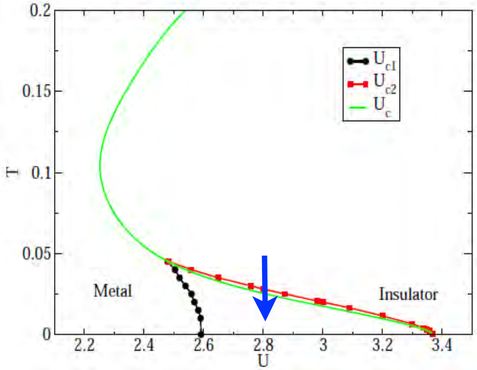


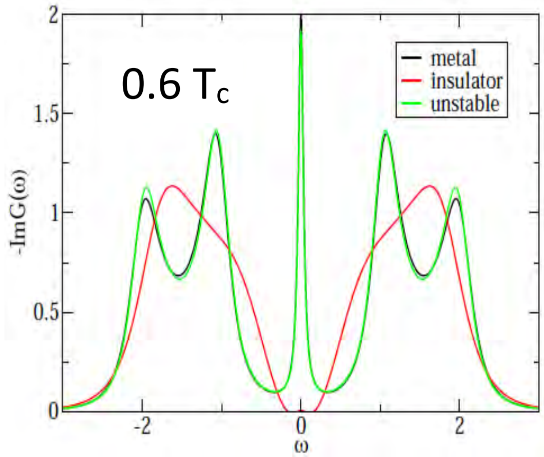


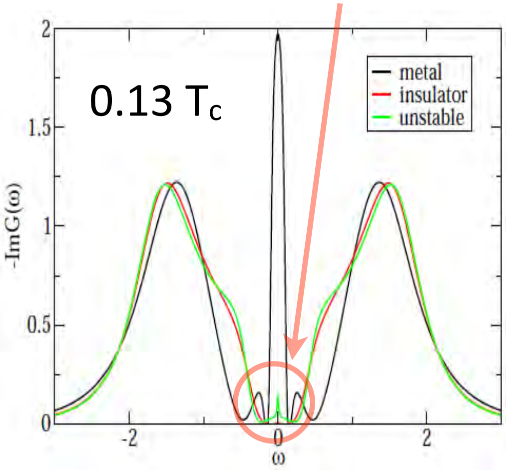
# The unstable solution at constant U

(Tsung-Han Lee, J. Vučičević, D. Tanasković,E. M., V. Dobrosavljević, in preparation)

Local DOS: clear quasiparticle peak even at lowest T



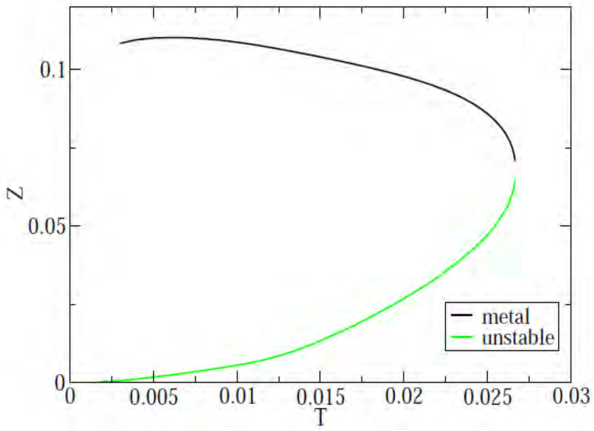


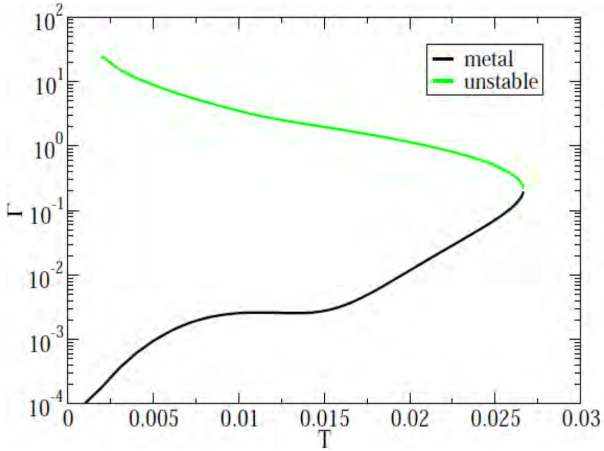


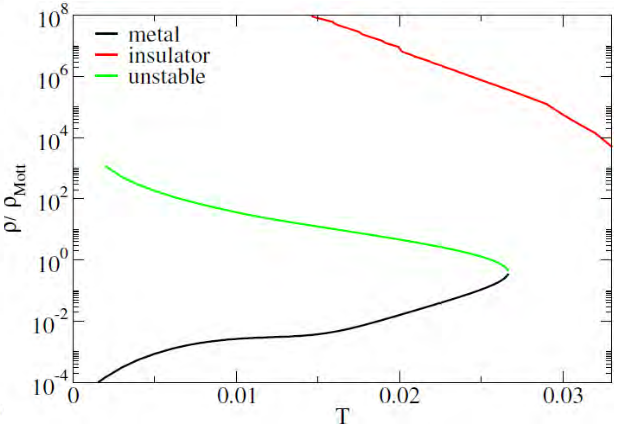
$$G_{loc}(\omega) = \frac{1}{\omega - \Delta(\omega) - \Sigma(\omega)}$$

$$\Sigma(\omega) \approx \left(1 - Z^{-1}\right)\omega - i\Gamma$$

- Resilient quasiparticles (Z<<1) (W. Xu *et al.*, PRL **111**, 03642 (2013))
- Scattering rate >> Mott limit: very bad conductor







# Strongly correlated disordered superconductors

High $T_c$ superconductors are known to be fairly insensitive to disorder
(H. Alloul *et al.*, RMP **81**, 45 (2009))

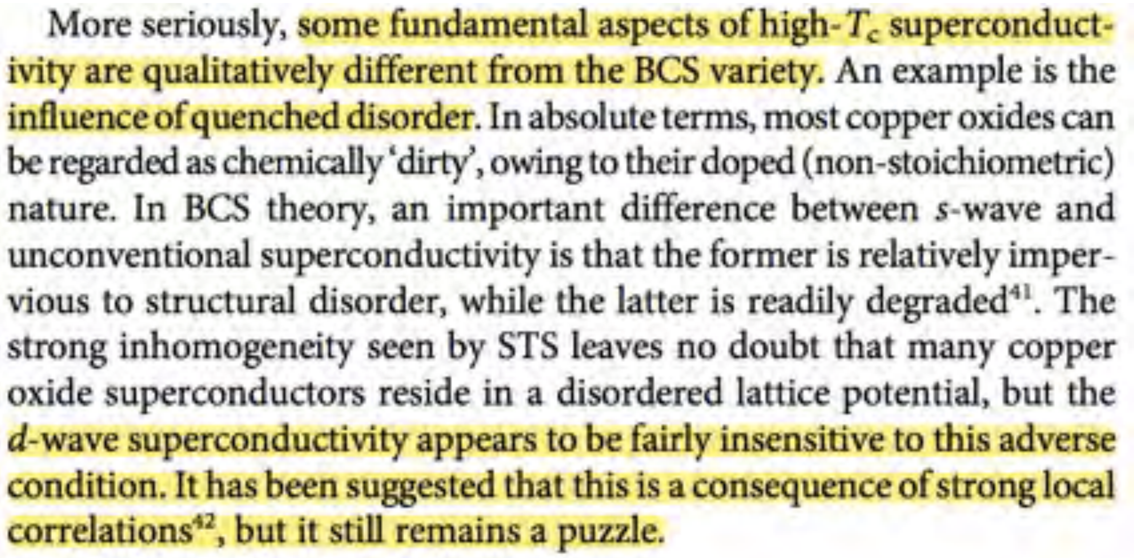
REVIEW

The 'consensus' document

doi:10.1038/nature14165

## From quantum matter to high-temperature superconductivity in copper oxides

B. Keimer[1], S. A. Kivelson[2], M. R. Norman[3], S. Uchida[4] & J. Zaanen[5]



More seriously, some fundamental aspects of high-$T_c$ superconductivity are qualitatively different from the BCS variety. An example is the influence of quenched disorder. In absolute terms, most copper oxides can be regarded as chemically 'dirty', owing to their doped (non-stoichiometric) nature. In BCS theory, an important difference between *s*-wave and unconventional superconductivity is that the former is relatively impervious to structural disorder, while the latter is readily degraded[41]. The strong inhomogeneity seen by STS leaves no doubt that many copper oxide superconductors reside in a disordered lattice potential, but the *d*-wave superconductivity appears to be fairly insensitive to this adverse condition. It has been suggested that this is a consequence of strong local correlations[42], but it still remains a puzzle.

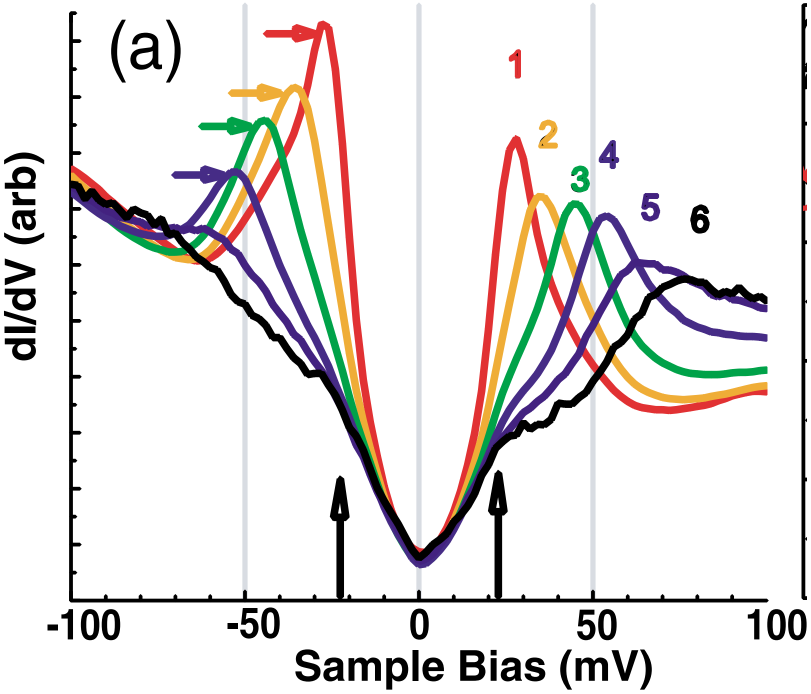
# Strongly correlated disordered superconductors

High $T_c$ superconductors are known to be fairly insensitive to disorder

(H. Alloul *et al.*, RMP **81**, 45 (2009))

## SC Density of States

K. McElroy et al., PRL **94**, 197005 (2005)



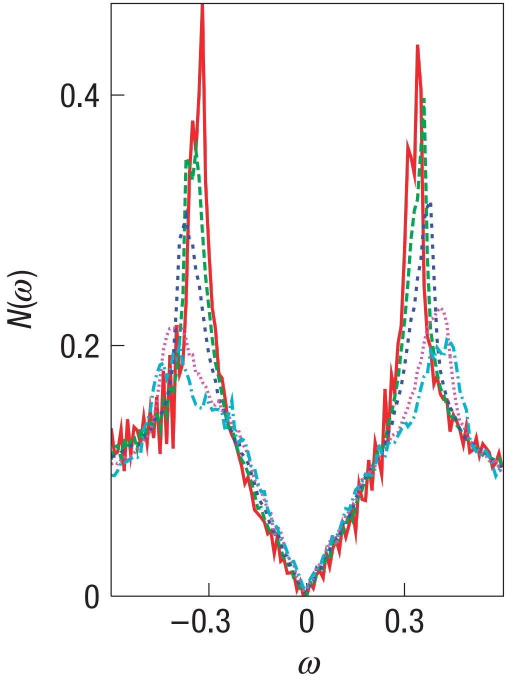
Low-energy (nodal) quasiparticles are weakly affected by disorder

# Strongly correlated disordered superconductors

$$H_{tJ} = -\sum_{i,j,\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + J \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{i,\sigma} \varepsilon_i c^{\dagger}_{i\sigma} c_{i\sigma}; \ \ (n_i \leq 1)$$

Projected RVB/slave bosons (Anderson 1987; Kotliar & Liu, Zhang *et al.*, Suzumura *et al.*, Yokoyama & Shiba 1988; Paramekanti *et al.* 2001)

$$\begin{aligned} c_{i\sigma} &= b_i f_{i\sigma} \approx r_i f_{i\sigma}; \ \text{(holons; spinons)} \\ \varepsilon_i &\to \varepsilon_i + \lambda_i; \text{(renormalized energies)} \\ \chi_{ij} &= \Sigma_\sigma \left\langle f^{\dagger}_{i\sigma} f_{j\sigma} \right\rangle; \ \text{(singlet bonds)} \\ \Delta_{ij} &= \left\langle f_{i\uparrow} f_{j\downarrow} - f_{i\downarrow} f_{j\uparrow} \right\rangle; \ \text{(SC order parameter)} \end{aligned}$$



Remarkable similarities with the experiments
(Garg *et al.*, Nature Phys. **4**, 762 (2008))

Note: no gauge flucuations, just saddle point!

# Can we gain some analytical insight into this result?

(S. Tang, E.M., V. Dobrosavljevic, PRB **91**, 020501(R) (2015)

Work to linear order in disorder $\varepsilon_i$

$$\begin{array}{rcl} r_i & = & r_0 + \delta r_i \approx r_0 + M^{r}_{ik}\varepsilon_k \\ \lambda_i & = & \lambda_0 + \delta\lambda_i \approx \lambda_0 + M^{\lambda}_{ik}\varepsilon_k \\ \chi_{ij} & = & \chi_0 + \delta\chi_{ij} \approx \chi_0 + M^{\chi}_{ij;k}\varepsilon_k \\ \Delta_{ij} & = & \Delta_0 + \delta\Delta_{ij} \approx \Delta_0 + M^{\Delta}_{ij;k}\varepsilon_k \end{array}$$

- $\delta\chi_{ij}$ fluctuations are negligible.
- Integrate out $\delta\lambda_i$ fluctuations and get:

$$\delta\Delta_{ij} = \chi^{pc}_{ij;k}\left(-2r_k\delta r_k\right) = \chi^{pc}_{ij;k}\delta n_k$$

where we used: $n_k = 1 - r_k^2 \Rightarrow -2r_k\delta r_k = \delta n_k$

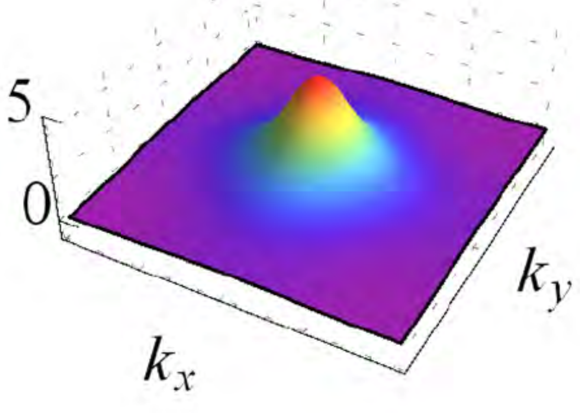
- But the charge-pair susceptibility is featureless and of order 1: $\chi^{pc} \sim \mathcal{O}\left(1\right)$
- Gap and charge fluctuations are suppressed by strong correlations for the same reason!

$\Rightarrow$ **‘Mottness’-induced healing**

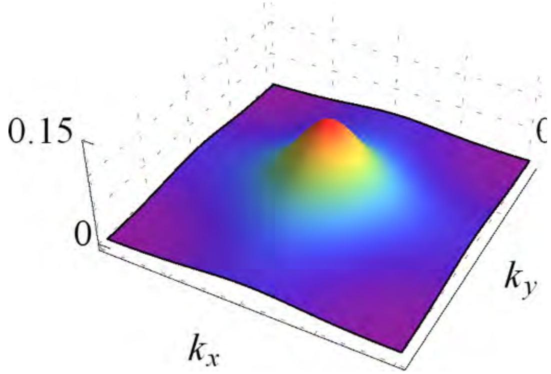
# Can we gain some analytical insight into this result?

(S. Tang, E.M., V. Dobrosavljevic, PRB **91**, 020501(R) (2015)

$$\frac{\delta\Delta\left(\mathbf{k}\right)|_{loc}}{\varepsilon\left(\mathbf{k}\right)}$$



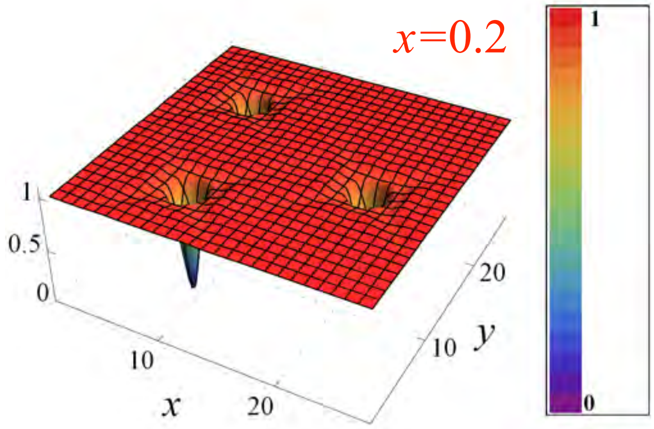
$$\frac{\delta n\left(\mathbf{k}\right)|_{loc}}{\varepsilon\left(\mathbf{k}\right)}$$



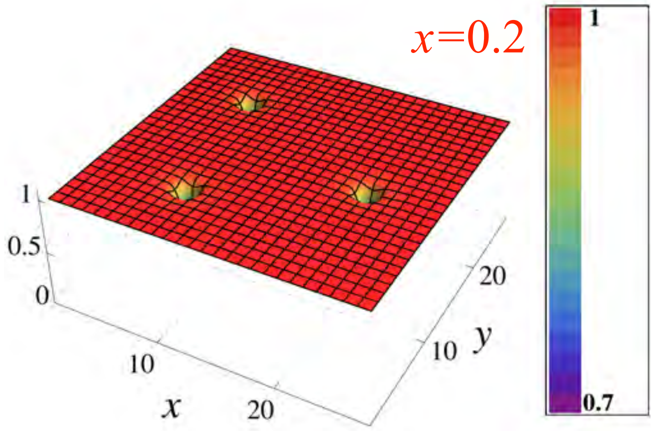
# Mottness-induced healing

(S. Tang, E.M., V. Dobrosavljevic, PRB **91**, 020501(R) (2015))

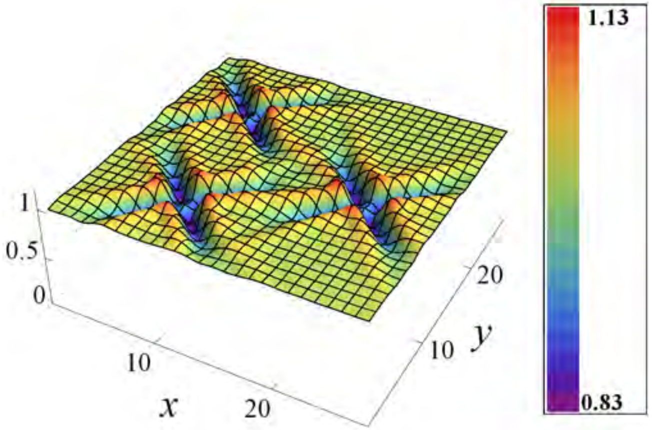
## Gap fluctuations for 3 impurities

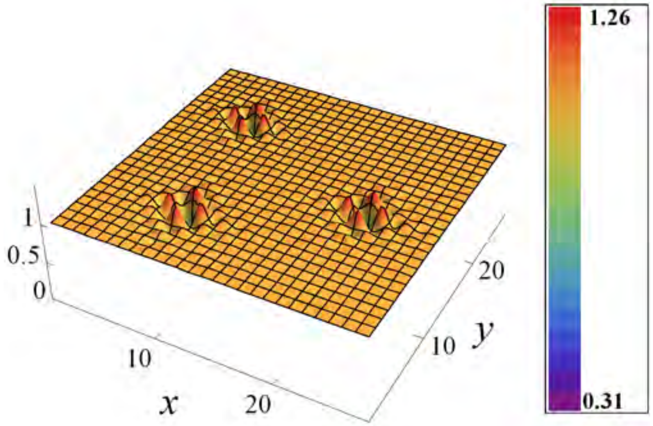


## Charge fluctuations for 3 impurities



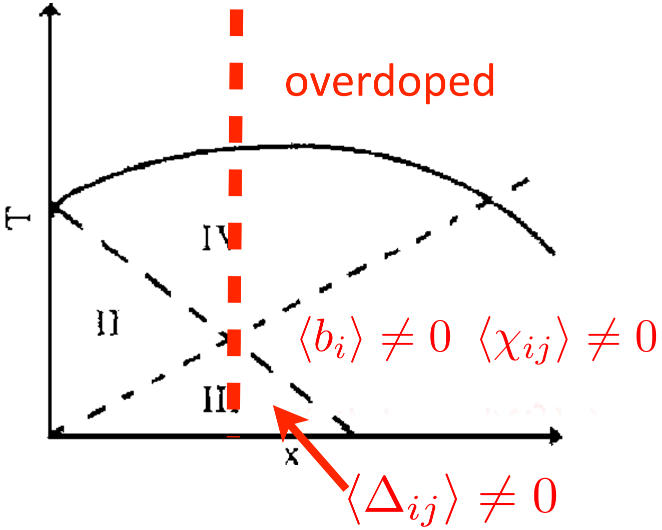
## When strong correlations are turned off:





# What about $T_c$ suppression by disorder?

(S. Tang, E.M., V. Dobrosavljevic, in preparation)



- In the overdoped region, $T_c$ is determined by the onset of pairing $\langle \Delta_{ij} \rangle \neq 0$
- At $T_c$, the other fields are well condensed: we take them at their T=0 value.
- We can extend Abrikosov-Gorkov theory of $T_c$ suppression by non-magnetic impurities to this strongly correlated superconductor.

$$\ln \frac{T_{c0}}{T_c} = \psi\left(\frac{1}{2} + \frac{\alpha}{2}\right) - \psi\left(\frac{1}{2}\right)$$

$$\alpha = \frac{1}{2\pi T_c \tau_{pb}}$$

The pair-breaking scattering rate: the pairs do not scatter off the bare impurity but the dressed disturbance created by the local (charge) rearrangements

$$\frac{1}{\tau_{pb}} \equiv \frac{nm^*}{2\pi} \int_0^{2\pi} \left| T_{renorm}(\theta) \right|^2 (1 - \cos 2\theta)\, d\theta$$

Angular dependence of the gap function

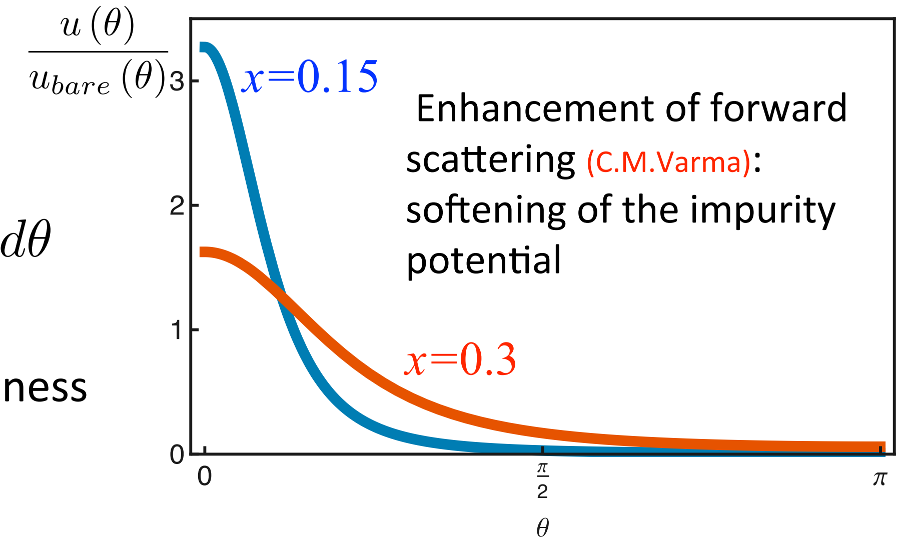
# What about $T_c$ suppression by disorder?

(S. Tang, E.M., V. Dobrosavljevic, in preparation)

$$T^{f}_{renorm}(\theta) = x u(\theta)$$

$$\frac{1}{\tau_{pb}} \equiv x^2 \frac{nm^*}{2\pi} \int_0^{2\pi} |u(\theta)|^2 (1-\cos 2\theta)\, d\theta$$

Pair-breaking scattering suppressed by $x^2$: Mottness

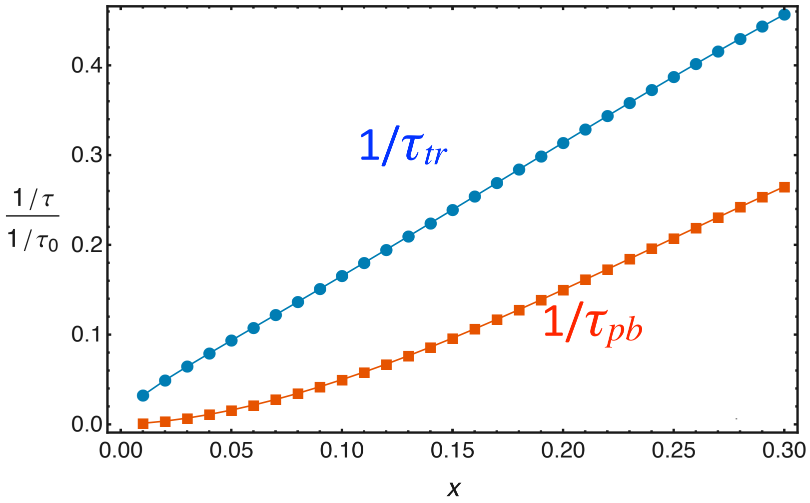


Enhancement of forward scattering (C.M.Varma): softening of the impurity potential

$$T^{e}_{renorm}(\theta) = u(\theta)$$

$$\frac{1}{\tau_{tr}} = x \frac{nm^*}{2\pi} \int_0^{2\pi} |u(\theta)|^2 (1-\cos \theta)\, d\theta$$

Normal state transport scattering suppressed by $x$



# Conclusions

- ★ Inhomogeneous Mott state:
  - ★ Behavior at the domain wall is important.
  - ★ Domain wall center: incoherent/bad metal or insulator.
- ★ Strongly correlated superconductor with non-magnetic impurities:
  - ★ Mottness-induced healing of gap fluctuations.
  - ★ Softening of the impurity potential, robustness of $T_c$ .

# Thanks!