Optical Conductivity in the Cuprates: Unparticles and Scale Invariance

Thanks to: NSF, EFRC (DOE)

Brandon Langley

Garrett Vanacore

Kridsangaphong Limtragool

Quantum critical behaviour in a high-T_c superconductor

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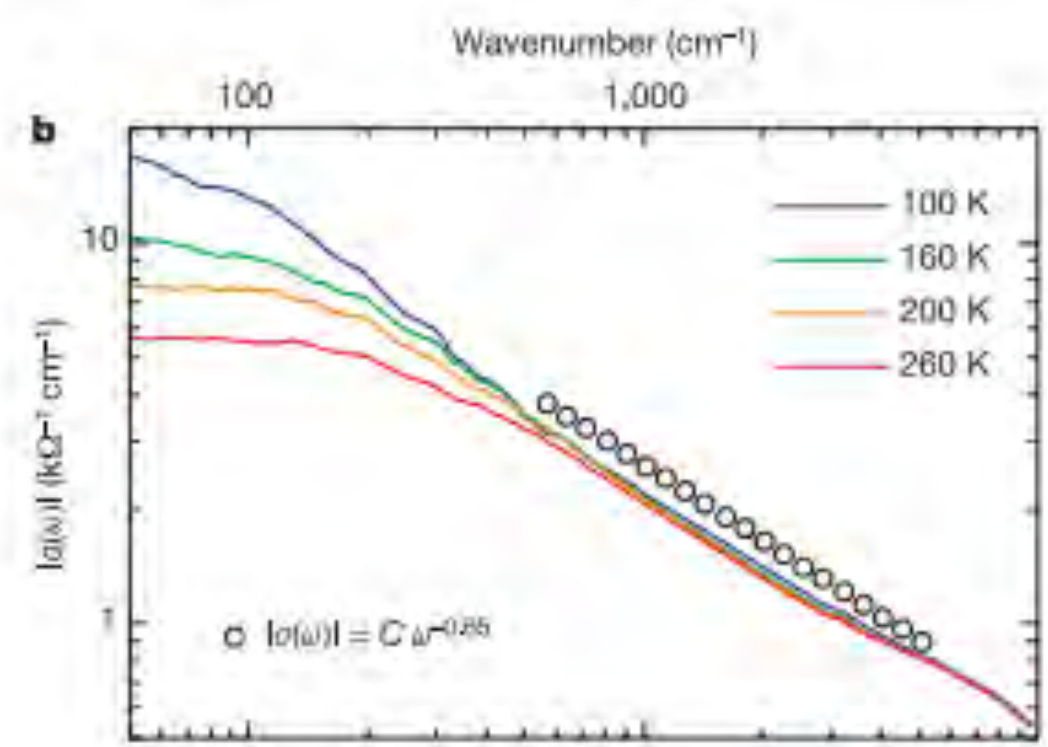
Drude conductivity

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$



$$\sigma(\omega) = C\omega^{-\frac{2}{3}}$$

criticality

scale invariance

power law correlations

scale-invariant propagators

$$\left(\frac{1}{p^2}\right)^{\alpha}$$

scale-invariant propagators

$$\left(\frac{1}{p^2}\right)^{\alpha}$$

Anderson: use Luttinger Liquid propagators

$$G^R \propto \frac{1}{(\omega - v_s k)^{\eta}}$$

compute conductivity without vertex corrections (PWA)

is flawed. In fact, in the Luttinger liquid such direct calculations are not to be trusted very firmly, since it is the nature of the Luttinger liquid that vertex corrections, if they must be included, will be singular; conventional transport theory is not applicable, and special methods such as the above are necessary.

$$\sigma(\omega) \propto \frac{1}{\omega}\int dx \int dt G^e(x,t)G^h(x,t)e^{i\omega t} \propto (i\omega)^{-1+2\eta}$$

problems

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1.) cuprates are not 1-dimensional

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2.) vertex corrections matter

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2.) vertex corrections matter

$$\left.\begin{aligned} &\sigma \propto G^2\Gamma^\mu\Gamma^{\mu\nu} \\ &[G] = L^{d+1-2d_U} \\ &[\Gamma^\mu] = L^{2d_U-d} \\ &[\Gamma^{\mu\nu}] = L^{2d_U-d+1} \end{aligned}\right\} \quad [\sigma] = L^{3-d}$$

problems

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2.) vertex corrections matter

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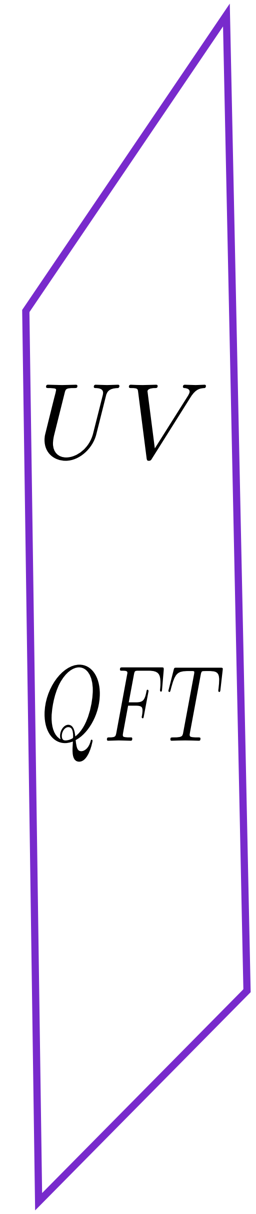
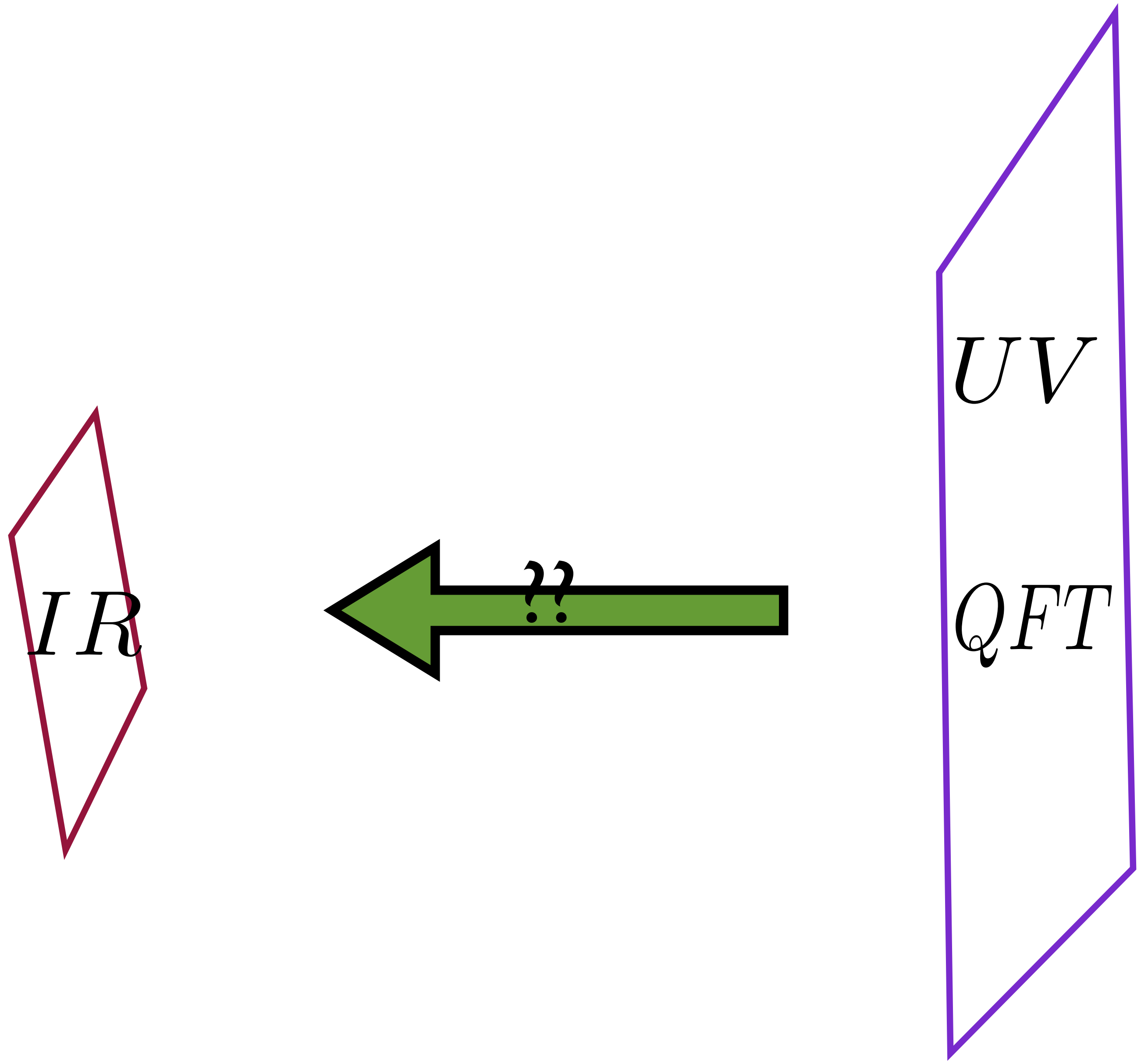
independent of d_U

$\boxed{\text{power law?}}$

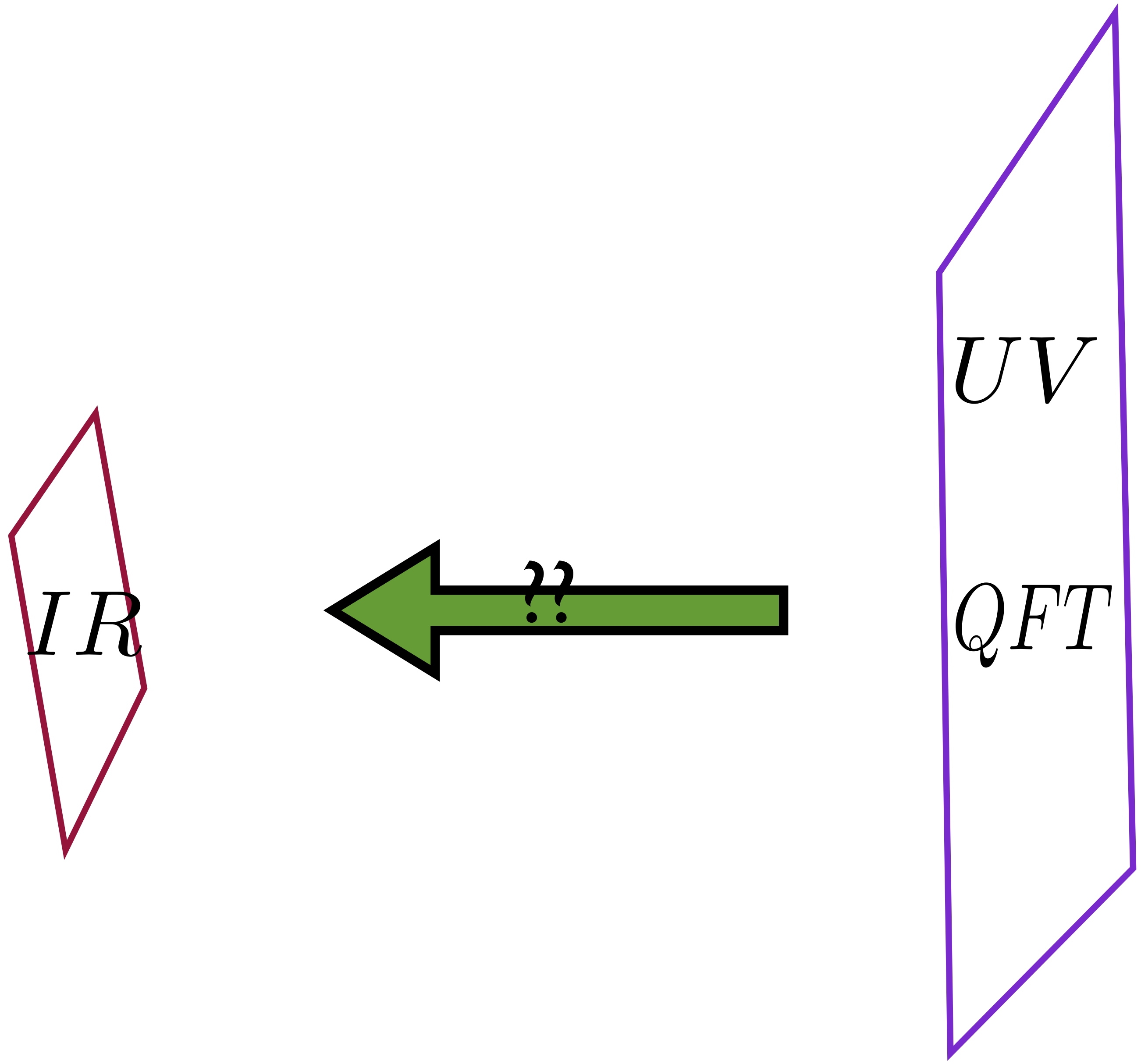
power law?

Could string theory be the answer?


UV
QFT

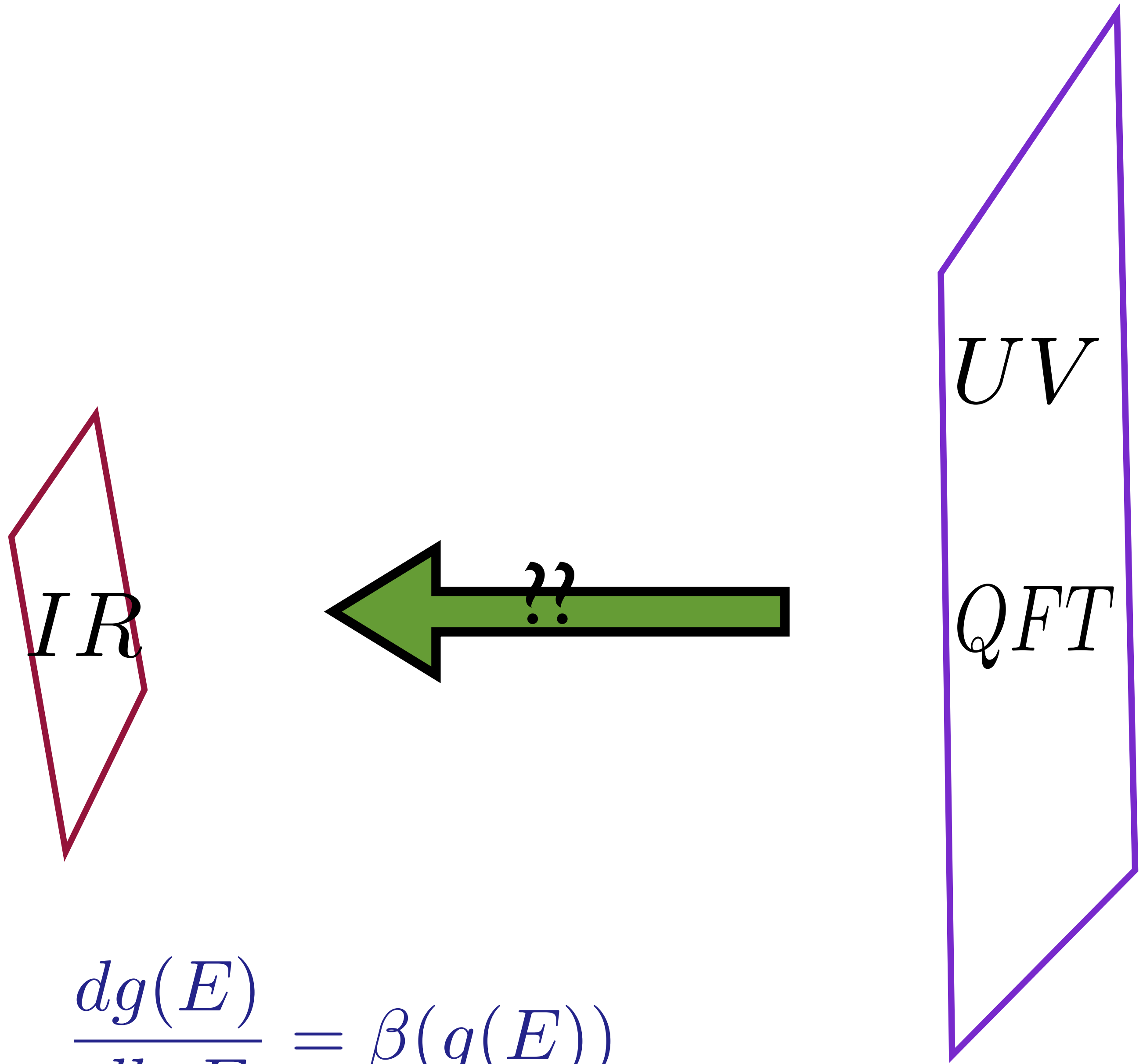


IR
??
UV
QFT



coupling constant

$g = 1/\text{ego}$



coupling constant

$g = 1/\text{ego}$

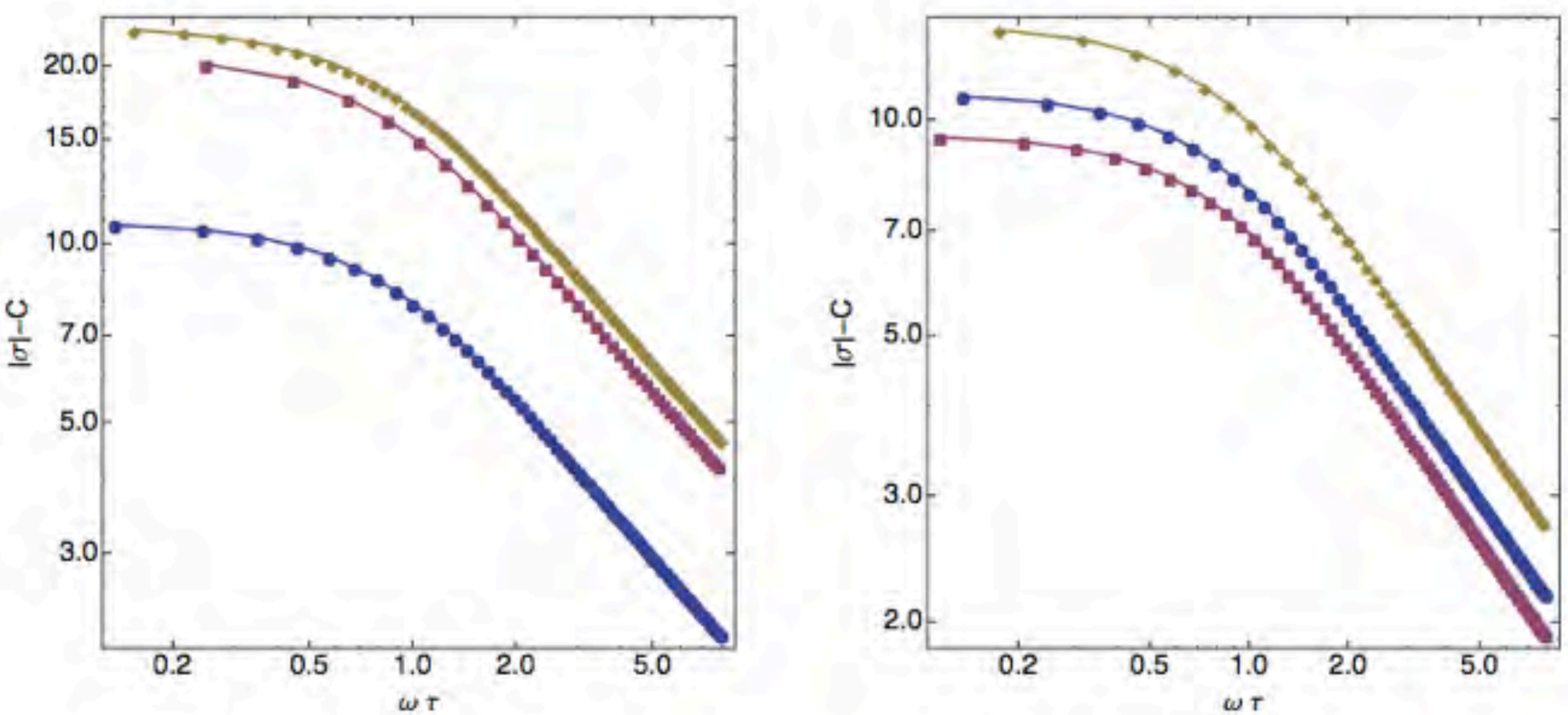
$$\frac{dg(E)}{dlnE} = \beta(g(E))$$

locality in energy

optical conductivity from a gravitational lattice

G. Horowitz *et al.*, Journal of High Energy Physics, 2012

optical conductivity from a gravitational lattice



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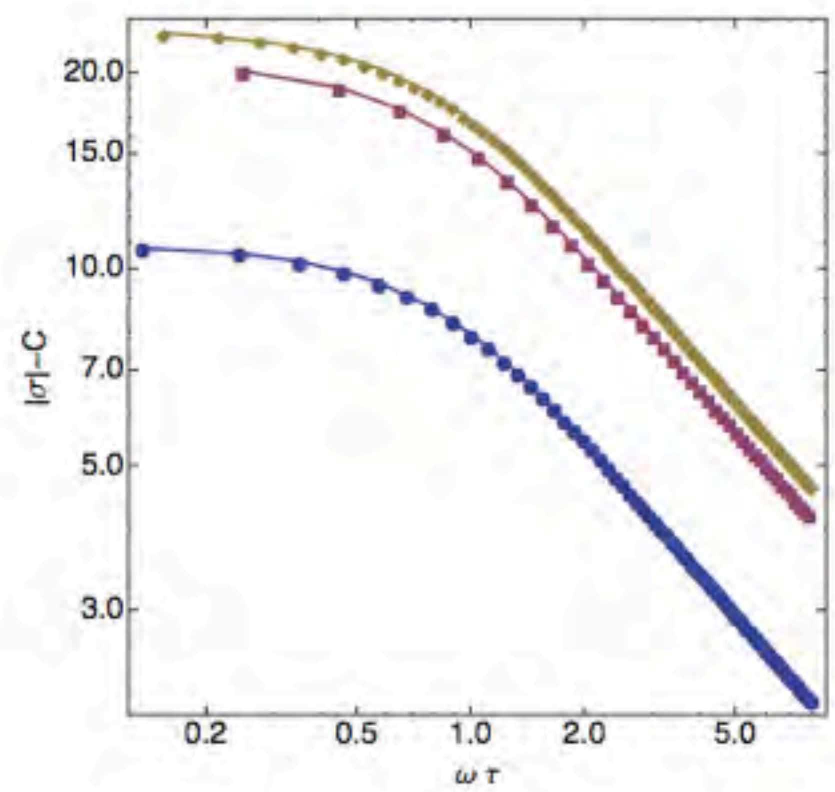

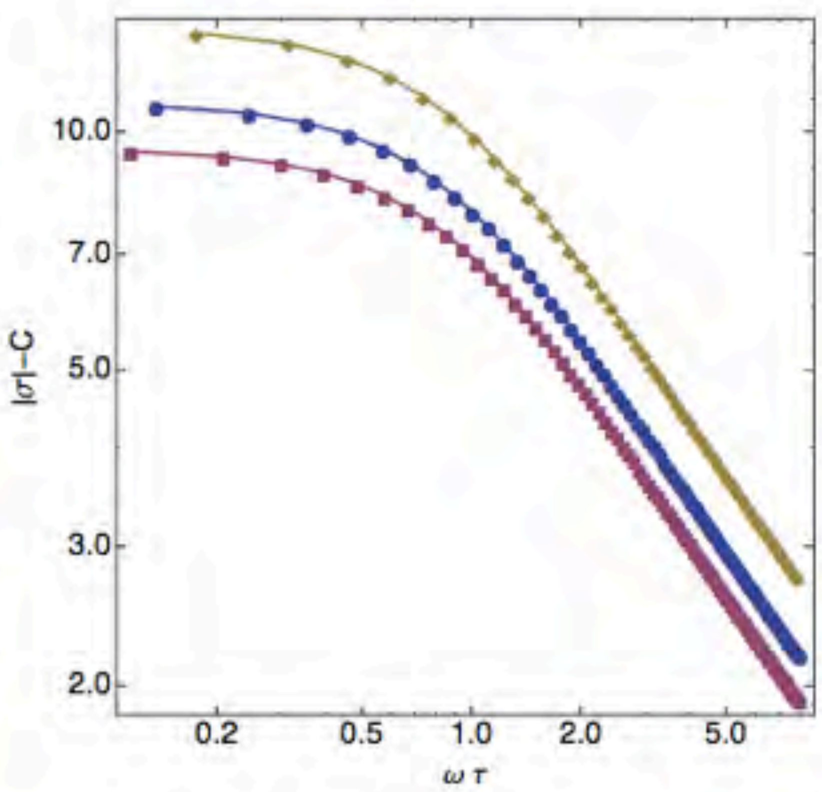

log-log plots for various parameters

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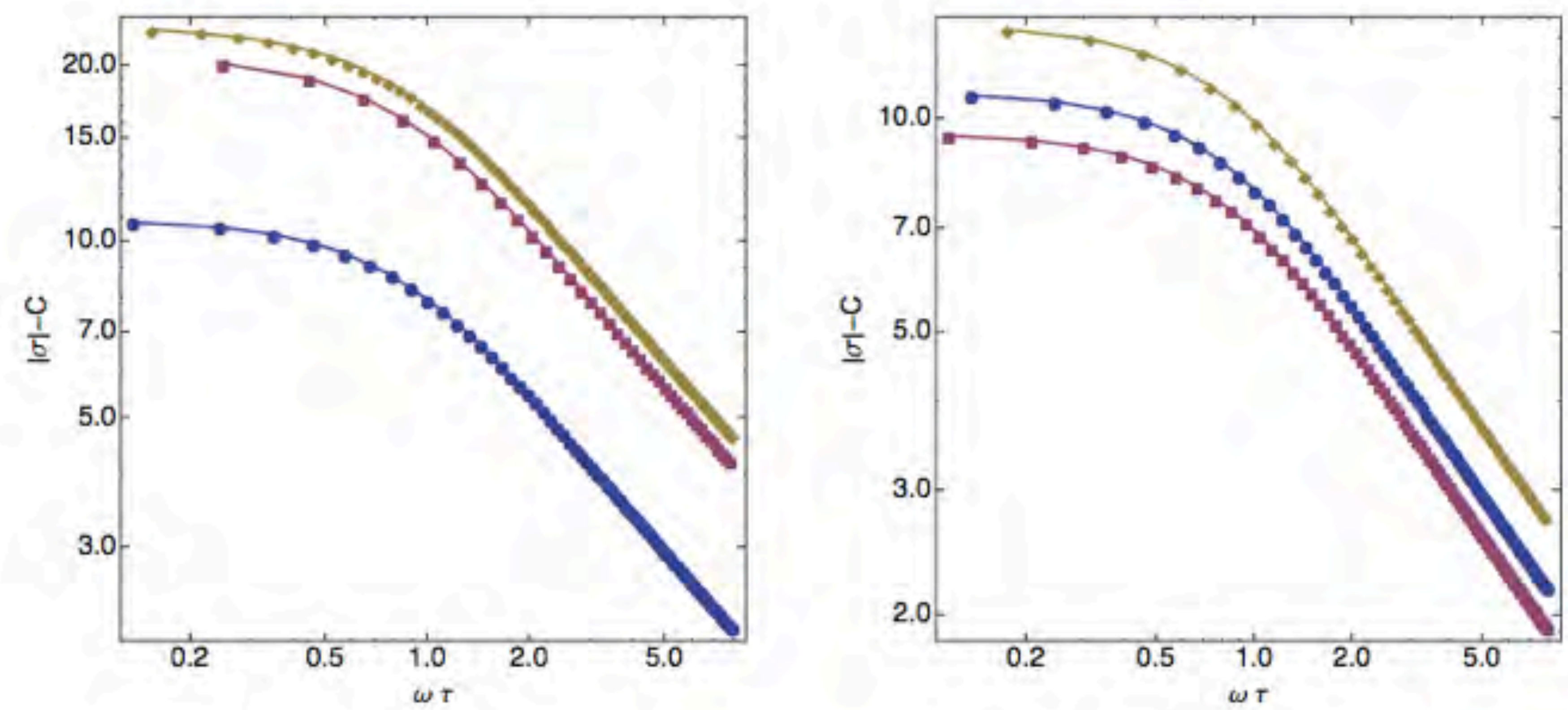

log-log plots for various parameters

$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

for $0.2 \lesssim \omega\tau \lesssim 0.8$

G. Horowitz *et al*., Journal of High Energy Physics, 2012

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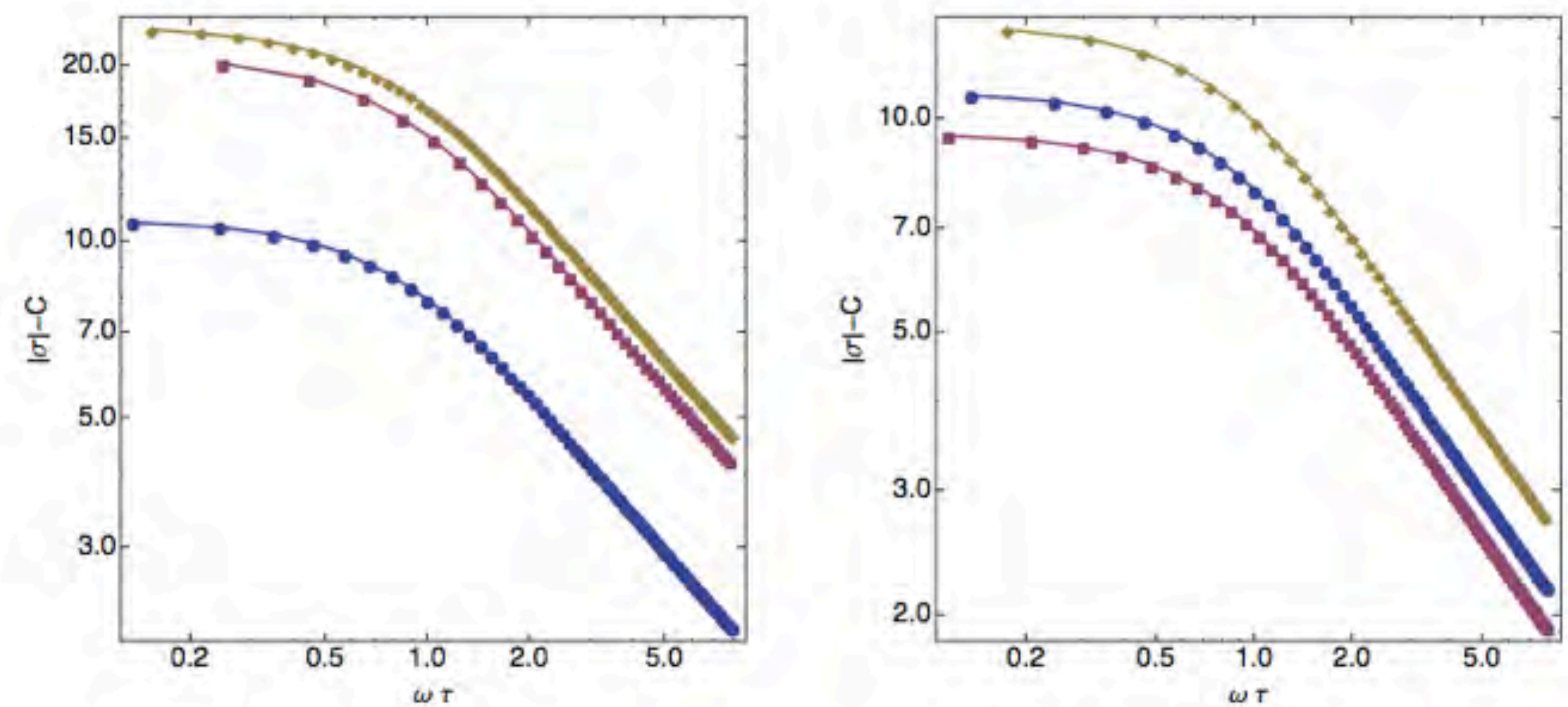

log-log plots for various parameters

$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

for $0.2 \lesssim \omega\tau \lesssim 0.8$

a remarkable claim!
replicates features of the strange metal? how?

G. Horowitz *et al.*, Journal of High Energy Physics, 2012 new equation!

Einstein-Maxwell equations $+$ non-uniform charge density $= B\omega^{-2/3}$

not so fast!

Donos and Gauntlett
(gravitational crystal)

Drude conductivity

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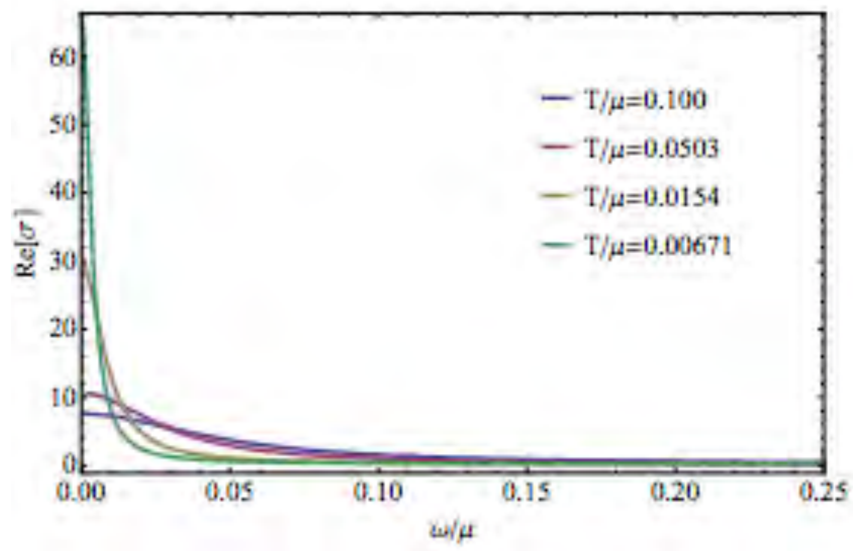

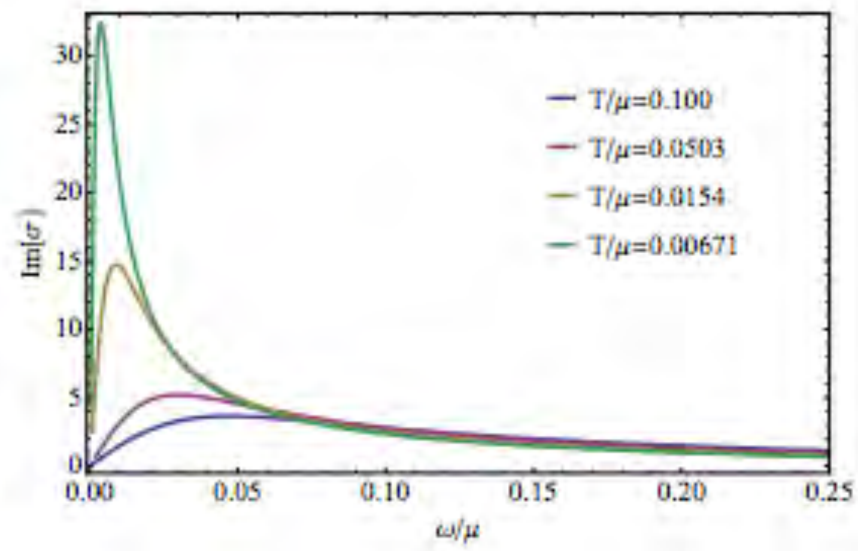

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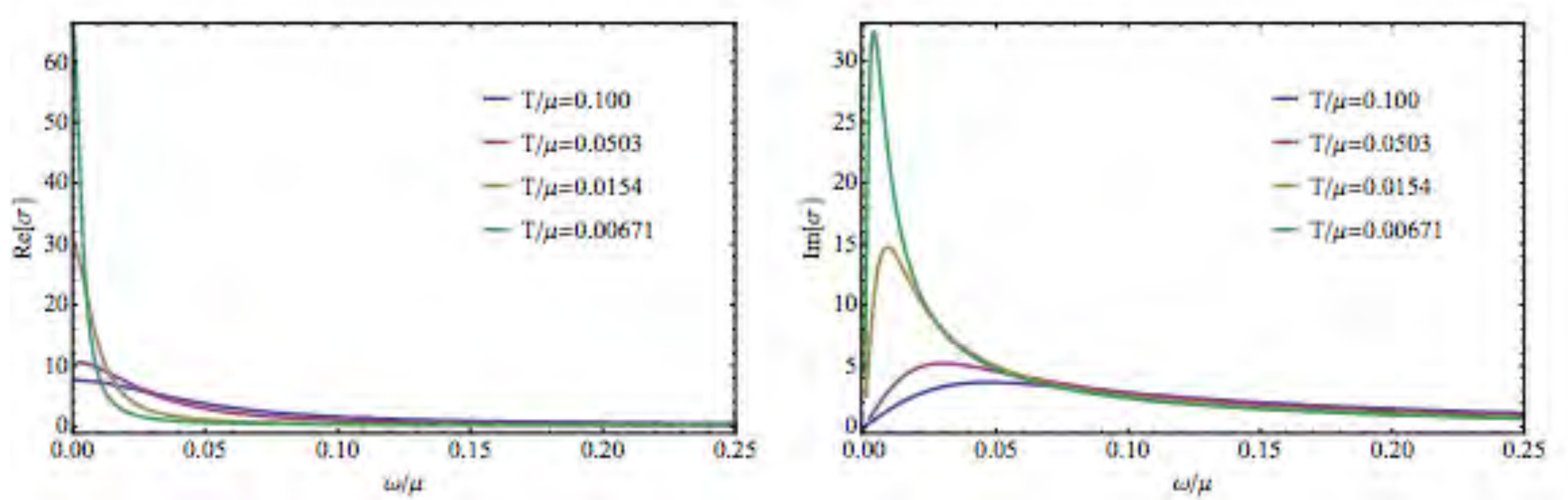

no power law!!

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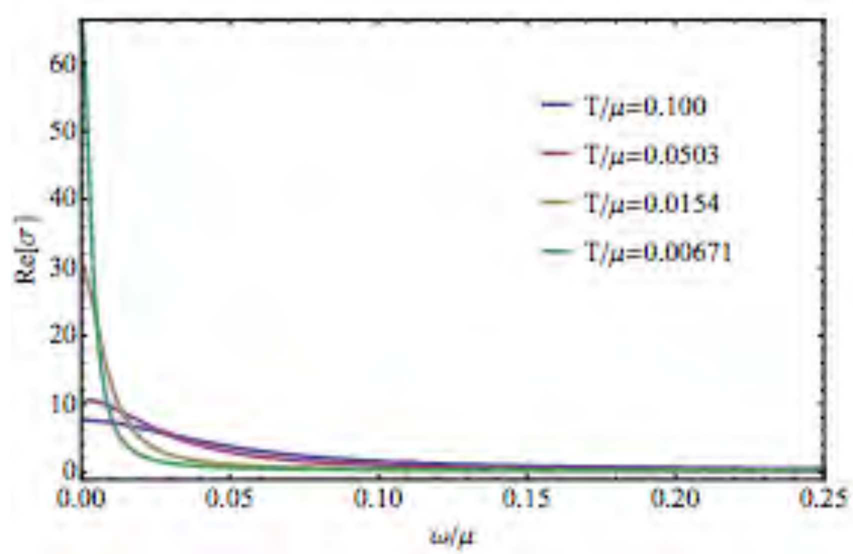

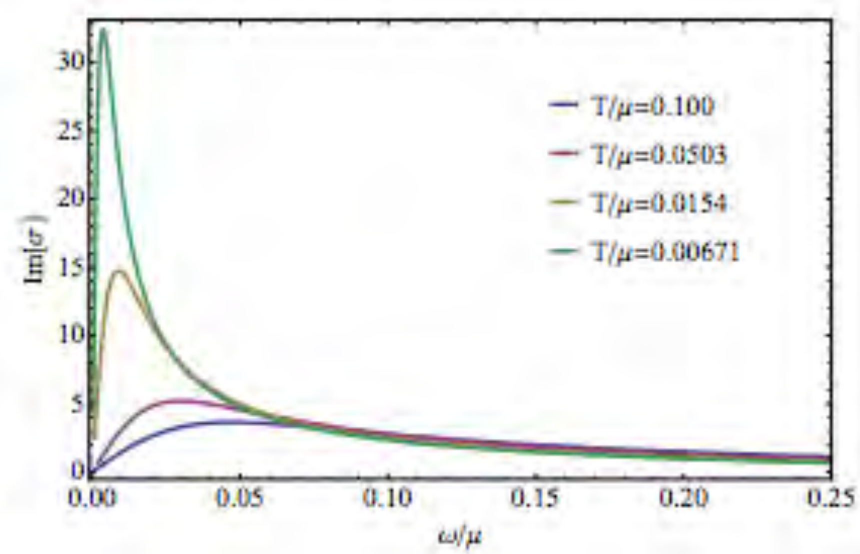

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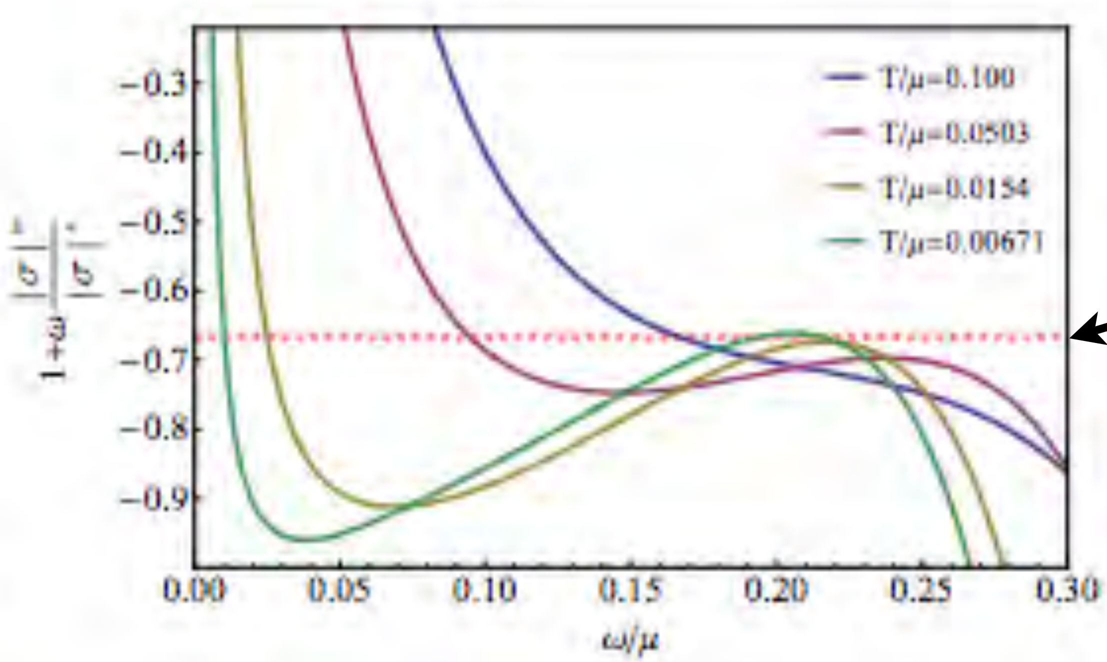


$B\omega^{-2/3}$

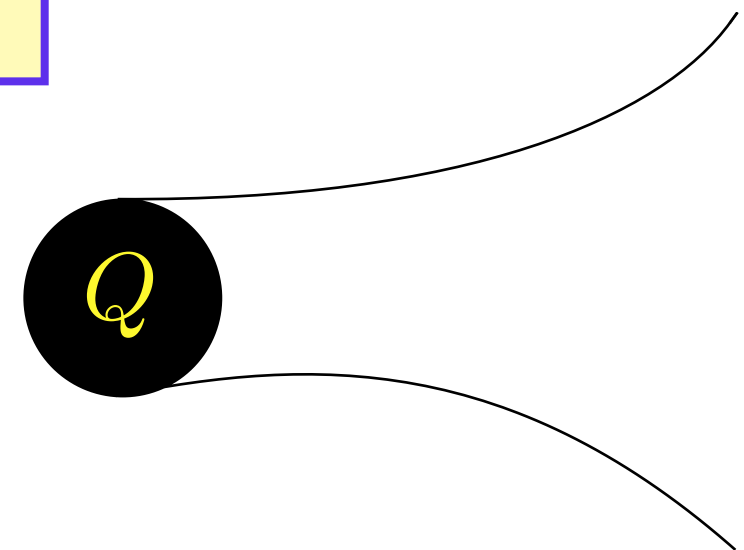
who is correct?

who is correct?

let's redo the calculation

conductivity
within AdS

$$(g_{\rm ab}, V(\Phi), A_t)$$
$$(\text{metric}\,,\text{potential}\,,\text{gaugefield})$$



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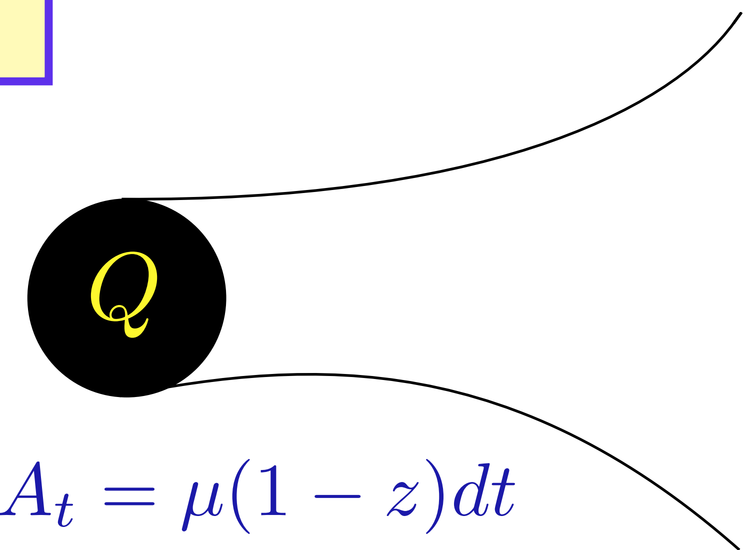

$$A_t = \mu(1-z)dt$$

$$\rho = \lim_{z\to 0}\sqrt{g}F^{tz}$$

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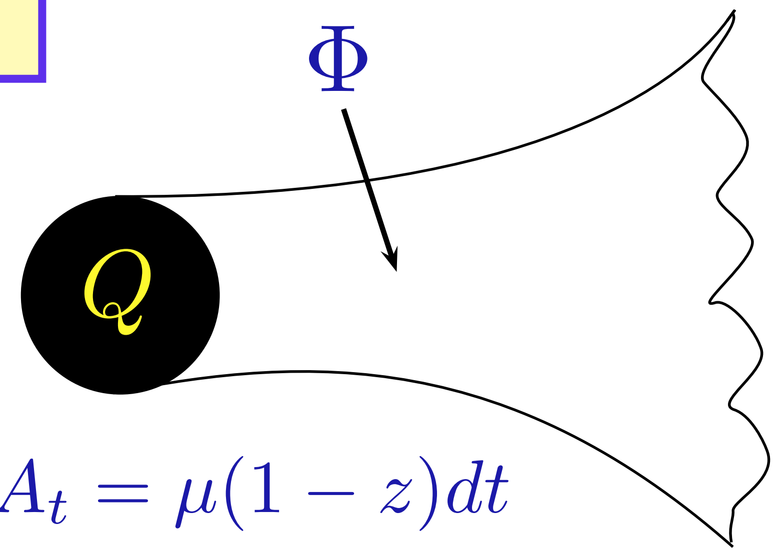

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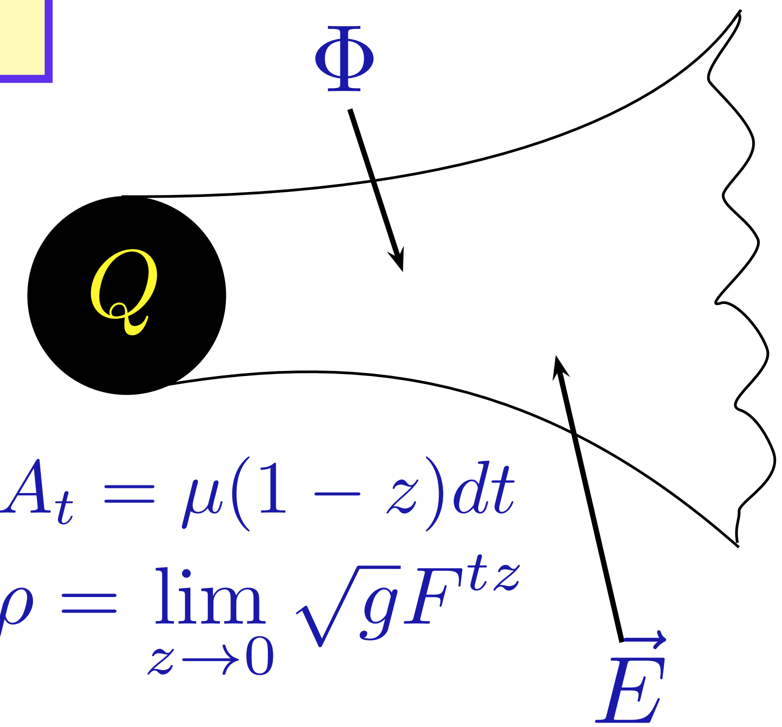
perturb with electric field



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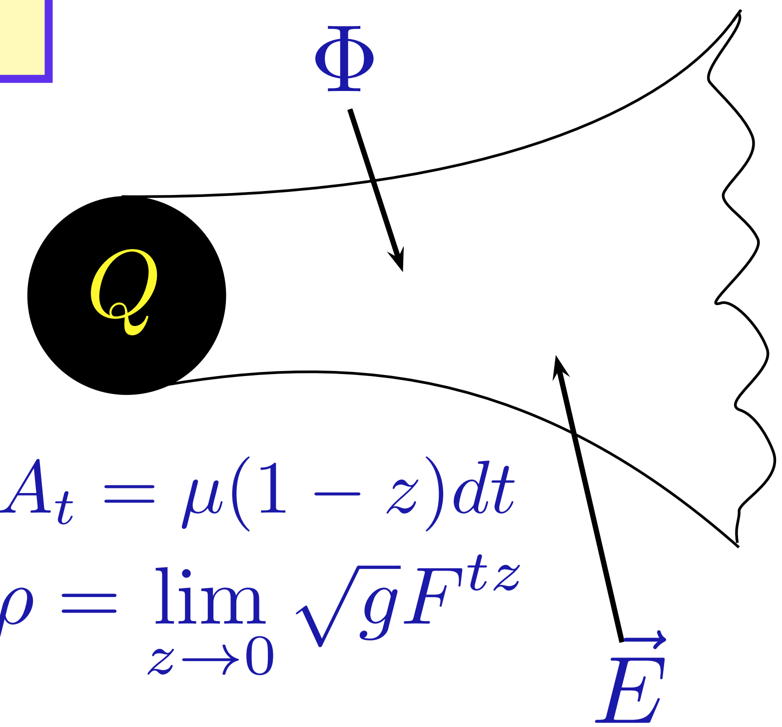

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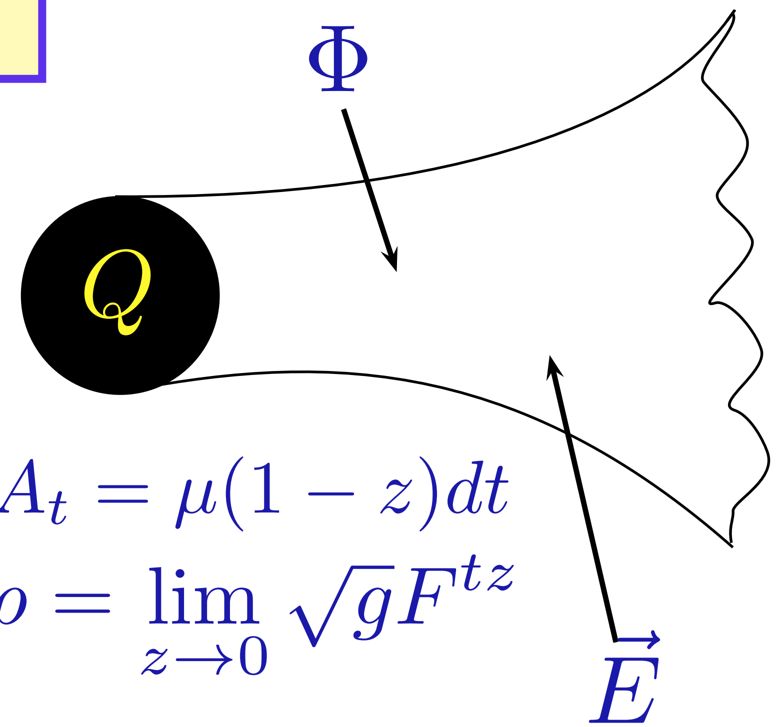

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$$\delta A_x = \frac{E}{i\omega} + J_x(x,\omega)z + O(z^2)$$

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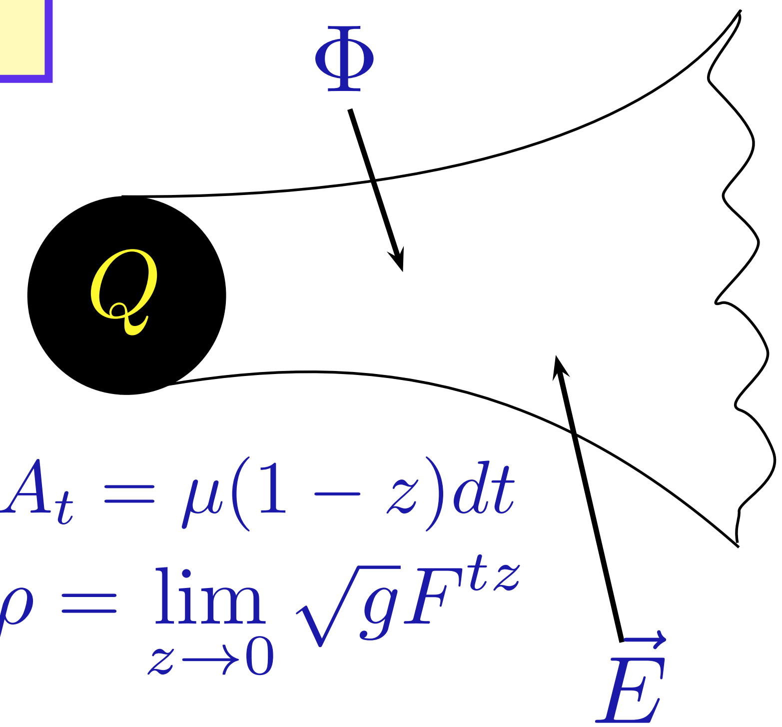

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solve equations of motion with gauge invariance

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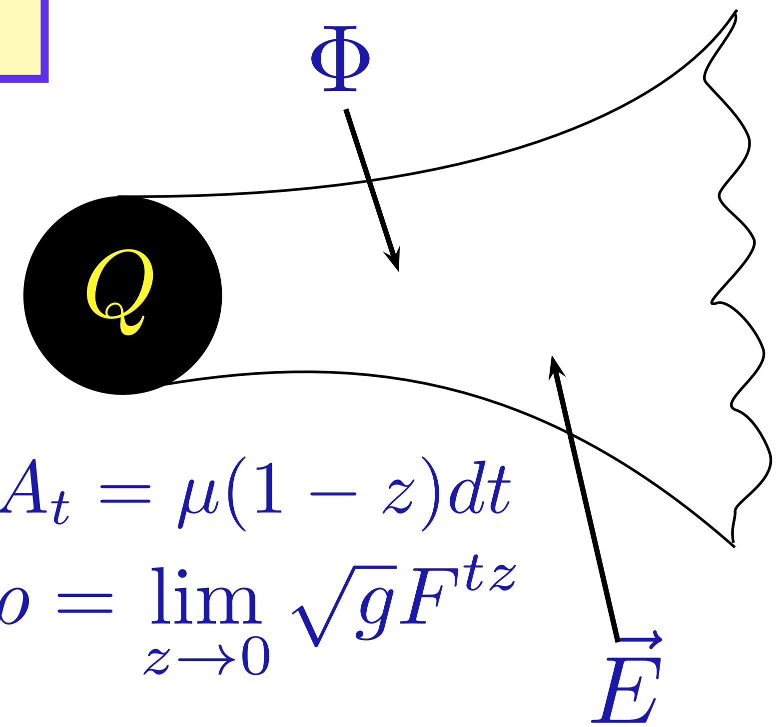

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solve equations of motion with gauge invariance

$$\sigma = J_x(x,\omega)/E$$

HST vs. DG

HST vs. DG

Horowitz, Santos, Tong (HST)

$$V(\Phi) = -\Phi^2/L^2$$

$$\Phi = z\Phi^{(1)} + z^2\Phi^{(2)} + \cdots \, ,$$
$$\Phi^{(1)}(x) = A_0 \cos(kx)$$

inhomogeneous in x

$$m^2 = -2/L^2$$

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DG

$$V\left(|\Phi|^2\right)$$

$$\Phi(z,x) = \phi(z)e^{ikx}$$

no inhomogeneity in x

$$m^2 = -3/(2L^2)$$

HST vs. DG

Horowitz, Santos, Tong (HST)

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de Donder gauge

radial gauge

Our Model

$$\mathcal{L}_\Phi = (\nabla\Phi_1)^2 + (\nabla\Phi_2)^2 + 2V(\Phi_1) + 2V(\Phi_2)$$

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$$\Phi_2 = \qquad z\Phi_2^{(1)} + z^2\Phi_2^{(2)} + \cdots\,, \quad \Phi_2^{(1)}(x) = A_0\cos\left(kx + \frac{\theta}{2}\right).$$

Our Model

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$$\theta = 0$$

$$\boxed{\text{HST}}$$

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$\theta = 0$ $\qquad\qquad$ $\theta = \frac{\pi}{2}$

HST $\qquad\qquad$ DG

Einstein-De Turck EOM

$$G^H_{ab} = G_{ab} - \nabla_{(a}\xi_{b)},$$

$$\xi^a = g^{cd}\left(\Gamma^a_{cd}(g) - \Gamma^a_{cd}(\bar{g})\right).$$

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reference metric

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metric ansatz

reference metric

$$ds^2 = \frac{L^2}{z^2}\left[-(1-z)P(z)Q_{tt}dt^2 + \frac{Q_{zz}dz^2}{(1-z)P(z)} + Q_{xx}(dx + z^2 Q_{zx}dz)^2 + Q_{yy}dy^2\right],$$

$$P(z) = 1 + z + z^2 - \frac{\mu_1^2}{2}z^3.$$

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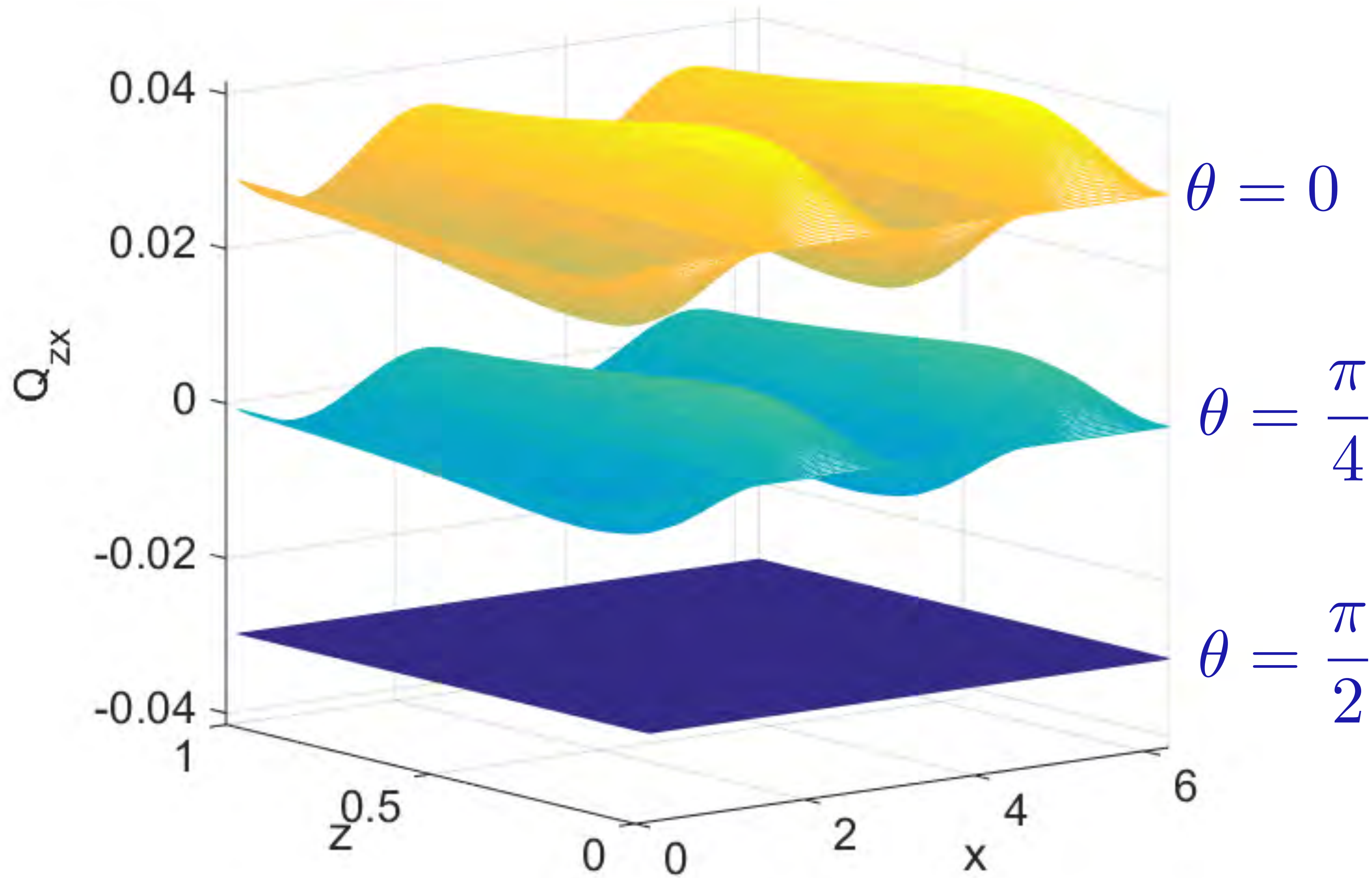
RN-AdS when

$$Q_{tt} = Q_{zz} = Q_{yy} = 1 \qquad \Phi = 0 \qquad a_t = \mu_1 = \mu$$

$A_0 = 0.75,\ k = 1,\ \mu = 1.4,\ T/\mu = 0.115$



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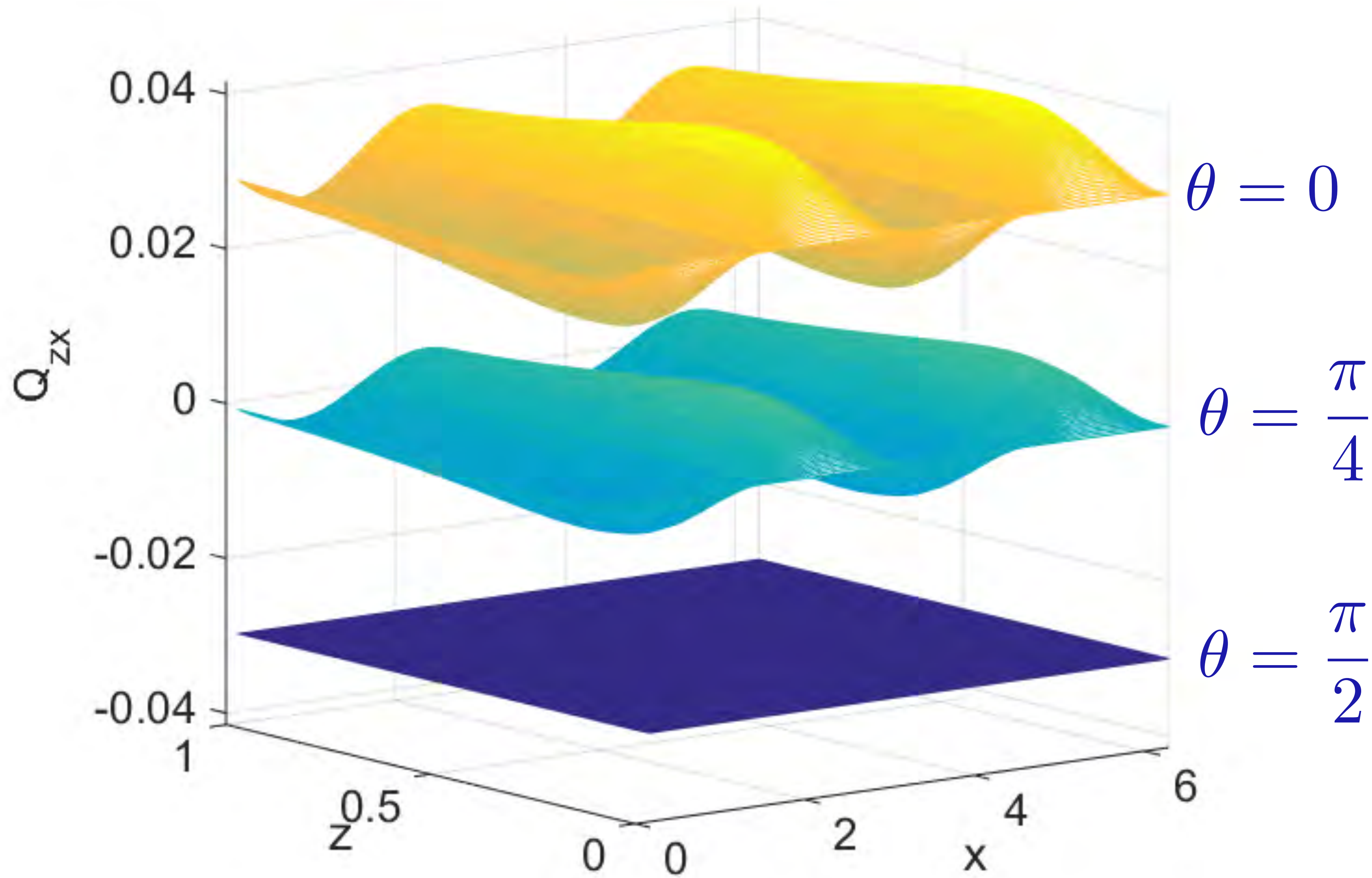
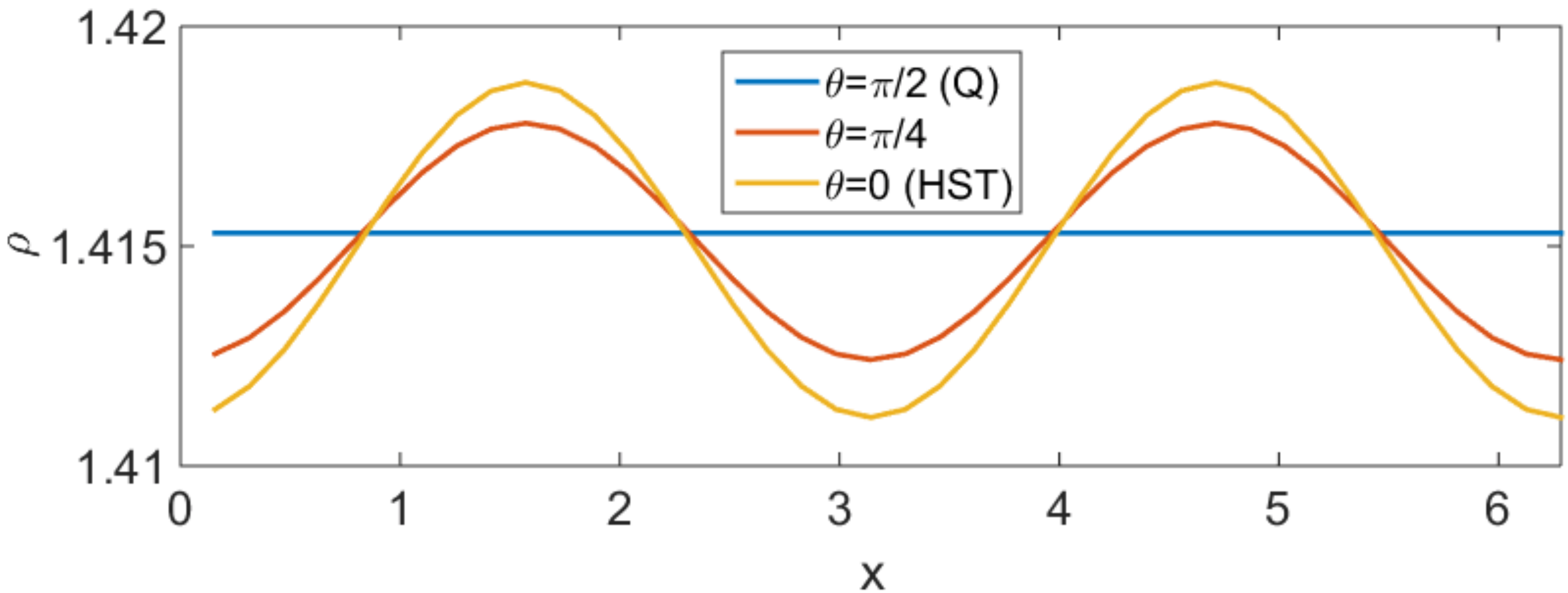


translational invariance is broken in metric in multiples of 2k

charge density

$$\rho = \lim_{z \to 0} \sqrt{-g} F^{tz}$$



perturb with electric field

$$g_{\mathrm{ab}} = \bar{g}_{\mathrm{ab}} + h_{\mathrm{ab}}$$
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gauge invariance

$$\delta g_{ab} + \mathcal{L}_\zeta \overline{g}_{ab} = 0,$$
$$\delta A_a + \mathcal{L}_\zeta \overline{A}_a + \nabla_a \Lambda = e^{-i\omega t} \mu_x^J,$$
$$\delta\Phi + \mathcal{L}_\zeta \overline{\Phi} = 0,$$

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solve equations without mistakes!!

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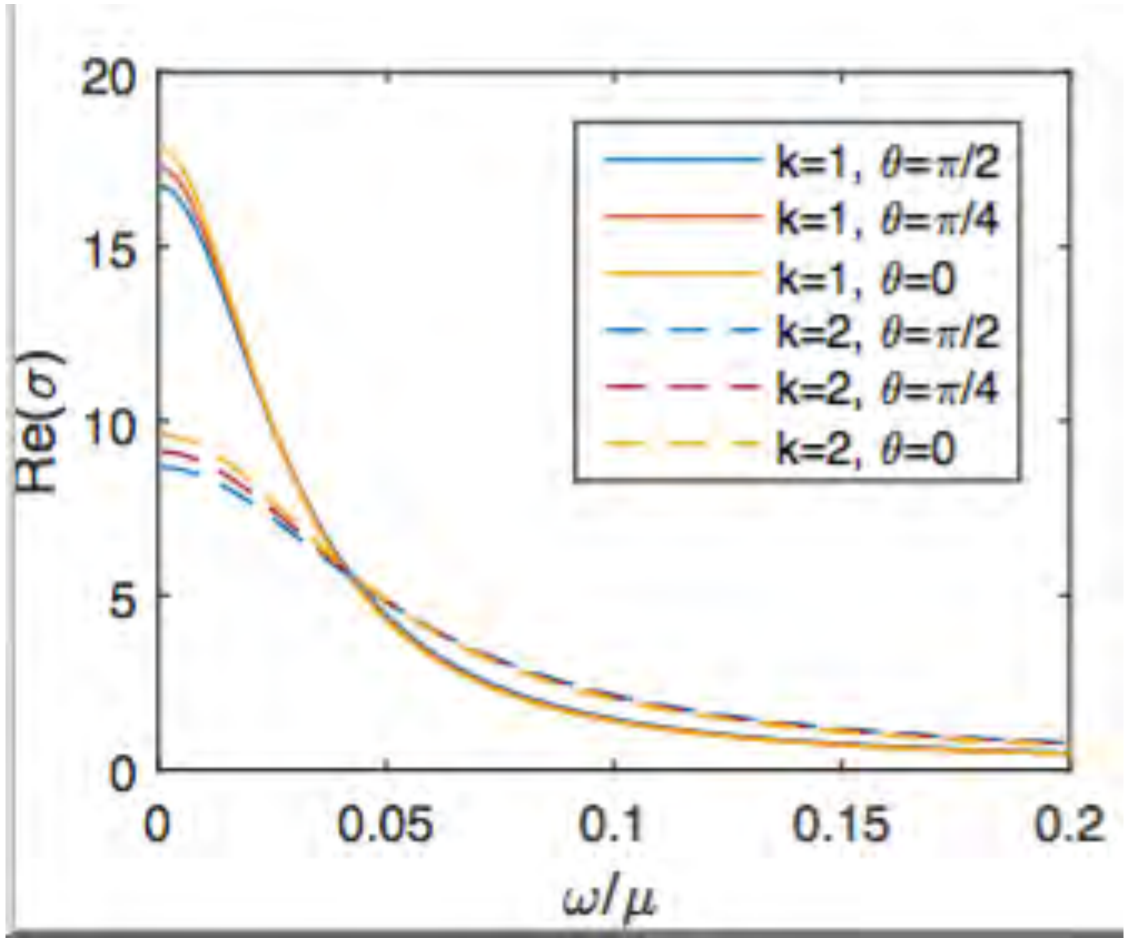
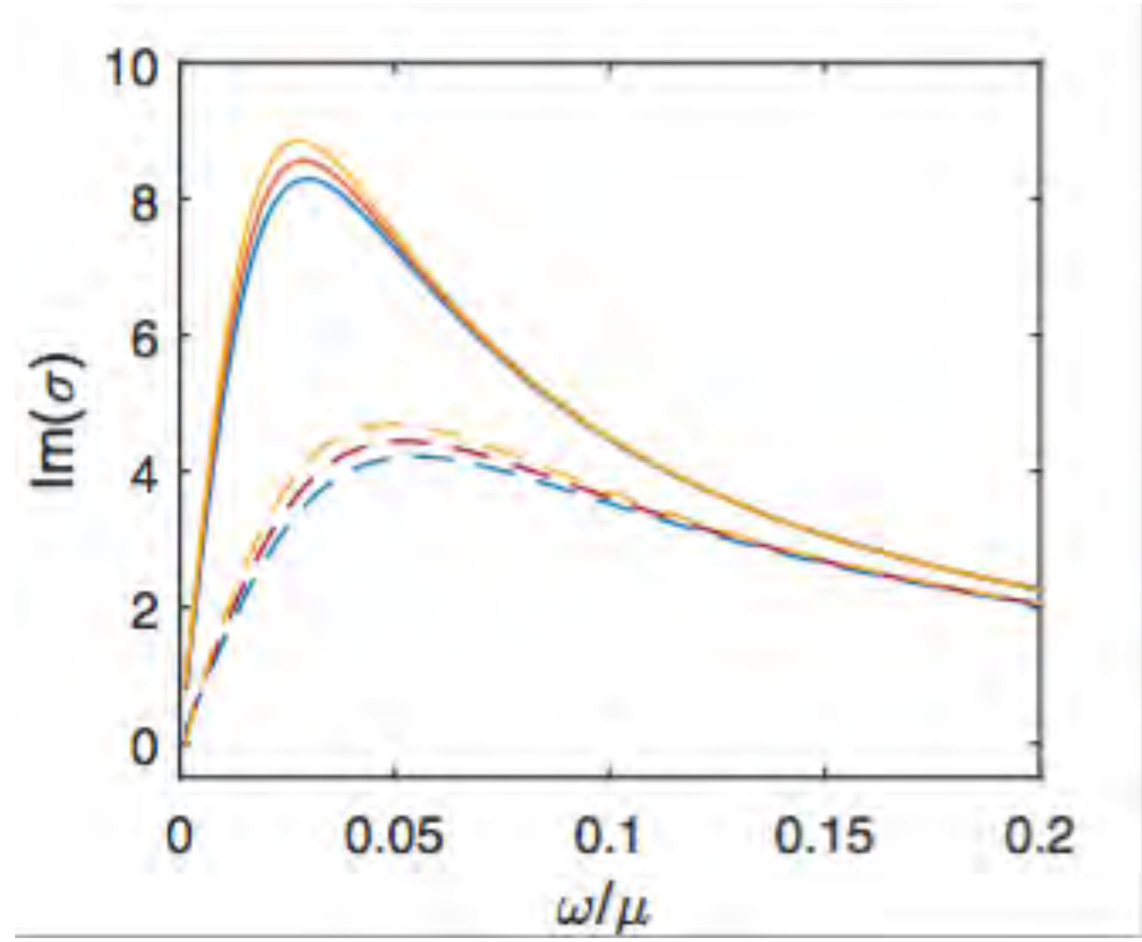
Brandon Langley

solve equations without mistakes!!

conductivity





- high-frequency behavior is identical
- low-frequency RN has $\mathrm{Re}(\sigma) \sim \delta(\omega),\ \mathrm{Im}(\sigma) \sim 1/\omega$
- low-frequency lattice has Drude form

is there a power law?

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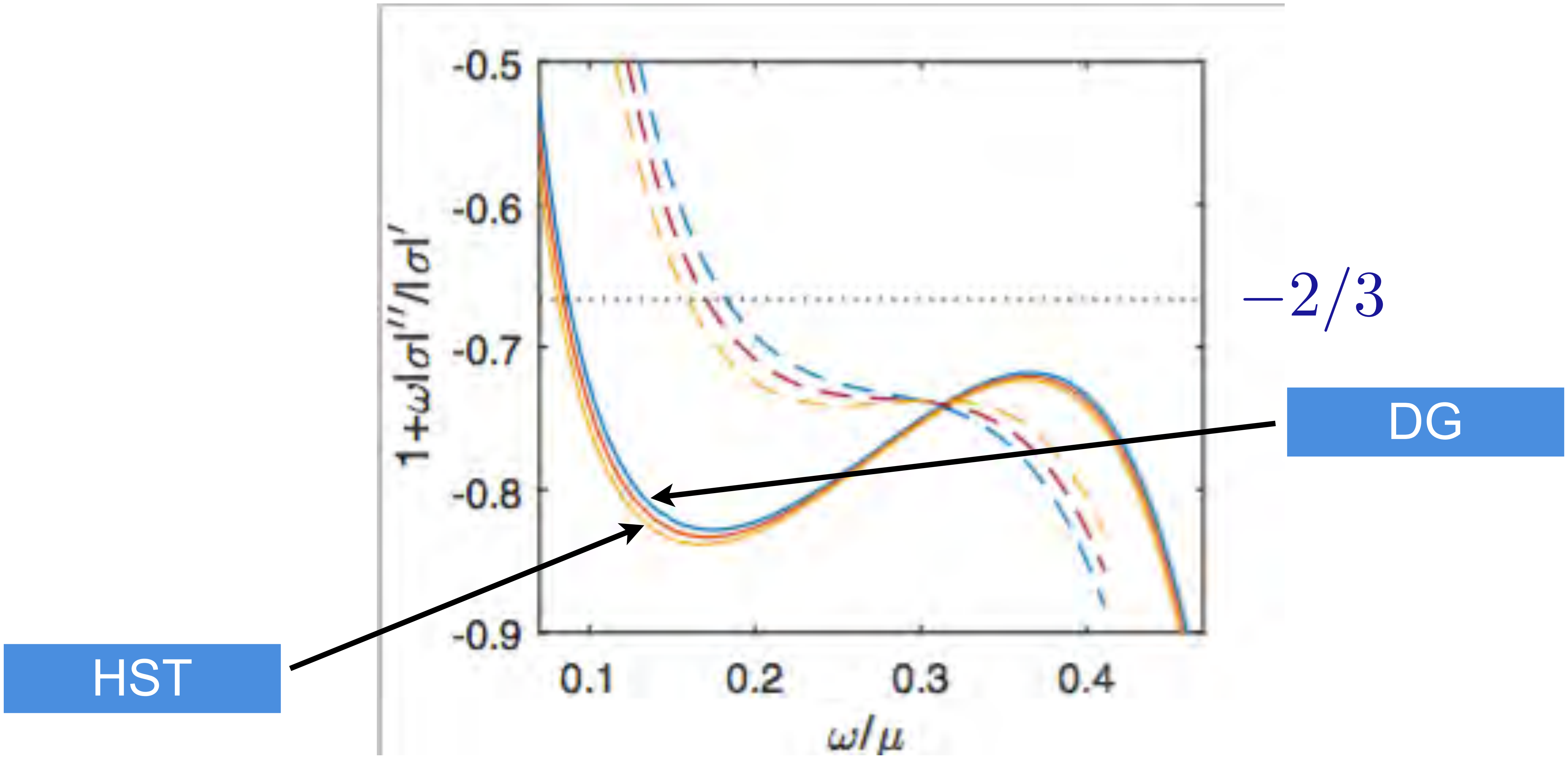
Results



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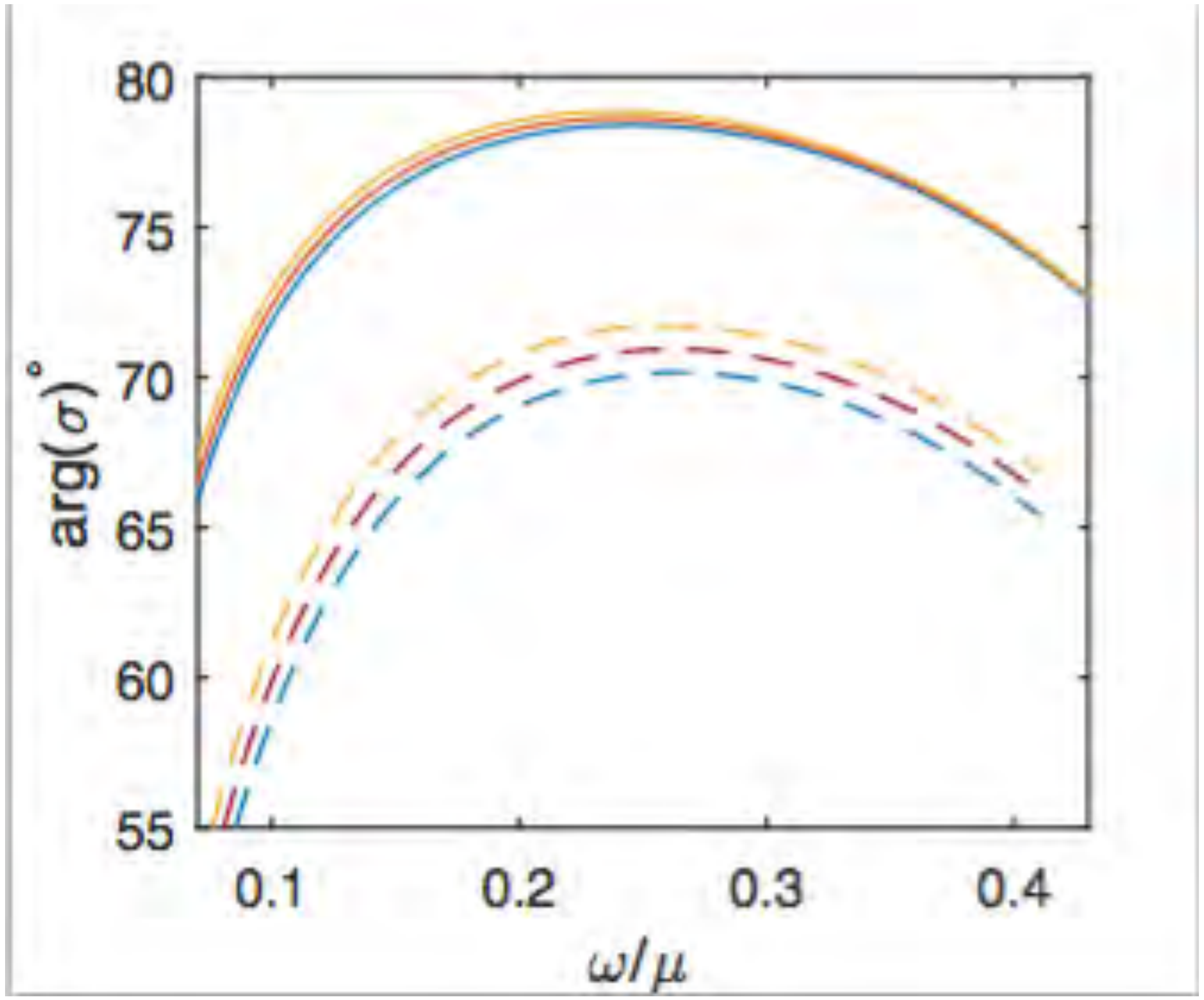
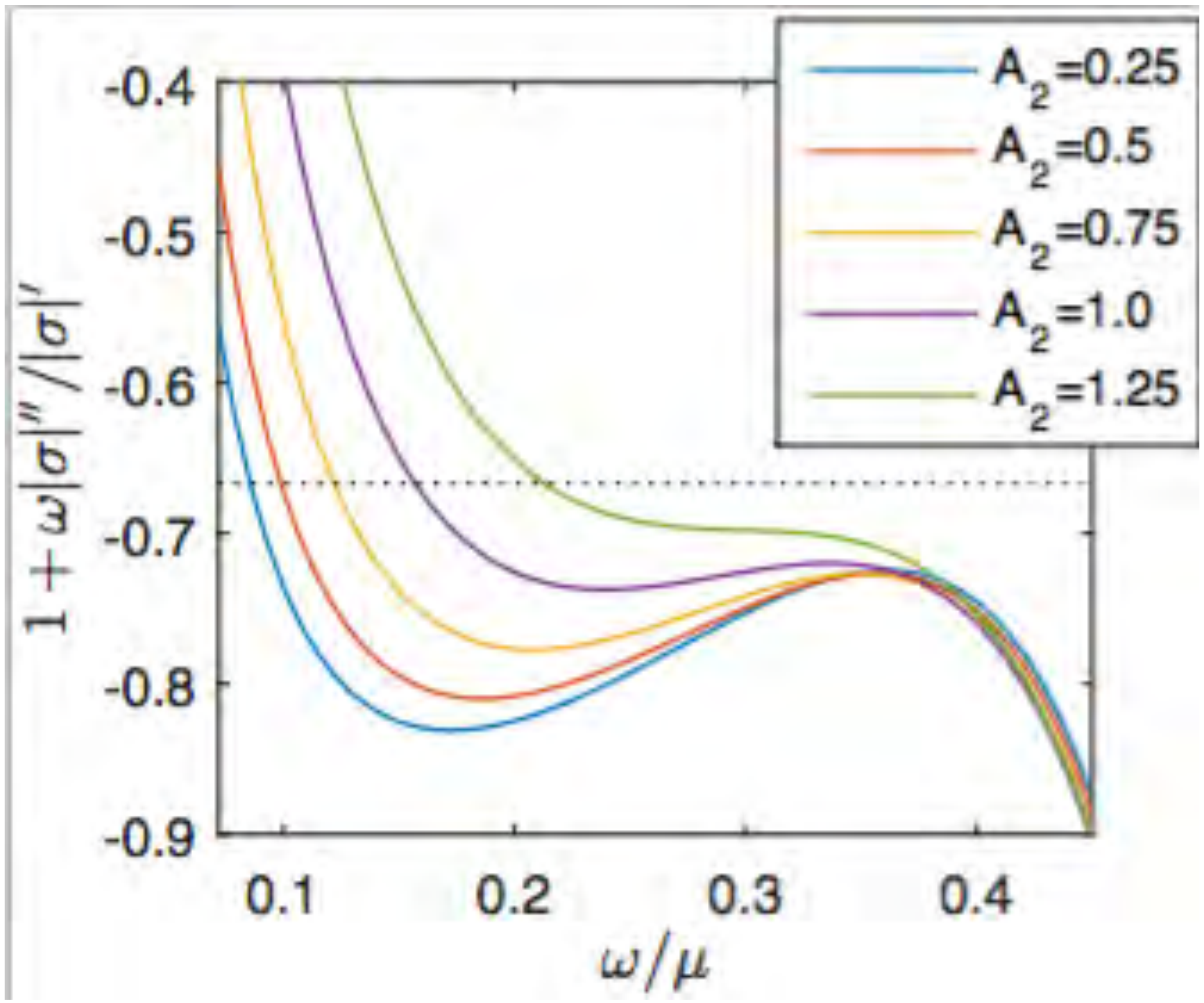


Results

$$A_1 = 0.75, k_1 = 2, k_2 = 2, \theta = 0, \mu = 1.4, T/\mu = 0.115$$



No!

origin of power law?

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phenomenology

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scale-invariant propagators

$$(p^2)^{d_U - d/2}$$

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no well-defined mass

$$\mathcal{L}_{\text{eff}} = \int_0^\infty \mathcal{L}(x, m^2) dm^2$$

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incoherent stuff (all energies)

$$\mathcal{L} = \quad \left(\partial^\mu \phi(x,m)\partial_\mu\phi(x,m) + m^2\phi^2(x,m)\right)$$

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theory with all possible mass!

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theory with all possible mass!

$$\phi \to \phi(x, m^2/\Lambda^2)$$
$$x \to x/\Lambda$$
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unparticles

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flavors

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$\phi(x, m^2) \longleftrightarrow e^2(m)$

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flavors

multi-bands

use unparticle propagators to calculate conductivity

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assume Gaussian action

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$$G(p) \sim \frac{i}{(-p^2 + i\epsilon)^{\frac{d+1}{2} - d_U}}$$

gauge unparticles

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Wilson line

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$$\sigma(i\omega_n)=(\frac{d+1}{2}-d_U)\sigma_0(i\omega_n)$$

what went wrong?

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$\alpha > 0$ convergence of integral

last attempt

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take experiments seriously

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variable masses for everything

$$\rho(m) = \rho_0 \frac{m^{a-1}}{M^a}$$

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Karch, 2015

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Karch, 2015

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Karch, 2015

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perform integral

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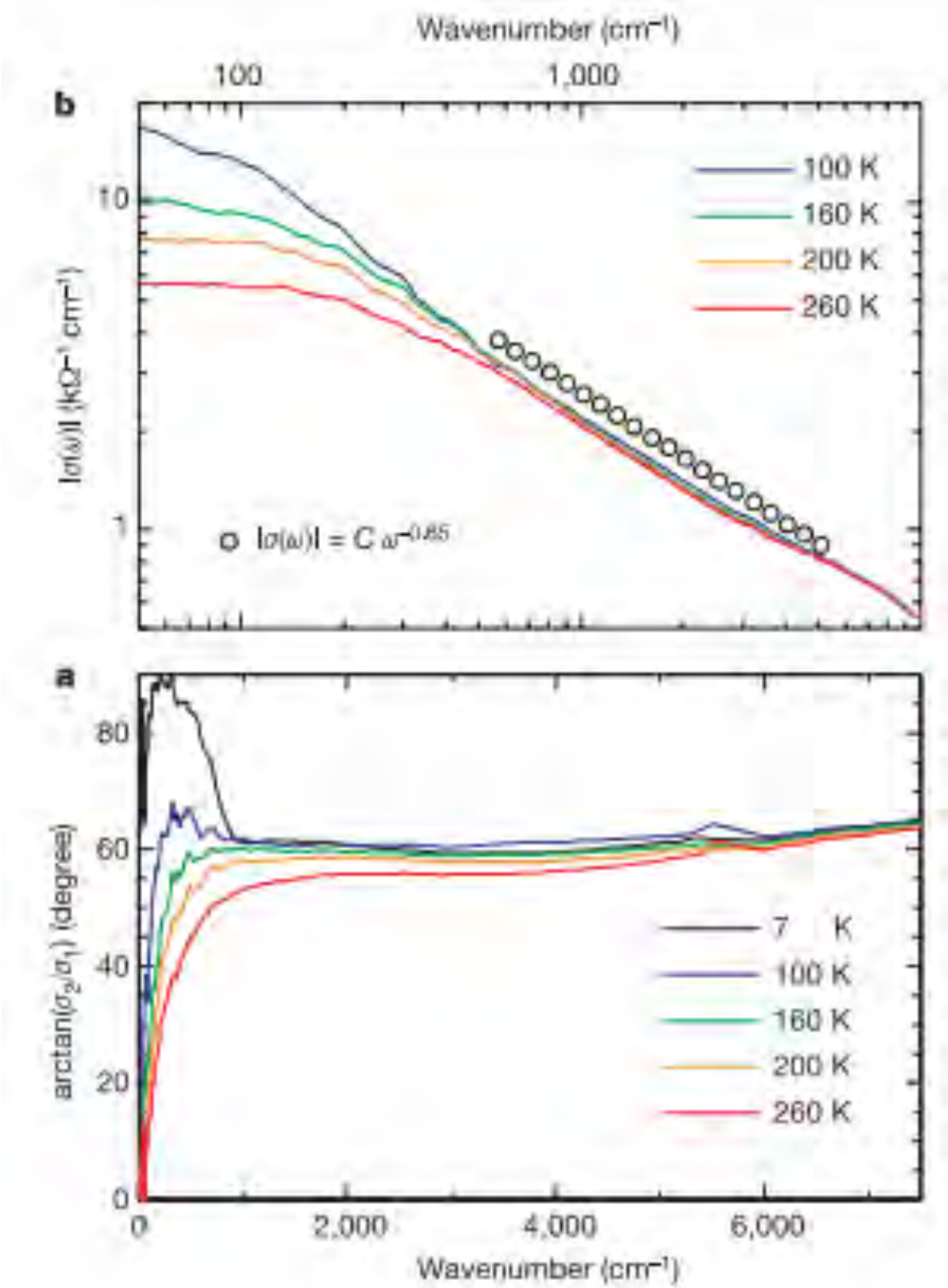
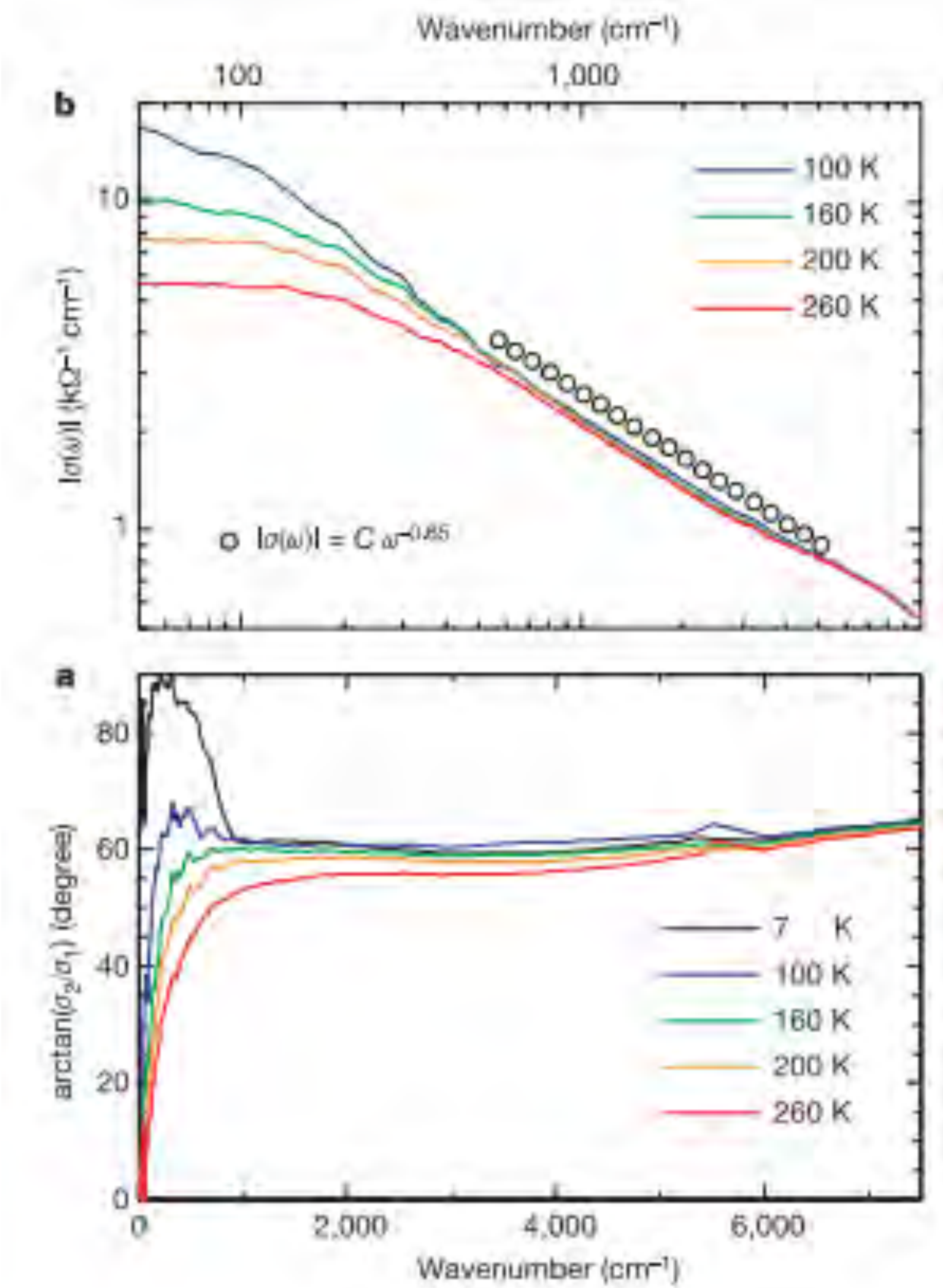
$$\boxed{\begin{array}{c}\tan\sigma = \sqrt{3} \\ 60^\circ\end{array}}$$

experiments



$$\boxed{\begin{gathered}\sigma(\omega) = C\omega^{\gamma-2}e^{i\pi(1-\gamma/2)} \\ \gamma = 1.35\end{gathered}}$$

experiments



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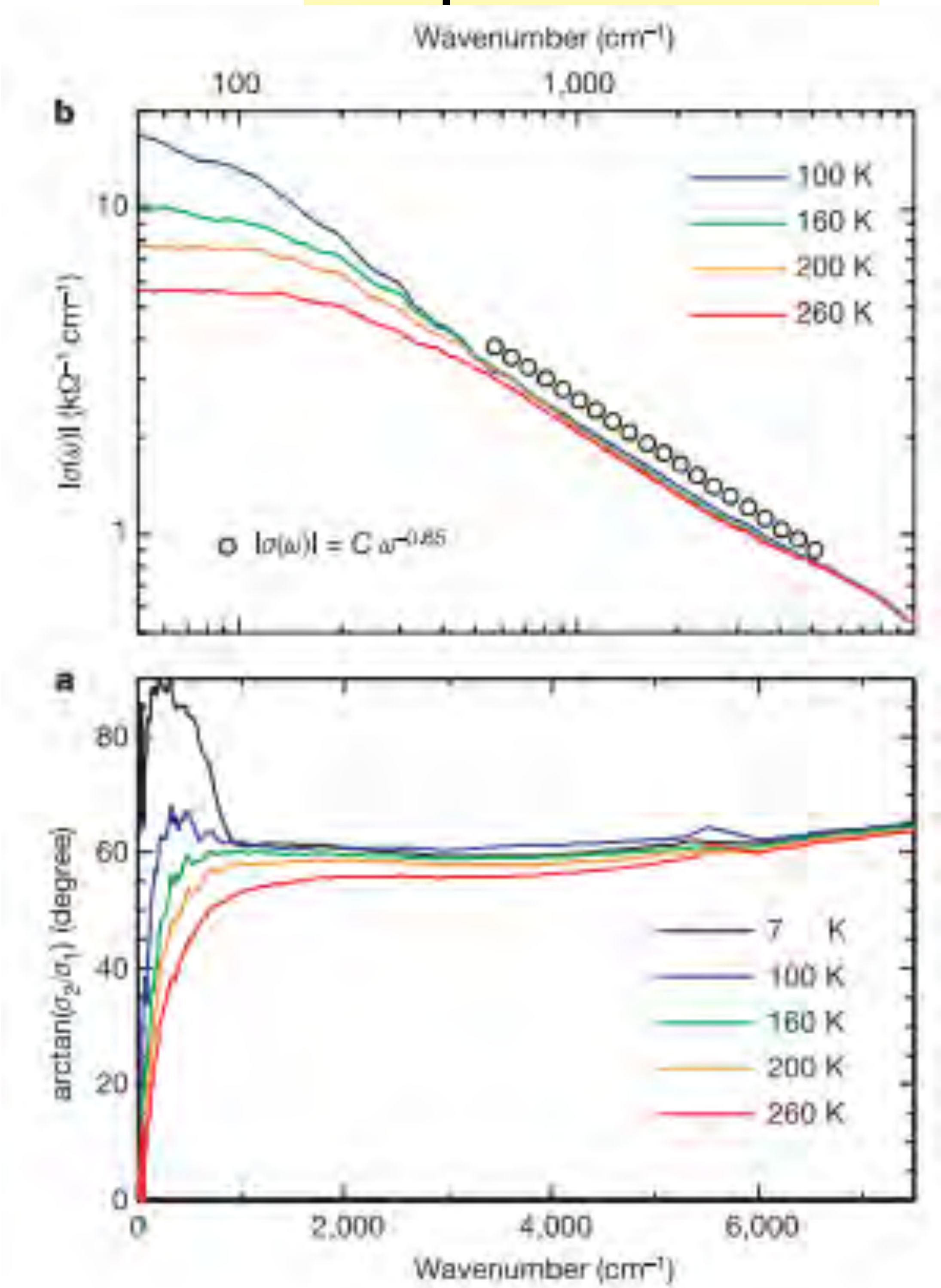

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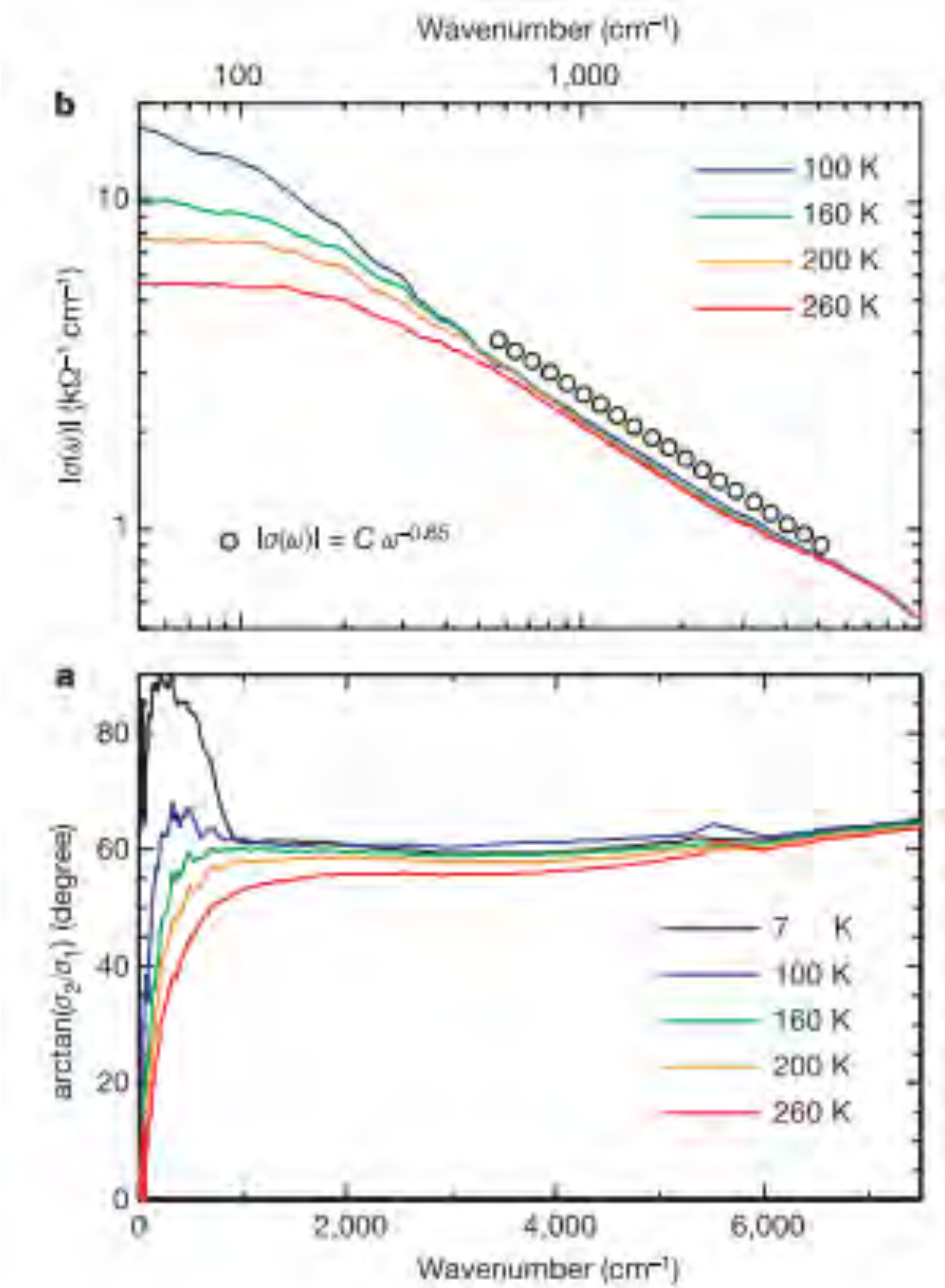

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victory!!

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are anomalous dimensions necessary

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$$a + 2b = 2/3 \quad \Rightarrow \quad b = 0, \quad a = 2/3$$

not
necessarily

not
necessarily

but they are
a possibility