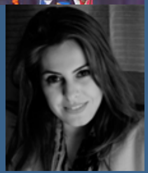
Glassy dynamics
in geometrically frustrated Coulomb liquids

Louk Rademaker, KITP, Santa Barbara

Bad Metals Conference, 30 June 2015, Mainz

Thanks to...

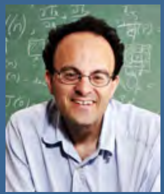
Vladimir Dobrosavljevic (FSU, USA)



Samiyeh Mahmoudian (FSU, USA)

Simone Fratini (Institut Neel, FR)

Arnaud Ralko (Institut Neel, FR)



Zohar Nussinov (Wash U, USA)

Miguel Ortuno (Murcia, Spain)

Leon Balents (KITP, UCSB, USA)

Introduction into glass

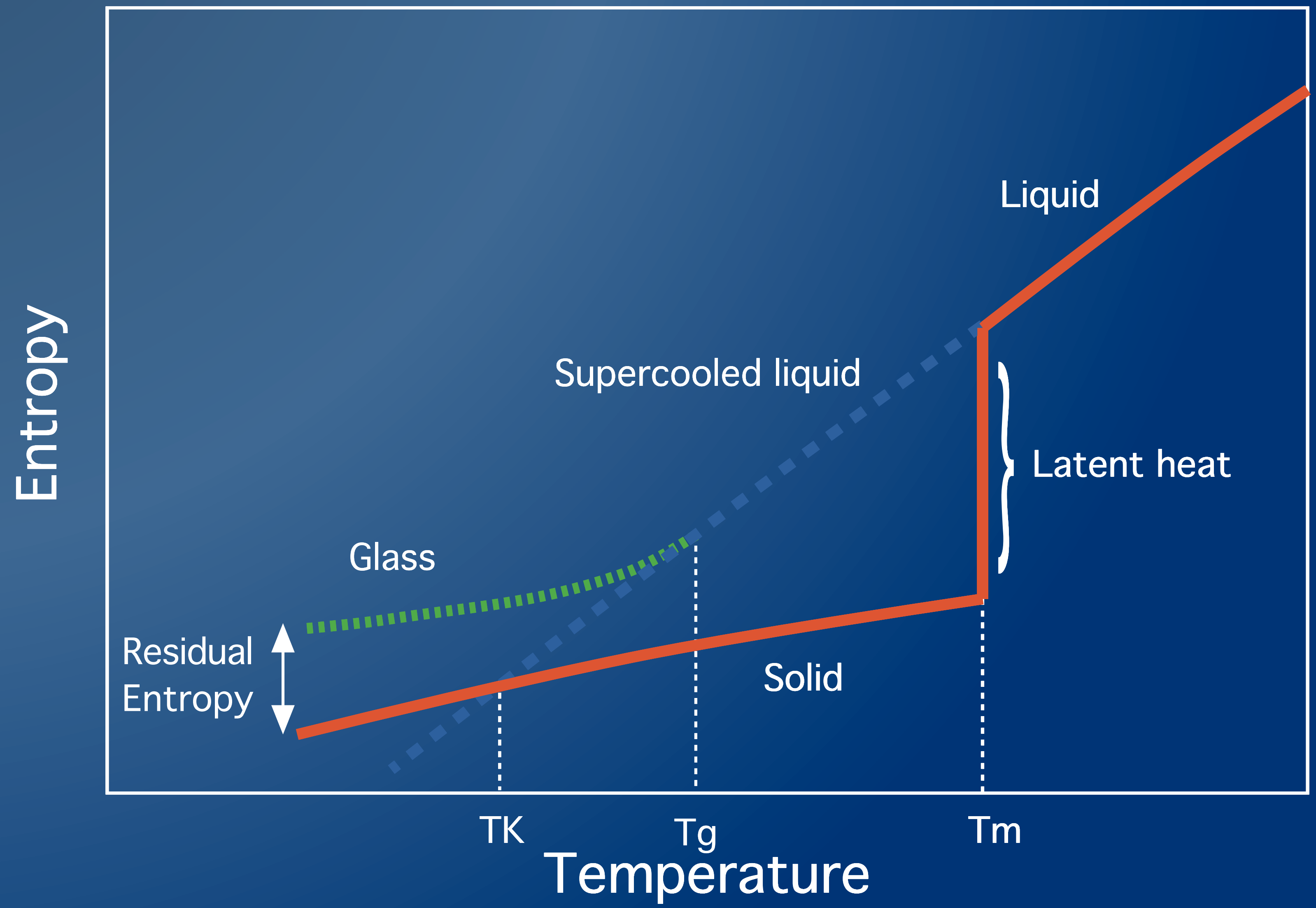
Slow dynamics

Wild energy landscape

Soft excitation gap

Short-range correlations

Supercooling a liquid

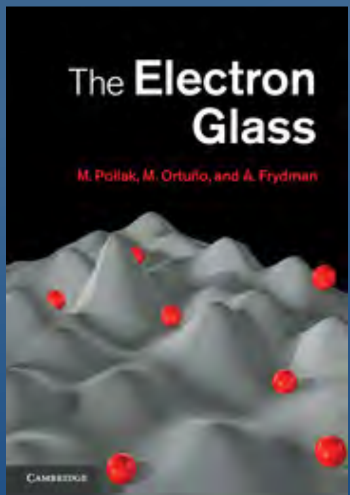


Electronic Glasses

Free Fermi **gas**, Landau Fermi **liquid**

Solid: Wigner crystal, charge-density-wave

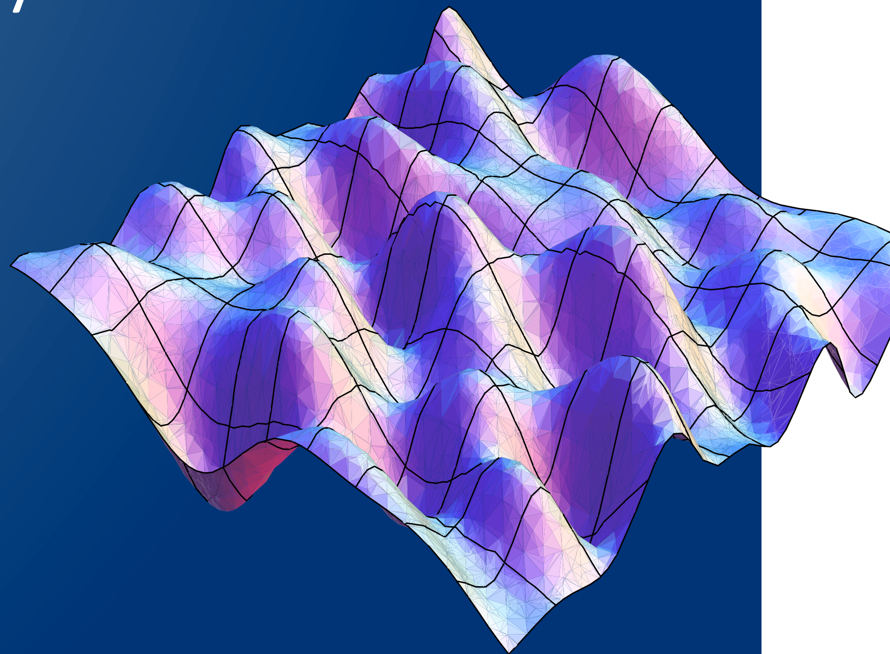
Electron glass: Intrinsic **Disorder!**



Anderson Localization

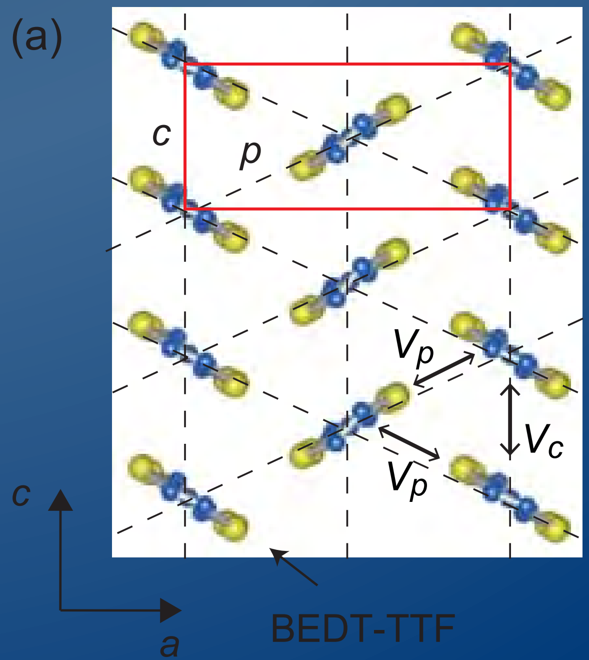
Many Body Localization

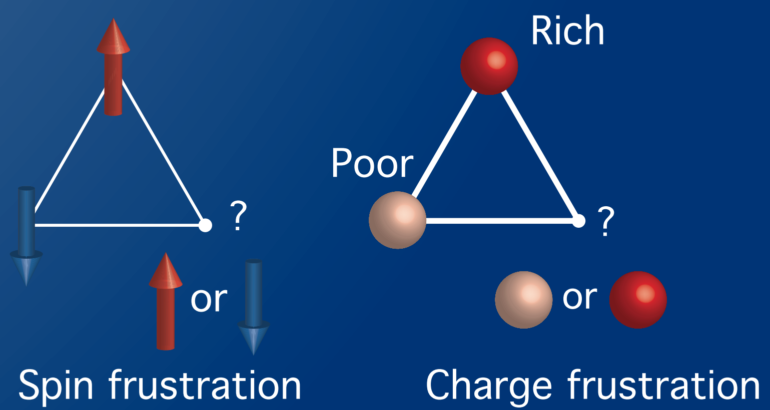
Coulomb glass



Supercool electron glass?

- Kagawa et al., Nat. Phys. 9, 413 (2013)
  (and Sato et al, JPSJ 2014; Sato et al, PRB 2014; ….)
- Material: **Clean** θ-$(BEDT\text{-}TTF)_2RbZn(SCN)_4$

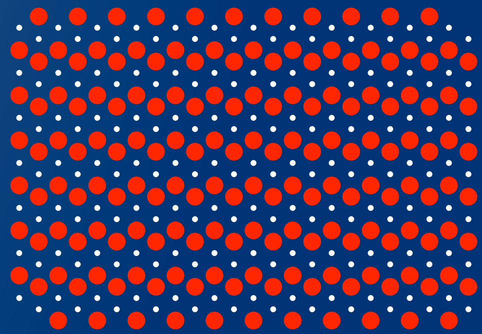


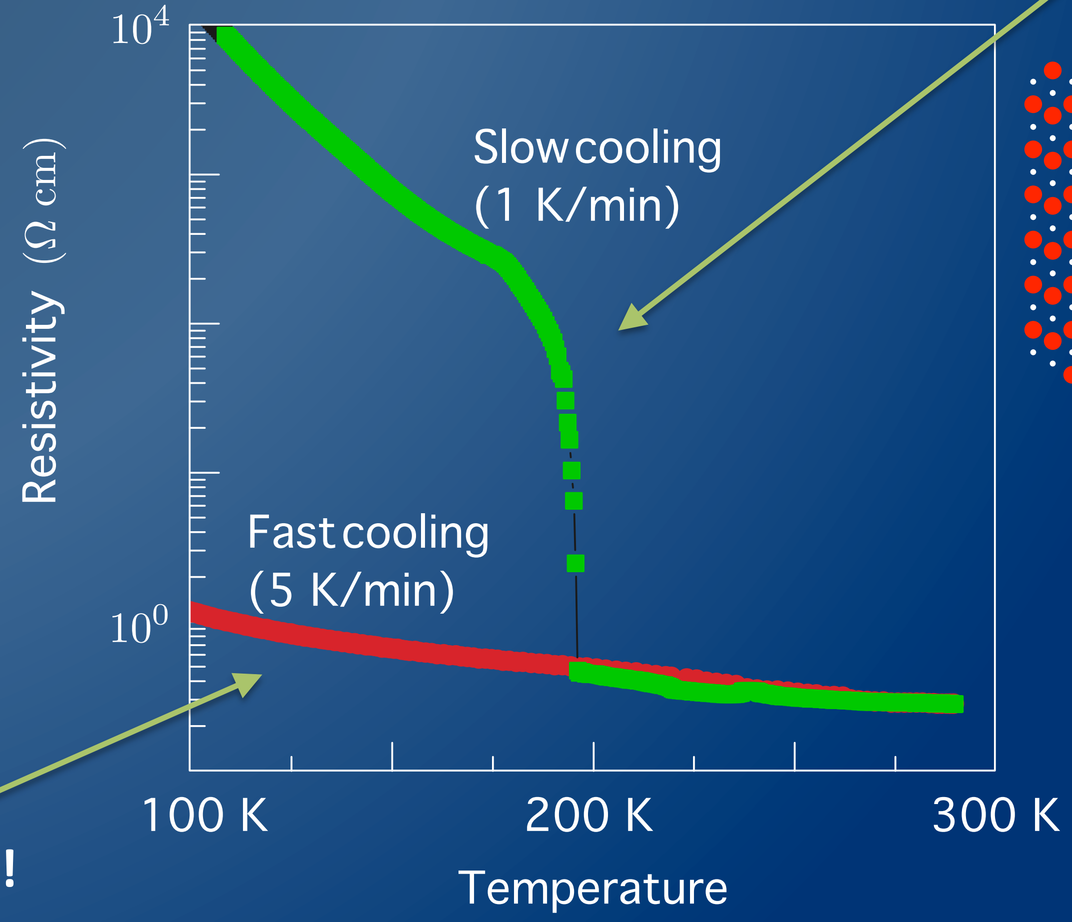


See talk by Simone Fratini, yesterday

Bad metal

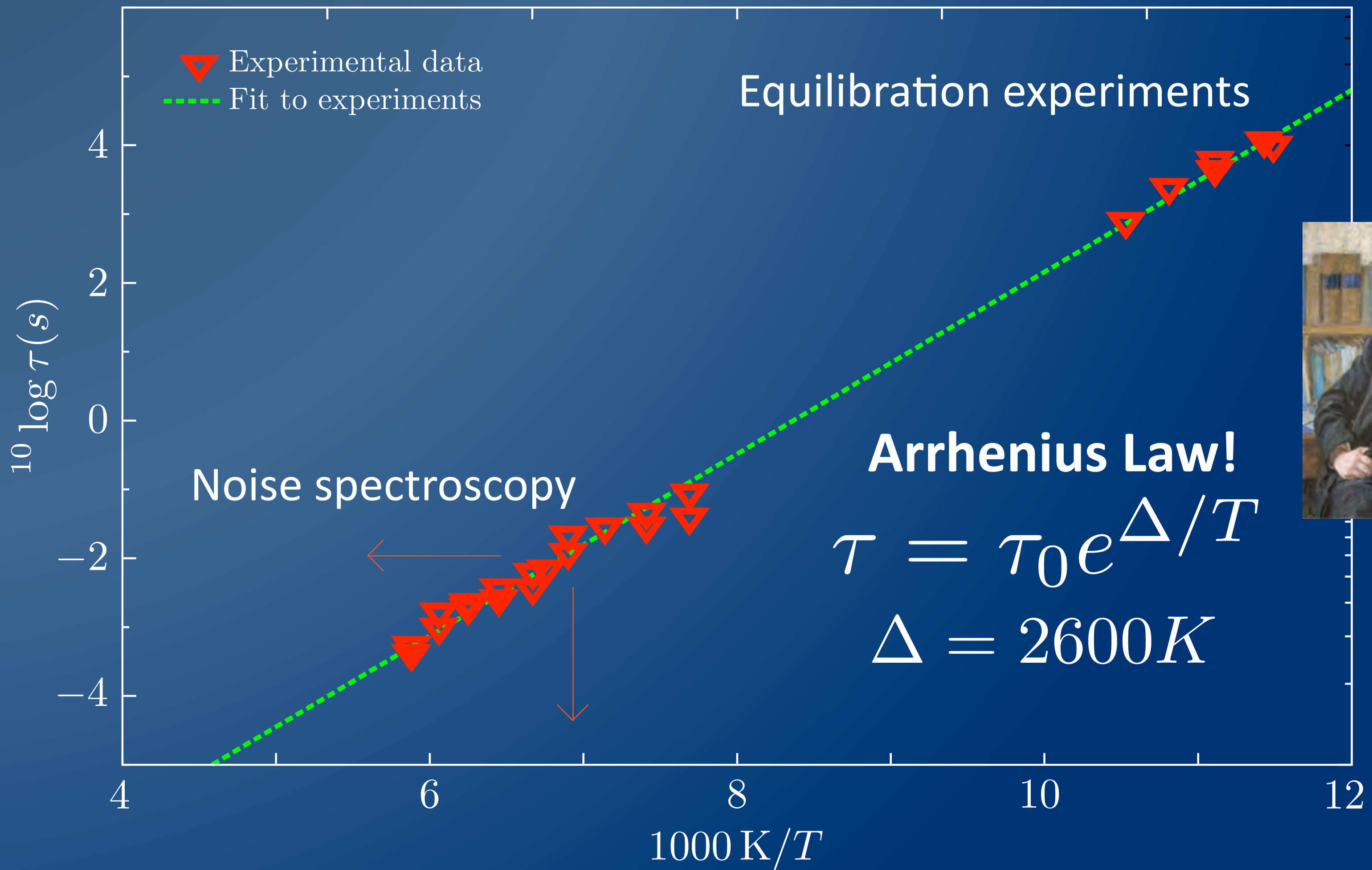
Electron solid:
Stripe order



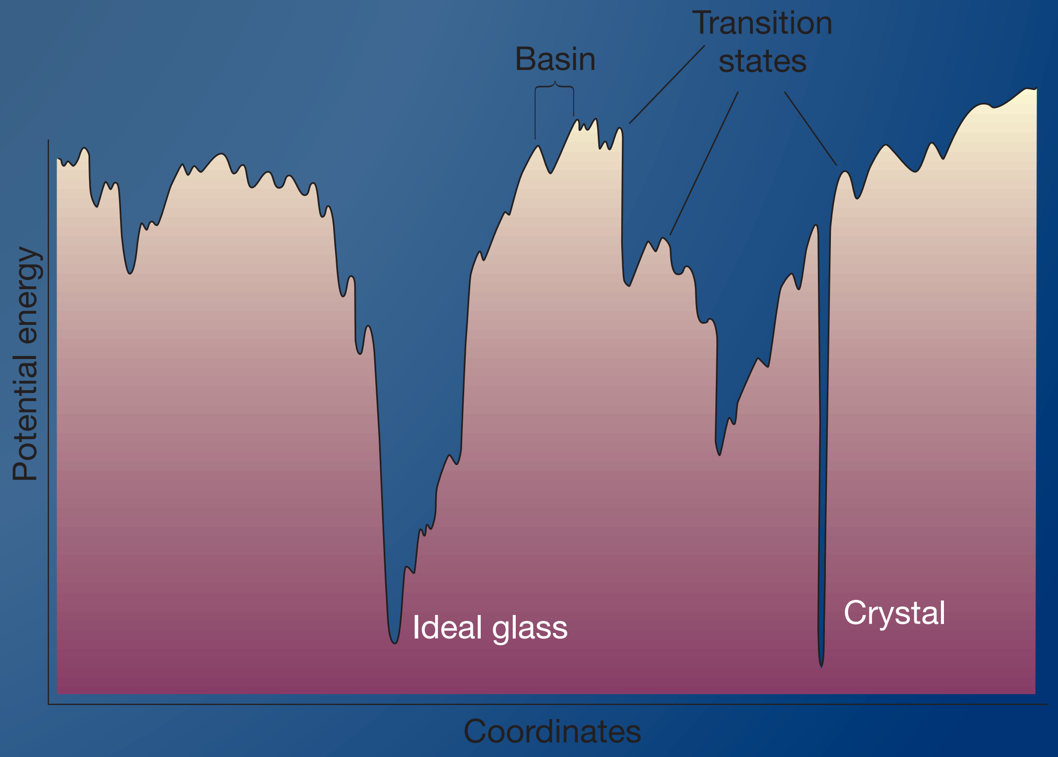


Supercool electron liquid!

Slow dynamics



Complex energy landscape



Debenedetti & Stillinger, Nature 2001

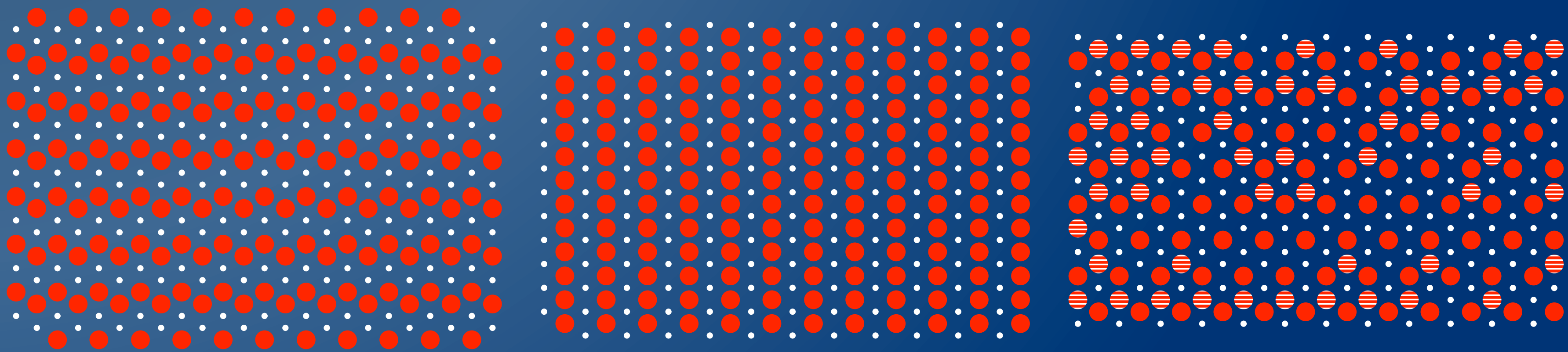
But: Electrons can **tunnel**!

Need **frustration**

Geometric Frustration

Triangular **Ising** Model $H = \frac{1}{2} \sum_{ij} V_{ij} \left(n_i - \frac{1}{2} \right) \left(n_j - \frac{1}{2} \right)$

Exponentially many Ground States $E = -V/4$



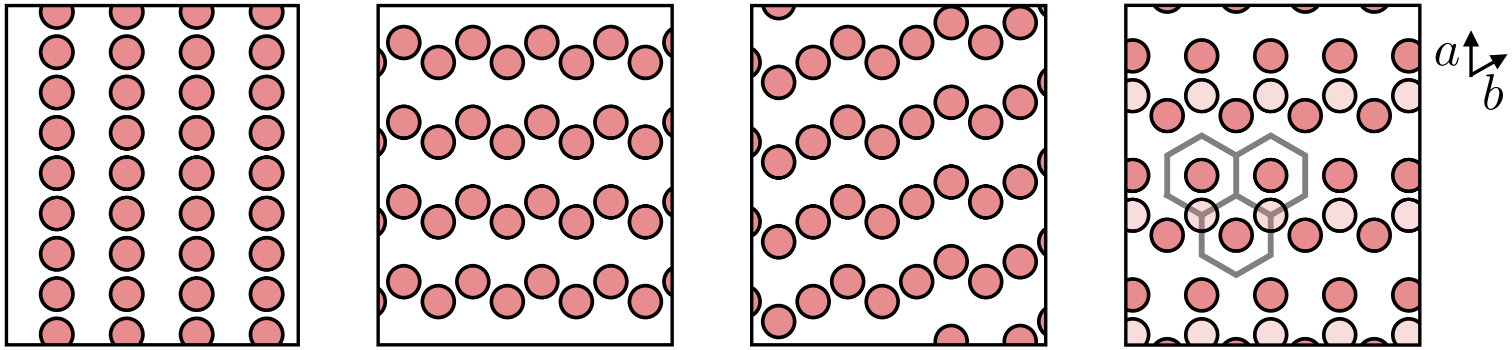
Zigzag Stripes Straight Stripes Three sublattice structure

... but not **metastable**!

Long-range interactions

Including **Long-Range Coulomb** interactions lifts degeneracy

$$V_{ij}^{\mathrm{LR}} = \frac{V}{|r_i - r_j|}$$



$a)\ -0.17556V$ $b)\ -0.17100V$ $c)\ -0.17253V$ $d)\ -0.16230V$

Macroscopically many states metastable!

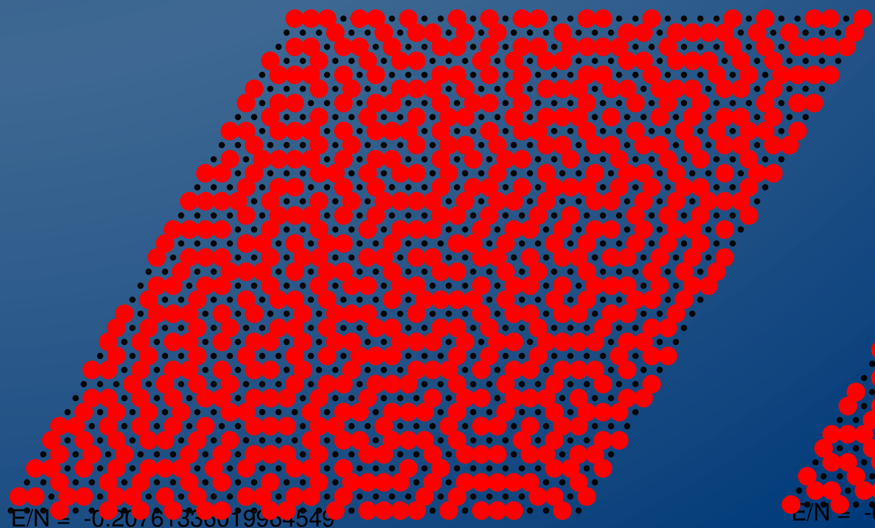
Glassiness due to long-range interactions: Schmalian&Wolynes, PRL 85 (2000)

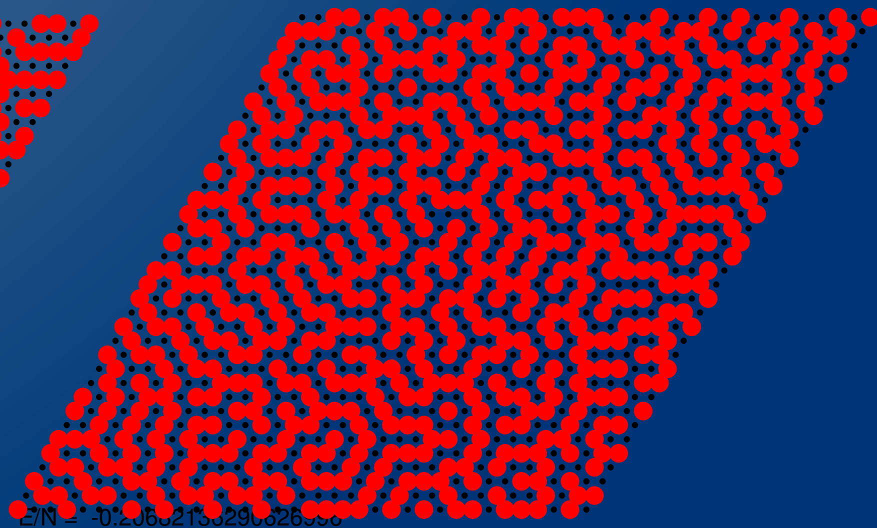
Counting of Metastable States

Stability criterion (move particle from i to j)

$$\Delta E = \epsilon_j - \epsilon_i - \frac{V}{|r_{ij}|} > 0$$

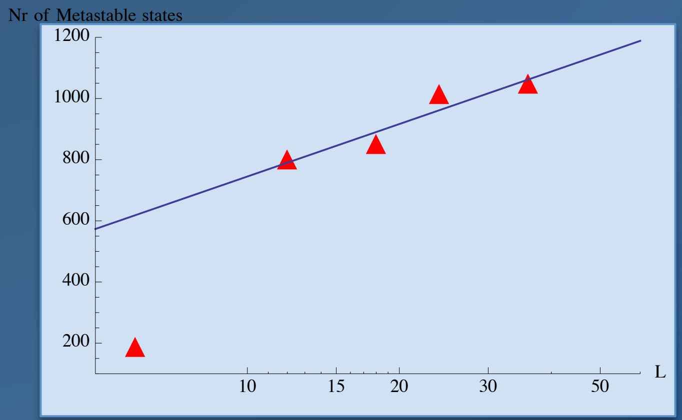
Exponentially many metastable states



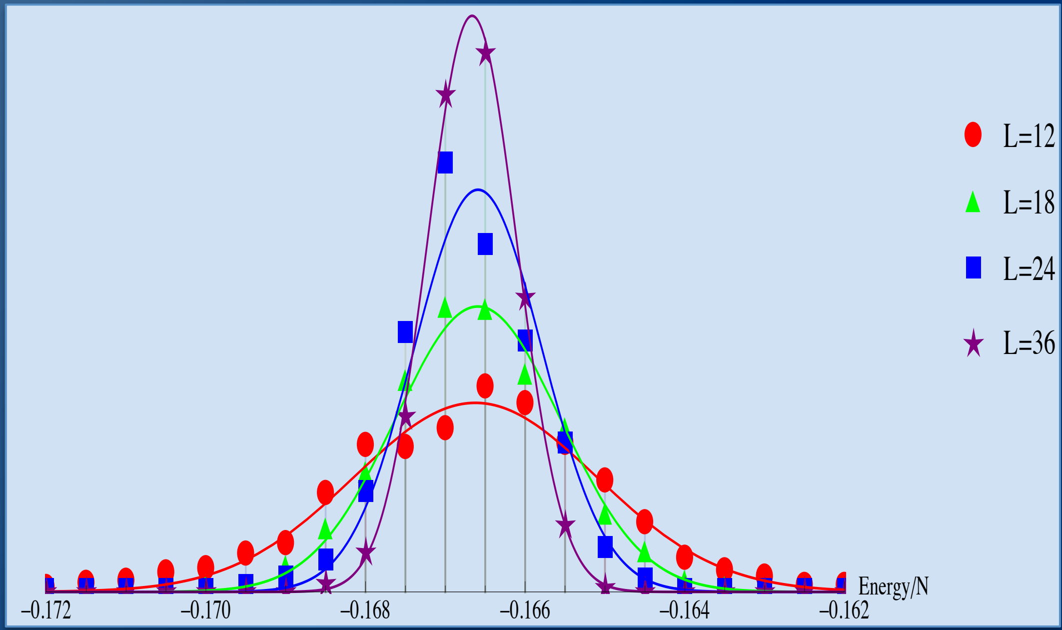


‘Inherent’ states

Only a small set of MS is relevant



L	N_{ms} (1440 sweeps)	expected total N_{ms}
6	183	185
12	695	798
18	708	849
24	775	1012
36	778	1046

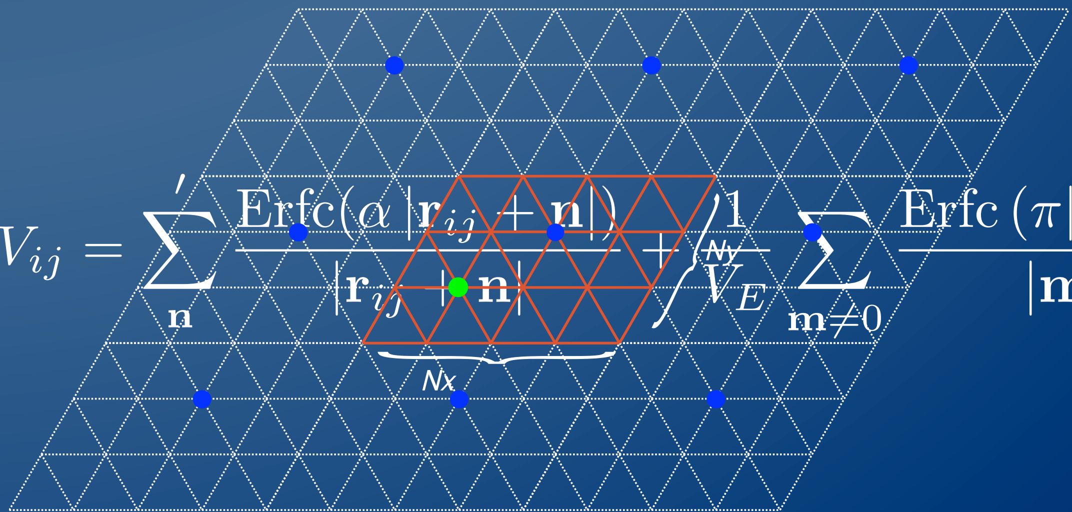


Higher energy than the ground state

Metastable states are indistinguishable in thermodynamic limit

Finite Temperature Monte Carlo

- **Metropolis**: accept local move with probability $P = \min(e^{-\Delta E/T}, 1)$
- Infinite range interactions on finite lattice? **Ewald summation**:



$$V_{ij} = \sum_{\mathbf{n}}{}' \frac{\mathrm{Erfc}(\alpha\,|\mathbf{r}_{ij} + \mathbf{n}|)}{|\mathbf{r}_{ij} + \mathbf{n}|} + \frac{1}{V_E} \sum_{\mathbf{m} \neq 0} \frac{\mathrm{Erfc}\,(\pi|\mathbf{m}|/\alpha)}{|\mathbf{m}|} \cos(2\pi \mathbf{m} \cdot \mathbf{r}_{ij}) - \frac{2\alpha}{\sqrt{\pi}} \delta_{ij}$$

$$E = \frac{1}{2} \sum_{i,j} \sum_{\mathbf{n}}{}' \frac{q_i q_j}{|\mathbf{r}_{ij} + \mathbf{n}|}$$

Relaxation Time

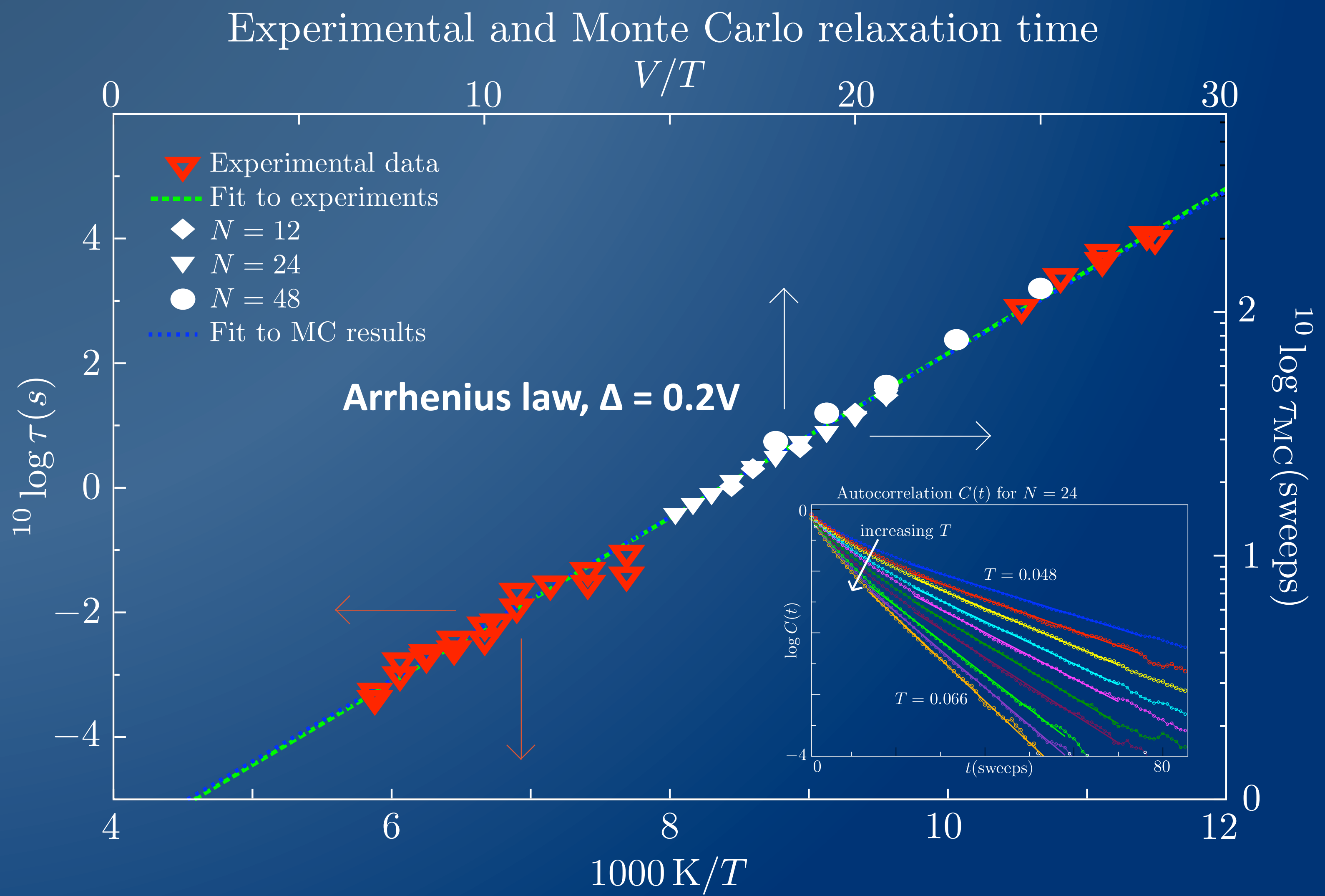
Autocorrelation function

$$C(t+t_w,t_w) = \frac{2}{N}\sum_i \langle \delta n_i(t+t_w)\delta n_i(t_w)\rangle$$

If system equilibrates $\quad C(t+t_w,t_w) \to C(t)$

Exponential tail of C(t) gives relaxation time

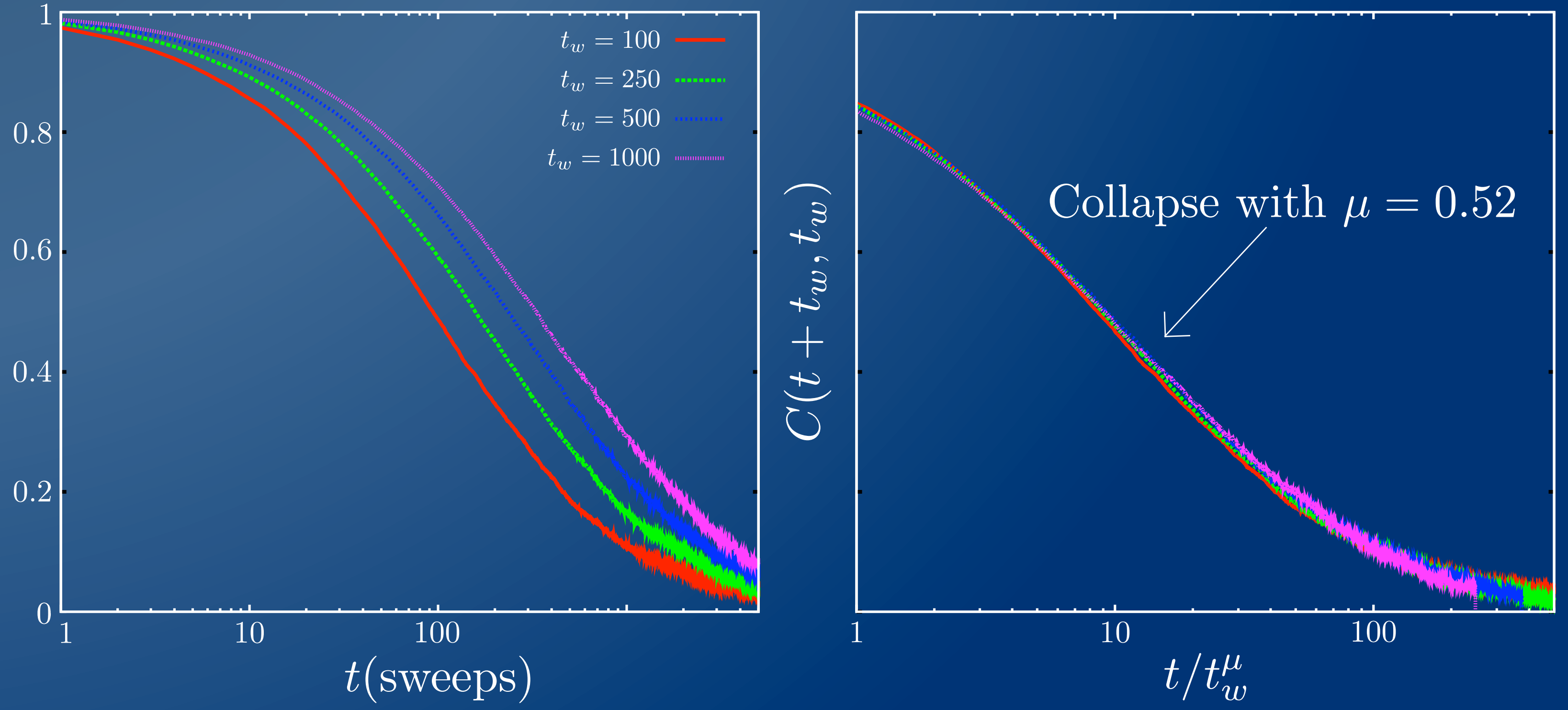
Relaxation Time



Aging & Memory

- Below T_s aging scaling $C(t+t_w, t_w) \sim F(t/t_w^{\mu})$

Aging behavior in the autocorrelation function, $T = 0.03V$

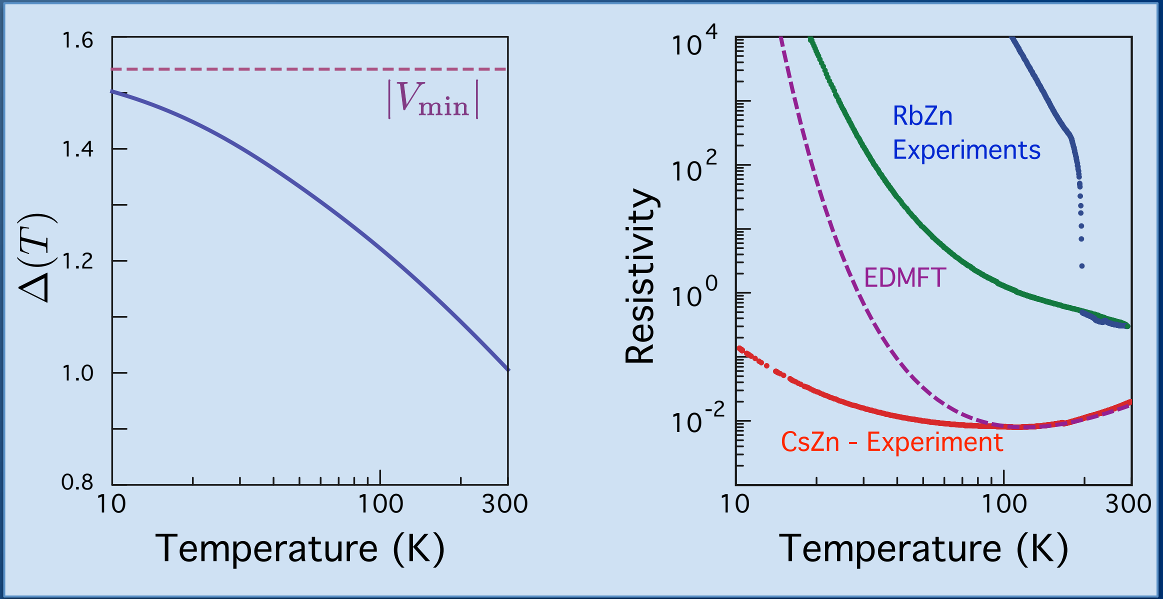


Conductivity

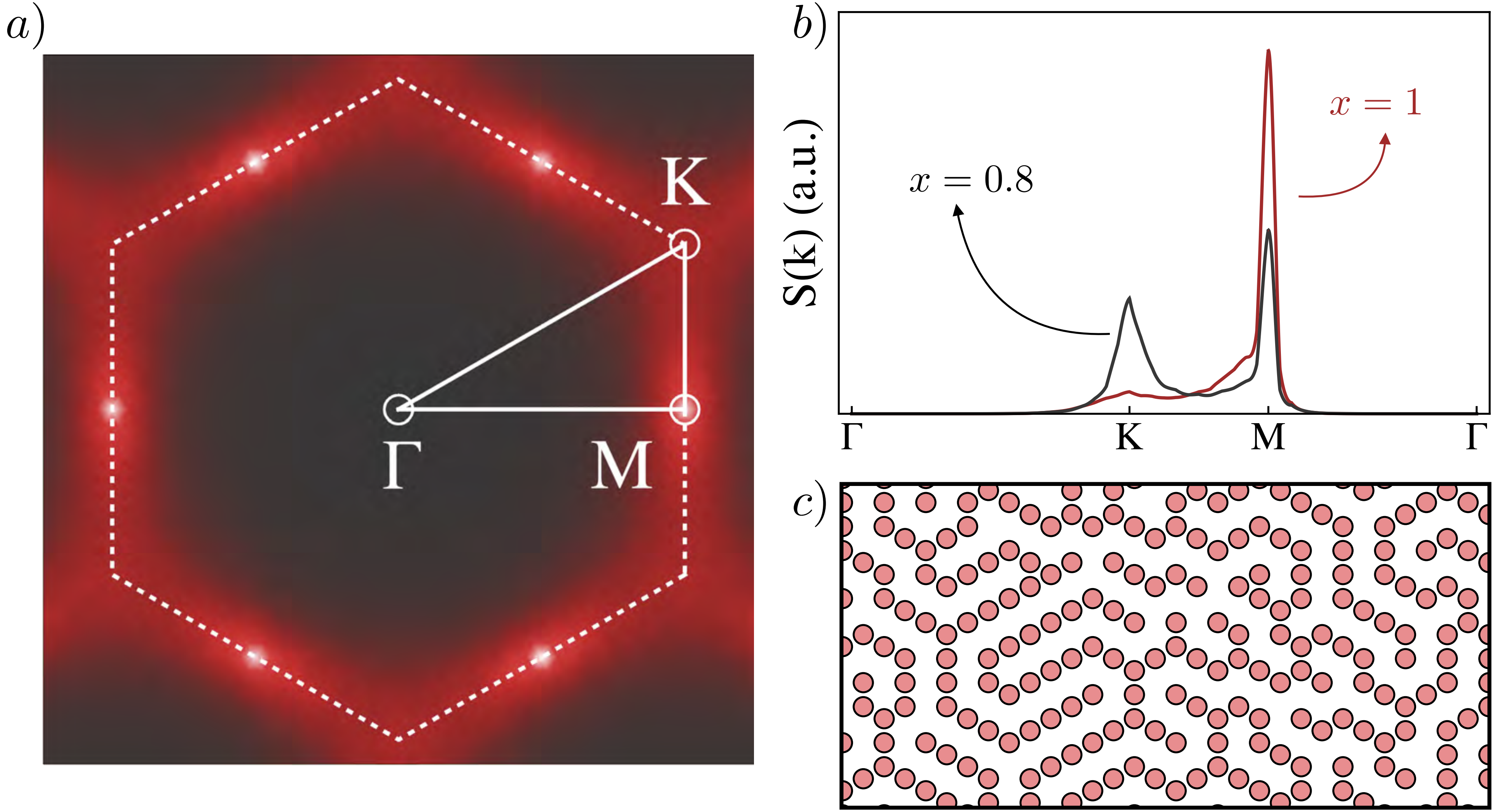
Extended Dynamical Mean Field Theory:

$$\Pi_{\mathbf{k}} = \sum_{\mathbf{r}_{ij}} \langle (n_i - \overline{n})(n_j - \overline{n}) \rangle e^{i\mathbf{k}\cdot\mathbf{r}_{ij}} = \frac{1}{(\overline{n} - \overline{n}^2)^{-1} + \beta(\Delta + V_{\mathbf{k}})}$$

$$\sigma_{DC} = \frac{\pi e^2}{\hbar a d} \int_{-\infty}^{\infty} d\omega \, \frac{\rho^2(\omega)}{4T \cosh^2 \frac{\omega}{2T}} = \frac{\beta e^2}{8\hbar a d} \sqrt{\frac{\beta\pi}{\Delta}} e^{-\frac{1}{4}\beta\Delta}$$



Density Correlations



Efros-Shklovskii gap (1)

Energy to remove or add a particle:

$$\epsilon_i = \mu_i + \frac{1}{2}\sum_{j \neq i} \frac{V}{|r_{ij}|}(n_j - \overline{n})$$

What is the **distribution** $g(\epsilon)$ of energies ϵ_i ?

(in the ground state of disordered system)

Stability when electron moves from i to j:

$$\Delta E = \epsilon_j - \epsilon_i - \frac{V}{|r_{ij}|} > 0$$

Efros-Shklovskii gap (2)

Distance between states close to Fermi level

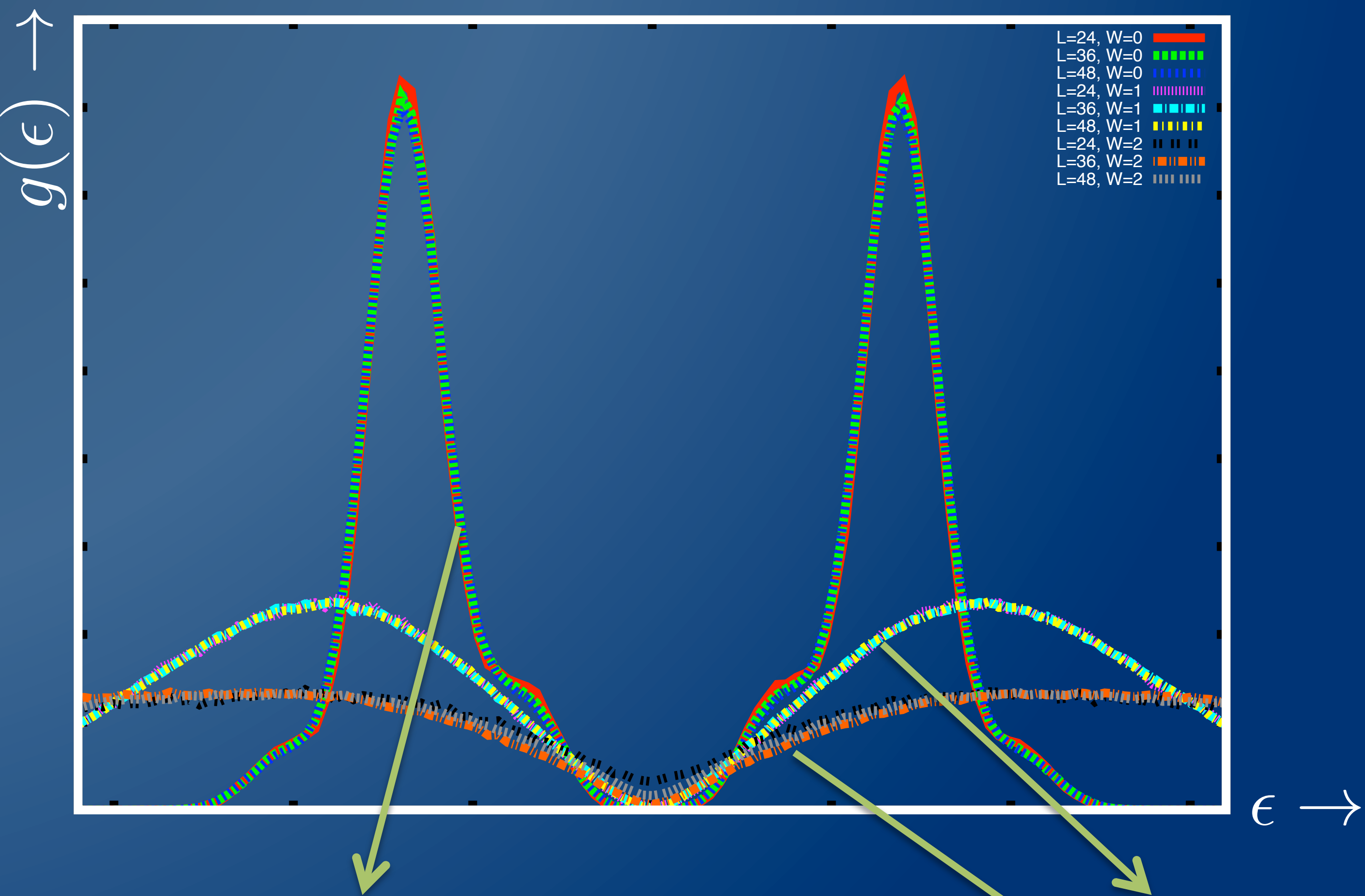
$$|r_{ij}| > \frac{V}{\epsilon_j - \epsilon_i}$$

Stability sets upper **bound** for density of states

$$g(\epsilon) \leq \frac{d}{2\pi} |\epsilon|^{d-1}$$

Efros Shklovskii 1975

Coulomb gap without disorder



Stronger Coulomb gap in absence of disorder!

$$g(\epsilon) \sim e^{-\Delta/|\epsilon|}$$

Linear ES gap with disorder

Gap & Correlations (1)

Density correlations: $\Pi_{ij} = \langle (n_i - \overline{n})(n_j - \overline{n}) \rangle$

Chance to find electron at r: $P_e(\vec{r}) = \frac{1}{2} - 2\Pi(\vec{r})$

Relate local density of states: $P_e(\vec{r}) = \int_{-\infty}^{0} d\epsilon' g_{\vec{r}}(\epsilon')$

Assumptions:

Density correlation decay like $\widetilde{\Pi}(r) = \frac{3A}{\sqrt{2\pi r}} e^{-r/\xi}$

At long distances, LDOS becomes DOS:

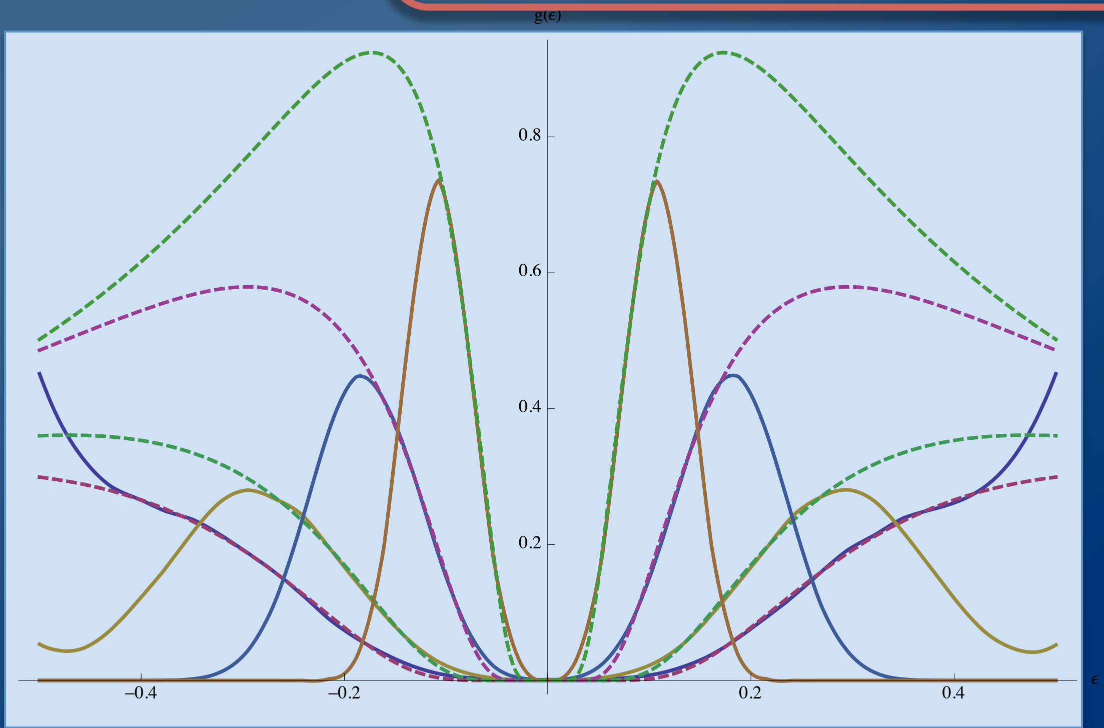
$$\int_{-\infty}^{-\frac{V}{|r|}} d\epsilon' \widetilde{g}_{\vec{r}}(\epsilon') \rightarrow \int_{-\infty}^{-\frac{V}{|r|}} d\epsilon' g(\epsilon')$$

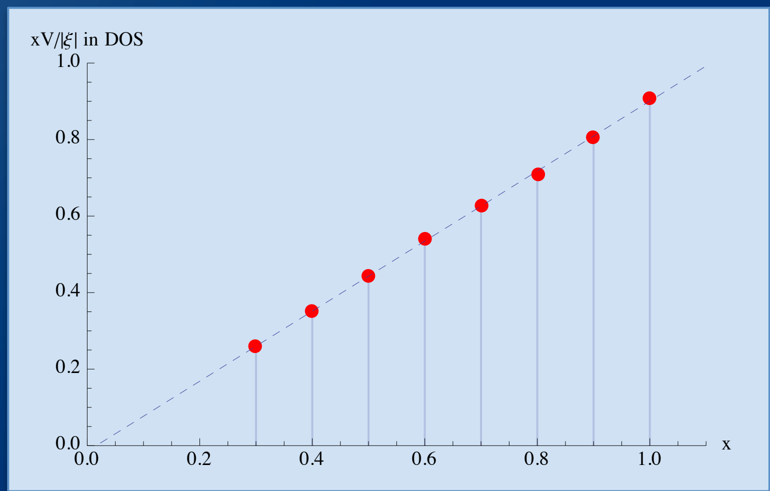
Gap & Correlations (2)

Hence DOS related to density correlations

$$-2\mathcal{D}_r\left[\ln\tilde{\Pi}(r)\right] = \frac{V}{r^2}\, g\left(\frac{V}{|r|}\right)$$

$$g(\epsilon) \sim \epsilon^{-3/2} e^{-V/\xi|\epsilon|}$$





Conclusions & Outlook

Slow dynamics ✓

?

?

Wild energy landscape ✓

Soft excitation gap ✓

?

Just discussed

Short-range correlations ✓

Reference: Mahmoudian, LR, Ralko, Fratini, Dobrosavljevic, arXiv1412.4441, PRL (2015)