Singular quasi-particles at a magnetic quantum critical point

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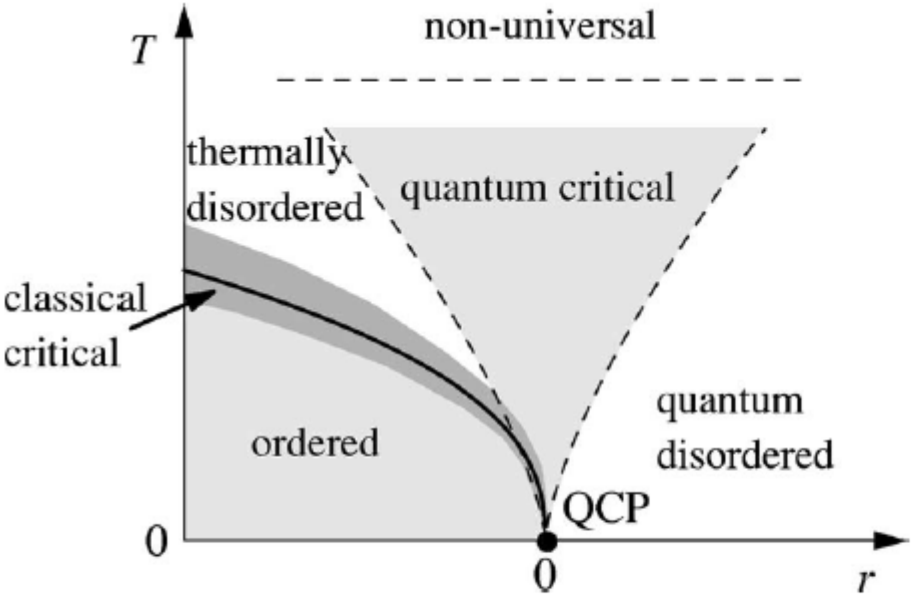
Strong coupling theory of heavy fermion criticality
Elihu Abrahams, Jörg Schmalian, Peter Wölfle
Phys. Rev. B **90**, 045105 (2014)

Quasi-particles near AF-critical points



H. v. Löhneysen, A. Rosch, M. Vojta, and P. Wölfle, RMP (2007)

correlation length of a soft, bosonic degree of freedom diverges

$$\xi \propto r^{-\nu}, T^{-1/z}$$

→ motivation for purely bosonic theories (Hertz, Moriya, Lonzarich, Moriya...)

$$\chi(\mathbf{Q}+\mathbf{q}) \propto \frac{1}{\xi^{-2} + q^2 - i\gamma\omega}$$

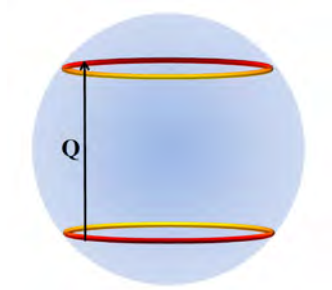
Q: What happens to fermions?

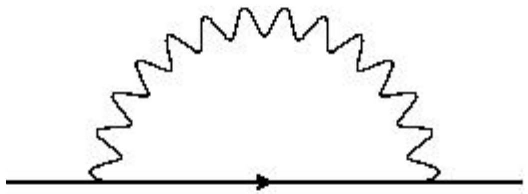
Q: Feedback of fermion criticality to the bosonic dynamics?

What happens to fermions?
(previous results)

1. Hot and cold regions of the Fermi surface, with singular behavior in hot parts

   Ar. Abanov, AV Chubukov, J Schmalian, Adv. in Phys 52 (2003);
   Ar. Abanov and A. V. Chubukov, PRL 93 (2004);
   M. A. Metlitski and S. Sachdev, PRB 82 (2010).

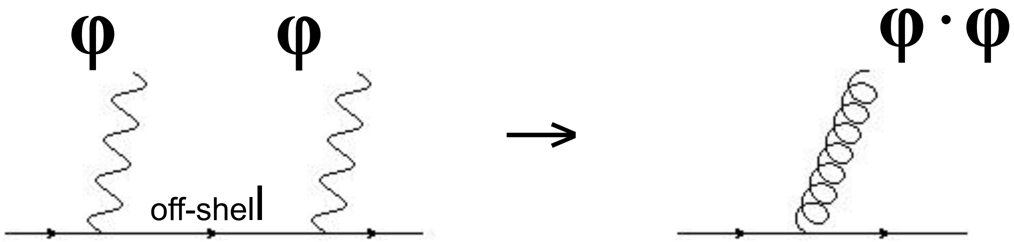


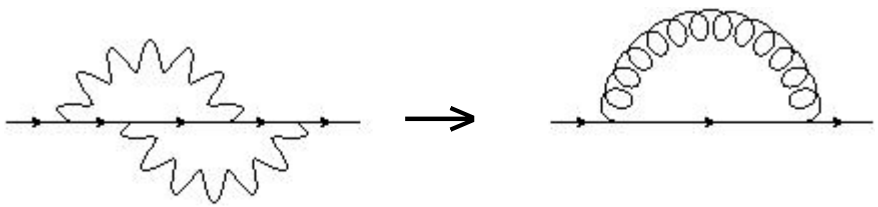


$$\Rightarrow \quad \Sigma_{k_{hot}}(\omega) \propto i\omega|\omega|^{\frac{d-3}{2}}$$

2. Cold regions: corrections due to composite energy-density fluctuations

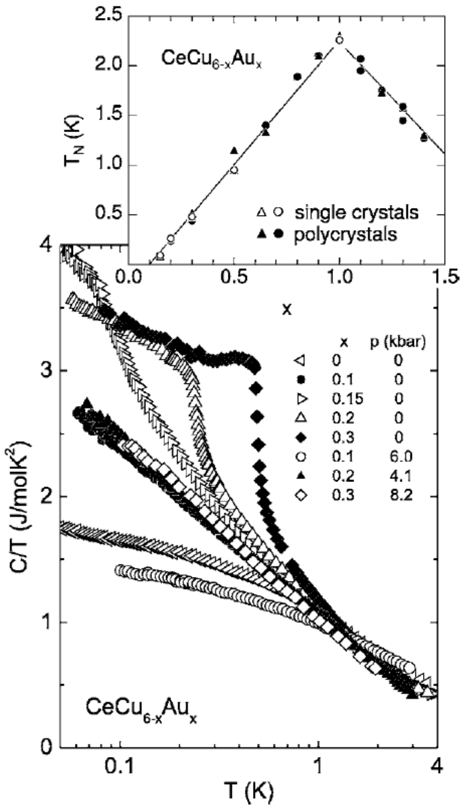
   S. A. Hartnoll, D. M. Hofman, M. A. Metlitski, and S. Sachdev, PRB B 84 (2011)



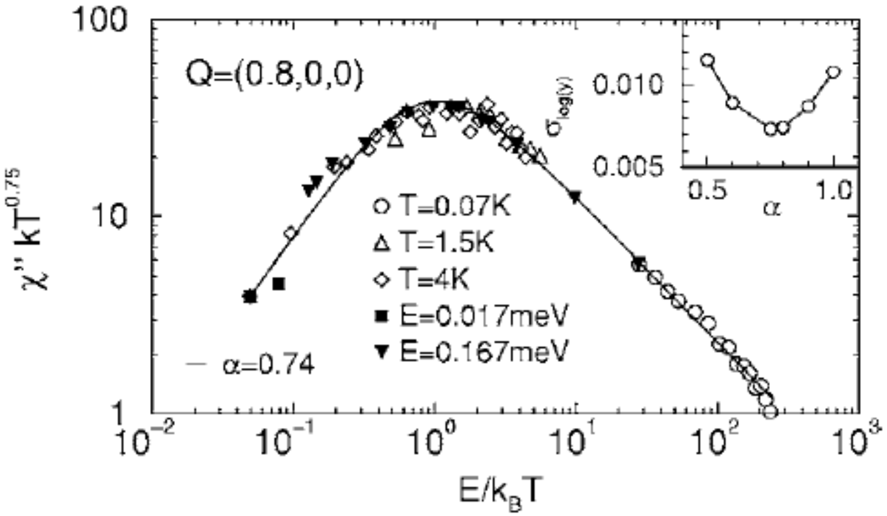


$$\Rightarrow \quad \Sigma_{k_{cold}}(\omega) \propto i\omega|\omega|^{d-3/2}$$

Theory does not account for experiment in systems like $CeCu_{6-x}Au_x$ or $YbRh_2Si_2$



H. v. Löhneysen J. Phys. CM 8 (1998);
H. v. Löhneysen et al. JMMM 177-181 (1998)



A. Schröder et al. PRL 80 (1998)

$$\chi(\mathbf{Q}+\mathbf{q}) \propto \frac{1}{T^{\alpha} + q^{2} + \left(i\gamma \operatorname{sign}(\omega)|\omega|\right)^{\alpha}}$$

$$\alpha \approx 0.74 \quad \Rightarrow z \approx 2.7$$

Critical quasi-particles

Karlsruhe Institute of Technology

quasi-particle concept beyond Fermi liquid theory

C.M. Varma et al, Phys. Rev. Lett. 63 (1989);
T. Senthil, PRB 78 (2008);
P. Wölfle and E. Abrahams, PRB 84 (2011).

$$G(\mathbf{k},\omega)=\frac{Z_{\mathbf{k}}}{\omega+i\Gamma_{\mathbf{k}}(\omega)-\varepsilon^*_{\mathbf{k}}}+G_{inc}(\mathbf{k},\omega)$$

$$\downarrow$$

$$G(\mathbf{k},\omega)=\frac{Z_{\mathbf{k}}(\omega)}{\omega+i\Gamma_{\mathbf{k}}(\omega)-\varepsilon^*_{\mathbf{k}}},\quad Z_{\mathbf{k}}(\omega)\propto|\omega|^{\eta},\quad 0\le\eta<1$$

- well defined Fermi surface: $n_{\mathbf{k}}-n^{\mathbf{0}}_{\mathbf{k}}\propto\left(\mathbf{v}_{\mathbf{k}}\cdot(\mathbf{k}-\mathbf{k}_F)\right)^{\frac{1-\eta}{\eta}}$
- dynamic scaling exponent: $z_F=\frac{1}{1-\eta}$ $\qquad C/T\propto T^{-\eta}$
- marginal Fermi liquid: $\eta\to 0$ $\qquad C/T\propto\log T$

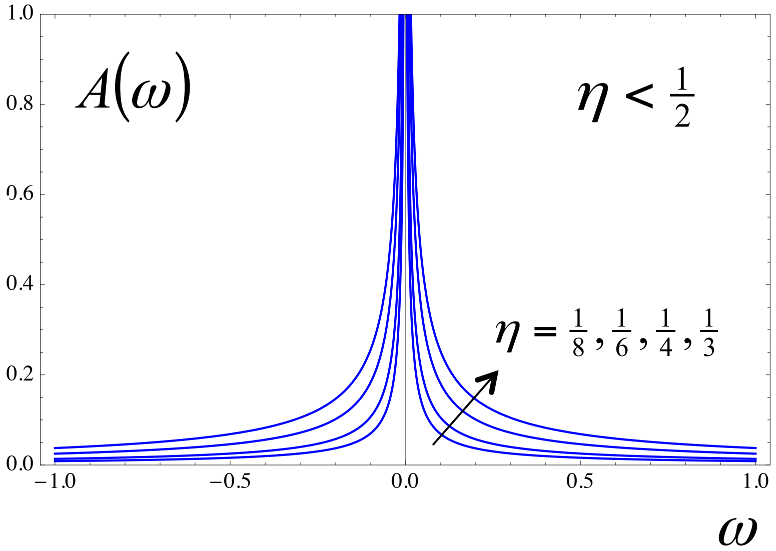
Critical quasi-particles

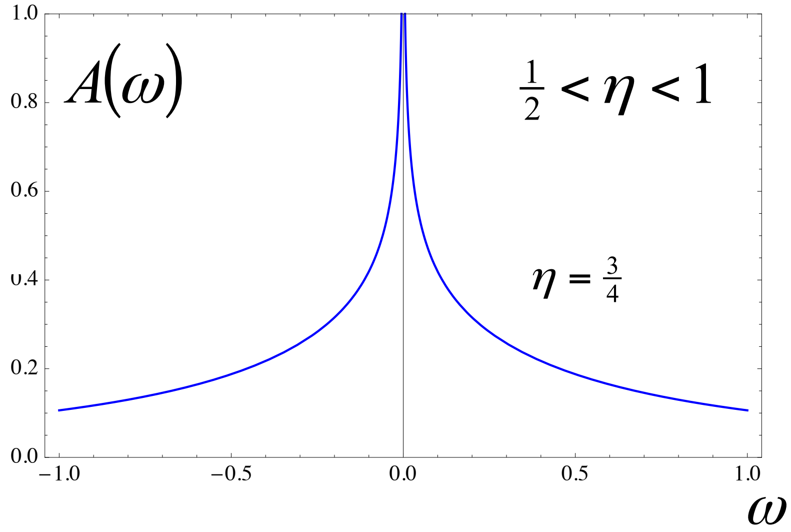
the self energy

$$Z(\omega) \propto |\omega|^{\eta} \quad \Leftrightarrow \quad \Sigma(\omega + i0^{+}) = -A|\omega|^{1-\eta}\left(\mathrm{sign}(\omega)\cot\left(\tfrac{\pi\eta}{2}\right) + i\right)$$

If $\eta < \frac{1}{2}$: the width of the peak is smaller than its position

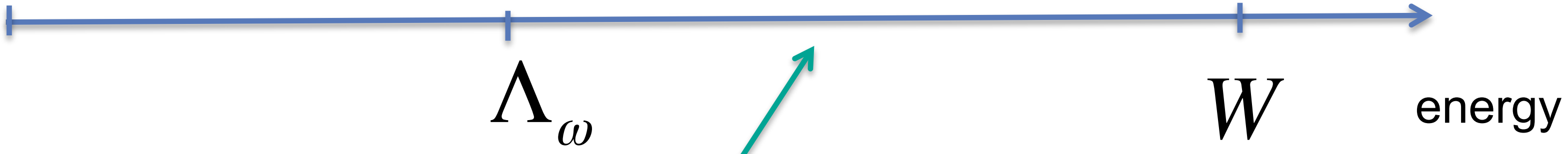
the spectral function





Self-consistency approach

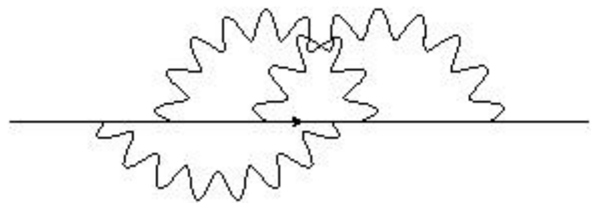
$$S = -\int \left(\psi^{+}\left(i\omega - \varepsilon\right)\psi + g\psi^{+}\boldsymbol{\sigma}\psi\cdot\boldsymbol{\varphi}\right) + \int r_0 \boldsymbol{\varphi}\cdot\boldsymbol{\varphi} + S_{\text{other channels}}$$



elimination of high energy degrees of freedoms

$$S_{low} = -\sum_{i=h,c}\int \left(\psi_i^{+}\left(i\omega - vk - \Sigma_i^{>}\right)\psi_i + \Gamma_{g,i}^{>}\psi_i^{+}\boldsymbol{\sigma}\psi_i\cdot\boldsymbol{\varphi}\right) + \int\left(r_0 - \Pi^{>}\right)\boldsymbol{\varphi}\cdot\boldsymbol{\varphi}$$

$$+\int \Gamma_\lambda^{>}\psi_c^{+}\psi_c\boldsymbol{\varphi}\cdot\boldsymbol{\varphi} + S_{\text{low}}^{\text{other channels}}$$



Self-consistency approach

self-energy and vertex corrections

$$\Sigma^{>}_{h,c}(k,\omega) = (1 - Z_{h,c,\omega})\, i\omega + v\,(Z_{h,c,k} - 1)\, k$$

$$\begin{aligned}
\Gamma^{>}_{h,g}(k,q,\omega,\Omega) &= gZ_{h,g} \\
\Gamma^{>}_{c,g}(k,q,\omega,\Omega) &= gZ_{c,g} \\
\Gamma^{>}_{\lambda}(k,q,\omega,\Omega) &= \lambda Z^{2}_{c,g} Z_{\lambda}
\end{aligned}$$

- weak momentum dependence within the hot and cold regions $Z_{h,k} = Z_{c,k} = 1$
- small-q scattering with energy density fluctuations $Z_{\lambda} = Z_{\omega}$

- power-law ansatz

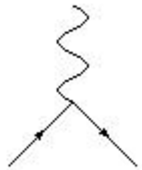
$$Z(\omega) = Z_{\omega}^{-1} \propto \omega^{\eta} \qquad \text{(q.p. weight)}$$

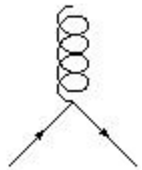
$$Z_g(\omega) \propto \omega^{-\phi} \qquad \text{(interaction vertex)}$$

Self-consistency approach

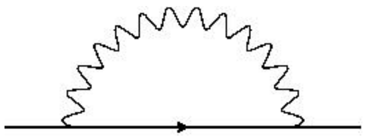
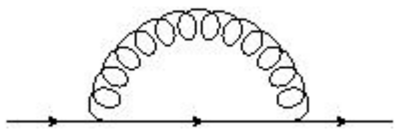
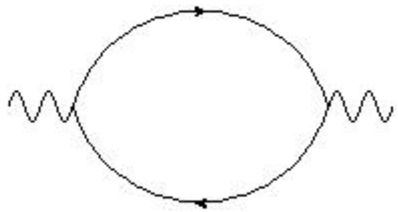
quasi-particle theory

$\psi_r = \psi / Z^{1/2}$ $\quad v^* = vZ$ $\quad g^* = vZ_g Z$ $\quad \lambda^* = \frac{g^2}{W} Z_g^2$





perturbation theory of renormalized quasiparticles at low energies

$\Sigma^{\text{q.p.}}_{\text{hot}} =$  $\Sigma^{\text{q.p.}}_{\text{cold}} =$  $\Pi =$ 

Self-consistency approach

Matching at intermediate scales:

intermediate scale is arbitrary:
same self energy as we approach Λ_ω
from above and below

$$\omega = \Lambda_\omega \qquad \Sigma^{q.p.}(\omega; Z) = Z(\omega)\Sigma^{>}(\omega)$$

- **Singular low energy behavior (even formally sub-leading) is boosted by high energy behavior**
- **There is always a weak coupling solutions:** $Z = const. \sim \mathcal{O}(1)$

Self-consistency approach

$$\Sigma^{q.p.}(\omega; Z) = Z(\omega)\Sigma^{>}(\omega) \quad \text{with} \quad Z(\omega)^{-1} = 1 - \frac{\partial \Sigma^{>}(\omega)}{\partial \omega}$$

$$\Rightarrow \eta_c = \frac{2d-3}{6}(1-2\phi) \qquad \eta_h = \frac{3-d}{2} + 2\phi(d-1)$$

Identical resuls follow from a field theoretic RG

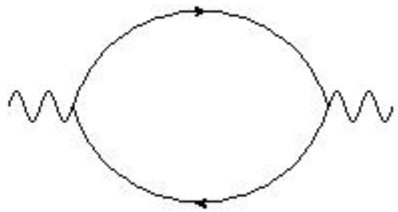
$$\Sigma(\omega) = \alpha(\mu) i\omega \qquad \alpha_c(\mu) \propto \mu^{\frac{2d-3}{2}} \frac{Z_{cg}^4 Z_\lambda}{Z_{c,\omega} Z_{ck}} \left(\frac{Z_{h,g}}{Z_{h,k}}\right)^{2d-5}$$

$$\beta = -\frac{\partial \alpha}{\partial \log \mu} \qquad \eta = \beta \frac{\partial \log Z_\omega}{\partial \alpha}$$

Self-consistency approach

$$\Sigma^{q.p.}(\omega;Z) = Z(\omega)\Sigma^{>}(\omega) \quad \text{with} \quad Z(\omega)^{-1} = 1 - \frac{\partial\Sigma^{>}(\omega)}{\partial\omega}$$

$$\Rightarrow \eta_c = \frac{2d-3}{6}(1-2\phi) \qquad \eta_h = \frac{3-d}{2} + 2\phi(d-1)$$

bosonic syn. scaling exponent $\Pi =$ 

$$\Pi(Q,\omega) - \Pi(Q,0) \propto iZ_g^2\omega \propto (i\,sign(\omega)|\omega|)^{1-2\phi}$$

$$z = \frac{2}{1-2\phi}$$

vertex corrections are responsible for $z > 2$

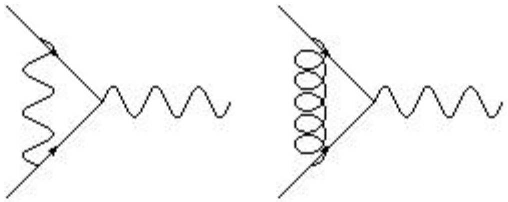
Self-consistency approach

$$\Sigma^{q.p.}(\omega; Z) = Z(\omega)\Sigma^{>}(\omega) \quad \text{with} \quad Z(\omega)^{-1} = 1 - \frac{\partial \Sigma^{>}(\omega)}{\partial \omega}$$

$$\Rightarrow \eta_c = \frac{2d-3}{6}(1-2\phi) \qquad \eta_h = \frac{3-d}{2} + 2\phi(d-1)$$

condition for vertices: bosonic syn. scaling exponent $z = \frac{2}{1-2\phi}$

- spin fermion model:



$\phi = 0$

→ No anomalous behavior of the spin-dynamics $z = 2$

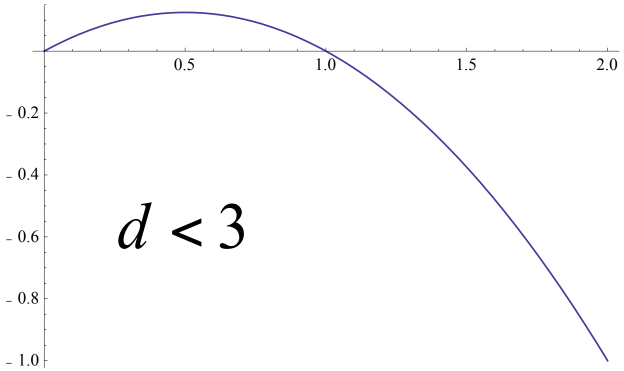
→ Known behavior for hot parts of the FS $\eta_h = \frac{3-d}{2}$

→ Critical FS elsewhere (strong coupling fixed point) $\eta_c = \frac{2d-3}{6}$

Renormalization group flow

hot-regions of the FS

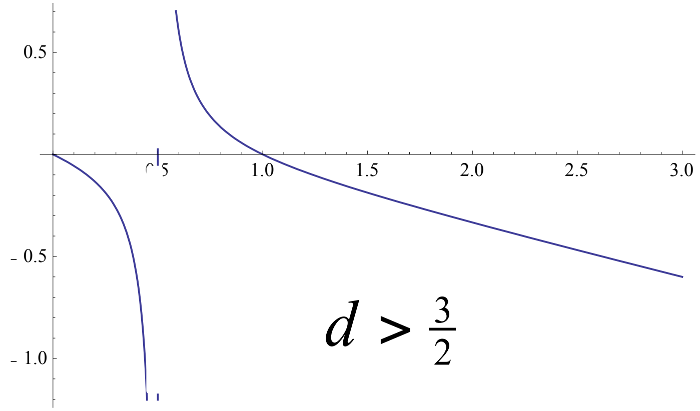
$$\beta_h = -\frac{\partial \alpha_h}{\partial \log \mu}$$



$d < 3$

cold-regions of the FS

$$\beta_c = -\frac{\partial \alpha_c}{\partial \log \mu}$$



$d > \frac{3}{2}$

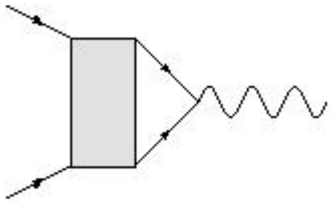
Self-consistency approach

$$\Sigma^{q.p.}(\omega;Z) = Z(\omega)\Sigma^{>}(\omega) \quad \text{with} \quad Z(\omega)^{-1} = 1 - \frac{\partial\Sigma^{>}(\omega)}{\partial\omega}$$

$$\Rightarrow \eta_c = \frac{2d-3}{6}(1-2\phi) \qquad \eta_h = \frac{3-d}{2} + 2\phi(d-1)$$

condition for vertices: bosonic syn. scaling exponent $z = \dfrac{2}{1-2\phi}$

- effects of other channels



conjecture: $\phi = \eta_c$ (large-Q Ward-Identity)

$$\Rightarrow \eta_c = \frac{2d-3}{4d}, \quad \eta_h = \frac{3+d}{4d} \quad \Rightarrow \quad z = \frac{4d}{3}$$

Scaling theory

$$\chi(\mathbf{Q}+\mathbf{q}) \propto \frac{1}{\xi^{-2}+q^2+i\gamma\omega/Z^2} \qquad Z(\omega,T,x-x_c)=b^{-\eta z}Z\left(b^z\omega,b^zT,b^{1/\nu}(x-x_c)\right)$$

field tuned transition $\quad [\xi^{-2}]=[H-H_c]-[Z] \qquad \left.\frac{\partial\Pi}{\partial H}\right|_{H=H_c} \propto (H-H_c)/Z$

P. Wölfle, E. Abrahams (2011)

$$\Rightarrow \nu = \frac{1}{2+z\eta} = \frac{3}{3+2d} \xrightarrow{d=3} \frac{1}{3}$$

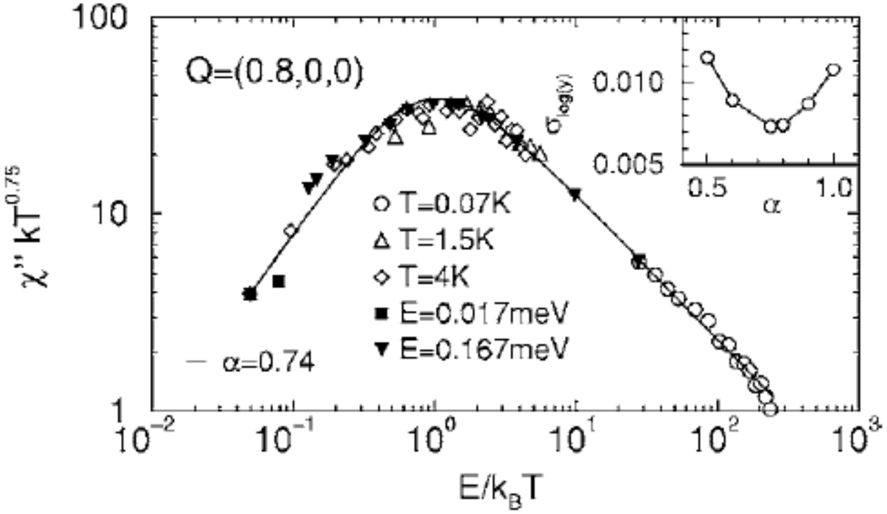
$$\gamma = 1+\nu\eta z = \frac{4d}{3+2d} \xrightarrow{d=3} \frac{4}{3} \qquad \beta = \nu d/2 \xrightarrow{d=3} \frac{1}{2} \qquad \delta = \frac{11}{3}$$

Note, there are two critical length scales: $\xi \propto T^{-1/z} \qquad \xi_F \propto T^{-1/z_F} >> \xi$

hyperscaling for critical fermions: $f(T)=b^{-(1+z_f)}f\left(b^{z_f}T\right)$

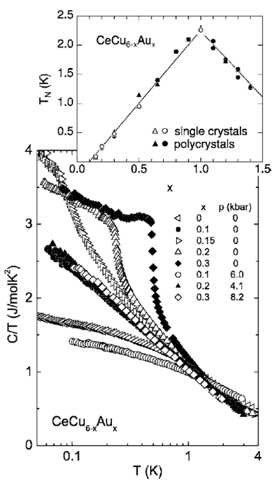
$$\Rightarrow C/T \propto T^{-\eta}$$

comparison with $CeCu_{6-x}Au_x$ (d=2)

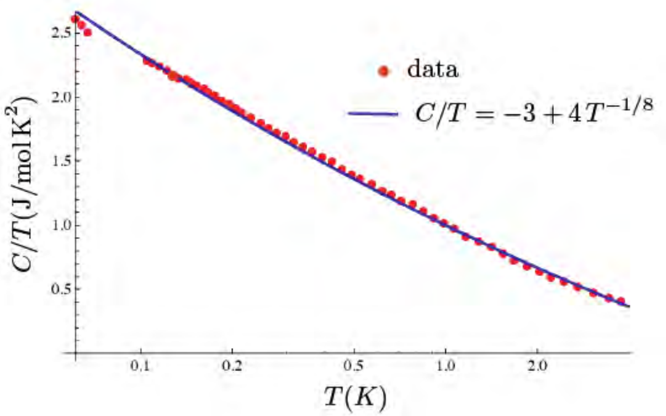


$$\chi(\mathbf{Q}+\mathbf{q}) \propto \frac{1}{q^2 + \left(i\gamma \operatorname{sign}(\omega)|\omega|\right)^{\alpha}}$$

$$\alpha(d=2) = 0.75 \qquad \alpha_{\exp} \approx 0.74$$



$$C/T \propto T^{-1/8}$$

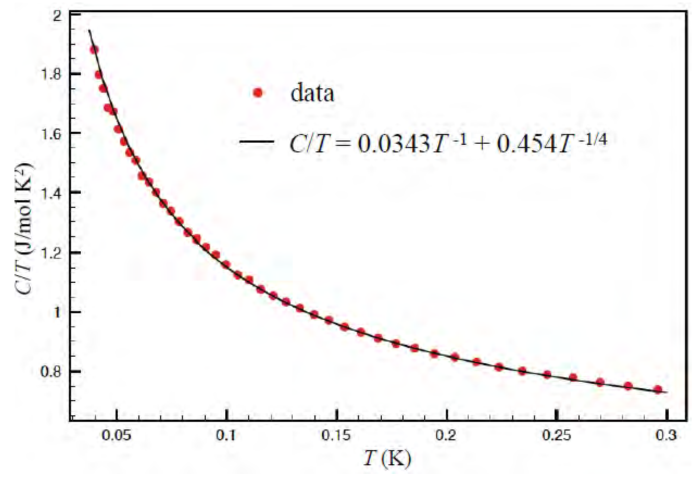


comparison with $YbRh_2Si_2$ (d=3)

Inside the critical cone:

$$C \propto T^{3/4}$$

E. Abrahams, PW, PNAS (2012)

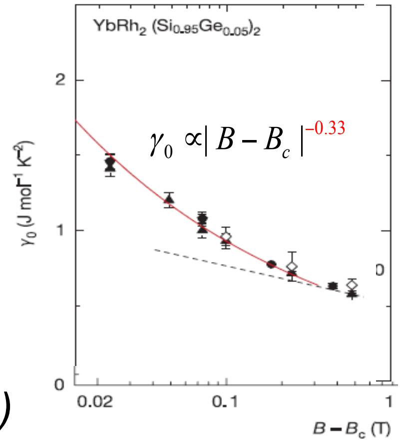


N. Oeschler et al., Physica B 403, 1254 (2008)

Outside the critical cone:

$$C \propto |H - H_c|^{-1/3}\, T$$

Custers et al., Nature (2003)



Conclusion

- Critical quasi-particles might be a powerful concept to combine Fermi liquid theory and quantum criticality.

$$G(\mathbf{k},\omega)=\frac{Z_{\mathbf{k}}(\omega)}{\omega+i\Gamma_{\mathbf{k}}(\omega)-\varepsilon_{\mathbf{k}}^{*}}, \qquad Z_{\mathbf{k}}(\omega)\propto|\omega|^{\eta}$$

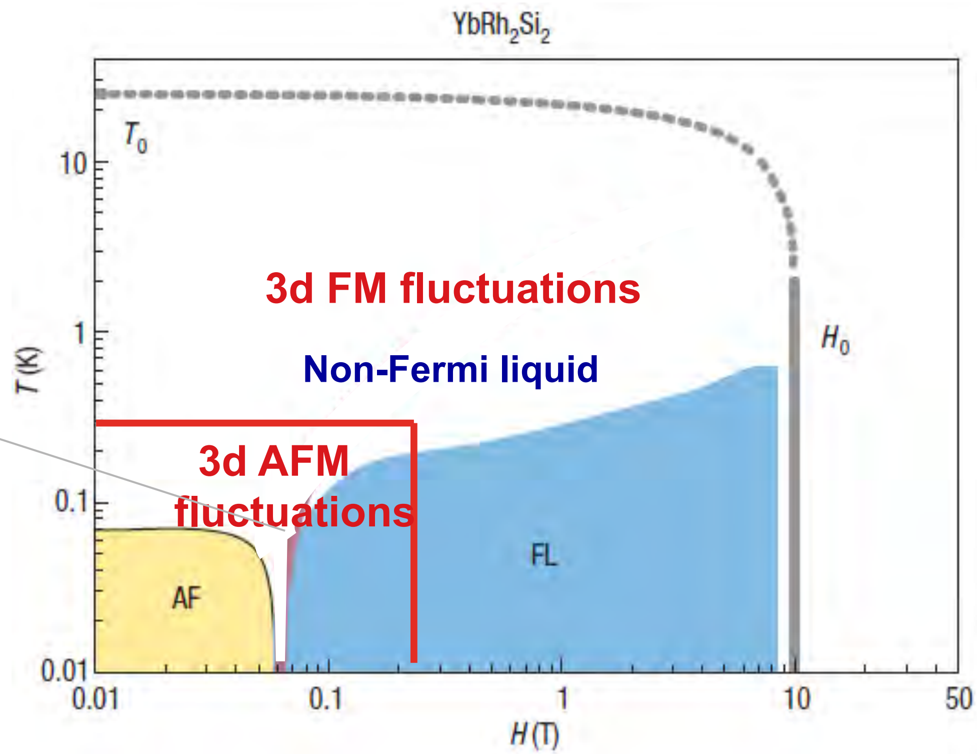
- Phenomenological approach: key assumption scale matching (high energy dynamics boosts singularities at low temperatures)

- Coupling to energy density fluctuations (higher loop spin fluctuations) yields explicit example for non-trivial critical quasi-particles

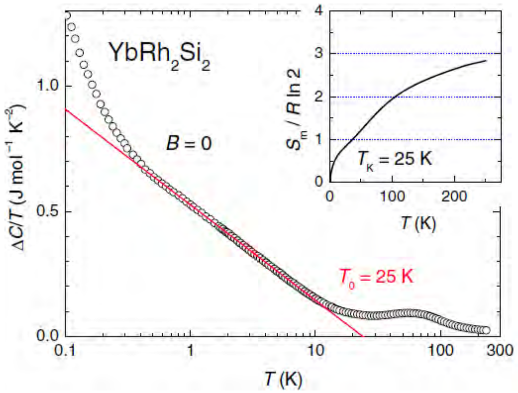
- Conjecture of a Ward-Identity: Results in good agreement with experiments on $YbRh_2Si_2$ (3-d fluctuations) and $CeCu_{6-x}Au_x$ (quasi-2-d fluctuations).

comparison with $YbRh_2Si_2$ (d=3)

$$z = 4/3, \quad \nu = 1/3$$



F. Steglich group (2000-present)
C. Broholm group (n-scatt.) (2012)



Trovarelli et al., PRL 85, 626 (2000)

Magnetic Grüneisen Ratio $YbRh_2Si_2$

Zhu, Garst, Rosch, Si (2003)

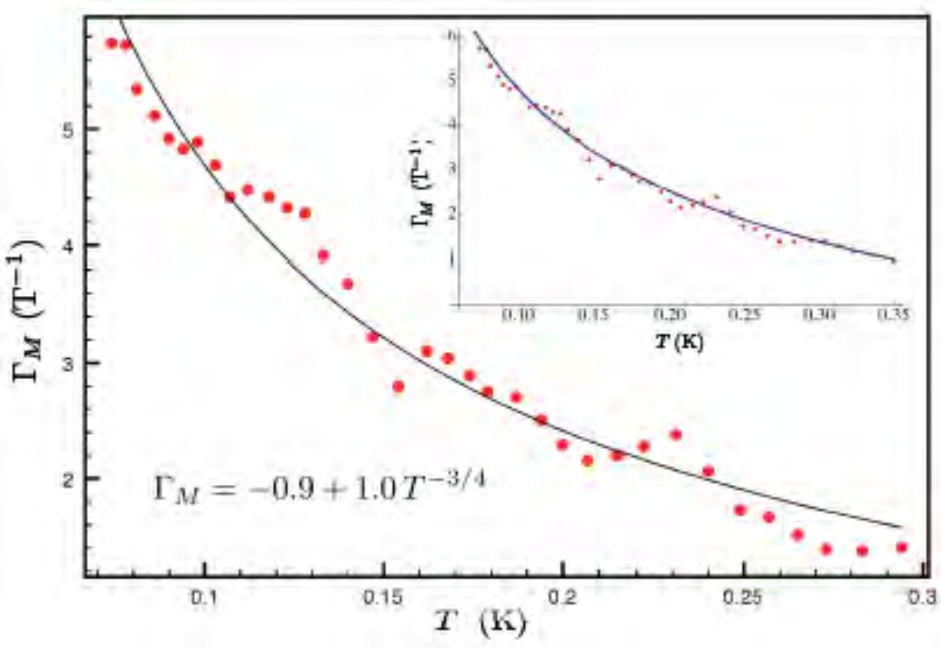
$$\Gamma_M = -\frac{(\partial M/\partial T)_H}{C_H} = -\frac{(\partial S/\partial H)_T}{C_H}$$

Inside the critical cone:

$$\Gamma_M(r=0,T) \propto T^{-3/4}$$

Outside the critical cone:

$$\Gamma_M(r,T=0) = \frac{\nu(z-d)}{H-H_c} = \frac{1/3}{H-H_c}$$



Tokiwa et al, PRL 2009

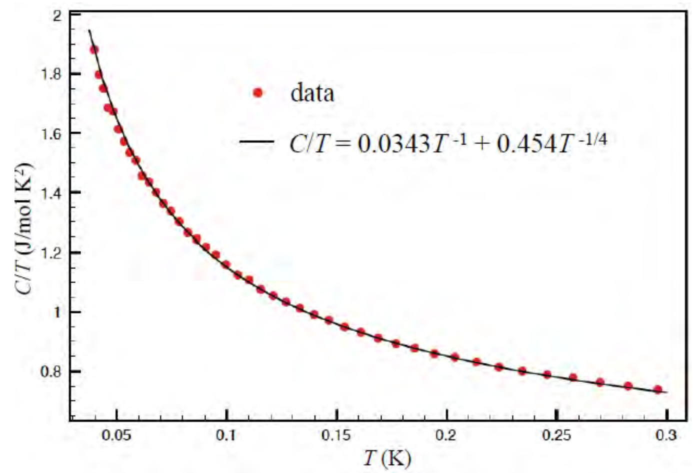
measured value of this universal coefficient: ≈ 0.3

comparison with $YbRh_2Si_2$ (d=3)

Inside the critical cone:

$$C \propto T^{3/4}$$

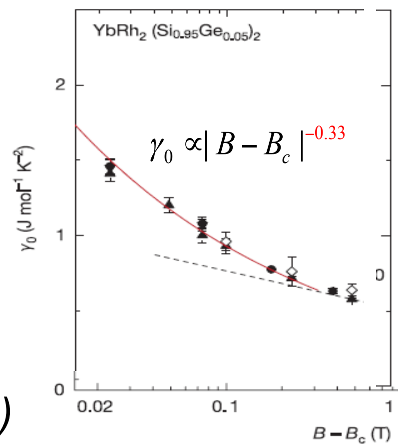
E. Abrahams, PW, PNAS (2012)



N. Oeschler et al., Physica B 403, 1254 (2008)

Outside the critical cone:

$$C \propto |H - H_c|^{-1/3}\, T$$



Custers et al., Nature (2003)