Holographic duality in condensed matter physics.

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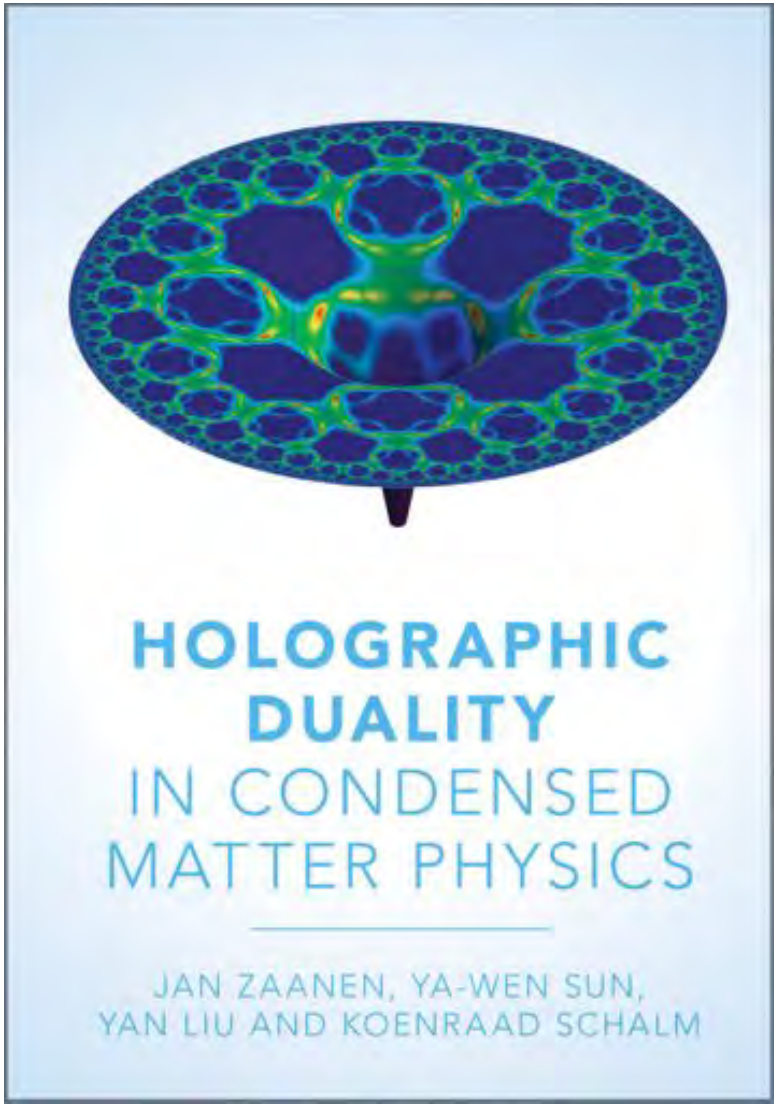
FOM

Book sales …



Cambridge University Press

Release: October 28 2015.

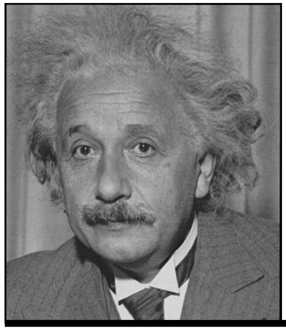
It is 600 pages and only € 80!

Plan of course.

1. Overview of AdS/CMT in pictures (slides).

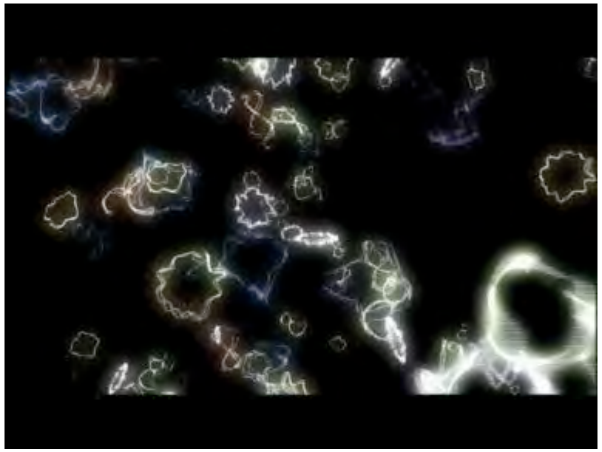
2. How the computations actually work: from GR and metrics to free energies and propagators with the GKPW rule (blackboard).

3. Physics highlights: strange metals, holographic superconductivity and Fermi liquids, transport, entanglement (slides).

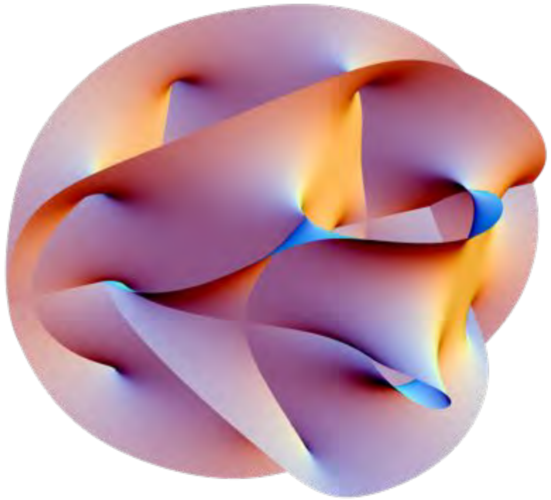


String theory

$$\mathcal{S} = \frac{T}{2}\int d^2\sigma\sqrt{-h}h^{ab}g_{\mu\nu}(X)\partial_a X^\mu(\sigma)\partial_b X^\nu(\sigma)$$



Quantum physics of strings



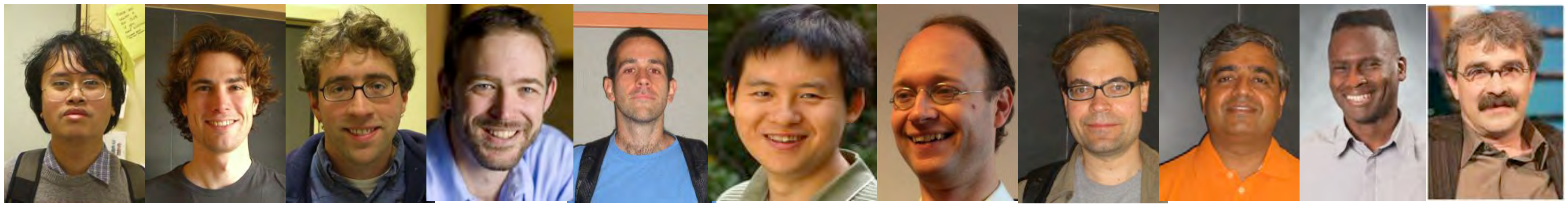
Beautiful mathematics (Calabi-Yau, …)

Quantum space-time (big bang, …)

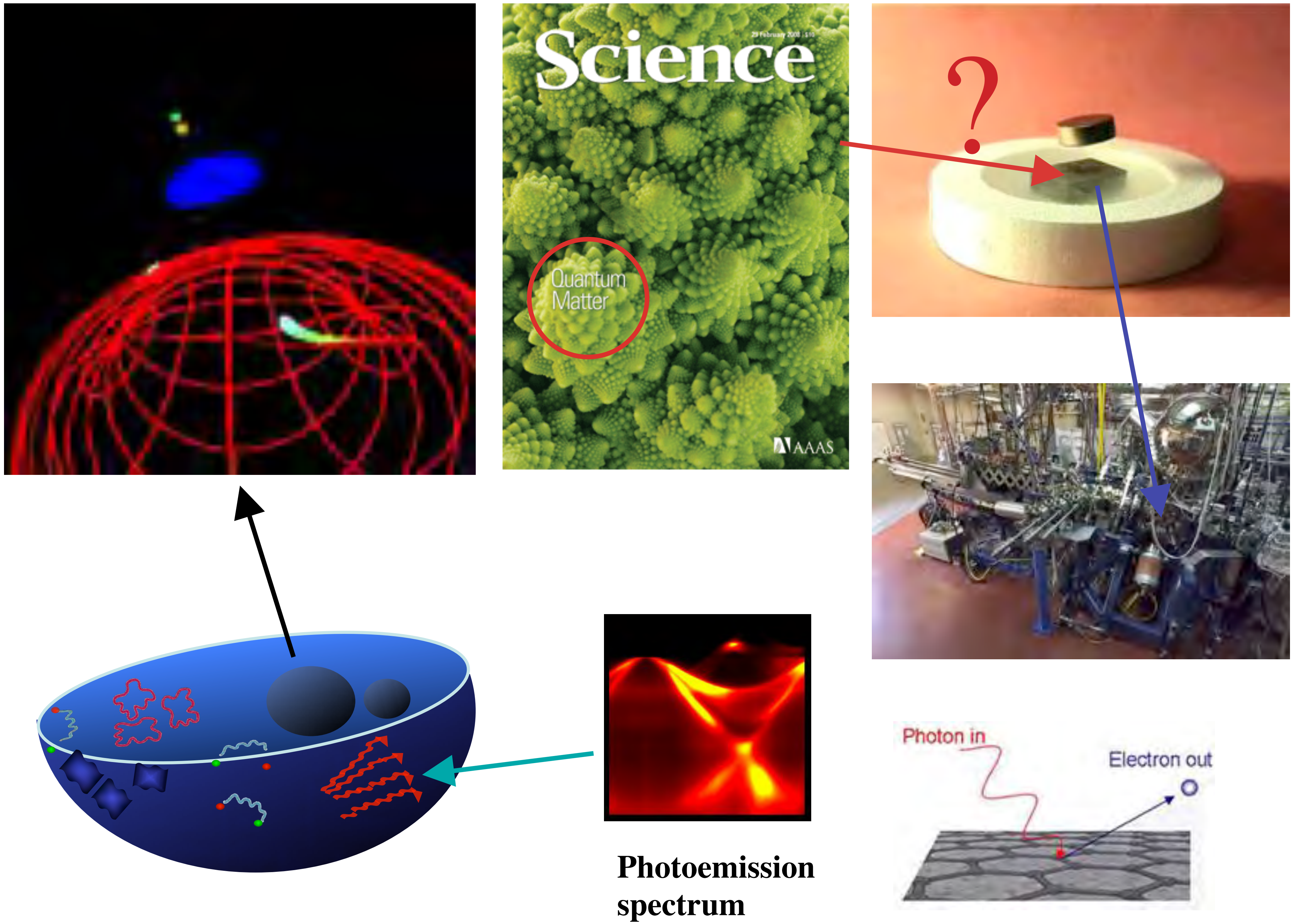
String theory: what is it really good for?

- Quantum matter: heavy fermion systems, high Tc superconductors, Quark-gluon plasma …

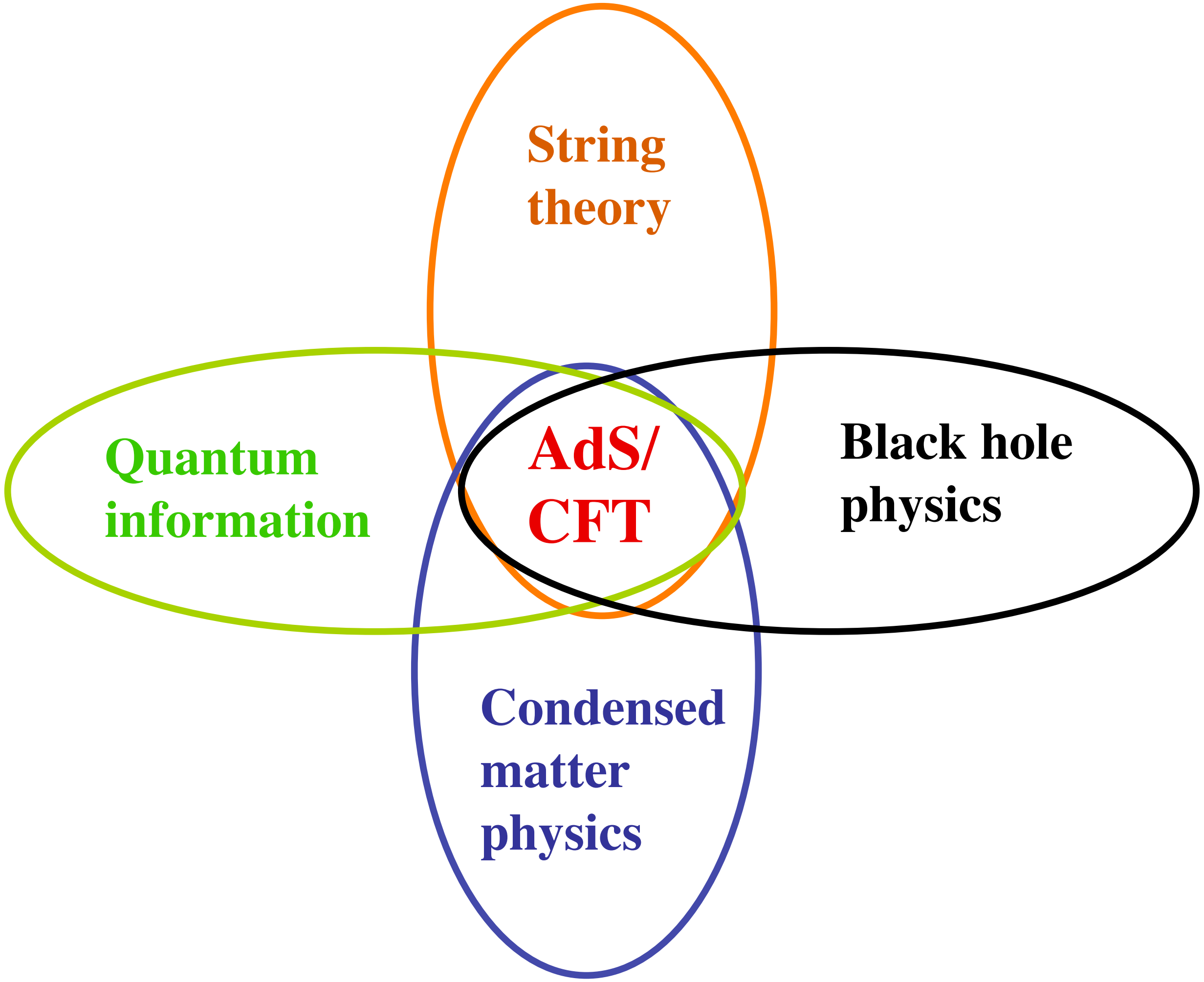
Polchinski Kachru Starinets Erdmenger Gubser Horowitz Silverstein Gauntlett Policastro Tong



Son Hartnoll Herzog Adams McGreevy Liu Schalm Karch Sachdev Phillips Zaanen


Science
Quantum Matter
AAAS
?
Photon in
Electron out
Photoemission spectrum

The cloverleaf of matter ….

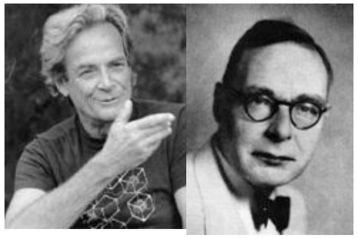


Quantum field theory = Statistical physics.

$$Z = \sum_{configs.} e^{-\frac{E_{config}}{k_B T}}$$

Path integral mapping ↓ **"Thermal QFT", Wick rotate:**

$$t \to i\tau$$



$$Z_\hbar = \sum_{worldhistories} e^{-\frac{S_{history}}{\hbar}}$$

But generically: the quantum partition function is not probabilistic: "sign problem", no mathematical control!

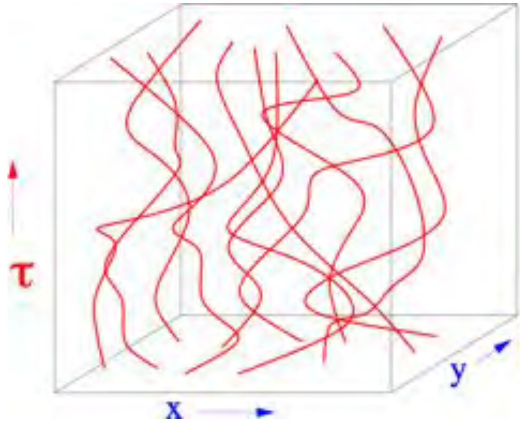
$$Z_\hbar = \sum_{worldhistories} (-1)^{history} e^{-\frac{S_{history}}{\hbar}}$$

Fermions at a finite density: the sign problem.

Imaginary time first quantized path-integral formulation

$$\mathcal{Z} = \mathrm{Tr}\exp(-\beta\hat{\mathcal{H}}) = \int \mathrm{d}\mathbf{R}\rho(\mathbf{R},\mathbf{R};\beta)$$

$$\mathbf{R} = (\mathbf{r}_1, \ldots, \mathbf{r}_N) \in \mathbb{R}^{Nd}$$



$$\rho_{B/F}(\mathbf{R},\mathbf{R};\beta) = \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\rho_D(\mathbf{R},\mathcal{P}\mathbf{R};\beta)$$
$$= \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\int_{\mathbf{R}\to\mathcal{P}\mathbf{R}}\mathcal{D}\mathbf{R}(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^{\hbar/T}\mathrm{d}\tau\left(\frac{m}{2}\dot{\mathbf{R}}^2(\tau)+V(\mathbf{R}(\tau))\right)\right\}$$

Boltzmannons or Bosons:

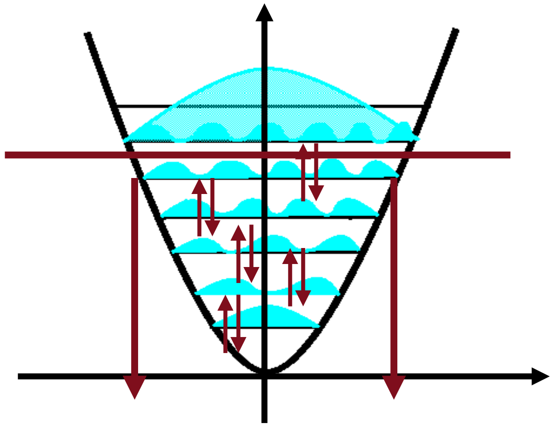
- integrand non-negative
- probability of equivalent classical system: (crosslinked) ringpolymers

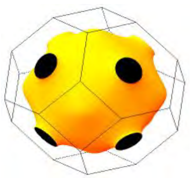
Fermions:

- negative Boltzmann weights
- non probablistic: NP-hard problem (Troyer, Wiese)!!!

Fermions: the tiny repertoire …

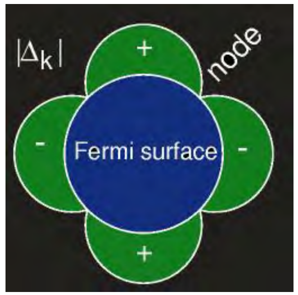
Fermiology

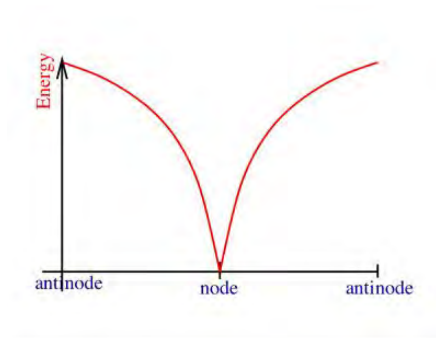




BCS superconductivity

$$\Psi_{BCS} = \Pi_k \left(u_k + v_k c^+_{k\uparrow} c^+_{-k\downarrow} \right) \left| vac. \right\rangle$$





29 years later: the "consensus document".

REVIEW

doi:10.1038/nature14165

From quantum matter to high-temperature superconductivity in copper oxides

B. Keimer[1], S. A. Kivelson[2], M. R. Norman[3], S. Uchida[4] & J. Zaanen[5]

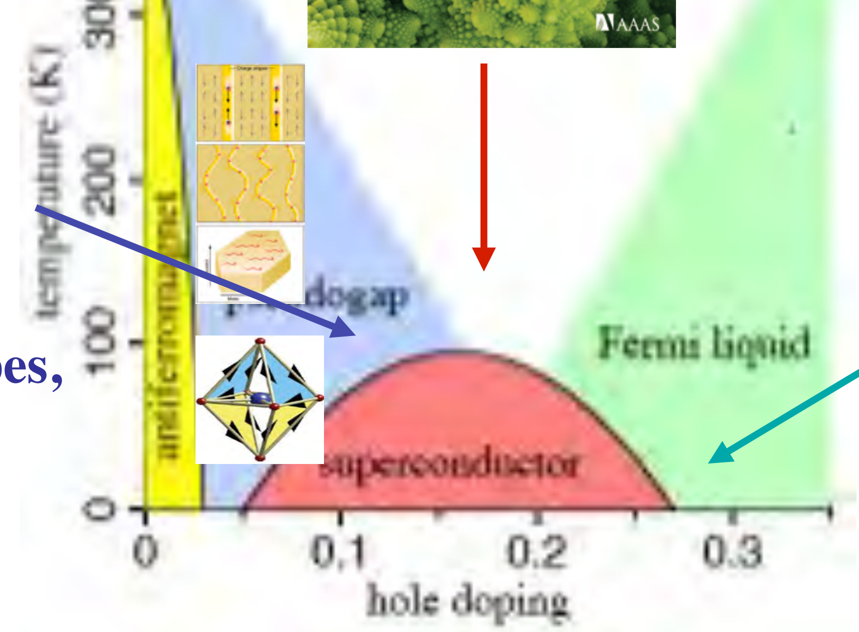
The discovery of high-temperature superconductivity in the copper oxides in 1986 triggered a huge amount of innovative scientific inquiry. In the almost three decades since, much has been learned about the novel forms of quantum matter that are exhibited in these strongly correlated electron systems. A qualitative understanding of the nature of the superconducting state itself has been achieved. However, unresolved issues include the astonishing complexity of the phase diagram, the unprecedented prominence of various forms of collective fluctuations, and the simplicity and insensitivity to material details of the 'normal' state at elevated temperatures.

The high Tc enigma.

The clash: the quantum critical metal

… which is good for superconductivity!

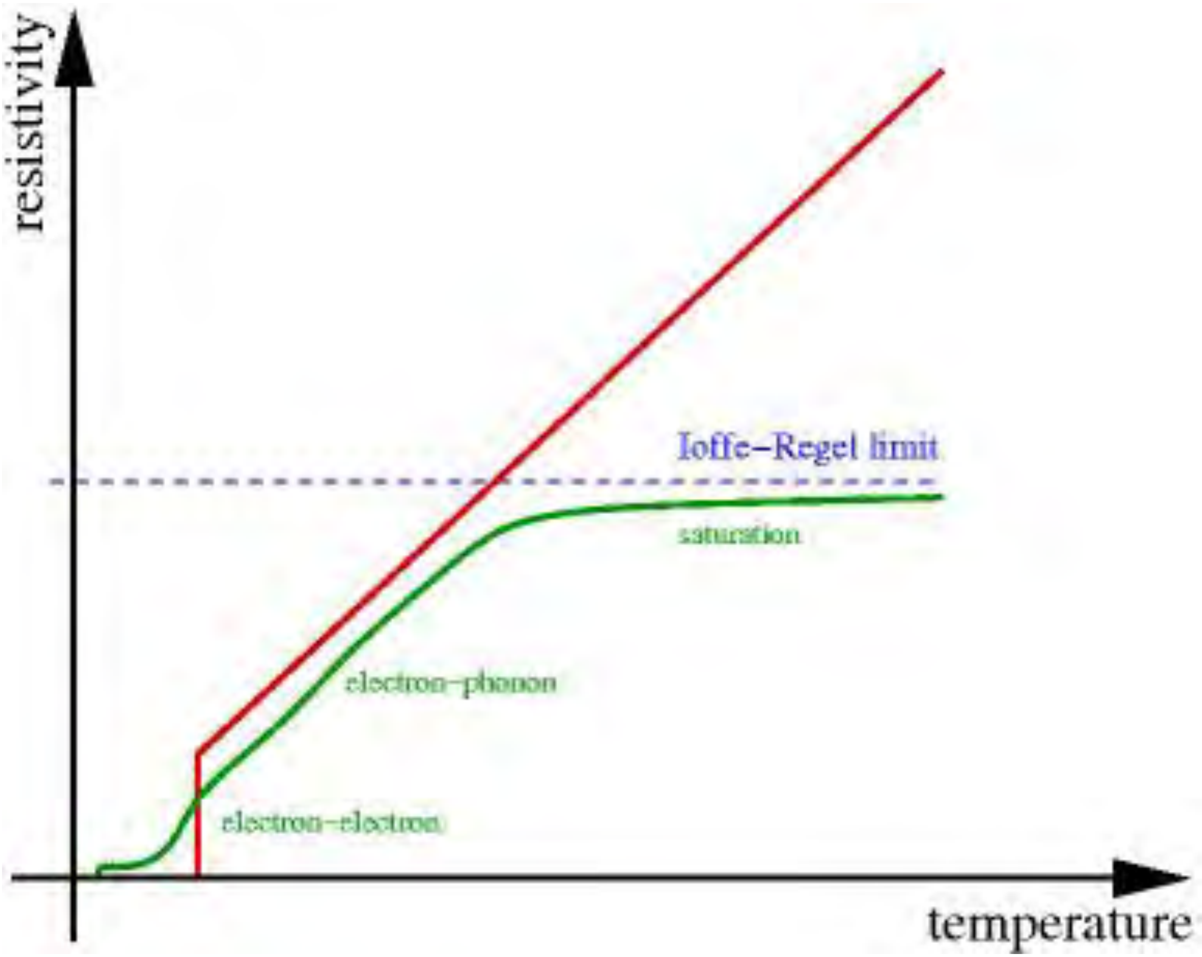
The quantized traffic jam



Exotic orders: stripes, orbital currents, nematics …

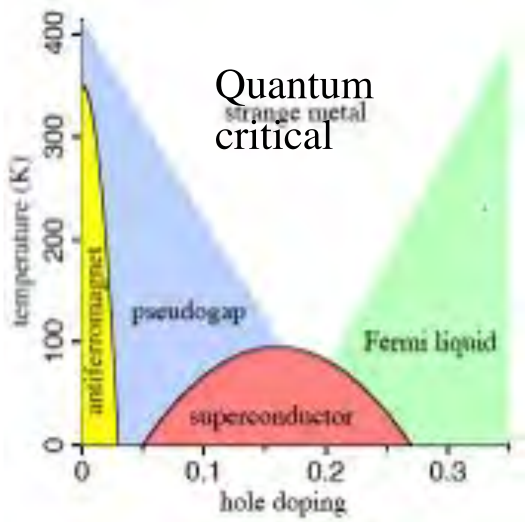
The quantum fog (Fermi gas) returns

Divine resistivity

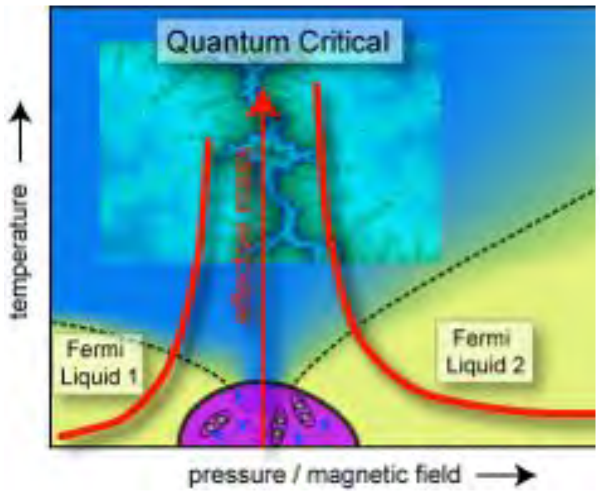


A universal phase diagram

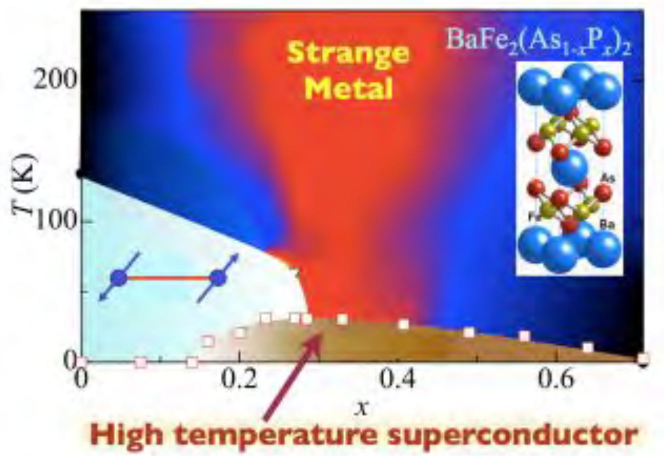
High Tc superconductors



Heavy fermions



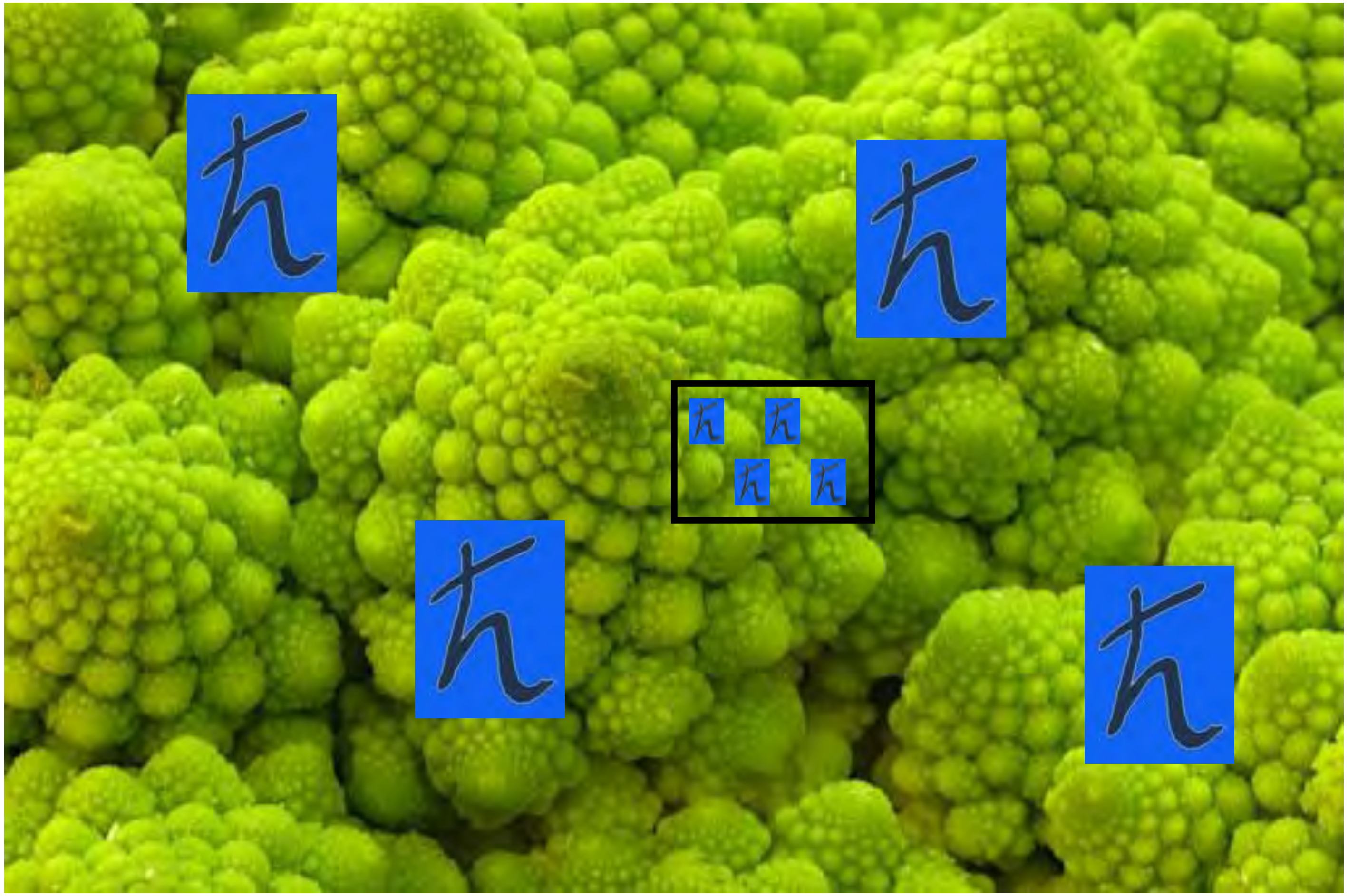
Iron superconductors (?)



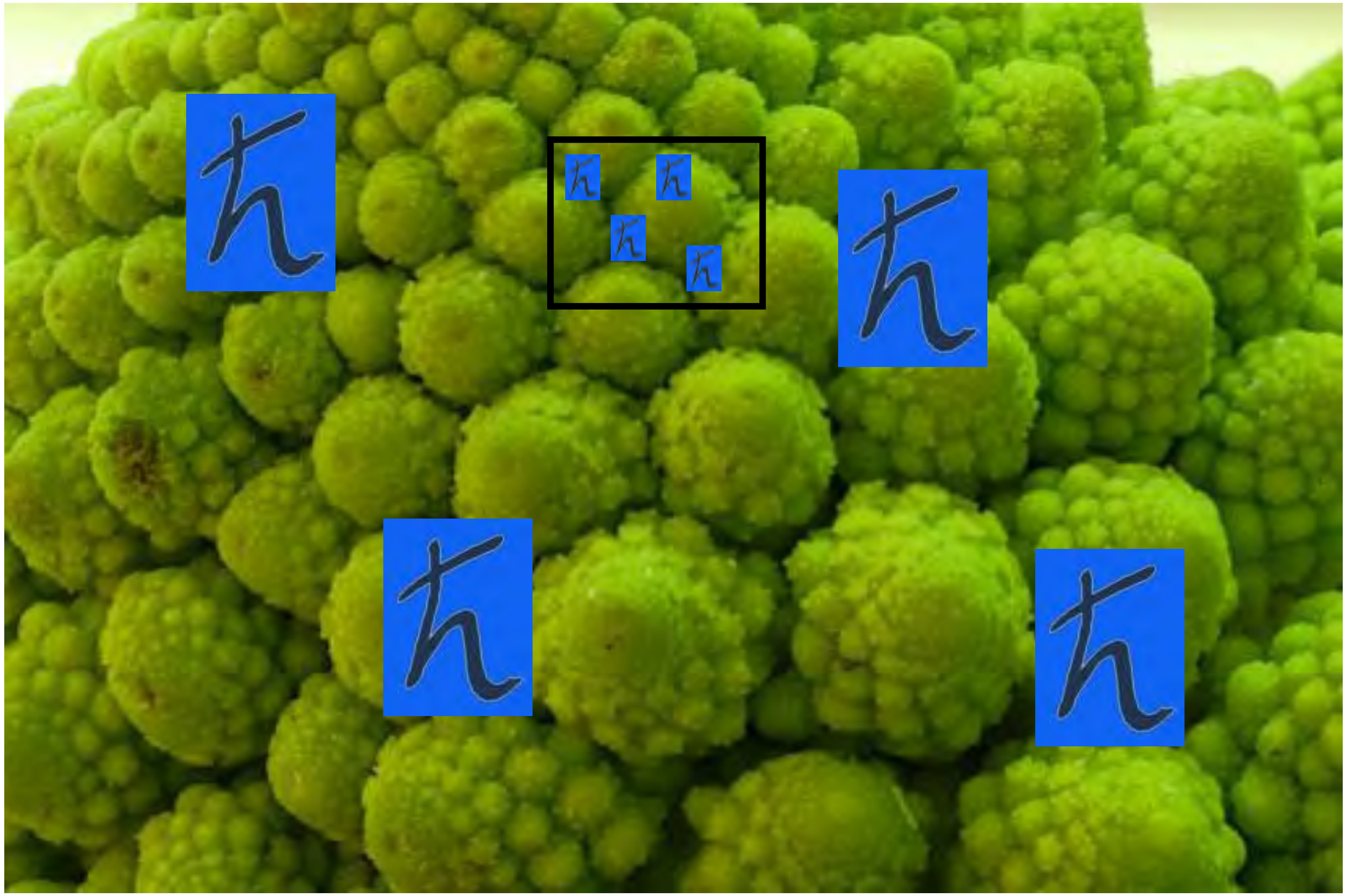
Fractal Cauliflower (romanesco)

Quantum Critical Cauliflower

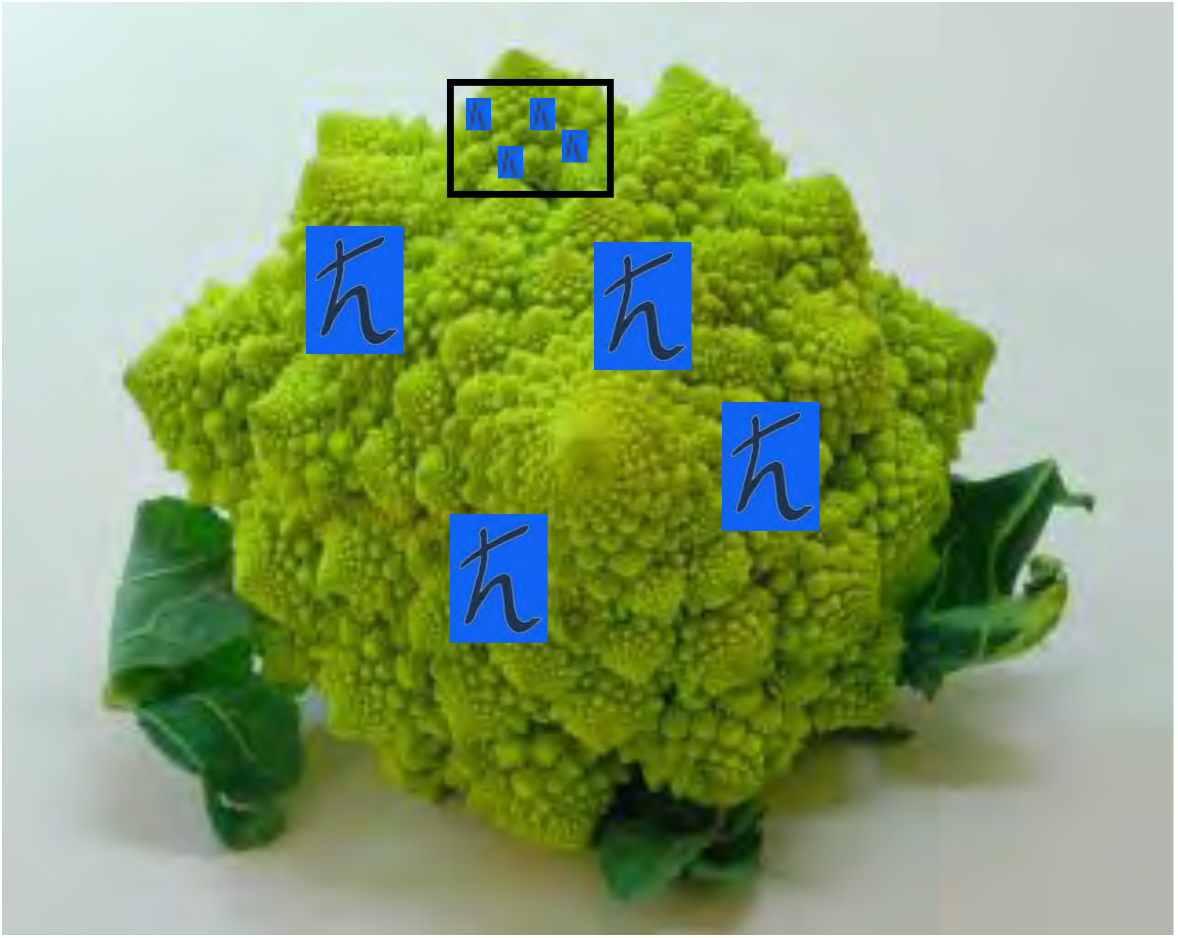
Quantum Critical Cauliflower



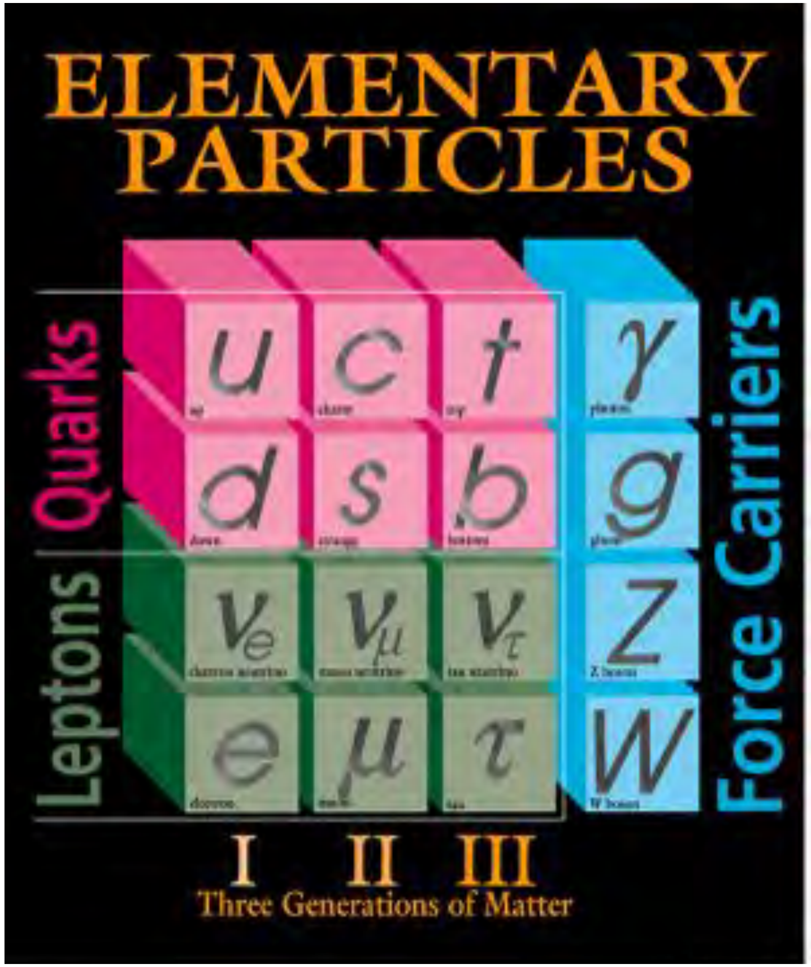
Quantum Critical Cauliflower

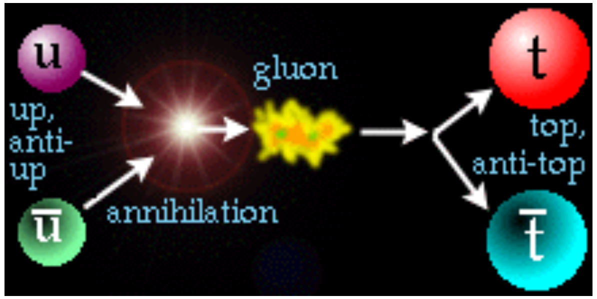


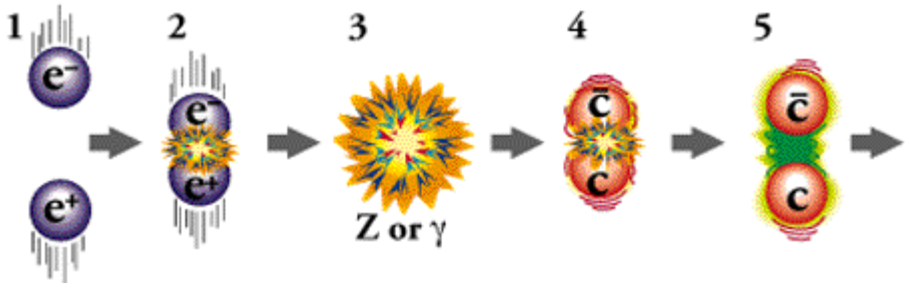
Quantum Critical Cauliflower



High energy physics: elementary particles.

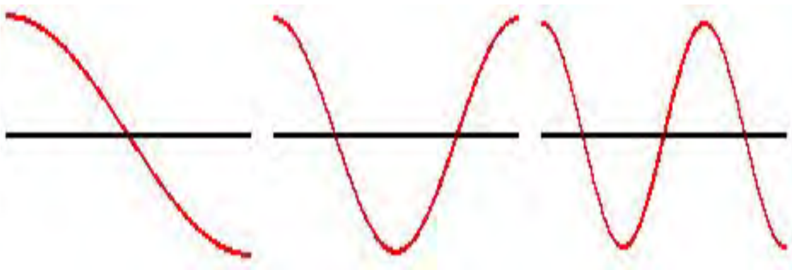






Particles as string vibrations (1980's)

String vibration modes→ Different particles

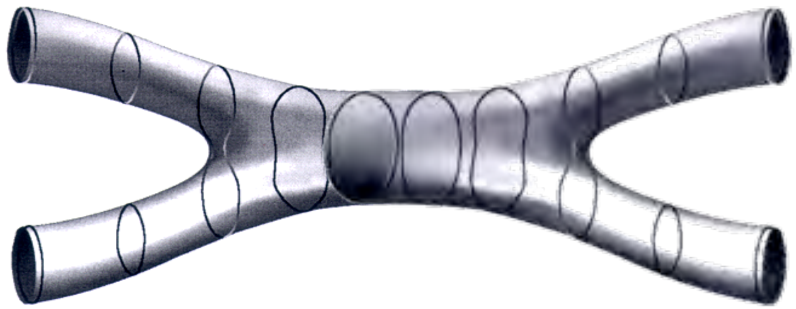


up quark

down quark

electron

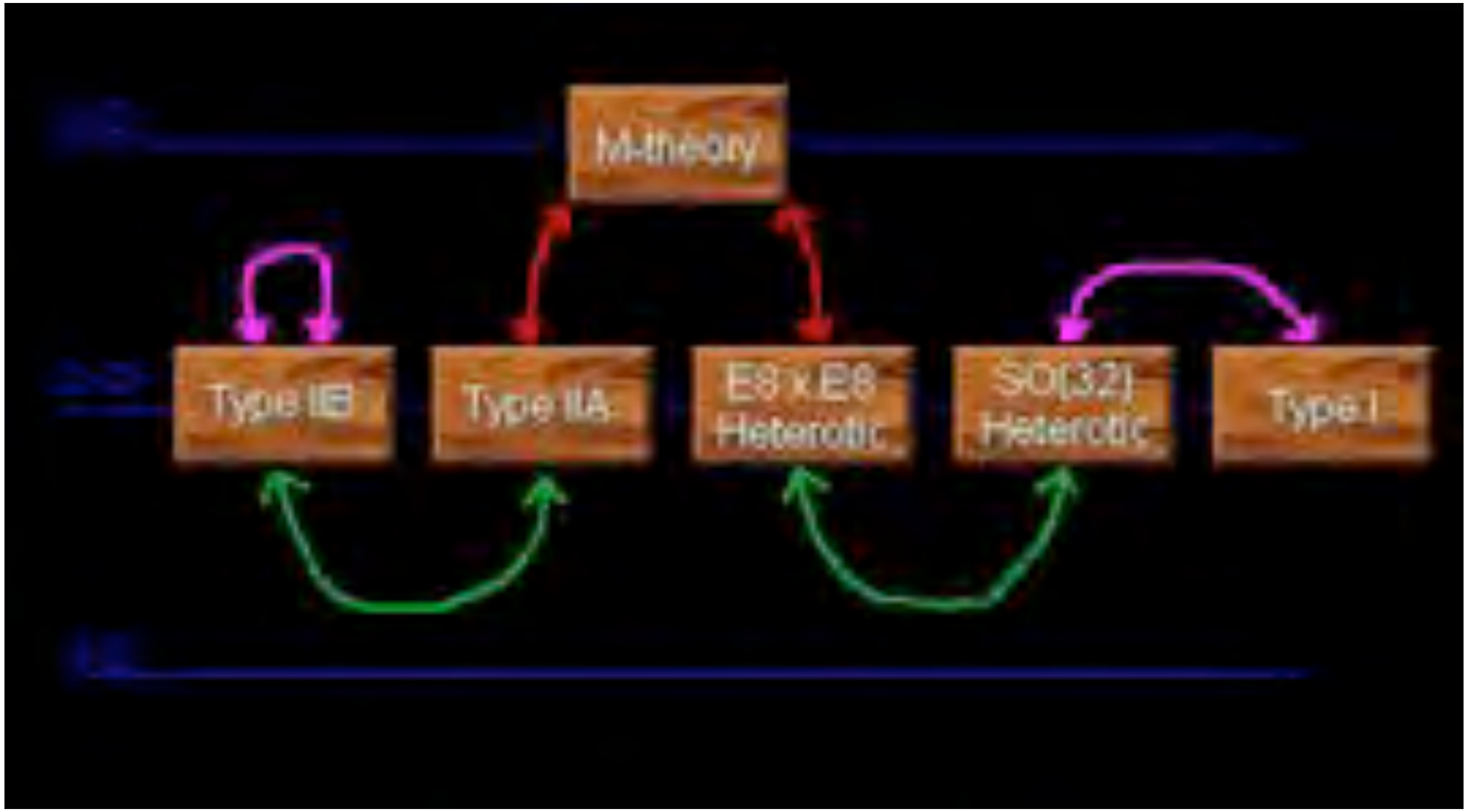
Morphing strings → Particle interactions



=>Unified theory: one string = all particles

=> Vibrations of "closed strings" describe gravitons (quantum particles carrying gravitational force).

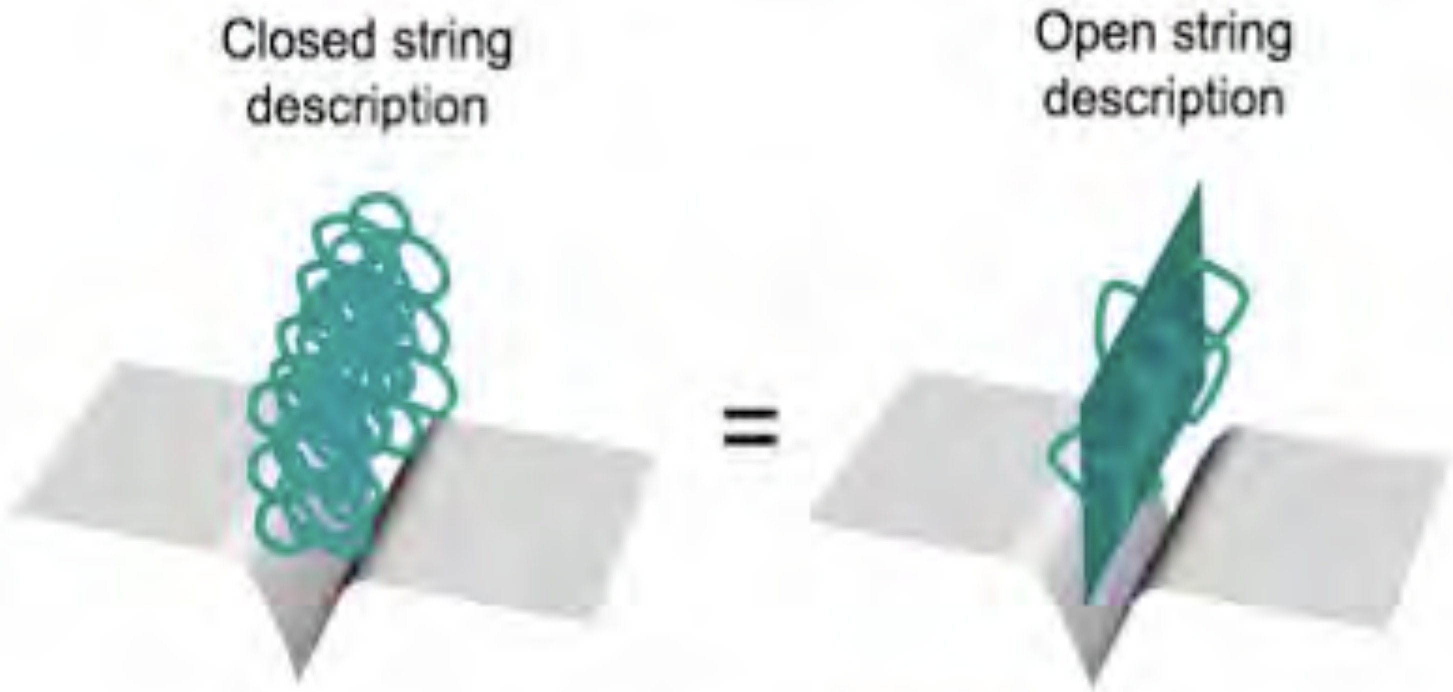
The "second string revolution" (1995)



Dualities

AdS/CFT correspondence

We have two different descriptions for same object!



Especially, in the case of D3-brane, at low energy these two description will be approximated by ....

AdS/CFT correspondence

Closed string description

=

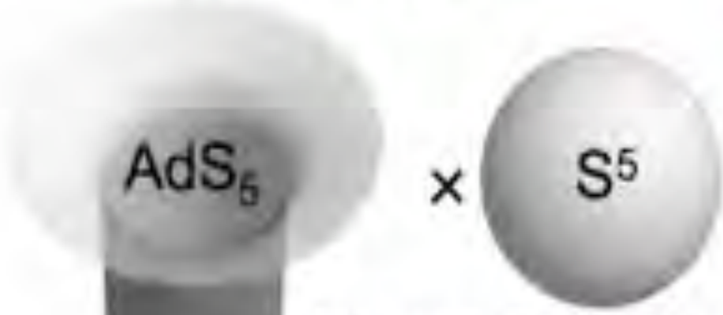
Open string description

Low energy limit with large N

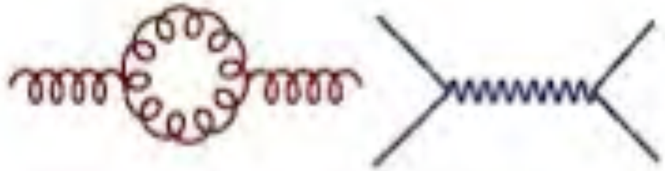
Supergravity on $AdS_5 \times S^5$

⟺ dual

D=4 $\mathcal{N}$=4 U(N) Super Yang-Mills



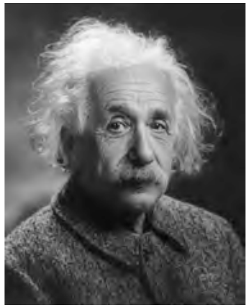
Good approximation for strong coupling
$\lambda = g_{YM}^2 N \gg 1$

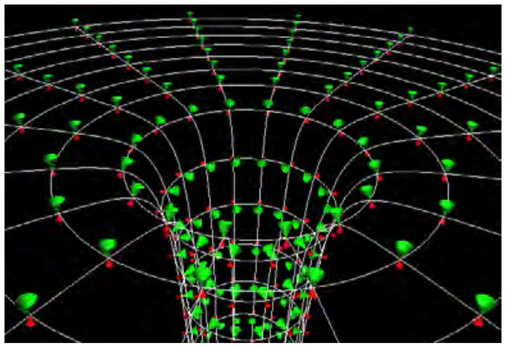


Good approximation for weak coupling
$\lambda = g_{YM}^2 N \ll 1$

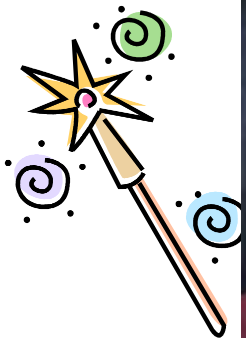
General relativity "=" quantum field theory

General relativity





'AdS/CFT'

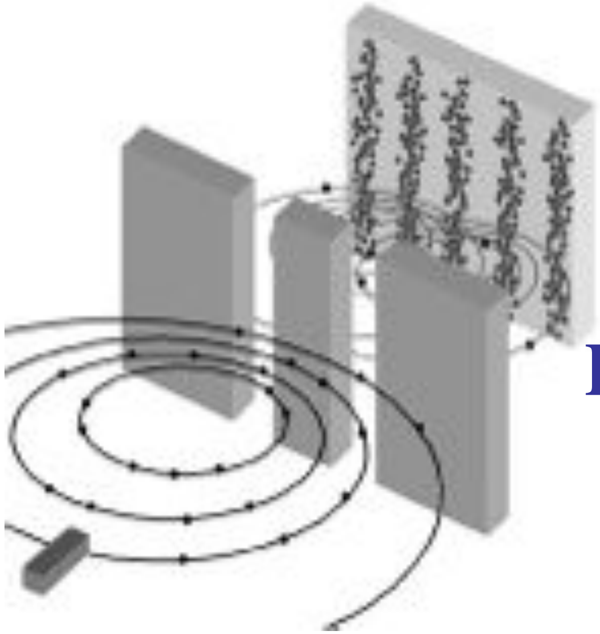


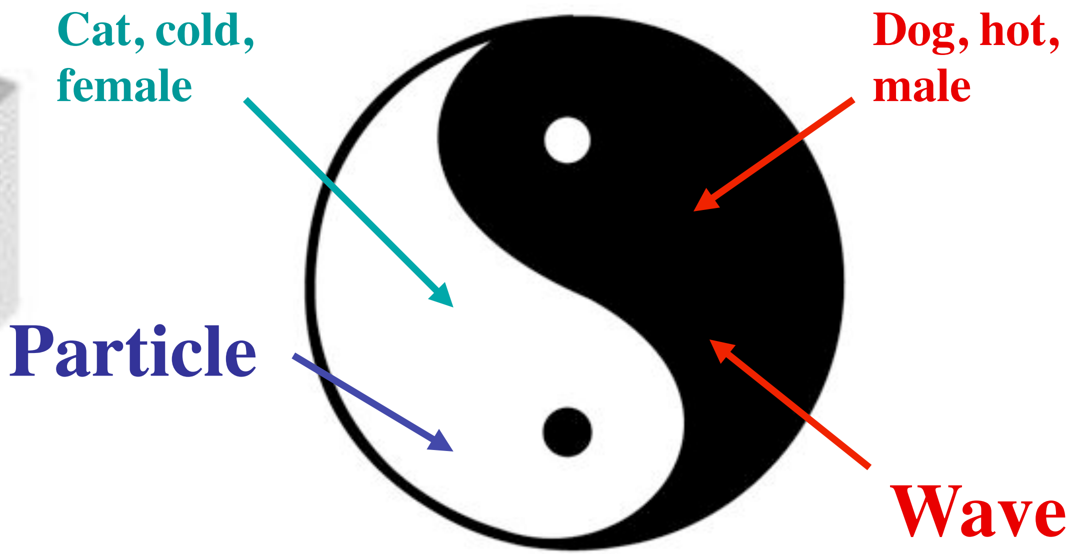
Maldacena 1997

=

Quantum fields

Particle-wave duality.





"The exact opposites form an indivisable whole"

Heisenberg uncertainty relation: $\Delta p \times \Delta x \geq \hbar$

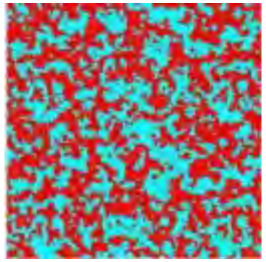
Weak-Strong or Kramers-Wannier duality

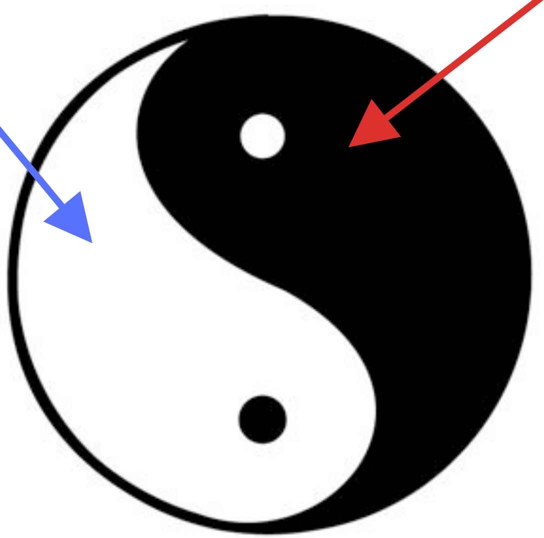
Kramers Wannier

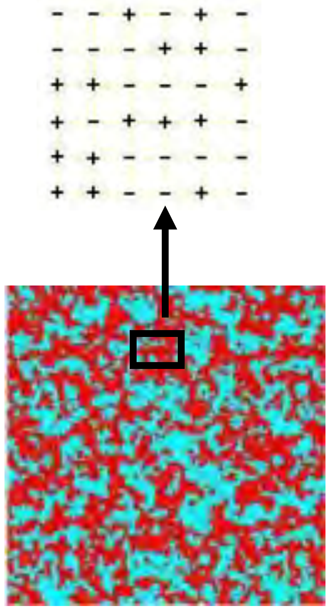
Low temperature or weakly coupled

$$H = \frac{J}{k_B T} \sum_{<ij>} S_i^z S_j^z$$

High temperature or strongly coupled

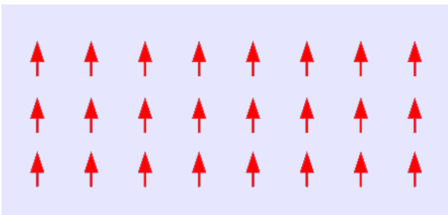






= domain wall condensate =

$$H = \frac{k_B T}{J} \sum_{<ij>} \tilde{S}_i^z \tilde{S}_j^z$$



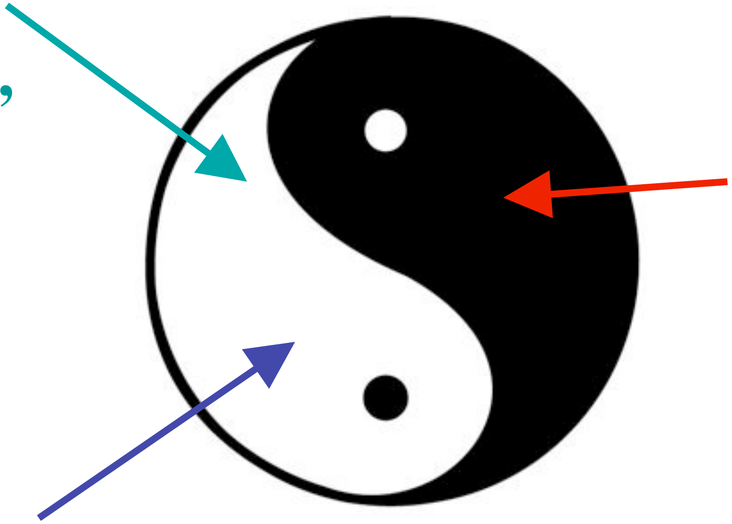
Self-duality special to 2D: e.g. in 3D global Ising dual to Ising gauge theory.

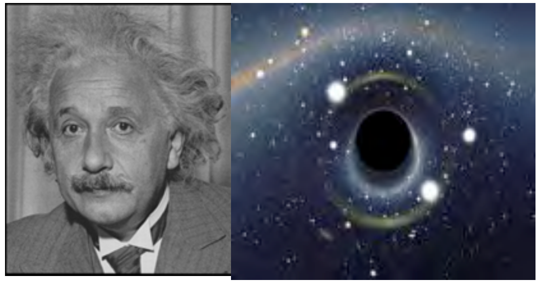
The Grand Unified "holographic" duality.

"AdS/CFT"

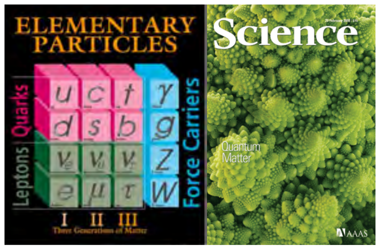
the exact opposites form an indivisible whole

Thermal matter, dissipation





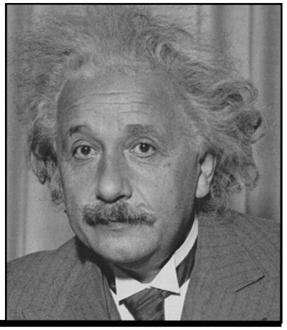
General relativity



Kramers/Wannier-, Local-gobal duality.

Quantum fields- Quantum matter

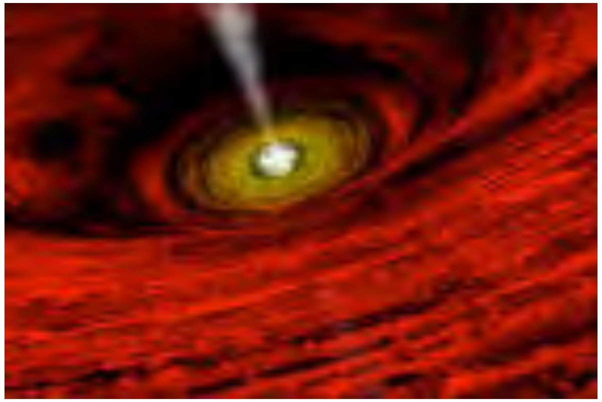
Einstein



Einstein equations (theory of general relativity):

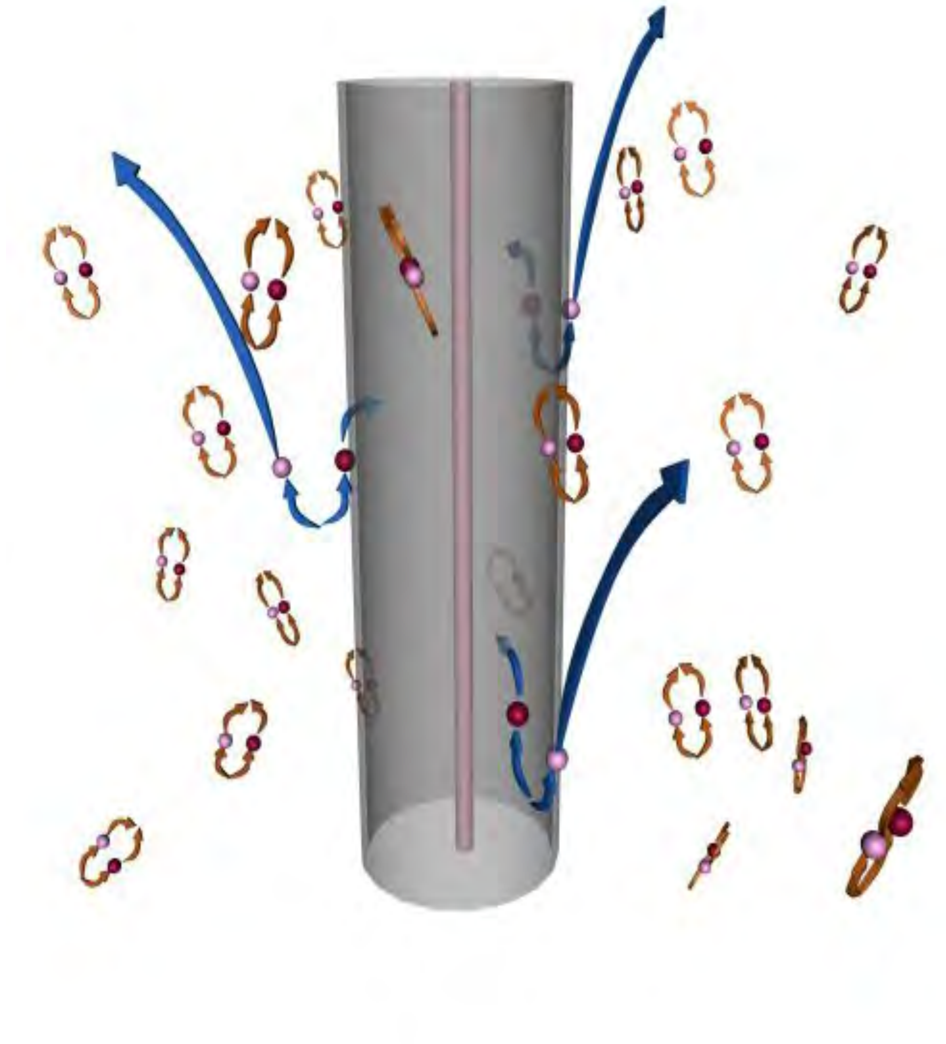
$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

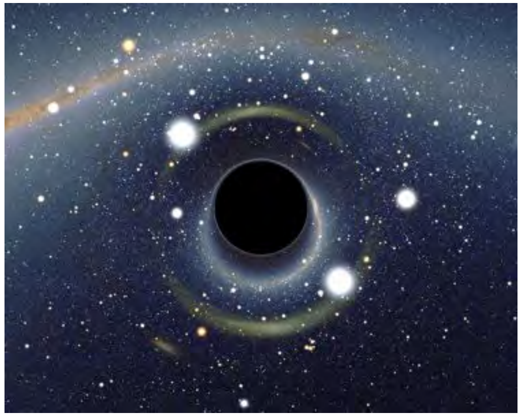
Space time as 'fabric'



Matter and energy

Hawking radiation: space-time turns into material stuff …

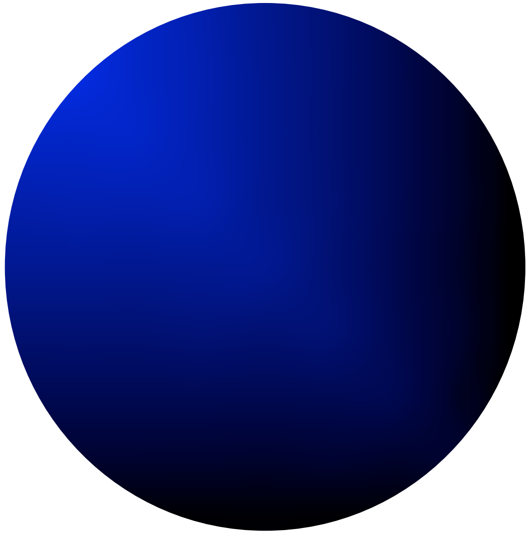




‘t Hooft’s holographic principle

Hawking Temperature: $T = \frac{\hbar g}{2\pi kc}$

g = acceleration at horizon

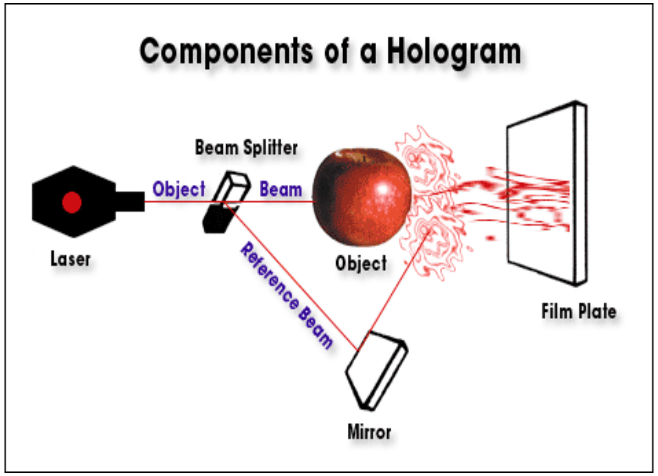


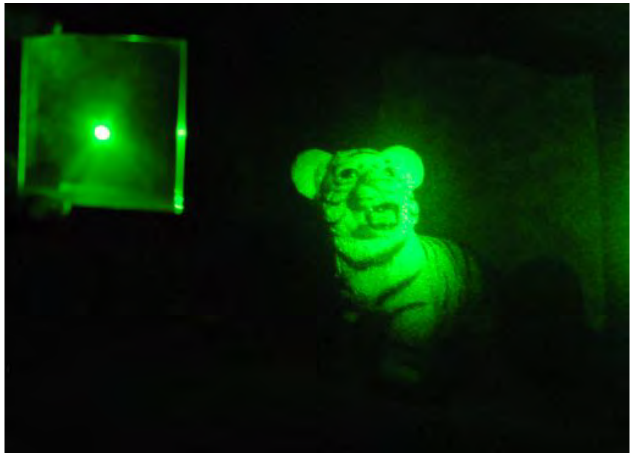
BH entropy: $S = \frac{kc^3 A}{4\hbar G}$

***A* = area of horizon**

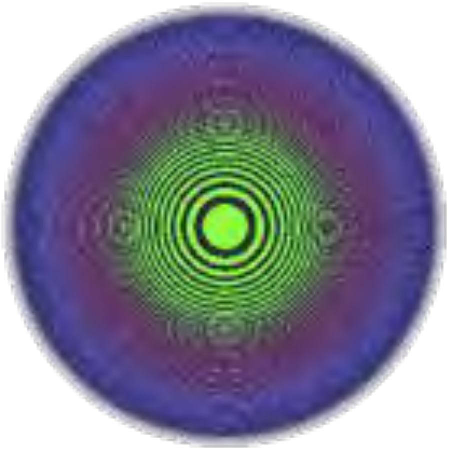
Number of degrees of freedom (field theory) scales with the area and not with the volume (gravity)

Holography with lasers





Three dimensional image



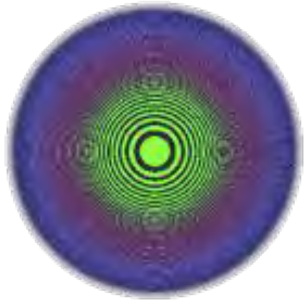
Encoded on a two dimensional photographic plate

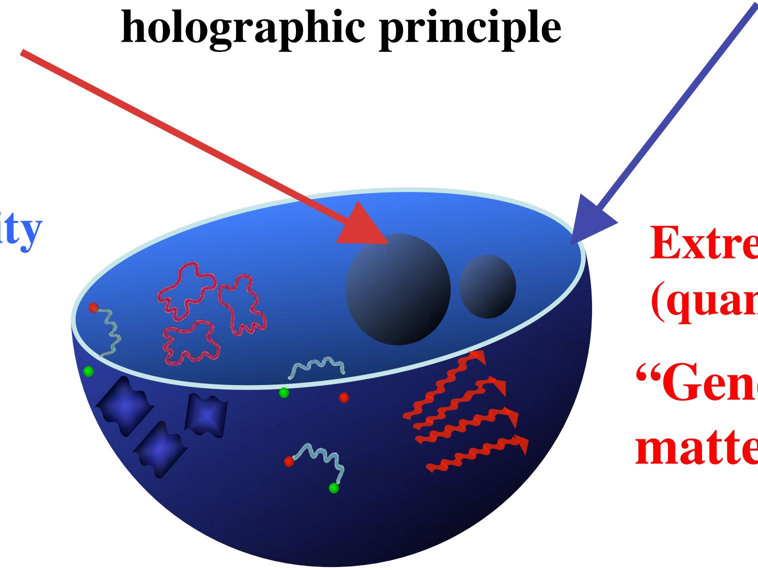
Holographic gauge-gravity duality

Einstein Universe "AdS"

't Hooft-Susskind holographic principle

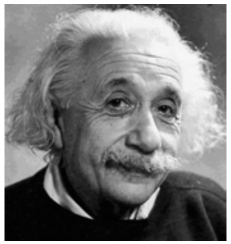
Quantum field world "CFT"





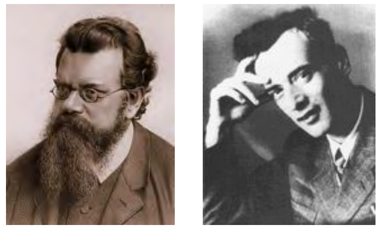
Classical general relativity

Uniqueness of GR solutions

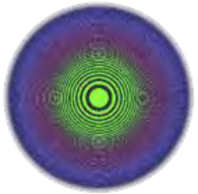


Extremely strongly coupled (quantum) matter

"Generating functional of matter emergence principle"

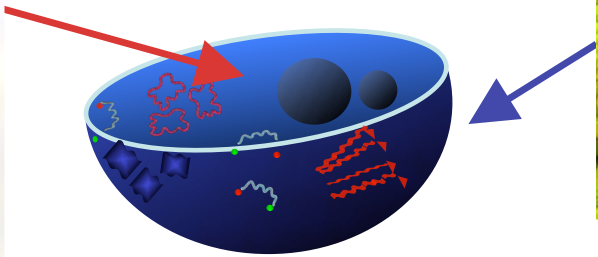


Holography and scale invariance.

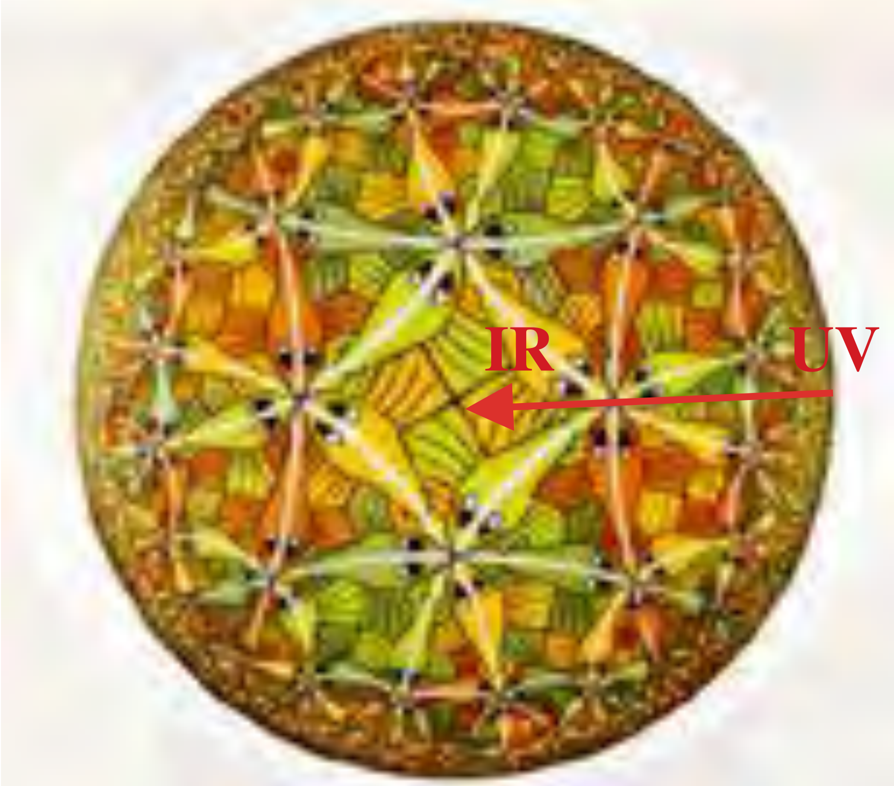


Einstein universe "AdS" =
Anti de Sitter universe

Quantum field theory
"CFT"= quantum critical



"GeneralRelativity = RenormalizationGroup"

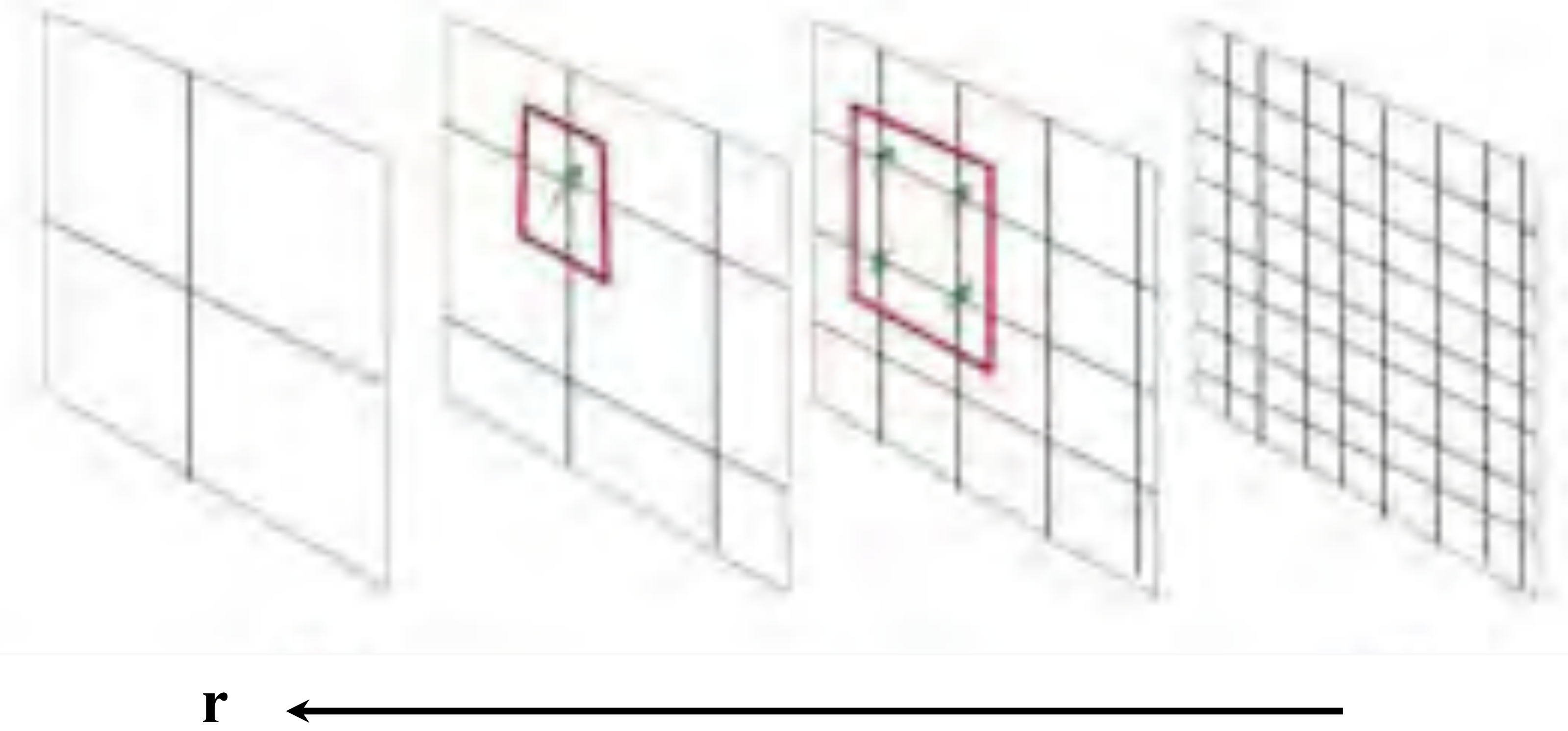


Extra radial dimension of the bulk <=> scaling "dimension" in the field theory

Bulk AdS geometry = **scale invariance of the field theory**

$$dr^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$F(r) = -\Lambda r^2 + 1, \qquad \Lambda < 0$$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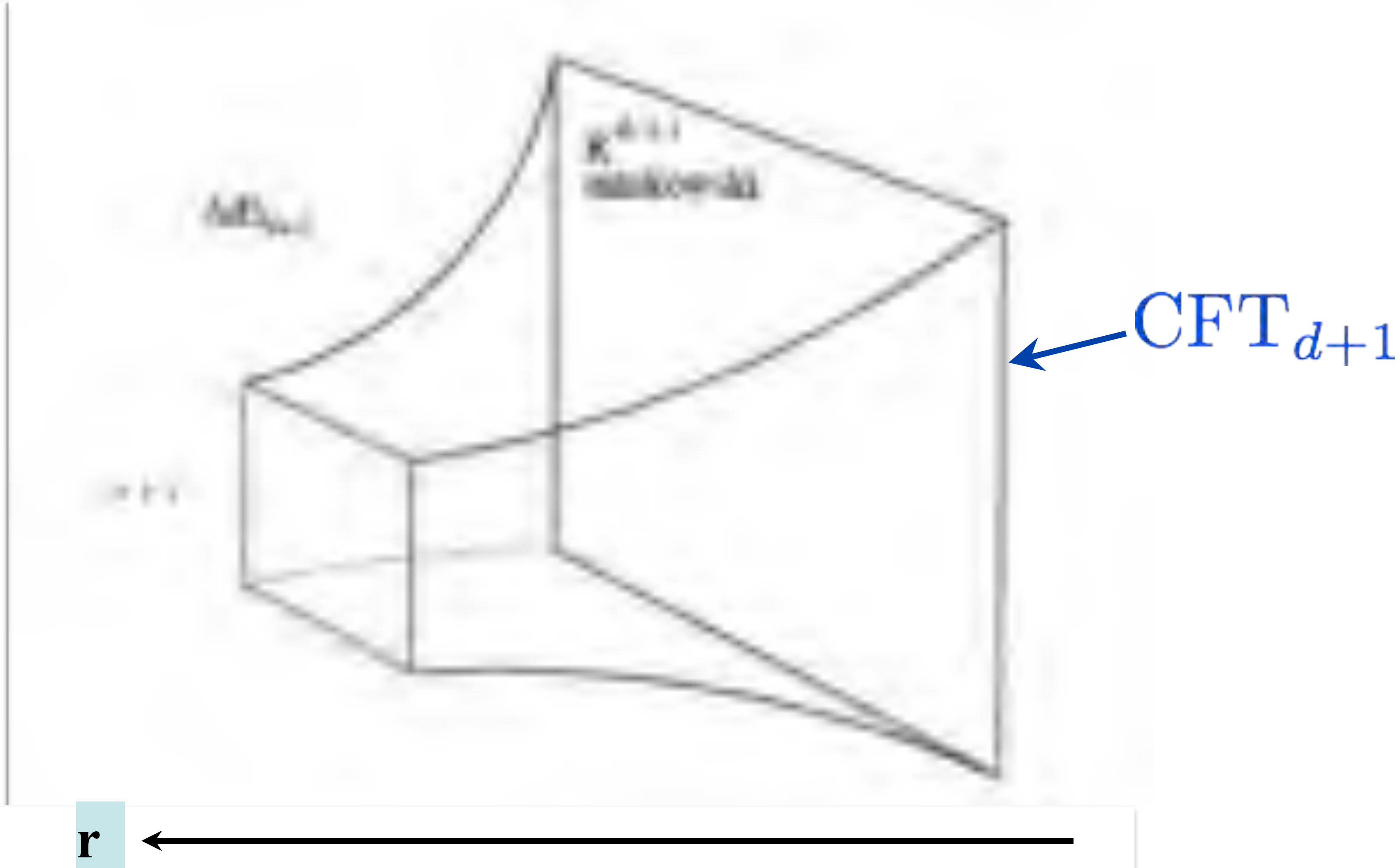


J. McGreevy, arXiv0909.0518

GKPW rule: propagators in QFT are classical waves in AdS

$$Z_{\mathrm{CFT}}(N) = \int \mathcal{D}\phi \, e^{iN^2 S_{\mathrm{AdS}}(\phi)}$$

$$\langle e^{\int d^{d+1}x J(x)\mathcal{O}(x)} \rangle_{\mathrm{QFT}} = \int \mathcal{D}\phi \; e^{iS_{\mathrm{bulk}}(\phi(x,r))|_{\phi(x,r=\infty)=J(x)}}$$

$$g_{YM}^2 N = \frac{R^4}{\alpha} \qquad g_{YM}^2 = g_s$$

Only in the *large N* limit the strongly coupled boundary field theory becomes dual to *classical* gravity!

SUSY Einstein-Maxwell in AdS <==> SUSY Yang-Mills CFT

AdS/CFT dictionary:

<u>E-field</u>
D transverse E-field <=> D-1 electric field
D radial E-field <=> D-1 charge density

<u>B-field</u>
D radial B-field <=> D-1 magnetic field
D transverse B-field <=> D-1 current density

<u>spatial metric perturb.</u>
D transverse gradient <=> D-1 distortion
D radial gradient <=> D-1 stress tensor

<u>temporal metric perturb</u>.
D transverse gradient <=> D-1 temperature gradient
D radial gradient <=> D-1 heat flow


WELCOME TO HELL
MATRIX LARGE N


UV INDEPENDENCE

Blackboard break.

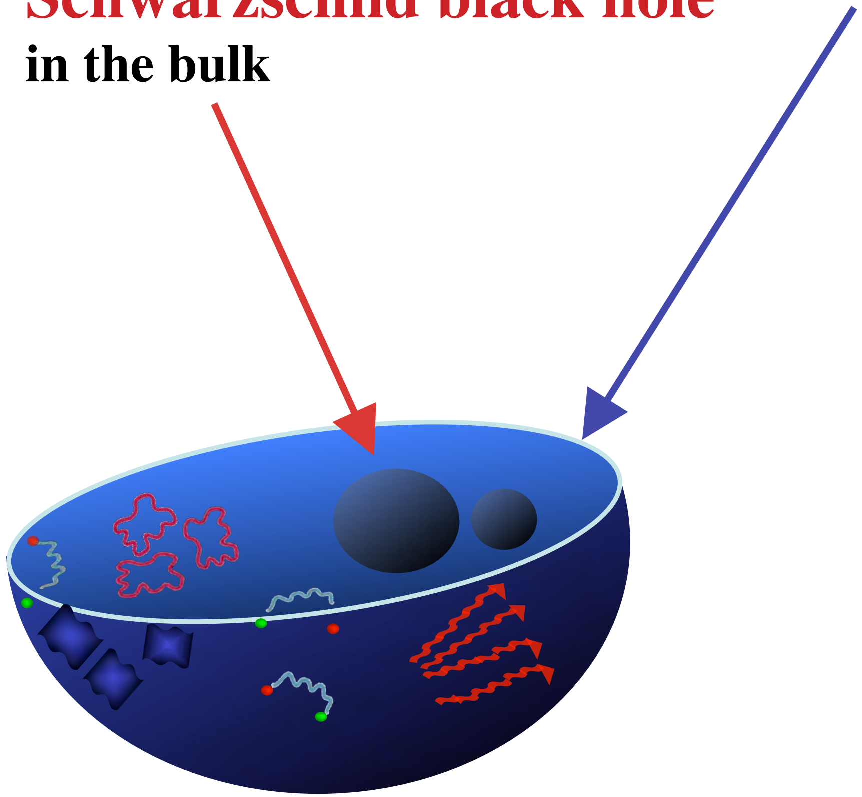
- GKPW rule (i): computing CFT propagators using GR.

- GKPW rule (ii): Schwarzschild black holes and finite temperature.

- The mysterious matrix large N mean field limit …

The triumph: gravitational encoding of all thermal physics!

Schwarzschild black hole
in the bulk



Boundary: the emergence theories of finite temperature matter.

- **All of thermodynamics! Caveat: phase transitions are mean field (large N limit).**

- **Precise encoding of Navier-Stokes hydrodynamics! Right now used to debug complicated hydrodynamics (e.g. superfluids).**

- **For special "Planckian dissipation" values of parameters (quantum criticality):**

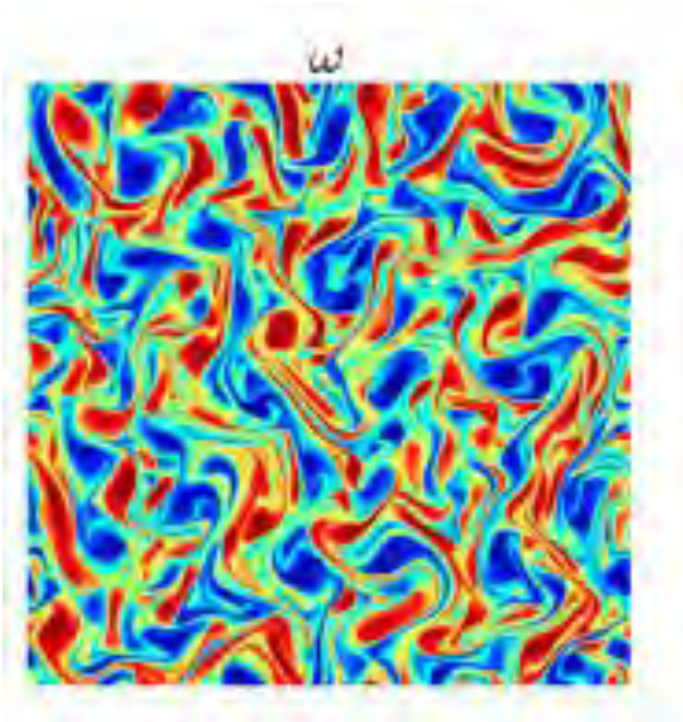
$$\tau_{\hbar} = const.\frac{\hbar}{k_B T}, \quad const. = O(1)$$

Turbulence and fractal black hole horizons.

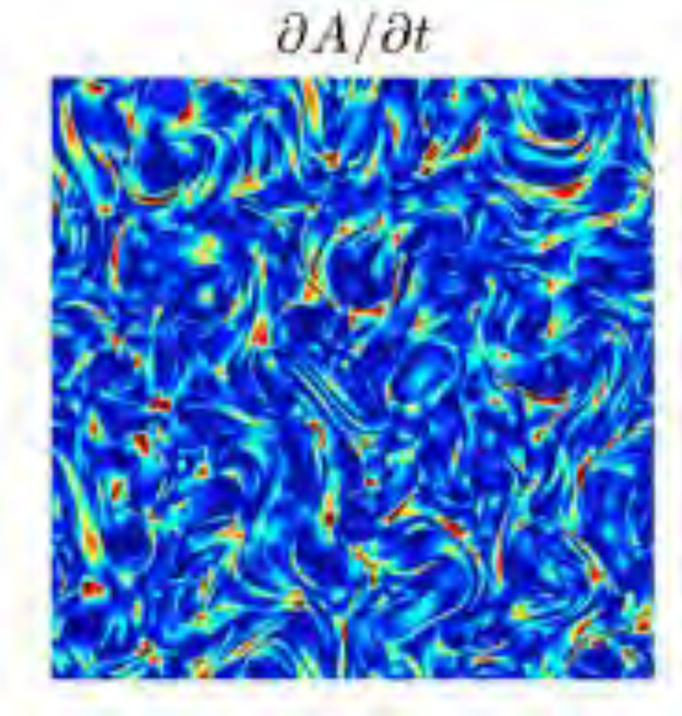
Chesler

Yaffe

Holography and numerical GR



Vorticity in the liquid (Kolmagorov scaling)



Near horizon geometry (fractal)

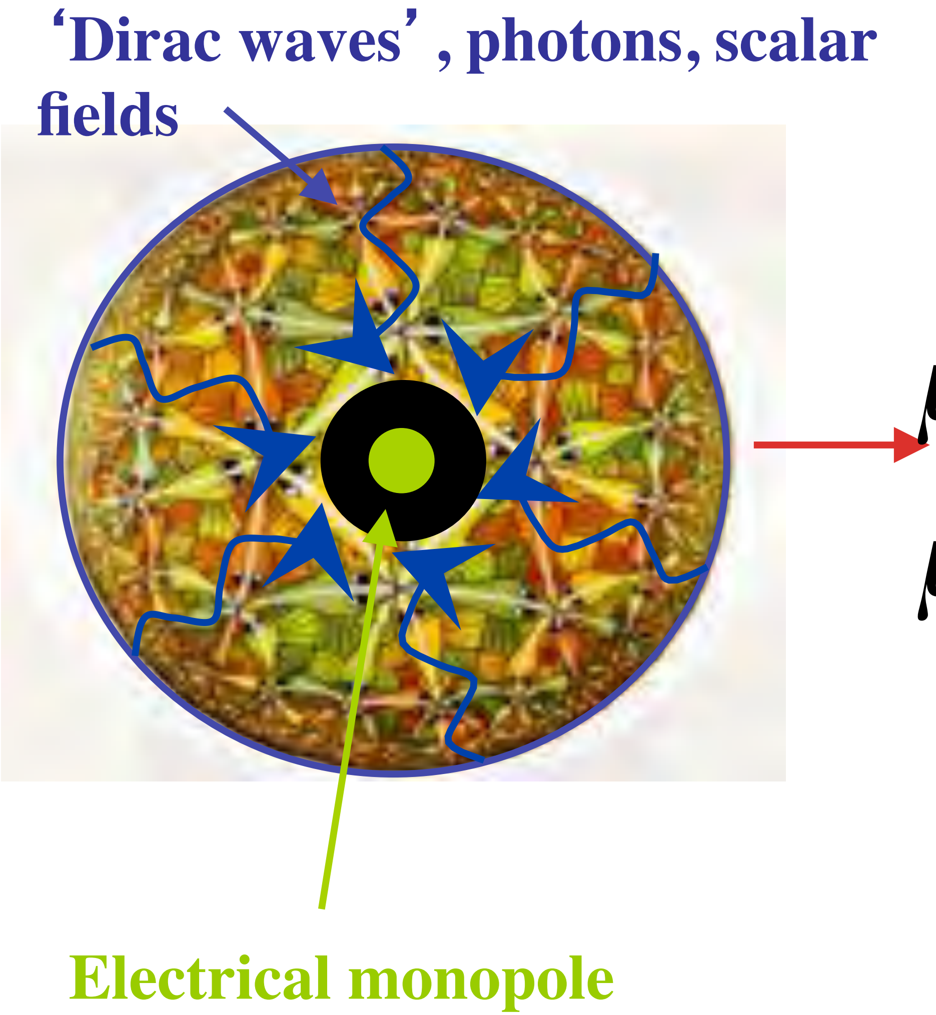
Plan of course.

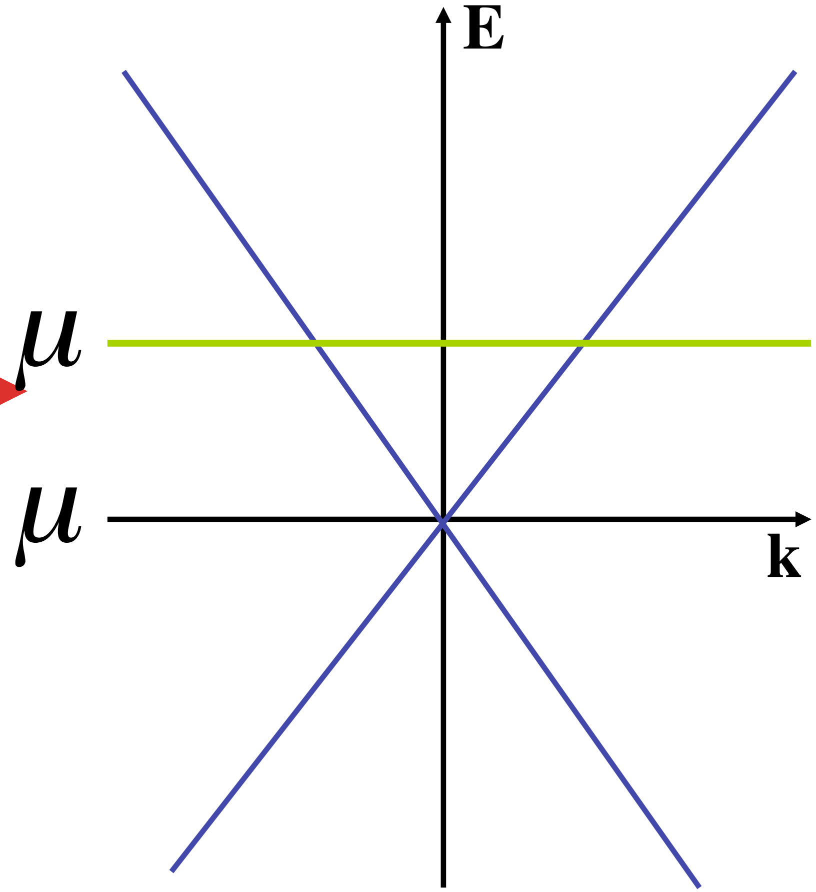
1. Overview of AdS/CMT in pictures (slides).

2. How the computations actually work: from GR and metrics to free energies and propagators with the GKPW rule (blackboard).

3. Physics highlights: strange metals, holographic superconductivity and Fermi liquids, transport, entanglement (slides).

Finite density: Reissner-Nordstrom Black Hole.

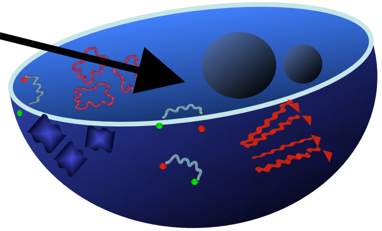


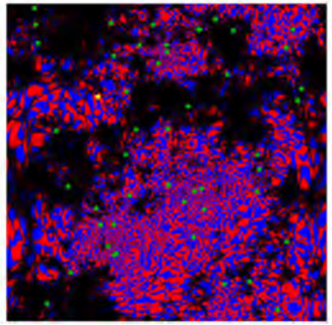


The charged back hole encoding for finite density (2008 - ????)

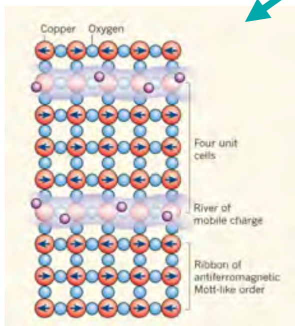
Anti de Sitter universe.

Finite density quantum matter:





Holographic strange metals



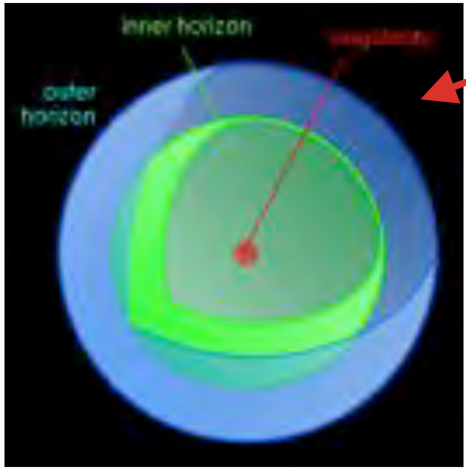
Charged black hole in the middle

Stripy pseudogap orders

High Tc superconductors

Emergent Fermi liquids

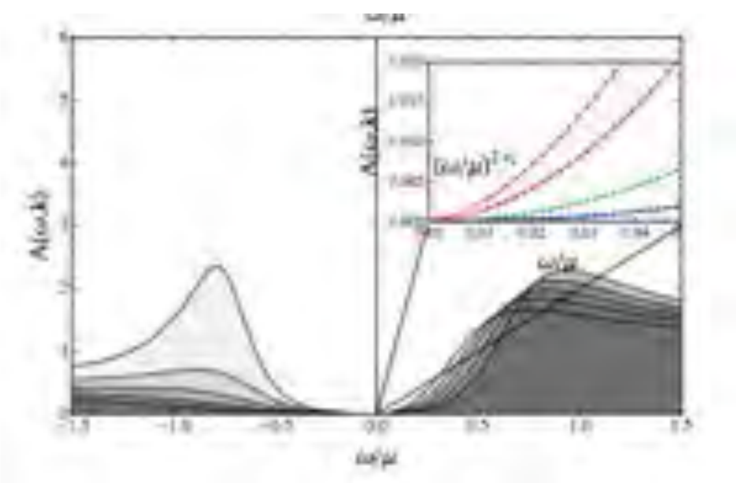
Finite density: the Reissner-Nordstrom strange metals (Liu et al.).



Near-horizon geometry of the extremal RN black hole:

- Space directions: flat, codes for simple Galilean invariance in the boundary.

- Time-radial(=scaling) direction: emergent AdS_2, codes for emergent temporal scale invariance!



Fermion spectral functions:

$$A(k,\omega) \propto G''_{AdS_2}(k,\omega) \propto \omega^{2\nu_k}$$

$$\nu_k = \frac{1}{\sqrt{6}}\sqrt{k^2 + \frac{1}{\xi^2}}$$

"Un-particle physics!"

"Scaling atlas" of holographic quantum critical phases.

Deep interior geometry sets the scaling behavior in the emergent deep infrared of the field theory. Uniqueness of GR solutions:

1. "Cap-off geometry" = confinement: conventional superconductors, Fermi liquids ….

2. Geometry survives: "hyperscaling violating geometries" (Einstein – Maxwell- Dilaton – Scalar fields –Fermions).

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\vartheta/(d-\theta)} dr^2 + dx_i^2 \right)$$

$$x_i \rightarrow \zeta x_i, \quad t \rightarrow \zeta^z t, \quad ds \rightarrow \zeta^{\theta/d} ds$$

$$S \propto T^{(d-\theta)/z}$$

Quantum critical phases with unusual values of:

$z =$ **Dynamical critical exponent**

$\theta =$ **Hyperscaling violation exponent**

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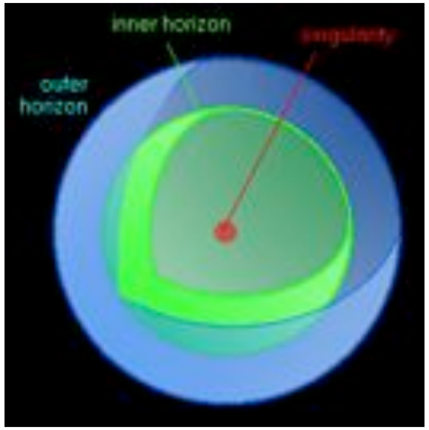
The holographic Fermi-liquid: uncollapsing in the "electron star".

(Reissner-Nordstrom)
"Black hole like object"

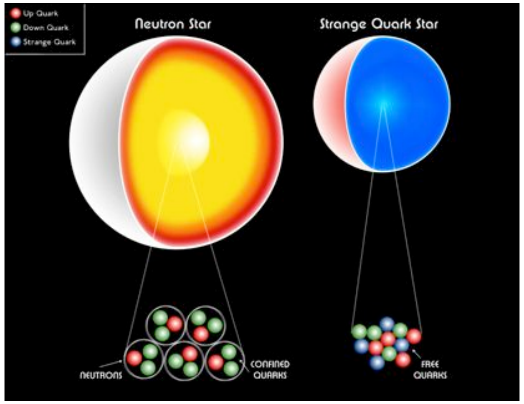


"fractionalized", "unstable":
strange metal

"uncollapse"

Phase transition

"star like object"



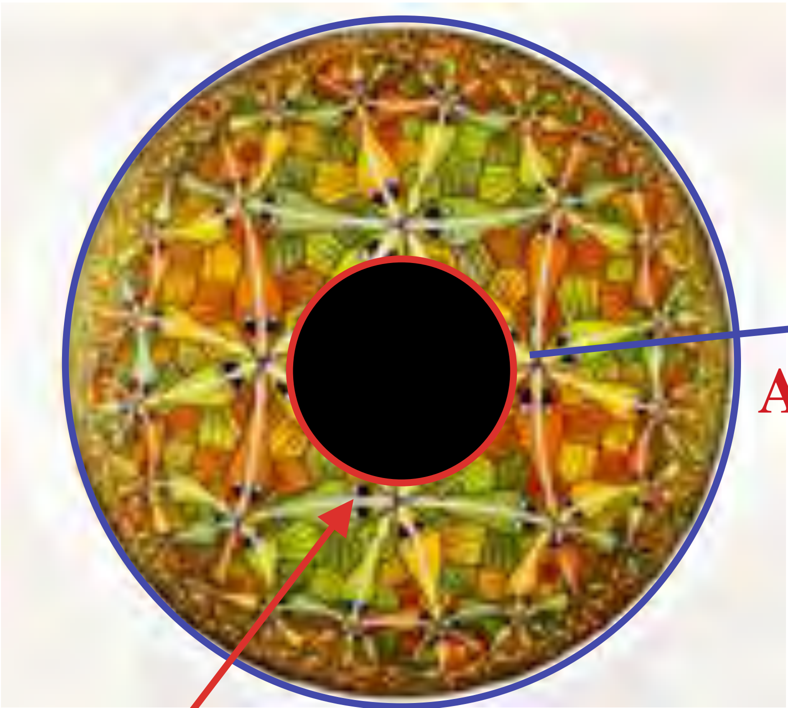
"Cohesive state":

Symmetry breaking:
superconductor, crystal
("scalar hair")

Fermi-liquid ("electron star")

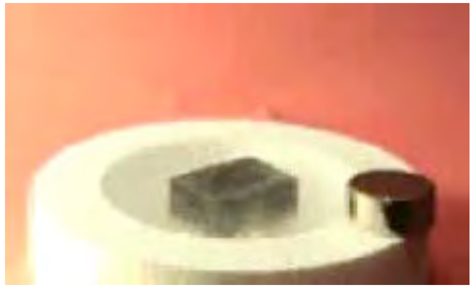
The holographic superconductor

Hartnoll, Herzog, Horowitz, arXiv:0803.3295



Condensate (superconductor, …) on the boundary!

AdS-CFT



'Super radiance' : in the presence of matter the extremal BH is unstable => zero T entropy always avoided by low T order!!!

(Scalar) matter 'atmosphere'

The hairy black hole …

Minimal model: $V(|\psi|) = -2\psi^2$, **the dual operator** Ψ **can have conformal dimensions** $\Delta = 1, 2$

The Reissner-Nordstrom BH describes the normal state, but it goes unstable at a $T < T_c \simeq \sqrt{\rho}$ **because** $m_{eff}^2 \approx m^2 - q^2 A_0^2$ **turns negative, "violation of the BF bound".**

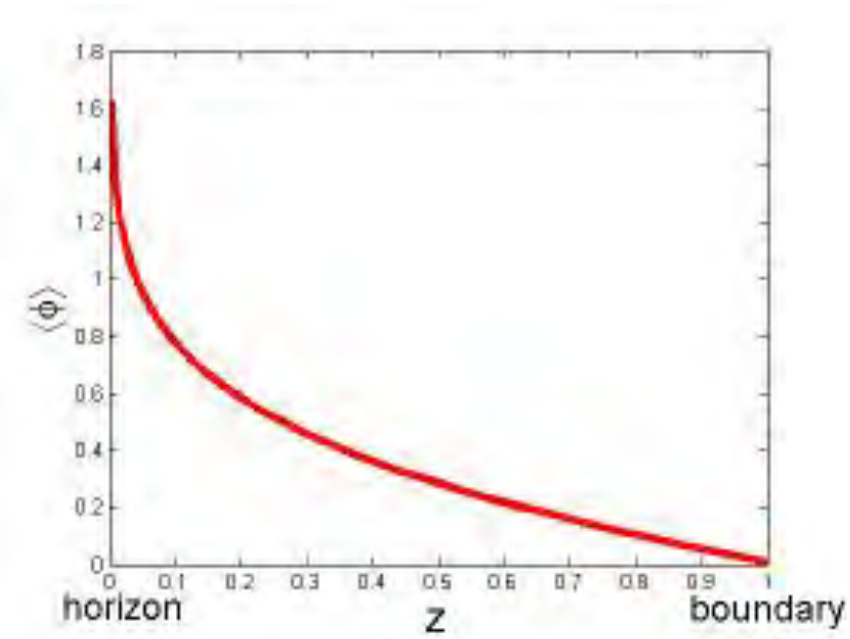
Below T_c **the black hole gets hair in the form of a "scalar atmosphere": via the dictionary, a VEV emerges in the field theory in the absence of a source.**

The global U(1) symmetry of the CFT is spontaneously broken into a superfluid!

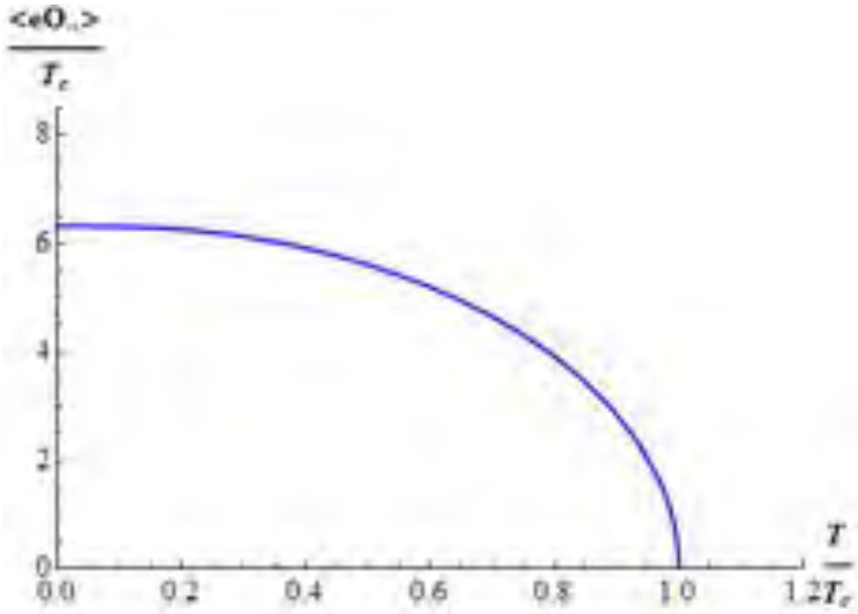
The Bose-Einstein Black hole hair

Hartnoll Herzog Horowitz

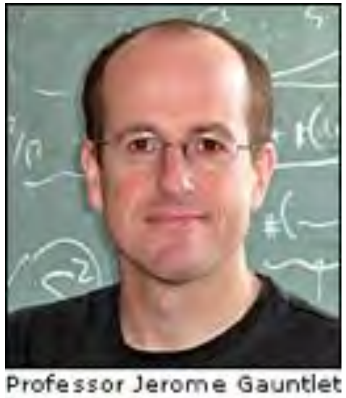
Scalar hair accumulates at the horizon



Mean field thermal transition.



The top-down holographic superconductors



Erdmenger et al.:

D3/D7 brane intersections, (arXiv:0810.2316)

Gubser et al.:

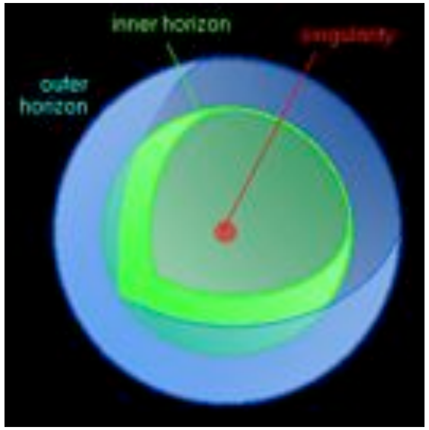
type II sugra (arXiv:0907.3510)

Gauntlett et al.:

M-theory, Sasaki-Einstein (arXiv:0907.3796).

The holographic Fermi-liquid: uncollapsing in the "electron star".

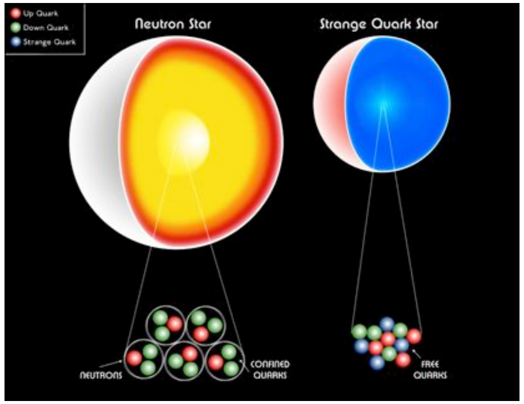
(Reissner-Nordstrom)
"Black hole like object"



"uncollapse"

Phase transition

"star like object"



"fractionalized", "unstable":
strange metal

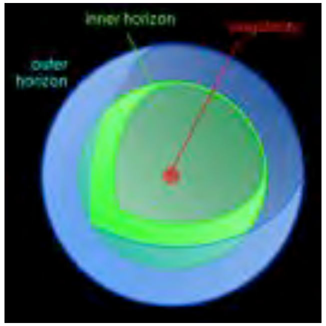
"Cohesive state":

Symmetry breaking:
superconductor, crystal
("scalar hair")

Fermi-liquid ("electron star")

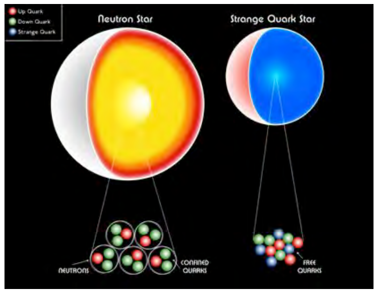
The holographic Fermi-liquid: uncollapsing in the “electron star”.

Reissner-Nordstrom BH



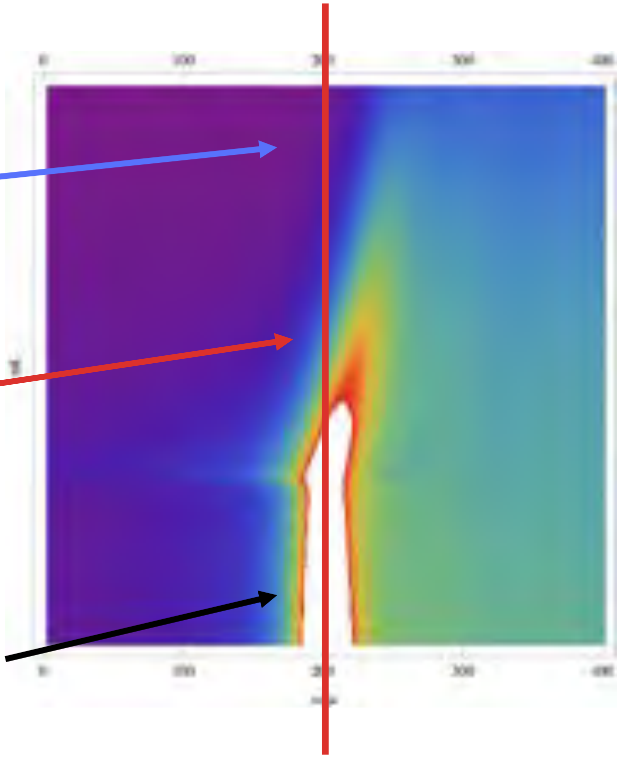
uncollapse

“charged neutron star”



Hartnoll et al., Schalm et al.

“Photoemission” in the boundary



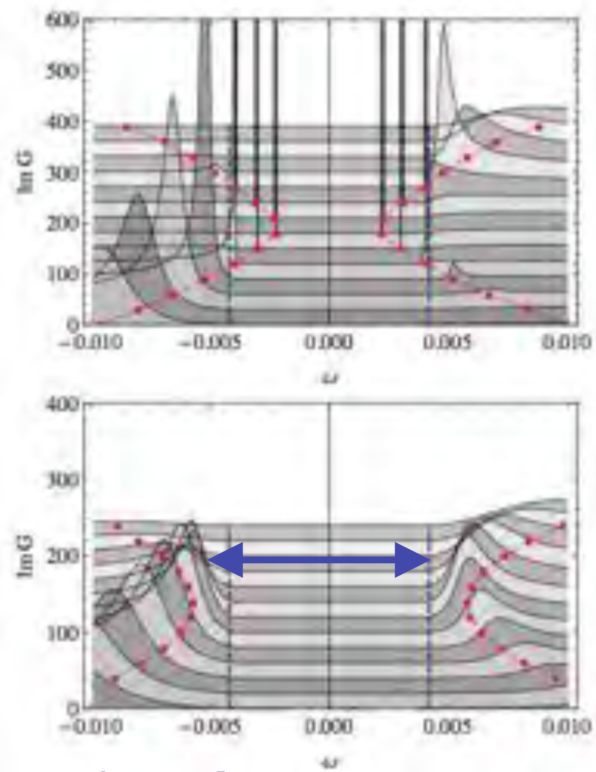
Strange metal

Relaxational Fermi surface

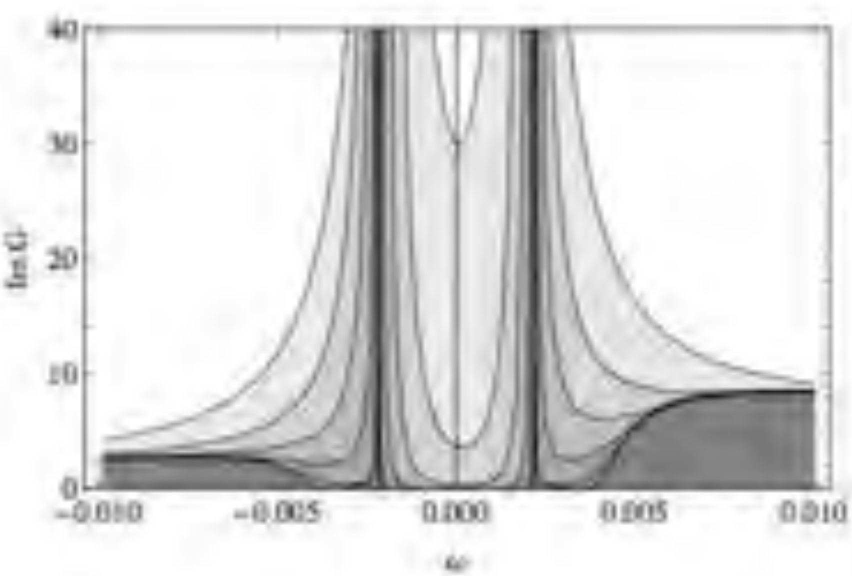
Fermi-surface (Fermi liquid)

Holographic superconductivity: stabilizing the fermions.

Fermion spectrum for scalar-hair black hole (Faulkner et al., 911.340):



'BCS' Gap in fermion spectrum !!



"Pseudogap" Temperature dependence

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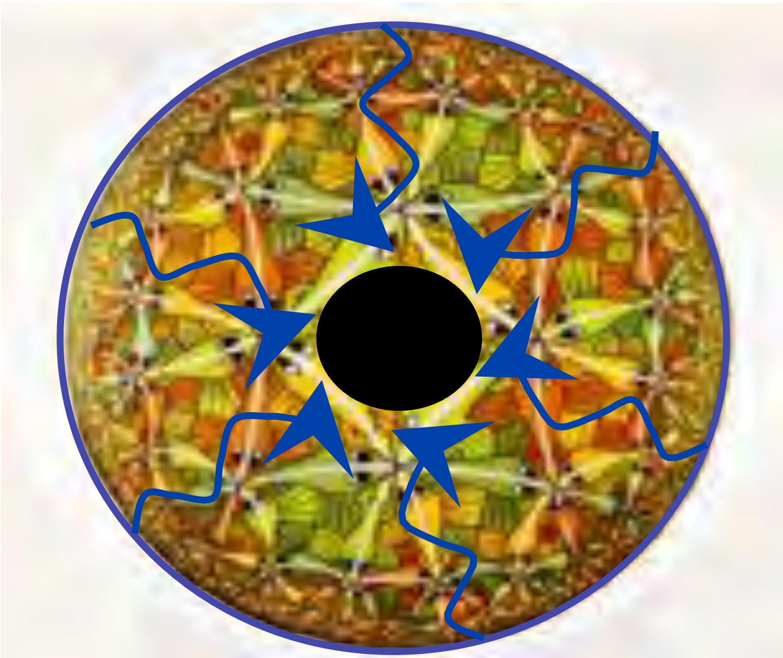
Dissipation = absorption of classical waves by Black hole!



Hartnoll-Son-Starinets (2002):

Viscosity: absorption cross section of gravitons by black hole

$$\eta = \frac{\sigma_{abs}(0)}{16\pi G}$$

= area of horizon (GR theorems)

Entropy density s: Bekenstein-Hawking BH entropy = area of horizon

Universal viscosity-entropy ratio for CFT's with gravitational dual limited in large N by:

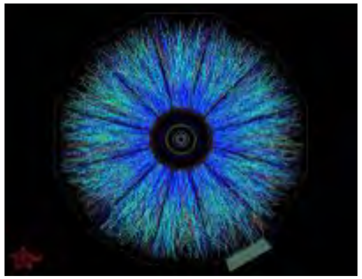
$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

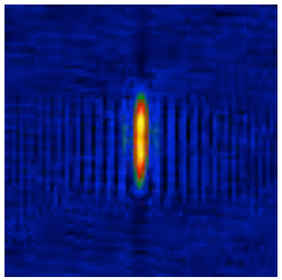
Planckian dissipation

Universal entropy production time in QC system: $\tau = \tau_{\hbar} \approx \frac{\hbar}{k_B T}$

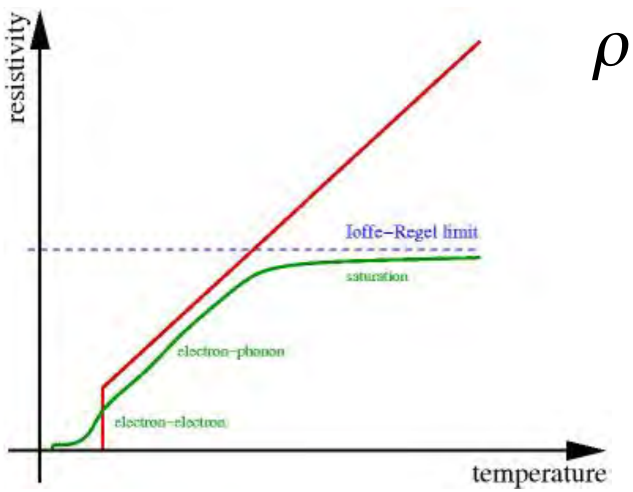
Observed in Quark gluon plasma (heavy ion colliders RIHC, LHC) and cold atom "unitary fermi gas":

$$\frac{\eta}{s} = T\tau_{\hbar} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$





Since early 1990's recognized as responsible for strange metal properties, also linear resistivity high Tc metals ??:

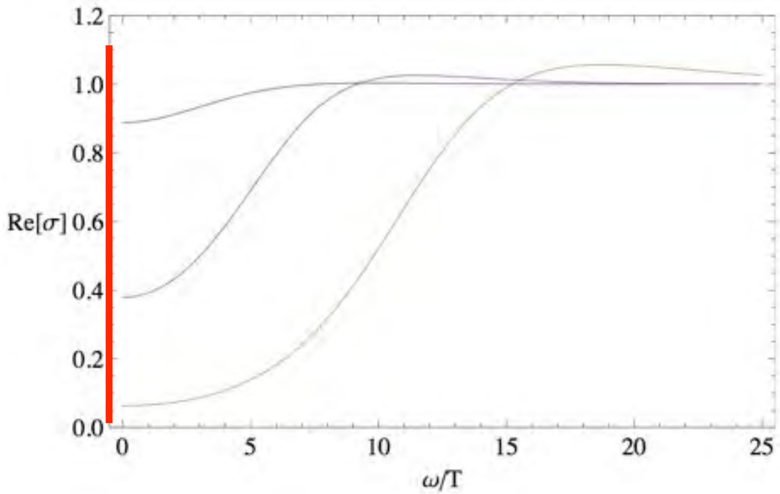


$$\rho \propto \frac{1}{\tau_{\hbar}} \propto k_B T$$

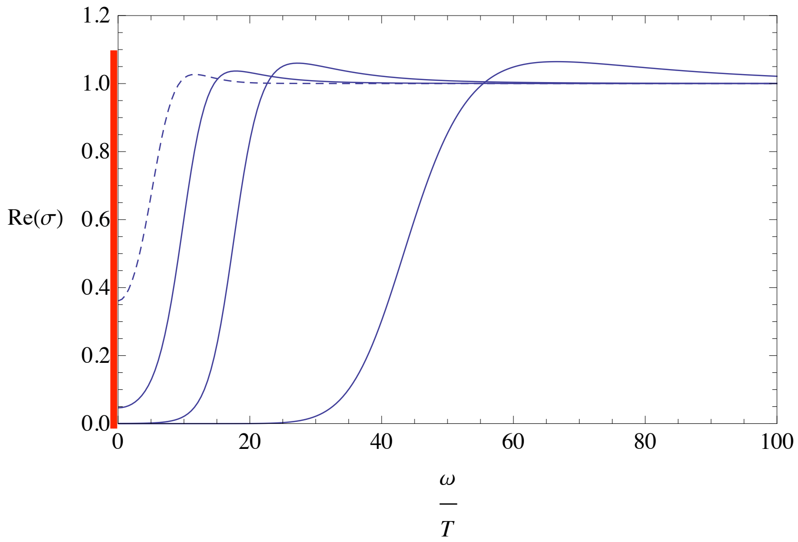
Holographic optical conductivity (2+1D).

Optical conductivity in finite density systems:

Strange metal.



Holographic superconductor.



Momentum conservation in Galilean continuum: everything is a perfect conductor!

Quasiparticle versus "Unparticle" transport.

Hydrodynamics is based on momentum conservation: how about the broken translational symmetry in metals?

Fermi liquid: the charge carriers are quantum-mechanical waves diffracting against the lattice while the Fermi-momentum is of order of the Umklapp momentum,

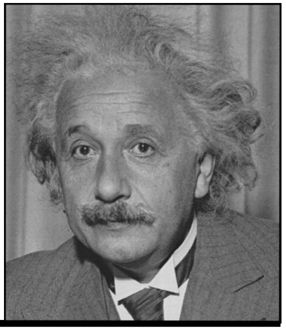
$$\frac{1}{\tau_{coll}} \simeq \frac{(k_B T)^2}{\hbar E_F} \qquad \frac{1}{\tau_K} = C \frac{1}{\tau_{coll}} \qquad \eta \simeq (n E_F) \tau_{coll} \sim \frac{1}{T^2}$$

Quantum critical metals:

- QC fluctuations are not waves, these do not diffract against the periodic lattice!

- "Planckian dissipation": extremely rapid equilibration, only after hydrodynamics is established momentum relaxes !?

Black holes with a corrugated horizon



Charged Black Hole: describes finite density strange metal .

Breaking translational symmetry in the boundary:

Corrugate the black hole horizon ….

Not a favorite thing of general relativity -- hard work, still in progress!

Holographic quenched disorder.

David Vegh

Dictionary entry "number one":

Global translational invariance in the boundary (energy-momentum conservation)

<==>

General covariance in the bulk (Einstein theory)

Breaking of Galilean invariance in the boundary = elastic scattering (?)

<==>

Fix the (spatial) frame in the bulk = "Massive gravity"

$$S = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\left(\mathcal{R} + \frac{6}{L^2} - \frac{L^2}{4}F_{\mu\nu}F^{\mu\nu} + m^2\left(\alpha \mathrm{Tr}\left(\mathcal{K}\right) + \beta\left(\mathrm{Tr}\left(\mathcal{K}\right)^2 - \mathrm{Tr}\left(\mathcal{K}^2\right)\right)\right)\right)$$

$$\mathcal{K}^{\mu}_{\alpha}\mathcal{K}^{\alpha}_{\nu} = g^{\mu\alpha}f_{\alpha\nu}$$

Couple the metric g_{ab} to a fixed metric $f_{xx}=f_{yy}=1$

Holographic linear resistivity.

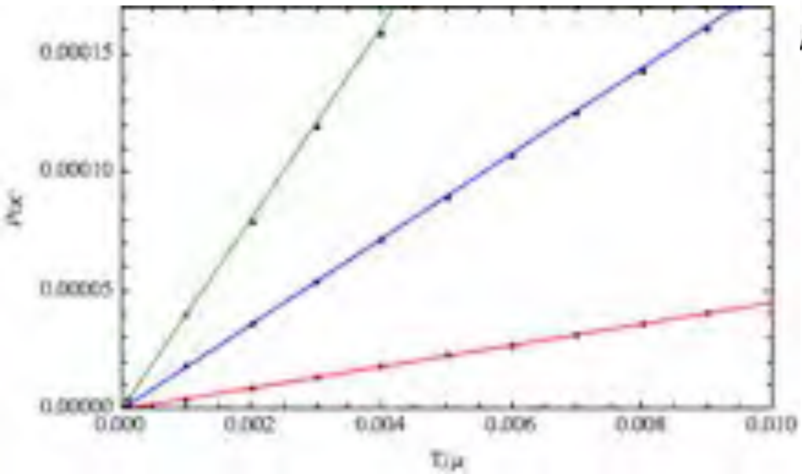
Richard Davison

Steve Gubser

"Champion" strange metal: Einstein-Maxwell-dilaton (consistent truncation), local quantum critical, marginal Fermi-liquid (3+1D), susceptible to holo. superconductivity, healthy thermodynamics: unique ground state, Sommerfeld thermal entropy.

David Vegh

Breaking of Galilean invariance (finite conductivities) due to quenched disorder: "massive gravity" = fixing space-like diffeomorphisms in the bulk.



$$S = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\Big[\mathcal{R} - \frac{1}{4}e^{\phi}F_{\mu\nu}F^{\mu\nu} - \frac{3}{2}\partial_\mu\phi\partial^\mu\phi + \frac{6}{L^2}\cosh\phi - \frac{1}{2}m^2\left(\mathrm{Tr}\left(\mathcal{K}\right)^2 - \mathrm{Tr}\left(\mathcal{K}^2\right)\right)\Big]$$

Explicit holographic construction explaining linear resistivity!

The secret of the linear resistivity …

Davison

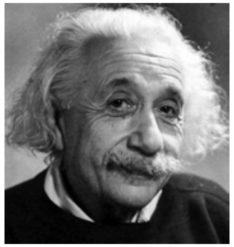
Hartnoll

Planckian dissipation = very rapid local equilibration: a hydrodynamical fluid is established before it realizes that momentum is non conserved due to the lattice potential (not true in Fermi-gas: Umklapp time is of order collision time).

Stokes

Resistivity in hydrodynamics

$$\rho(T) \propto \frac{1}{\tau_{rel}} = \frac{D}{l^2}$$



Einstein

Einstein relation:

$$D = \frac{\eta}{m_e n_e}$$

Sachdev

Son

Planckian viscosity

$$\eta = A \frac{\hbar}{k_B} s$$

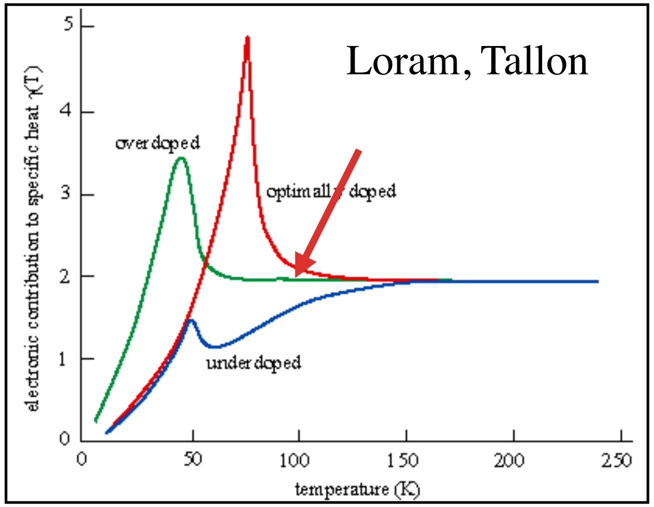
$$\rho(T) = \frac{1}{\omega_p^2 \tau_{rel}} = A \frac{\hbar}{\omega_p^2 l^2 m_e} \frac{S}{k_B}$$

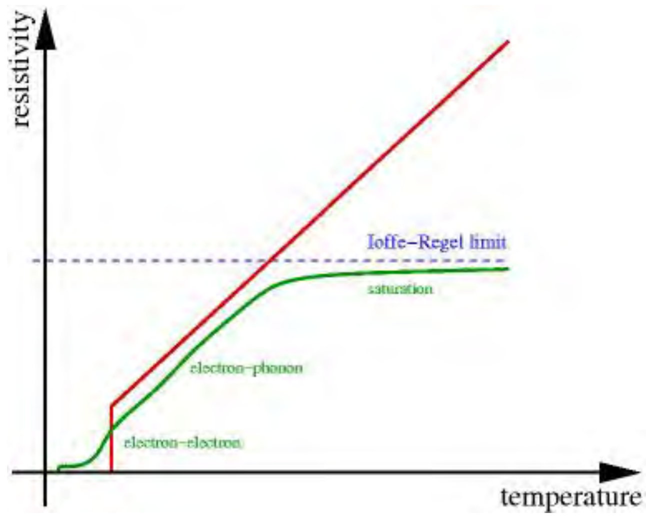
Entropy versus transport: optimal doping

Optimally doped

$$C = \gamma T \Rightarrow S = T / \mu$$

$$\rho \propto \frac{1}{\tau_{rel}} \propto S \propto T$$





Plugging in numbers: "mean-free path" $l \approx 10^{-9} m$

Quite dirty but no residual resistivity since the fluid becomes perfect at T =0 !

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Quantum matter.

"Macroscopic stuff that can quantum compute all by itself"

$$|\Psi\rangle = \sum_{configs} A_{configs} |configs\rangle$$

- Topological incompressible systems, no low energy excitations but the whole carries quantum information: fractional quantum Hall, top. Superconductors/insulators (Majorana' s, theta vacuum, ..)

- Compressible systems: are the strange metals of this kind??

Strongly interacting fermions at finite density: the fermion signs as entanglement resource!

Entanglement entropy

Bipartite von Neumann entropy: measures entanglement = quantum information of Bell pairs.

Trace the full density matrix over B:

$$\rho_A = Tr_B \rho$$

Compute the entropy associated with the reduced density matrix:

$$S_{vN,A} = Tr\left[\rho_A \ln \rho_A\right]$$

Universal measure of two bit entanglement:

$$|Bell\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B\right) \qquad S_{vN,A} = \sqrt{2}$$

$$|\mathrm{Pr}\, od\rangle = \frac{1}{2}\left(|0\rangle + |1\rangle\right)_A \otimes \left(|0\rangle + |1\rangle\right)_B \qquad S_{vN,A} = 0$$

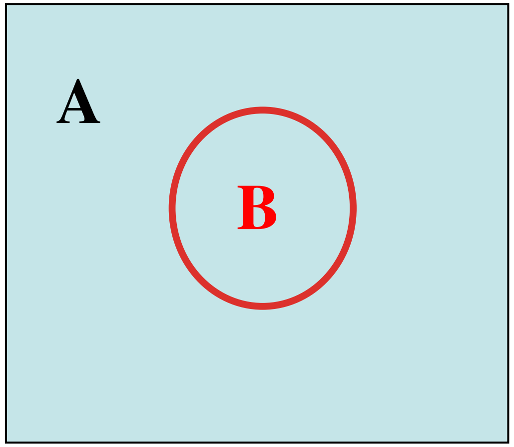
Bipartite entanglement entropy and quantum field theory.


Wilczek

$$\rho_A = Tr_B[\,\rho\,]$$

$$S_{vN} = Tr[\rho_A \ln \rho_A]$$

Measure of entanglement of degrees of freedom in spatial volume B with those in A.

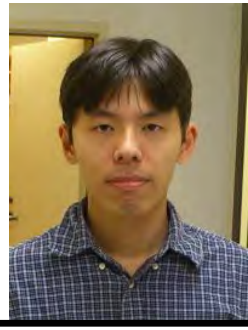


Generic energy eigenstates: S_{vN} scales with volume L^d of B.

Ground states of bosonic systems: S_{vN} scales with the area L^{d-1} of B.

Fermi gas: longer ranged "signful" entangled $S_{vN} \sim L^{d-1} \ln(L^{d-1})$

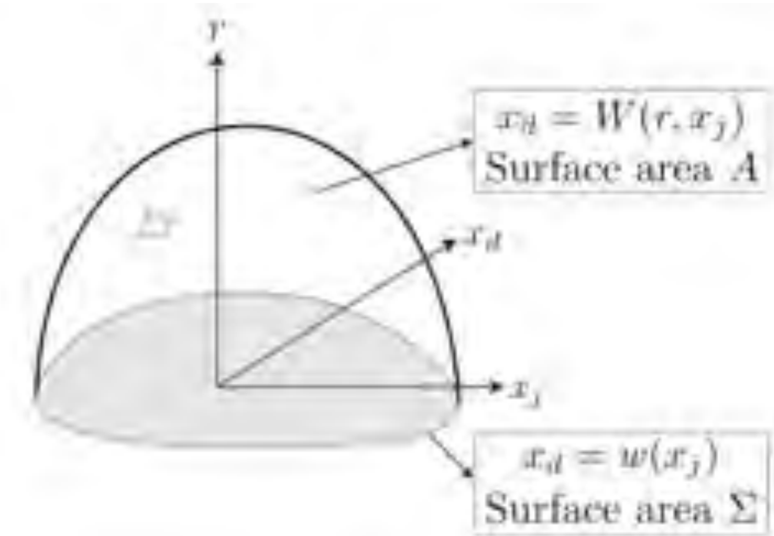
Entanglement entropy versus AdS/CFT.

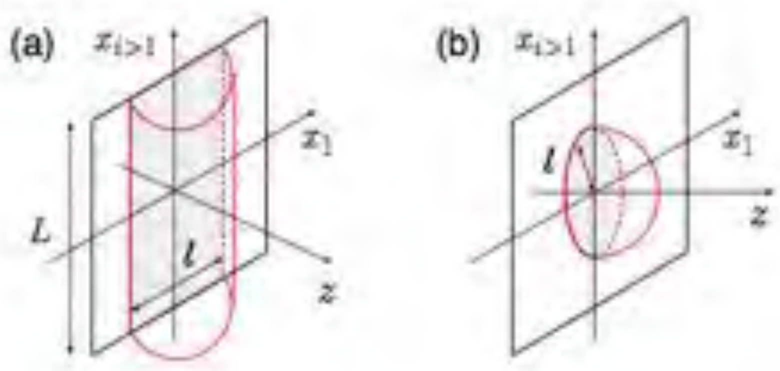



Takayanagi Ryu

$$\rho_A = Tr_B[\rho] \qquad S_{vN} = Tr[\rho_A \ln \rho_A]$$

The spatial bipartite entanglement entropy in the boundary is dual to the area of the minimal surface in the bulk, bounded by the cut in the space of the boundary





Holographic strange metal entanglement entropy.

Huijse Swingle Sachdev

Einstein-Maxwell-Dilaton bulk => "hyperscaling violating geometry" (Kiritsis et al.):

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\vartheta/(d-\theta)}dr^2 + dx_i^2 \right)$$

Boundary: interpolating between "normal" and RN strange metals.

$$x_i \to \zeta x_i, \quad t \to \zeta^z t, \quad ds \to \zeta^{\theta/d} ds \qquad\qquad S \propto T^{(d-\theta)/z}$$

Entanglement entropy:

$$S_{vN} \propto L^{d-1} \qquad , \theta < d-1$$ **Bosonic fields**

$$S_{vN} \propto L^{d-1} \ln L^{d-1}, \theta = d-1$$ **Fermi liquid-like**

$$S_{vN} \propto L^{\theta} \qquad , d-1 < \theta < d$$ **But this is longer ranged !**

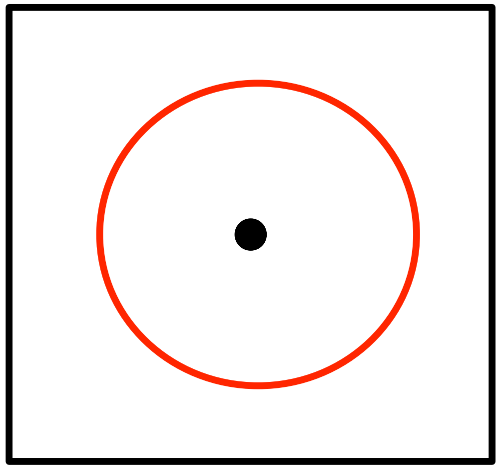
Hyperscaling violation.

Bosonic field theories: single point of masslessness in momentum space.

$$\theta = 0$$

Fermi gas: surface of masslessness with dimension d-1 in momentum space (Fermi-surface).

$$\theta = d - 1$$


momentum space

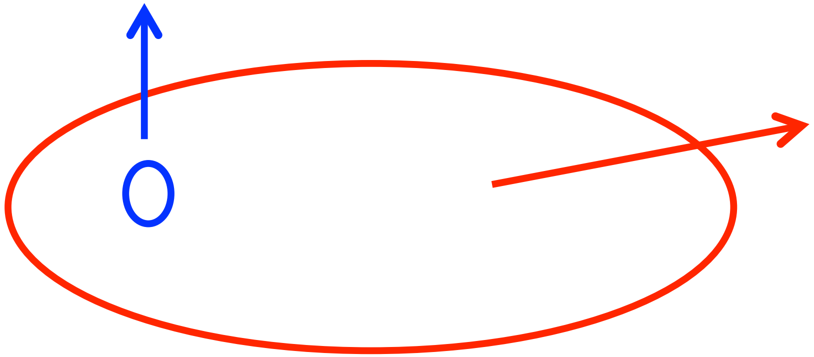
Very exotic in Ginzburg-Landau-Wilson classical/bosonic physics. For Fermi gas rooted in "poor man's" antisymmetrization entanglement!

$$|k_1 k_2 \cdots k_n\rangle = \frac{1}{\sqrt{N}} \sum_P \eta_P |k_1(\mathcal{P})1\rangle |k_2(\mathcal{P}2)\rangle \cdots |k_N(\mathcal{P}N)\rangle$$

Fermion signs and dense entanglement …


Grover Fisher
arXiv:1412.3534

S_{vN} area law: ground states of "sign-free" systems (bosons, tensor product states ..)



Energy eigenstates:

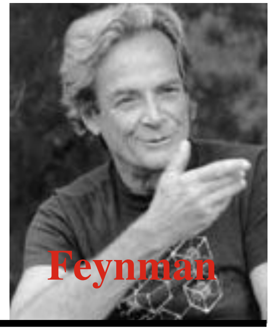
$$|\Psi_i\rangle = \sum_{conf} (-1)^{i,conf} |A^i_{conf}| |conf\rangle$$

$$(-1)^{i,conf} =$$

- **Antisymmetrization => area log area S_{vN} (Fermi gas)**
- **Random => Volume S_{vN} (typical excited states)**

The *quantum critical metallic phases* of holography are characterized by dense "sign driven" entanglement as characterized by the hyperscaling violation exponent!

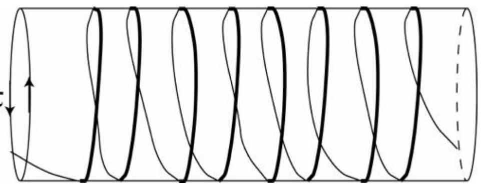
Quantum statistics and path integrals

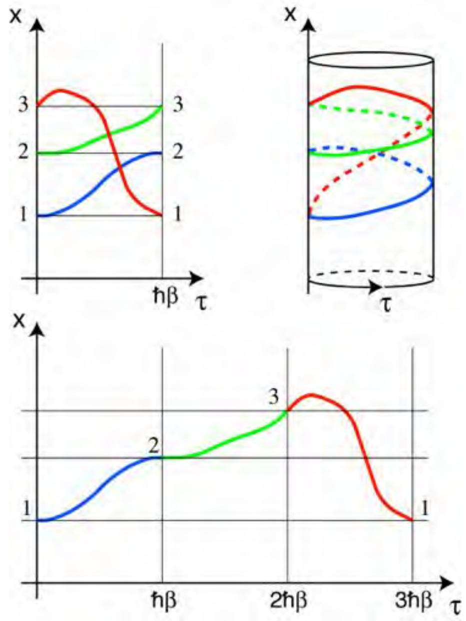


$$\rho_{B/F}(\mathbf{R},\mathbf{R};\beta) = \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\rho_D(\mathbf{R},\mathcal{P}\mathbf{R};\beta)$$

$$= \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\int_{\mathbf{R}\to\mathcal{P}\mathbf{R}}\mathcal{D}\mathbf{R}(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^{\hbar/T}d\tau\left(\frac{m}{2}\dot{\mathbf{R}}^2(\tau)+V(\mathbf{R}(\tau))\right)\right\}$$

Bose condensation: Partition sum dominated by infinitely long cycles





Cycle decomposition

Fermions: infinite cycles set in at T_F, but cycles with length w and w +1 cancel each other approximately. Free energy pushed to E_F!

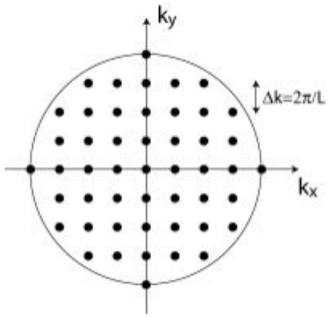
The nodal hypersurface

Antisymmetry of the wave function

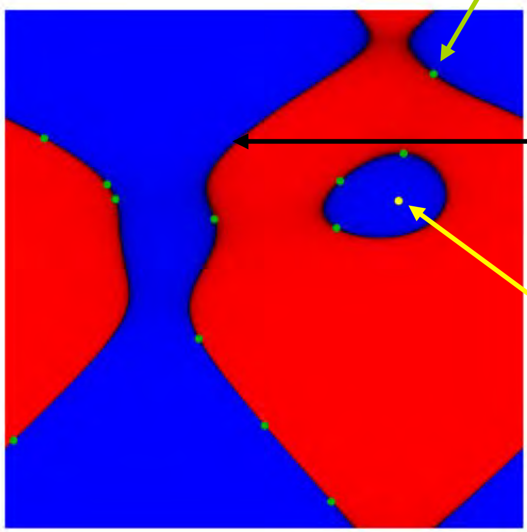
$$\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_i, \ldots, \mathbf{r}_j, \ldots, \mathbf{r}_N) = -\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_j, \ldots, \mathbf{r}_i, \ldots, \mathbf{r}_N)$$

Free Fermions

$$\Psi_0(\mathbf{R}) \sim \mathrm{Det}\left(e^{i\mathbf{k}_i \mathbf{r}_j}\right)_{ij}$$



d=2



Pauli hypersurface

$$P = \bigcup_{i \neq j} P_{ij}$$

$$P_{ij} = \{\mathbf{R} \in \mathbb{R}^{Nd} | \mathbf{r}_i = \mathbf{r}_j\}$$

$$\dim P = Nd - d$$

Nodal hypersurface

$$\Omega = \{\mathbf{R} \in \mathbb{R}^{Nd} | \Psi(\mathbf{R}) = 0\}$$

$$\dim \Omega = Nd - 1$$

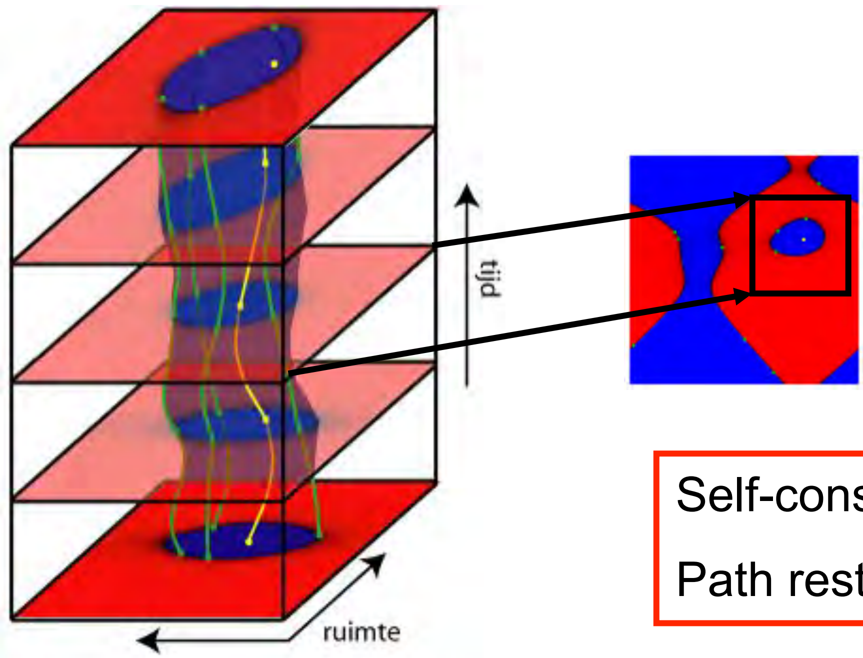
Test particle

Constrained path integrals

Formally we can solve the sign problem!!

$$\rho_F(\mathbf{R},\mathbf{R};\beta) = \frac{1}{N!}\sum_{\mathcal{P},\mathrm{even}} \int_{\gamma:\mathbf{R}\to\mathcal{P}\mathbf{R}}^{\gamma\in\Gamma(\mathbf{R},\mathcal{P}\mathbf{R})} \mathcal{D}\mathbf{R}(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^{\hbar/T} \mathrm{d}\tau\left(\frac{m}{2}\dot{\mathbf{R}}^2(\tau)+V(\mathbf{R}(\tau))\right)\right\}$$

$$\Gamma(\mathbf{R},\mathbf{R}') = \{\gamma:\mathbf{R}\to\mathbf{R}'|\rho_F(\mathbf{R},\mathbf{R}(\tau);\tau)\neq 0\}$$



Ceperley, J. Stat. Phys. (1991)

Self-consistency problem:

Path restrictions depend on ρ_F !

Reading the worldline picture

Fermi-energy: confinement energy imposed by local geometry

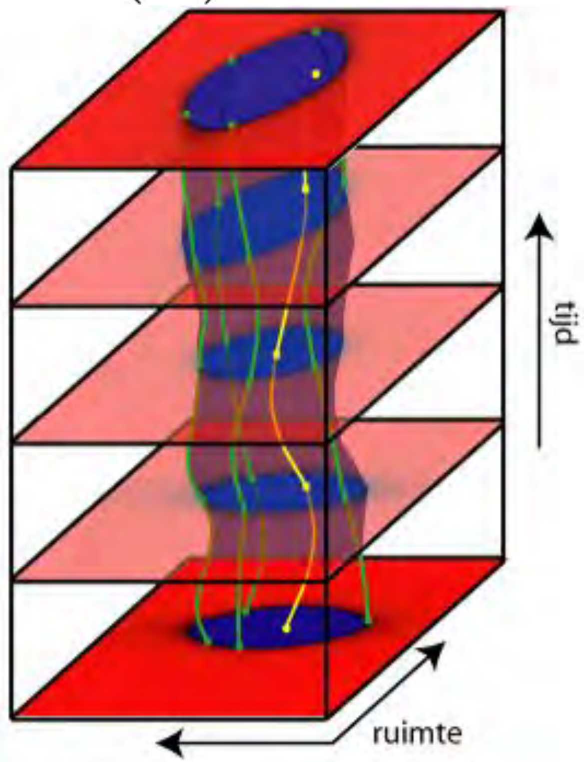
$$l^2(\tau) = \langle(\mathbf{r}_i(\tau) - \mathbf{r}_i(0))^2\rangle = 2d\mathcal{D}\tau = 2d\frac{\hbar}{2m}\tau$$

$$l^2(\tau_c) \simeq r_s^2 \rightarrow \tau_c \simeq \frac{1}{2d}\frac{2m}{\hbar}n^{-2/d}$$

$$\hbar\omega_c = \frac{\hbar}{\tau_c} \simeq d\frac{\hbar^2}{2m}n^{2/d} \simeq E_F$$

Average node to node spacing

$$\sim r_s = \left(\frac{V}{N}\right)^{1/d} = n^{-1/d}$$



Fermi surface encoded globally: $\rho_F = Det\left(e^{ik_i r_j}\right) = 0$

Change in coordinate of one particle changes the nodes everywhere

Finite T: $\rho_F = (4\pi\lambda\beta)^{-dN/2} Det\left[\exp\left(-\frac{(r_i - r_{j0})^2}{4\lambda\tau}\right)\right]$

$\lambda = \hbar^2/(2M)$

Non-locality length: $\lambda_{nl} = v_F\tau_{inel} = v_F\left(\frac{E_F}{k_BT}\right)\left(\frac{\hbar}{k_BT}\right)$

Key to fermionic quantum criticality


Kruger JZ

Phys. Rev. B **78**, 035104 (2008)

At the QCP scale invariance, no E_F → **Nodal surface has to become fractal !!!**

Turning on the backflow

Nodal surface has to become fractal !!!

↓

Try backflow wave functions

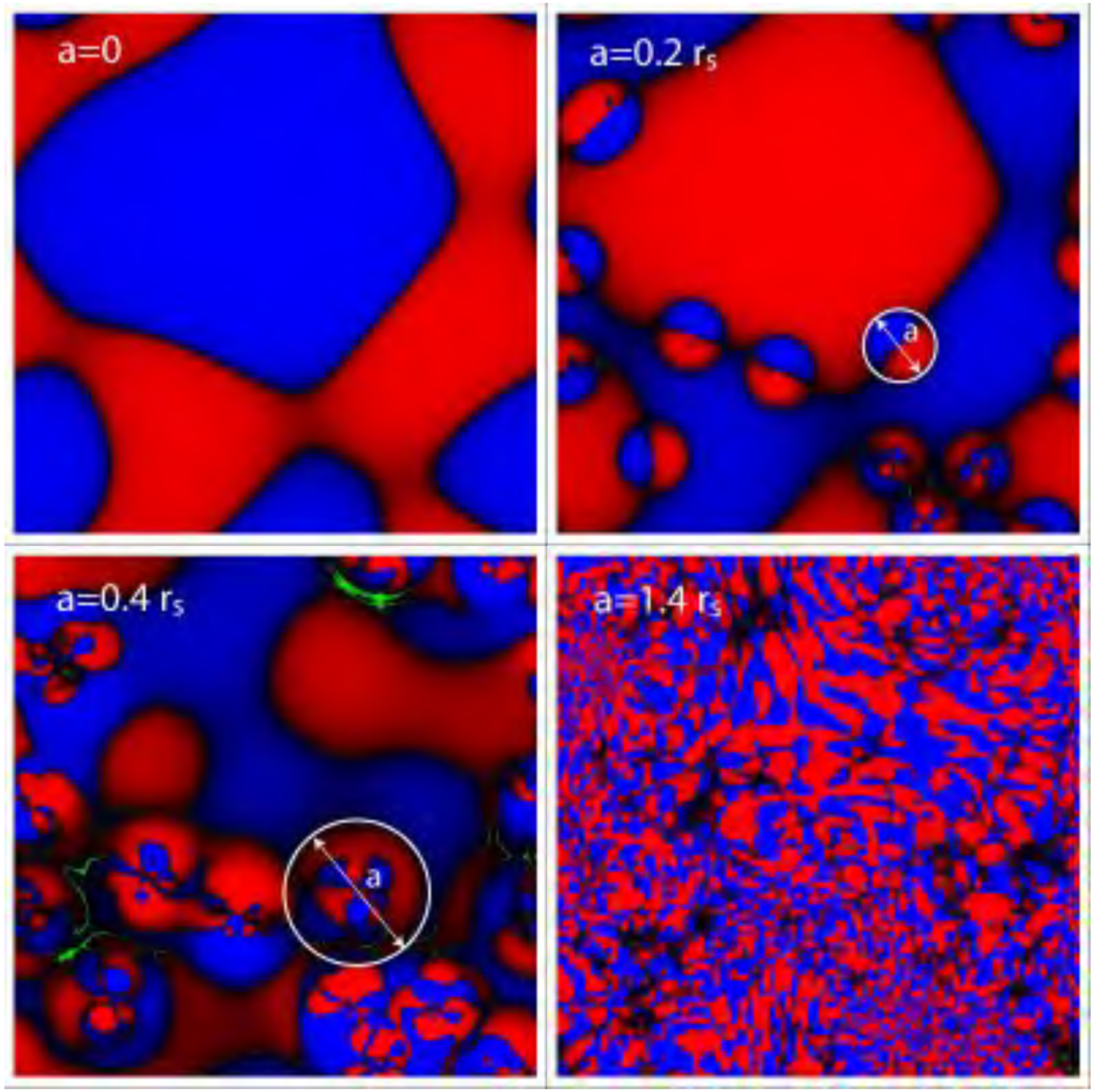
$$\psi_{bf}(\mathbf{R}) \sim \text{Det}\left(e^{i\mathbf{k}_i \tilde{\mathbf{r}}_j}\right)_{ij}$$

$$\tilde{\mathbf{r}}_j = \mathbf{r}_j + \sum_{l(\neq j)} \eta(r_{jl})(\mathbf{r}_j - \mathbf{r}_l)$$

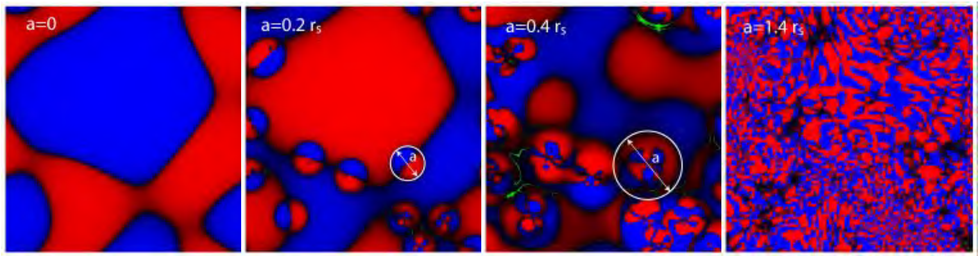
$$\eta(r) = \frac{a^3}{r^3 + r_0^3}$$

Collective (hydrodynamic) regime:

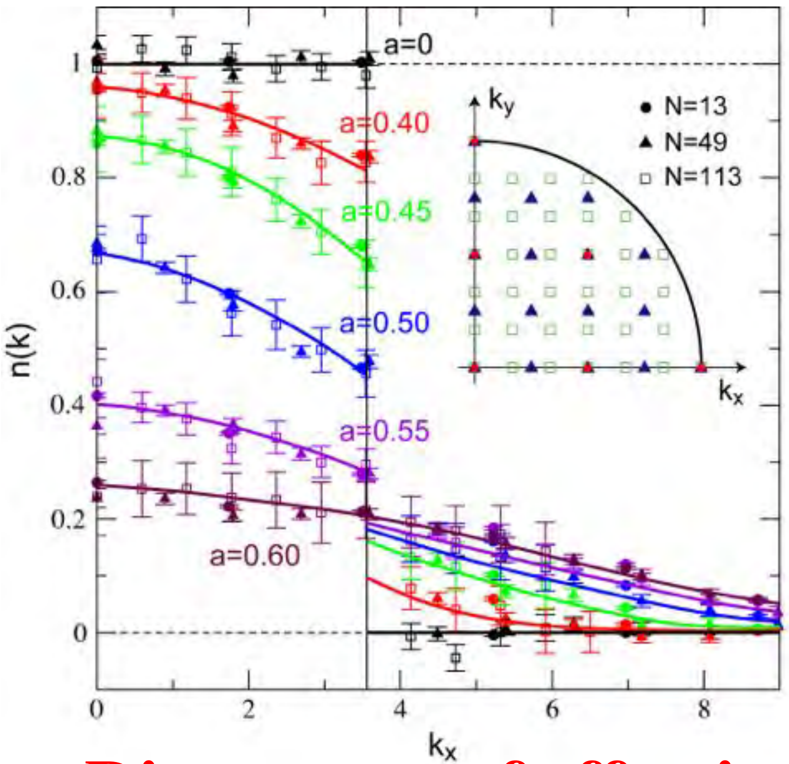
$$a \gg r_s$$

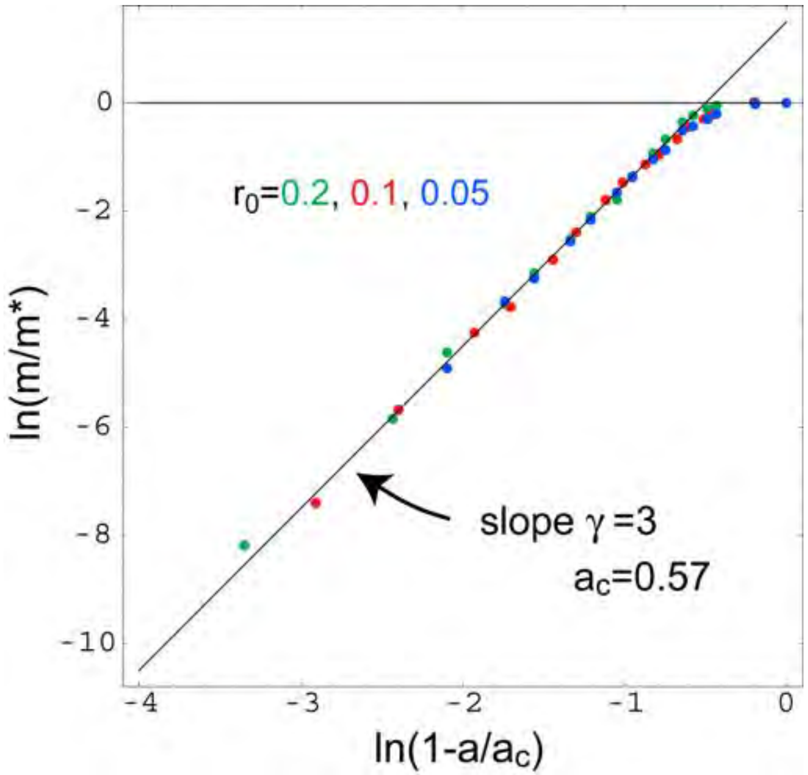


MC calculation of n(k)



$$\frac{m}{m^*} \propto \left(1 - \frac{a}{a_c}\right)^3$$



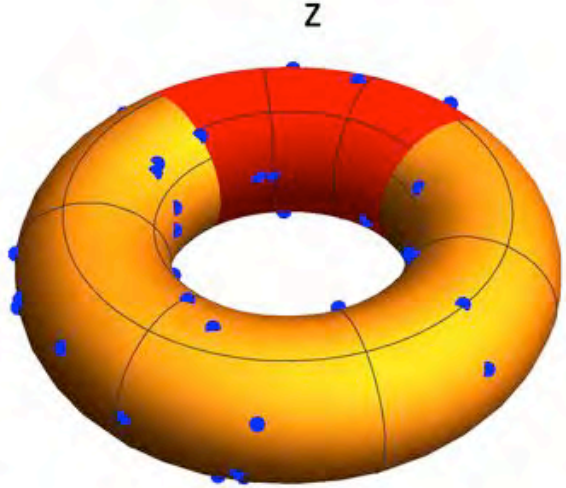


Divergence of effective mass as $a \rightarrow a_c$

Fractal nodes and entanglement entropy.

Grover Kaplis Kruger

Second Renyi entropy: leading contribution scales like vN entropy.



$$S^q(z) = \frac{\ln(Tr\rho_A^q)}{1-q}, \quad q=2$$

$$\rho(\mathbf{R},\mathbf{R}') = \psi^*(\mathbf{R})\psi(\mathbf{R}')$$

$$\psi_{bf}(\mathbf{R}) \sim \mathrm{Det}\left(e^{i\mathbf{k}_i\tilde{\mathbf{r}}_j}\right)_{ij}$$

$$\tilde{\mathbf{r}}_j = \mathbf{r}_j + \sum_{l(\neq j)} \eta(r_{jl})(\mathbf{r}_j - \mathbf{r}_l)$$

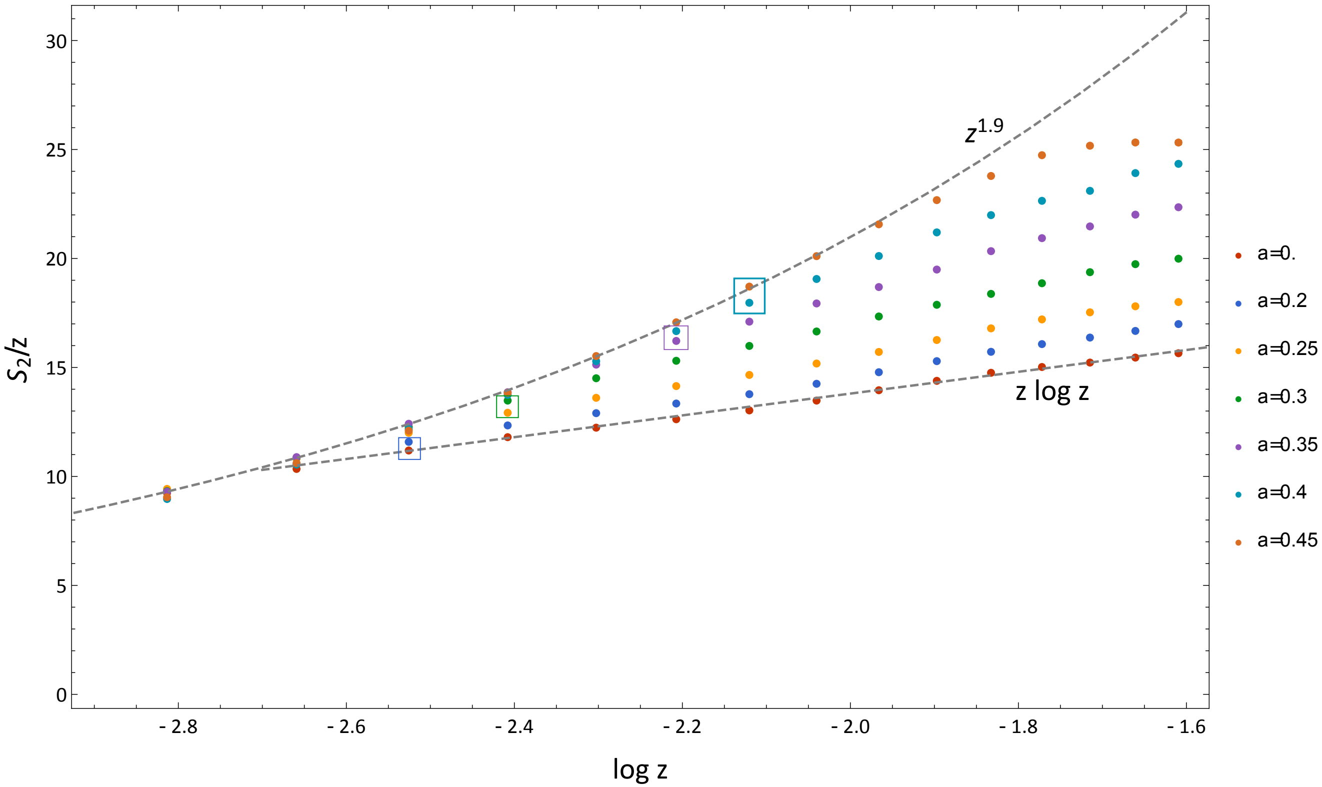
Backflow range exponent "eta" (=3 for hydro backflow) : $\eta(r) = \frac{a^\eta}{r^\eta + r_0^\eta}$

Fractal nodes and entanglement entropy.

Grover Kaplis Kruger

Second Renyi entropy: $S^q(z) = \frac{\ln(Tr\rho_A^q)}{1-q}, \quad q = 2$

Hydrodynamical backflow, for increasing backflow length a (a_c =0.5):



Fractal nodes and entanglement entropy.

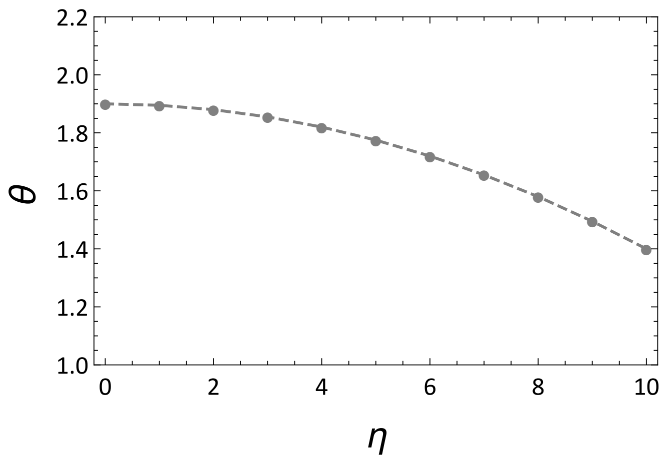
Grover Kaplis Kruger

$$\psi_{bf}(\mathbf{R}) \sim \mathrm{Det}\left(e^{i\mathbf{k}_i\tilde{\mathbf{r}}_j}\right)_{ij}$$

$$\tilde{\mathbf{r}}_j = \mathbf{r}_j + \sum_{l(\neq j)} \eta(r_{jl})(\mathbf{r}_j - \mathbf{r}_l)$$

$$\eta(r) = \frac{a^\eta}{r^\eta + r_0^\eta}$$

Tentative result: by varying the backflow range exponent *eta* the Haussdorff dimension of the nodal surface is changing, and thereby the range of the entanglement entropy, as for the holographic strange metals!



$$S^{(2)}(z) \sim S_{vN}(z) \sim z^\theta$$

Conclusions.

- Non-Fermi-liquids as densely entangled states of quantum matter: "signs" are a blessing!

- Holographic principle: non-FL quantum liquids at finite density are quantum critical phases characterized by non-Wilsonian scaling properties (space vs. time, "hyperscaling violation", …).

- Conjecture 1: the nodal surface (zero's of the full density matrix) forms the universal measure of "signful" infinite party entanglement.

- Conjecture 2: the nodal surface is either smooth or fractal and the vacuum is therefore either a Fermi-liquid or a quantum critical strange metal phase.

The tip of the iceberg.

- Electron systems in solids: "black holes teach us to think differently"
High quality "Smoking gun" predictions are lying ready to be tested in the laboratory (strange metals, holographic SC).

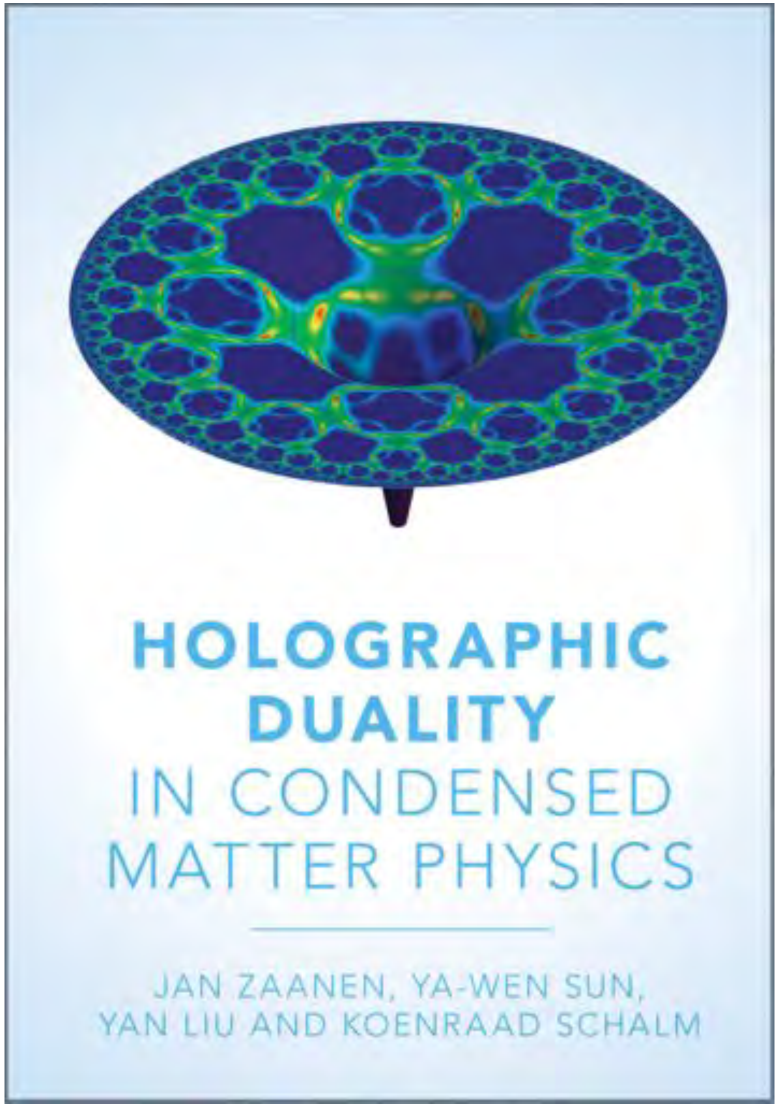
- Decoding the holographic answers in the field theoretical language: Wick-rotation, quantum information, generalized quantum statistics!?

- A wealth of more involved correspondences are lying ready to be explored ("top-downs": Dp/Dq branes, etc.)

- Time dependent AdS/CFT: non-equilibrium physics of field theoretical systems.

- What does this all mean for the greater quantum-gravity agenda?

Book sales …



Cambridge University Press

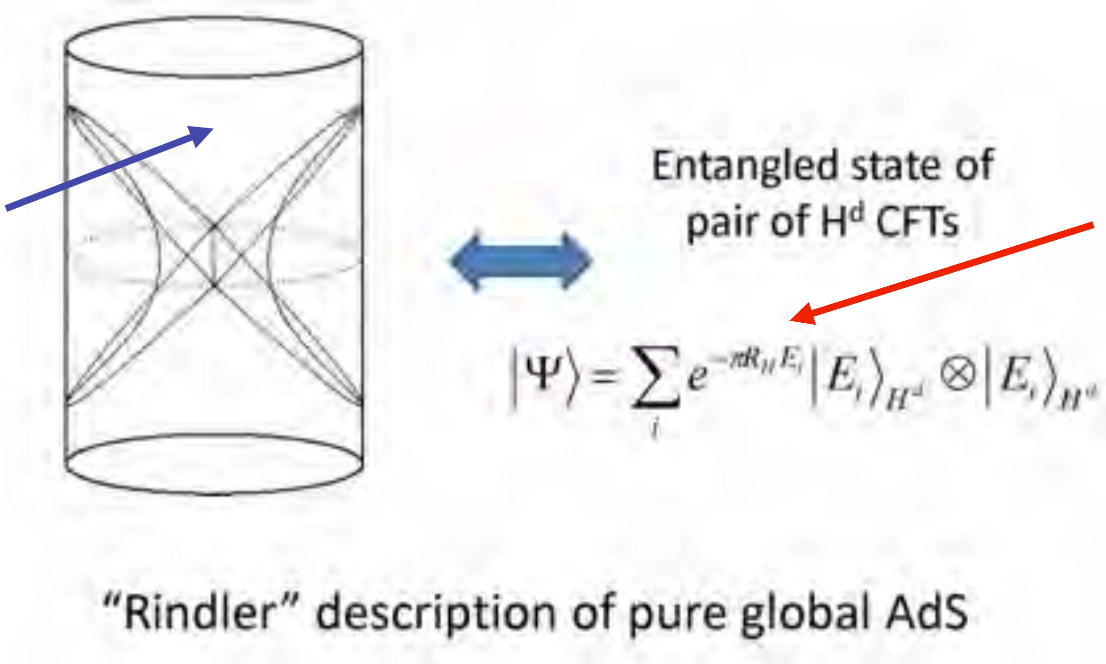
Release: October 28 2015.

It is 600 pages and only € 80!

Its from quantum bits ?

Van Raamsdonk

Classical space time in bulk …



Encoded by quantum info (entanglement spectrum) in boundary

also

Hubeny Myers Swingle

• • • • •

Magnitude of momentum relaxation.

According to the cuprate optical conductivity the momentum relaxation rate is:

$$\frac{1}{\tau_{\exp}} \approx \frac{k_B T}{\hbar}$$

According to "massive gravity", the RN strange metal has a momentum relaxation rate:

$$\frac{1}{\tau_{\exp}} = A\frac{\hbar}{l^2 m_e}\frac{S}{k_B} = A\frac{\hbar^2}{\mu l^2 m_e}\frac{k_B T}{\hbar} \qquad \textbf{assuming} \qquad \frac{S}{k_B} = \frac{k_B T}{\mu}$$

It follows for the microscopic mean free path:

$$l = \hbar\sqrt{\frac{A}{\mu m_e}} \approx 10^{-9} m$$

Holography’s Predictive power

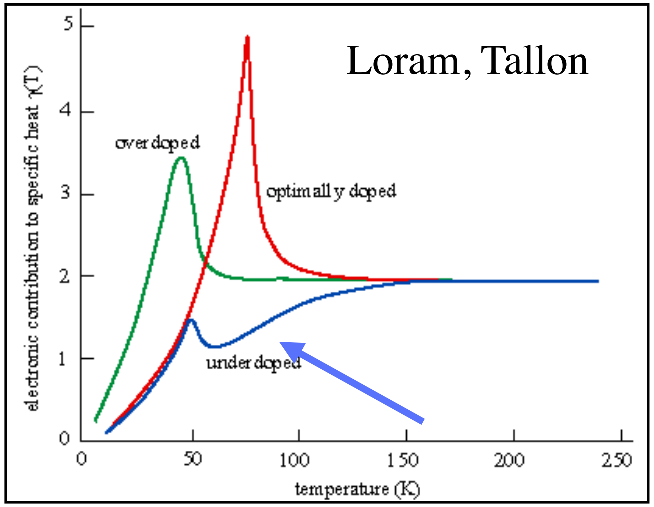
Richard Davison

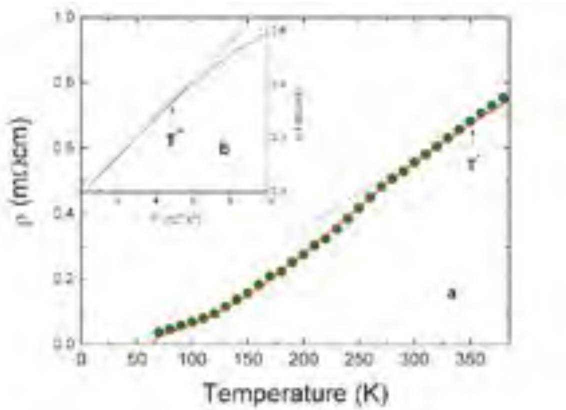
Pseudogap regime

$$C \propto T^2 \Rightarrow S \propto T^2 (?)$$

Massive gravity:

$$\rho \propto \frac{1}{\tau_{rel}} \propto S \propto T^2$$





DC resistivity

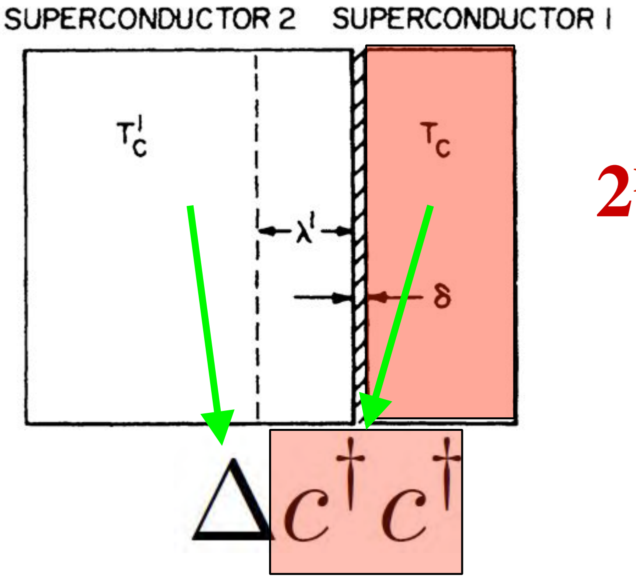
In addition:

- Violation of Wiedemann - Franz at low T.

- absence of anomalous skin effect at low T.

This would imply that the pseudogap phase is an “order induced” CFT (=> holography)!

Observing the origin of the pairing mechanism



$T_c' > T > T_c$

2nd order Josephson effect

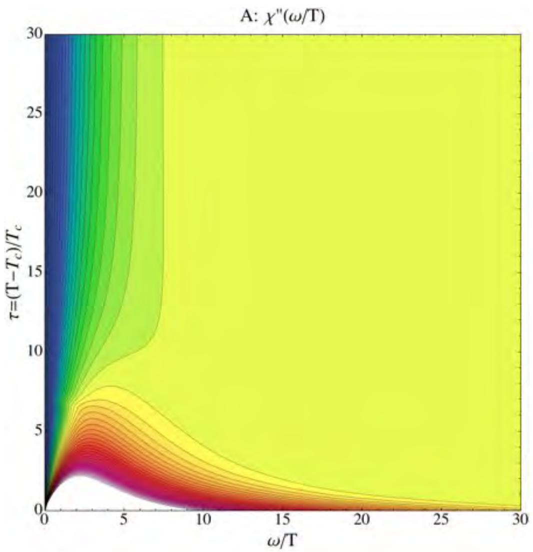
Ferrell Scalapino

1969 1970

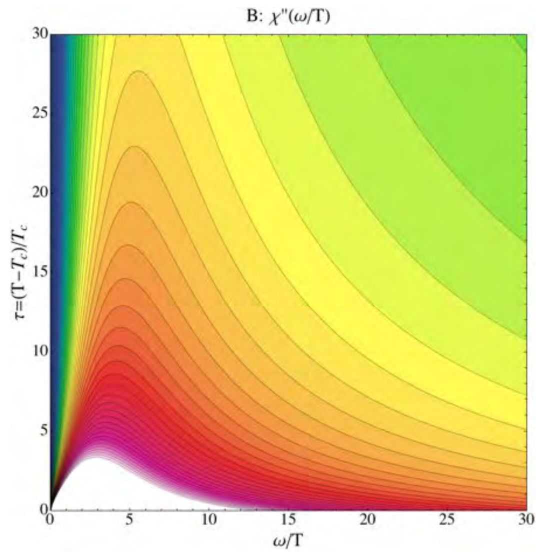
$$I_s(\mathbf{H}, V) \sim \frac{1}{R_N^2} \mathrm{Im}\chi_{\mathrm{pair}}(\mathbf{k}, \omega)$$

$$\omega = 2eV$$

Standard BCS



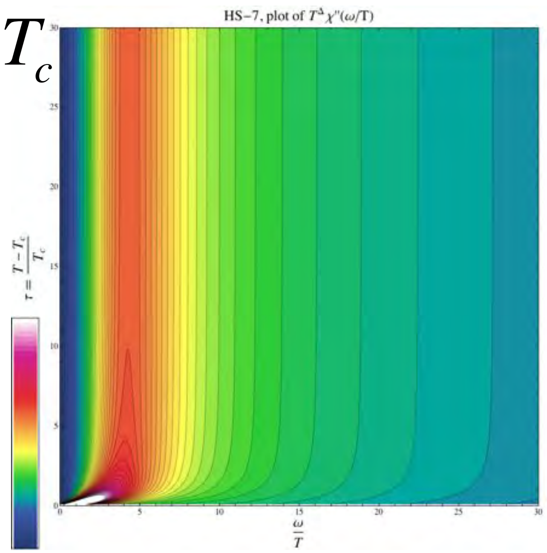
"Critical glue"



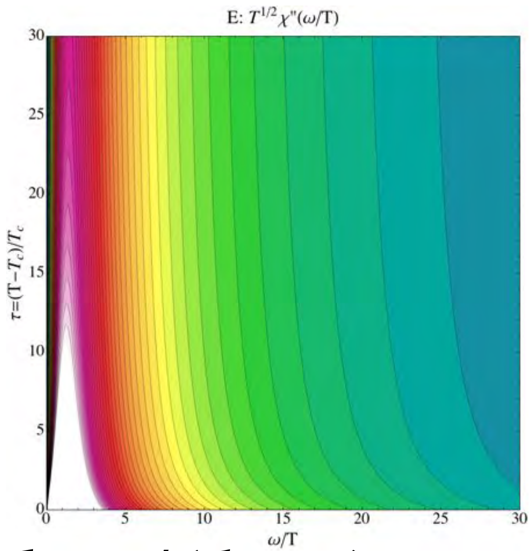
$$T^{\Delta}\chi_p^{''}\left(\frac{\hbar\omega}{k_B T}\right)$$

Holographic SC (AdS4)

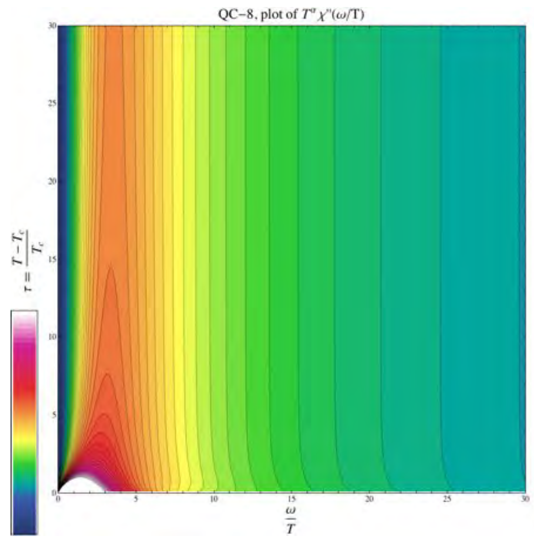
$T - T_c$



Holographic SC (AdS2)



QC-BCS

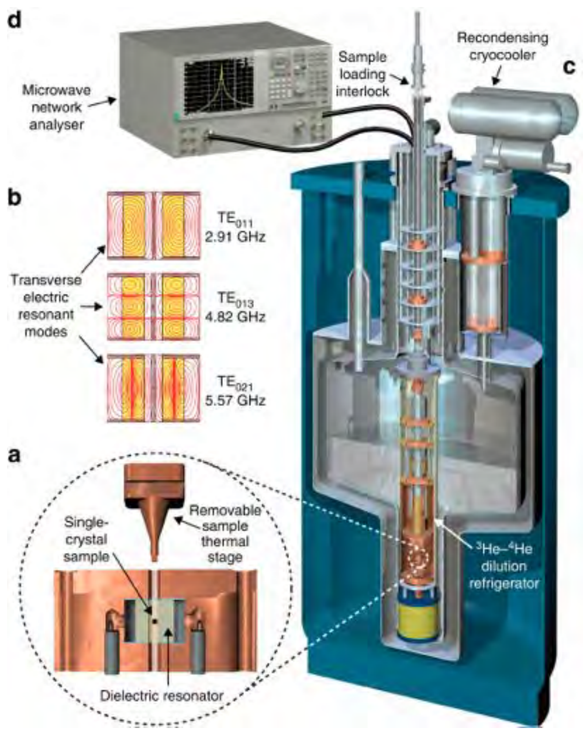


$\hbar\omega/(k_B T)$

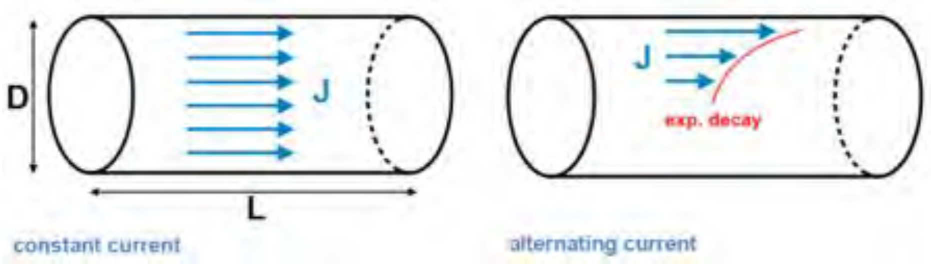
Skin effect in metals …

Measure impedance of sample in microwave cavity:

$$Z(\omega) = Z'(\omega) + iZ''(\omega)$$



AC currents penetrate over a skin depth $\delta \simeq Z'(\omega)$



Fermi-liquid at high temperature:
"classical" skin effect

$$\delta \simeq \sqrt{\frac{\rho(T)}{\omega}}$$

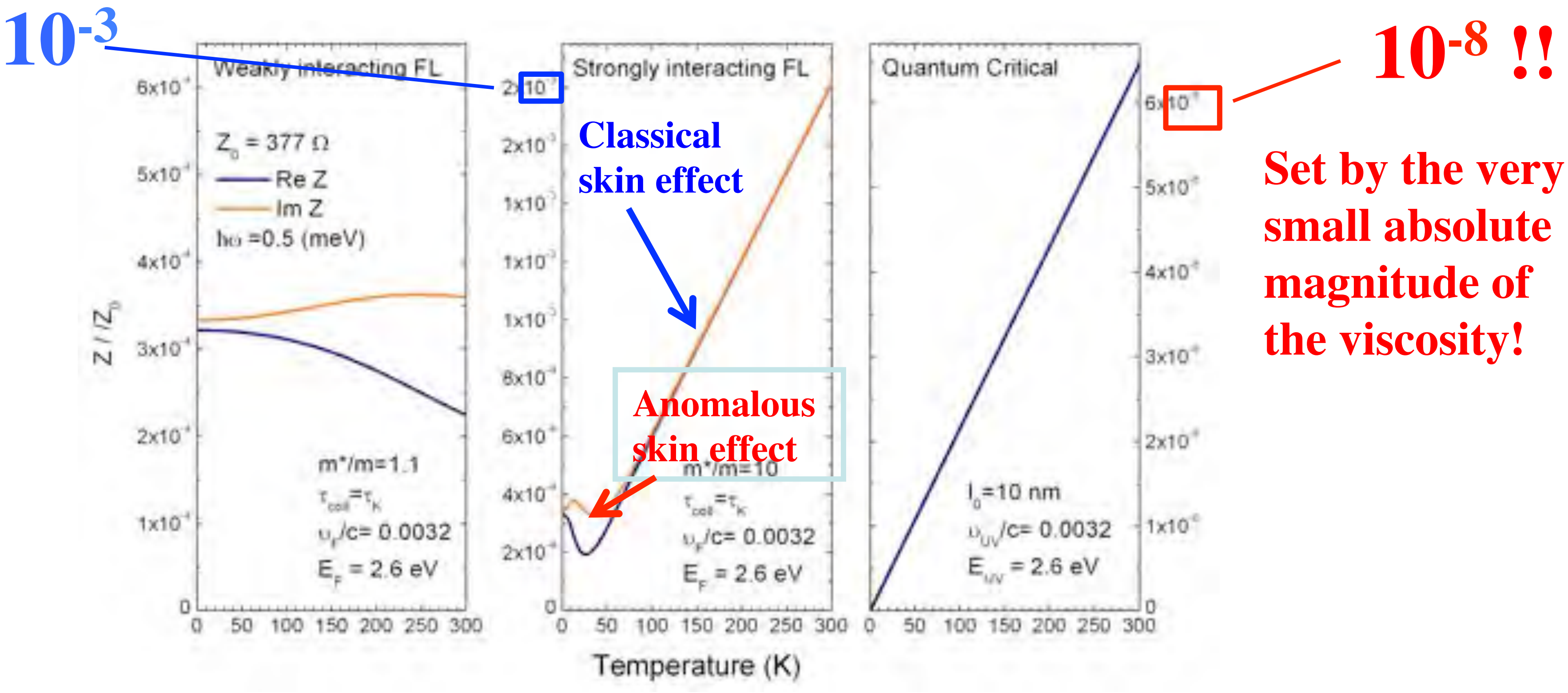
Collision-less regime at low temperature:
"anomalous" skin effect

$$\delta \simeq \frac{1}{\omega^{1/3}}$$

Skin effect and viscosity…

Forcella, JZ, Valentinis, van der Marel, arXiv:1406.1356.

Reformulation skin effect in magneto-hydrodynamical language: set by propagation of transversal sound, sensitive to viscosity!



Empty.

Empty.

Empty.

Bulk geometry: AdS Reissner-Nordstrom black hole

Finite temperature and finite charge density: AdS RN black hole

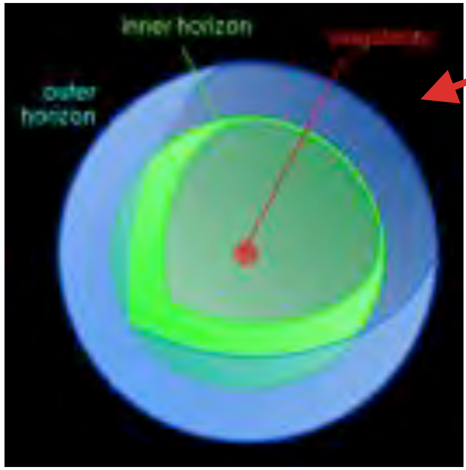
$$ds^2 = -g(r)dt^2 + \frac{dr^2}{g(r)} + r^2\left(dx^2 + dy^2\right)$$

where $$g(r) = r^2 - \frac{1}{r}\left(r_+^3 + \frac{\rho^2}{4r_+}\right) + \frac{\rho^2}{4r^2}$$

Scalar potential: $$A_0 = \rho\left(\frac{1}{r_+} - \frac{1}{r}\right)$$

Hawking temperature: $$T = \frac{12r_+^4 - \rho^2}{16\pi r_+^3}$$

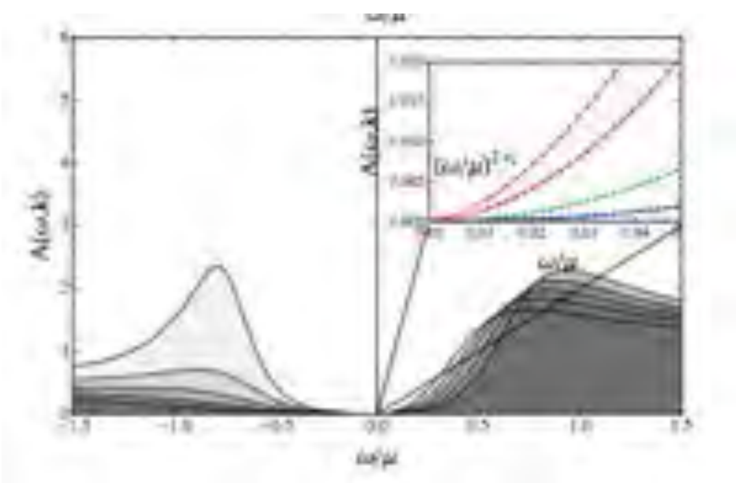
Finite density: the Reissner-Nordstrom strange metals (Liu et al.).



Near-horizon geometry of the extremal RN black hole:

- Space directions: flat, codes for simple Galilean invariance in the boundary.

- Time-radial(=scaling) direction: emergent AdS_2, codes for emergent temporal scale invariance!



Fermion spectral functions:

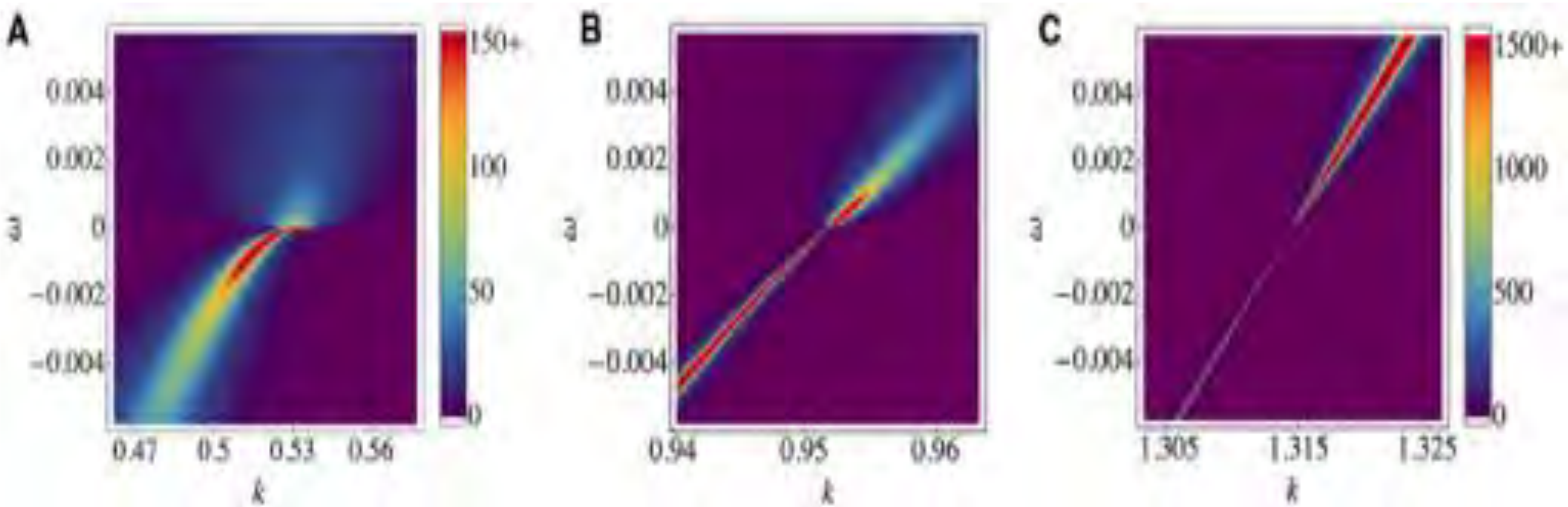
$$A(k,\omega) \propto G''_{AdS_2}(k,\omega) \propto \omega^{2\nu_k}$$

$$\nu_k = \frac{1}{\sqrt{6}}\sqrt{k^2 + \frac{1}{\xi^2}}$$

"Un-particle physics!"

AdS/ARPES for the Reissner-Nordstrom non-Fermi liquids

Fermi surfaces but no quasiparticles!



Critical FL

Marginal FL

Non Landau FL

Cubrovic, Schalm, JZ;
Liu, McGreevy et al.