Macroscopic manipulation of Majorana fermions with superconducting circuits

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Plan

Main question:
When Majoranas in superconducting devices are made, how to braid them, and how to use them for quantum computing?

Plan:

- Cooper pair box, a building block for coupling Majoranas.
- Minimal braiding setup.
- A possible QC register.

(Re)introduction to Majorana fermions

1. Majorana fermion is a particle equal to its antiparticle
$$\gamma = \gamma^\dagger$$
2. Two Majoranas may store an electron
$$c^\dagger = \frac{1}{\sqrt{2}}(\gamma_1 + i\gamma_2), \;\; c = \frac{1}{\sqrt{2}}(\gamma_1 - i\gamma_2)$$
3. Energy cost of two Majoranas
$$H = \varepsilon c^\dagger c = 2\varepsilon i \gamma_1 \gamma_2$$
vanishes if they are separated. (Kitaev, 2000)
The electron stays completely hidden.
4. Majoranas can be created by cleverly combining a mesoscopic system, superconductor, and time-reveral symmetry breaking.

What's the big deal with Majoranas

1. Take 4 Majoranas
2. Prepare in state $|\psi\rangle = c_{12}^\dagger c_{34}^\dagger |\text{vac}\rangle$

γ_1 γ_2

$|\text{filled}\rangle$

$|\text{filled}\rangle$

γ_4 γ_3

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3. Pair Majoranas differently…

$$\begin{array}{ccc} \gamma_1 & & \gamma_2 \\ \bullet & |\text{filled}\rangle & \bullet \\ & & \\ \hline & & \\ \bullet & |\text{filled}\rangle & \bullet \\ \gamma_4 & & \gamma_3 \end{array}$$

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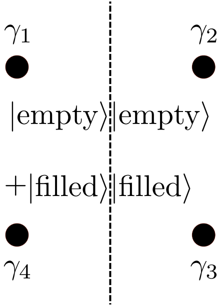
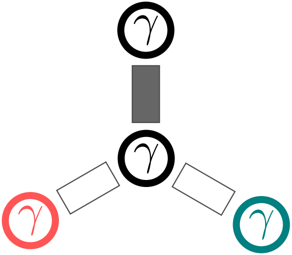
3. Pair Majoranas differently. . .
4. $|\psi\rangle = \frac{1}{\sqrt{2}}(1 + c^\dagger_{13} c^\dagger_{34})|\text{vac}\rangle$



Discrete version of braiding



Clarke, Sau, Tewari

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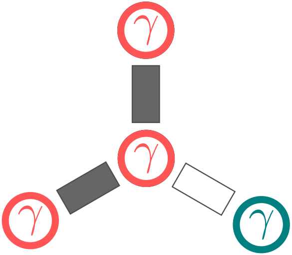

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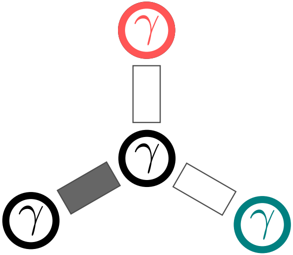

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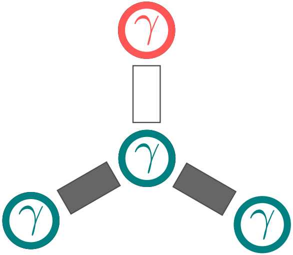

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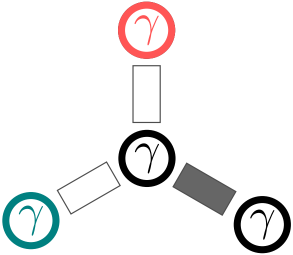

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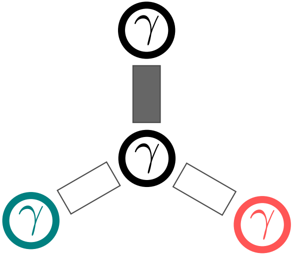

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Are Majoranas useful?

Braiding is nice, but:

1. It only allows to implement a finite set of unitaries.
2. Even if you couple two Majoranas, they are still non-interacting fermions (and so not a universal QC)

Yes, they are: 'Magic state' distillation

Majorana-based quantum computer (Bravyi, Kitaev):

1. Braiding
2. Measure if two Majoranas store a fermion:
$$(1 - 2c^\dagger c) = 2i\gamma_1\gamma_2$$
3. Phase gate (coupling two Majoranas):
$$\exp(i\alpha c^\dagger c)$$
4. Measure four Majoranas (= fermion parity):
$$(1 - 2c_1^\dagger c_1)(1 - 2c_2^\dagger c_2) = 4\gamma_1\gamma_2\gamma_3\gamma_4$$

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Only one error-prone operation (phase gate) with 11% error tolerance.

Coupling Majoranas

Two ways:

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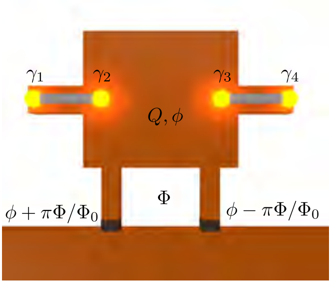
Coupling Majoranas

Two ways:

1. Bring Majorana close, make wave functions overlap
   Used in "piano keyboard" braiding and many readout schemes.
2. Put Majorana on a small capacitor, use Coulomb energy
   - *Required* to measure four Majoranas
     (single particle Hamiltonian not enough)
   - Can be used as the only ingredient for universal Majorana manipulation.
   - Was used to implement topological phases with Majoranas
     (Fu&Xu, Terhal&Hassler&DiVincenzo)

Cooper pair box (CPB)

A small superconducting island, coupled to a bulk superconductor



$$H = -E_{J1}\cos(\phi + \pi\Phi/\Phi_0) - E_{J2}\cos(\phi - \pi\Phi/\Phi_0))$$

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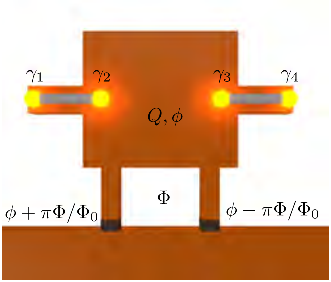

$$H = -E_J(\Phi)\cos\phi$$

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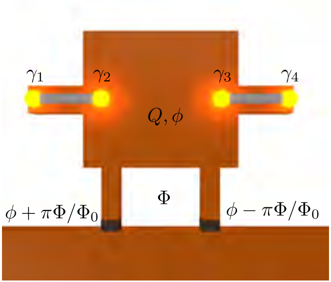

$$H = -E_J(\Phi)\cos\phi + E_C(n - CV/e)^2$$

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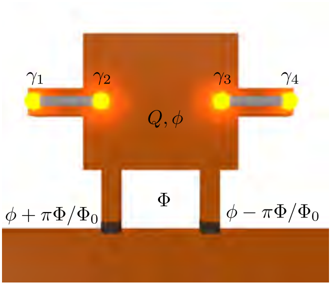
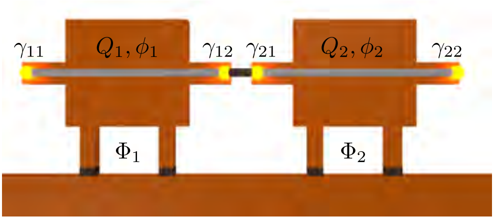


$$H = -E_J(\Phi)\cos\phi + E_C(n - CV/e)^2$$

$$n = \frac{\partial}{\partial\phi}, \quad (-1)^n = \prod_n \sqrt{2} i\gamma_n \equiv \mathcal{P}$$

Majorana Josephson junction

For $\mathcal{P}$ to not be constant, Majoranas from different islands must couple.



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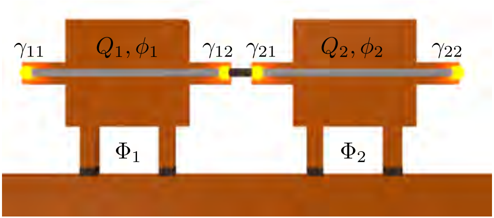

$$H = iE_M \gamma_{12}\gamma_{21} \cos(\phi_1/2 - \phi_2/2)$$

Flips $\mathcal{P}$ and changes n by ± 1 simultaneously.

Effective Hamiltonian of a CPB network

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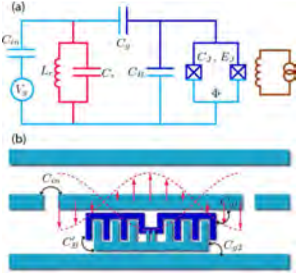
2. Set $E_J \gg E_C$, then $\phi \approx 0$.
   - $E_n = \bar{E}_n - U_n\mathcal{P}, \quad U_0 = 16\left(\frac{E_C E_J^3}{2\pi^2}\right)^{1/4} e^{-\sqrt{8E_J/E_c}} \cos(CV/e)$
   - $E_1 - E_0 \sim \mathcal{P}$

Readout of Majorana parity in a CPB

- Splitting $E_1 - E_0$ was measured using a transmission line
  Cavity resonance frequency
  $\omega = \omega_0 + g^2 \mathcal{P}(U_1 - U_0)/\hbar$



(See PRA 76, 042319 for review)

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- Splitting $E_1 - E_0$ was measured using a transmission line
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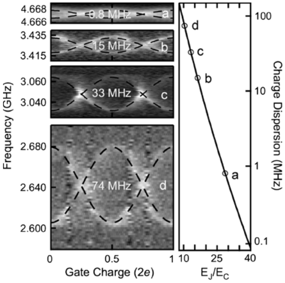
- Exponential suppression of U by flux was demonstrated.



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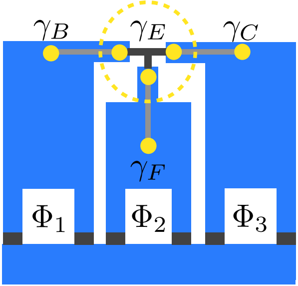
Tri-junction Majorana

When $E_C \ll E_M$, the Majoranas at the tri-junction hybridize, forming

$$\gamma_E = \frac{\cos\alpha_{23}\gamma_1 + \cos\alpha_{13}\gamma_2 + \cos\alpha_{12}\gamma_3}{\sqrt{\cos^2\alpha_{23} + \cos^2\alpha_{13} + \cos^2\alpha_{12}}}$$



A setup for demonstration of non-Abelian statistics

A CPB can be used to couple and readout Majoranas, how to demonstrate braiding in a minimal fashion?

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- 6 Majoranas (2 control + 2 braid + 2 ancillas for braiding)

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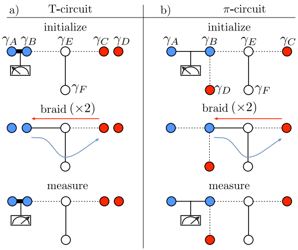
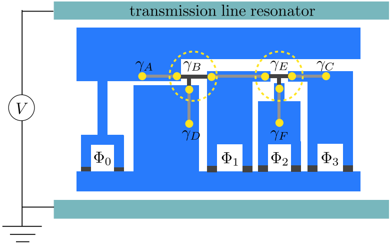
- 6 Majoranas (2 control + 2 braid + 2 ancillas for braiding)
- 4 adjustable couplings (3 to braid, 1 for readout)
- 1 tri-junction + 1 controlled Majorana coupling or 2 tri-junctions



Braiding in a π-circuit

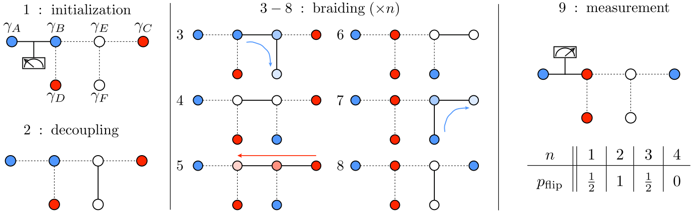
a)



b)

time step	Φ_0	Φ_1	Φ_2	Φ_3
0	0	0	$-\Phi_{\mathrm{max}}$	0
1	Φ_{max}	0	0	0
2	0	0	$-\Phi_{\mathrm{max}}$	0
3	0	Φ_{max}	$-\Phi_{\mathrm{max}}$	0
4	0	Φ_{max}	0	0
5	0	Φ_{max}	0	Φ_{max}
6	0	0	0	Φ_{max}
7	0	0	$-\Phi_{\mathrm{max}}$	Φ_{max}
8	0	0	$-\Phi_{\mathrm{max}}$	0
9	Φ_{max}	0	0	0

c)



n	1	2	3	4
p_{flip}	$\frac{1}{2}$	1	$\frac{1}{2}$	0

Majorana qubit

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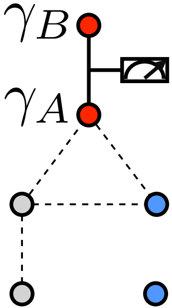

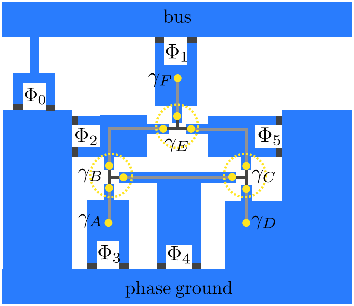

RAMM — Random Access Majorana Memory

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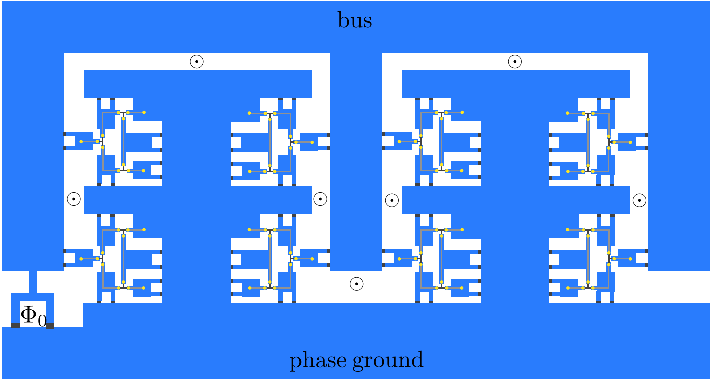

Summary

1. Coulomb coupling is a necessary component of a Majorana quantum computer.
2. It can be controlled with exponential range by using transmon qubits.
3. Transmon circuits allow to implement braiding and quantum computing registers using minimal number of control parameters.

Summary

Thank you all.
The end.