Anyonic defects: A new paradigm for non-Abelian Statistics

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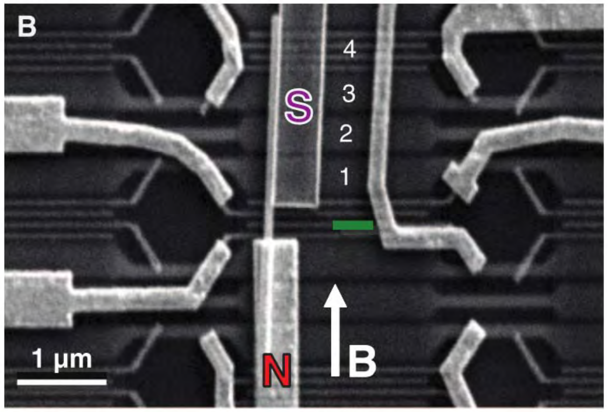
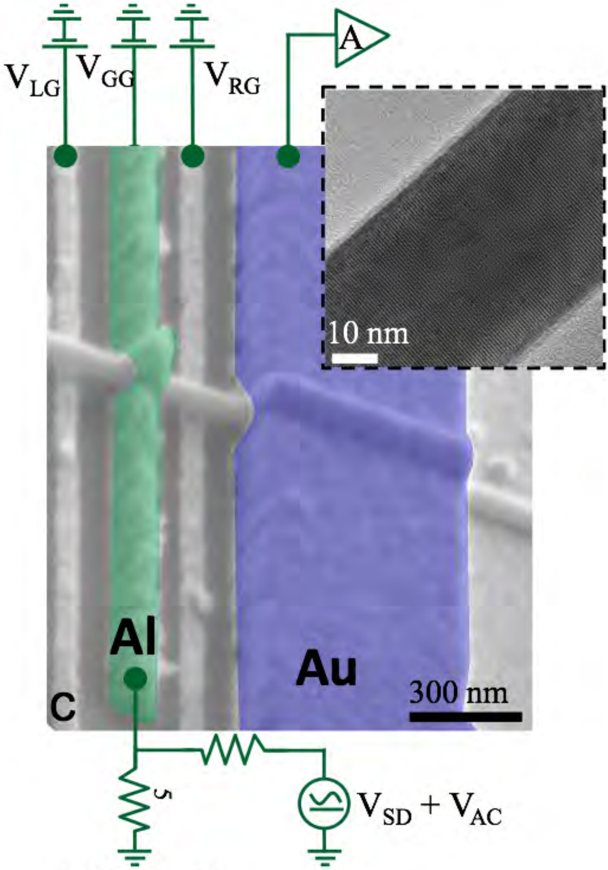
Majorana zero modes

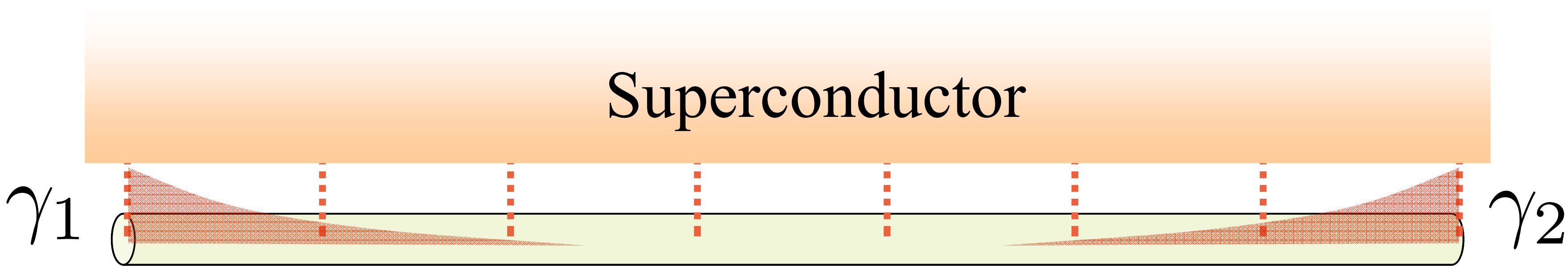


Mourik et al. (2012)



Das et al. (2012)

Majorana zero modes in a topological superconductor

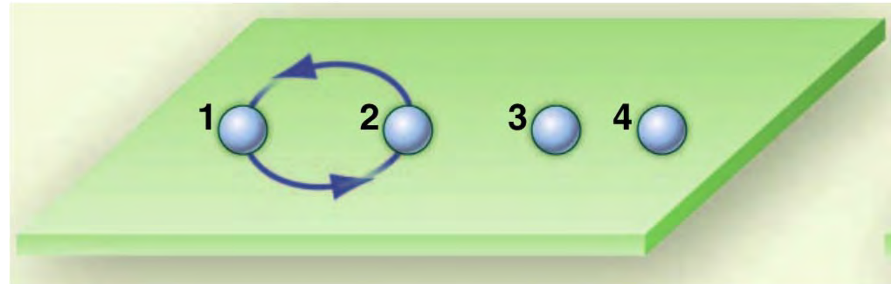


- **Gapped system, two degenerate ground states,** characterized by having a different **fermion parity**
- **Defects** (in this case, the edges of the system) carry protected **zero modes**
- Ground state degeneracy is "**topological**": no local measurement can distinguish between the two states!
- **Useful as a "quantum bit"?**

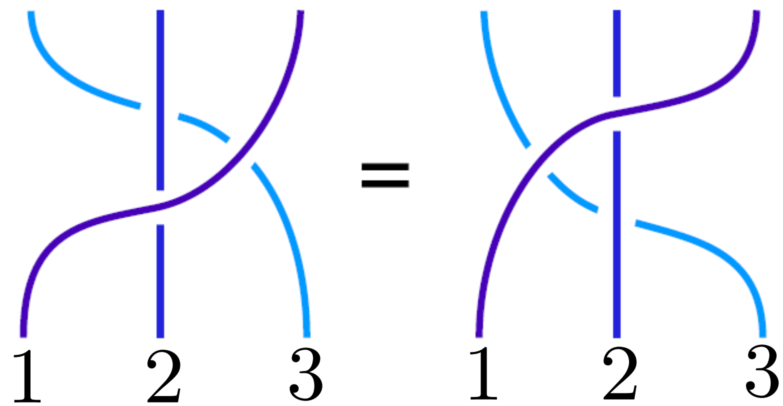
Kitaev (2001), Oreg (2009), Lutchyn (2009),…

Non-Abelian Statistics: Braiding

$$|\psi_i\rangle \rightarrow \sum_j U_{ij} |\psi_j\rangle$$



Braid group:



Majorana Fermions: $$e^{(\pi/4)\gamma_1\gamma_2} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

2D vortices: Ivanov, Read & Green,…
1D wire network: Alicea et.al (2010)

Outline

- "Fractionalized Majoranas" on fractional quantum Hall edges
  - Fractionalized 1D superconductors
  - Twist defects
- Anyonic defects in non-Abelian systems

- **Can we get something richer than Majorana fermions in 1D ?**

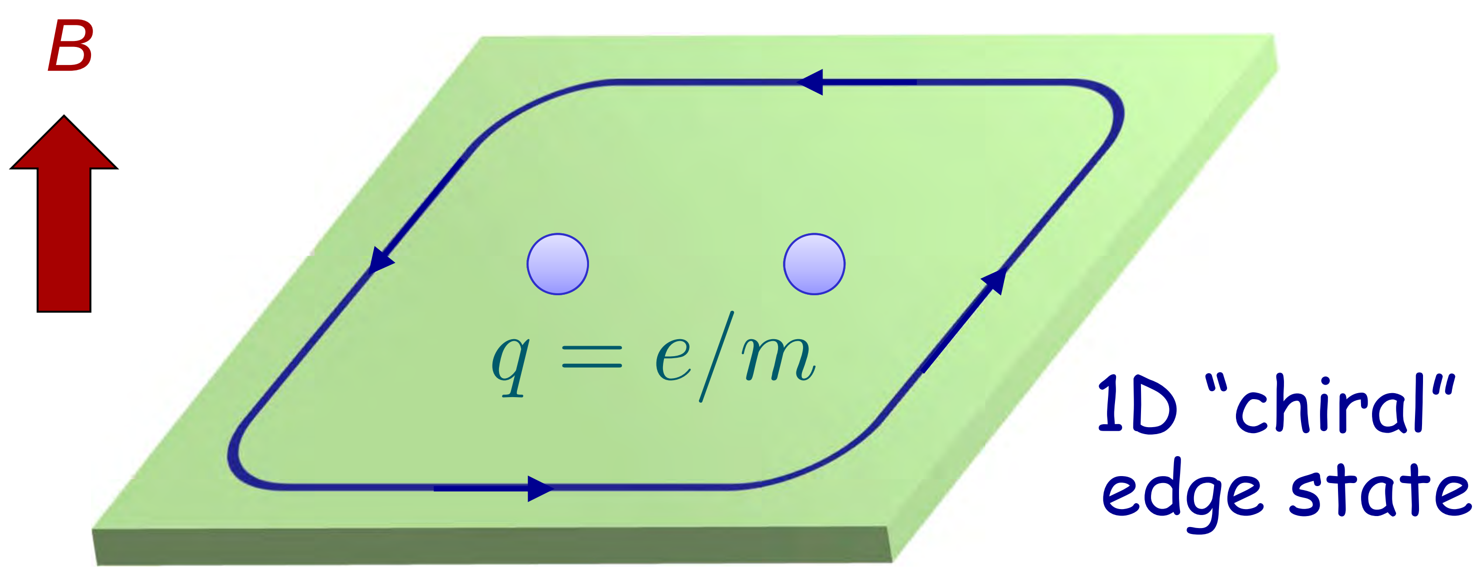
"Theorem" (Fidkowsky, 2010; Turner, Pollmann, and EB, 2010):

Gapped, local Hamiltonians of fermions or bosons in 1D, can give (at best) Majorana zero modes.

Beyond Majorana fermions

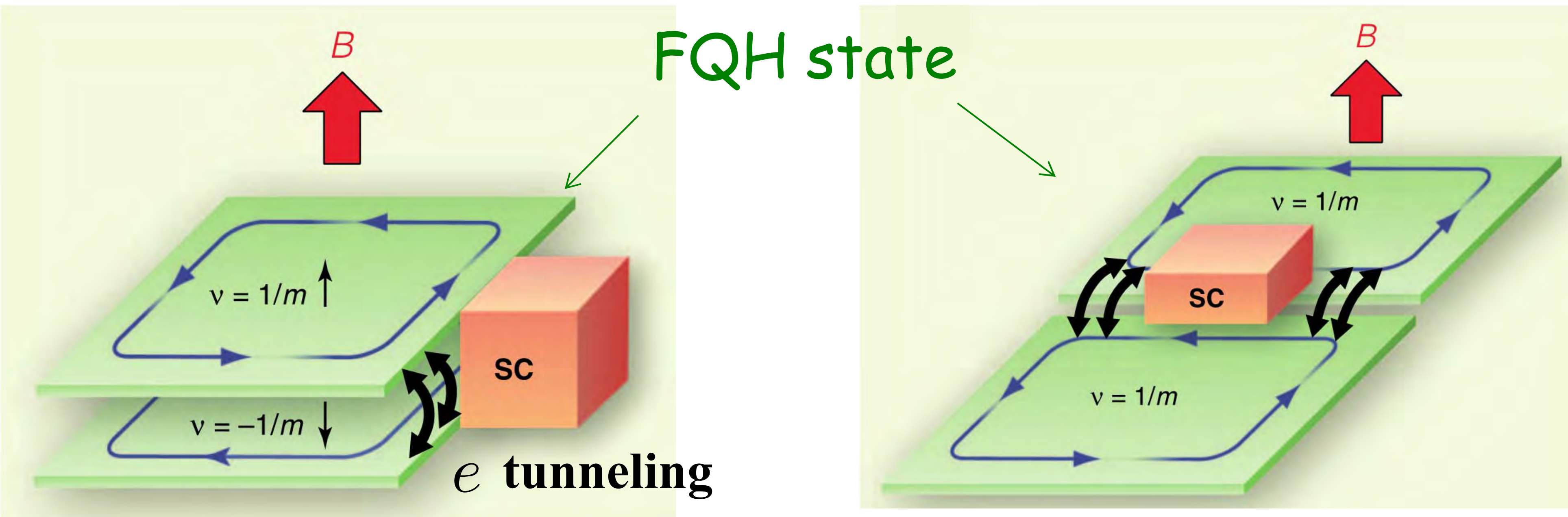
Consider the effectively 1D boundaries
of 2D a topological phase
which supports (abelian) anyons.

For example:
ν = 1/m Fractional Quantum Hall (Laughlin) state



Beyond Majorana fermions

Setups for fractionalized Majorana zero modes:



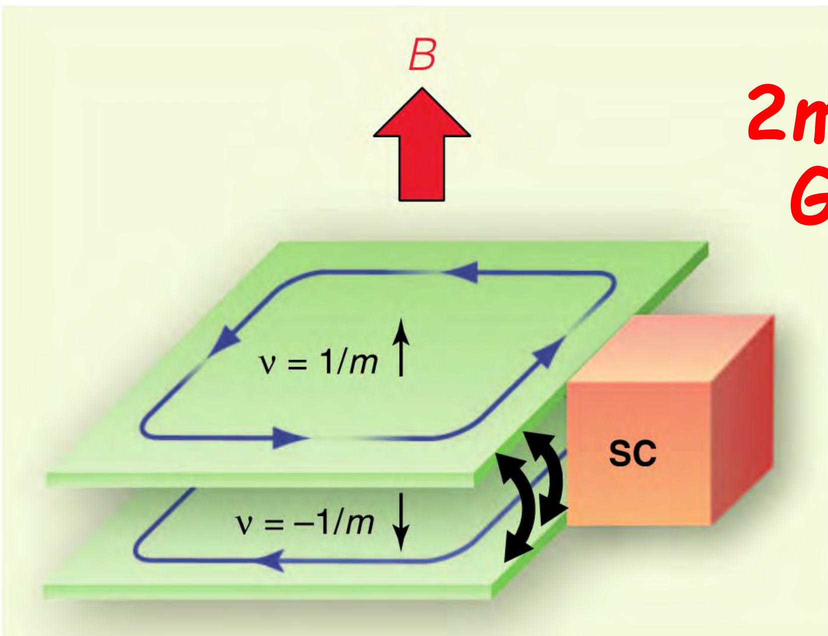
Lindner, EB, Stern, Refael (PRX, 2012);
Clarke, Alicea, Shtengel (Nature Comm., 2013);
Cheng (PRB, 2013)

Ground state degeneracy

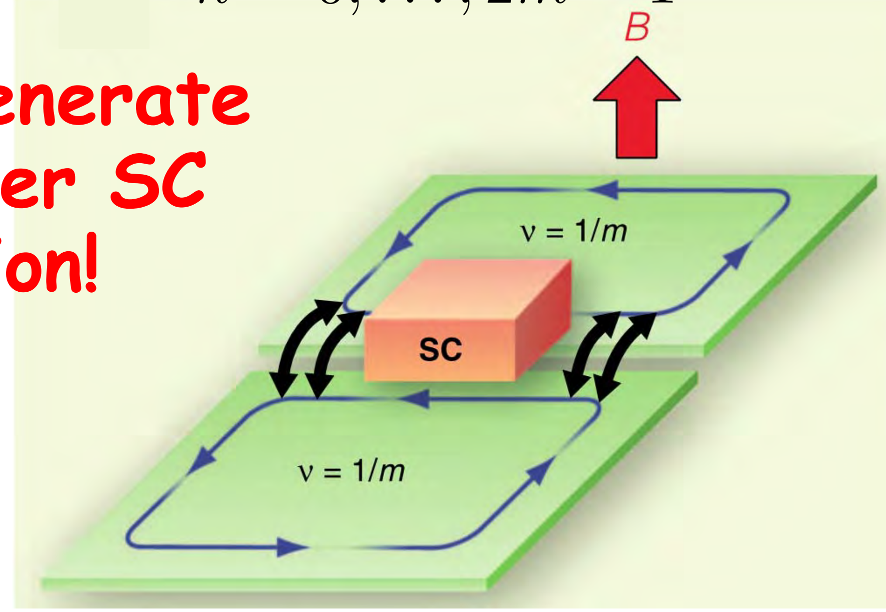
Fermion parity conservation in SC region: $[e^{i\pi\hat{Q}}, H] = 0$

On the edge of a fractional quantum Hall phase, Q is fractional: $\hat{Q} = \frac{n}{m}$

$n = 0, \ldots, 2m-1$



2m degenerate G.S. per SC region!



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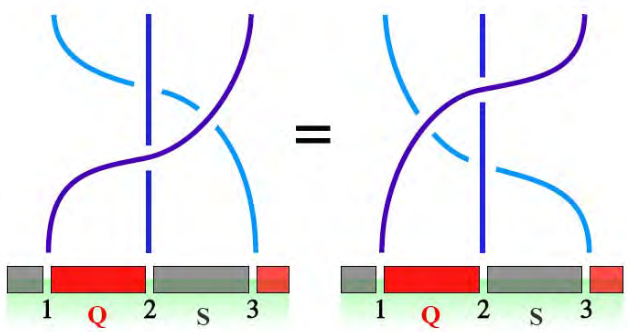
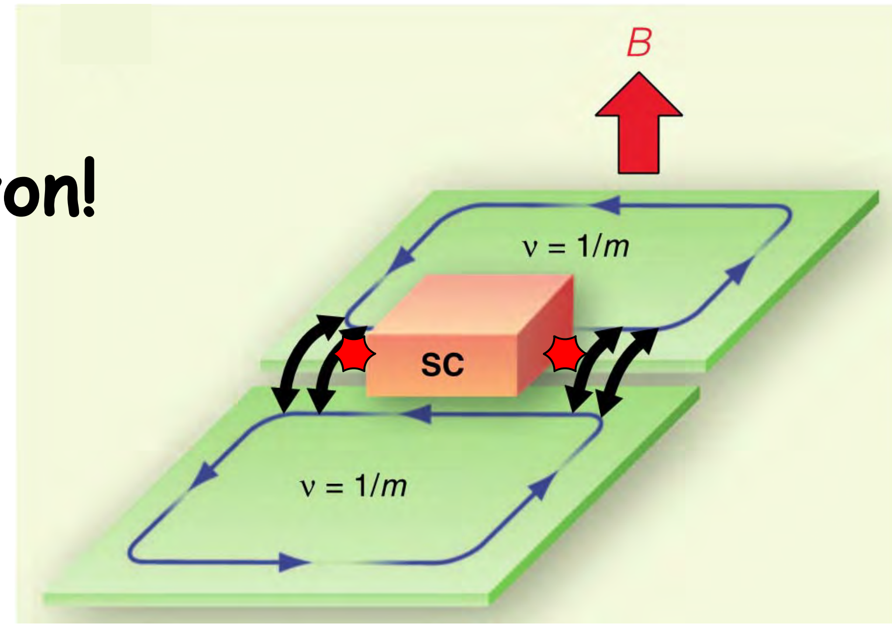
Fractionalized Majorana zero modes

"non-Majorana" zero mode,
Carrying a degeneracy of $\sqrt{2m}$

Braiding rule: $U_{12} = e^{i\frac{\pi m}{2}\hat{Q}^2}$

New type of non-abelian anyon!





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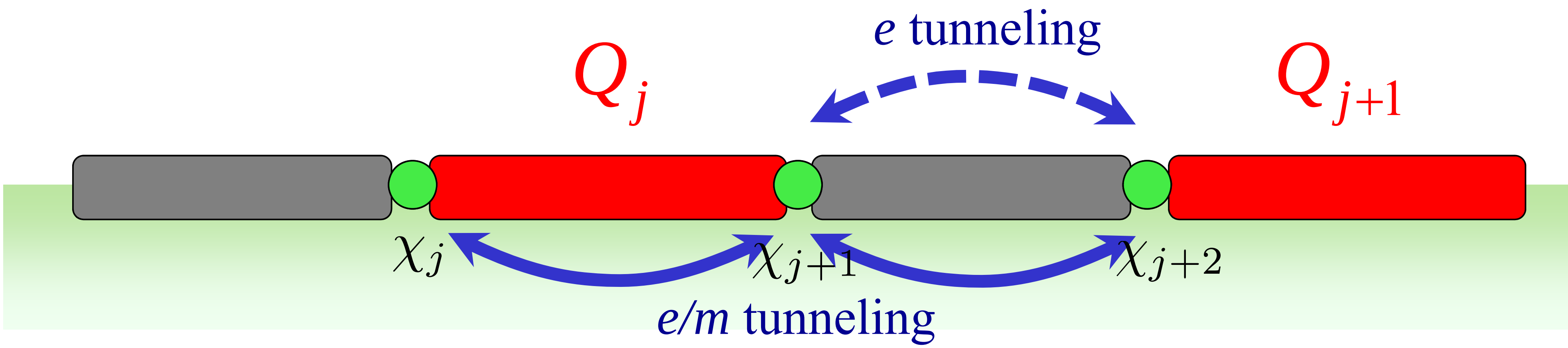
Prafermion (fractionalized Majorana) zero modes



"Parafermionic" exchange relations: $\chi_i \chi_j = \chi_j \chi_i e^{\frac{i\pi \mathrm{sgn}(j-i)}{m}}$

Fradkin, Kadanoff (1980); Fendley (2012)

e tunneling: $H_{ij,e} = -t(\chi_i^{+})^m (\chi_j)^m + H.c.$

e/m tunneling: $H_{ij,e/m} = -t\chi_i^{+}\chi_j + H.c.$

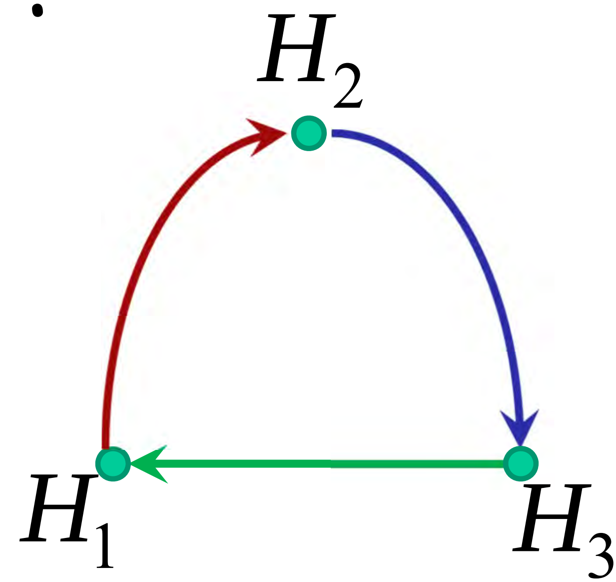
Chains of interacting parafermions: Motruk, EB, Turner, Pollmann (2013)

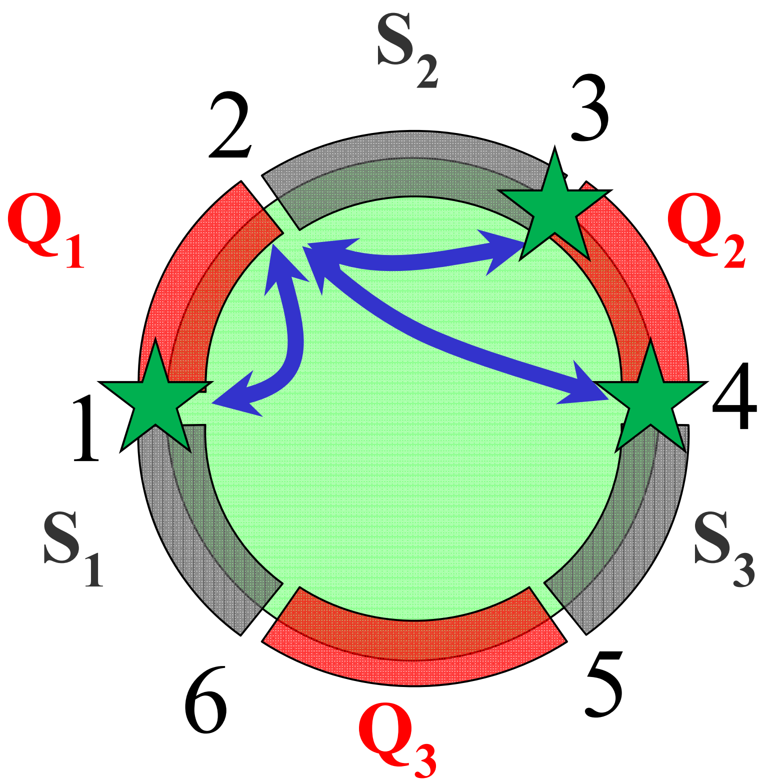
2D arrays: Mong, Clarke, Alicea, … , EB, … (2014)

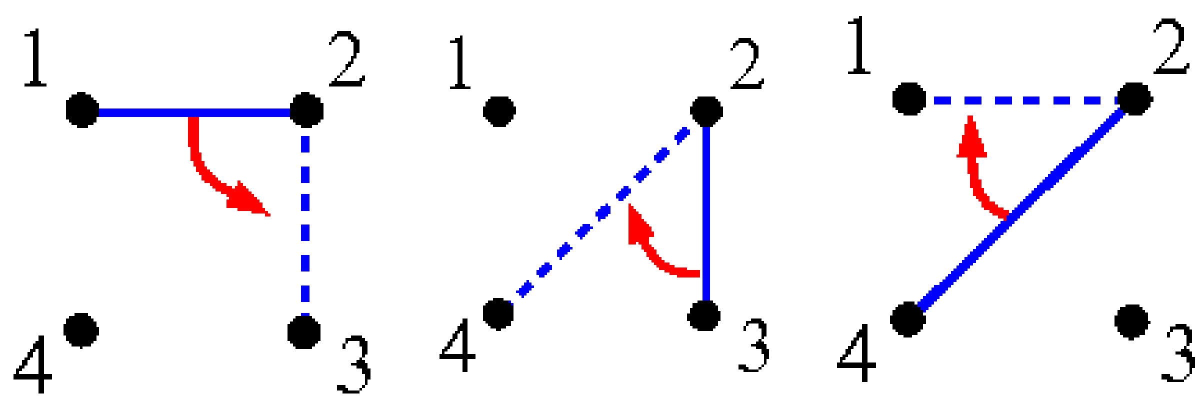
Braiding

Braiding interfaces ★ :

$$H(t) = -\sum_{ij} \lambda_{ij} \chi_i^{+} \chi_j + H.c.$$



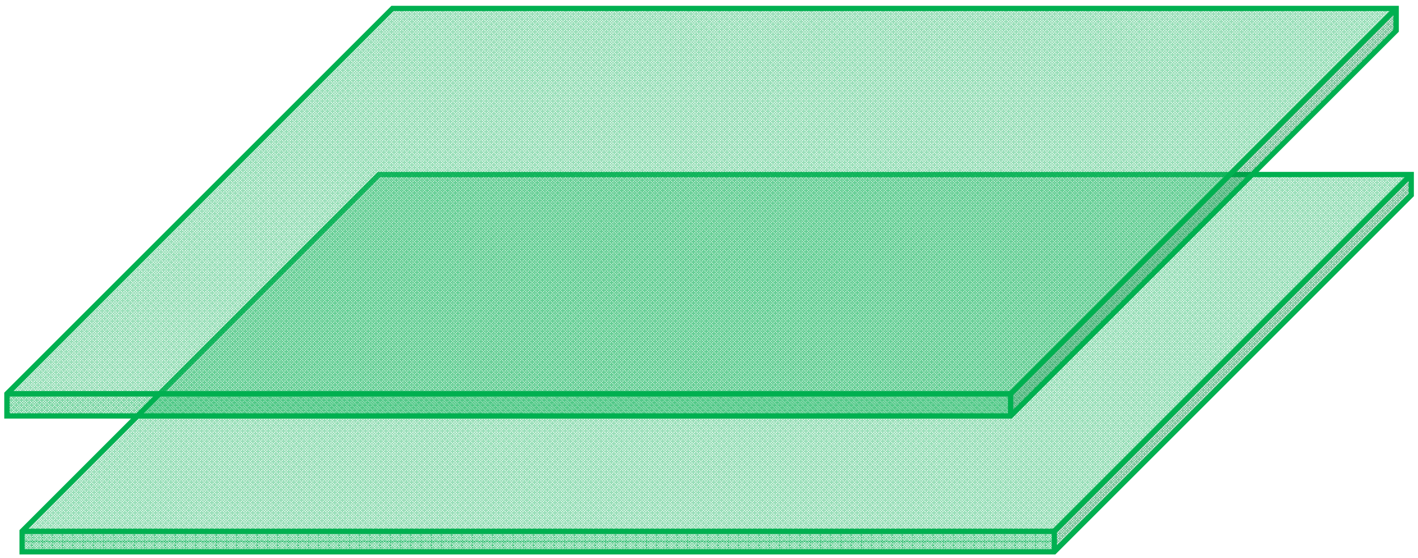




$$U_{34} = e^{\frac{i\pi m}{2} \hat{Q}_2^2}$$

Fractionalized zero modes at "twist defects" in topological phases

Another example: ν=1/3 bilayer

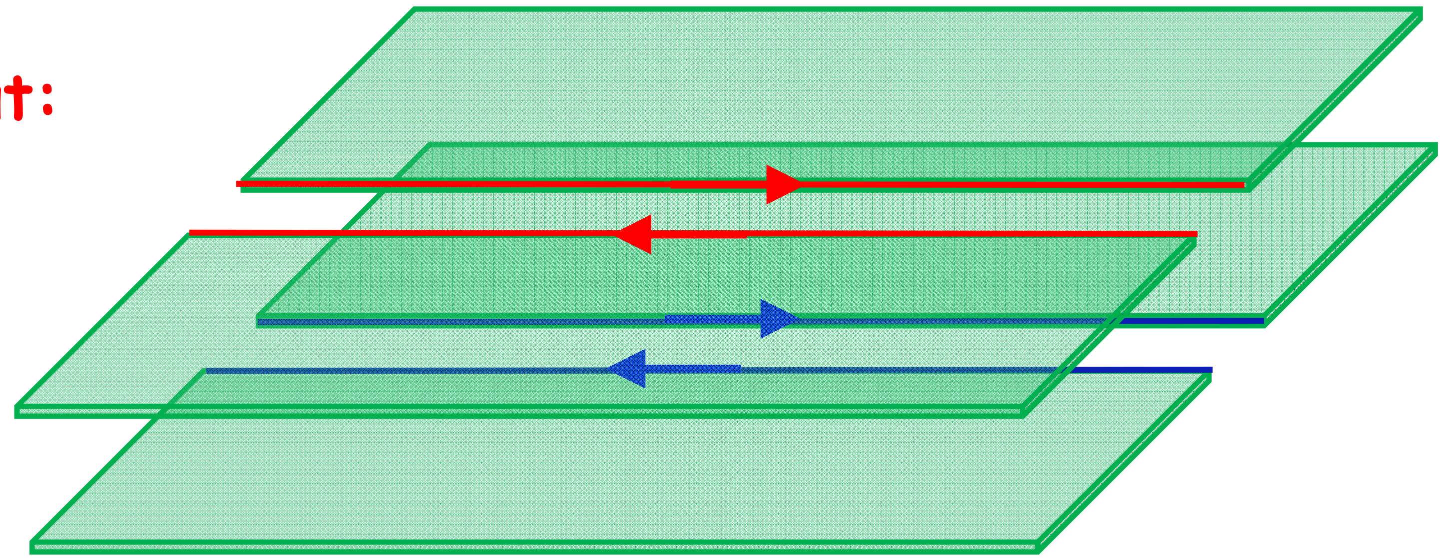


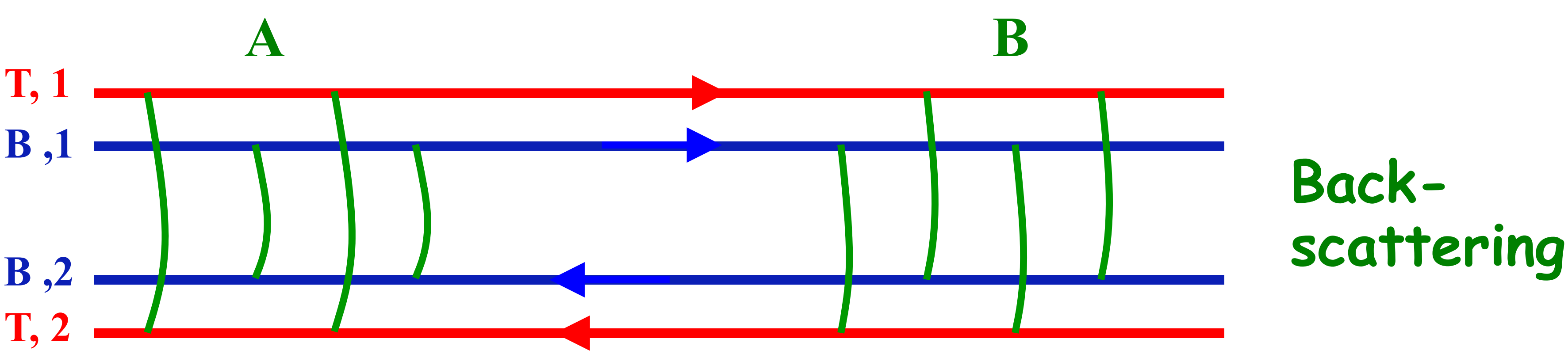
Barkeshli, Jian, Qi (2013; 2014)

Fractionalized zero modes at "twist defects" in topological phases

Another example: ν=1/3 bilayer

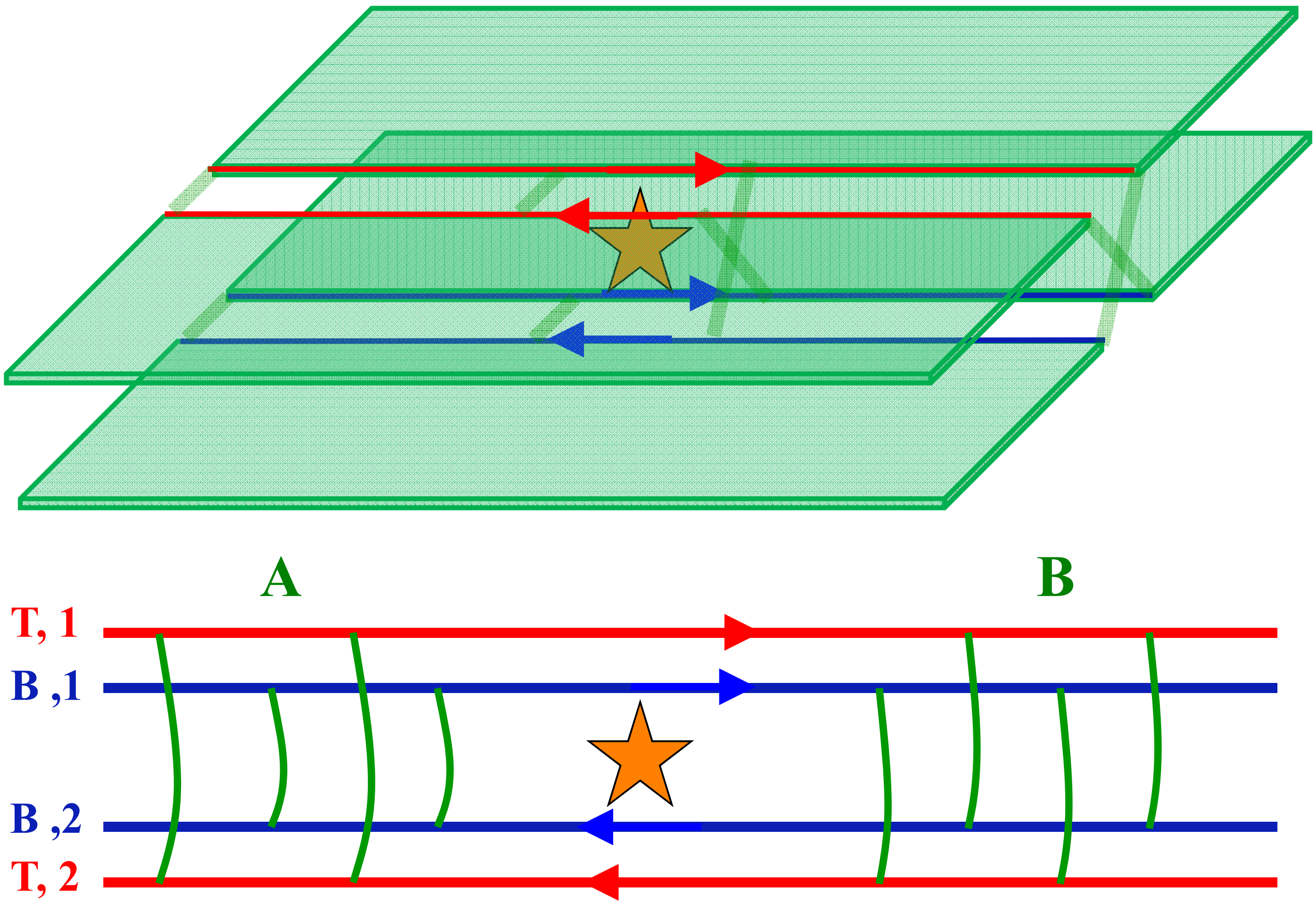
Cut:





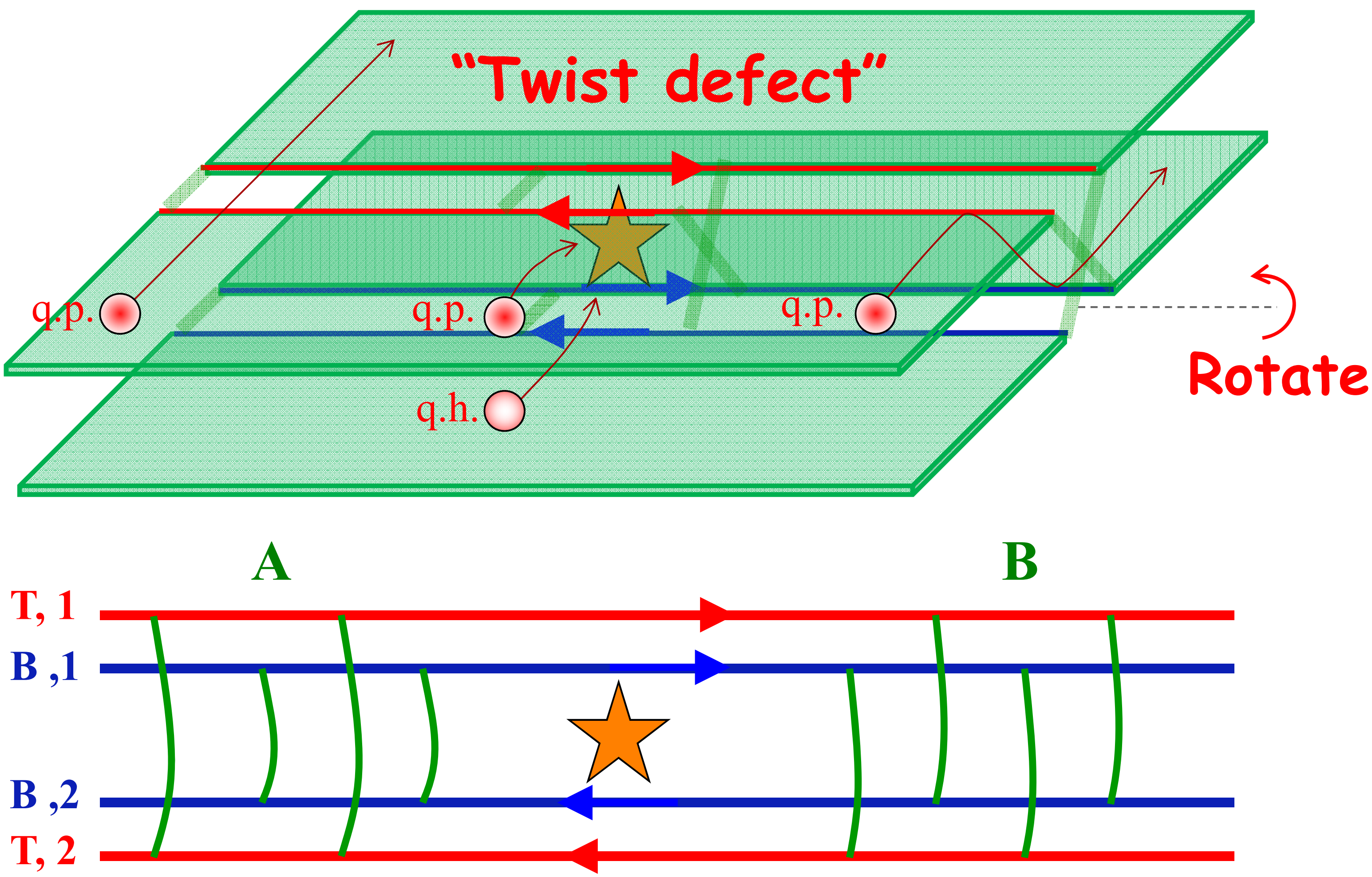
Fractionalized zero modes at "twist defects" in topological phases

ν=1/3 bilayer

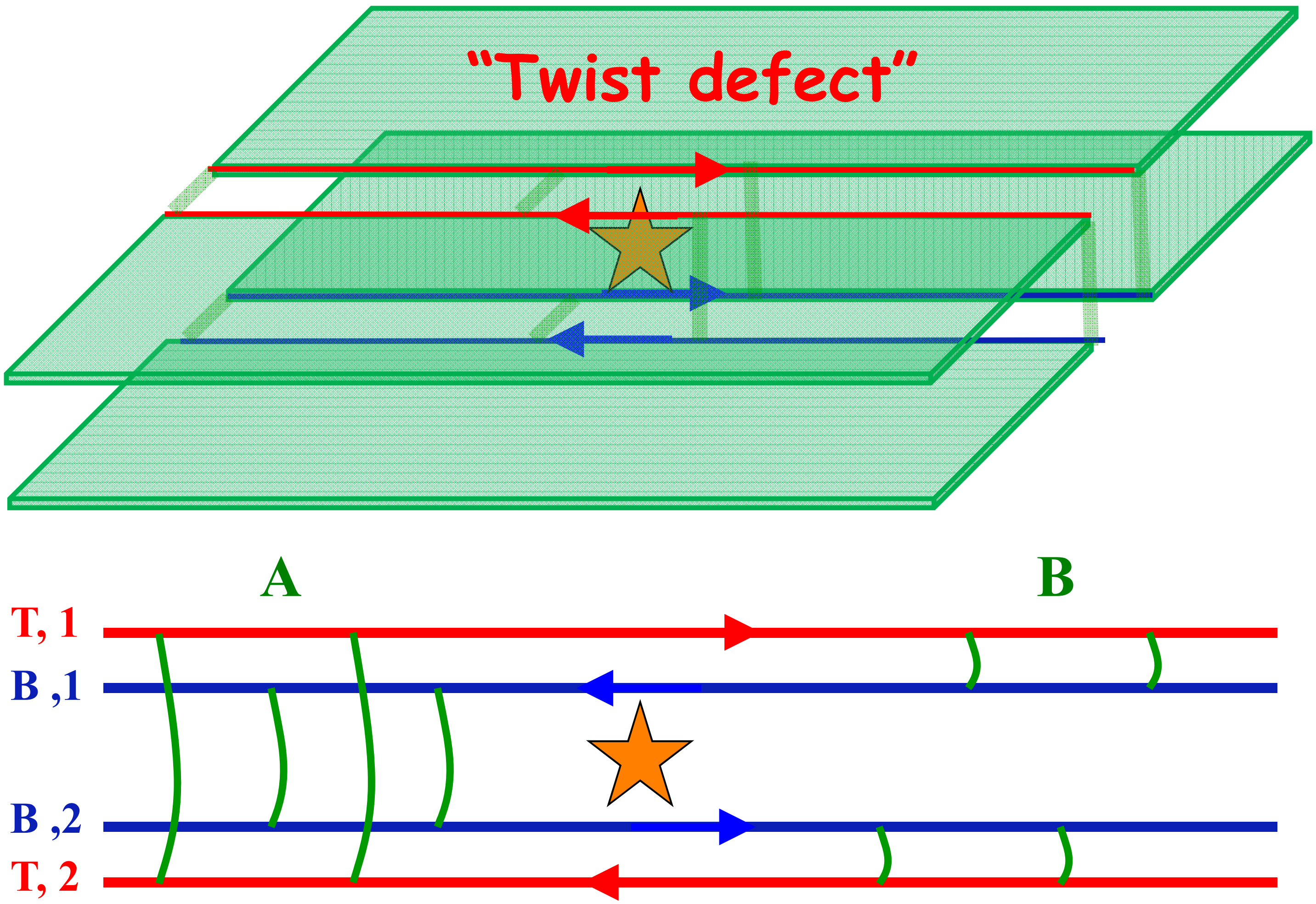


Fractionalized zero modes at "twist defects" in topological phases

ν=1/3 bilayer



Fractionalized zero modes at "twist defects" in topological phases

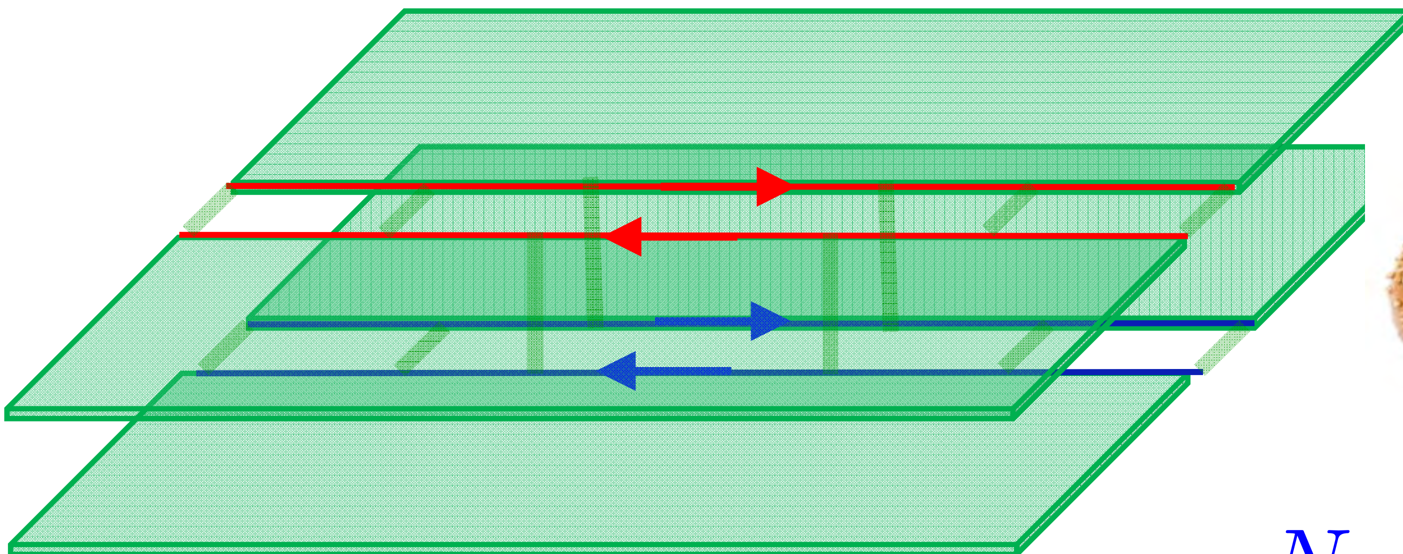


Fractionalized zero modes at “twist defects” in topological phases

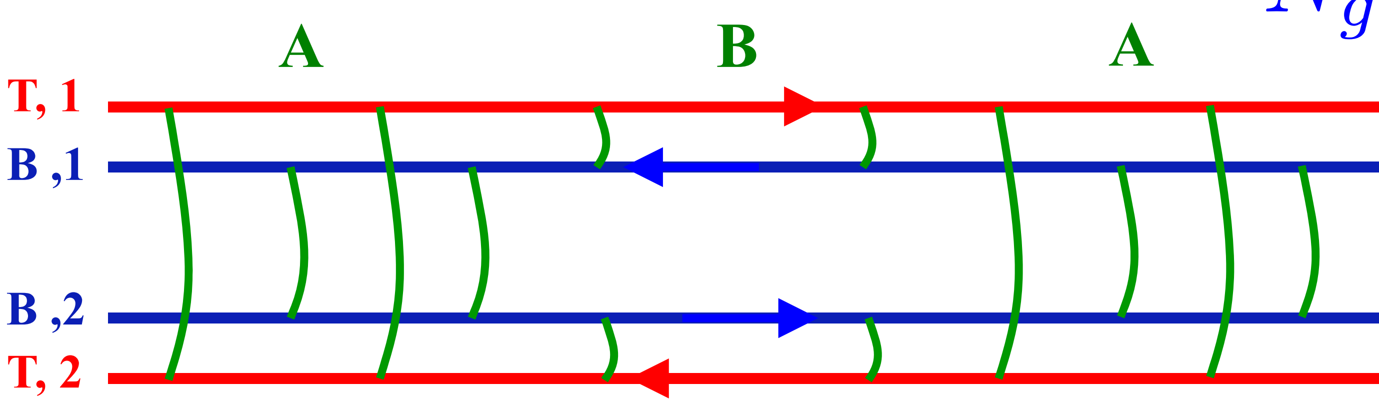
Alternate A,B domains:

Parafermions without superconductivity!

High genus surface



$$N_{gs} = 3^{N_{\text{holes}}}$$



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Enriching non-Abelian phases by defects

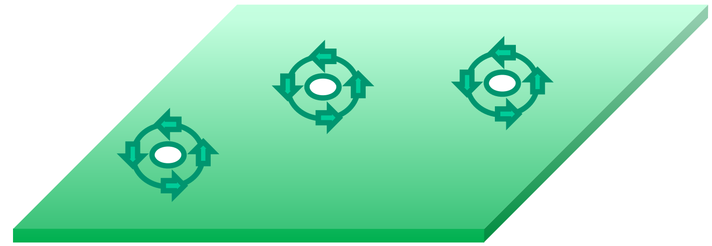
Defects in Abelian phases (e.g. FQH) have non-Abelian properties.

However, the non-Abelian statistics of defects in Abelian phases is never universal for TQC.

Begin with a non-Abelian phase and "enrich" its properties by defects?

Ising anyons

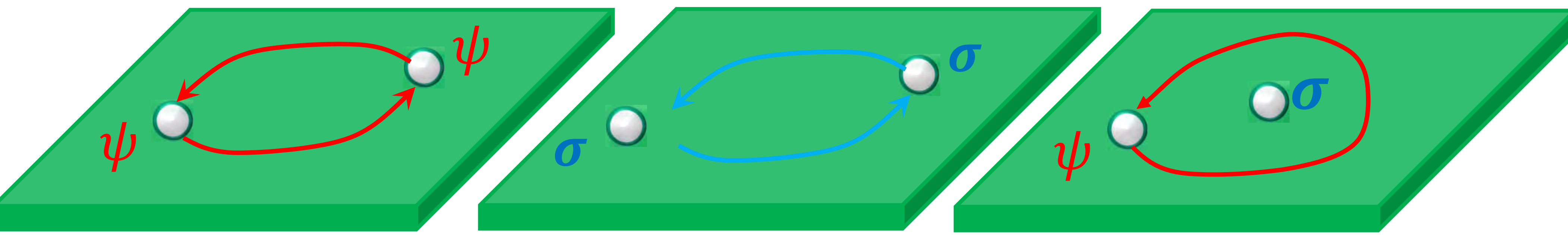
ν = 5/2 QHE
px+ipy Superconductors
Kitaev's hexagonal spin model



Three types of particles: I (vacuum), ψ (fermion), σ (vortex)

Fusion rules: $\psi \times \psi = I \qquad \sigma \times \psi = I$

$$\sigma \times \sigma = I + \psi$$



$$R^{I}_{\psi\psi} = -1$$

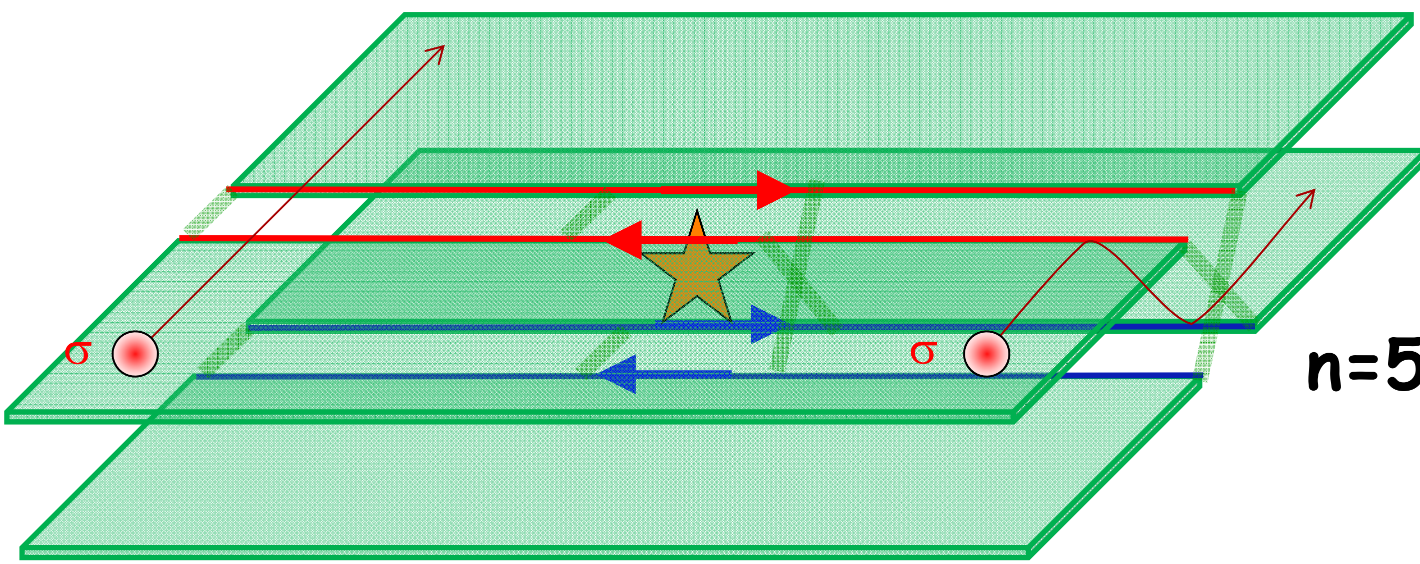
$$R^{I}_{\sigma\sigma} = e^{-i\pi/8}$$

$$R^{\psi}_{\sigma\sigma} = e^{3i\pi/8}$$

$$\left(R_{\sigma\psi}\right)^2 = -1$$

Defects in a bilayer Ising phase

- What is the mathematical description of the zero modes associated with the defects?
- Can the zero modes realize universal TQC even though the host Ising phase is not universal?



Bilayer of n=5/2/p+ip SC/…

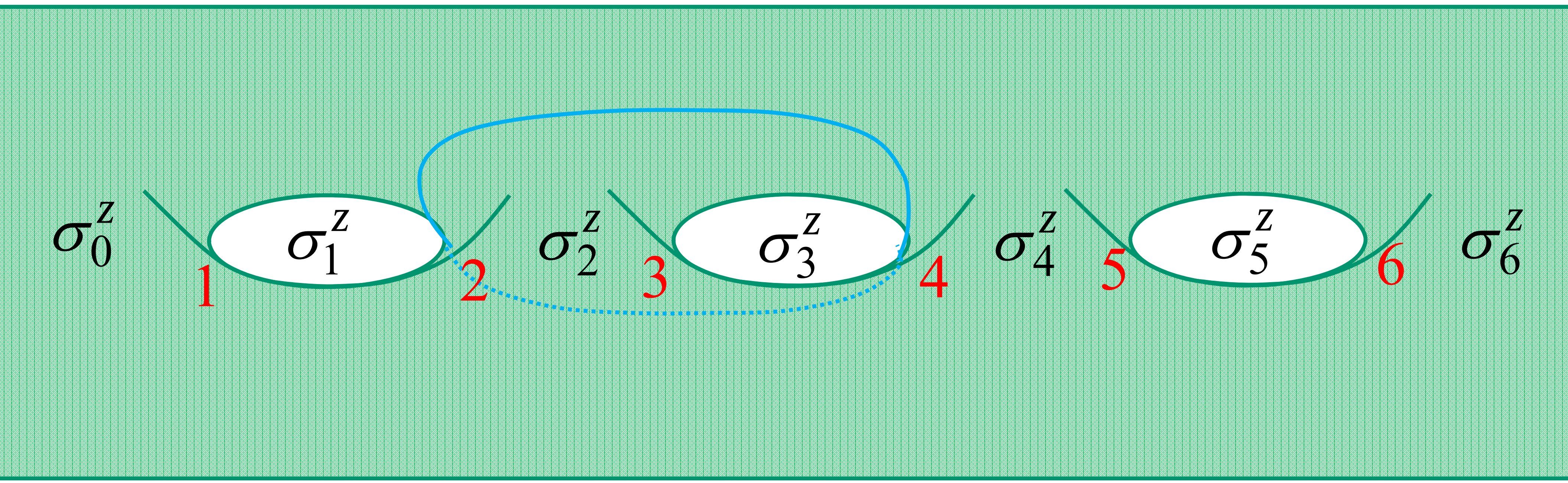
Tunneling operators

Tunneling of σ quasiparticles between zero modes: W_{mn}

Generalization of the parafermion algebra:

$$\boxed{W_{mn} = e^{i\pi/8}\left(W_{mk}W_{kn} + h.c\right)}$$

"tri-algebra"

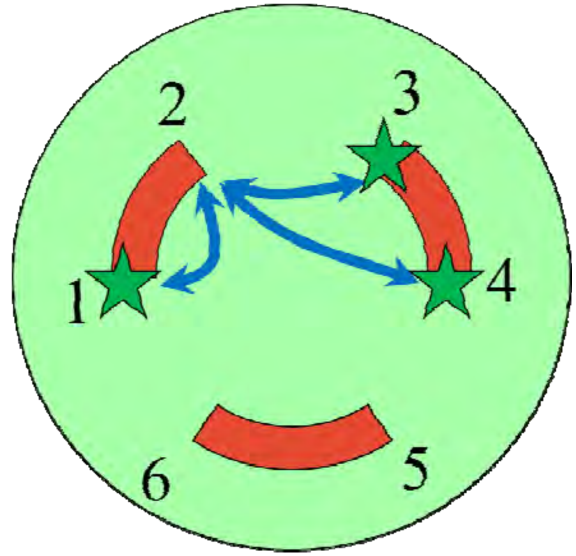


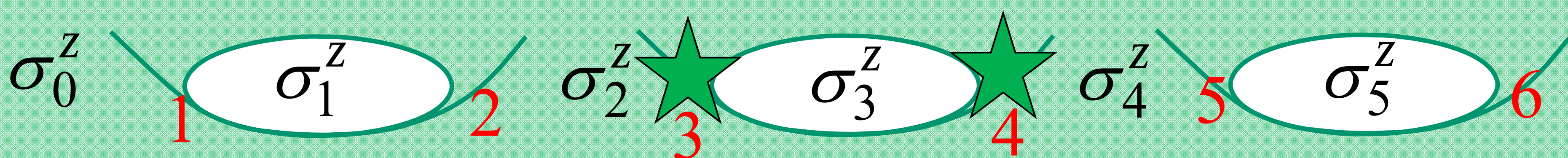
Braiding

EB, Lindner, Stern (in preparation)

$$U_{34} = \left(\frac{1+\sigma_3^z}{2}\right)\sigma_2^x\sigma_4^x + \left(\frac{1-\sigma_3^z}{2}\right)e^{i\pi/4}$$

Phase gate needed to make Ising theory universal!





Conclusions

In Abelian phases they harbor parafermion (fractional Majorana) zero modes.

In a non-Abelian Ising phase they realize new zero modes that enrich the non-Abelian statistics of the host phase.

Pure Ising anyons (Kitaev spin model) + defects: universal for TQC.

Bilayer of n=5/2 is not, however… Other physical realization?

Thank you.