

# Towards a Non-equilibrium Bethe Ansatz for the Kondo Model

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# Introduction

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- The Heisenberg Hamiltonian,

$$H = \sum_{i=1}^L \vec{\sigma}_i \vec{\sigma}_{i+1} + \Delta \sigma_i^z \sigma_{i+1}^z,$$

and many other models may be diagonalized by the Bethe ansatz.

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and many other models may be diagonalized by the Bethe ansatz.

- $[H, \sum_i \sigma_i^z] = 0$ , thus eigenstates are labelled by the Magnon number,  $M = \frac{L}{2} - \sum_i \langle \sigma_i^z \rangle$ .

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and many other models may be diagonalized by the Bethe ansatz.

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- An eigenstate has the form:

$$\vec{\psi}^E = \sum_{i_1, i_2, \dots, i_n} \psi_{i_1, i_2, \dots, i_M}^E \times \left| \begin{array}{ccccccc} & i_1 & & i_2 & & \dots & i_M \\ \uparrow, \dots, & \uparrow, & \downarrow, & \uparrow, \dots, & \uparrow, & \dots, & \uparrow, \\ \end{array} \right\rangle$$

# The Bethe Ansatz

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- for  $M = 1$ ,  $\psi_j^{E(k)} = e^{\imath k j}$

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- for  $M = 1$ ,  $\psi_j^{E(k)} = e^{\imath k j}$
- If  $|i - j| > 1$  then the Hamiltonian does not produce an interaction between  $i$  and  $j$

$$\left| \uparrow, \dots, \overset{i}{\uparrow}, \downarrow, \overset{j}{\uparrow}, \dots, \uparrow, \downarrow, \uparrow, \dots \right\rangle,$$

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$$\left| \uparrow, \dots, \overset{i}{\uparrow}, \downarrow, \uparrow, \dots, \overset{j}{\uparrow}, \downarrow, \uparrow, \dots \right\rangle,$$

thus for  $M = 2$ :

$$\psi_{i,j}^{E(k_1)+E(k_2)} = A e^{\imath(k_1 i + k_2 j)} + B e^{\imath(k_1 j + k_2 i)}$$

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- In general:  $\psi_{i_1, \dots, i_M} = \sum_P A(P) e^{\imath \sum_j i_j k_{P(j)}}$

# Statement of the Problem

- The wave,  $\sum_P A(P) e^{i \sum_j i_j k_{P(j)}}$ , function contains  $M!$  terms.

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- How are we then to compute objects such as  $\langle E_1 | \mathcal{O} | E_n \rangle$ ?

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- How are we then to compute objects such as  $\langle E_1 | \mathcal{O} | E_n \rangle$ ?
- These objects are important in physics immediately when one starts to speak about dynamics. E.g.,

$$\langle J \rangle = \sum_{i_1, i_2} \langle I | E_2 \rangle \langle E_2 | J | E_1 \rangle \langle E_1 | I \rangle e^{\frac{\imath(E_1 - E_2)t}{\hbar}}$$

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# Slavnov Overlaps

# Slavnov Overlaps

- Denote Bethe states

$$|\lambda\rangle = B(\lambda_1)B(\lambda_2)\dots B(\lambda_N)|\Omega\rangle.$$

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 $|\lambda\rangle = B(\lambda_1)B(\lambda_2)\dots B(\lambda_N)|\Omega\rangle.$
- $|a\rangle$  satisfies the Bethe equations with inhomogeneities,  $w$ .
- $|b\rangle$  is generic.

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- $|a\rangle$  satisfies the Bethe equations with inhomogeneities,  $w$ .

- $|b\rangle$  is generic.

- Slavnov (extending Korepin, Gaudin) ( $Q_\alpha(x) = \prod_i (x - \alpha_i)$ ):

$$\begin{aligned} \langle a | b \rangle = & \det_{i,j} \frac{1}{a_i - b_j} - \frac{1}{a_i - b_j + \eta} + \\ & + \frac{Q_w(b_j - \eta) Q_a(b_j + \eta)}{Q_w(b_j) Q_a(b_j - \eta)} \left[ \frac{1}{a_i - b_j} - \frac{1}{a_i - b_j - \eta} \right] \end{aligned}$$

# Advantages and Disadvantages

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- +  $\langle i | \mathcal{O} | f \rangle$ ,  $\langle i | \mathcal{O} | i \rangle$ , are usually Slavnov overlaps  
 $\langle i | (\mathcal{O} | f \rangle)$ .

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 $\langle i|(\mathcal{O}|f)\rangle$ .
- A determinant is computationally difficult to compute.
- The form is not very illuminating – even asymptotics are hard to extract.
- Formation of densities of rapidities in thermo' limit do not have a direct interpretation.

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- Formation of densities of rapidities in thermo' limit do not have a direct interpretation.
- ⇒ Slavnov det may serve as a starting point, on which additional formalism must be built.

# Alternative Approaches

- An axiomatic approach may be taken instead of a direct approach

F. A. Smirnov *Form Factors Completely Integrable Models QFT.*

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- An axiomatic approach may be taken instead of a direct approach F. A. Smirnov *Form Factors Completely Integrable Models QFT*.
- In certain cases overlaps satisfy enough conditions to fix them entirely. Worked through in the case of Sine-Gordon, qKdV and reductions.

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- In certain cases overlaps satisfy enough conditions to fix them entirely. Worked through in the case of Sine-Gordon, qKdV and reductions.
- This approach may be combined with semiclassics (classical KdV) to obtain results.

Babelon,Bernard, Smirnov, Comm. Math. Phys. 182,186 (1996). Smirnov hep-th/9802132

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# Functional-analytic Approach to Slavnov Determinants

# Sutherland Limit

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- A symmetric I. Kostov, Y. Matsuo, JHEP 1210 (2012) 168 representation can be given EB, I. Kostov JPhysA 47(2014) 25401.  
Take  $\mathbf{u} = \mathbf{a} \cup \mathbf{b}$ ,  $Q_{\mathbf{a}}(x) = \prod_i (x - a_i)$ :

$$\langle \mathbf{a} | \mathbf{b} \rangle = \det_{i,j} \left( \delta_{i,j} + \frac{Q_{\mathbf{z}}(u_i) Q_{\mathbf{u}}(u_i + \eta)}{Q_{\mathbf{z}}(u_i + \eta) Q'_{\mathbf{u}}(u_i)} \frac{1}{u_i - u_j + \eta} \right)$$

# Sutherland Limit

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- We must take det of  $\mathbb{1} + K$ , where  
 $K = \frac{Q_{\mathbf{z}}(u_i) Q_{\mathbf{u}}(u_i + \eta)}{Q'_{\mathbf{u}}(u_i) Q_{\mathbf{z}}(u_i + \eta)} \frac{1}{u_i - u_j + \eta}$

# Functional Representation

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- $\vec{\psi} \rightarrow \psi(x) = \sum \frac{\psi_j}{x - u_j},$

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- $\vec{\psi} \rightarrow \psi(x) = \sum \frac{\psi_j}{x - u_j}, \quad K\vec{\psi} \rightarrow \mathcal{K}\psi = \sum \frac{(K\vec{\psi})_j}{x - u_j}$

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- $\vec{\psi} \rightarrow \psi(x) = \sum \frac{\psi_j}{x - u_j}, \quad K\vec{\psi} \rightarrow \mathcal{K}\psi = \sum \frac{(K\vec{\psi})_j}{x - u_j}$
- Then  $\sum_j \frac{\psi_j}{x - u_j + \eta} = \psi(x + \eta)$ .
- $\frac{Q_{\mathbf{z}}(x)Q_{\mathbf{u}}(x+\eta)}{Q'_{\mathbf{u}}(x)Q_{\mathbf{z}}(x+\eta)} \psi(x + \eta)$  is a candidate for  $\mathcal{K}\psi$ .

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- $K = \frac{Q_z(u_i)Q_u(u_i+\eta)}{Q'_u(u_i)Q_z(u_i+\eta)} \frac{1}{u_i - u_j + \eta}$
- $\vec{\psi} \rightarrow \psi(x) = \sum \frac{\psi_j}{x - u_j}, \quad K\vec{\psi} \rightarrow \mathcal{K}\psi = \sum \frac{(K\vec{\psi})_j}{x - u_j}$
- Then  $\sum_j \frac{\psi_j}{x - u_j + \eta} = \psi(x + \eta)$ .
- $\frac{Q_z(x)Q_u(x+\eta)}{Q'_u(x)Q_z(x+\eta)} \psi(x + \eta)$  is a candidate for  $\mathcal{K}\psi$ .
- More precisely:

$$(\mathcal{K}\psi)(y) = \oint_u \frac{1}{y - x} \frac{Q_z(x)Q_u(x + \eta)}{Q'_u(x)Q_z(x + \eta)} \psi(x + \eta)$$

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$$(\mathcal{K}\psi)(y) = \oint_{\mathbf{u}} \frac{1}{y-x} \frac{Q_z(x)Q_u(x+\eta)}{Q_u(x)Q_z(x+\eta)} \psi(x+\eta)$$

The Slavnov matrix is just  $1 + \mathcal{K}$  with:

$$\mathcal{K} = \mathcal{P} e^{-\Phi} e^{\eta \partial} e^\Phi$$

with

$$e^\Phi = \frac{Q_u}{Q_z}, \quad (\mathcal{P}f)(x) = \oint \frac{f(y)}{x-y}$$

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- We consider the Kondo model with the Hamiltonian:

$$\int \psi_\sigma^\dagger(x) (-i\hbar\partial_x) \psi_\sigma(x) + g\psi_\sigma^\dagger(0)\vec{\sigma}_{\sigma\sigma'}\psi_{\sigma'}(0) \cdot \vec{S}.$$

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- Standard Representation:  $\tilde{\psi}_{\sigma \otimes s}(x)$ .  
 $\sigma \otimes s = (\sigma_1, \sigma_2, \dots, \sigma_N, s)$ .

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- Standard Representation:  $\tilde{\psi}_{\sigma \otimes s}(x)$ .  
 $\sigma \otimes s = (\sigma_1, \sigma_2, \dots, \sigma_N, s)$ .
- Let  $Q$  order  $x$ :  $x_{Q(1)} < x_{Q(2)} < \dots < x_{Q(N)}$ .  
Define Non-standard representation:  $\psi_{\sigma \otimes s}(x)$

$$\psi_{Q\sigma \otimes s}(x) = \tilde{\psi}_{\sigma \otimes s}(x),$$

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- The Bethe Ansatz solution:

$$\psi_{\sigma \otimes s}(x) = \sum_{P \in S_N} \text{sign}(P) \Psi_{P \circ Q}(\sigma \otimes s) e^{ix \cdot Pk}.$$

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- In **Kondo**  $\Psi_P = \Psi$ , and factorizes:

$$\psi_{\sigma}(\mathbf{x}) = \left( \det_{i,j} e^{i k_i x_j} \right) \Psi(\sigma \otimes s).$$

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$$\psi_{\sigma}(\mathbf{x}) = \left( \det_{i,j} e^{i k_i x_j} \right) \Psi(\sigma \otimes s).$$

- $\Psi$  is an *inhomogeneous* Heisenberg Bethe ansatz wavefunction.

# Relation Between the Kondo and Heisenberg Problems

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- Denote Bethe states  $|\lambda\rangle$ , where  $\lambda_j = \frac{e^{\imath k_j - \imath}}{e^{\imath k_j + \imath}}$ .

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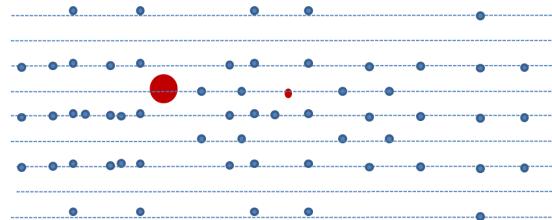
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- In the thermodynamic limit strings form:



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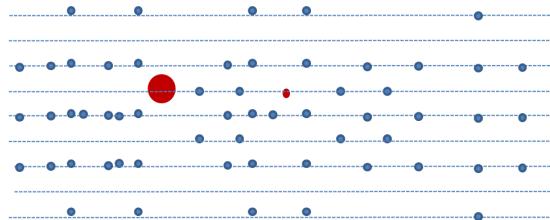
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- Denote Bethe states  $|\lambda\rangle$ , where  $\lambda_j = \frac{e^{\imath k_j - \imath}}{e^{\imath k_j + \imath}}$ .

- In the thermodynamic limit strings form:



- Independent density  $\sigma(\lambda)$  and dependent density  $\sigma_h(\lambda)$ .

# Thermodynamic Bethe Ansatz

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- The partition function

$$Z(T) = \sum_f e^{-\frac{E_f - E_i}{T}} = \int e^{-\beta F} \mathcal{D}\sigma$$

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- The partition function

$$Z(T) = \sum_f e^{-\frac{E_f - E_i}{T}} = \int e^{-\beta F} \mathcal{D}\sigma$$

- "Saddle point" Weigmann/Andrei-Lowenstein (1980)

$$\frac{\delta F}{\delta \sigma(\lambda)} = 0, \quad F = E(\sigma) - TS(\sigma)$$

# Thermodynamic Bethe Ansatz

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- The partition function

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- $E = \int \varepsilon(\lambda) \sigma(\lambda)$

$$S = \int (\sigma + \sigma_h) \log (\sigma + \sigma_h) - \sigma \log (\sigma) - \sigma_h \log (\sigma_h).$$

$$\frac{\delta \sigma_h(\lambda)}{\delta \sigma(\lambda')} = K(\lambda - \lambda')$$

# Quench Action Approach

- We wish to compute a non-equilibrium version, e.g., the amount of energy absorbed:

$$P(T) = \sum_f |\langle i | f \rangle|^2 e^{-\frac{E_f - E_i}{T}}.$$

# Quench Action Approach

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# Quench Action Approach

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- We write integral equations for  $\frac{\delta \log |\langle i | \sigma \rangle|^2}{\delta \sigma(\lambda)}$ , the '*Non-equilibrium source*'.

# Integral Eqs. for Non-equilibrium Source

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- $\frac{\delta \log \det(1+\mathcal{K})}{\delta \sigma} = \text{tr}(1 + \mathcal{K})^{-1} \frac{\delta \mathcal{K}}{\delta \sigma}$ , where  
 $\mathcal{K} = \mathcal{P} e^{-\Phi} e^{\eta \partial} e^{\Phi}, \quad \mathcal{P} = \oint \frac{1}{x-y}$

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- Find  $\mathcal{R} = (1 + \mathcal{K})^{-1}$  by solving EB, JPhysA (2015)

$$(1 + \mathcal{K})\mathcal{R}(x, y) = \frac{1}{x - y},$$

# Integral Eqs. for Non-equilibrium Source

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$$(1 + \mathcal{K})\mathcal{R}(x, y) = \frac{1}{x - y},$$

explicitly:

$$\mathcal{R}(x, y) - \oint_u \frac{Q_u(x' + i) Q_z(x')}{Q_u(x') Q_z(x' + i)} \frac{\mathcal{R}(x' + i, y)}{(x' - x)} =$$
$$\frac{1}{x - y}$$

# Summary

- Write  $\frac{\delta F}{\delta \sigma(\lambda)} = T \frac{\delta \log |\langle i | \sigma \rangle|^2}{\delta \sigma(\lambda)}$  as an integral equation with a source.

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- Write  $\frac{\delta F}{\delta \sigma(\lambda)} = T \frac{\delta \log |\langle i|\sigma \rangle|^2}{\delta \sigma(\lambda)}$  as an integral equation with a source.
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# Summary

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- Fast convergence? Validity? Numerics?

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# The End