

# Iron Based Superconductors: Lessons from Raman Spectroscopy

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## Collaboration:

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# Outline

## Part I

1. Intro to Fe based superconductors
2. Signatures of structural transition
3. Raman as a probe of orbital correlations
4. Critical slowdown of orbital fluctuations

## Part II

1. Family of iron(Fe)-based superconductors with only electron pockets:  $A_x\text{Fe}_{2-y}\text{Se}_2$ ,  $A = \text{K, Rb, Cs}$
2. Pairing symmetry: “s”, “d”, “s+−” and “s+id”
3. Raman in  $A_x\text{Fe}_{2-y}\text{Se}_2$ ,  $A = \text{K, Rb, Cs}$

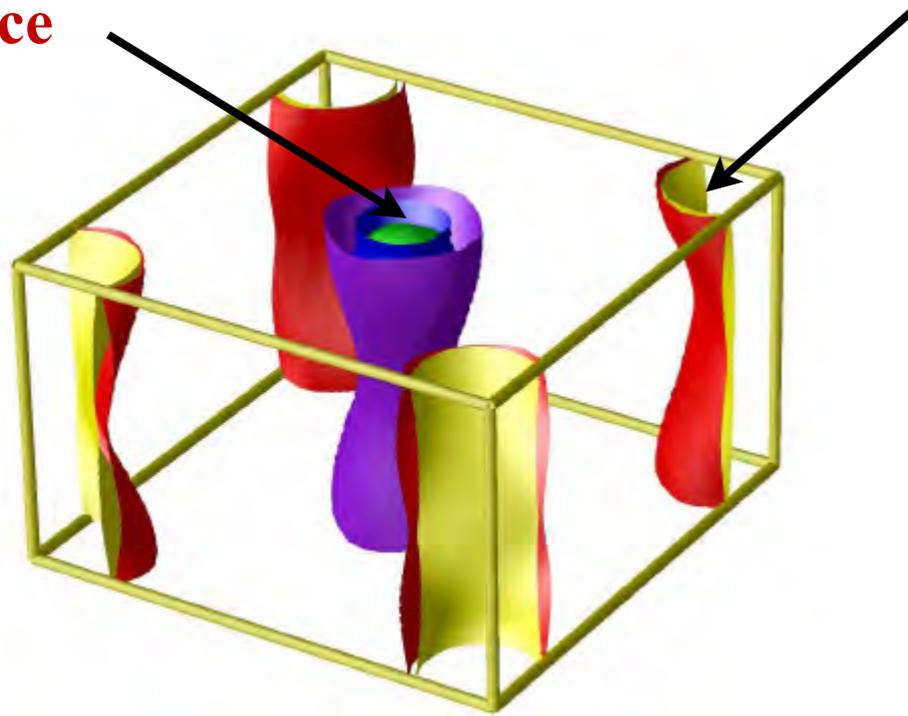
# I. Band structure

1. Multi-band multi-orbital metals
2. Quasi-two-dimensional materials
3. Conduction bands:  
partially filled  $dxz$   $dyz$  ( $dxy$ ) orbitals

# Band structure(cont)

**Hole Fermi surface**

**Electron Fermi surface**



2(3) hole pockets at  $(0, 0)$   
2 electron pockets at  $(\pi, \pi)$

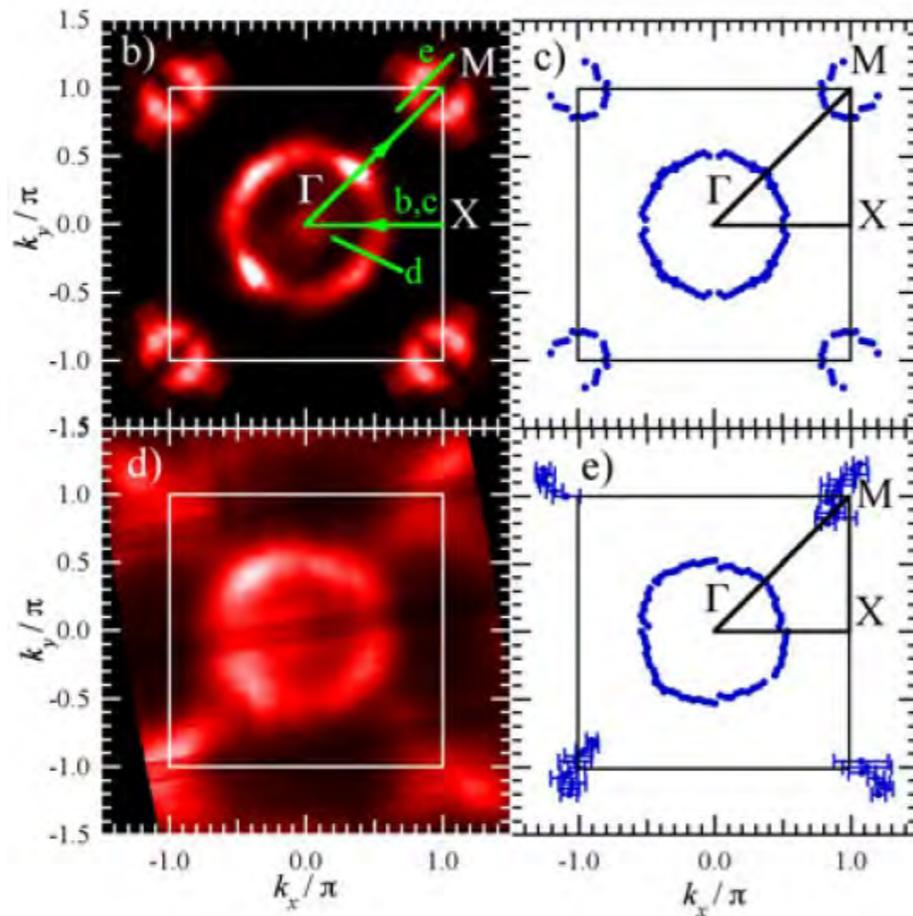
Lebegue, Mazin et al, Singh & Du, Cvetkovic & Tesanovic...

# Band structure (cont)

## ARPES

**NdFeAs(O<sub>1-x</sub>F<sub>x</sub>) (x=0.1)**

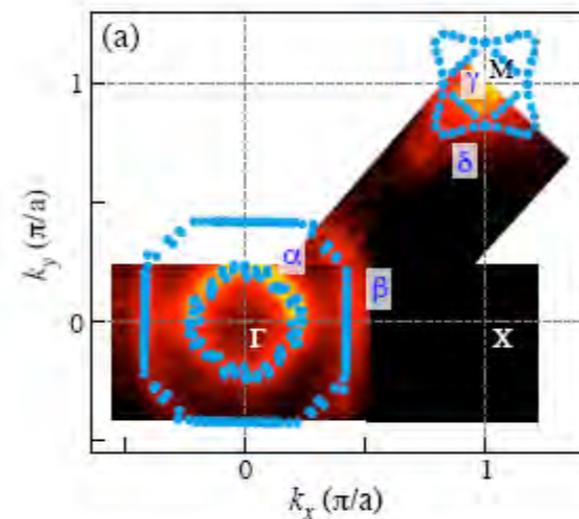
A. Kaminski et al.



Hole pockets near (0,0)  
Electron pockets near ( $\pi,\pi$ )

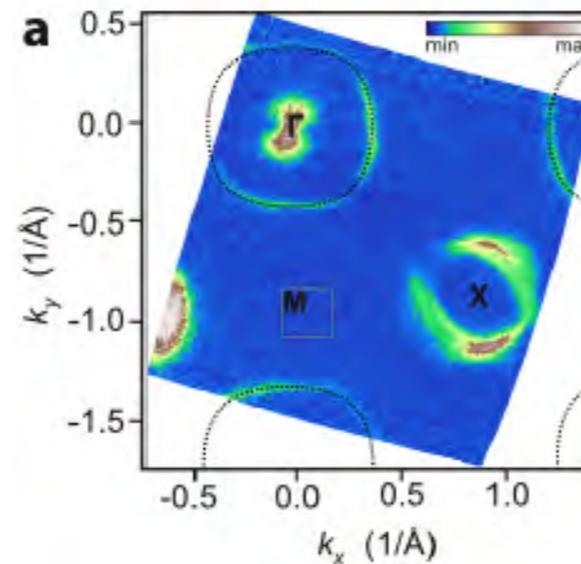
**Ba<sub>0.6</sub>K<sub>0.4</sub>Fe<sub>2</sub>As<sub>2</sub>**

H. Ding et al.



**LiFeAs**

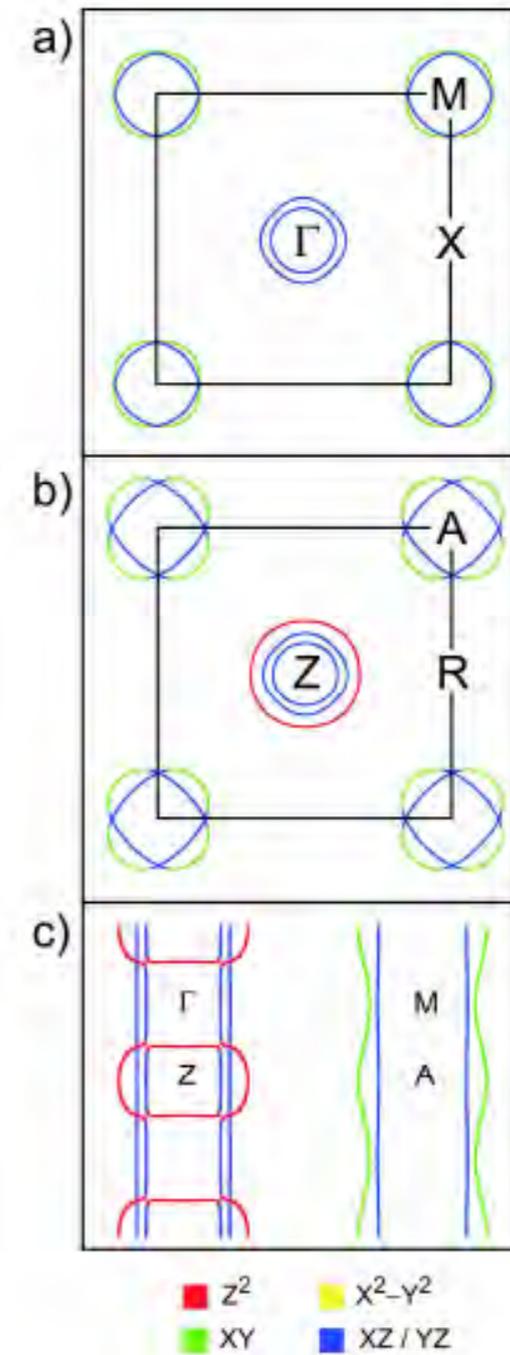
A. Kordvuk et al



## dHVa

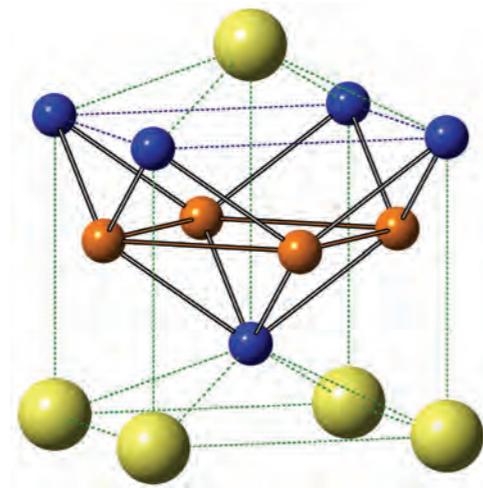
**LaFeOP**

A. Coldea et al,

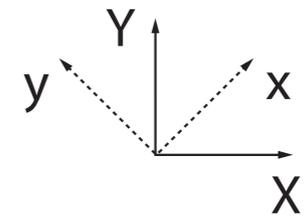
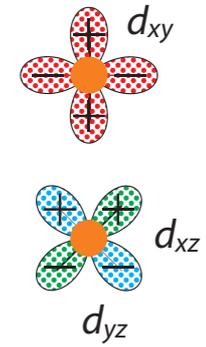
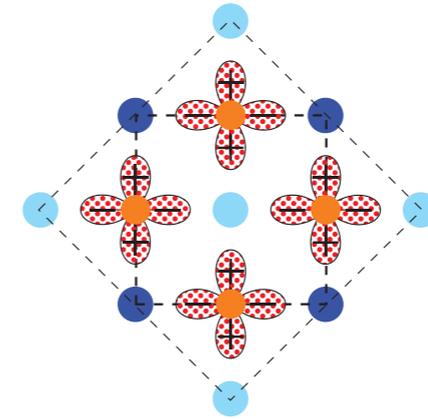
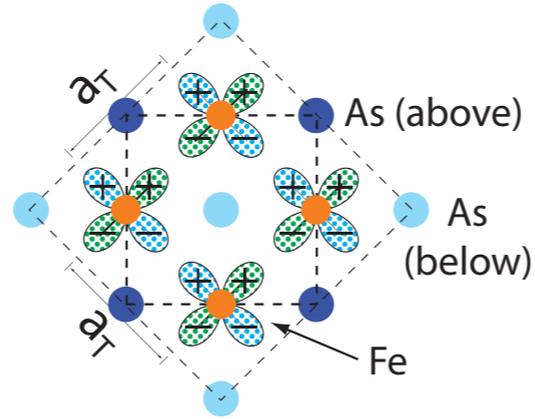
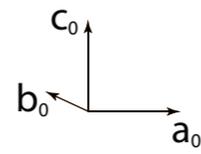


# NaFe<sub>1-x</sub>Co<sub>x</sub>As

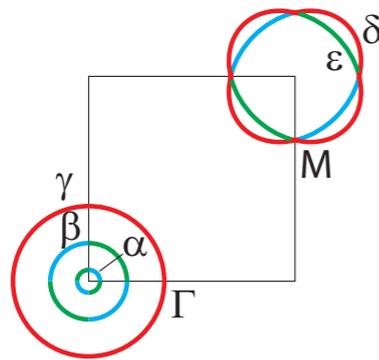
A



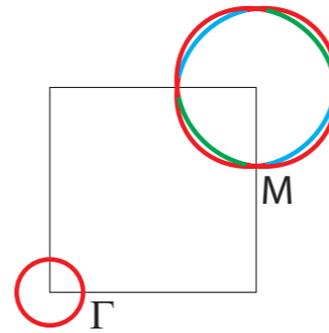
B



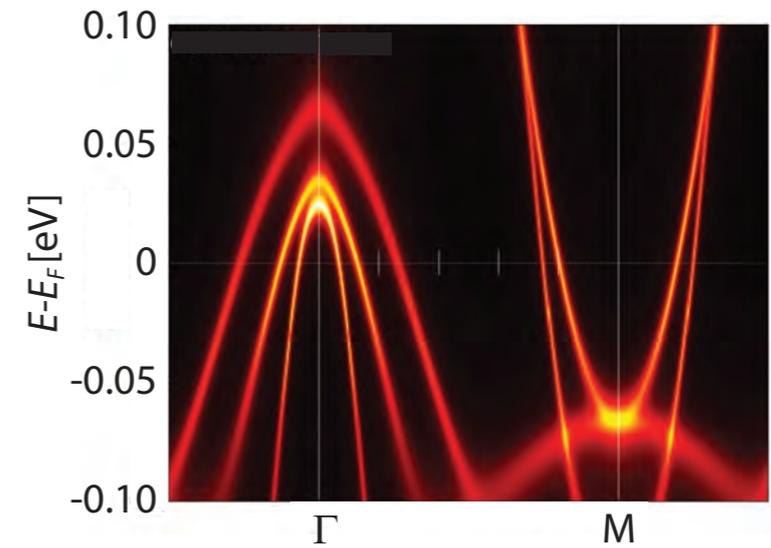
C



D



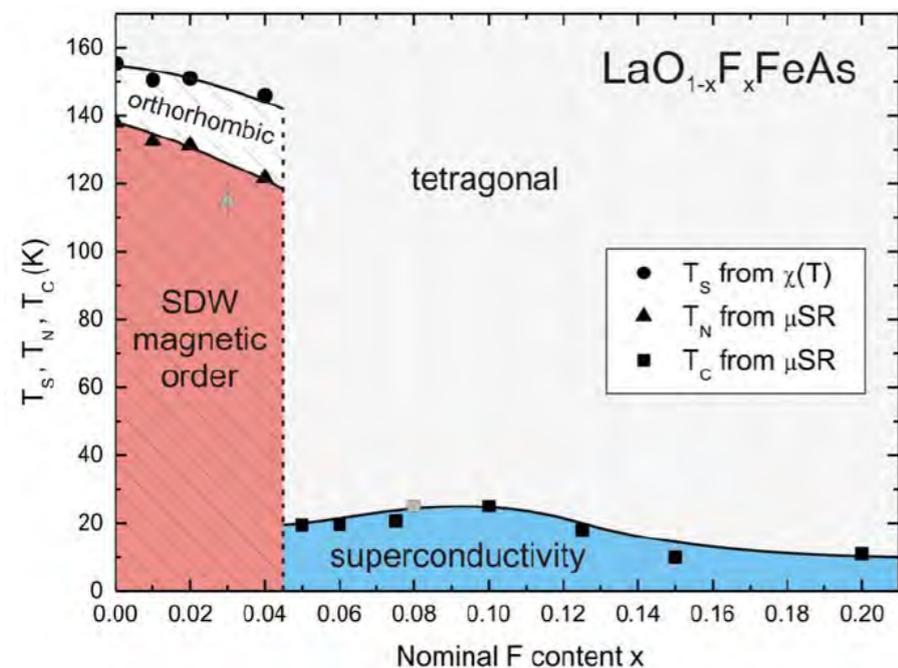
E



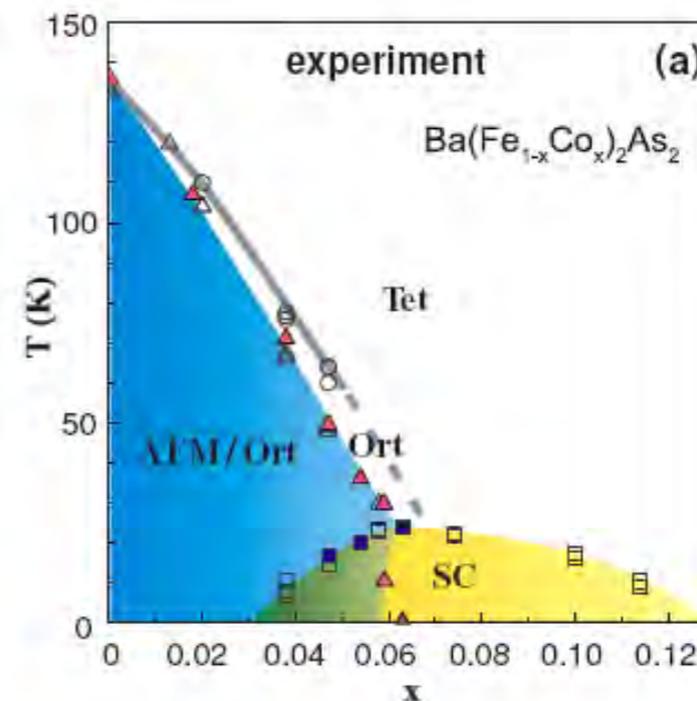
[arXiv:1410.6456](https://arxiv.org/abs/1410.6456)

V. K. Thorsmølle, MK, Z. P. Yin, Chenglin Zhang, S. V. Carr, Pengcheng Dai, G. Blumberg

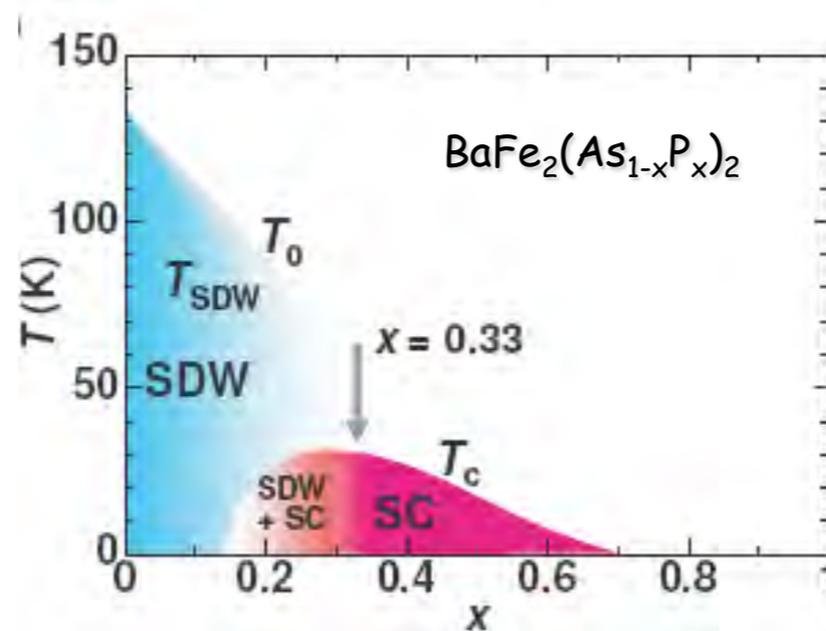
# II. Phase Diagram



Luetkens et al



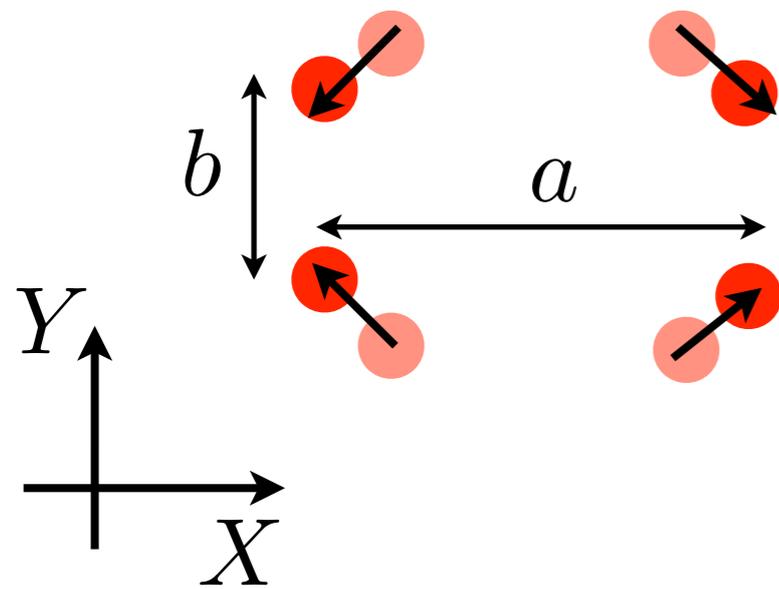
Fernandes et al



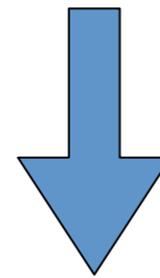
Matsuda et al

# Structural transition

## Orthorhombic distortion



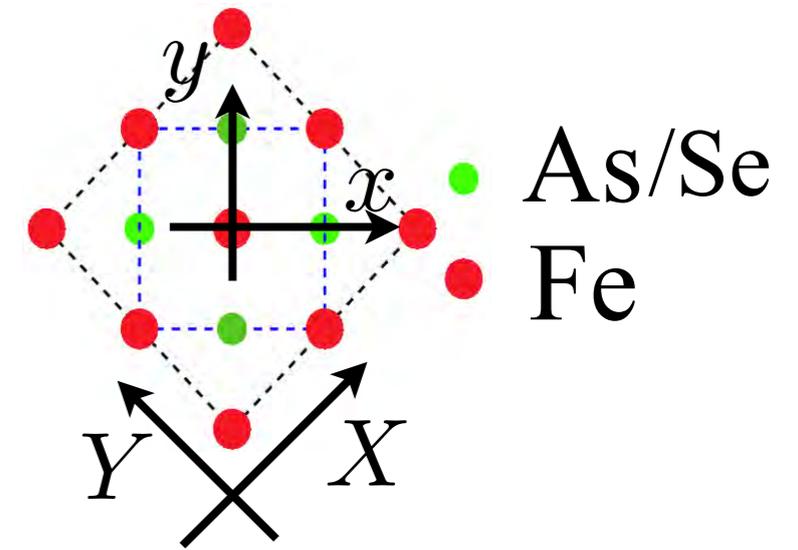
$a \neq b$  anisotropy



Discrete  $C_4 \rightarrow C_2$  symmetry breaking

$$\epsilon_6 = \frac{\partial u_X}{\partial X} - \frac{\partial u_Y}{\partial Y} \neq 0$$

Order Parameter



# Structural transition: lattice softening

$$\text{Shear stress} \longrightarrow \sigma_6 = C_{66} \epsilon_6 \longleftarrow \text{Shear strain}$$

(orthorhombic distortion)

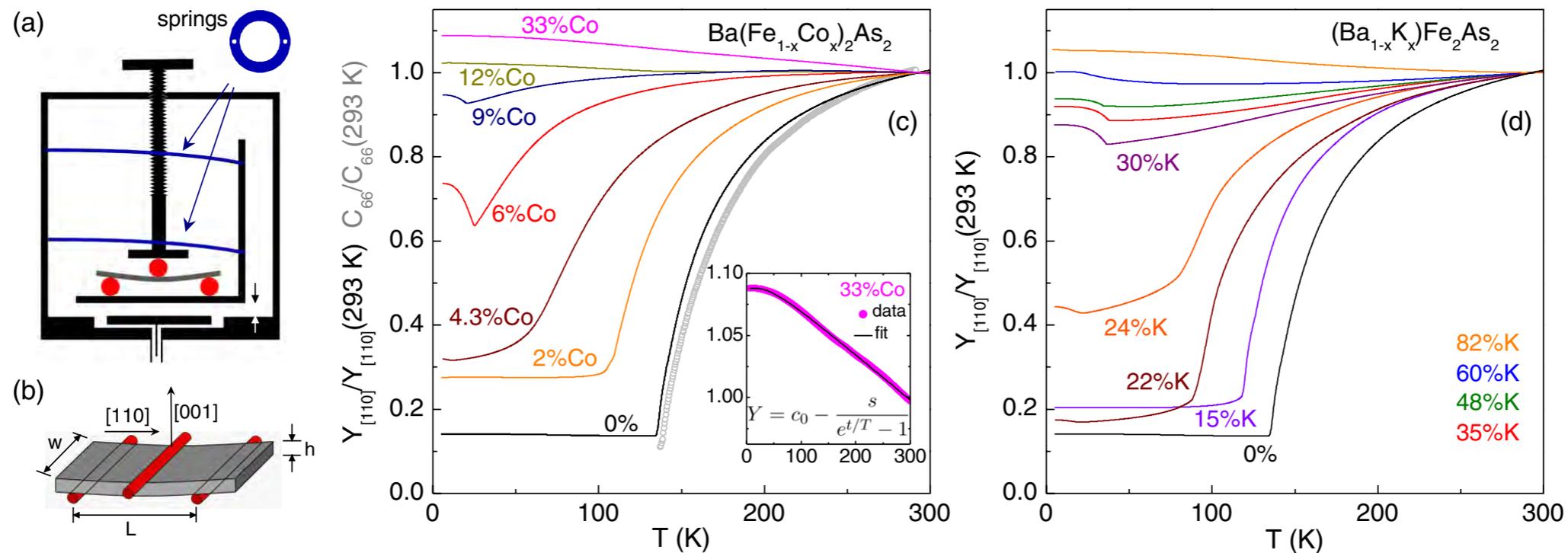
$$\downarrow$$

Elastic shear modulus

PRL 112, 047001 (2014)

PHYSICAL REVIEW LETTERS

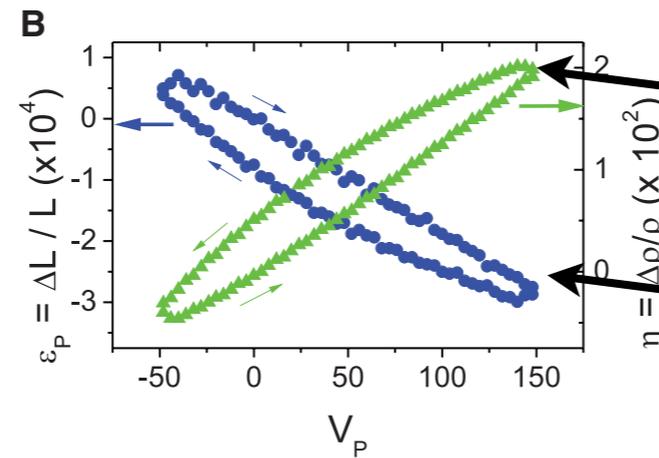
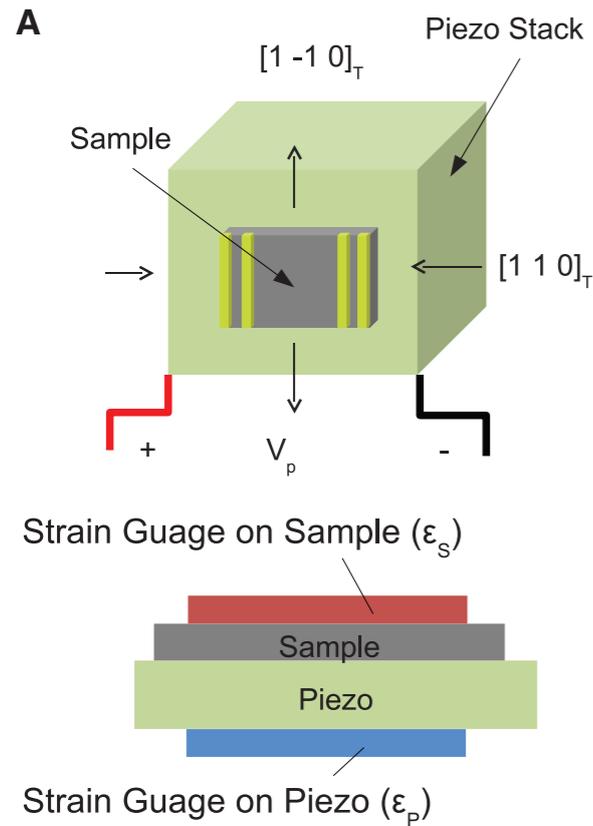
week ending  
31 JANUARY 2014



A. E. Böhmer, P. Burger, F. Hardy, T. Wolf, P. Schweiss, R. Fromknecht, M. Reinecker, W. Schranz, and C. Meingast

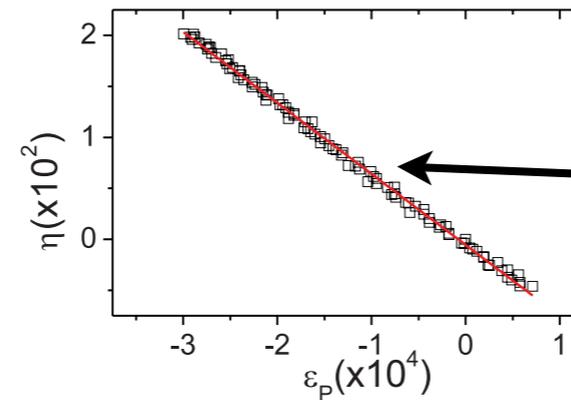
Towards transition:  $C_{66} \rightarrow 0$

# Structural transition: electrons vs lattice

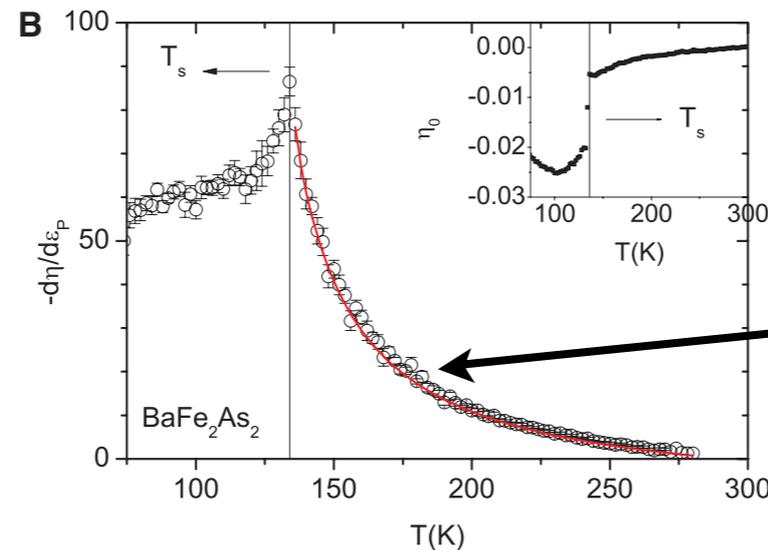
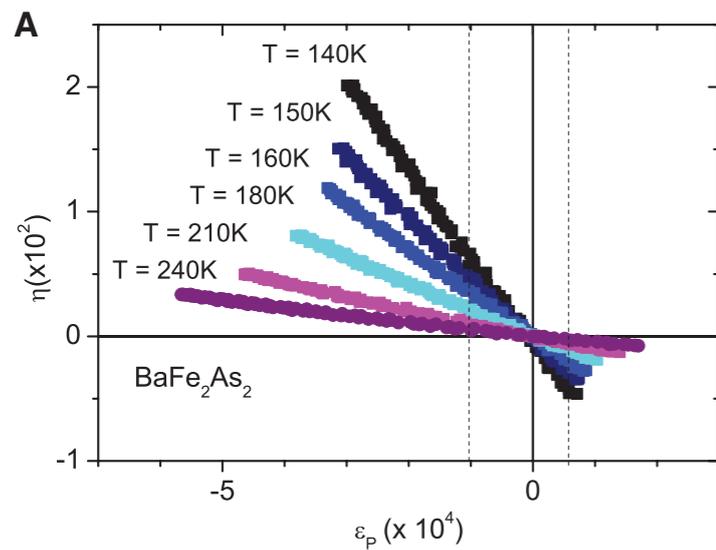


Resistivity anisotropy

Fractional length change



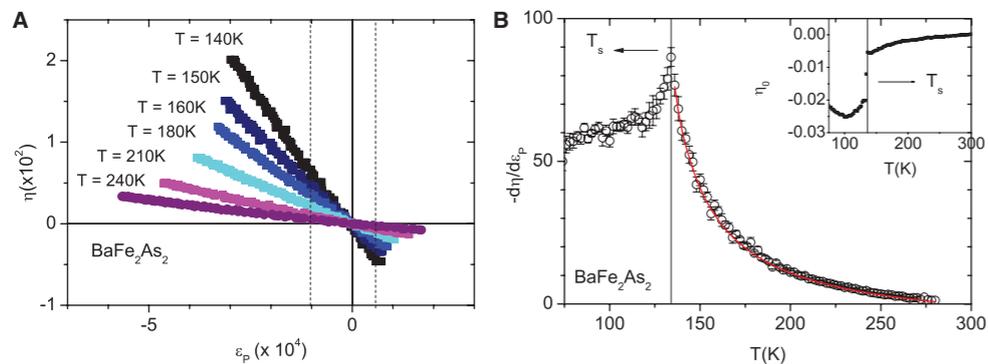
Resistivity is larger (smaller) along short (long) direction



Divergent anisotropy susceptibility!

$$\bar{T}_s < T_s$$

$$116\text{K} < 138\text{K}$$



$$\bar{T}_s < T_s$$

$$116\text{K} < 138\text{K}$$

$$F[\phi, \epsilon_6] = \frac{1}{2} \chi_\phi^{-1} \phi^2 - \lambda \epsilon_6 \phi + \frac{1}{2} C_{66,0} \epsilon_6^2 + \frac{B}{4} \phi^4$$

Strained controlled:  $\epsilon_6 \rightarrow 0$

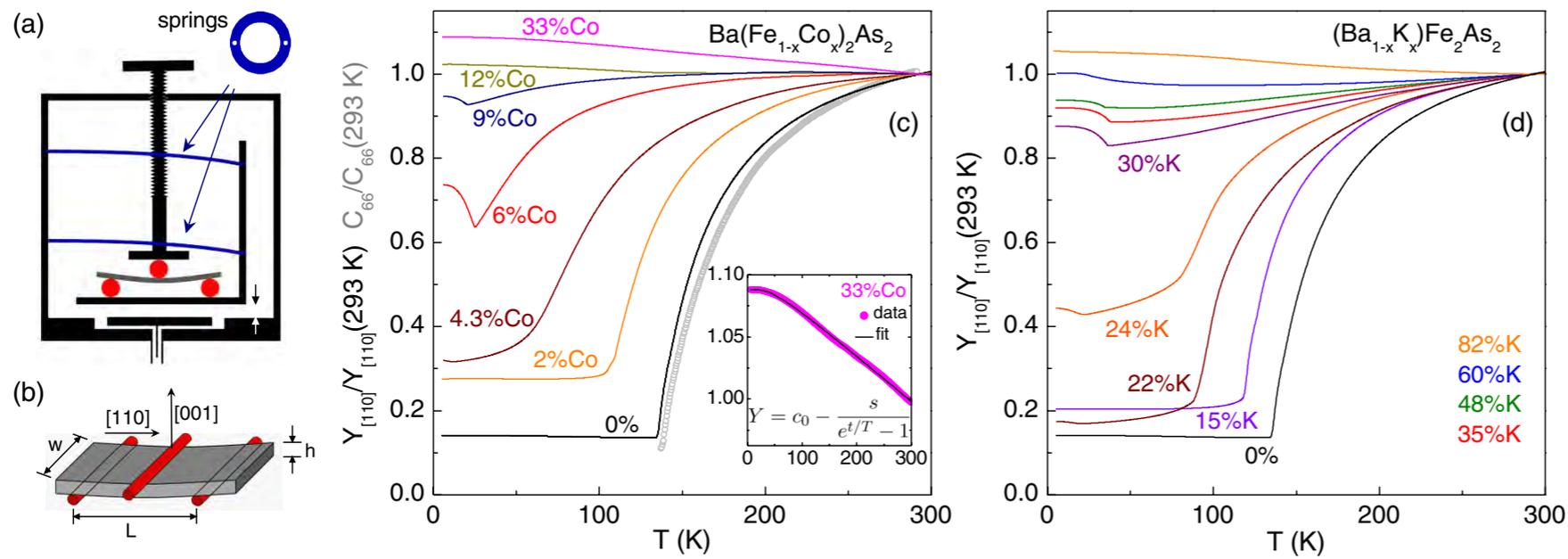
$$F[\phi, \epsilon_6 = 0] = \frac{1}{2} \chi_\phi^{-1} \phi^2 + \frac{B}{4} \phi^4 \quad \longrightarrow \quad \chi_\phi^{-1} = a(T - \bar{T}_s)$$

Strained free:

$$\frac{\partial F[\phi, \epsilon_6]}{\partial \epsilon_6} = 0 \quad \longrightarrow \quad \epsilon_6 = \frac{\lambda \phi}{C_{66,0}}$$

$$\longrightarrow \quad F = \frac{1}{2} a(T - T_s) \phi^2 + \frac{B}{4} \phi^4$$

$$T_s = \bar{T}_s + \frac{\lambda^2}{a C_{66,0}}$$



$$C_{66} = \frac{d^2 F}{d\epsilon^2}$$

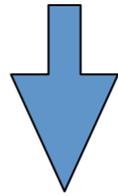
$$\frac{\partial F}{\partial \phi} [\phi(\epsilon), \epsilon] = 0$$

$$\frac{\partial F}{\partial \phi} = 0 \quad \Rightarrow \quad \frac{dF}{d\epsilon} = \frac{\partial F}{\partial \epsilon} \quad \Rightarrow \quad \frac{d^2 F}{d\epsilon^2} = \frac{\partial^2 F}{\partial \epsilon^2} + \frac{\partial^2 F}{\partial \phi \partial \epsilon} \frac{d\phi}{d\epsilon}$$

$$\frac{\partial F}{\partial \phi} = 0 \quad \Rightarrow \quad \frac{d\phi}{d\epsilon} = - \frac{\partial^2 F / (\partial \phi \partial \epsilon)}{\partial^2 F / \partial \phi^2}$$

# Phenomenology (cont):

$$F[\phi, \epsilon_6] = \frac{1}{2} \chi_\phi^{-1} \phi^2 - \lambda \epsilon_6 \phi + \frac{1}{2} C_{66,0} \epsilon_6^2 + \frac{B}{4} \phi^4$$



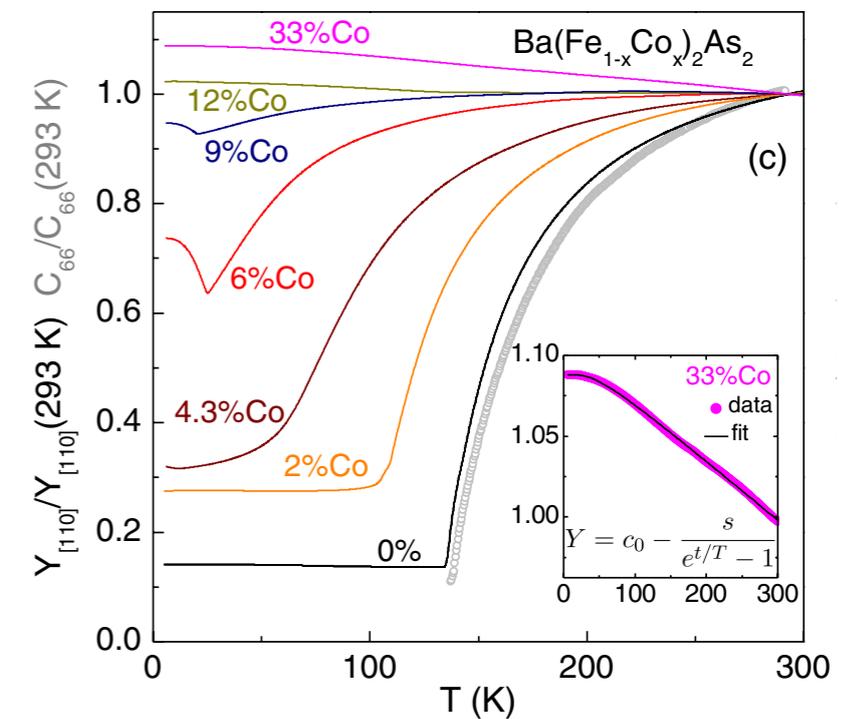
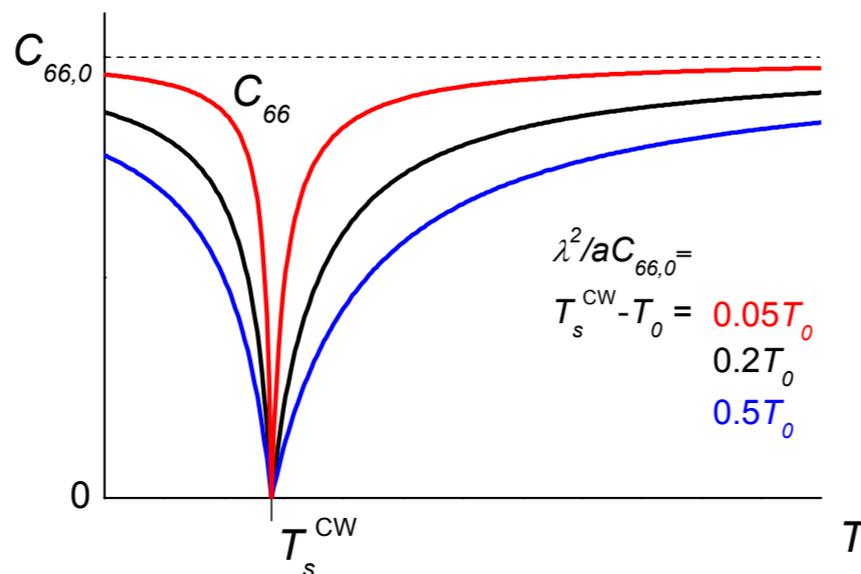
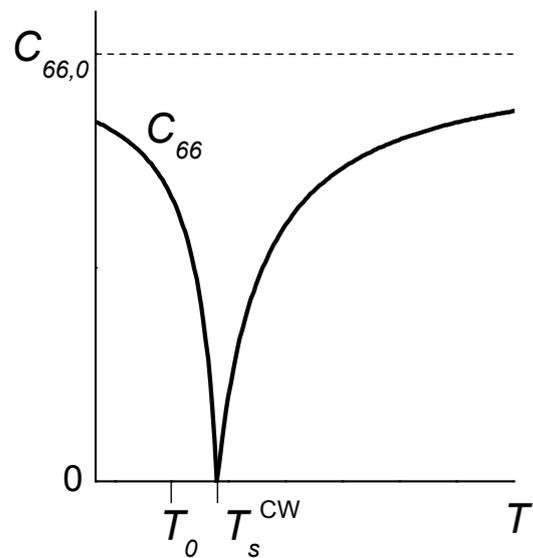
$$C_{66} = C_{66,0} + \frac{\lambda^2}{\chi_\phi^{-1} + 3B\phi^2}$$

$$C_{66} = C_{66,0} \frac{T - T_s}{T - \bar{T}_s}$$

Curie-Weiss

$$C_{66} = \frac{d^2 F}{d\epsilon_6^2}$$

$$\chi_\phi^{-1} = a(T - \bar{T}_s)$$



Shear modulus, resistivity: static induced anisotropy



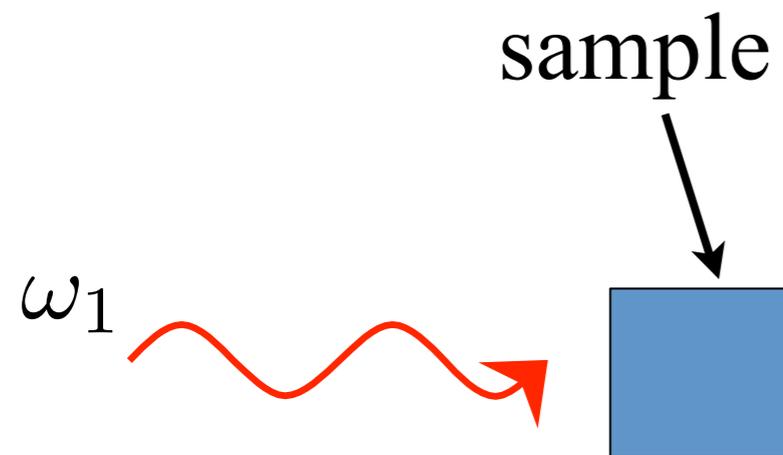
Weak dynamic probe of  $\phi$  is needed



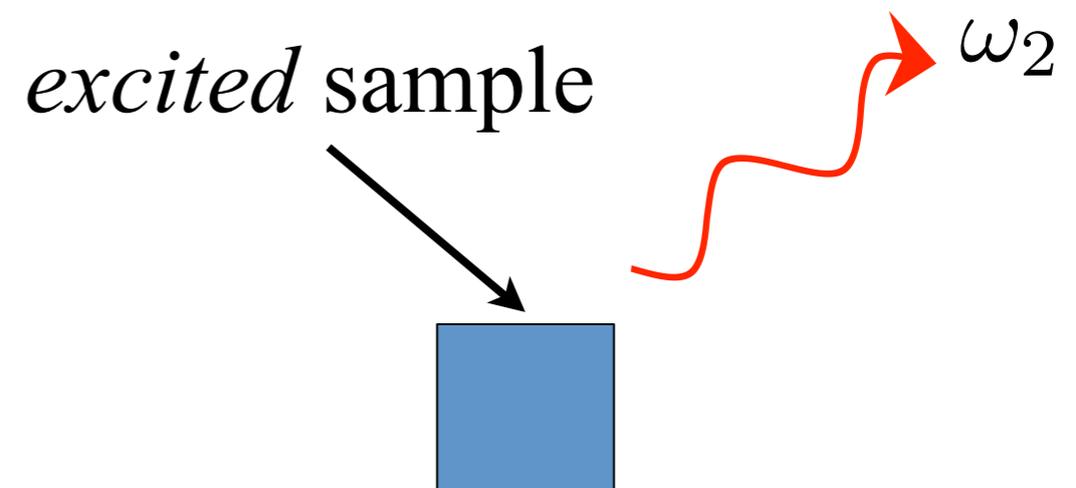
Raman spectroscopy

# Raman: inelastic light scattering

Initial state



Final state



Measure:

$$\frac{d\sigma}{d\Omega d\omega} = \frac{\text{Flux of photons scattered into } d\Omega \text{ with } \omega < \omega_2 < \omega + d\omega}{\text{Flux of incident photons}}$$

$$\omega_2 - \omega_1 = \text{Excitation energy} = \text{Raman shift}$$

# The case of electron plasma

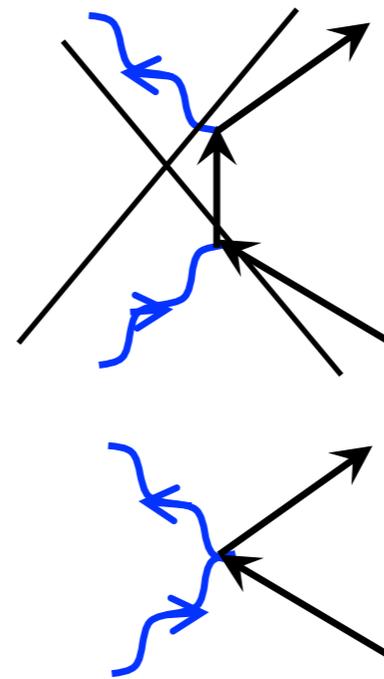
P.M. Platzmann,  
Phys. Rev. **139**, A379 (1965)

$$H = \sum_i \frac{1}{2m} \left[ \mathbf{p}_i - \frac{e}{c} \mathbf{A}(\mathbf{r}_i, t) \right]^2 + H_{int}$$

$$H = H_0 + V_1 + V_2$$

$$V_1 = -\frac{e}{mc} \sum_i \mathbf{p}_i \cdot \mathbf{A}(\mathbf{r}_i, t)$$

$$V_2 = \frac{e^2}{2mc^2} \sum_i \mathbf{A}^2(\mathbf{r}_i, t)$$



Stay off direct resonances:  
Energy denominators!

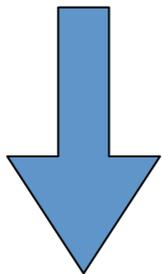
$$r_0 = \frac{e^2}{mc^2}$$

# The case of electron plasma (cont)

$$\hat{\rho}(\mathbf{r}) = \sum_i \delta(\mathbf{r} - \mathbf{r}_i)$$

$$V_2 = \frac{r_0}{2} \int d^d r \mathbf{A}^2(\mathbf{r}, t) \hat{\rho}(\mathbf{r})$$

Field Quantization:  $A(\mathbf{r}, t) \rightarrow \hat{A}(\mathbf{r}, t)$



Photons:  $\hbar\omega_k = c\hbar k$

# Field Quantization(cont)

$$H_{EM} = \frac{1}{8\pi} \int d^3r (\mathbf{E}^2 + \mathbf{B}^2) = \sum_{\mathbf{k}, \lambda} \hbar \omega_{\mathbf{k}} \left( a_{\mathbf{k}\lambda}^\dagger a_{\mathbf{k}\lambda} + 1/2 \right)$$

$$[a_{\mathbf{k}\lambda}, a_{\mathbf{k}\lambda}^\dagger] = 1$$

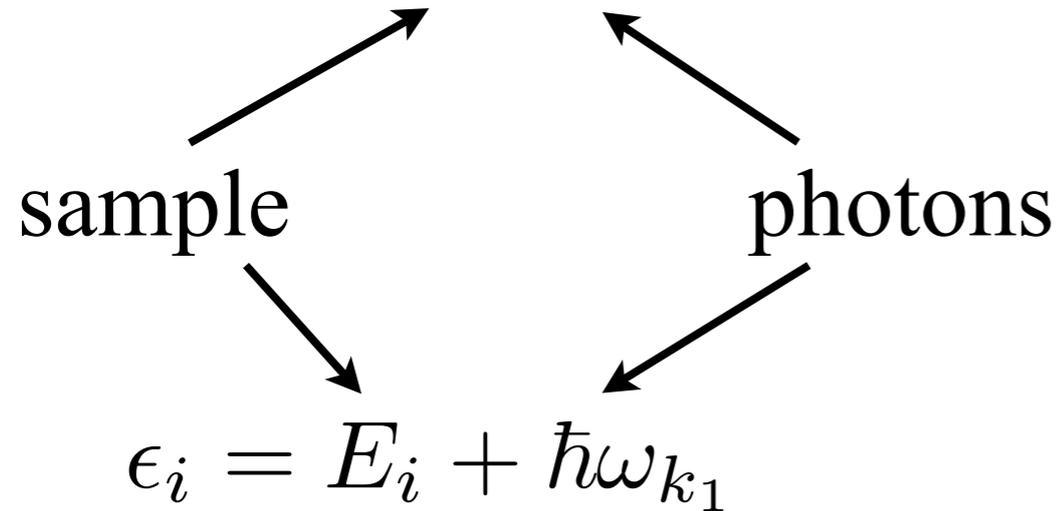
$$\hat{A}_\alpha(\mathbf{r}, t) = \sum_{\mathbf{k}\lambda} \left( \frac{2\pi\hbar^2 c}{V\omega_{\mathbf{k}}} \right)^{1/2} \mathbf{e}_\alpha^\lambda e^{i\mathbf{k}\mathbf{r}} \left( a_{\mathbf{k}\lambda} e^{-i\omega_{\mathbf{k}}t} + a_{-\mathbf{k}\lambda}^\dagger e^{i\omega_{\mathbf{k}}t} \right)$$



One photon in a volume V

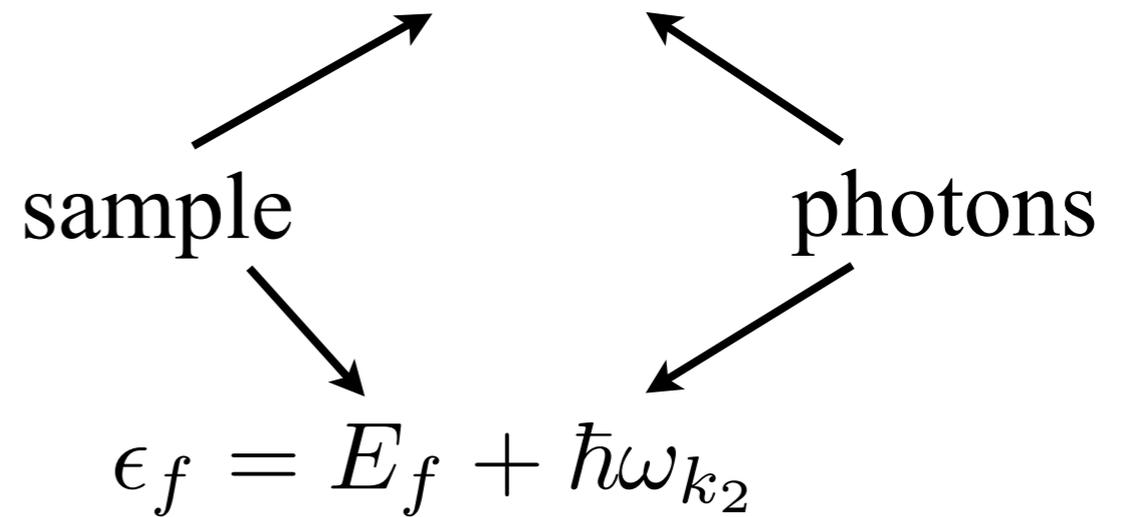
Initial state

$$|\psi_i\rangle = |i\rangle |\mathbf{k}_1 \lambda_1\rangle$$



Final state

$$|\psi_f\rangle = |f\rangle |\mathbf{k}_2 \lambda_2\rangle$$



Golden Rule:

$$\Gamma_{f \leftarrow i} = \frac{2\pi}{\hbar} |\langle \psi_i | V_2 | \psi_f \rangle|^2 \delta(\epsilon_i - \epsilon_f)$$

Scattered flux:

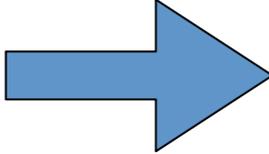
$$\frac{V}{(2\pi)^3} \sum_f k_2^2 dk_2 d\Omega \Gamma_{f \leftarrow i}$$

Incident flux:

$$\frac{c}{V}$$

$$\rho_{\mathbf{q}} = \int d^3r e^{-i\mathbf{q}\cdot\mathbf{r}} \hat{\rho}(\mathbf{r})$$

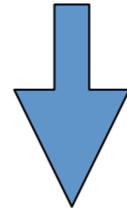
$$V_2 = \frac{r_0}{2} \frac{1}{V} \sum_{\mathbf{k}\mathbf{k}'} \sum_{\lambda\lambda'} \hat{\rho}_{\mathbf{k}+\mathbf{k}'} \frac{2\pi\hbar c^2}{\sqrt{\omega_{\mathbf{k}}\omega_{\mathbf{k}'}}} \sum_{\alpha} \mathbf{e}_{\mathbf{k}\lambda}^{\alpha} \mathbf{e}_{\mathbf{k}'\lambda'}^{\alpha} (a_{\mathbf{k}\lambda} + a_{-\mathbf{k}\lambda}^{\dagger})(a_{\mathbf{k}'\lambda'} + a_{-\mathbf{k}'\lambda'}^{\dagger})$$

Photon part of  $\langle \psi_i, \mathbf{k}_1 \lambda | V_2 | \psi_f, \mathbf{k}_1 \rangle$  

$$\mathbf{k}' = \mathbf{k}_1, \lambda' = \lambda_1 \quad \mathbf{k} = -\mathbf{k}_2, \lambda = \lambda_2$$

$$\mathbf{k}' = -\mathbf{k}_2, \lambda = \lambda_2 \quad \mathbf{k}' = \mathbf{k}_1, \lambda = \lambda_1$$

$$\Gamma_{f \leftarrow i} = \frac{1}{V^2} \frac{2\pi}{\hbar} r_0^2 \frac{(2\pi\hbar c^2)^2}{\omega_{k_1} \omega_{k_2}} |\langle f | \hat{\rho}_{\mathbf{k}_1 - \mathbf{k}_2} | i \rangle|^2 \left| \sum_{\alpha} e_{\lambda_1}^{\alpha} e_{\lambda_2}^{\alpha} \right|^2 \delta(E_f - E_i - (\omega_2 - \omega_1))$$



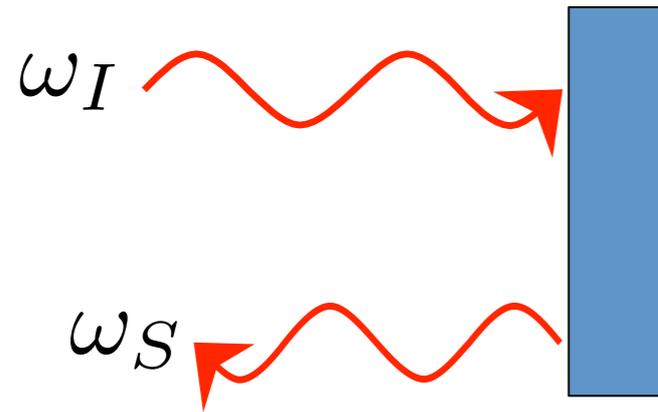
$$\frac{d\sigma}{d\Omega d\hbar\omega_2} = r_0^2 \frac{\omega_2}{\omega_1} \left| \sum_{\alpha} e_{\lambda_1}^{\alpha} e_{\lambda_2}^{\alpha} \right|^2 \underbrace{\sum_f |\langle f | \hat{\rho}_{\mathbf{k} - \mathbf{k}'} | i \rangle|^2 \delta(E_f - E_i - (\omega_2 - \omega_1))}_{\text{Dynamical Structure Factor}}$$

Dynamical Structure Factor

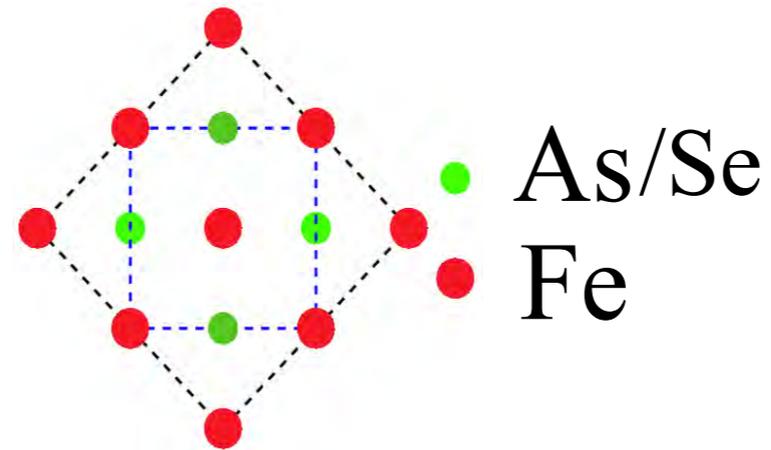
$$\frac{d\sigma}{d\Omega d\hbar\omega_2} = \frac{r_0^2}{2\pi} \frac{\omega_2}{\omega_1} \frac{1}{1 - e^{-\omega/T}} \chi''(\mathbf{k}_1 - \mathbf{k}_2, \omega_2 - \omega_1)$$

$$\chi(\mathbf{q}, \omega) = -i \int_0^{\infty} dt e^{i\omega t} [\rho_{-\mathbf{q}}(t), \rho_{\mathbf{q}}(0)] \quad \text{Raman susceptibility}$$

Experiment:



Quasi-backscattering geometry



Layered materials:  $\mathbf{q} = 0$      $\rho_{\mathbf{q}}(t) = \rho_{\mathbf{q}}(0)$      $\chi(0, \omega \neq 0) = 0$

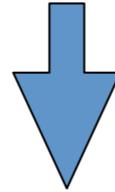
No Raman !?!?....

Raman coupling is different in multi-band/multi-orbital materials!

Shear modulus, resistivity: static induced anisotropy



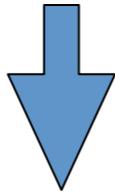
Weak dynamic probe of  $\phi$  is needed



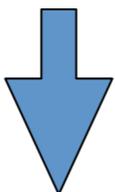
Raman spectroscopy

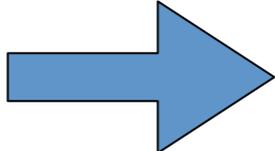


$m^{-1}$  too symmetric to couple to anything less trivial than density



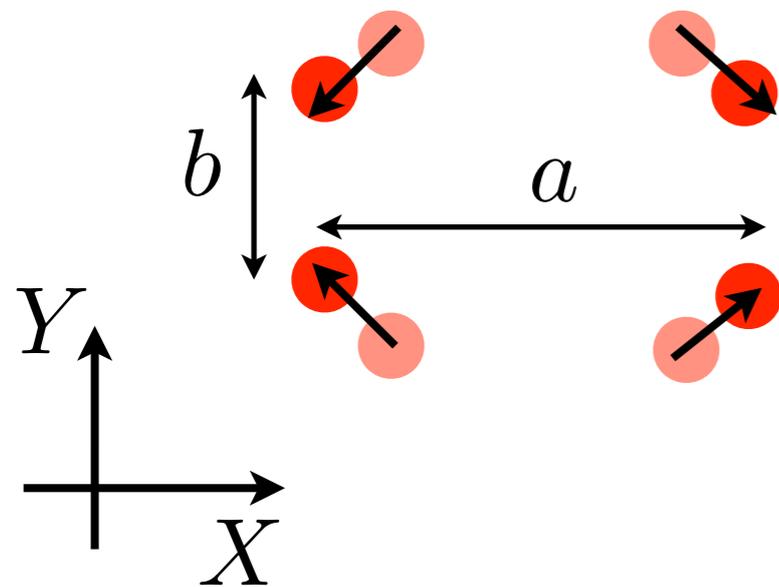
What is the symmetry of  $\phi$ ? Does the lattice help?



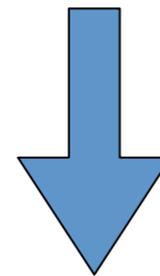
$F \sim -\lambda \epsilon_6 \phi$  scalar  Symmetry of  $\epsilon_6$  ?

# Structural transition

## Orthorhombic distortion



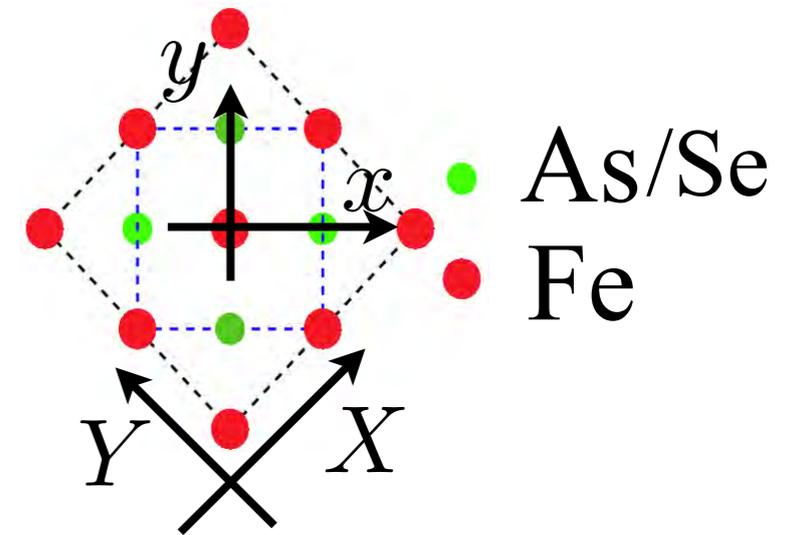
$a \neq b$  anisotropy



Discrete  $C_4 \rightarrow C_2$  symmetry breaking

$$\epsilon_6 = \frac{\partial u_X}{\partial X} - \frac{\partial u_Y}{\partial Y} \neq 0$$

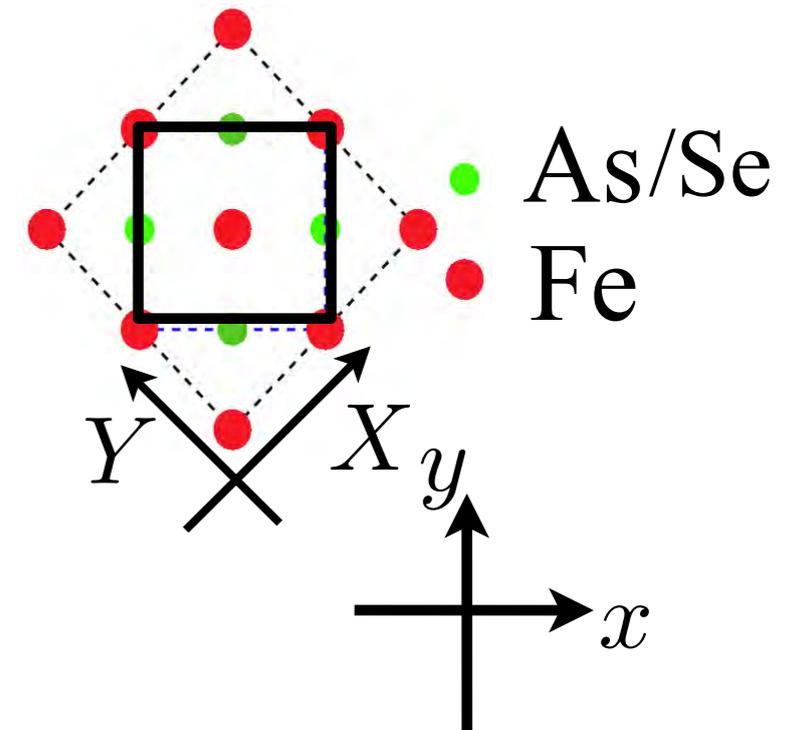
Order Parameter



# Structural transition: order parameter symmetry

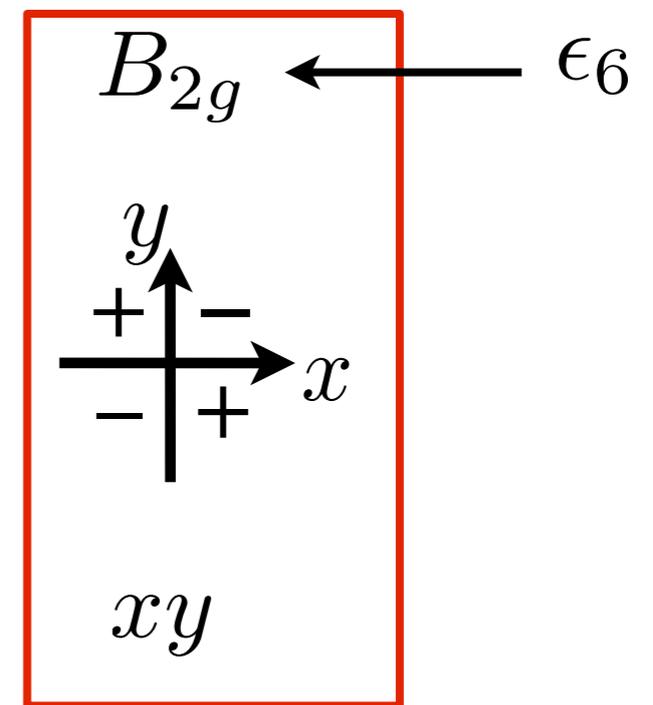
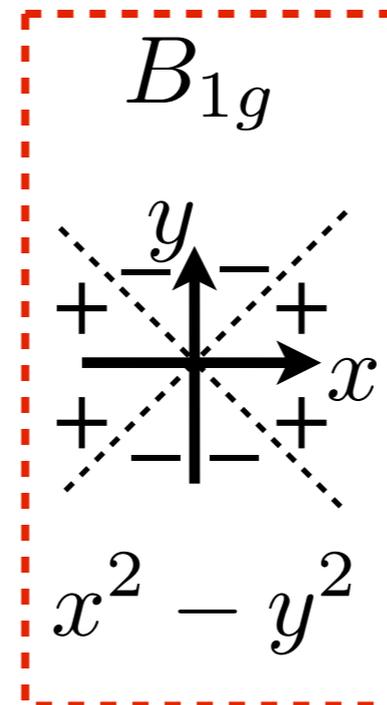
$$\epsilon_6 = \frac{\partial u_X}{\partial X} - \frac{\partial u_Y}{\partial Y} \sim X^2 - Y^2 = 2xy$$

$$D_{4h} = D_4 \otimes I = 8 * 2 = 16$$

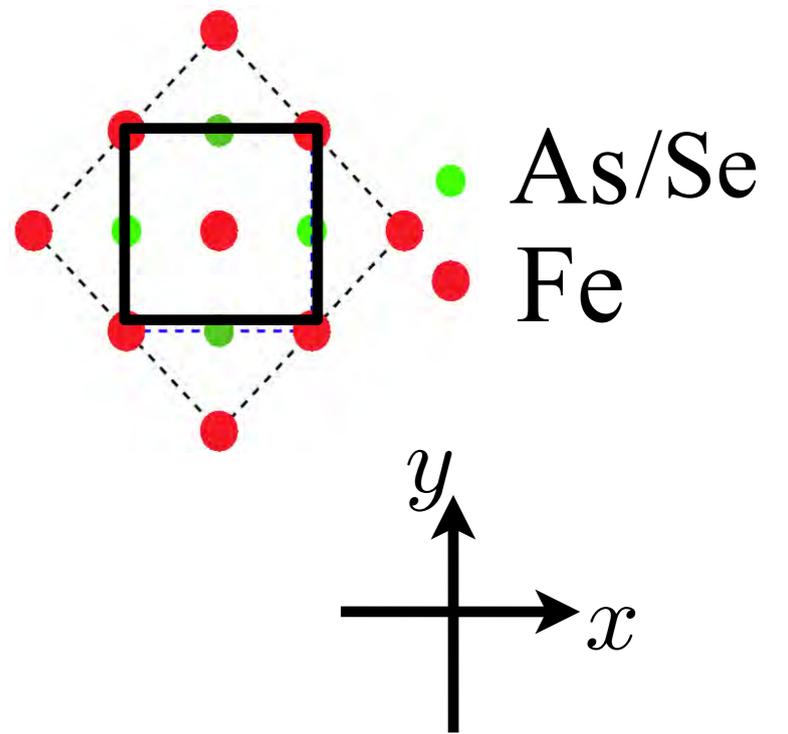


$A_{1g}$

Fully symmetric (scalar)



$$\epsilon_6 \quad \text{and} \quad \phi \sim xy = B_{2g}$$

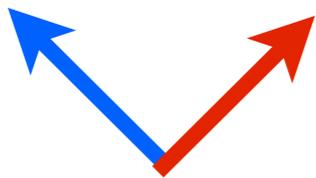


Raman?  $e^I \uparrow e^S \uparrow$

Tensor  $\rightarrow e_\alpha^I e_\beta^S$

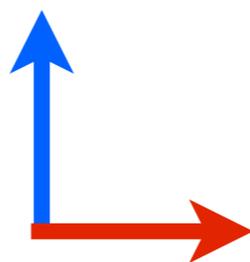
$$B_{1g}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



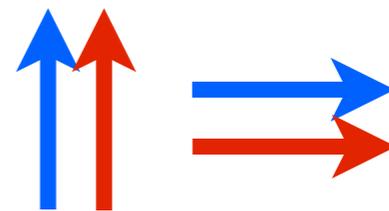
$$B_{2g} \sim xy$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

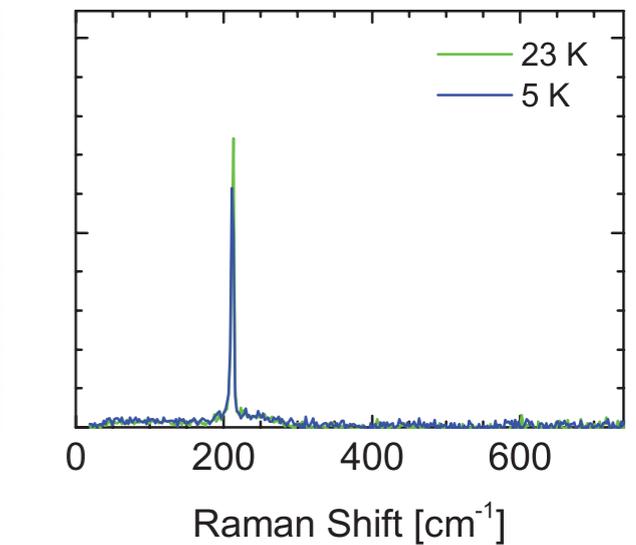
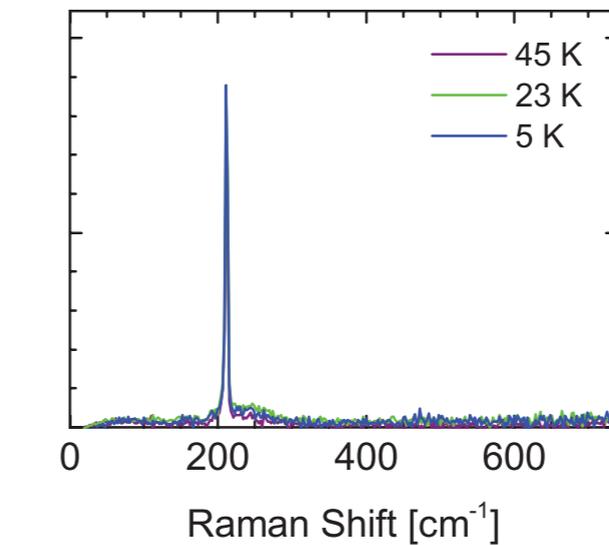
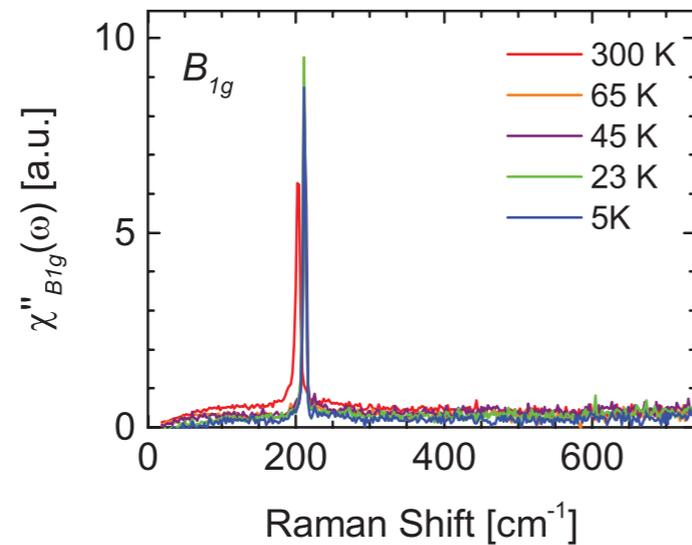
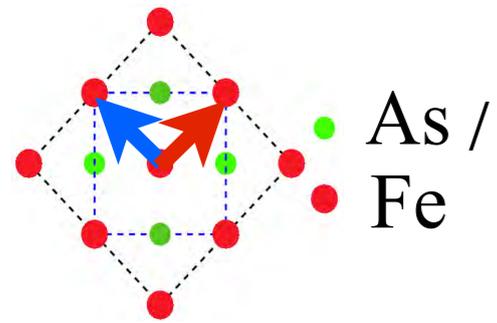
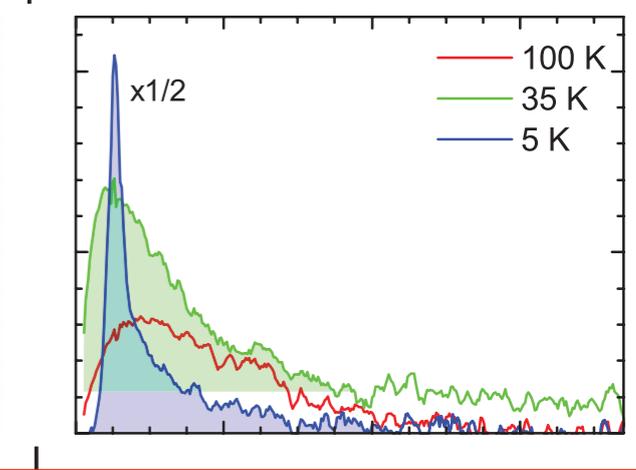
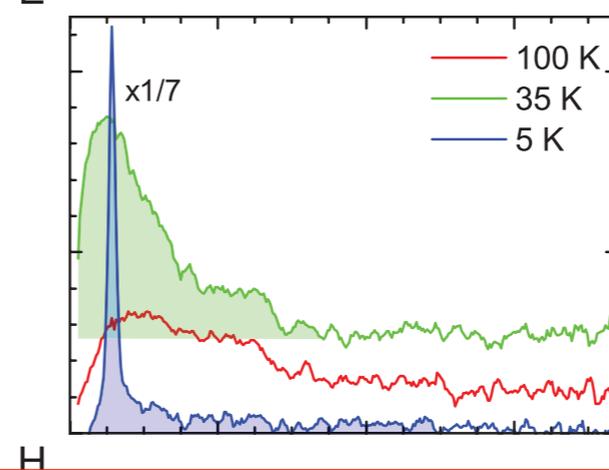
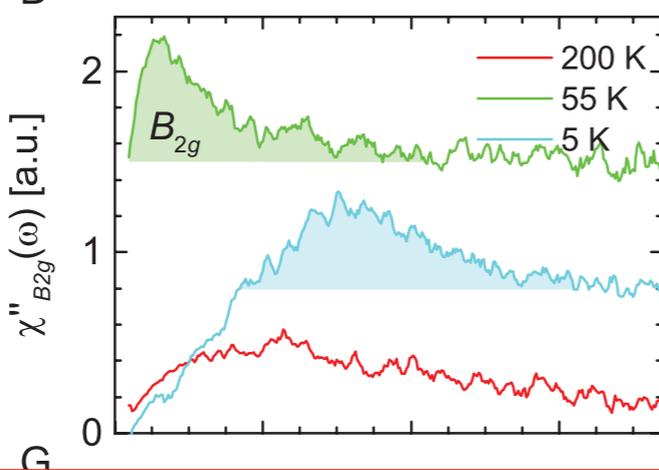
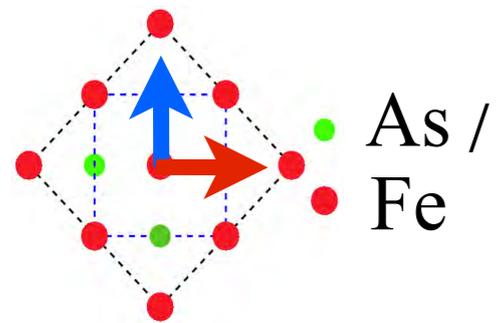
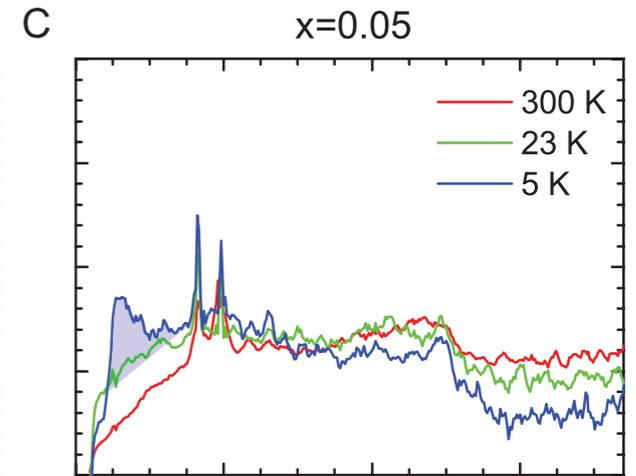
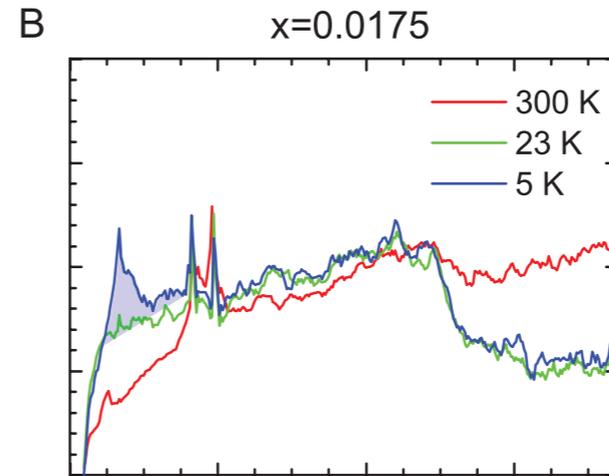
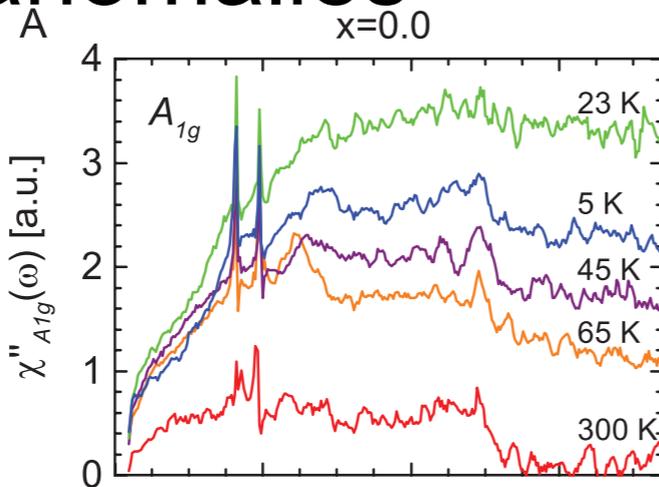
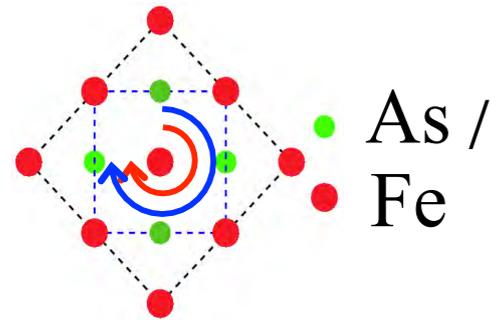


$$A_{1g}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



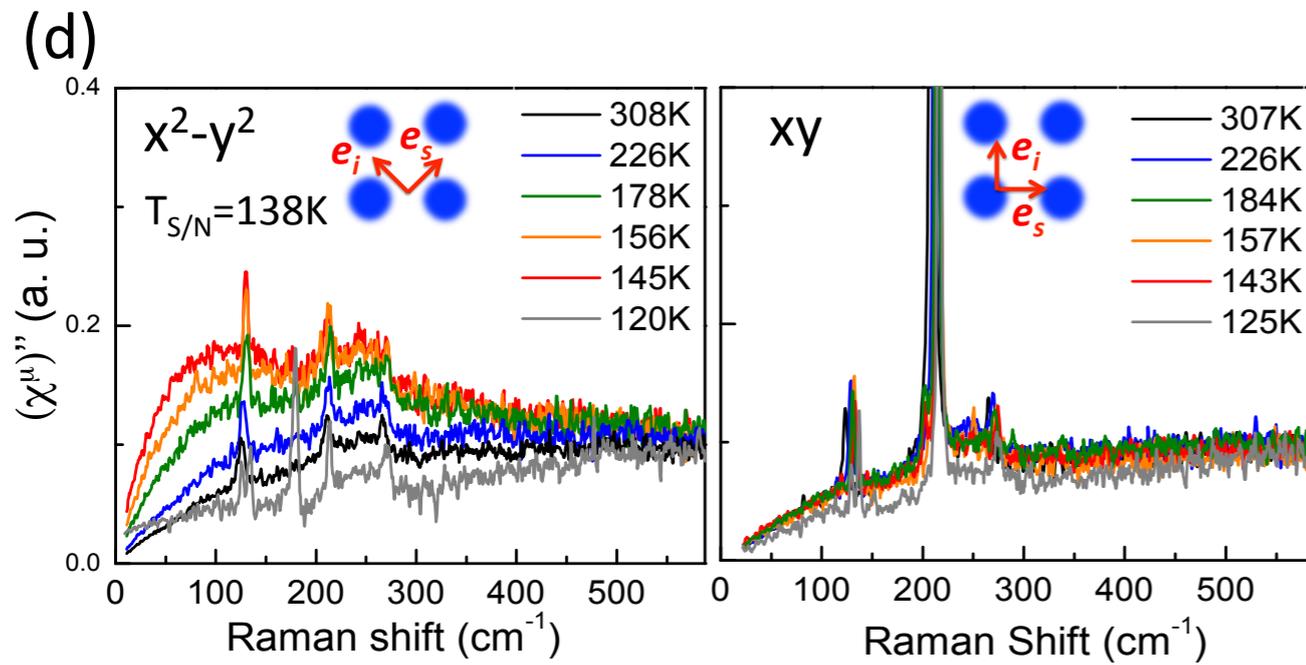
# Raman B<sub>2g</sub> anomalies



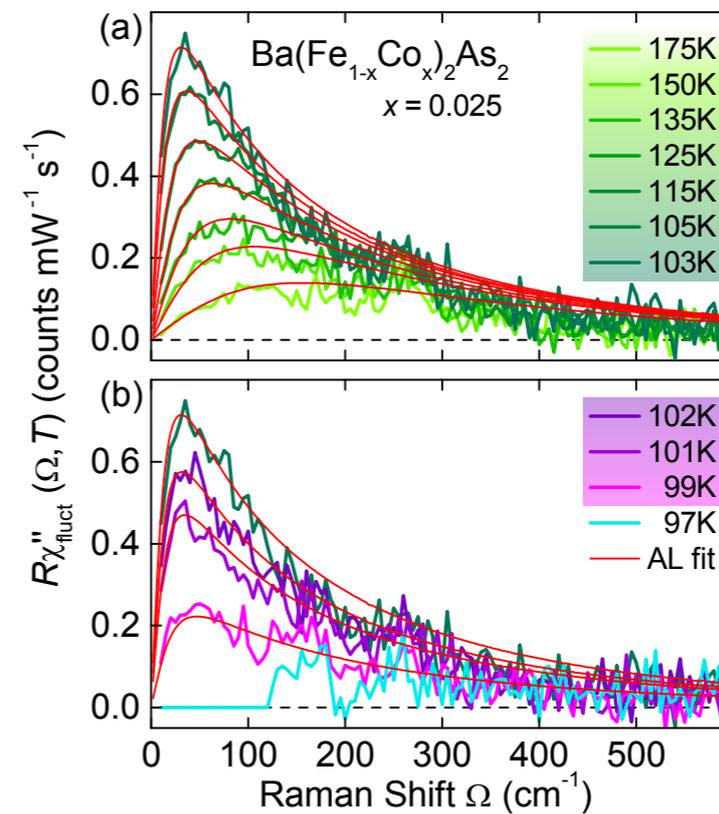
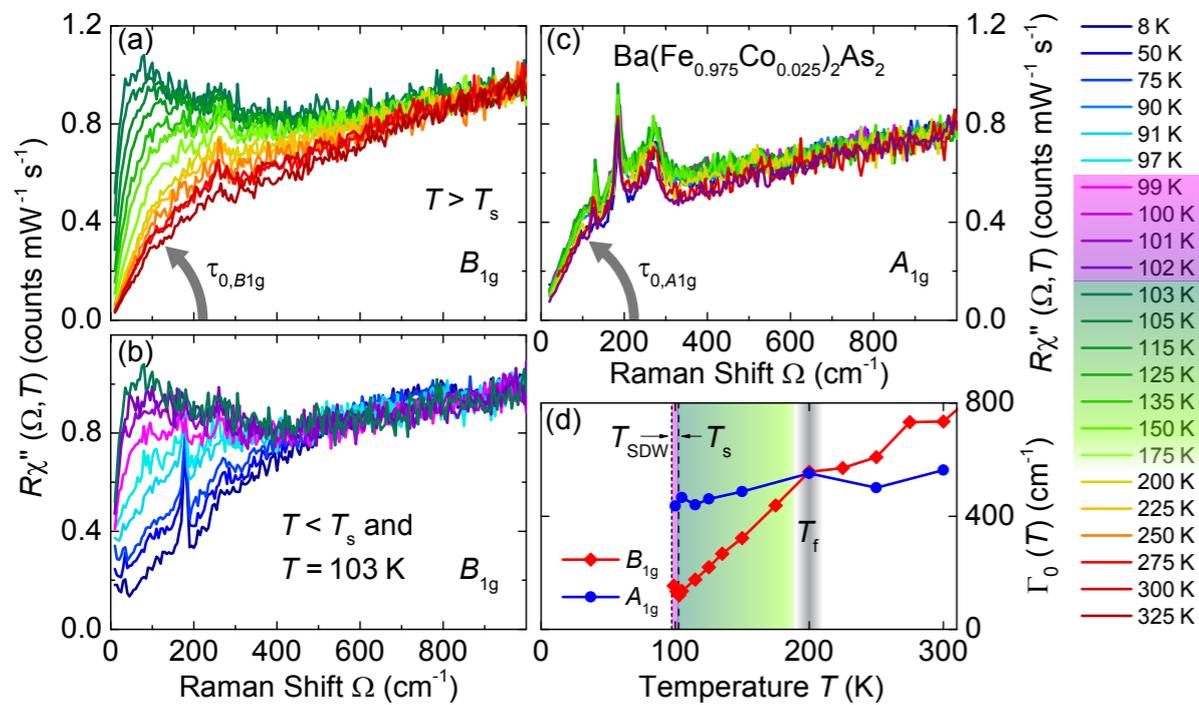
[arXiv:1410.6456](https://arxiv.org/abs/1410.6456)

**Critical Charge Fluctuations in Iron Pnictide Superconductors**

V. K. Thorsmølle, MK, Z. P. Yin, Chenglin Zhang, S. V. Carr, Pengcheng Dai, G. Blumberg



Y. Gallais, R. M. Fernandes, I. Paul, L. Chauviere, Y.-X. Yang, M.-A. Measson, M. Cazayous, A. Sacuto, D. Colson, A. Forget, Phys. Rev. Lett. 111, 267001 (2013)

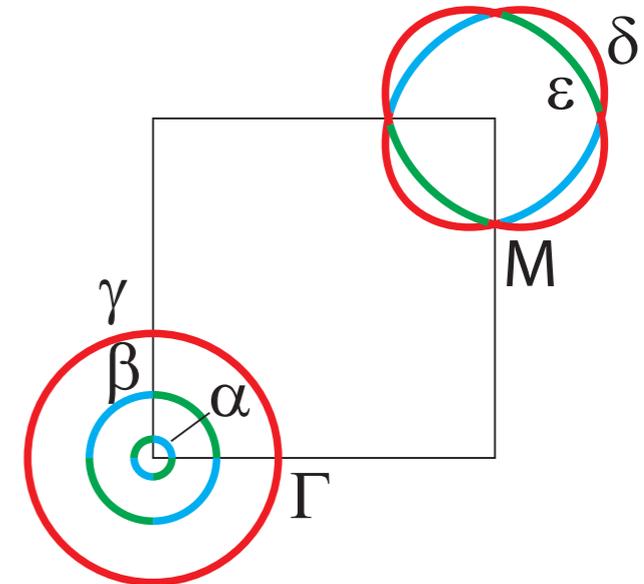
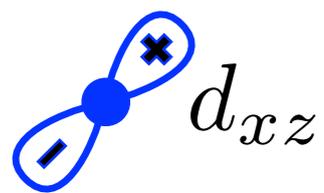


arXiv:1507.06116 Florian Kretzschmar, Thomas Böhm, Una Karahasanović, Bernhard Muschler, Andreas Baum, Daniel Jost, Joerg Schmalian, Sergio Caprara, Marco Grilli, Carlo Di Castro, James G. Analytis, Jiun-Haw Chu, Ian Randal Fisher, Rudi Hackl

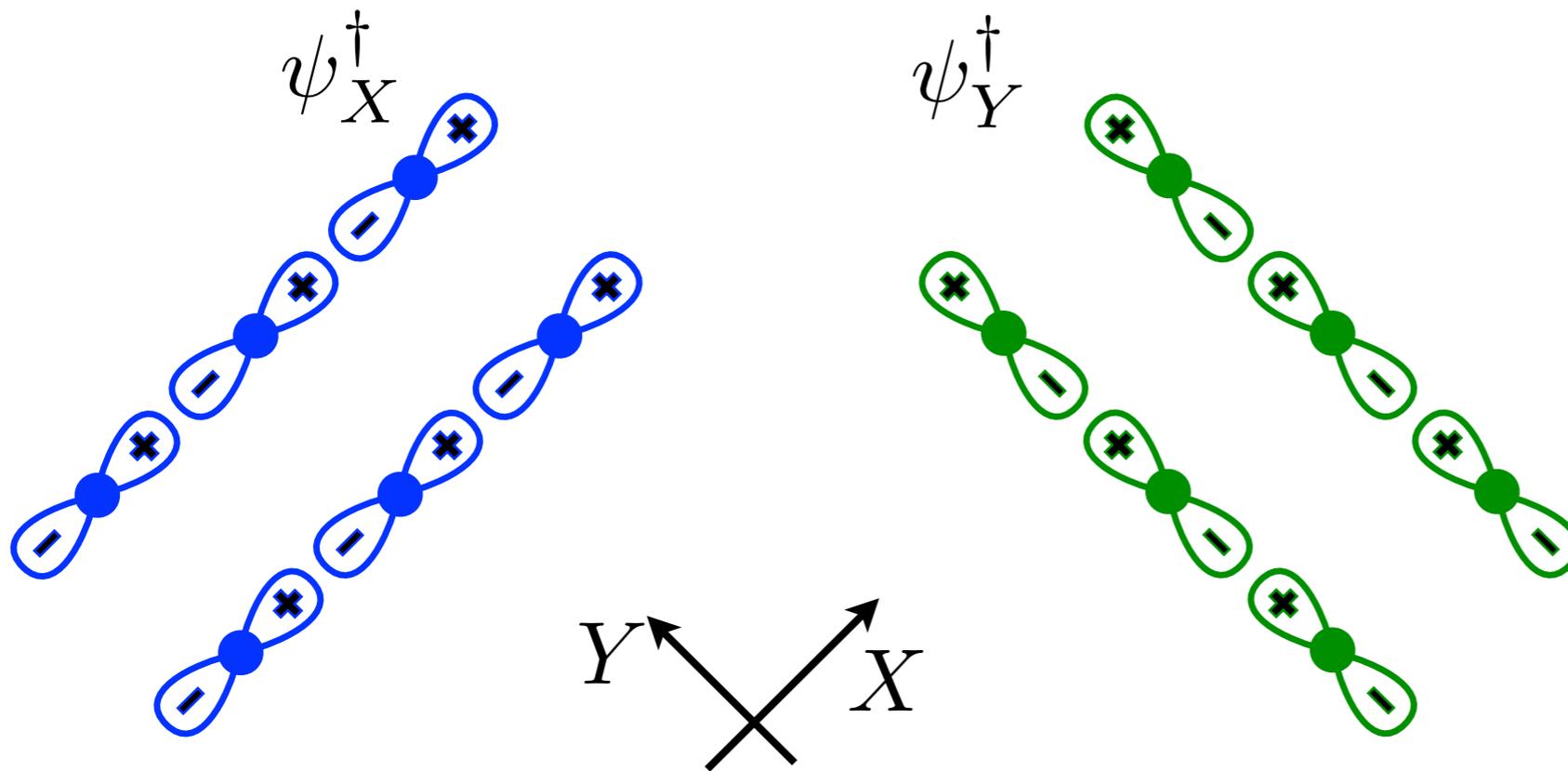
# What does Raman see?

Examine band structure

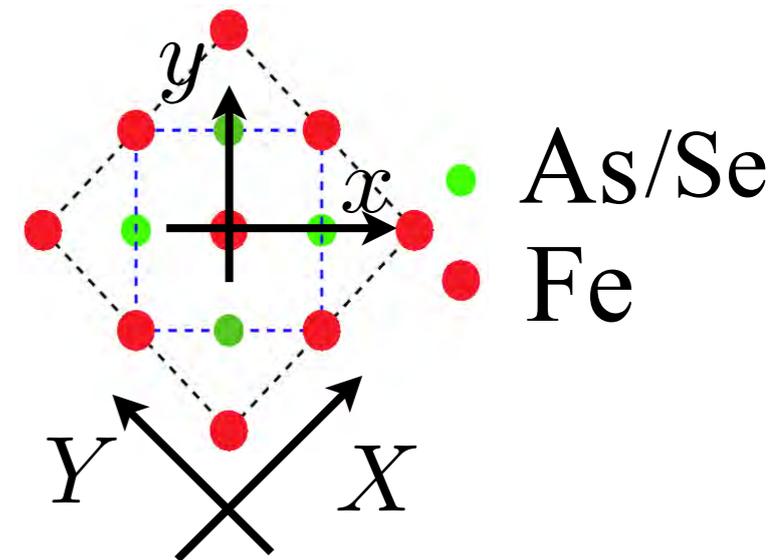
Focus on  $\Gamma$  point  $\alpha\beta$  bands



At  $\Gamma$  double degeneracy (no spin-orbit)



Iron only lattice



Slightly off  $\Gamma$  point

Method of invariants (Luttinger)

Symmetry allowed terms:

$$\sim (k_x^2 + k_y^2)(\psi_X^\dagger \psi_X + \psi_Y^\dagger \psi_Y)$$

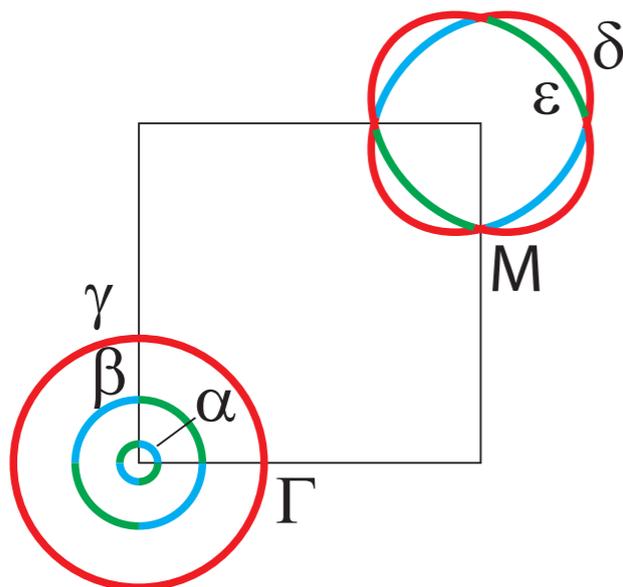
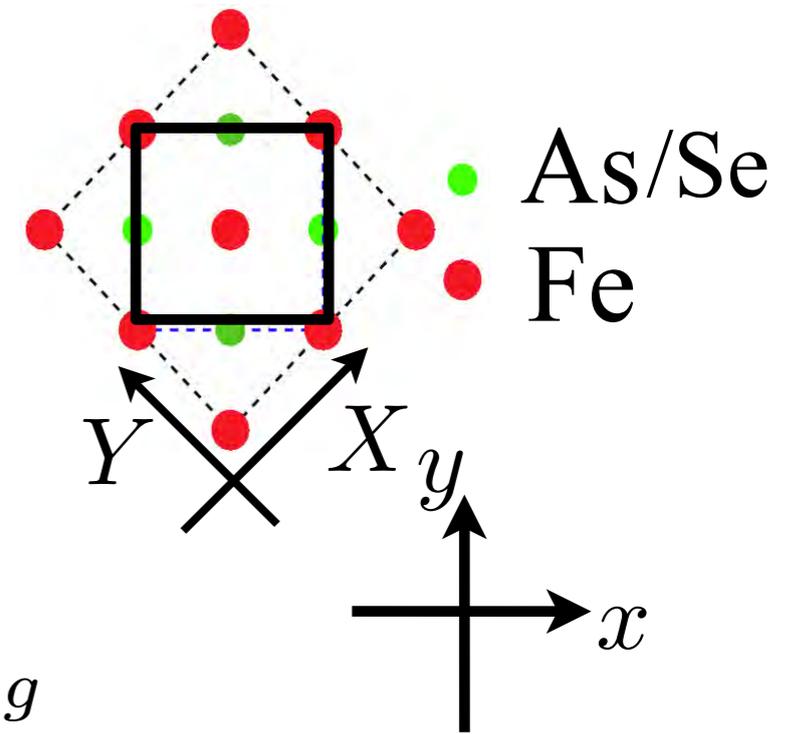
$$A_{1g} * A_{1g}$$

$$\sim (k_x^2 - k_y^2)(\psi_X^\dagger \psi_Y + \psi_Y^\dagger \psi_X)$$

$$B_{1g} * B_{1g}$$

$$\sim k_x k_y (\psi_X^\dagger \psi_X - \psi_Y^\dagger \psi_Y)$$

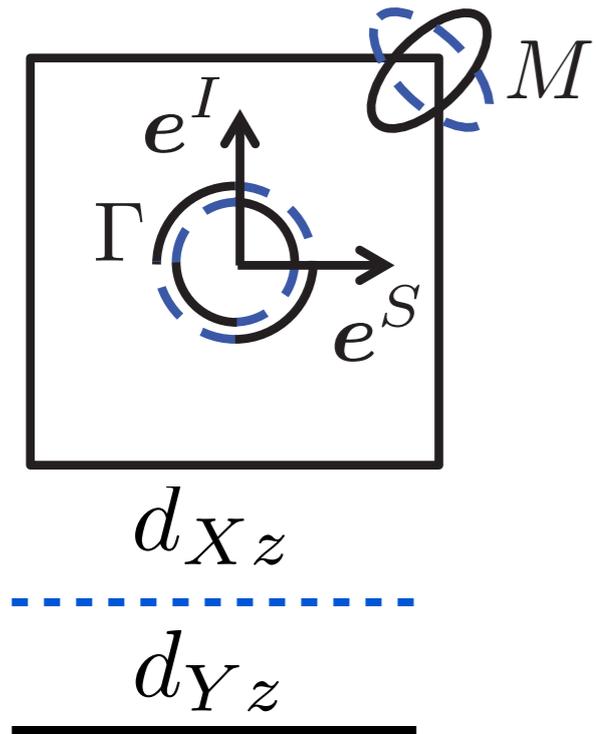
$$B_{2g} * B_{2g}$$



Holes  $\Gamma$

$$\mathcal{H}_{\mathbf{k}}^{\Gamma} = \begin{bmatrix} \epsilon_{\Gamma} + \frac{k^2}{2m_{\Gamma}} + ak_x k_y & \frac{c}{2}(k_x^2 - k_y^2) \\ \frac{c}{2}(k_x^2 - k_y^2) & \epsilon_{\Gamma} + \frac{k^2}{2m_{\Gamma}} - ak_x k_y \end{bmatrix}$$

# What does B2g Raman see?



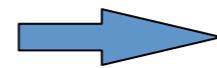
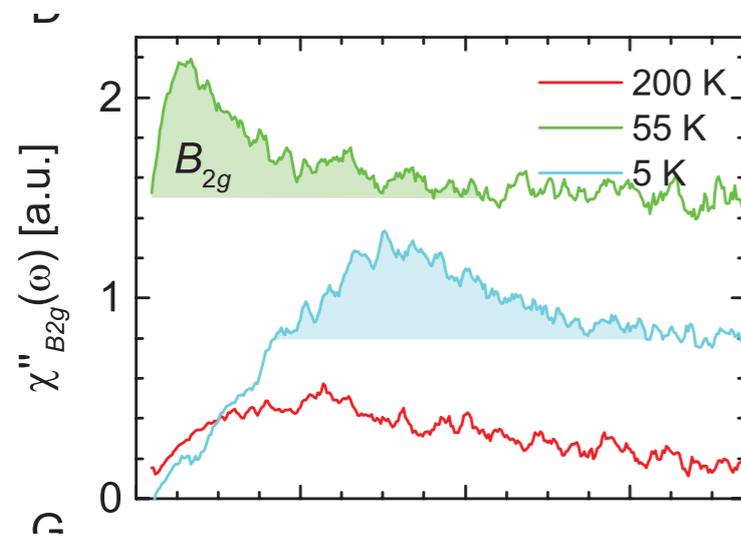
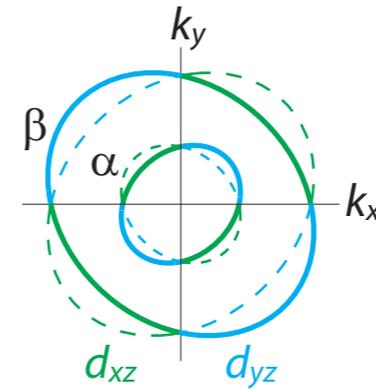
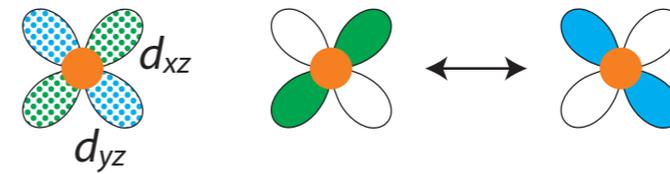
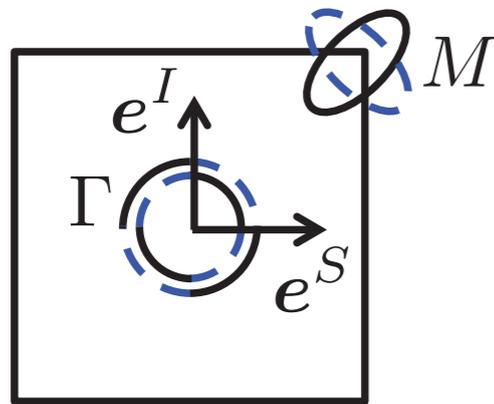
$$\mathcal{H}_{\mathbf{k}}^{\Gamma} = \begin{bmatrix} \epsilon_{\Gamma} + \frac{k^2}{2m_{\Gamma}} + ak_xk_y & \frac{c}{2}(k_x^2 - k_y^2) \\ \frac{c}{2}(k_x^2 - k_y^2) & \epsilon_{\Gamma} + \frac{k^2}{2m_{\Gamma}} - ak_xk_y \end{bmatrix}$$

$$\mathbf{k} \rightarrow \mathbf{k} + \mathbf{A} \quad \longrightarrow \quad r_{i,j}^{\Gamma(M)} = \sum_{\lambda\lambda'} e_{\lambda}^I e_{\lambda'}^S \frac{\partial^2 \mathcal{H}_{ij}^{\Gamma(M)}}{\partial k_{\lambda} \partial k_{\lambda'}}.$$

$$r^{\Gamma} = a \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \longleftrightarrow \quad \hat{r}^{\Gamma} \propto \psi_X^{\dagger} \psi_X - \psi_Y^{\dagger} \psi_Y$$

# B2g Raman (cont)

$$r^\Gamma = a \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \longleftrightarrow \hat{r}^\Gamma \propto \psi_X^\dagger \psi_X - \psi_Y^\dagger \psi_Y = n_{Xz} - n_{Yz}$$

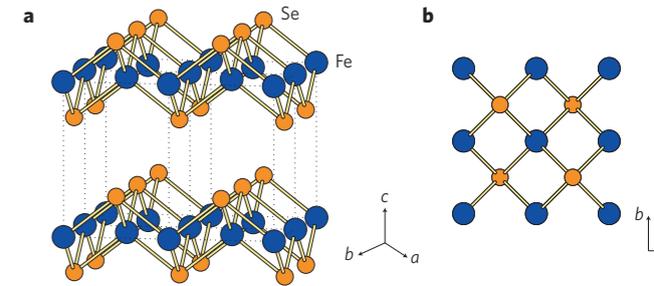


Tendency to orbital ordering

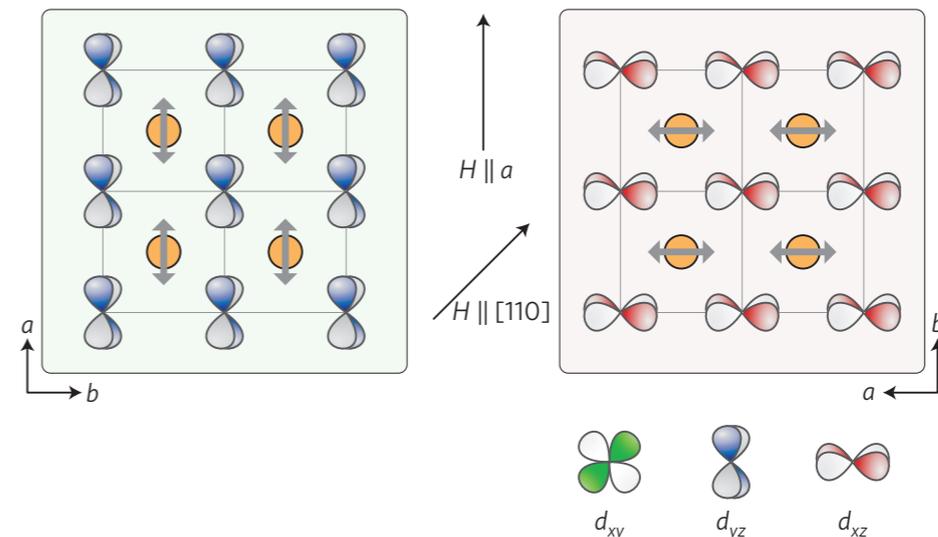
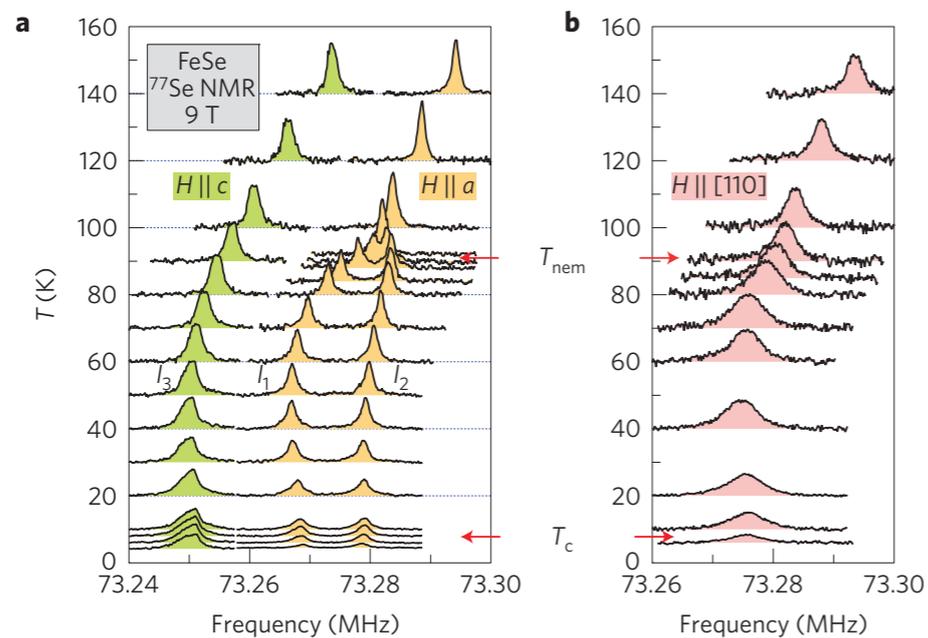
# Orbital-driven nematicity in FeSe

S-H. Baek<sup>1\*</sup>, D. V. Efremov<sup>1</sup>, J. M. Ok<sup>2</sup>, J. S. Kim<sup>2</sup>, Jeroen van den Brink<sup>1,3</sup> and B. Büchner<sup>1,3</sup>

A fundamental and unconventional characteristic of superconductivity in iron-based materials is that it occurs in the vicinity of two other instabilities. In addition to a tendency towards magnetic order, these Fe-based systems have a propensity for nematic ordering: a lowering of the rotational symmetry while time-reversal invariance is preserved. Setting the stage for superconductivity, it is heavily debated whether the nematic symmetry breaking is driven by lattice, orbital or spin degrees of freedom. Here, we report a very clear splitting of NMR resonance lines in FeSe at  $T_{\text{nem}} = 91\text{ K}$ , far above the superconducting  $T_c$  of 9.3 K. The splitting occurs for magnetic fields perpendicular to the Fe planes and has the temperature dependence of a Landau-type order parameter. Spin-lattice relaxation rates are not affected at  $T_{\text{nem}}$ , which unequivocally establishes orbital degrees of freedom as driving the nematic order. We demonstrate that superconductivity competes with the emerging nematicity.



$$\frac{1}{T_1 T} \sim \sum_{\mathbf{q}} |\gamma_n A(\mathbf{q})|^2 \frac{\text{Im} \chi(\mathbf{q}, f)}{f}$$

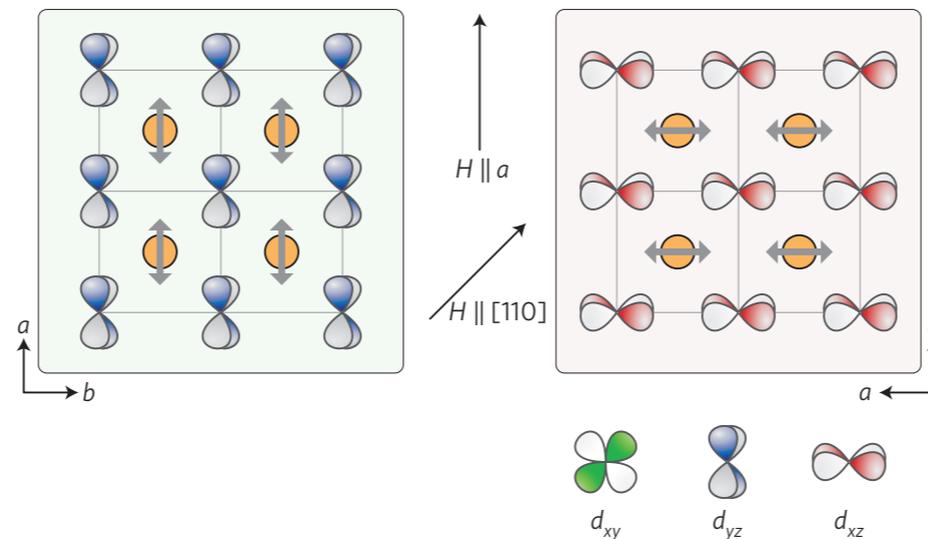


# Orbital order: C4 to C2 discrete symmetry breaking

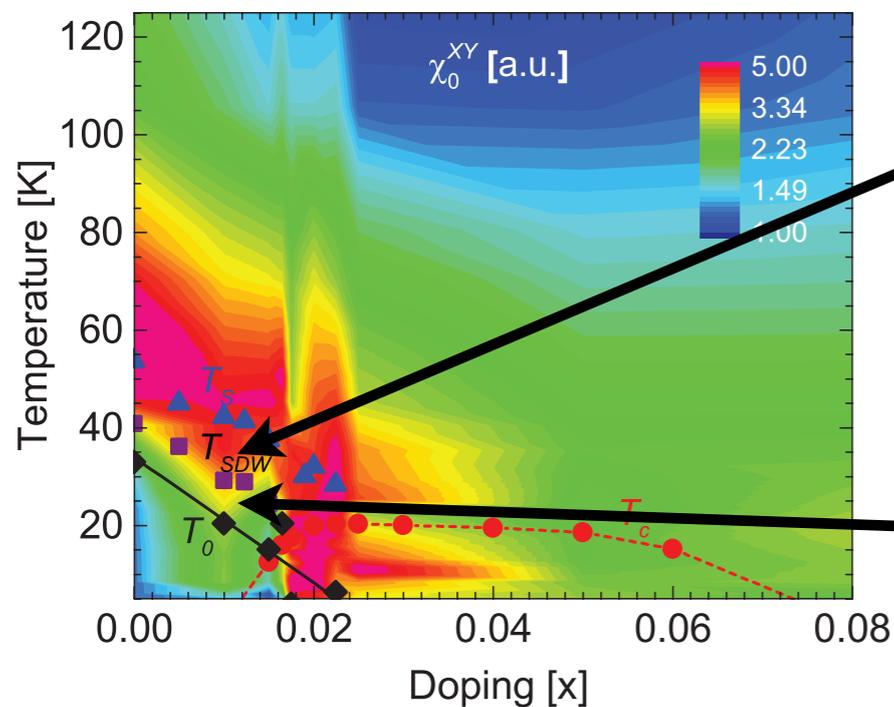


Ising nematic transition: difference in orbital occupation

$$n_{X_z} - n_{Y_z} \neq 0$$



# Alternative: Spin Ising Nematic Transition

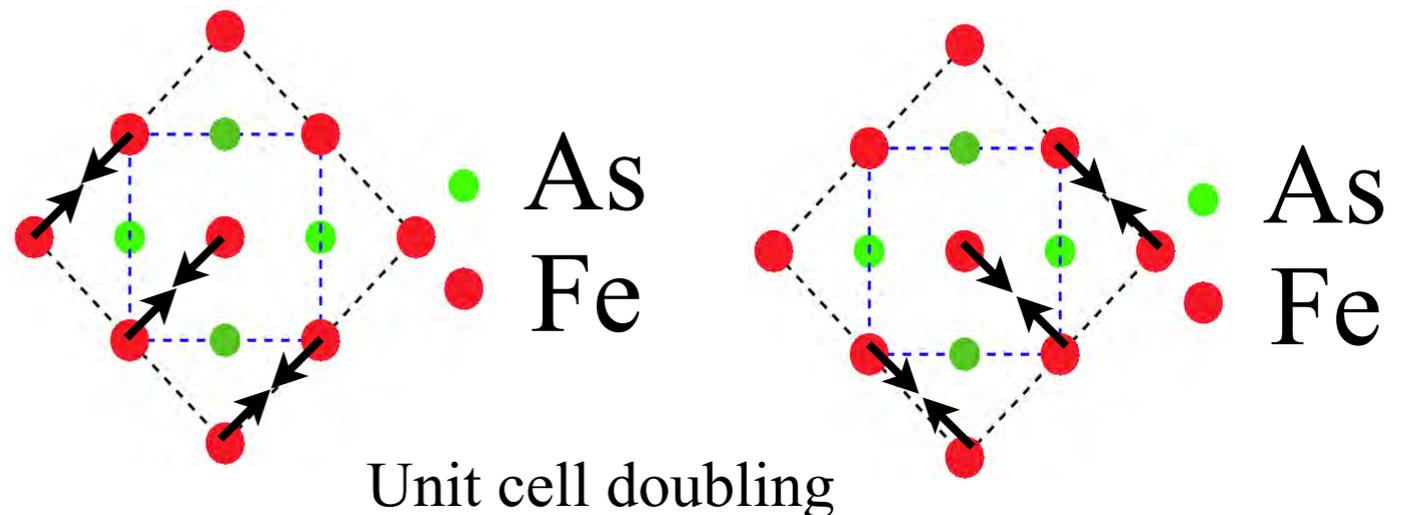


$$T_{SDW} < T < T_S$$

$$\langle \Delta_X^2 \rangle - \langle \Delta_Y^2 \rangle \neq 0$$

Stripe magnetizations  $\langle \Delta_{X,Y} \rangle = 0$

$$T < T_{SDW}$$



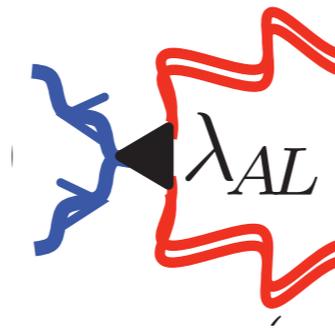
# What drives nematic order in iron-based superconductors?

R. M. Fernandes<sup>1\*</sup>, A. V. Chubukov<sup>2\*</sup> and J. Schmalian<sup>3\*</sup>

Although the existence of nematic order in iron-based superconductors is now a well-established experimental fact, its origin remains controversial. Nematic order breaks the discrete lattice rotational symmetry by making the  $x$  and  $y$  directions in the iron plane non-equivalent. This can happen because of a regular structural transition or as the result of an electronically driven instability — in particular, orbital order or spin-driven Ising-nematic order. The latter is a magnetic state that breaks rotational symmetry but preserves time-reversal symmetry. Symmetry dictates that the development of one of these orders immediately induces the other two, making the origin of nematicity a physics realization of the 'chicken and egg problem'. In this Review, we argue that the evidence strongly points to an electronic mechanism of nematicity, placing nematic order in the class of correlation-driven electronic instabilities, like superconductivity and density-wave transitions. We discuss different microscopic models for nematicity and link them to the properties of the magnetic and superconducting states, providing a unified perspective on the phase diagram of the iron pnictides.

## 2 electronic alternatives are not independent

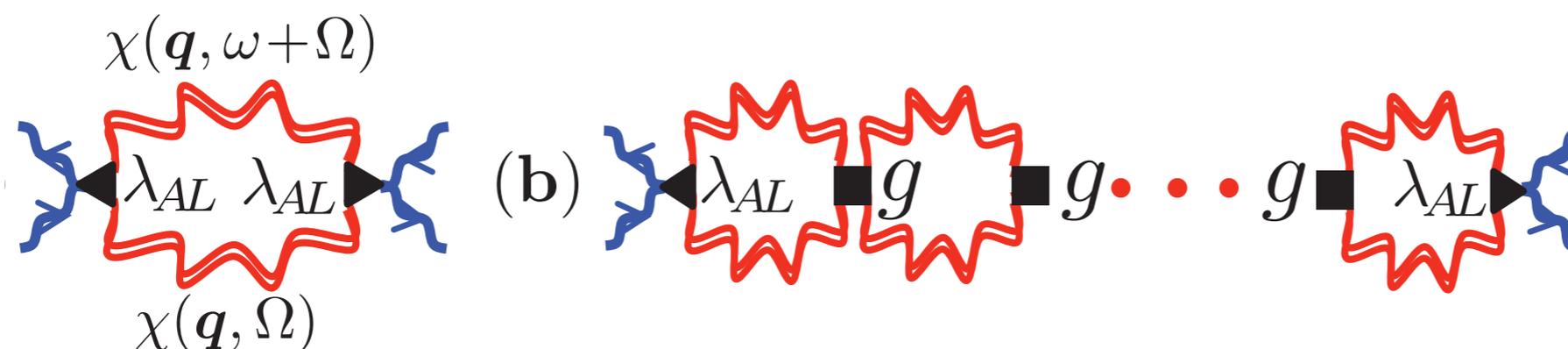
$$n_{Xz} - n_{Yz}$$



$$|\Delta_X|^2 - |\Delta_Y|^2$$

# Raman correlations in magnetic scenario

$$S_{\text{eff}} [\Delta_X, \Delta_Y] = r_0 (\Delta_X^2 + \Delta_Y^2) + \frac{u}{2} (\Delta_X^2 + \Delta_Y^2)^2 - \frac{g}{2} (\Delta_X^2 - \Delta_Y^2)^2 + v (\Delta_X \cdot \Delta_Y)^2$$

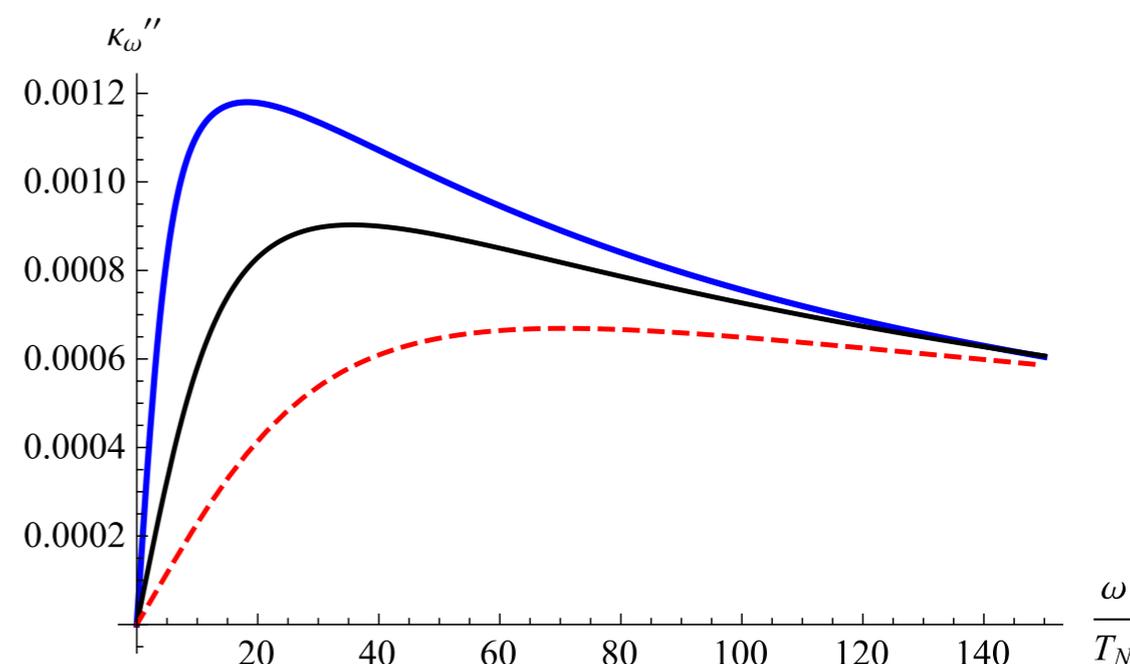


## Quasi-Elastic Scattering The case of critical slow-down

MK and A. Levchenko, Phys.Rev. B **91**, 235119 (2015)

[arXiv:1504.06841](https://arxiv.org/abs/1504.06841)

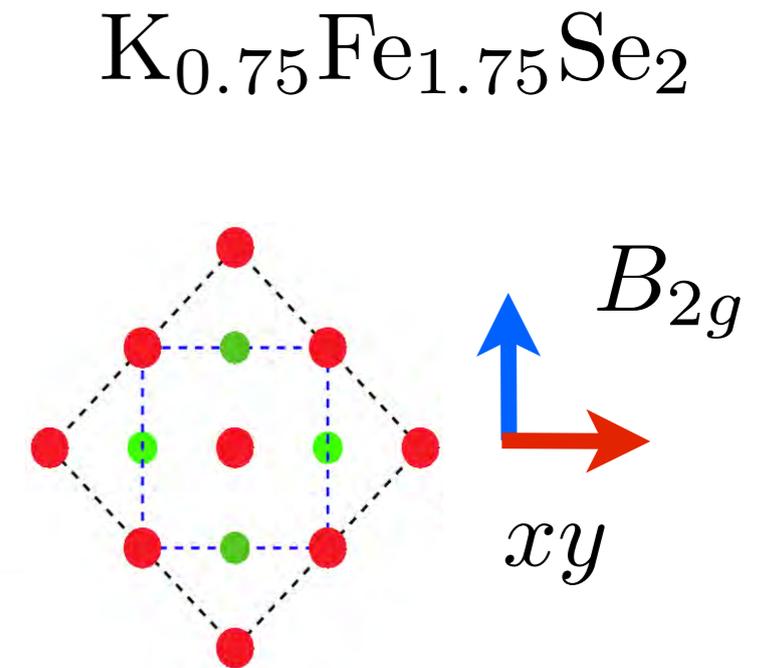
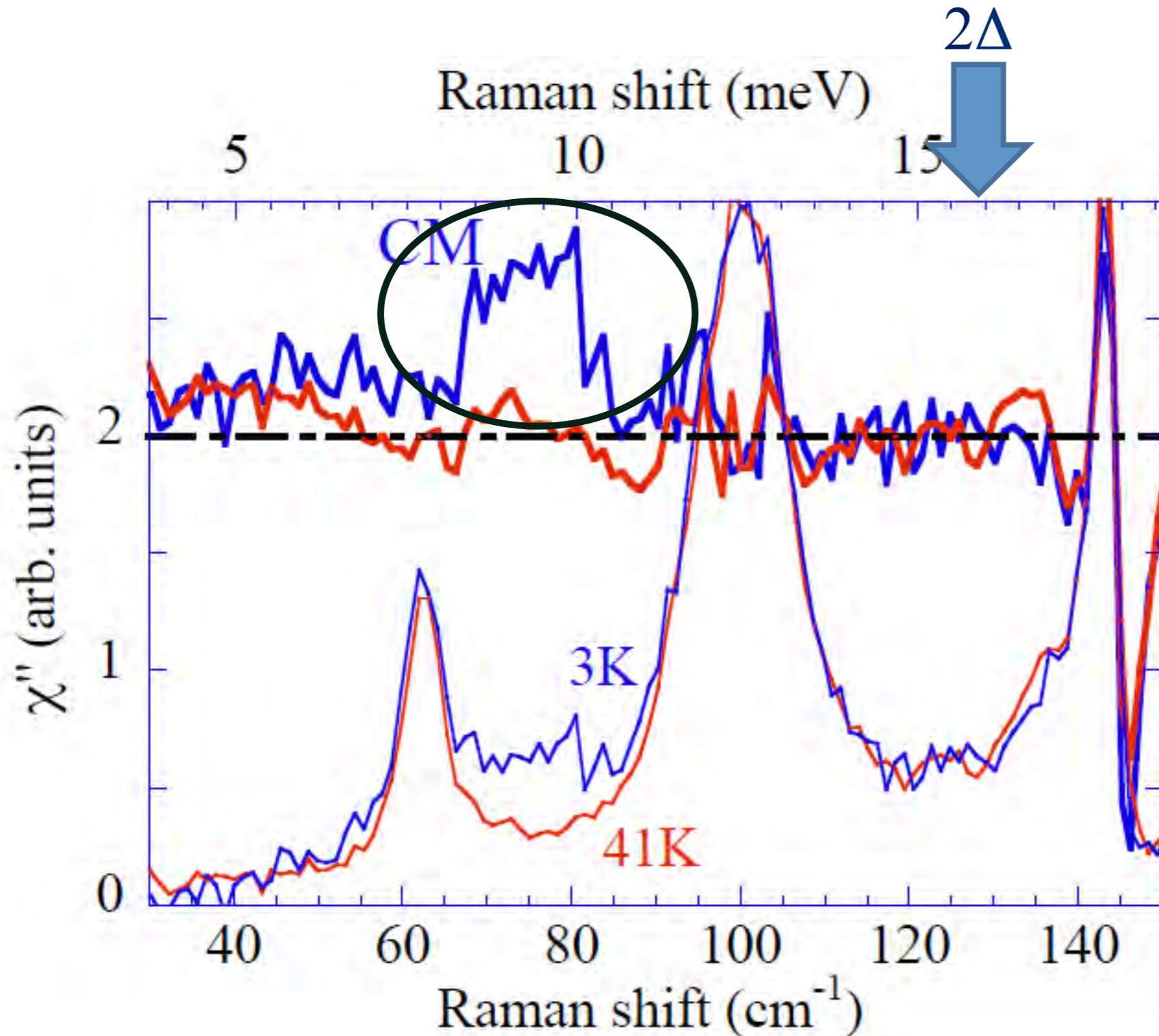
[Una Karahasanovic](#), [Florian Kretzschmar](#), [Thomas Boehm](#), [Rudi Hackl](#), [Indranil Paul](#), [Yann Gallais](#), [Joerg Schmalian](#)



## Part II

# Raman in Iron Selenides

A. Ignatov, A. Kumar, P. Lubik, R. H. Yuan, W. T. Guo, N. L. Wang, K. Rabe, and G. Blumberg,  
*Phys. Rev. B* **86**, 134107 (2012)



ARPES:

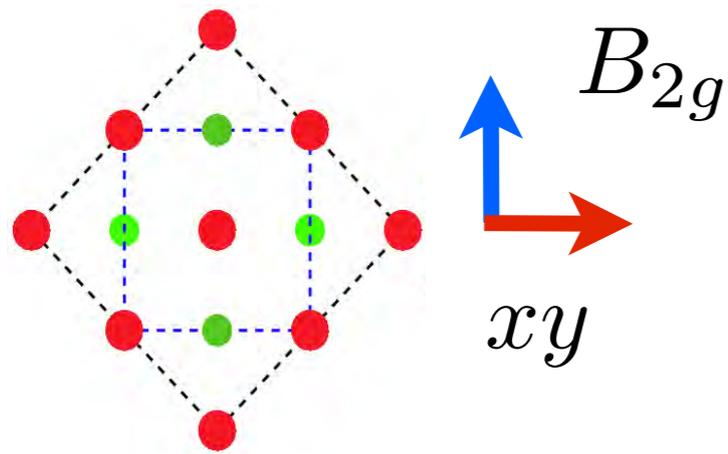
$$\Delta \approx 8\text{meV}$$

M. Xu, Q. Q. Ge, R. Peng, Z. R. Ye, J. Jiang,  
F. Chen, X. P. Shen, B. P. Xie, Y. Zhang,  
A. F. Wang, X. F. Wang, X. H. Chen,  
and D. L. Feng,  
*Phys. Rev. B* **85**, 220504 (2012)

# Raman-Scattering Detection of Nearly Degenerate $s$ -Wave and $d$ -Wave Pairing Channels in Iron-Based $\text{Ba}_{0.6}\text{K}_{0.4}\text{Fe}_2\text{As}_2$ and $\text{Rb}_{0.8}\text{Fe}_{1.6}\text{Se}_2$ Superconductors

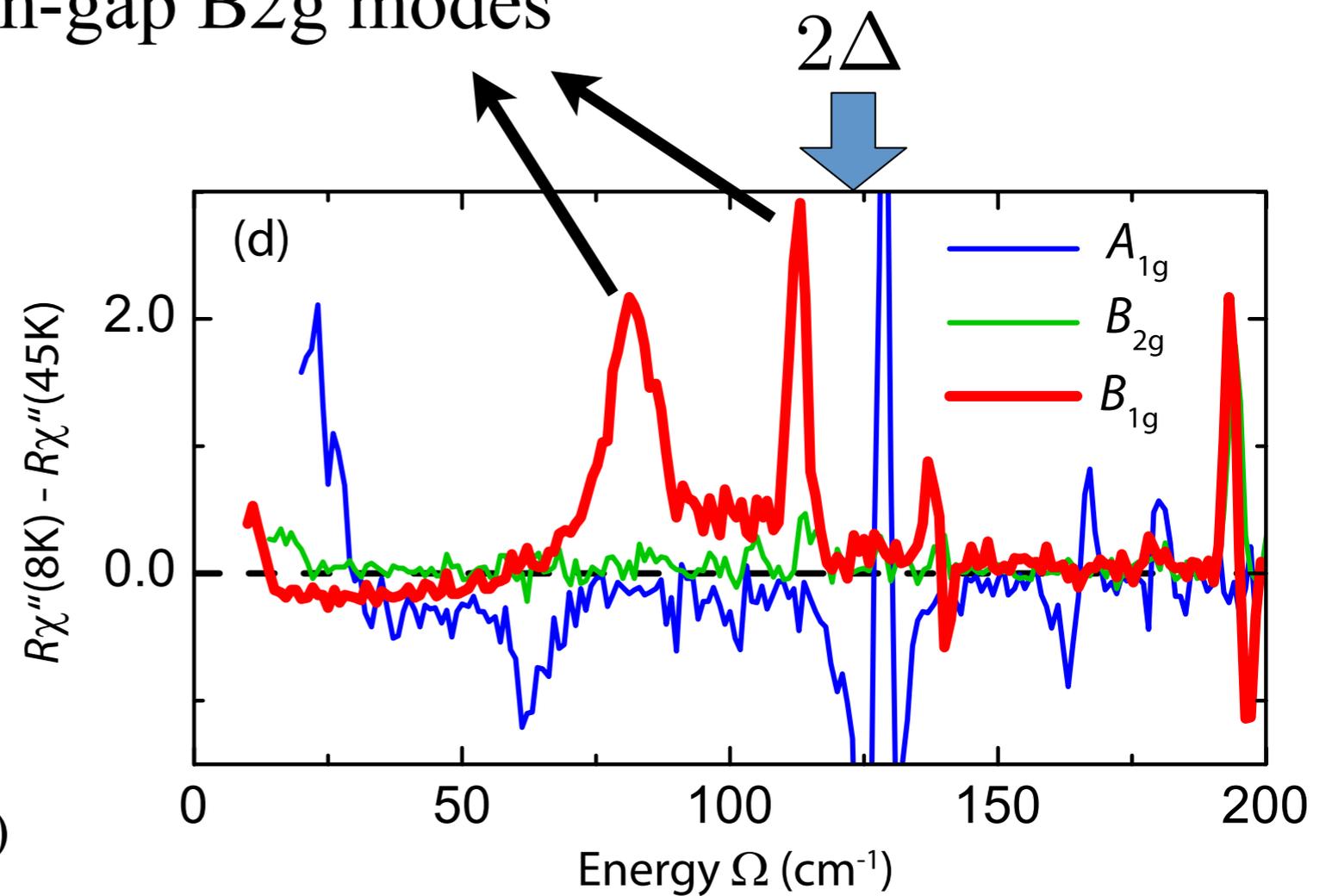
F. Kretzschmar,<sup>1</sup> B. Muschler,<sup>1</sup> T. Böhm,<sup>1</sup> A. Baum,<sup>1</sup> R. Hackl,<sup>1</sup> Hai-Hu Wen,<sup>2</sup>  
V. Tsurkan,<sup>3,4</sup> J. Deisenhofer,<sup>3</sup> and A. Loidl<sup>3</sup>

$\text{Rb}_{0.8}\text{Fe}_{1.6}\text{Se}_2$

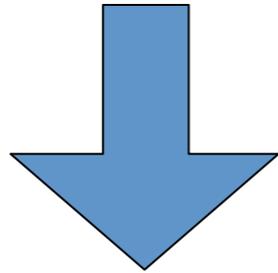


$B_{2g}$  (2Fe/unit cell) =  $B_{1g}$  (1Fe/unit cell)

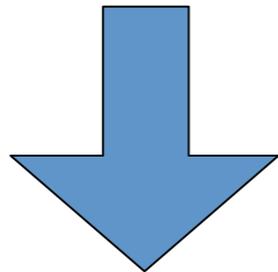
In-gap  $B_{2g}$  modes



Raman: In-gap B<sub>2g</sub> (d-wave) excitation



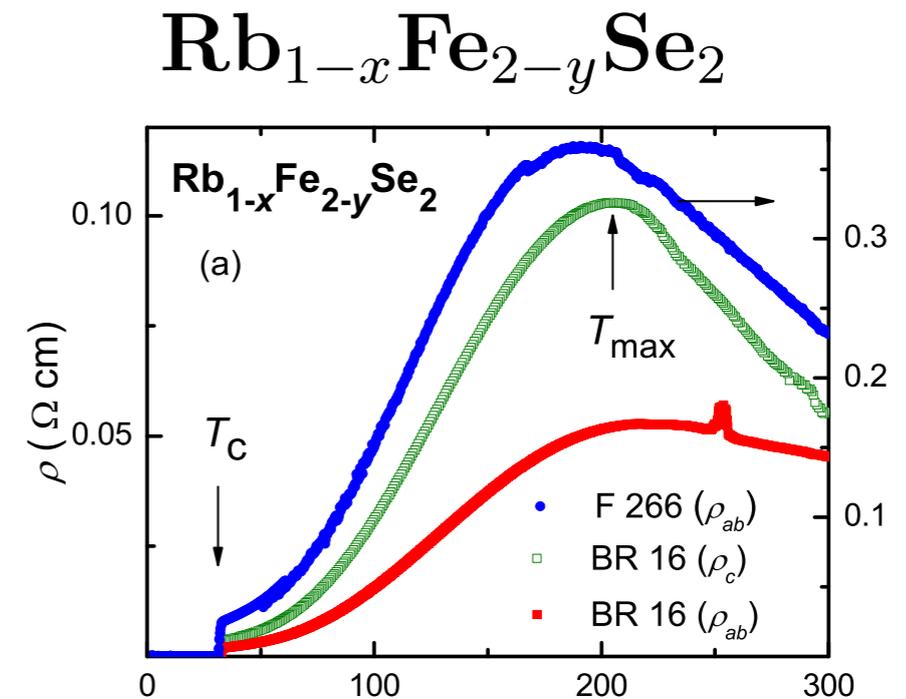
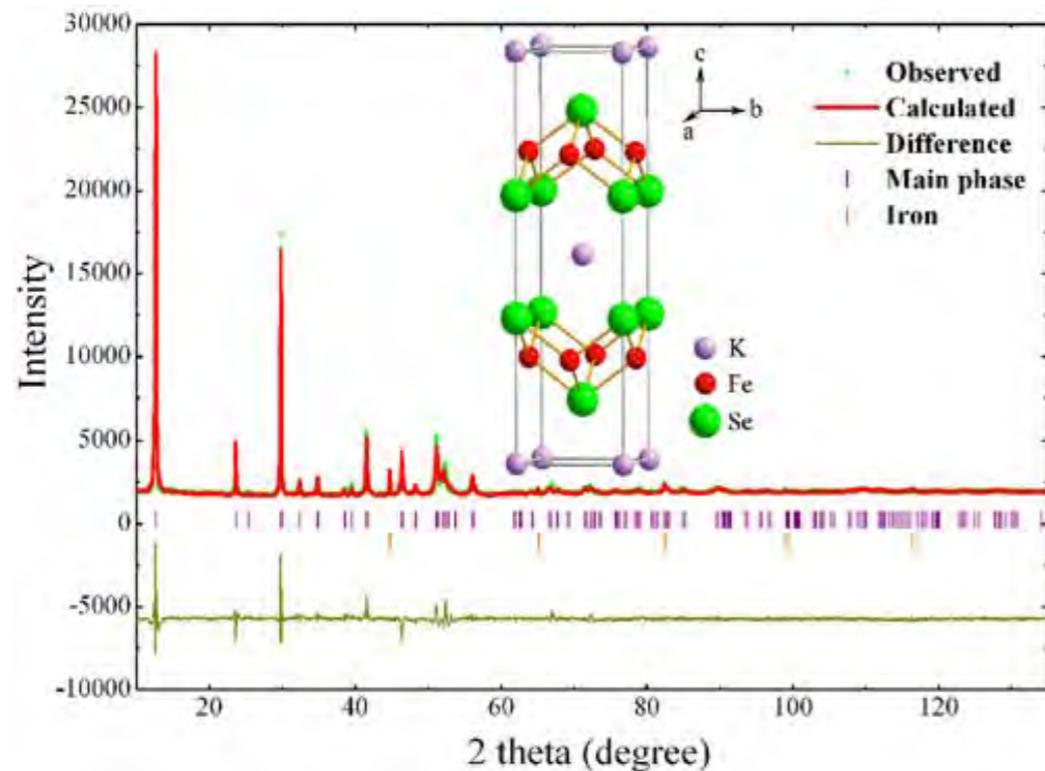
What about ground state ?



Take a closer look at selenides



$$T_c \simeq 30 - 40\text{K}$$



V. Tsurkan, et.al. (2011)

$$T_c \approx 32\text{K}$$

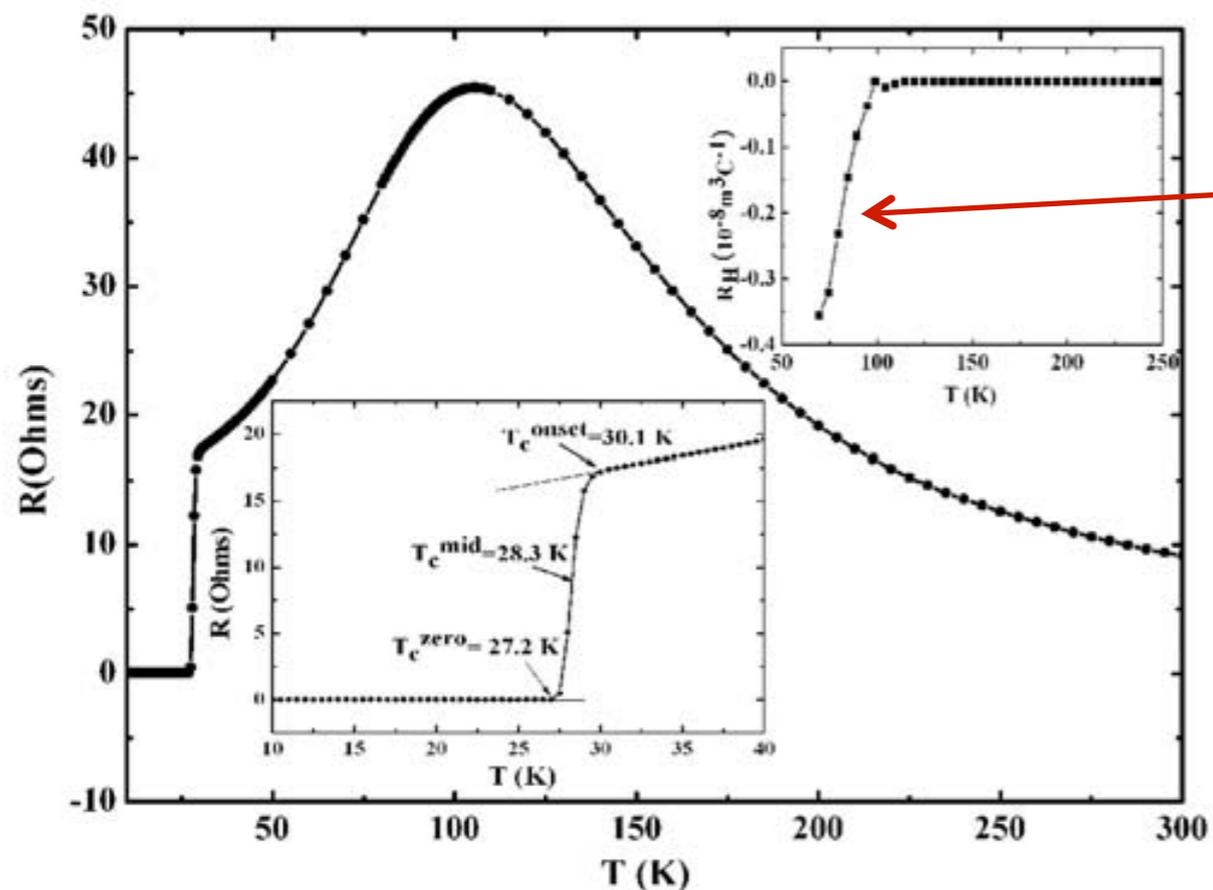
J. Guo et.al.  $\text{K}_x\text{Fe}_2\text{Se}_2$  ( $0 \leq x \leq 1$ )

Isostructural to  $\text{Ba}_x\text{Fe}_2\text{As}_2$  (122)

Intercalation of alkali metal A in between FeSe layers

# "Strongly electron doped" systems, $A_x\text{Fe}_{2-y}\text{Se}_2$ $A = \text{K}, \text{Rb}, \text{Cs}$

Alkali metal A donates electrons to FeSe layers



Negative Hall coefficient

$$R_H < 0$$



electrons

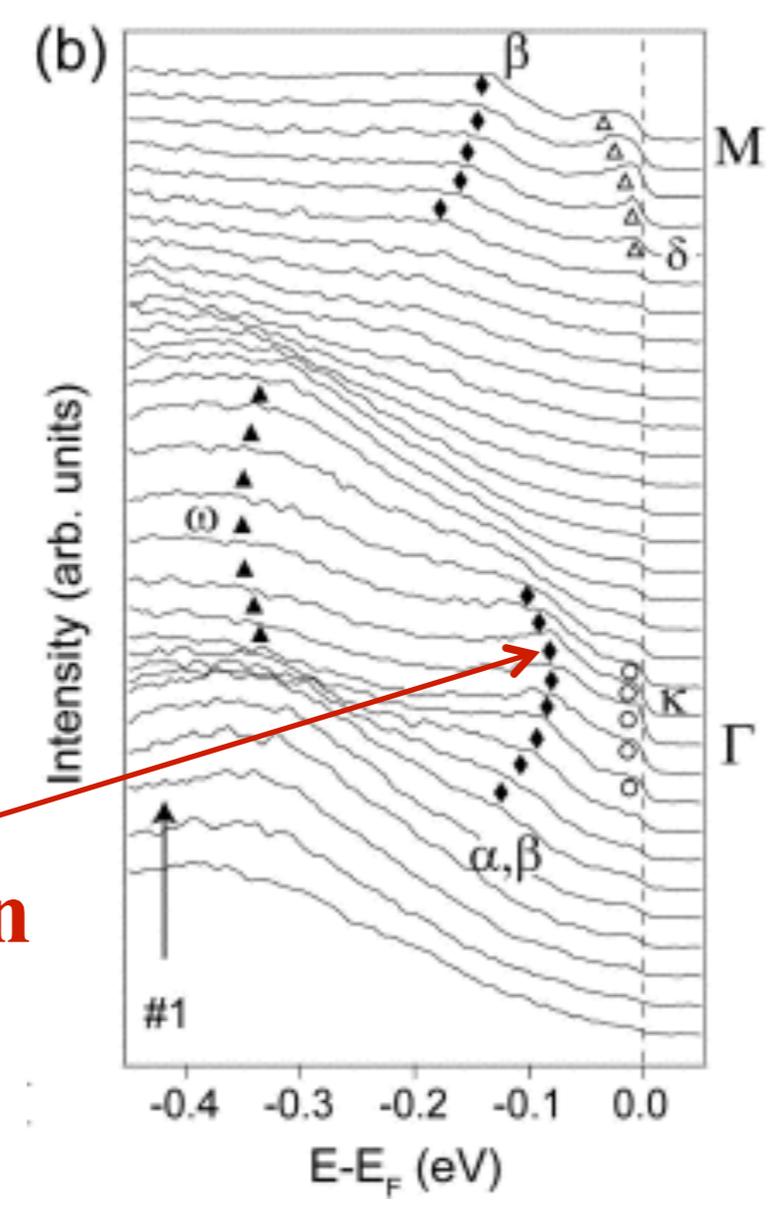
J. Guo et.al.  $\text{K}_x\text{Fe}_2\text{Se}_2$  ( $0 \leq x \leq 1$ )

“Strongly electron doped” systems,  $A_x\text{Fe}_{2-y}\text{Se}_2$   $A = \text{K}, \text{Rb}, \text{Cs}$

$T_c \sim 40\text{K}$

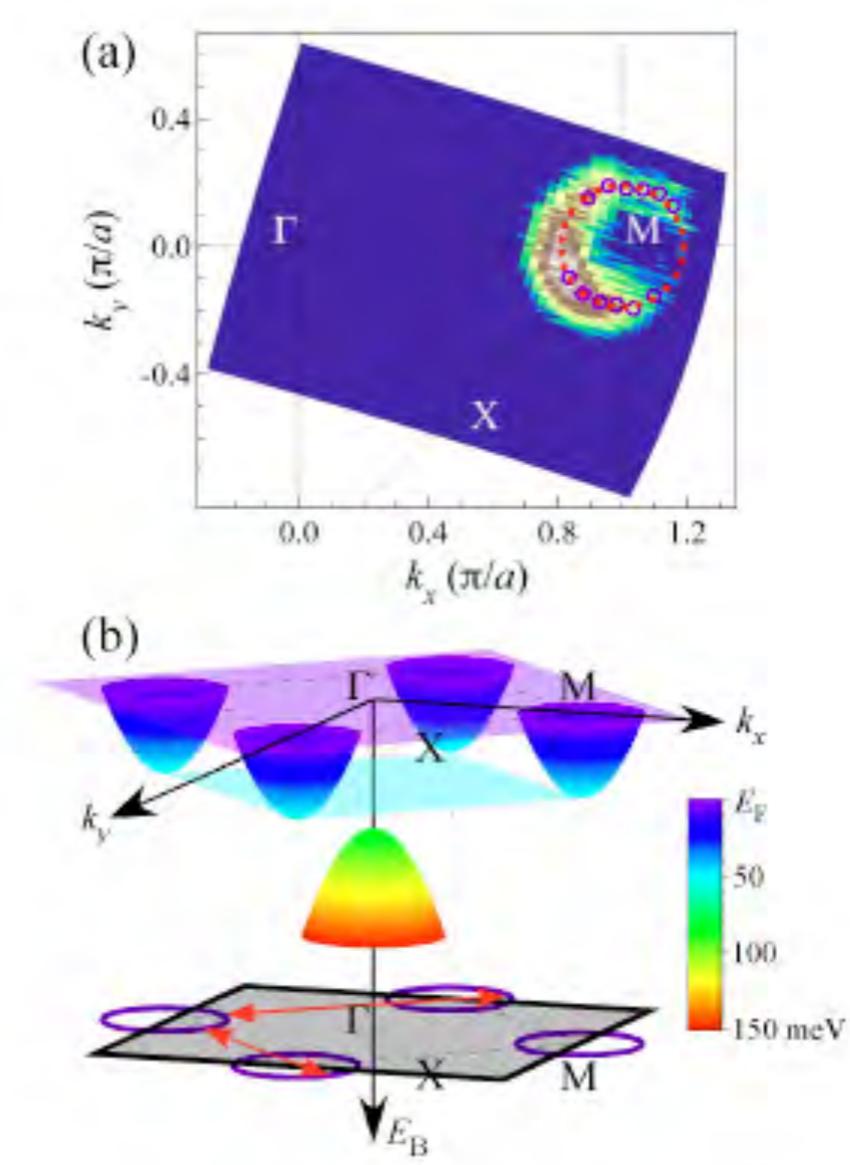
Only electron FSs are present

$\text{KFe}_2\text{Se}_2$



hole dispersion

Y. Zhang et al



Hong Ding et al

What is the pairing *symmetry* ?

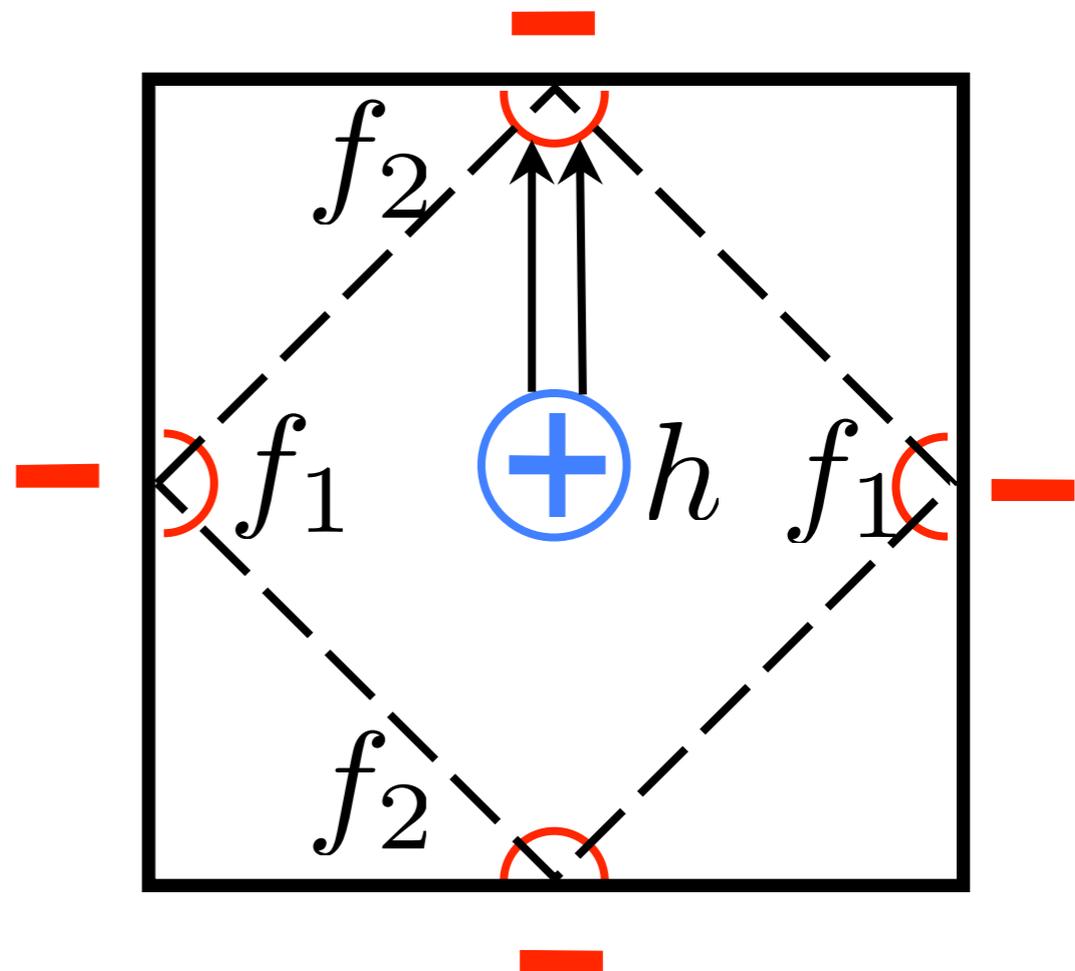
# Pairing: order parameter *symmetry* ?

Look at pnictides with both electron and *hole* pockets

$$\begin{cases} \Delta_h + u\Delta_e \log \frac{\Lambda}{T_c} = 0 \\ \Delta_e + u\Delta_h \log \frac{\Lambda}{T_c} = 0 \end{cases}$$

$$u > 0 \longrightarrow s^\pm$$

$\mathcal{S} \longrightarrow$  Invariant under crystal symmetry

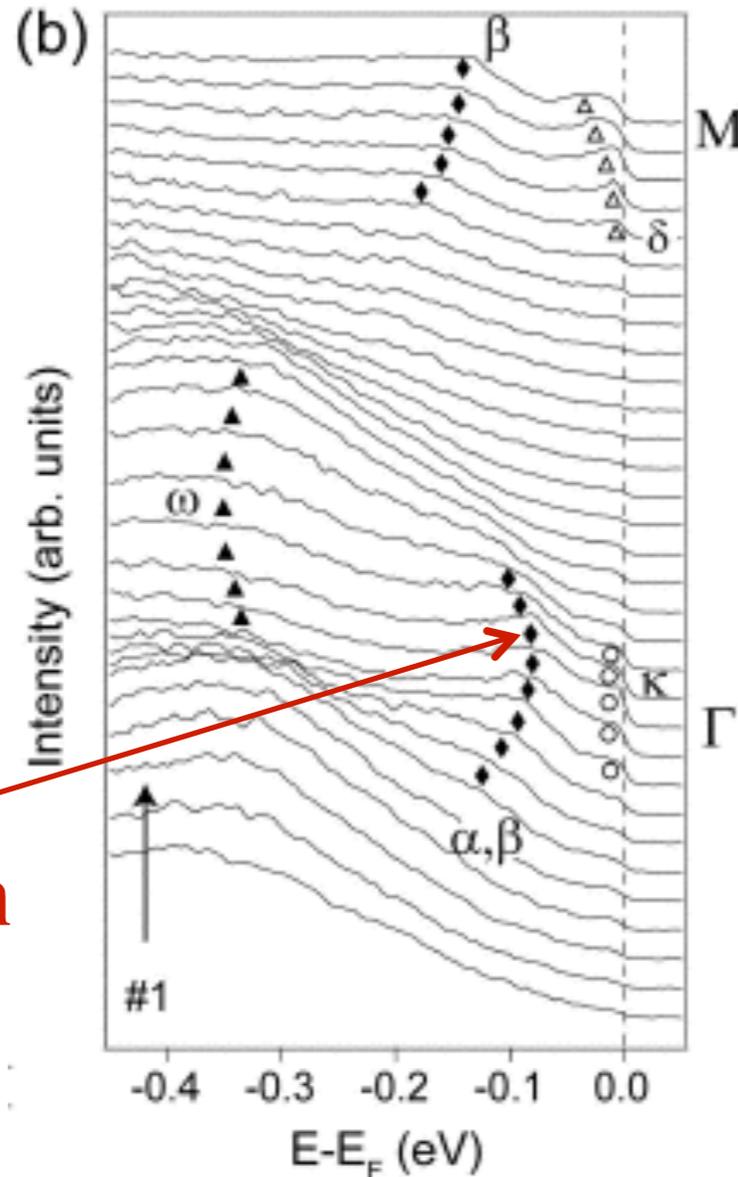


# “Strongly electron doped” systems, $A_x\text{Fe}_{2-y}\text{Se}_2$ $A = \text{K, Rb, Cs}$

$T_c \sim 40\text{K}$

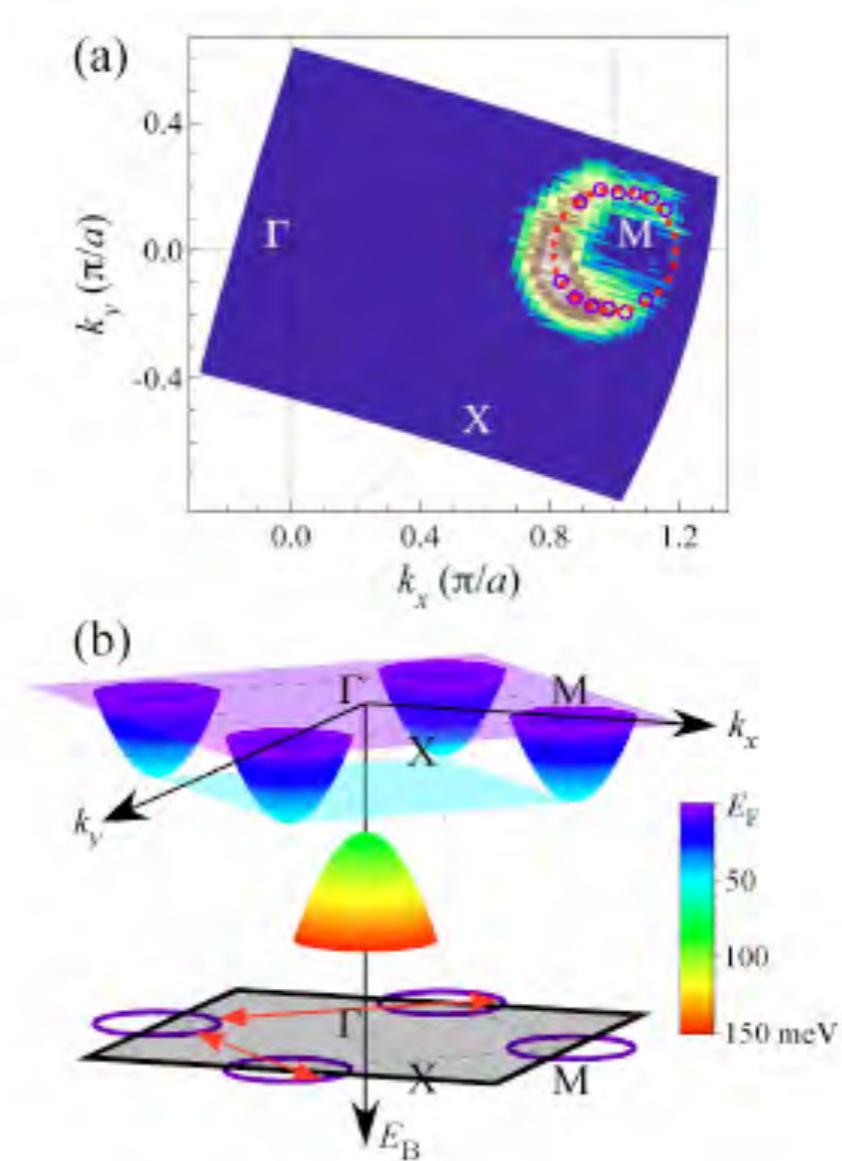
Only electron FSs are present

$\text{KFe}_2\text{Se}_2$



hole dispersion

Y. Zhang et al



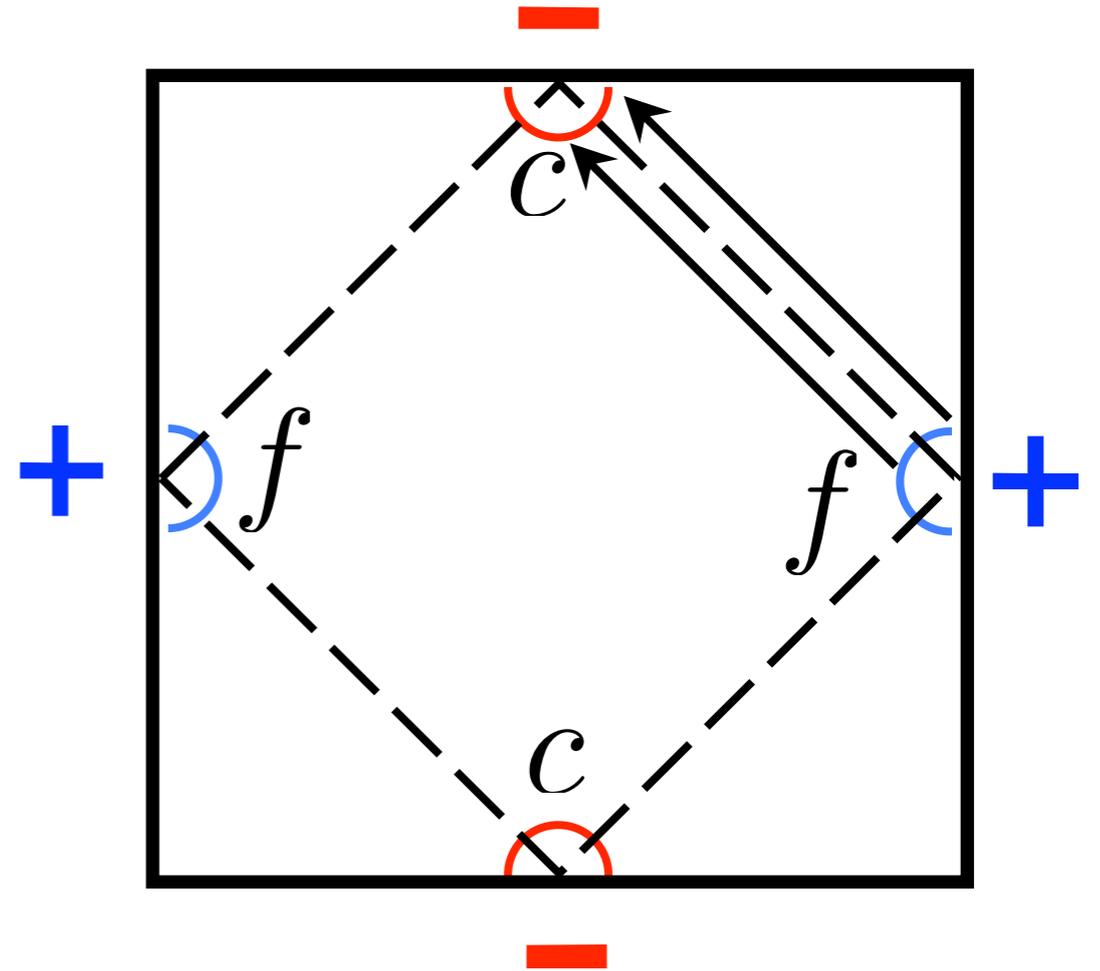
Hong Ding et al

Hole pockets are gapped – the driving force for s-wave SC is gone

Can this still work for  $AFe_2Se_2$  ?

Nodeless  $d$ ?

Different from pnictides  
with hole pockets!

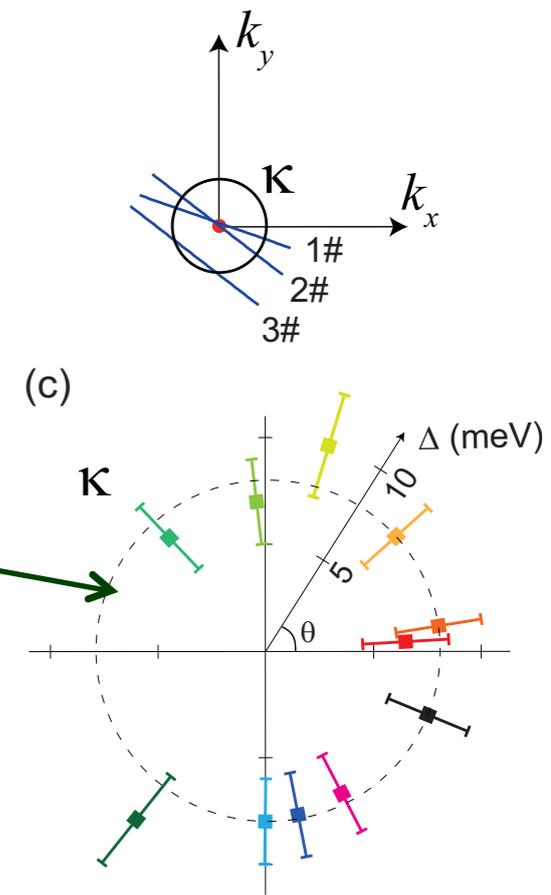
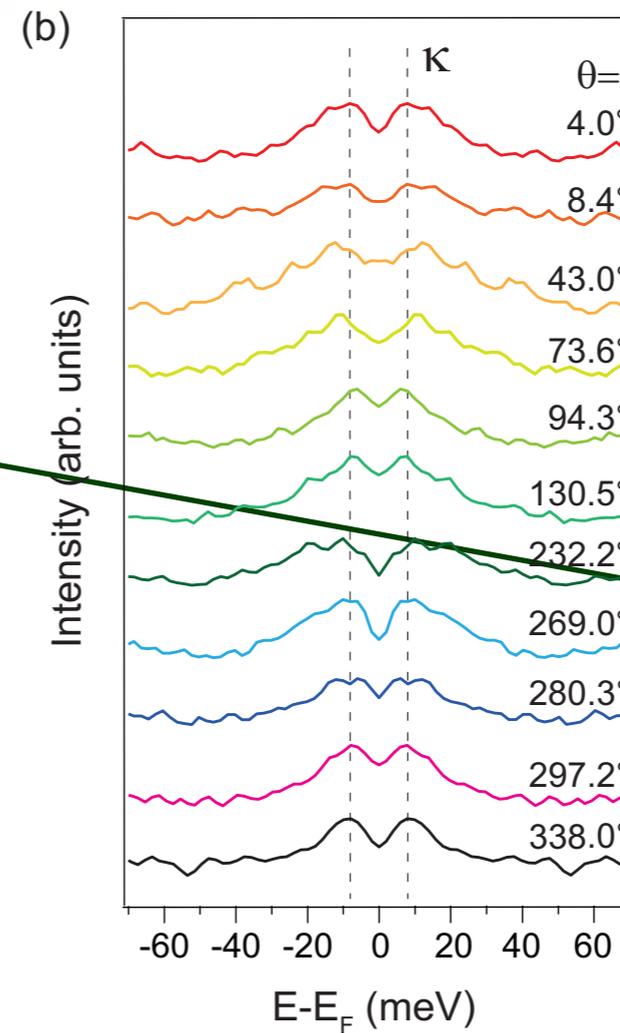
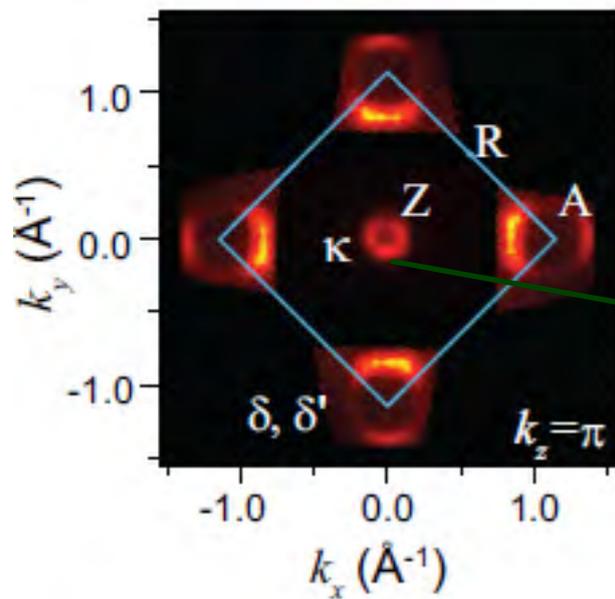
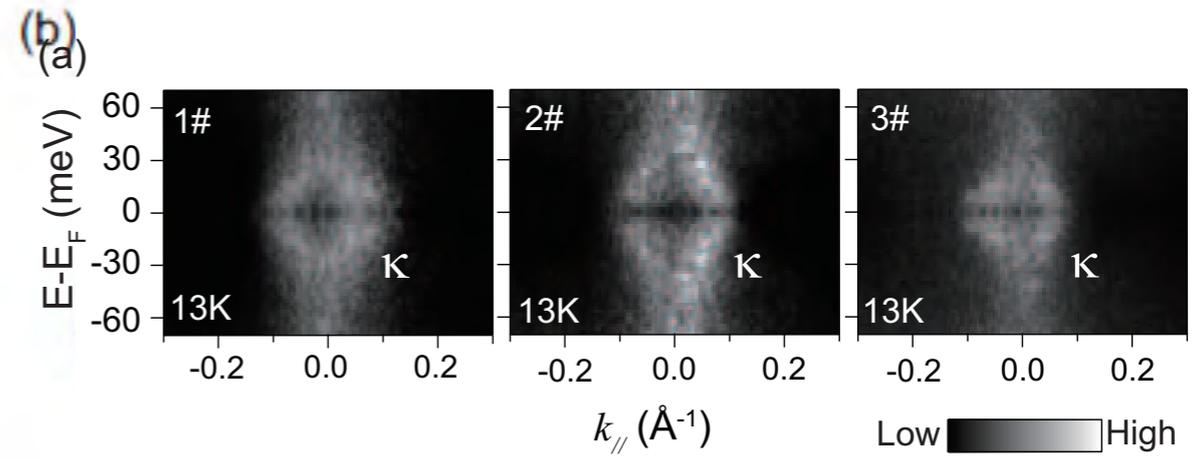
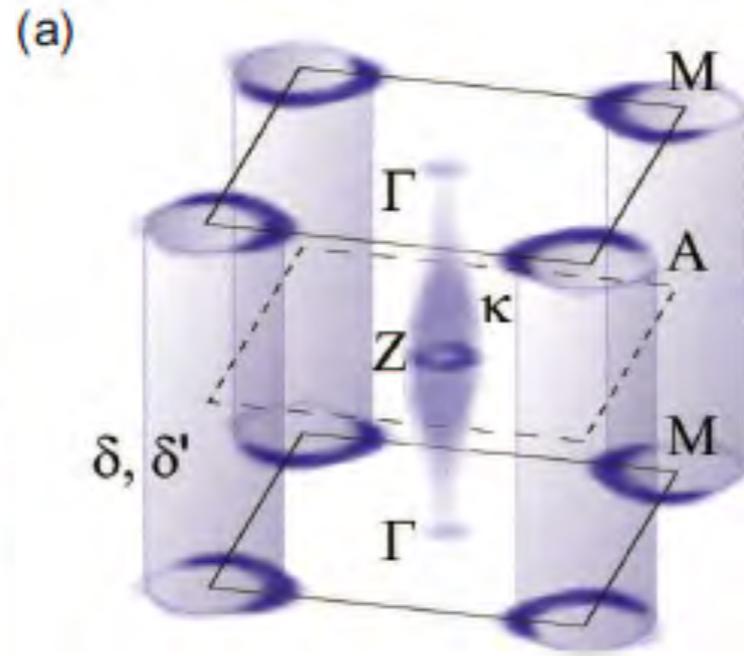
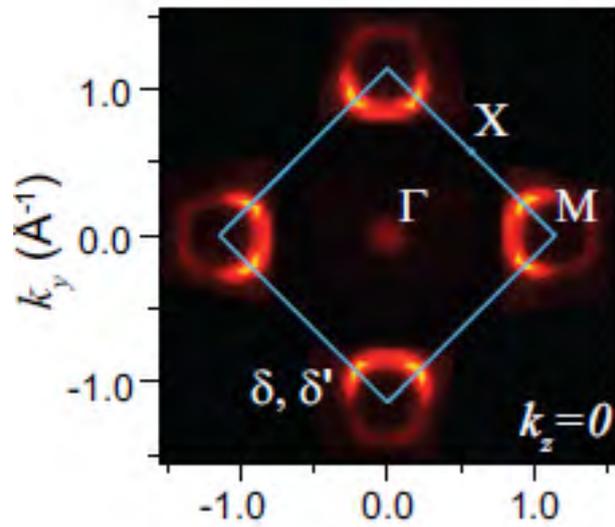


Experiment: ?

# Recent ARPES

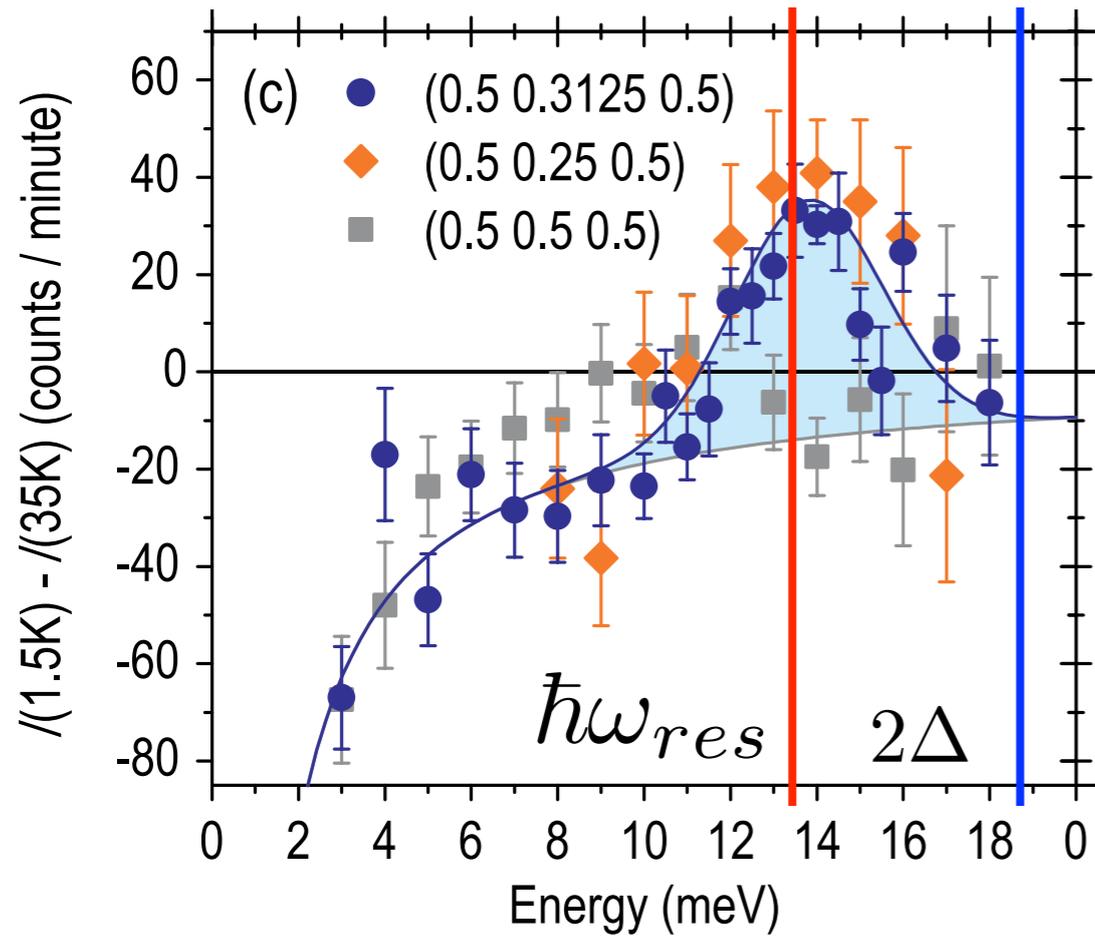
## Feng's group

Phys. Rev. B 85, 220504(R) (2012)



No nodal gap,  
inconsistent with d-wave

# What about regular s-wave?

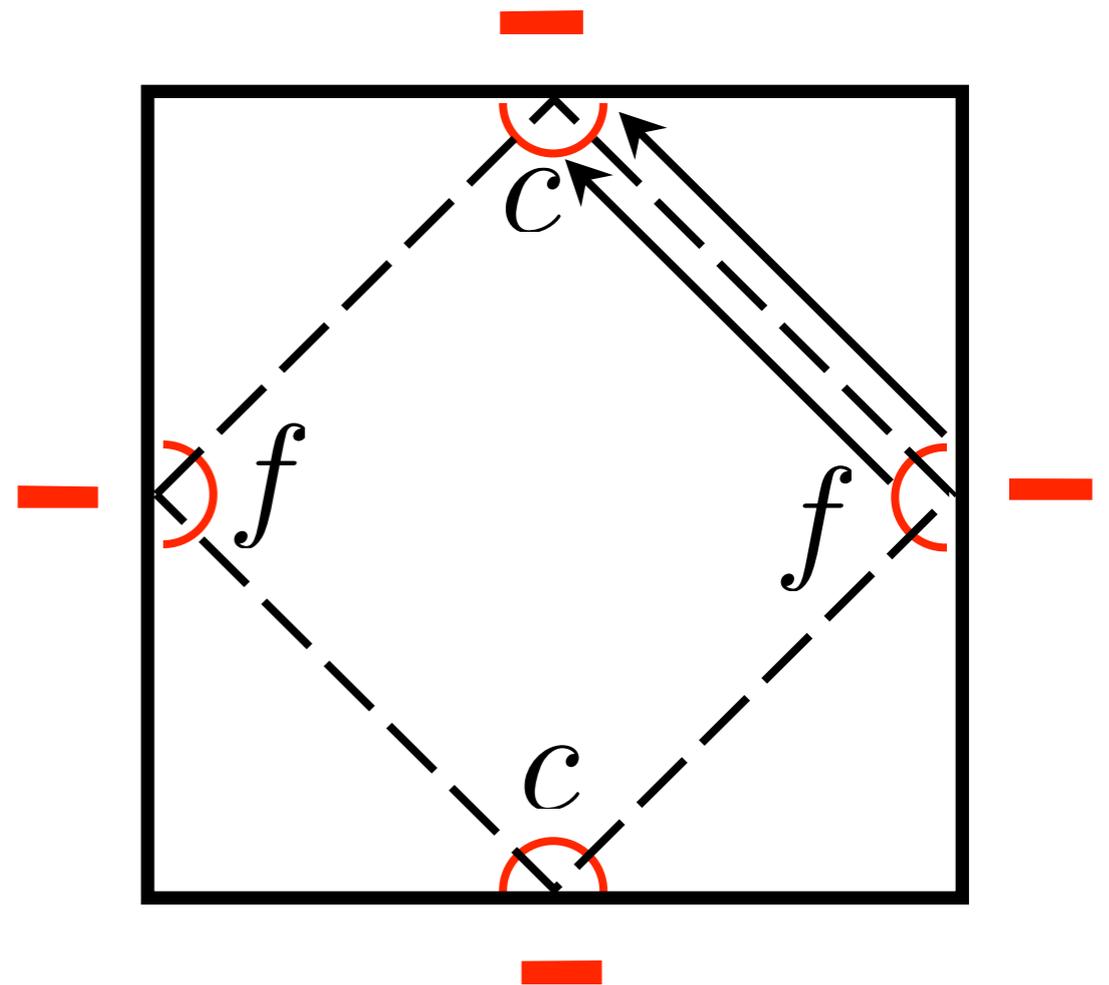


**Spin resonance  
below  $T_c$**

Inosov et al



Requires sign changing order parameter

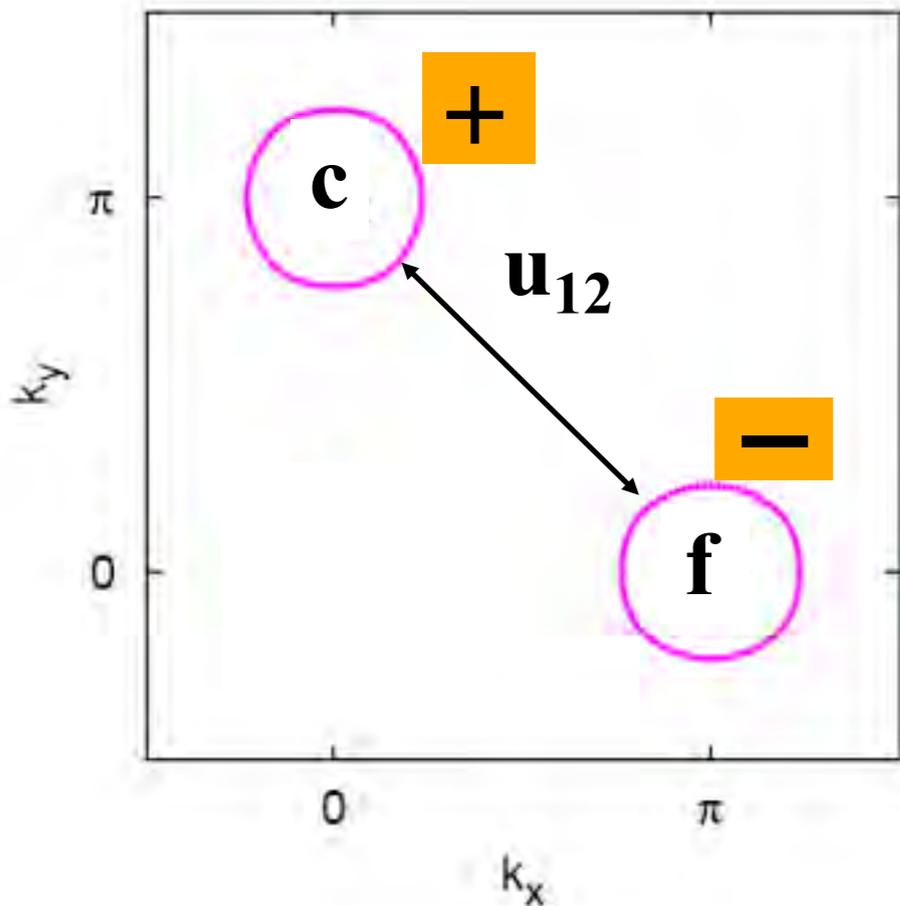


$$\frac{\hbar\omega_{res}}{2\Delta} \approx 0.7$$

**There is a third possibility**  
**“another s<sup>+</sup>-”**

**Consistent with both ARPES and neutrons**

## Let's go back to d-wave reasoning



Suppose FSs are circles  
(must be identical circles)

d-wave:  $\Delta_c = -\Delta_f$

$$\Delta_c = \langle c_{\uparrow}^{\dagger} c_{\downarrow}^{\dagger} \rangle, \quad \Delta_f = \langle f_{\uparrow}^{\dagger} f_{\downarrow}^{\dagger} \rangle$$

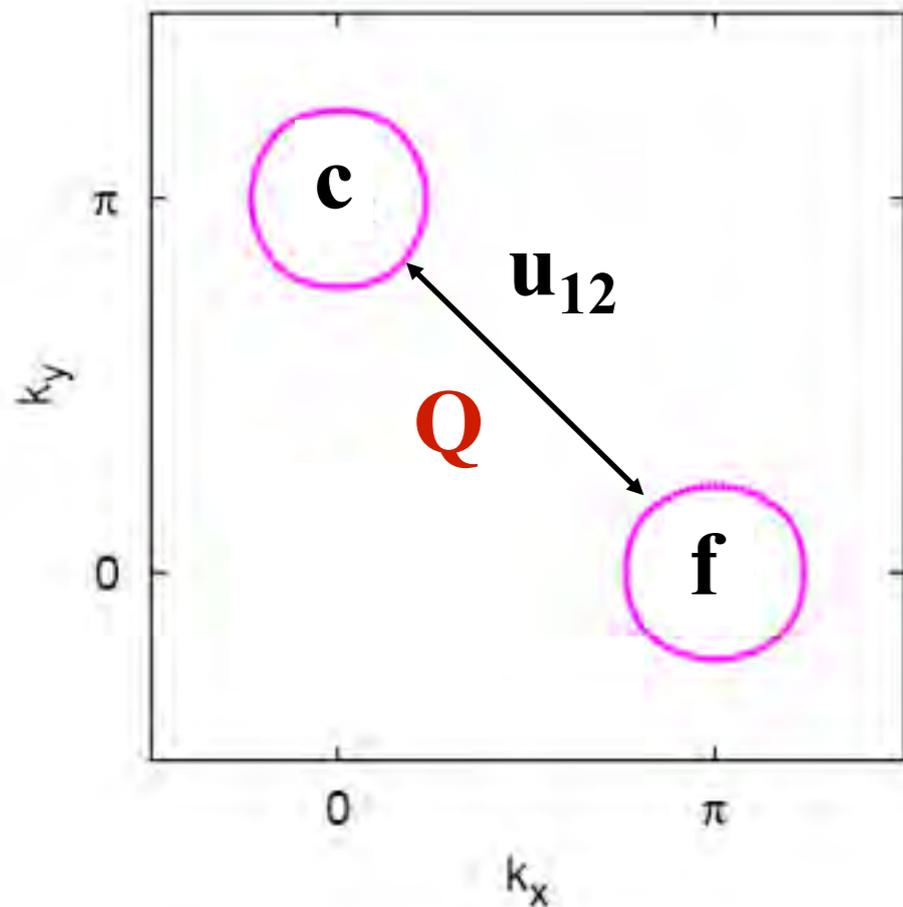
$$T_c : 1 = u_{12} \chi_{pp}(0)$$

$$\chi_{pp}(0) = \text{diagram}$$

$$= \iint_T \frac{d\omega d\varepsilon_k}{\omega^2 + \varepsilon_k^2} = \log \frac{E_F}{T}$$

**Cooper logarithm**

And what if we consider inter-pocket pairing ?



s-wave

Singlet

$$\langle c_{\uparrow}^{\dagger} f_{\downarrow}^{\dagger} \rangle = \Delta = \langle f_{\uparrow}^{\dagger} c_{\downarrow}^{\dagger} \rangle$$

$$T_c : 1 = u_{12} \chi_{pp} (Q)$$

$$\chi_{pp} (Q) = \text{diagram} = \int_T \frac{d\omega}{\omega^2} \frac{d\varepsilon_k}{\varepsilon_k^2} = \log \frac{E_F}{T} \quad \text{Cooper logarithm}$$

Exactly the same result as for plus-minus gap

**d-wave and s-wave pairing states are completely degenerate for circular electron pockets**

# Electron Pockets

1) not circular 2) hybridized



d-s degeneracy is lifted



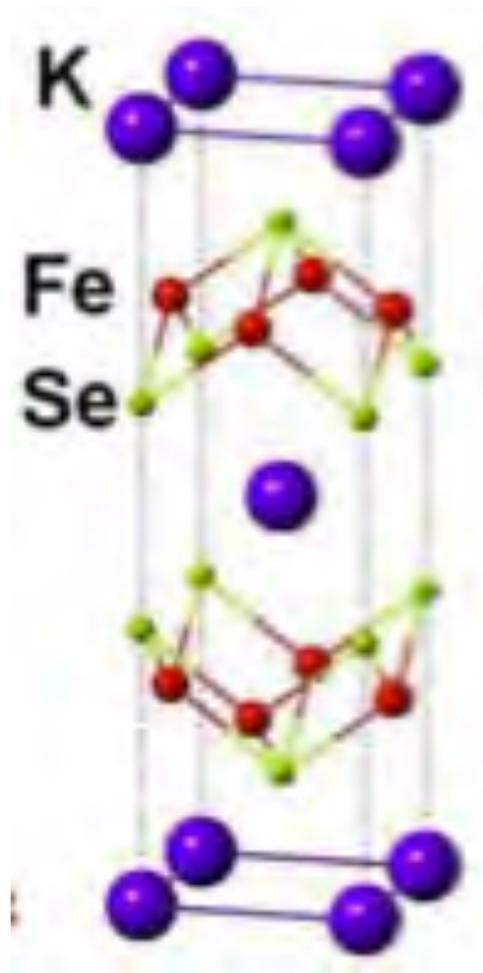
d-s competition



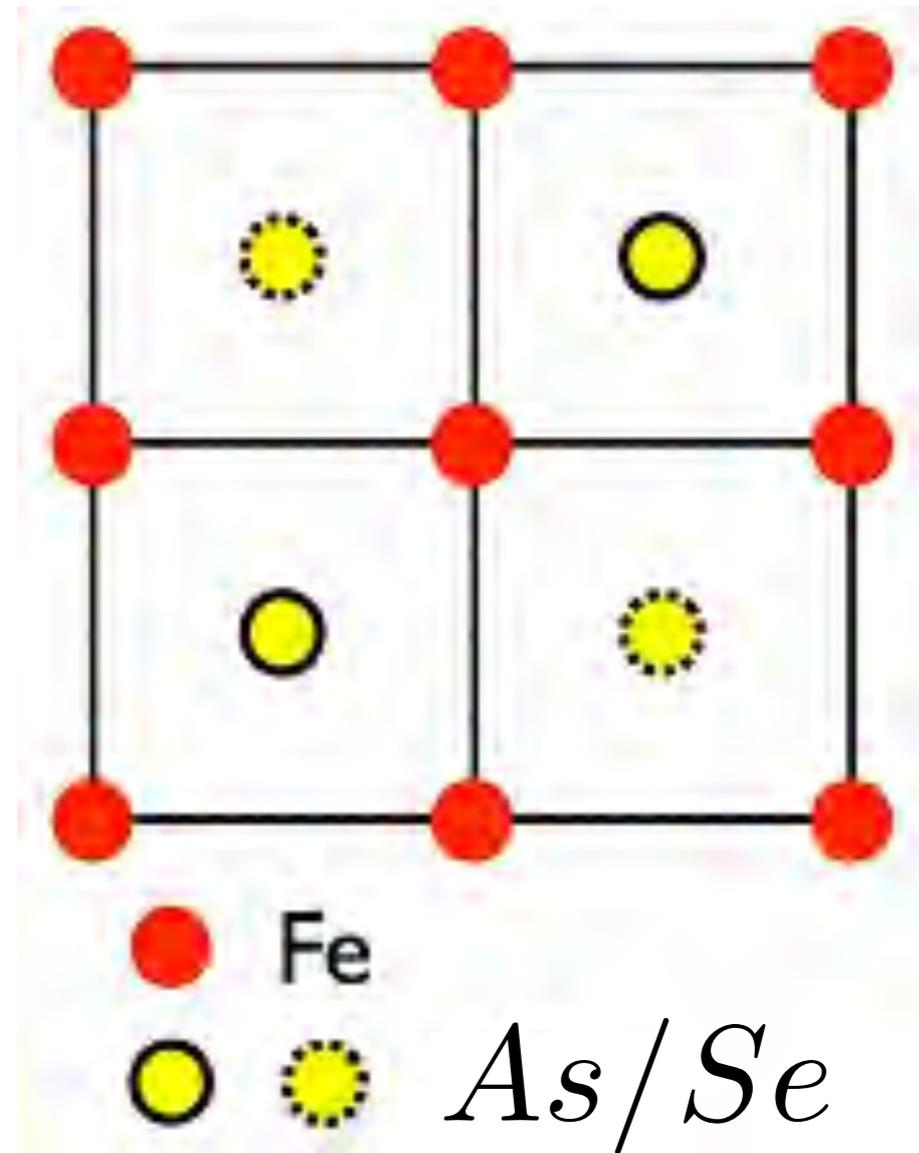
which way ?

# Hybridization

**Folding: pnictogen/chalcogen is above or below Fe plane**



Single FeSe layer

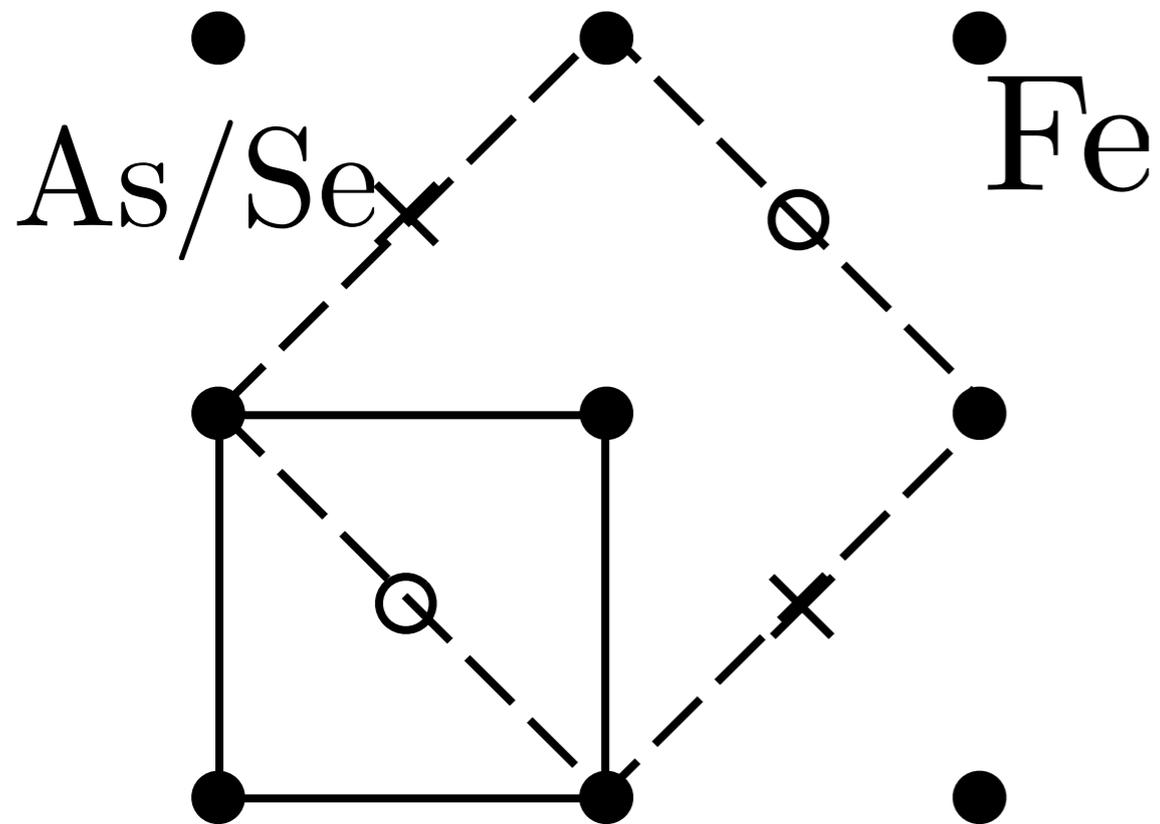


**Two non-equivalent positions of Fe – unit cell has 2 Fe atoms**

# Hybridized pockets

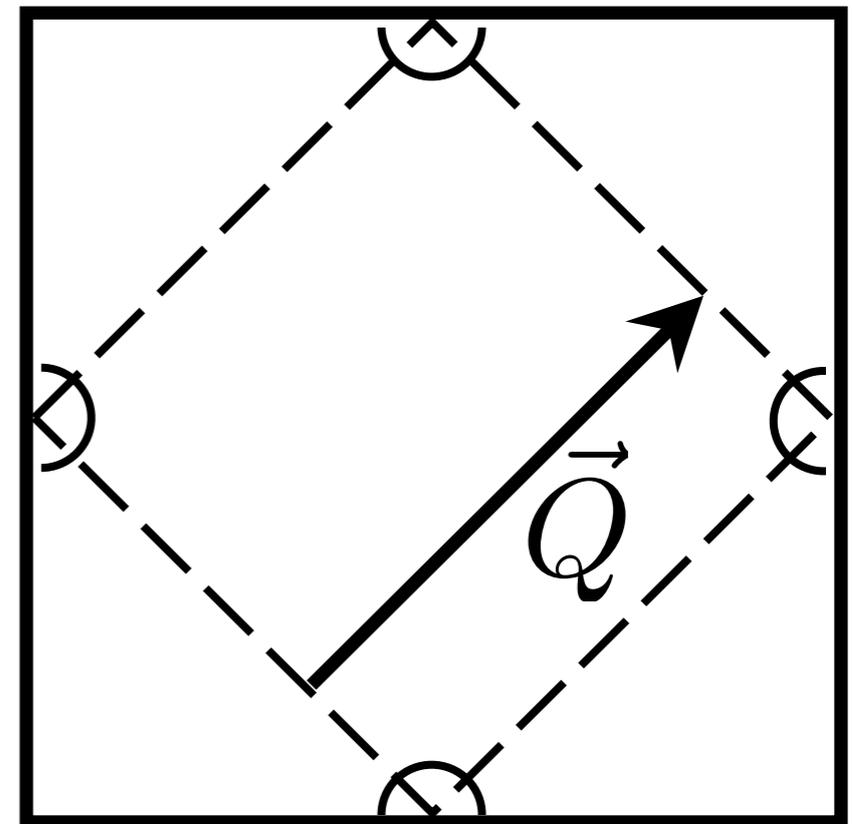
Single layer

Real space



Reciprocal lattice

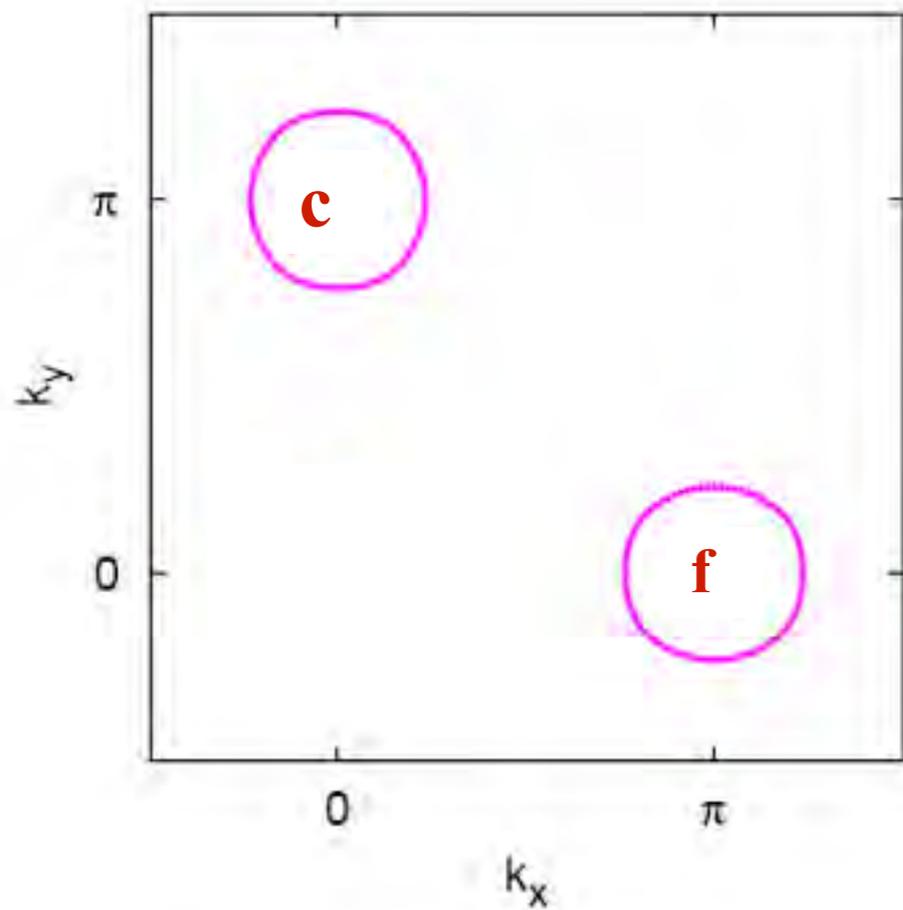
1FeBZ



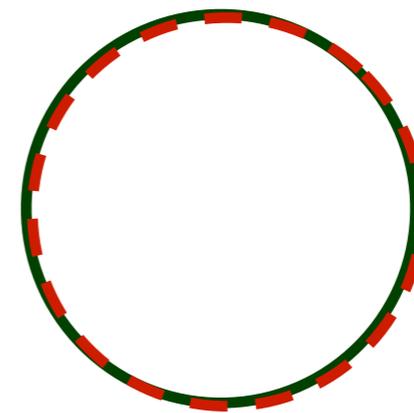
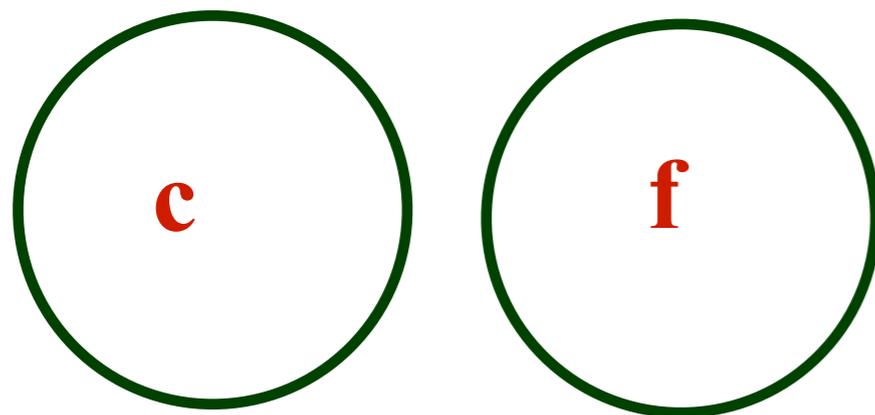
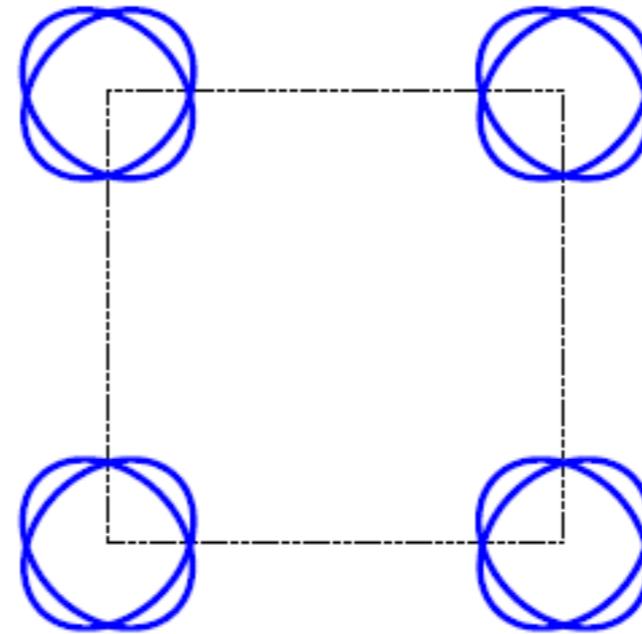
$$\vec{Q} = (\pi, \pi)$$

# A simple picture of folding – a shift of the position of the FSs

**unfolded zone**

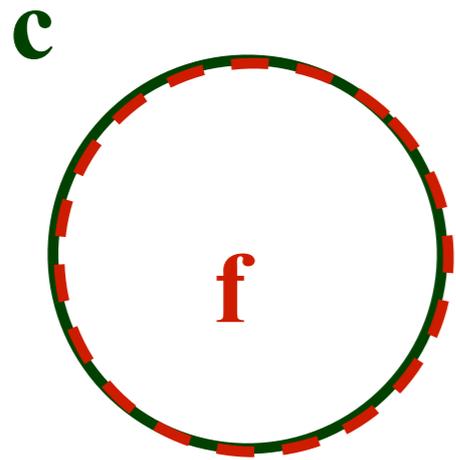


**folded zone**



Inter-pocket and intra-pocket pairing are degenerate

$$H_{hybr} = \lambda(c_k^\dagger f_{k+Q} + f_{k+Q}^\dagger c_k)$$



**no hybridization**

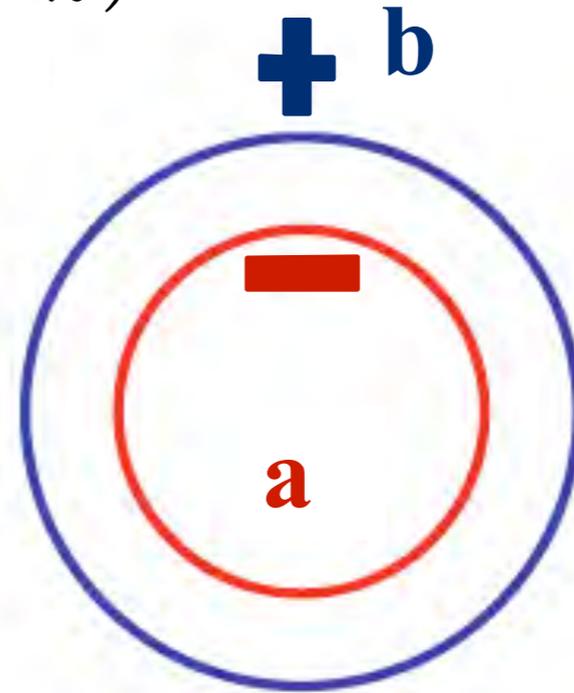
$$\langle c_{\uparrow}^\dagger c_{\downarrow}^\dagger \rangle = \Delta = -\langle f_{\uparrow}^\dagger f_{\downarrow}^\dagger \rangle$$

**d - wave**

$$\langle c_{\uparrow}^\dagger f_{\downarrow}^\dagger \rangle = \Delta = \langle f_{\uparrow}^\dagger c_{\downarrow}^\dagger \rangle$$

**s - wave**

**s-wave is inter-pocket pairing  
in terms of original fermions**



**s<sup>+-</sup> gap**

$$a = (c + f)/\sqrt{2}$$

$$b = (c - f)/\sqrt{2}$$

**hybridization**

$$\langle a_{\uparrow}^+ a_{\downarrow}^+ \rangle = \Delta = -\langle b_{\uparrow}^+ b_{\downarrow}^+ \rangle$$

**s - wave**



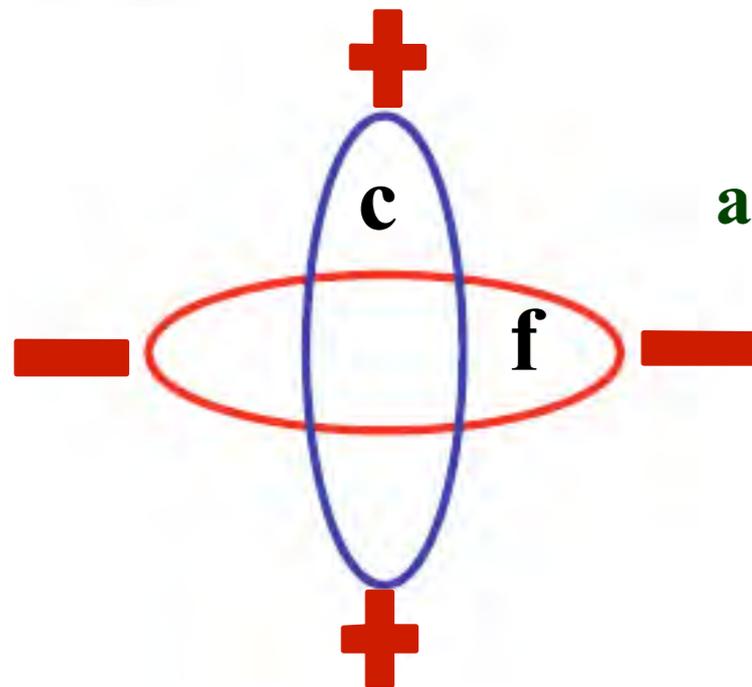
$$\langle a_{\uparrow}^+ b_{\downarrow}^+ \rangle = \Delta = \langle b_{\uparrow}^+ a_{\downarrow}^+ \rangle$$

**d - wave**

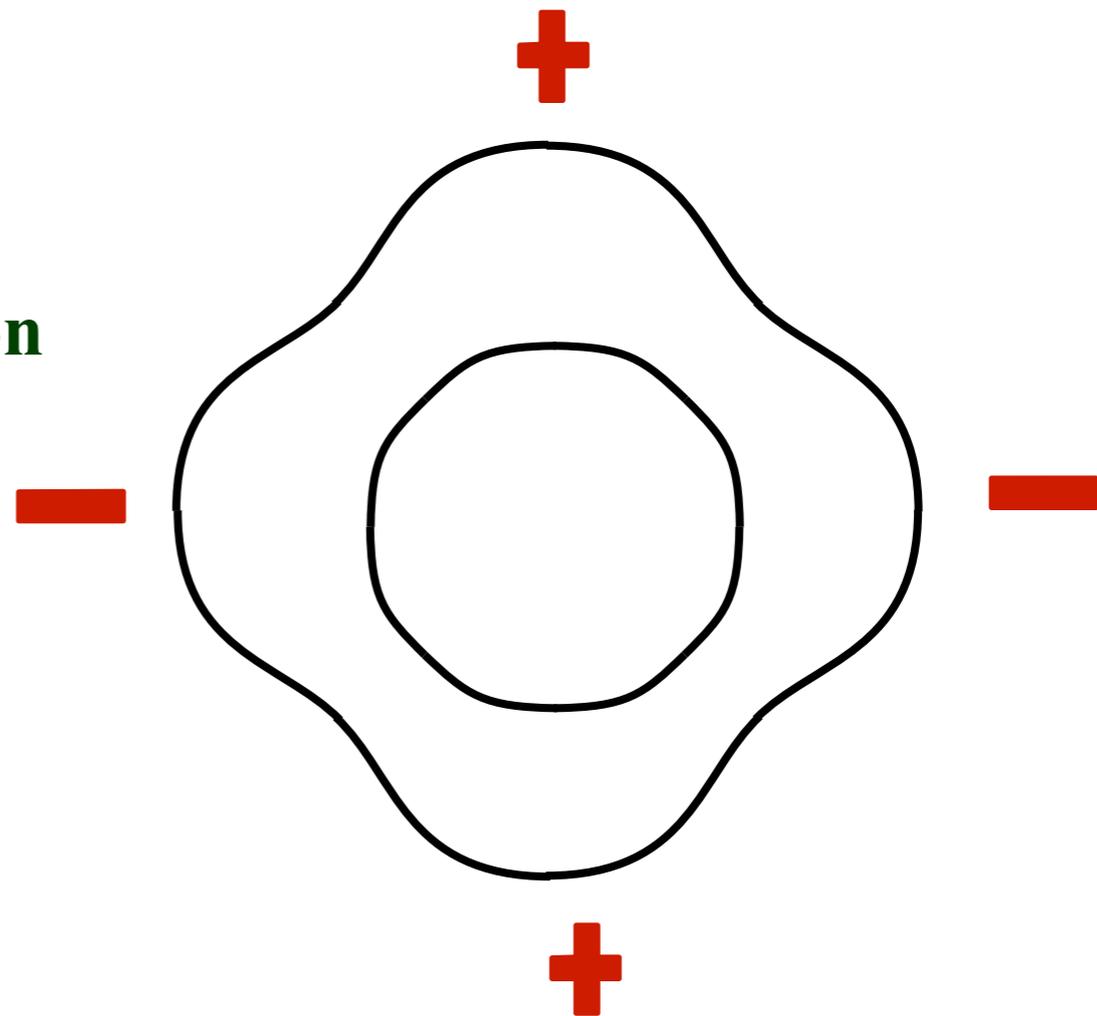
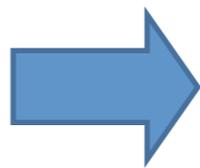
**d-wave is inter-pocket pairing  
in terms of hybridized fermions**

**A situation is somewhat different when electron pockets are ellipses**

**no hybridization**



**add hybridization**



**d-wave is better**

$$\langle c_{\uparrow}^{\dagger} c_{\downarrow}^{\dagger} \rangle = \Delta = -\langle f_{\uparrow}^{\dagger} f_{\downarrow}^{\dagger} \rangle$$

**d - wave**

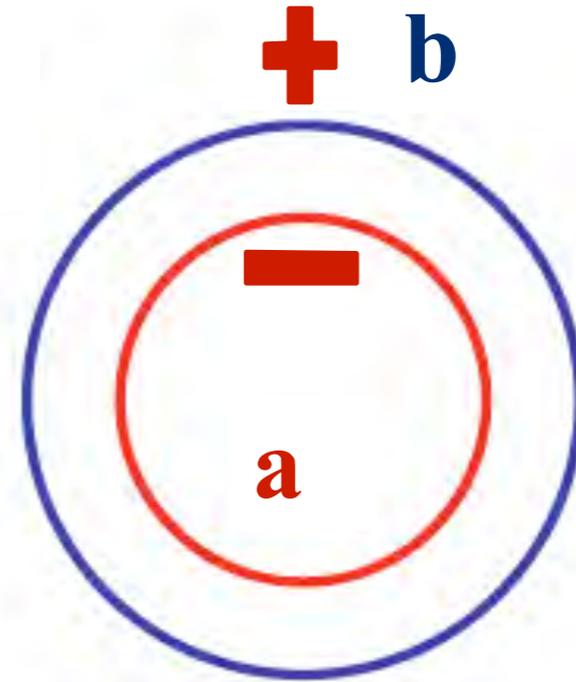
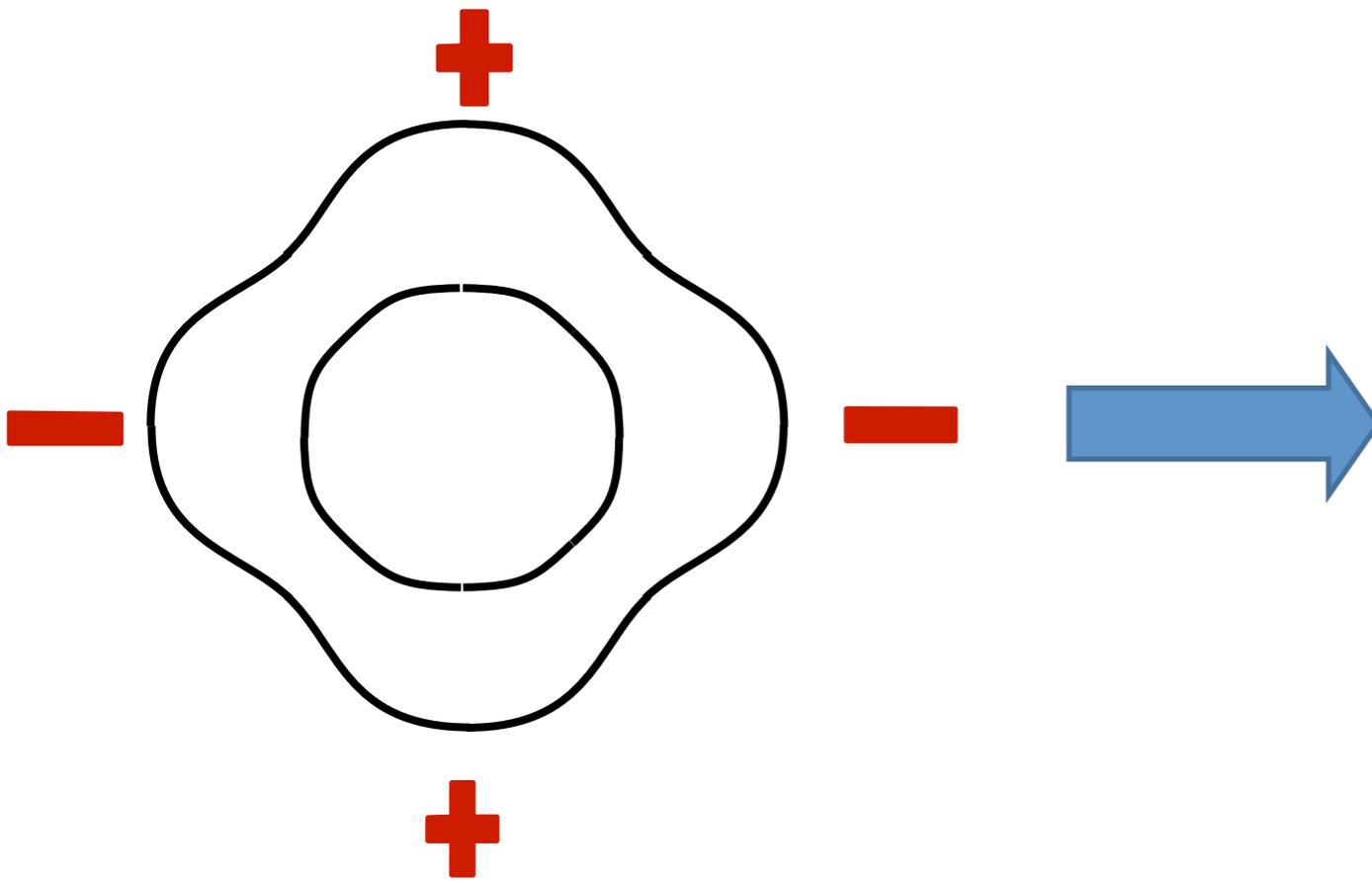
$$\langle c_{\uparrow}^{\dagger} f_{\downarrow}^{\dagger} \rangle = \Delta = \langle f_{\uparrow}^{\dagger} c_{\downarrow}^{\dagger} \rangle$$

**s - wave**

**for small hybridization, d-wave is still better.**

**This is a conventional, d-wave**

Let's increase hybridization further



Eventually s-wave  
wins

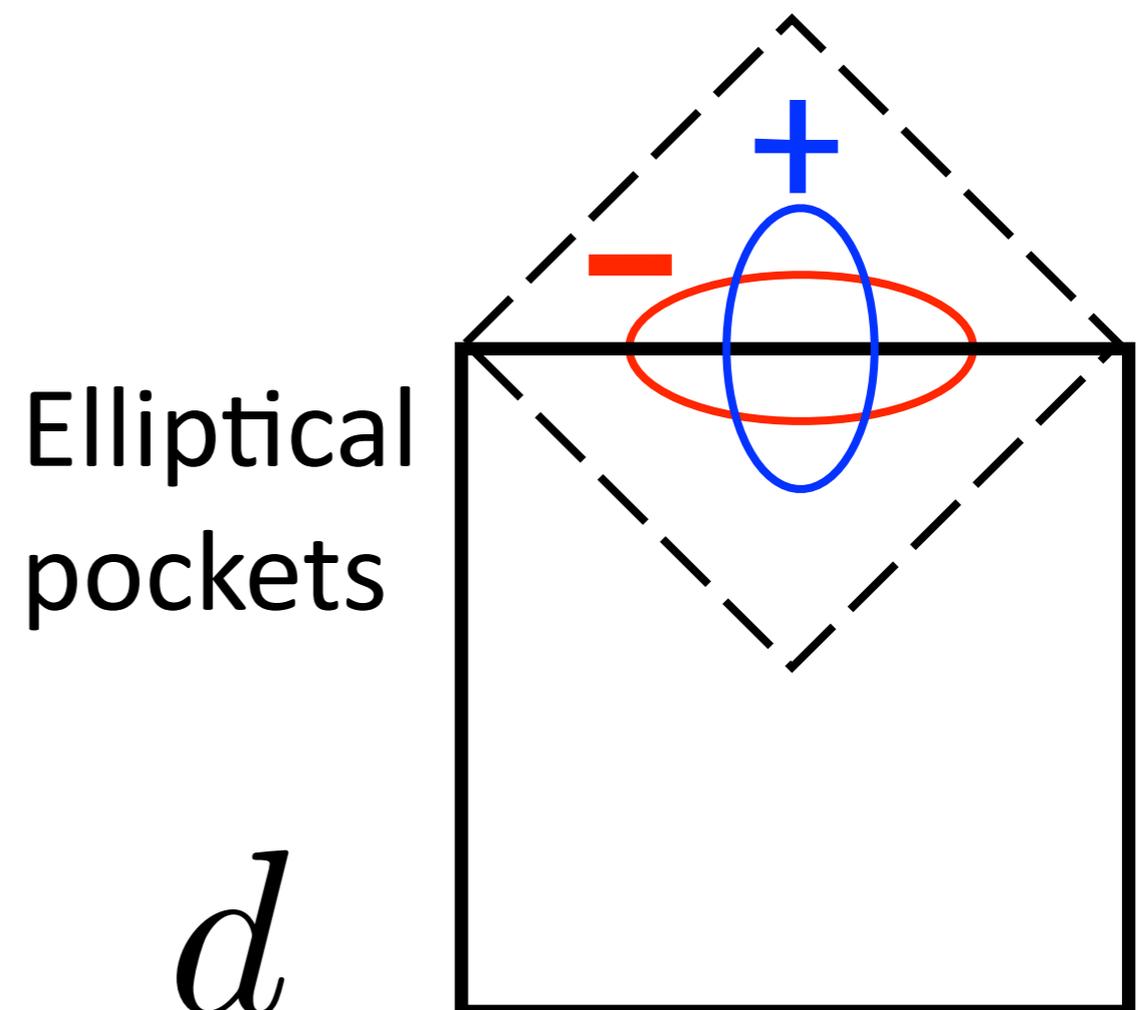
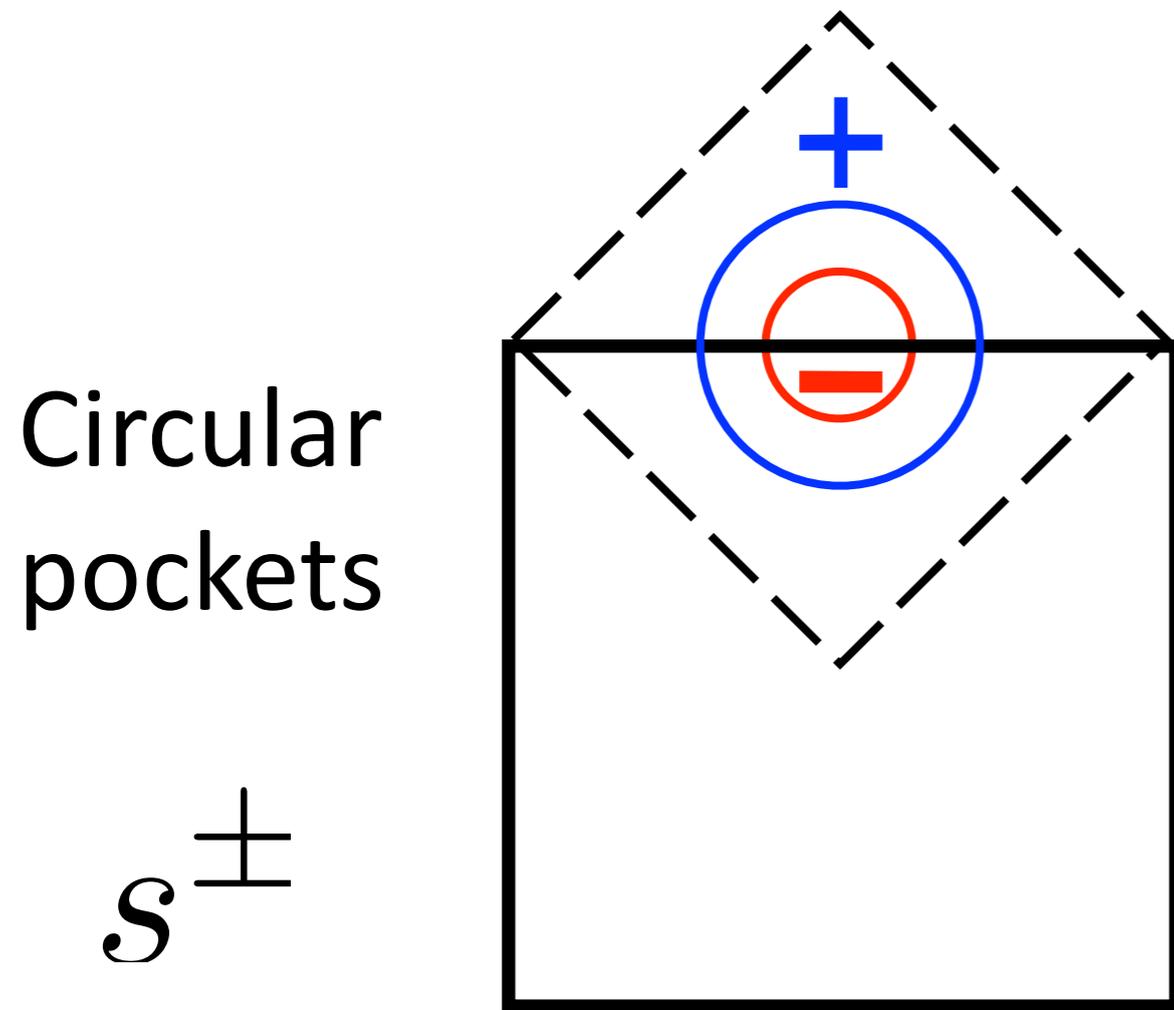
$$\langle a_{\uparrow}^+ a_{\downarrow}^+ \rangle = \Delta = - \langle b_{\uparrow}^+ b_{\downarrow}^+ \rangle$$

s - wave

$$\langle a_{\uparrow}^+ b_{\downarrow}^+ \rangle = \Delta = \langle b_{\uparrow}^+ a_{\downarrow}^+ \rangle$$

d - wave

# The effect of hybridization for different pocket configuration



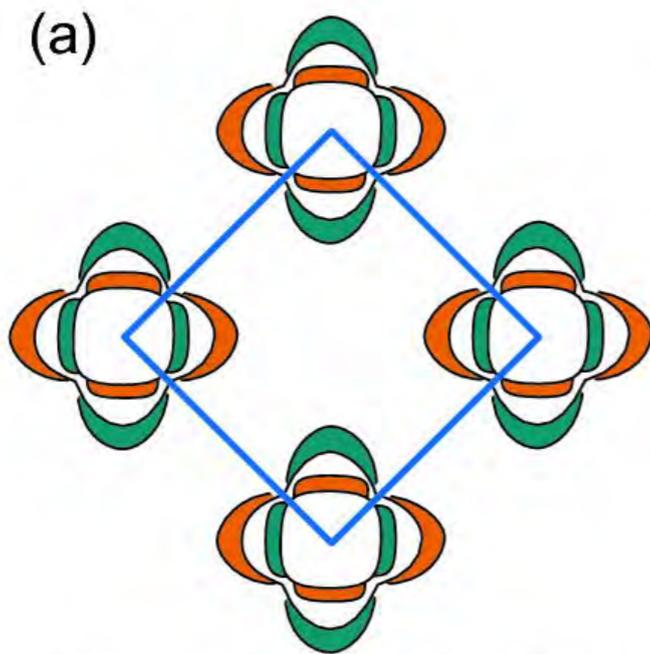
I. Mazin, PRB (2011)

Competition: hybridization vs. ellipticity

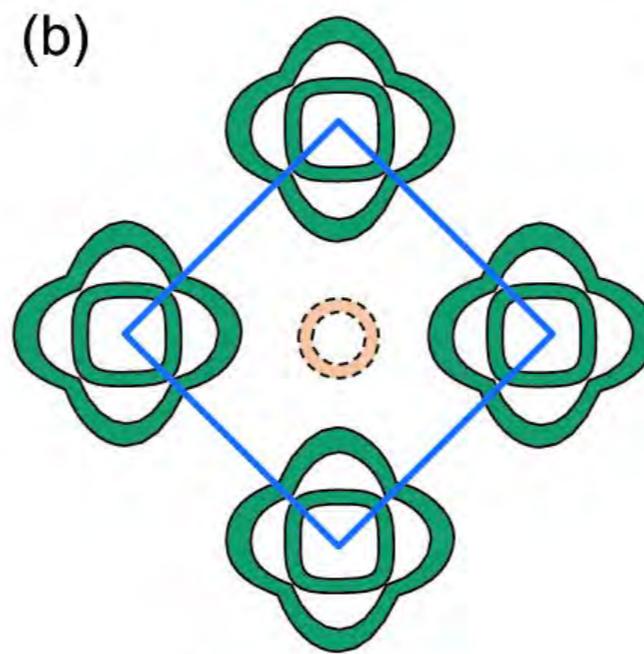
# “Gap symmetry and structure of Fe-based superconductors”

P. J. Hirschfeld, M. M. Korshunov, I. I. Mazin

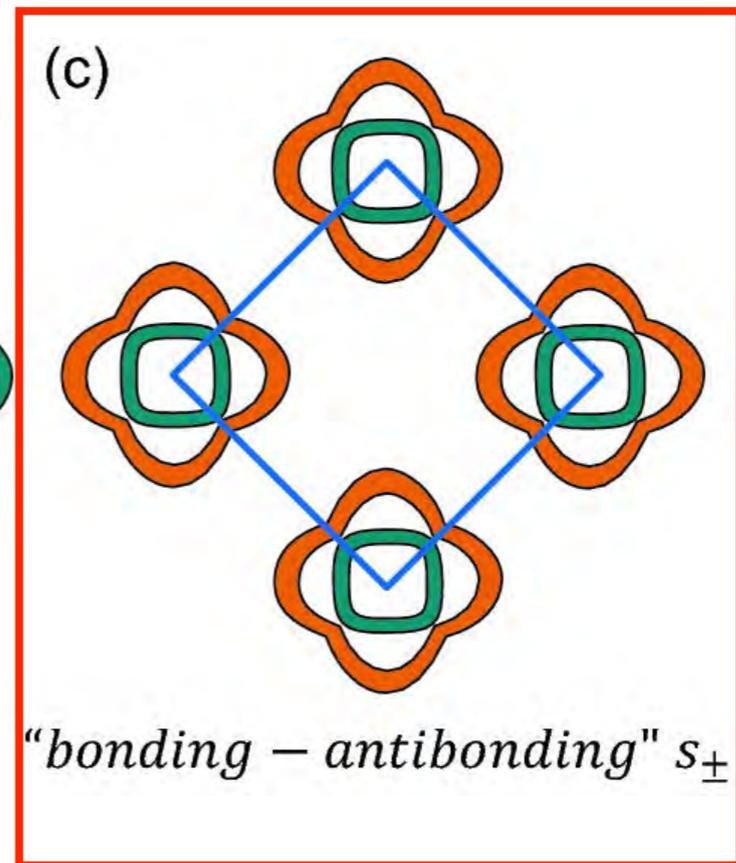
Rep. Prog. Phys. **74**, 124508 (2011)



“quasi – nodeless”  $d$

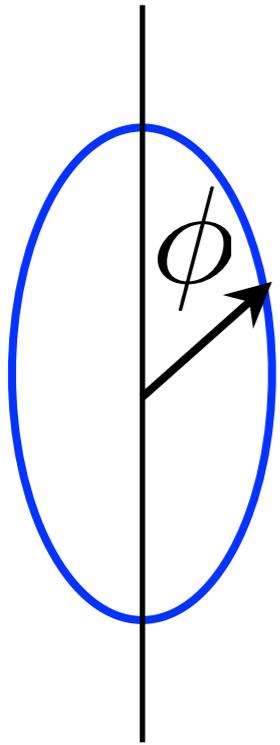


“incipient”  $s_{\pm}$



“bonding – antibonding”  $s_{\pm}$

# Hybridization vs. ellipticity (cont)



$$v_F(\phi) = v_F(1 + a \cos 2\phi)$$

$$k_F(\phi) = k_F(1 + \boxed{b} \cos 2\phi)$$

Anisotropy parameter

Small

$$\frac{\lambda}{v_F k_F}$$

Dimensionless hybridization

$$\kappa = \frac{\text{hybridization}}{\text{ellipticity}}$$

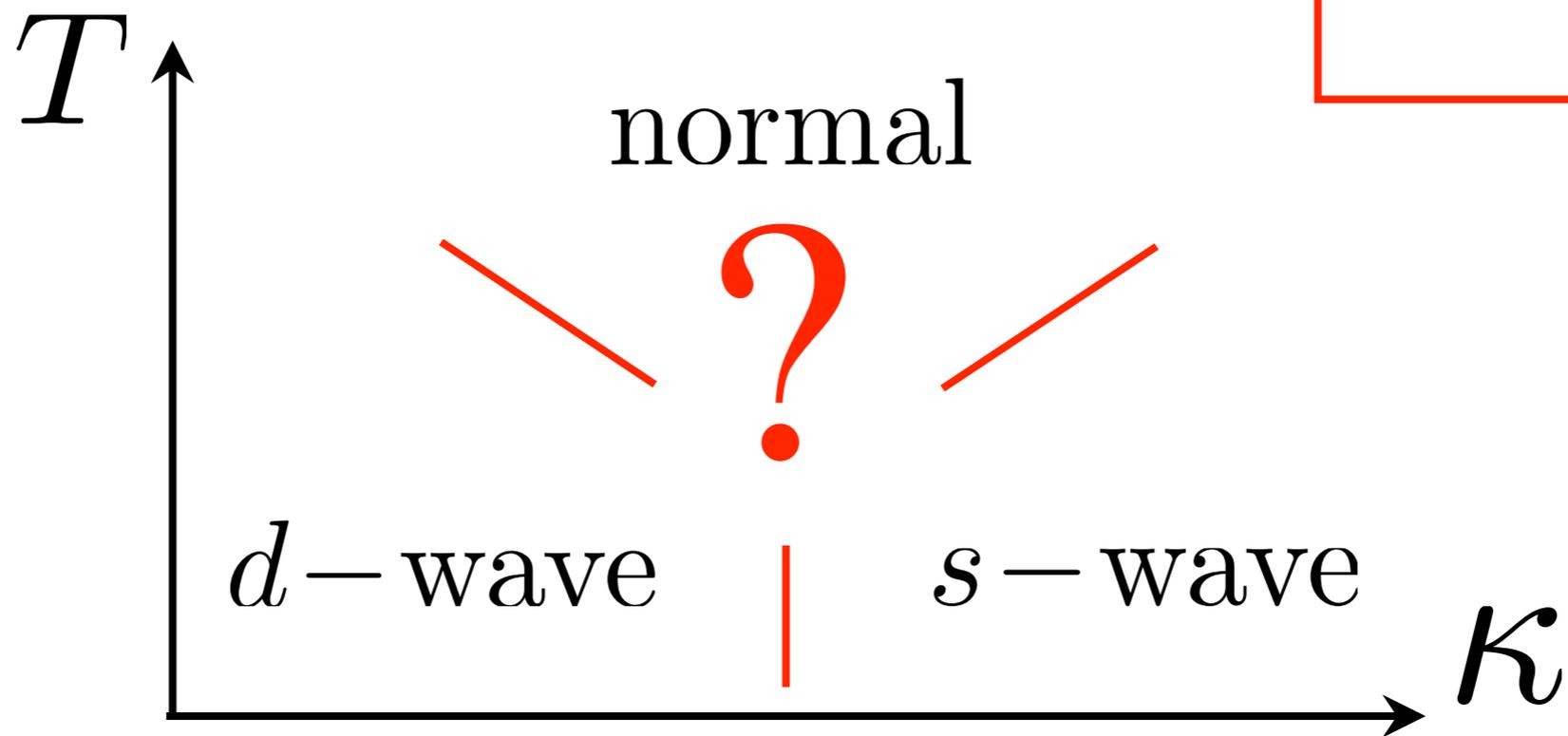
$$\kappa = \frac{\lambda}{v_F k_F |b|}$$

Can be anything

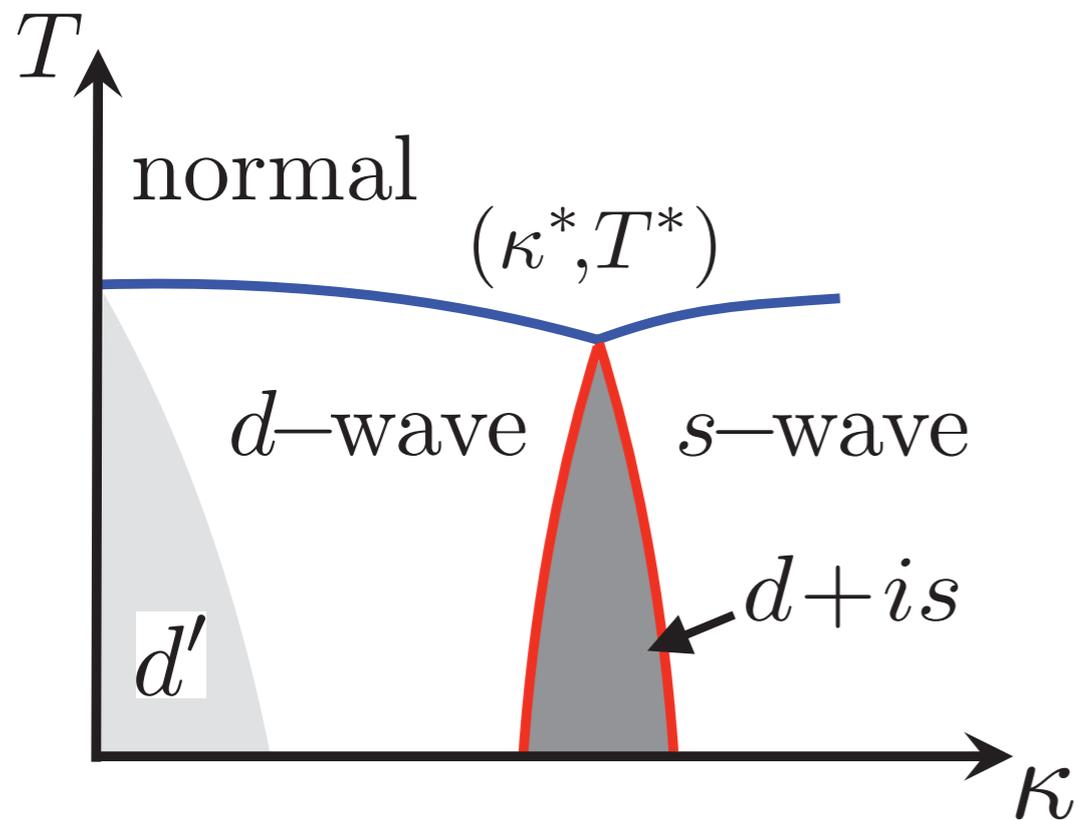
# Results

$$\kappa = \frac{\lambda}{v_F k_F |b|}$$

Question



$$T_c \ll \lambda (\approx 20 \text{meV})$$

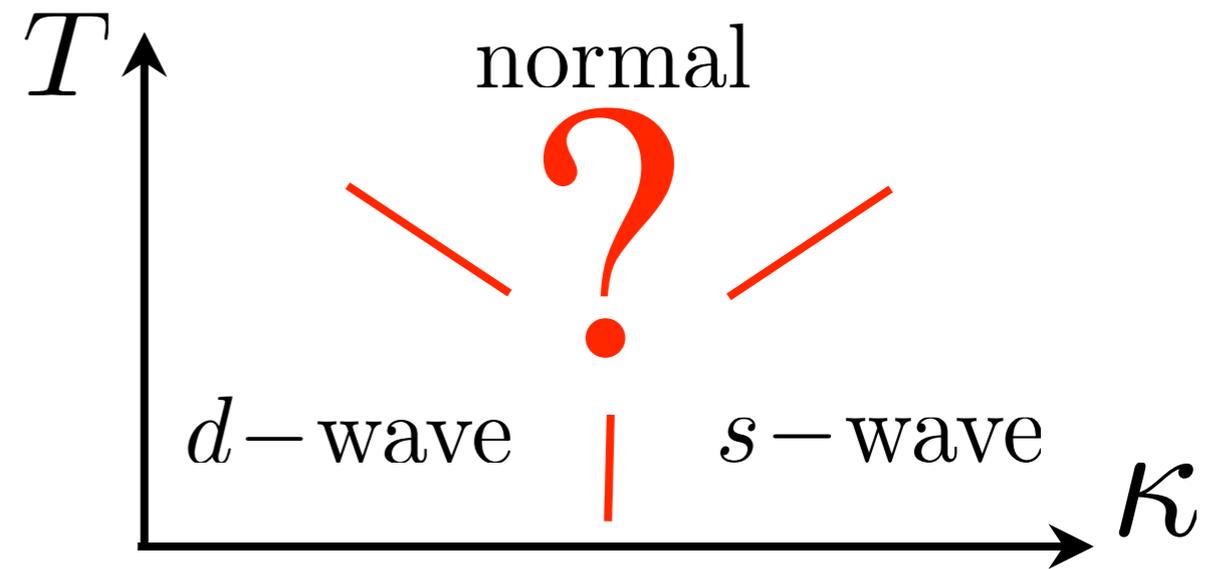


# Phase diagram

Ginzburg-Landau functional  
two order parameters

$\Delta_s$  s-wave

$\Delta_d$  d-wave



~~$\Delta_d^* \Delta_s$~~  ← symmetry

$$F_{GL} = A_s |\Delta_s|^2 + A_d |\Delta_d|^2 + \frac{B_s}{2} |\Delta_s|^4 + \frac{B_d}{2} |\Delta_d|^4 \\ + C |\Delta_s|^2 |\Delta_d|^2 + \frac{E}{2} [(\Delta_s \Delta_d^*)^2 + (\Delta_s^* \Delta_d)^2]$$

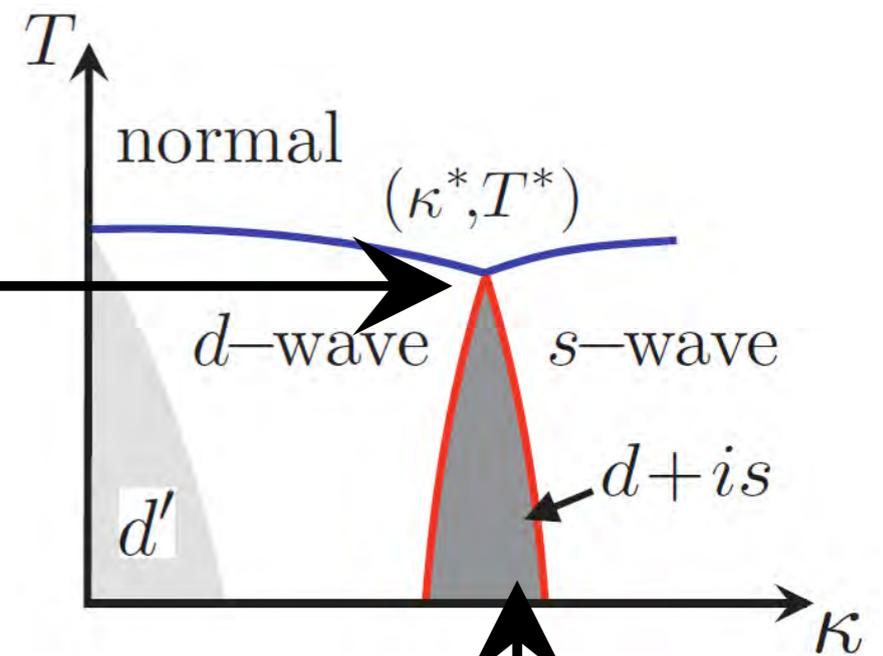
# Ginzburg-Landau functional treatment (cont)

$$F_{GL} = A_s |\Delta_s|^2 + A_d |\Delta_d|^2 + \frac{B_s}{2} |\Delta_s|^4 + \frac{B_d}{2} |\Delta_d|^4 + C |\Delta_s|^2 |\Delta_d|^2 + \frac{E}{2} [(\Delta_s \Delta_d^*)^2 + (\Delta_s^* \Delta_d)^2]$$

Tetracritical point:

$$A_s(\kappa^*, T^*) = A_d(\kappa^*, T^*)$$

$$\kappa^* \simeq 1$$



$$\lambda \gg T_c$$



$$B_s = B_d = B = \frac{10}{3} E$$

$$C = 2E$$



$$B+E > C$$

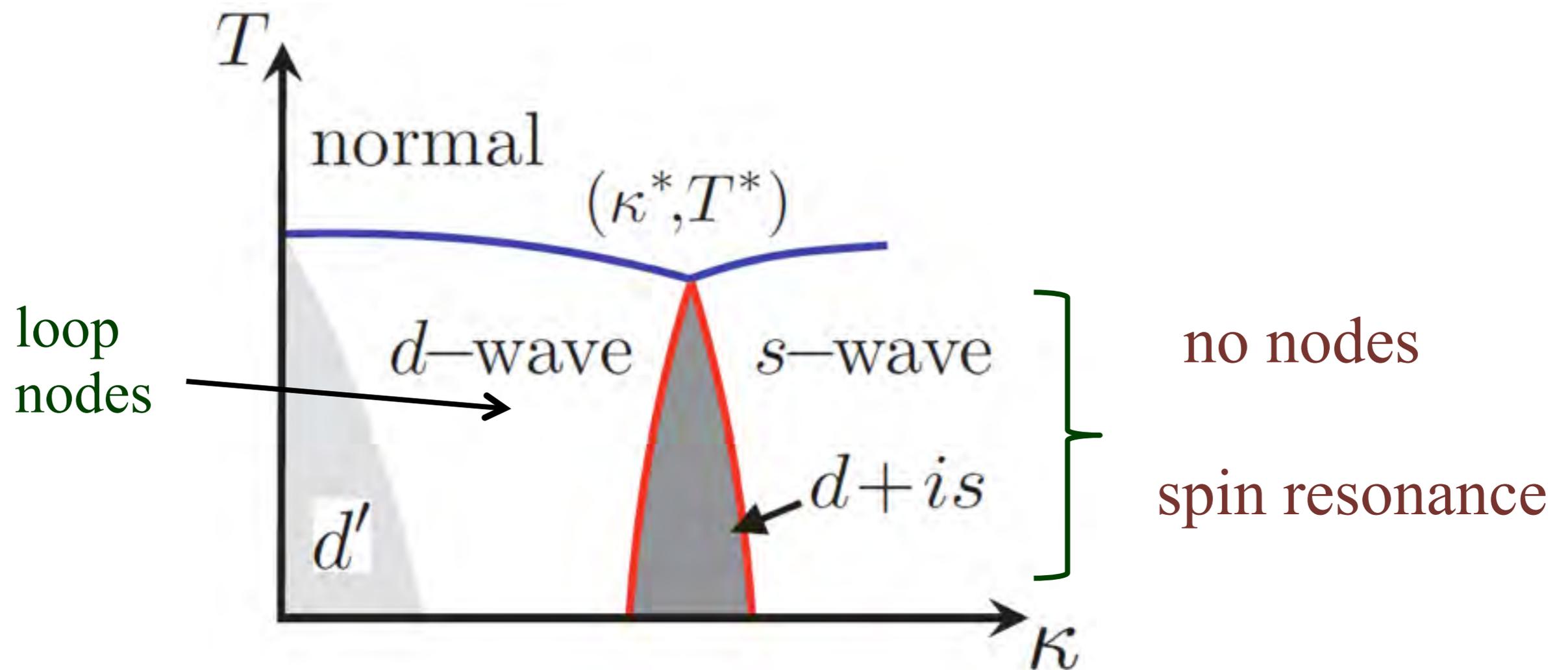


$$s + id$$

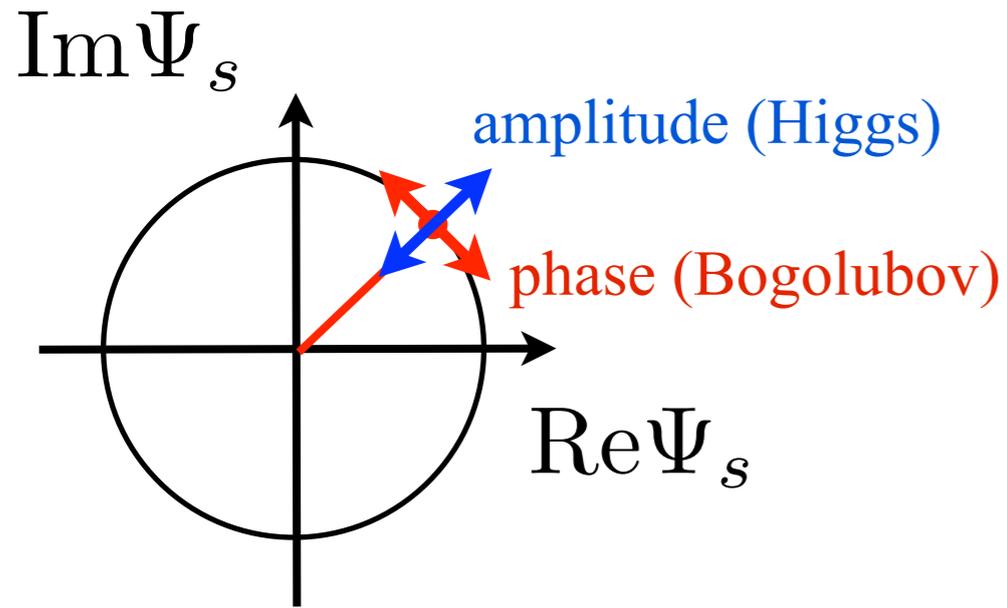
Time-reversal  
breaking

## Conclusions:

The pairing between electron pockets **MUST** include inter-pocket pairing on equal footing with intra-pocket pairing



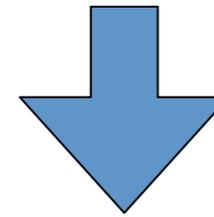
# Bardasis-Schrieffer Modes (1961)



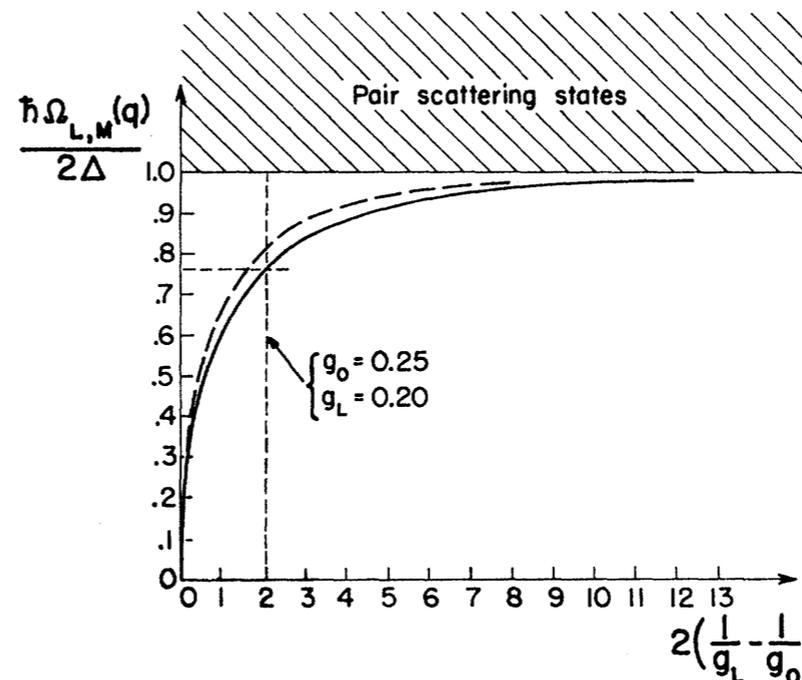
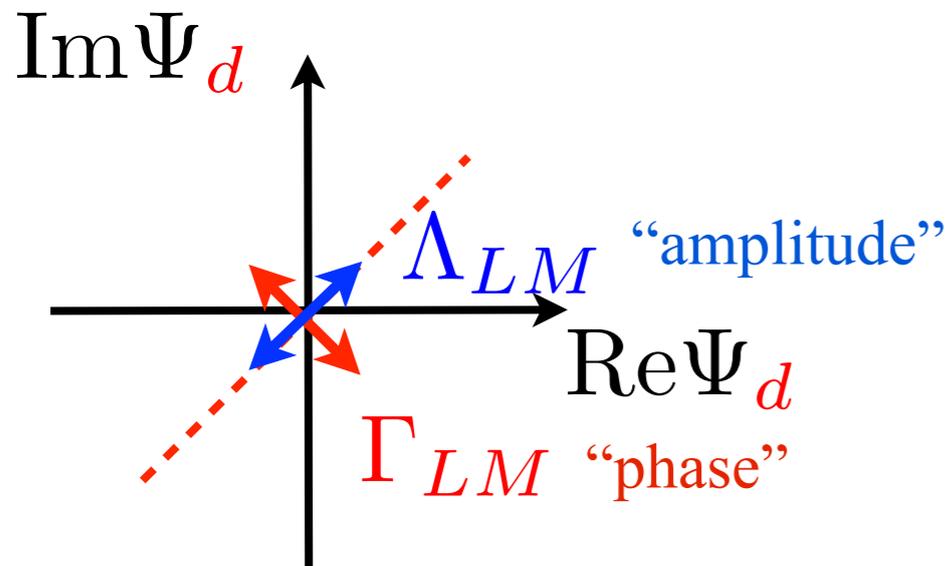
s-wave superconductor

$$|\Psi_s| \neq 0$$

Attraction in d-wave Cooper channel



1. In-gap "phase" mode
2. Damped decoupled "amplitude" mode

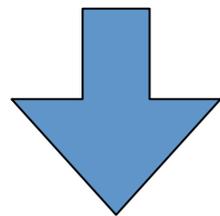


# Particle-Hole Exciton

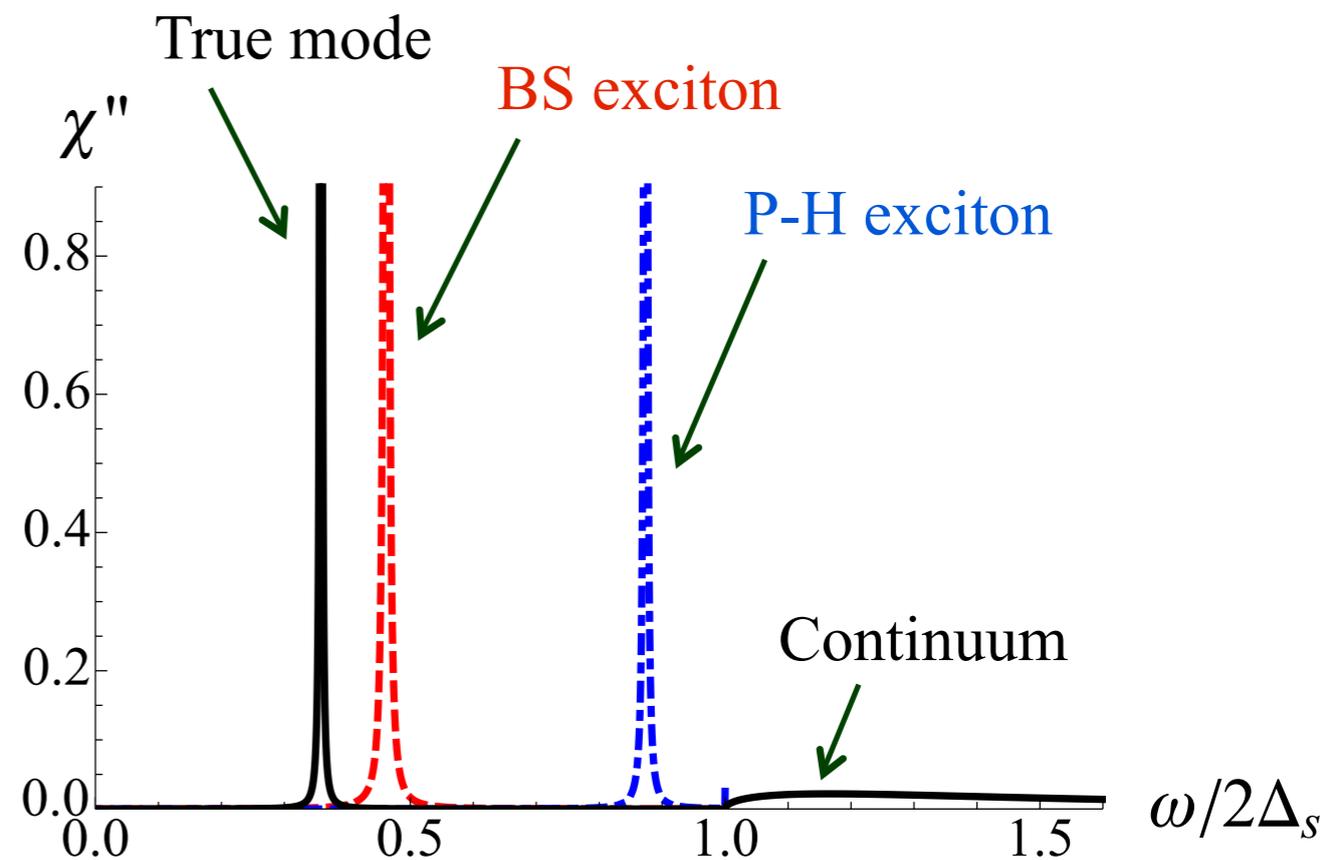
What if the P-H channel is attractive too?

Two excitons?? NO!

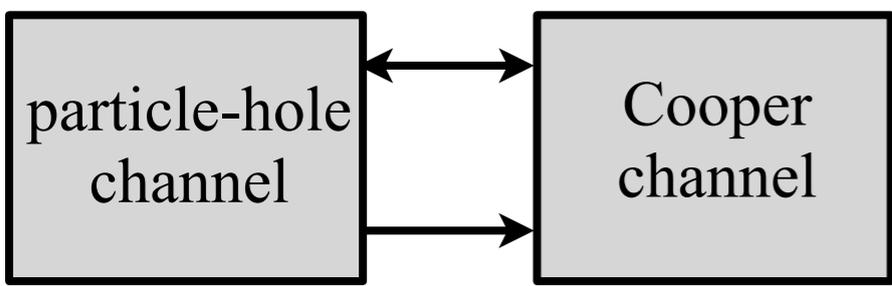
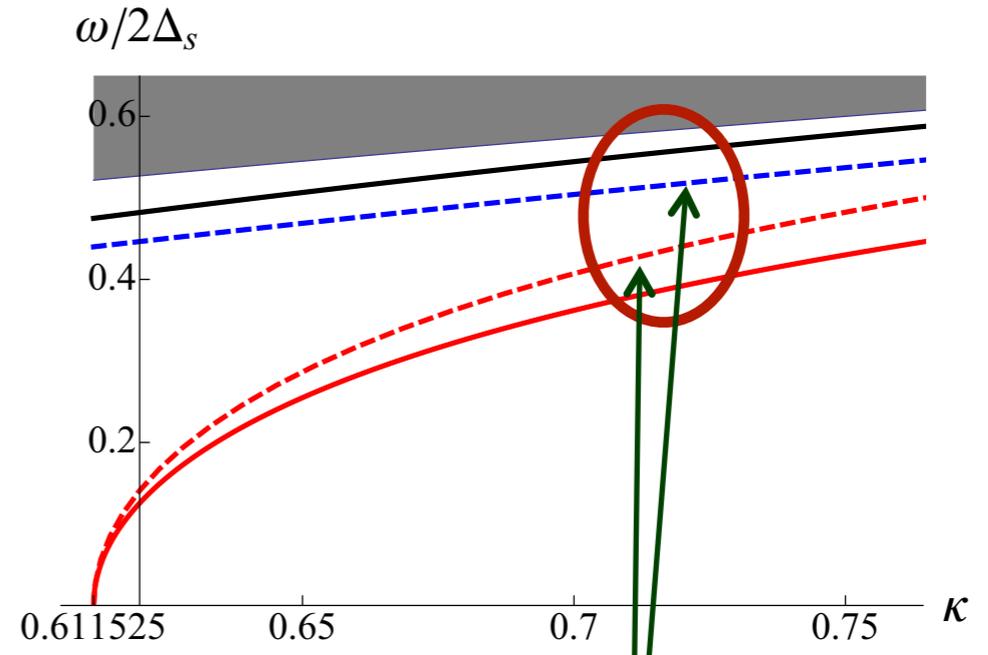
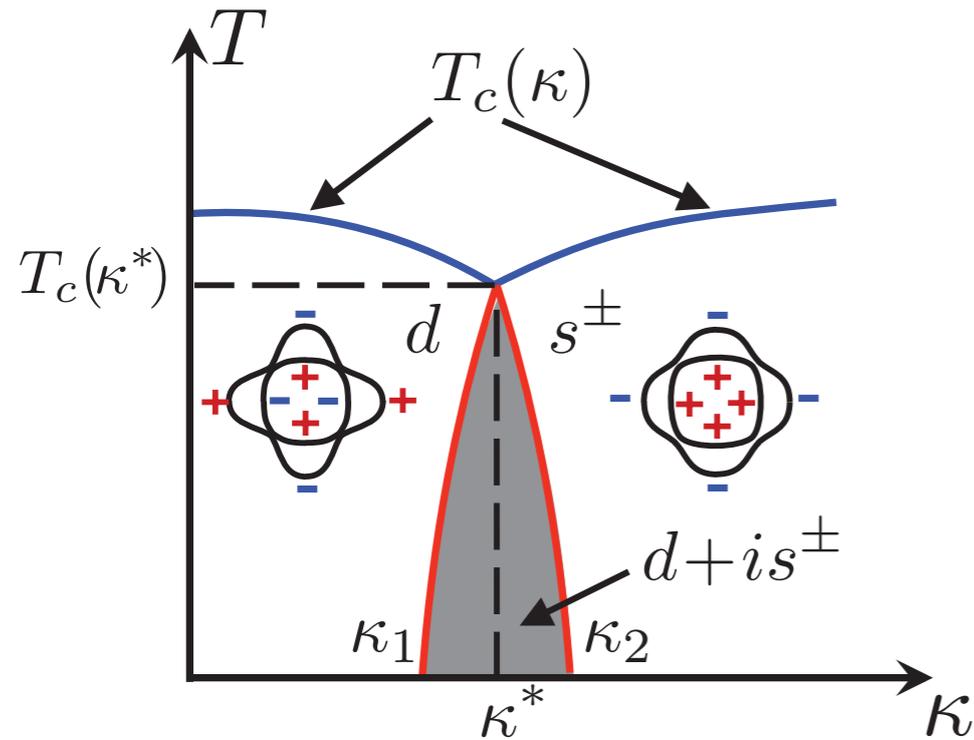
$$[\text{density}, \text{phase}]_- = i\hbar$$



**One mode**



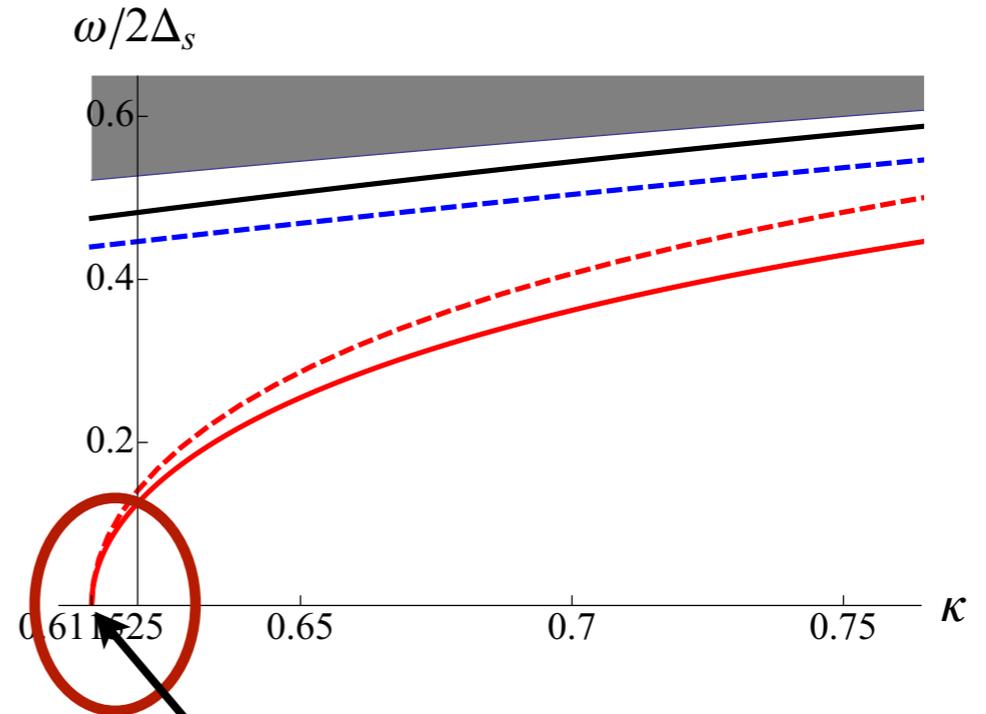
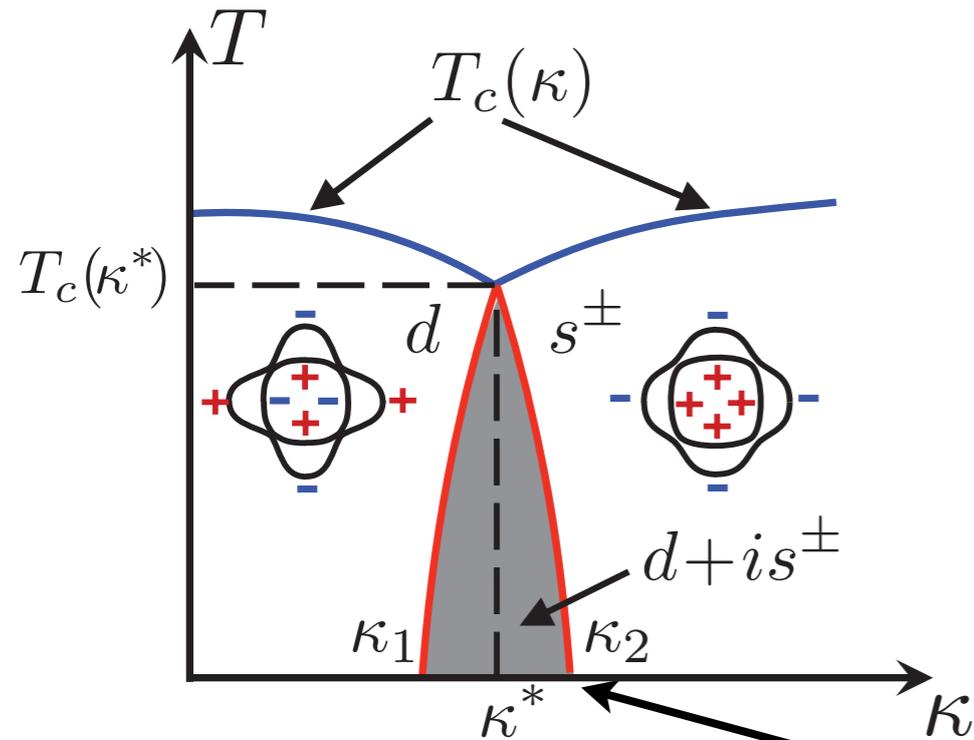
# Raman in selenides: Results



$\propto \Delta \Rightarrow$  No mixing  $\Rightarrow$  Two separate modes

↑  
Sign-Changing  
Order Parameter

# Raman in selenides: Results (cont)



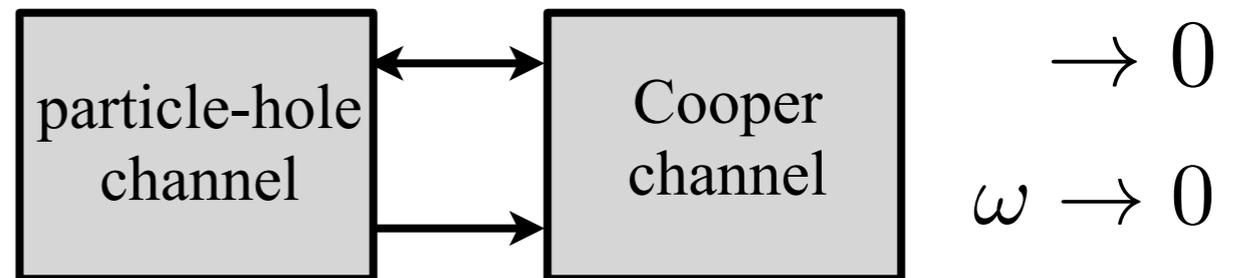
$$[\text{density}, \text{phase}]_- = i\hbar$$

$$[q, p]_- = i\hbar$$

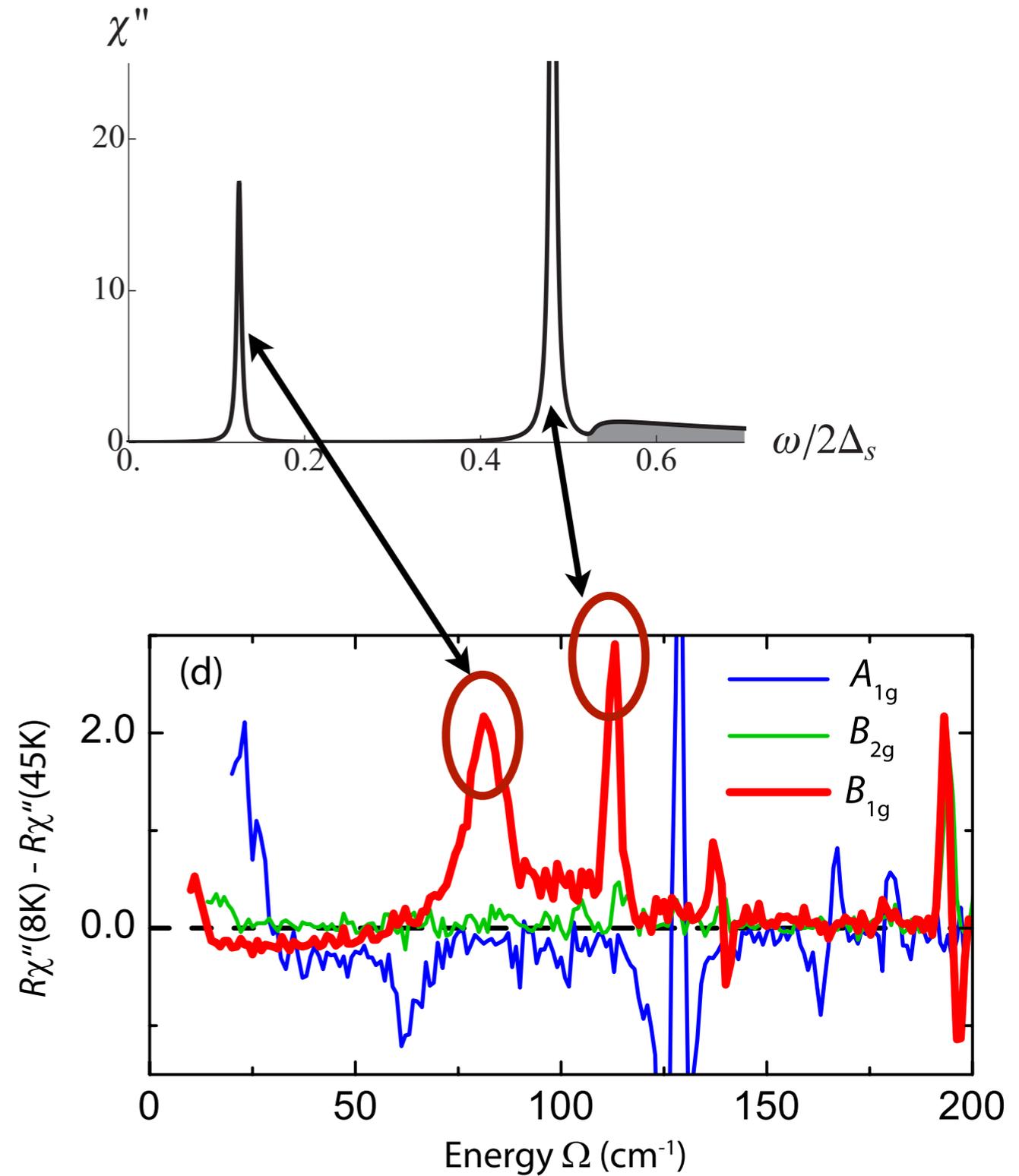
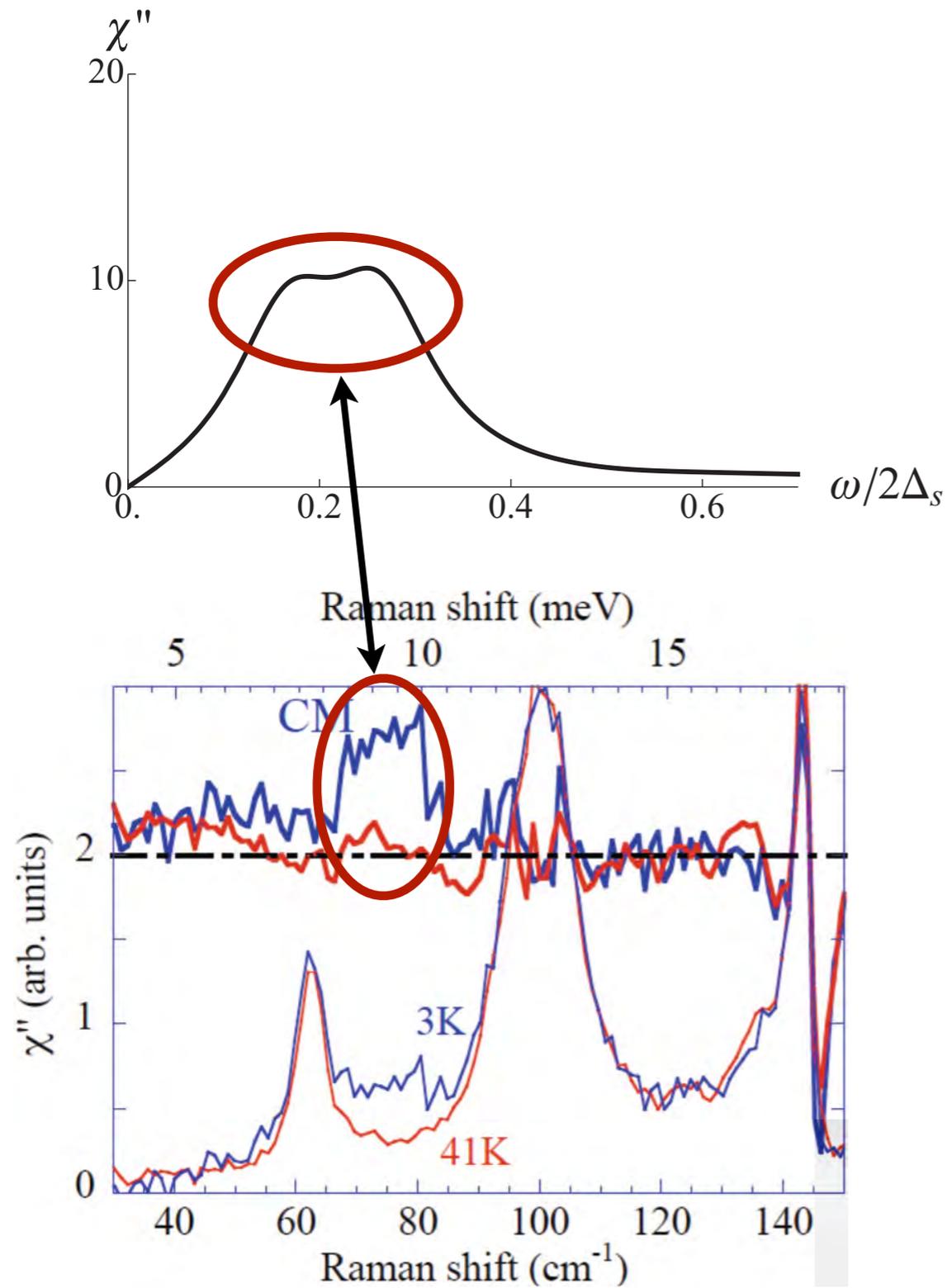
$$L = \boxed{pq} - H$$

$$\cancel{pq} \propto \omega$$

Decoupling at low frequencies



# Raman in selenides: Results (cont)



# Conclusions II

1. Time reversal symmetry breaking  $\rightarrow$  Raman active in-gap modes
2. Competing channels  $\rightarrow$  sharp in-gap modes
3. Sign-changing gap  $\rightarrow$  decoupling of Cooper and P-H channels

