

Superconductivity in the presence of spin-orbit coupling old dog, new tricks

Karen Michaeli





Stable finite momentum pairing

$Al^{3+}O_{2}^{4-}$
$La^{3+}O^{2-}$
$Al^{3+}O_2^{4-}$
$La^{3+}O^{2-}$
$Al^{3+}O_2^{4-}$
$La^{3+}O^{2-}$
$Al^{3+}O_2^{4-}$
$La^{3+}O^{2-}$
$Ti^{4+}O_2^{4-}$
$Sr^{2+}O^{2-}$
$Ti^{4+}O_2^{4-}$
$Sr^{2+}O^{2-}$

Oxide heterostructures

Peculiar s.c phasemagnetization relation



Critical magnetic field



Enhancement of T_c



2.1nm

H. Gardner, et al, 2011



Enhancement of T_c



 $\mu_0 H(T)$

H. Gardner, et al, 2011

Electronic spectrum









Magnetic field

The two Rashba bands in the presence of a Zeeman field:

 $\vec{B} = B\hat{x}$

$$\varepsilon_{k}^{\pm} = \frac{kk^{2}}{22m} \pm \frac{kk^{2}}{k} \sqrt{k_{x}^{2} + (k_{y} + \mu_{0}B/\alpha)^{2}}$$



Magnetic field

The two Rashba bands in the presence of a Zeeman field:

 $\vec{B} = B\hat{x}$

$$\varepsilon_k^{\pm} = \frac{k^2}{2m} \pm \alpha \sqrt{k_x^2 + (k_y + \mu_0 B/\alpha)^2}$$



Magnetic field

The two Rashba bands in the presence of a Zeeman field:

 $\vec{B} = B\hat{x}$

$$\varepsilon_{k}^{\pm} = \frac{k^{2}}{2m} \pm \alpha \sqrt{k_{x}^{2} + (k_{y} + \mu_{0}B/\alpha)^{2}}$$

$$F_{k}^{\pm} = \frac{k^{2}}{2m} \pm \alpha \sqrt{k_{x}^{2} + (k_{y} + \mu_{0}B/\alpha)^{2}}$$

$$\varepsilon_{k+q/2}^{\pm} \approx \frac{k^{2}}{2m} \pm \alpha |\vec{k}| + \frac{1}{2} (V_{F}q \pm 2\mu_{0}B) \sin \theta$$

$$\varepsilon_{-k+q/2}^{\pm} \approx \frac{k^{2}}{2m} \pm \alpha |\vec{k}| + \frac{1}{2} (V_{F}q \pm 2\mu_{0}B) \sin \theta$$

The FFLO state and spin-orbit

 k_{v}

k_x

$$\varepsilon_{k+q/2}^{\pm} \approx \frac{k^2}{2m} \pm \alpha |\vec{k}| + \frac{1}{2} \left(V_F q \pm 2\mu_0 B \right) \sin \theta$$
$$\varepsilon_{-k+q/2}^{\pm} \approx \frac{k^2}{2m} \pm \alpha |\vec{k}| + \frac{1}{2} \left(V_F q \pm 2\mu_0 B \right) \sin \theta$$
$$q = \frac{2\mu_0 B}{v_F}$$

The FFLO state and spin-orbit

$$\varepsilon_{k+q/2}^{\pm} \approx \frac{k^2}{2m} \pm \alpha |\vec{k}| + \frac{1}{2} \left(V_F q \pm 2\mu_0 B \right) \sin \theta$$
$$\varepsilon_{-k+q/2}^{\pm} \approx \frac{k^2}{2m} \pm \alpha |\vec{k}| + \frac{1}{2} \left(V_F q \pm 2\mu_0 B \right) \sin \theta$$
$$q = \frac{2\mu_0 B}{v_F}$$

Pairs of electrons in the - band are not affected by the magnetic field

Pairs of electrons in the + band feel the decoherence effect of the magnetic field



The critical field is mainly determined by the + band

k_x

The FFLO state and spin-orbit

$$\mu_0 B_c \sim \Delta_0 \left(\frac{\Delta_{so}}{\Delta_0}\right)^x$$

$$\Delta(\vec{r}) = \Delta e^{i\vec{q}\cdot\vec{r}}$$

The FFLO state

P. Fulde and R. A. Ferrell, 1964 A. I. Larkin and Yu. N. Ovchinnikov, 1964



Effect of disorder

KM, A. C. Potter, and P. A. Lee, PRL 2012



Free energy

Superconducting order parameter:

$$\Delta(\mathbf{q}) = U \sum_{\mathbf{k}} \left[\Psi_{\mathbf{k},\uparrow} \Psi_{-\mathbf{k}+\mathbf{q},\downarrow} - \Psi_{\mathbf{k},\downarrow} \Psi_{-\mathbf{k}+\mathbf{q},\uparrow} \right] \qquad \Delta = |\Delta| \exp^{i\Phi}$$

Magnetization:

$$\vec{M}(\mathbf{q}) = \mu_B \sum_{\mathbf{k}} \Psi_{\mathbf{k},\alpha}^{\dagger} \vec{\sigma}_{\alpha,\beta} \Psi_{\mathbf{k}+\mathbf{q},\beta}$$

$$F = \frac{\rho_s}{2} \left(\vec{\nabla} \Phi(\mathbf{r}) - 2e\vec{A}(\mathbf{r}) \right)^2 + \kappa \left[\left(\vec{\nabla} \Phi(\mathbf{r}) - 2e\vec{A}(\mathbf{r}) \right) \times \vec{H}_T(\mathbf{r}) \right] \cdot \hat{z}$$

Free energy





Current and magnetization

V.M. Edelstein, 1995

$$\vec{J} = -e\rho_s \left[\vec{\nabla}\Phi(\mathbf{r}) - 2e\vec{A}(\mathbf{r})\right] - e\kappa \left[\vec{H}_T \times \hat{z}\right]$$

$$ec{M} = rac{\kappa \chi}{\chi_s} \hat{z} \times \left[ec{
abla} \Phi(\mathbf{r}) - 2e ec{A}(\mathbf{r})
ight] + \chi ec{H}_{ext}$$
 $\chi =$

$\chi = \frac{\chi_s}{1 - \chi_s U/g^2 \mu_B^2}$

The superconducting current carries magnetization



Current and magnetization

V.M. Edelstein, 1995

$$\vec{J} = -e\rho_s \left[\vec{\nabla}\Phi(\mathbf{r}) - 2e\vec{A}(\mathbf{r})\right] - \frac{e\kappa}{H_T} \left[\vec{H}_T \times \hat{z}\right]$$

$$\vec{M} = \frac{\kappa \chi}{\chi_s} \hat{z} \times \left[\vec{\nabla} \Phi(\mathbf{r}) - 2e\vec{A}(\mathbf{r}) \right] + \chi \vec{H}_{ext}$$

$$\chi = \frac{\chi_s}{1 - \chi_s U/g^2 \mu_B^2}$$



S.-K. Yip, 2005 M. K. Kashyap and D. F. Agterberg, 2013



$$F = \frac{\rho_s}{2} \left(\vec{\nabla} \Phi(\mathbf{r}) - 2e\vec{A}(\mathbf{r}) \right)^2 + \kappa \left[\left(\vec{\nabla} \Phi(\mathbf{r}) - 2e\vec{A}(\mathbf{r}) \right) \times \vec{H}_T(\mathbf{r}) \right] \cdot \hat{z}$$

$$-\frac{\chi_s}{2}H_T^2 + \frac{U}{2\mu_B^2}M^2$$



$$\begin{split} F &= -\rho_s \sum_{\vec{i},\hat{\mu}} \cos\left(\Phi_{\vec{i}+\hat{\mu}} - \Phi_{\vec{i}}\right) + \kappa \sum_{\vec{i},\hat{\mu},\hat{\eta}} \varepsilon_{\mu\eta z} \sin\left(\Phi_{\vec{i}+\hat{\mu}} - \Phi_{\vec{i}}\right) \frac{H_{\vec{i}}^{\eta} + H_{\vec{i}+\hat{\mu}}^{\eta}}{2} \\ &- \frac{\chi_s}{2} H^2 + \frac{U}{2\mu_B^2} M^2 \end{split}$$

Free energy

$$F = -\rho_s \sum_{\vec{i},\hat{\mu}} \cos\left(\Phi_{\vec{i}+\hat{\mu}} - \Phi_{\vec{i}}\right)$$

$$-\kappa^2 \frac{\chi U}{2\chi_s \mu_B^2} \sum_{\vec{i},\hat{\mu}} \left[\sin\left(\Phi_{\vec{i}+\hat{\mu}} - \Phi_{\vec{i}}\right) + \sin\left(\Phi_{\vec{i}} - \Phi_{\vec{i}-\hat{\mu}}\right) \right]^2$$

 $\vec{S}_{\vec{i}} = \begin{pmatrix} \cos \Phi_{\vec{i}} \\ \sin \Phi_{\vec{i}} \end{pmatrix}$









Magnetic Field

$$\begin{split} F &= -\rho_s \sum_{\vec{i},\hat{\mu}} \left\{ \vec{S}_i \cdot \vec{S}_{i+\hat{\mu}} + \frac{\tilde{\kappa}}{\rho_s} \left[\vec{S}_{\vec{i}+\hat{\mu}} \times \vec{S}_{\vec{i}} + \vec{S}_{\vec{i}} \times \vec{S}_{\vec{i}-\hat{\mu}} \right]^2 \right\} \\ &+ \gamma \sum_{\vec{i},\hat{\mu}} \hat{z} \cdot \left(\hat{\mu} \times \vec{H}_{ext} \right) \vec{S}_{\vec{i}+\hat{\mu}} \times \vec{S}_{\vec{i}} \end{split}$$

$$\rho_s \to \sqrt{\rho_s^2 + (\gamma H_{ext})^2}$$

Helical phase for all values of $\ \widetilde{\kappa}$









$Al^{3+}O_{2}^{4-}$ $La^{3+}O_{2}^{4-}$ $Al^{3+}O_{2}^{4-}$ $La^{3+}O_{2}^{4-}$ $La^{3+}O_{2}^{4-}$ $Al^{3+}O_{2}^{4-}$ $La^{3+}O_{2}^{4-}$ $La^{3+}O_{2}^{4-}$ $La^{3+}O_{2}^{4-}$ $La^{3+}O_{2}^{4-}$
$La^{3+}O^{2-}$ $Al^{3+}O^{4-}_{2}$ $La^{3+}O^{2-}_{2}$ $Al^{3+}O^{4-}_{2}$ $La^{3+}O^{2-}_{2}$ $Al^{3+}O^{4-}_{2}$ $La^{3+}O^{2-}_{2}$ $La^{3+}O^{2-}_{2}$ $La^{3+}O^{2-}_{2}$
$Al^{3+}O_{2}^{4-}$ $La^{3+}O_{2}^{2-}$ $Al^{3+}O_{2}^{4-}$ $La^{3+}O_{2}^{2-}$ $Al^{3+}O_{2}^{4-}$ $La^{3+}O_{2}^{2-}$ $Ti^{4+}O_{2}^{4-}$
$Al^{3+}O_{2}^{4-}$ $La^{3+}O^{2-}$ $Al^{3+}O_{2}^{4-}$ $La^{3+}O^{2-}$ $Ti^{4+}O_{2}^{4-}$
$ La^{3+}O^{2-} \\ Al^{3+}O^{4-} \\ La^{3+}O^{2-} \\ Ti^{4+}O^{4-} \\ Ti^{4+}O^{4-} \\ $
$ \begin{array}{c} Al^{3+}O_2^{4-} \\ La^{3+}O^{2-} \\ Ti^{4+}O_2^{4-} \end{array} $
$\frac{La^{3+}O^{2-}}{Ti^{4+}O_2^{4-}}$
$Ti^{4+}O_2^{4-}$
<u> </u>
$Sr^{2+}O^{2-}$
$Ti^{4+}O_2^{4-}$
$Sr^{2+}O^{2-}$

Oxide heterostructures



Stable finite momentum pairing





Strongly correlated superconductors

Peculiar s.c phasemagnetization relation

