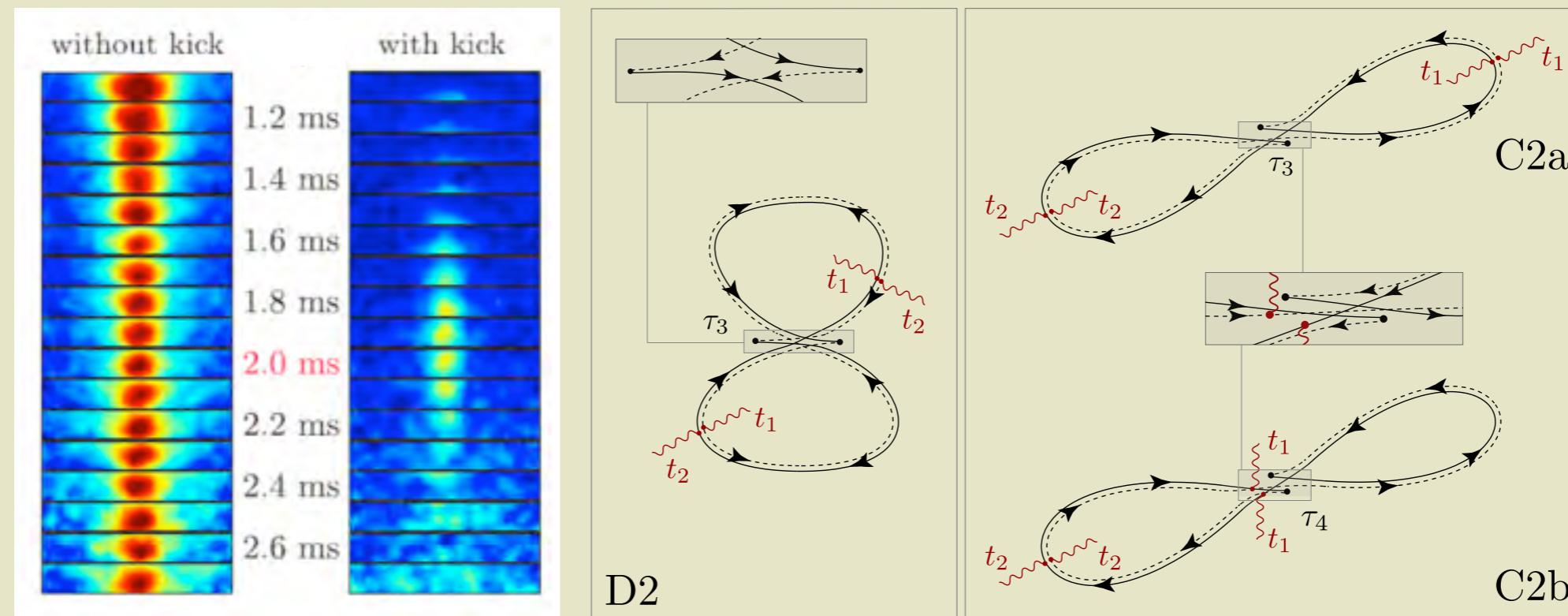


# aspects of Anderson Localization with Cold Atoms



Tobias Micklitz



Centro Brasileiro de  
Pesquisas Físicas

collaborations: A. Altland, C. Müller (theory)  
A. Aspect, V. Josse (experiment)

# Motivation

## Ultra cold atoms in a laser speckle: a quantum simulator for Anderson localization

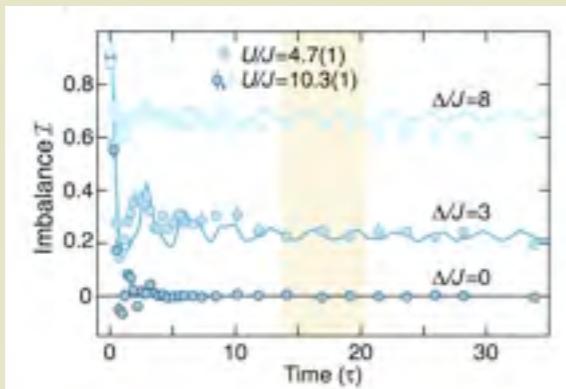
Alain Aspect – Institut d'Optique – Palaiseau

tunable system parameters:

- full control of the trapping potentials (lattice geometry, effective dimensionality), random potentials (disorder), interactions (by Feshbach resonances or density), type of quantum statistics (ultracold bosons or fermions)
- closed system (no coupling to bath, decoherence)

recent examples:

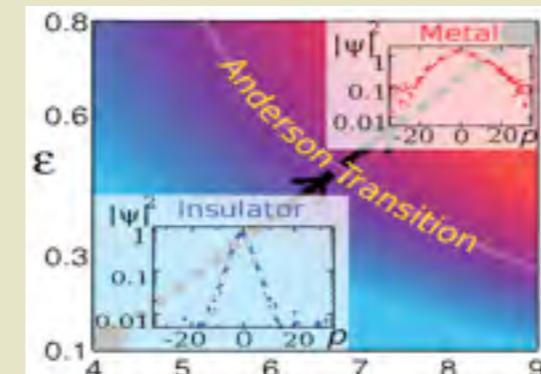
**observation of many-body localization?**



Observation of many-body localization of interacting fermions in a quasi-random optical lattice

Michael Schreiber<sup>1,2</sup>, Sean S. Hodgman<sup>1,2</sup>, Pranjal Bordia<sup>1,2</sup>, Henrik P. Lüschen<sup>1,2</sup>, Mark H. Fischer<sup>1</sup>, Ronen Vosk<sup>1</sup>, Ehud Altman<sup>1</sup>, Ulrich Schneider<sup>1,2</sup> and Immanuel Bloch<sup>1,2</sup>  
<sup>1</sup>Fakultät für Physik, Ludwig-Maximilians-Universität München, Scheinerstr. 4, 80799 Munich, Germany  
<sup>2</sup>Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1, 85748 Garching, Germany  
<sup>3</sup>Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel

**(dynamical) localization transition**

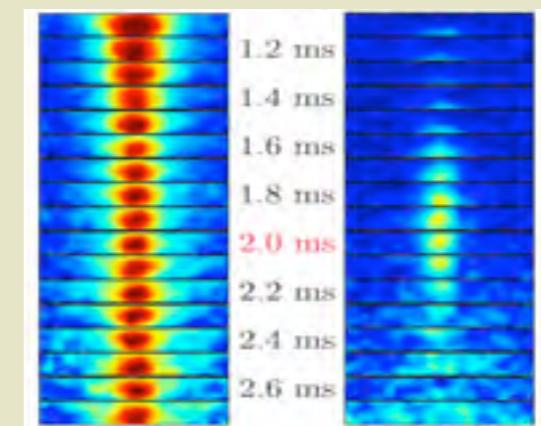


PRL 101, 255702 (2008) | Selected for a Viewpoint in Physics | PHYSICAL REVIEW LETTERS | work ending 14 DECEMBER 2008

Experimental Observation of the Anderson Metal-Insulator Transition with Atomic Matter Waves

Julien Chabé,<sup>1</sup> Gérard Lemaitre,<sup>2</sup> Béatrice Grémaud,<sup>2</sup> Dominique Delande,<sup>2</sup> Pascal Szriftgiser,<sup>1</sup> and Jean-Claude Garreau<sup>1</sup>  
<sup>1</sup>Laboratoire de Physique des Lasers, Atomes et Molécules, Université Paris-Sud, CNRS, (LAPLACE),  
F-91405 Villebon-sur-Yvette, France<sup>2</sup>  
<sup>2</sup>Laboratoire Kastler-Brossel, Université Pierre et Marie Curie-Paris 6, CNRS, 4 Place Jussieu, F-75005 Paris, France  
(Received 11 July 2008; revised manuscript received 11 September 2008; published 12 December 2008)

**weak-localization echo**



PRL 114, 205301 (2015) | PHYSICAL REVIEW LETTERS | work ending 22 MAY 2015

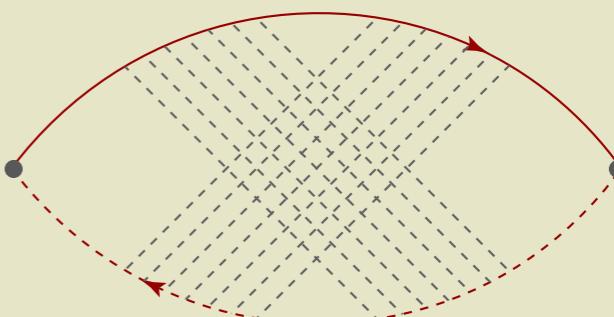
Suppression and Revival of Weak Localization through Control of Time-Reversal Symmetry

K. Müller,<sup>1</sup> J. Richard,<sup>2</sup> V. V. Micheli,<sup>3</sup> N. V. Dinechand,<sup>1</sup> P. Bouyer,<sup>2</sup> A. Aspect,<sup>1</sup> and V. Josse<sup>1,2</sup>  
<sup>1</sup>Laboratoire Charles Fabry, UMR 6530, Institut d'Optique, CNRS, Univ. Paris Sud 11, 2 Avenue Augustin Fresnel,  
91127 Palaiseau cedex, France  
<sup>2</sup>LP2N UMR 5298, Université Bordeaux I, Institut d'Optique et CNRS, 351 cours de la Libération, 33405 Talence, France  
(Received 10 October 2014; published 18 May 2015)

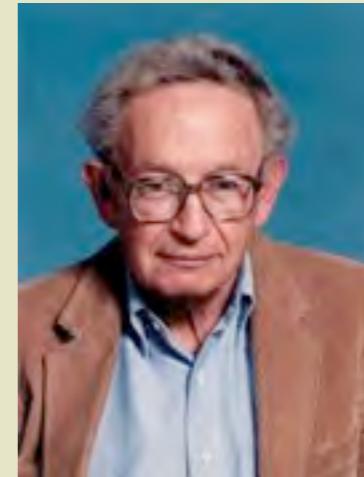
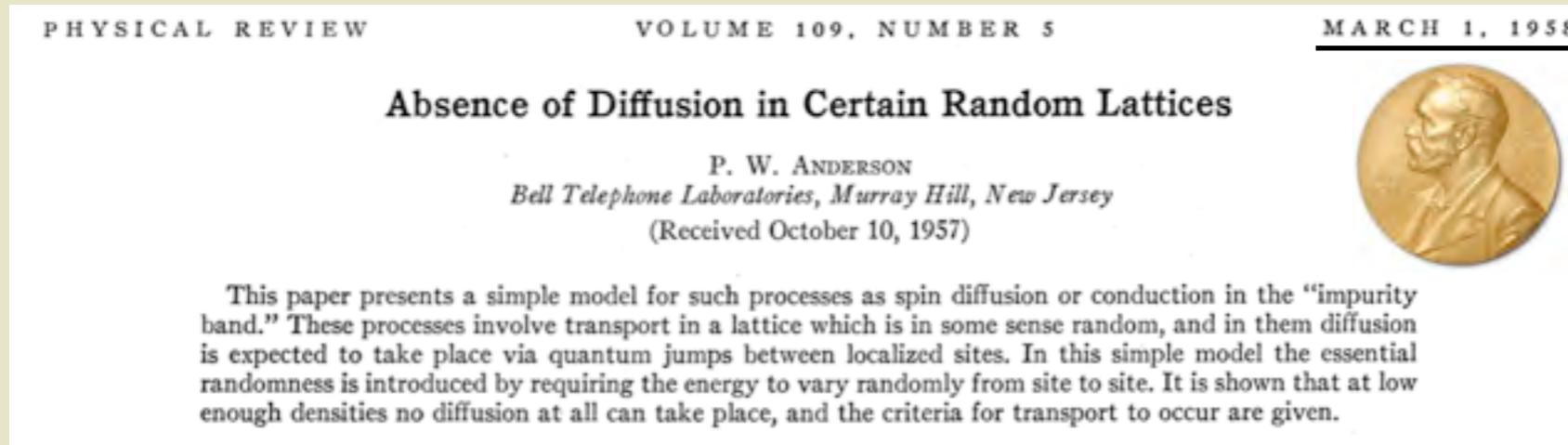
focus here: single-particle physics...

# Outline

- introduction Anderson localization (AL)
- quantum quench experiment
- forward scattering peak as a signature of strong AL
- echo-spectroscopy at the onset of AL



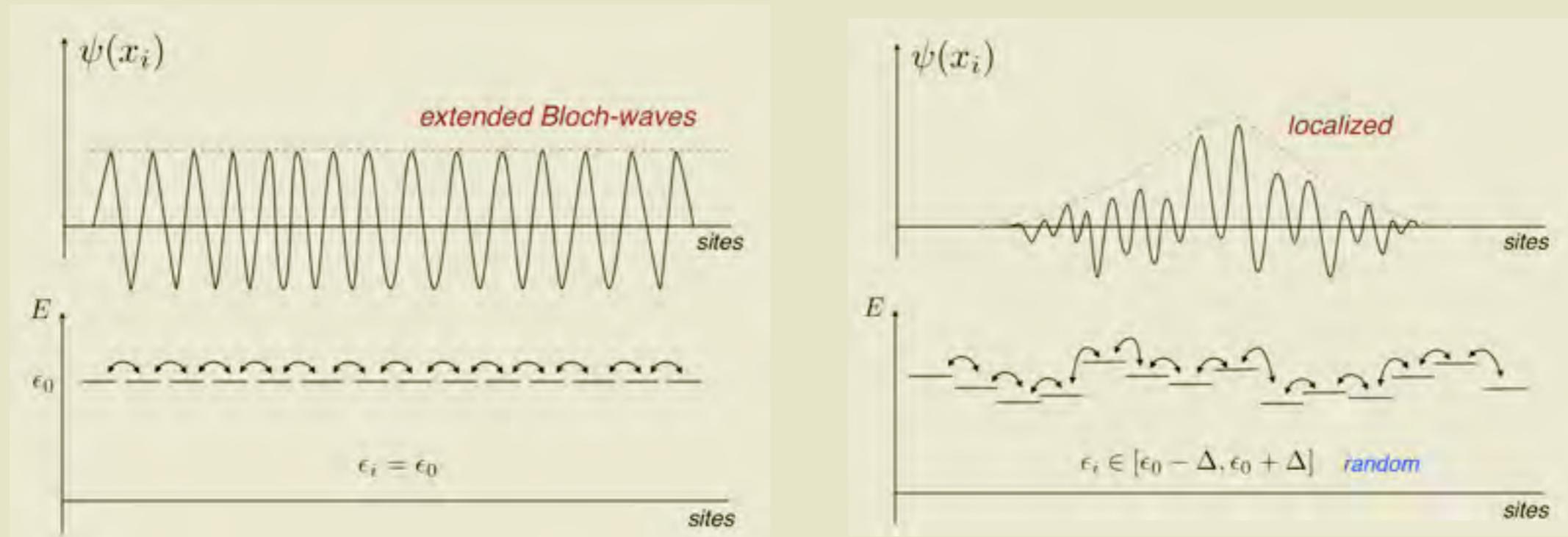
# Anderson Localization (AL)



single-particle wave-function  
in a (macroscopic) disordered  
quantum system can be  
exponentially localized

# “Absence of diffusion in certain lattices” (Anderson, 1958)

1d example:  $H = t \sum_{\langle ij \rangle} c_j^\dagger c_i + \sum_i \epsilon_i c_i^\dagger c_i$   $\epsilon_i \in [\epsilon_0 - \Delta, \epsilon_0 + \Delta]$  (random)



## role of dimensionality:

### “gang of four” (1979)

Abrahams, Anderson, Licciardello, Ramakrishnan

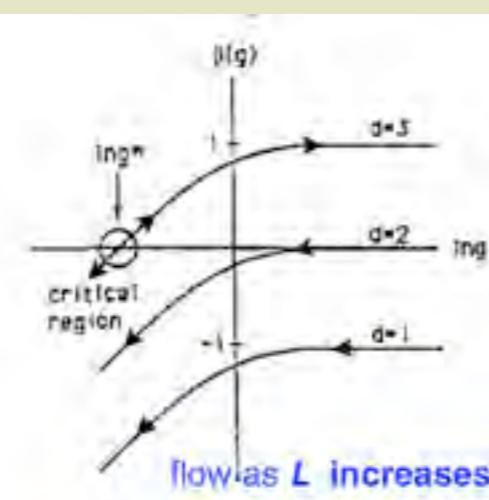
$$\beta(g) = \frac{d \ln g}{d \ln L} \quad \text{depends only on conductance itself!}$$

$g(bL) = f_b(g(L))$

#### qualitative behavior of $\beta$

$g \gg 1$  ohmic:  $g \sim L^{d-2} \rightarrow \beta(g) = d - 2$

$g \ll 1$  insulating:  $g \sim e^{-L/\xi} \rightarrow \beta(g) = \ln g$   
smooth interpolation by monotonic function



in an infinite system at T=0:

“all states are (Anderson) localized by disorder in  $d < 3$ ”

“localization of high-energy/short wave-length waves by random potentials”: different from classical trapping in a random potential...

## quantum vs. classical diffusion



classically: random walk

$$\langle r^2 \rangle = Dt \quad (\text{Markovian process})$$



quantum mechanically: interference

$$\langle r^2 \rangle \xrightarrow[t \rightarrow \infty]{} \text{const.} \quad (\text{system has memory})$$

...a quantum interference phenomenon!

# microscopic origin: the onset of localization

“What is the probability to propagate from  $A$  to  $B$  ?”



Semiclassical limit  $S_\gamma/\hbar \gg 1$

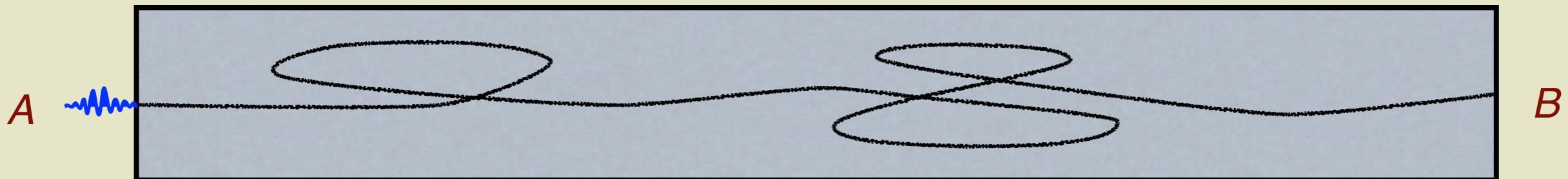
$$P_{A \rightarrow B} \simeq \sum_{\gamma} \left| A_{\gamma} \begin{smallmatrix} \text{classical} \\ \text{paths} \end{smallmatrix} \right|^2 + \sum_{\gamma\gamma'} \left( A_{\gamma} A_{\gamma'}^* \begin{smallmatrix} \text{different classical paths} \\ \text{with same action} \end{smallmatrix} \right) \quad (\text{quantum interference})$$

Q: What are the relevant classical paths giving rise to quantum-interference corrections?

A: self-intersecting paths, which exactly depends on the symmetries of the system...

# microscopic origin: the onset of localization

## self-intersecting paths



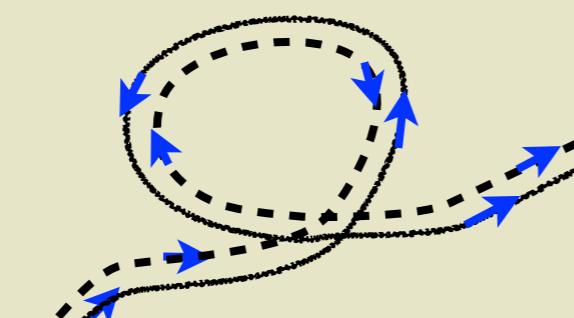
**role of symmetries:**

**symmetries/class:**

$$\sum_{\gamma\gamma'} \left( A_\gamma A_{\gamma'}^* \text{ } \begin{smallmatrix} \text{different classical paths with} \\ \text{same action} \end{smallmatrix} \right)$$

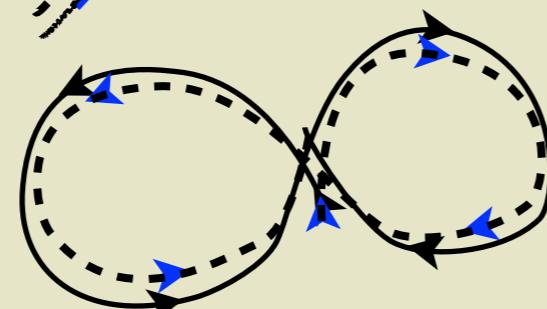
orthogonal (O)

time-reversal	spin-rotational
yes	yes



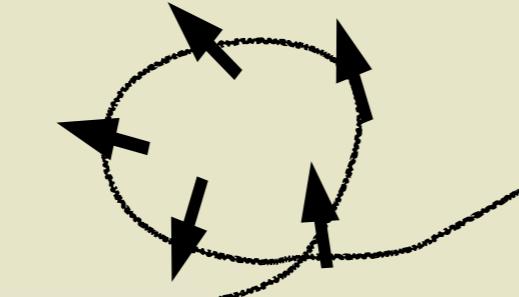
unitary (U)

time-reversal	spin-rotational
no	yes



symplectic (Sp)

time-reversal	spin-rotational
yes	no

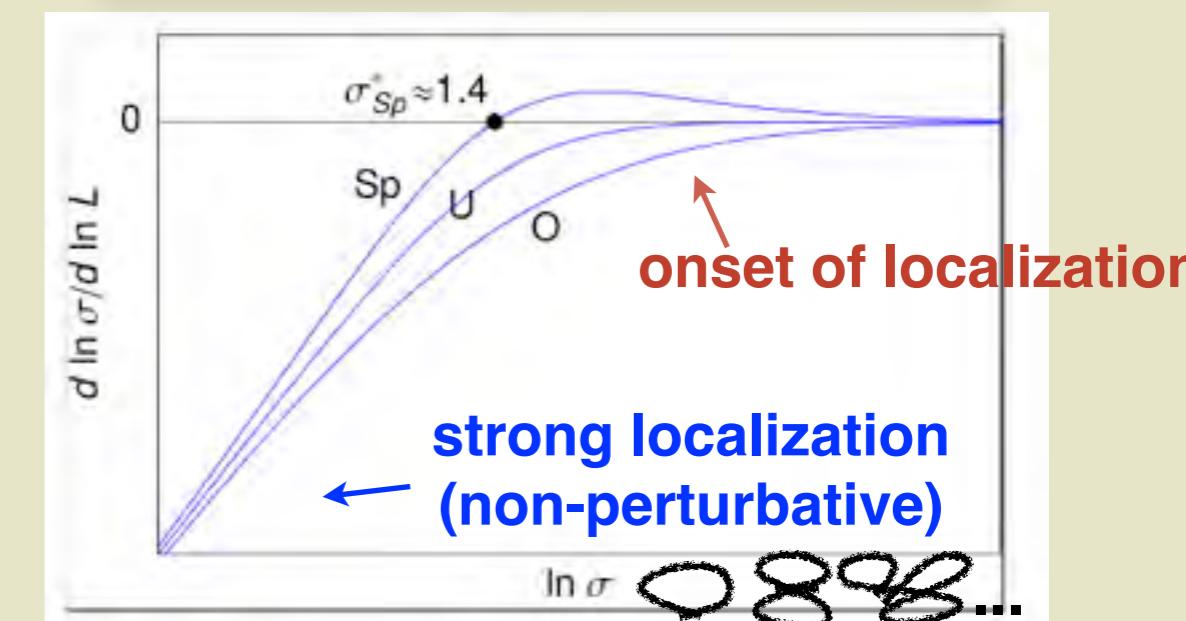


**perturbative RG  $d=2+\epsilon$ :**

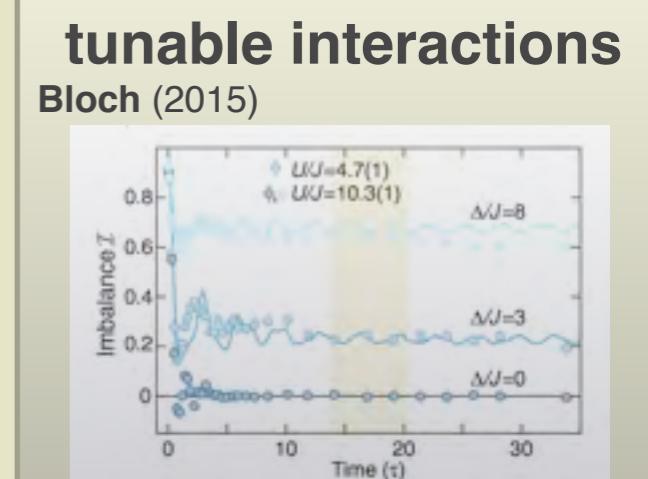
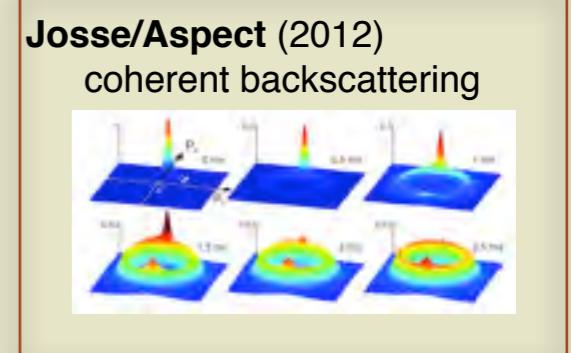
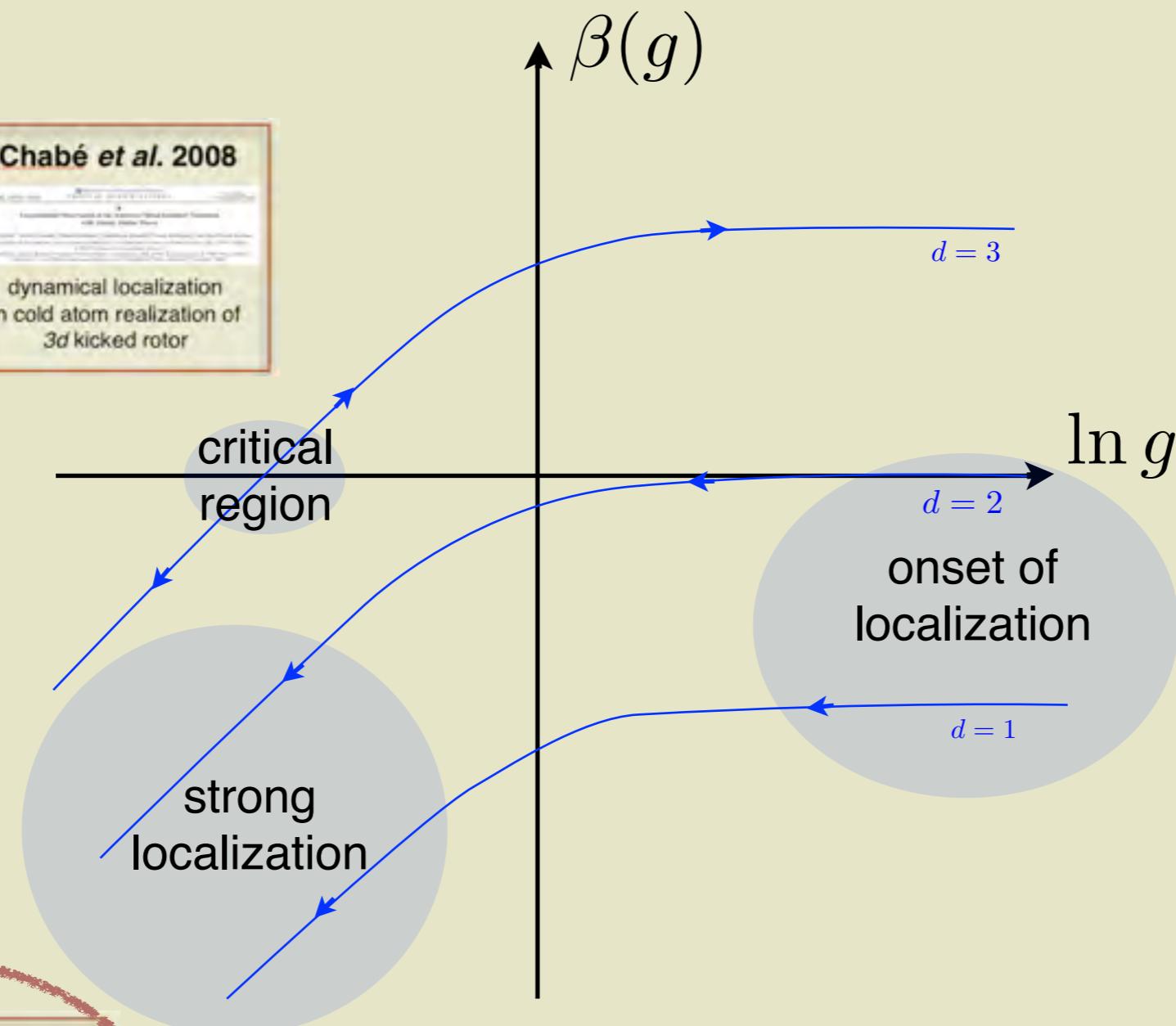
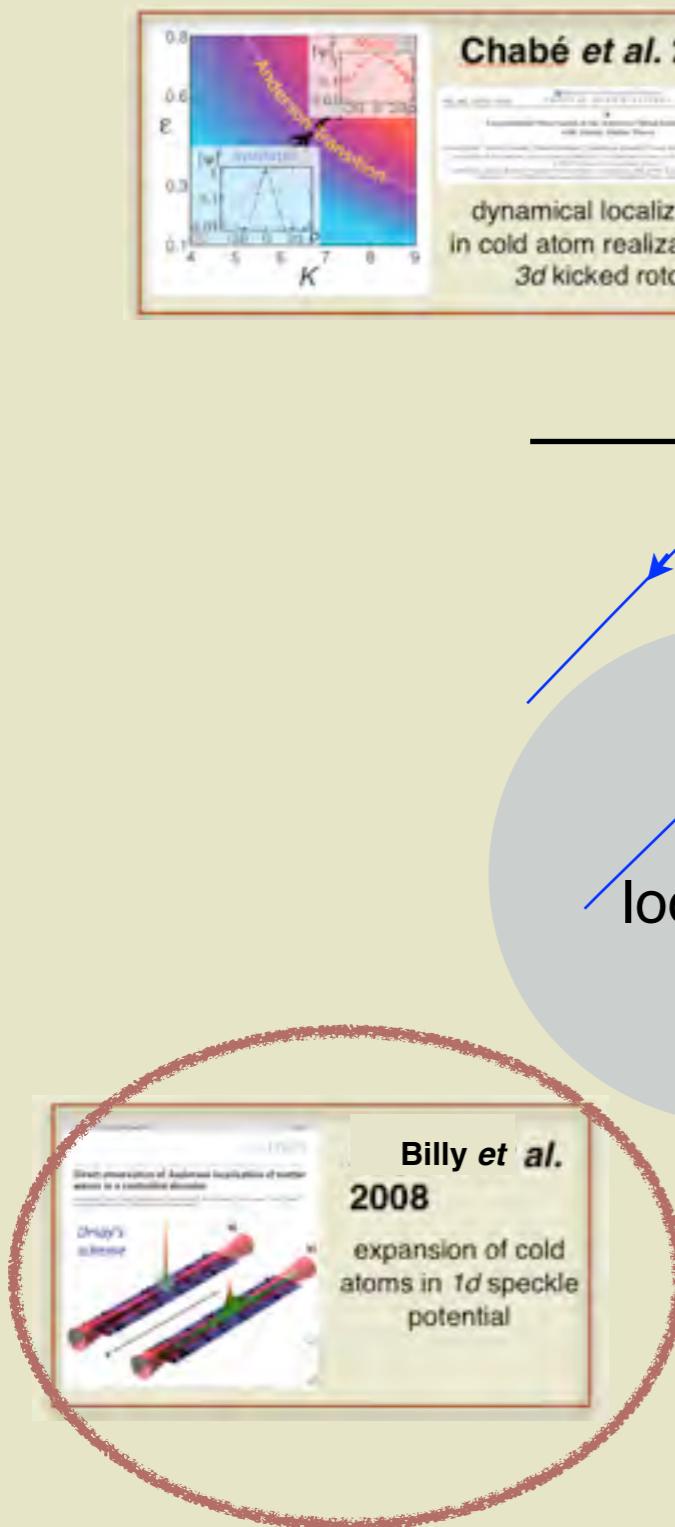
$$\beta(t) = \epsilon - t - \frac{3}{4}\zeta(3)t^4 + \mathcal{O}(t^5) \quad (\text{O})$$

$$\beta(t) = \epsilon - \frac{1}{2}t^2 - \frac{3}{8}t^4 + \mathcal{O}(t^6) \quad (\text{U})$$

$$\beta(t) = \epsilon + t - \frac{3}{4}\zeta(3)t^4 + \mathcal{O}(t^5) \quad (\text{Sp})$$



# some recent cold atom experiments



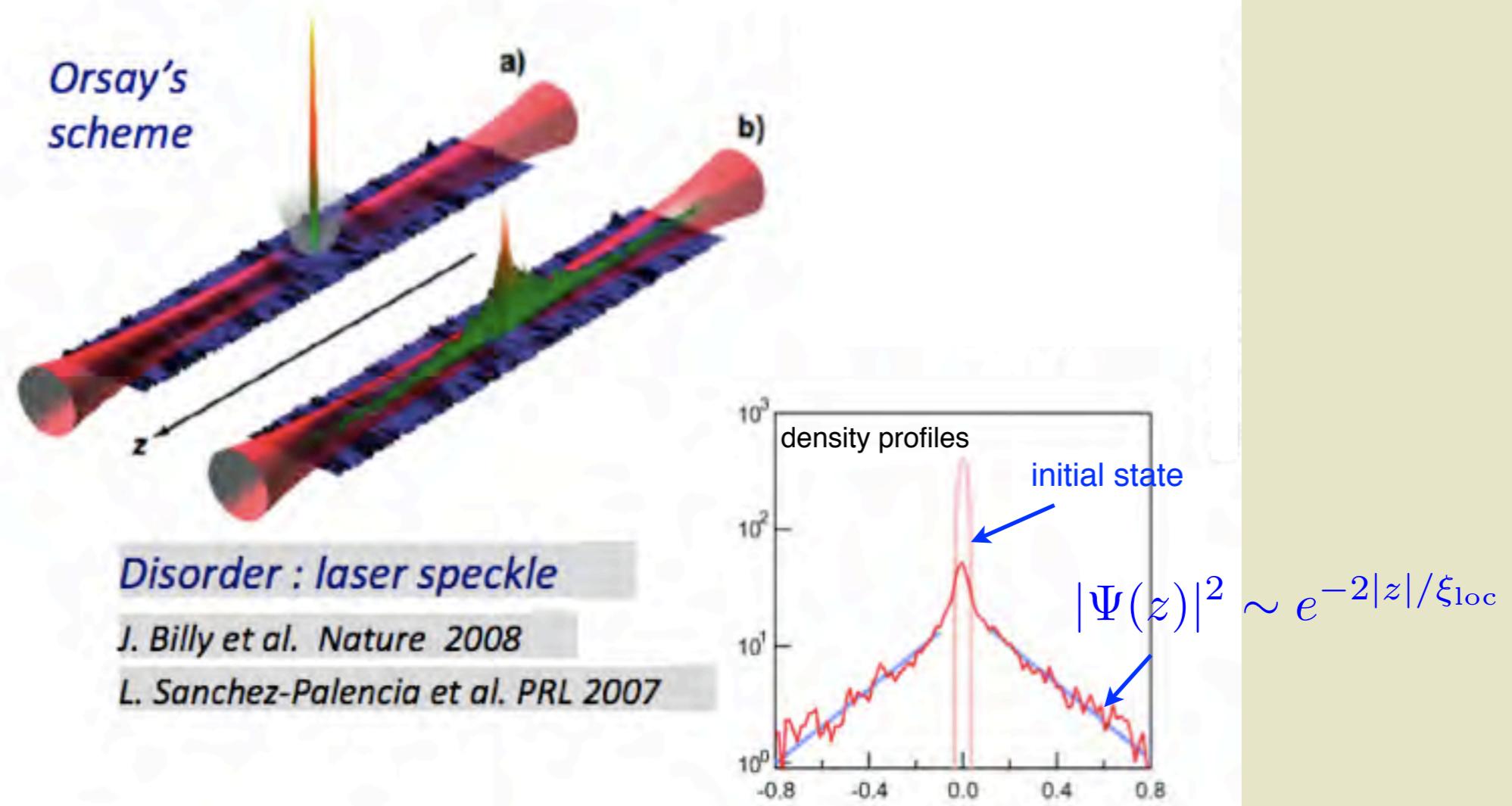
# 1d exponential localization of atomic matter wave

(direct observation of wave-function)

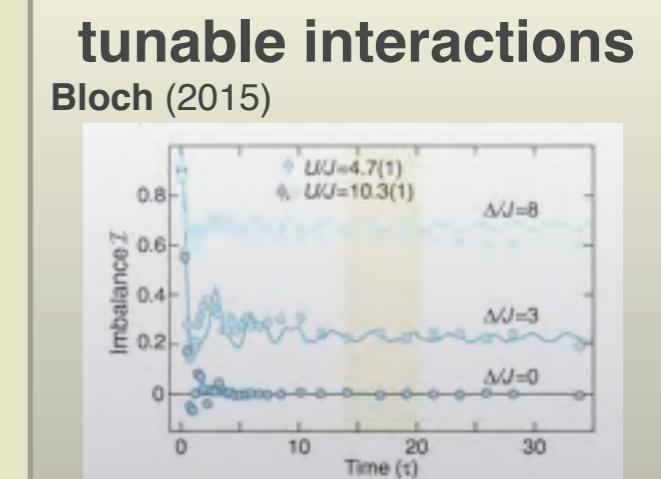
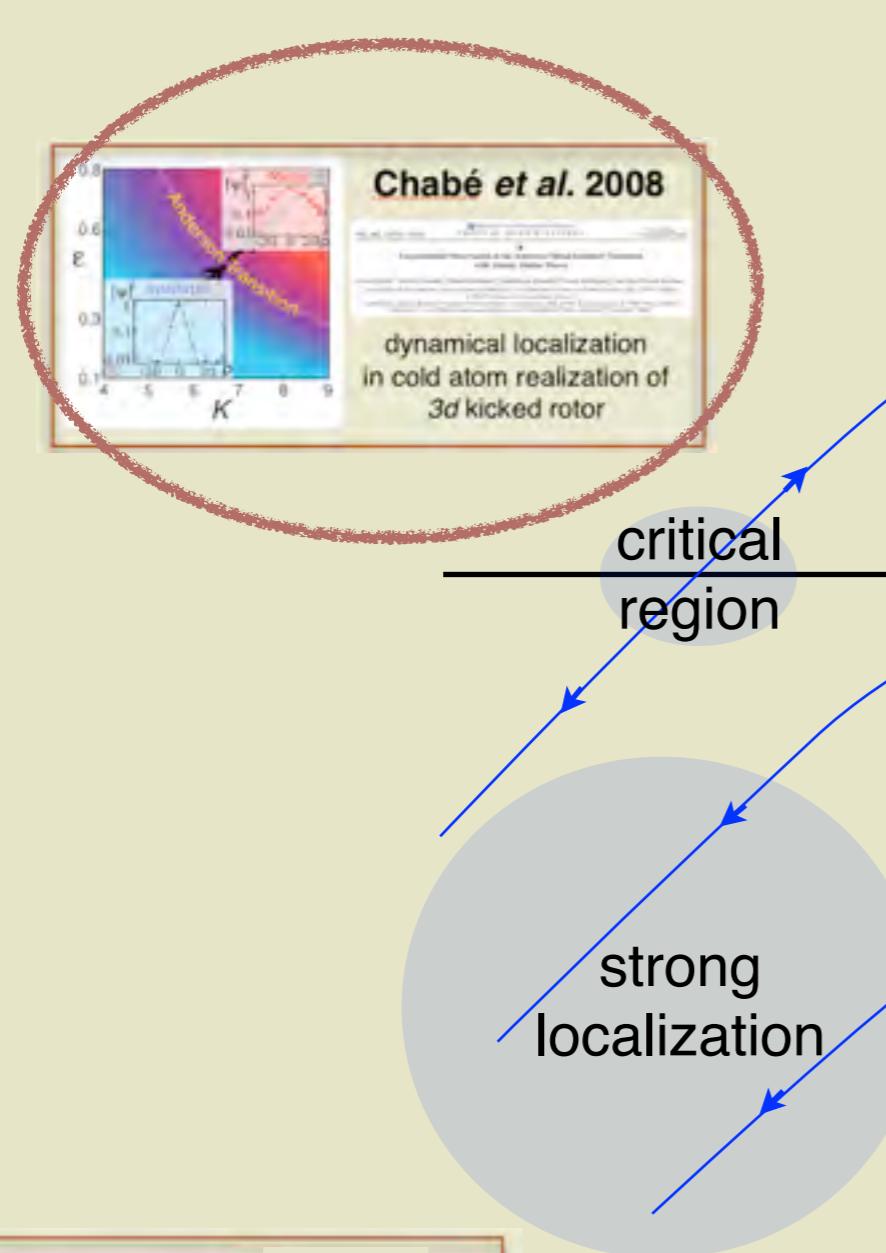


cold atoms released into 1d wave-guide in presence of disorder

2008 : 1D direct observation (real space)



# some recent cold atom experiments

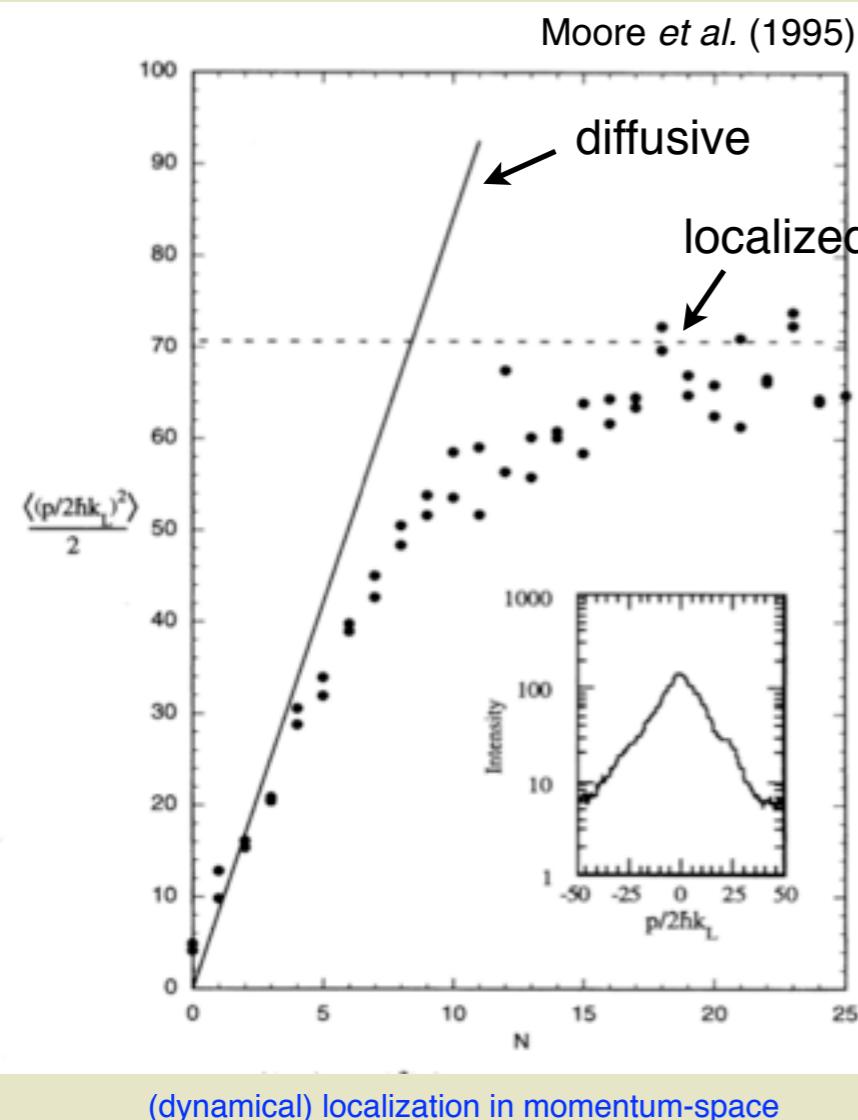


# 3d AL-transition in cold atom realization of kicked rotor

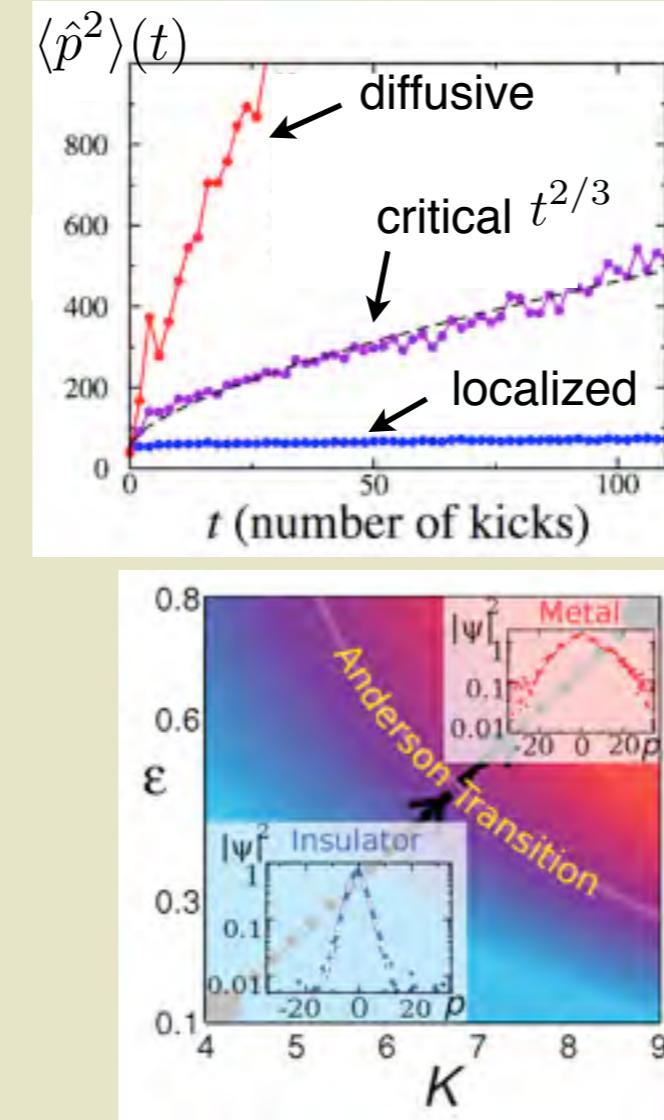
$$\hat{H} = \frac{\hat{p}^2}{2} + K \cos \hat{x} [1 + \epsilon \cos(\omega_2 t) \cos(\omega_3 t)] \sum_n \delta(t - n)$$

quasi-periodic kicked rotor

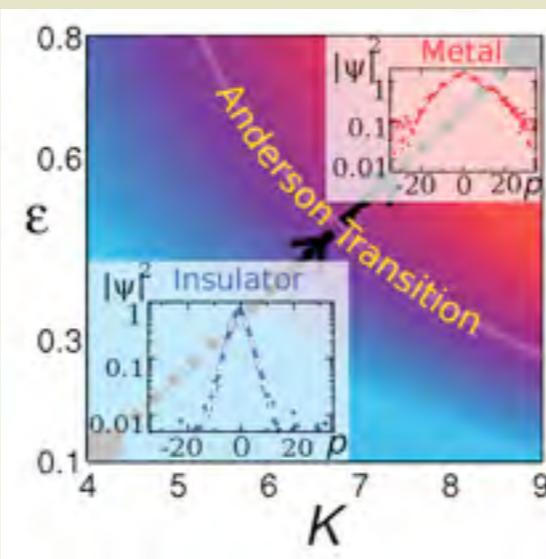
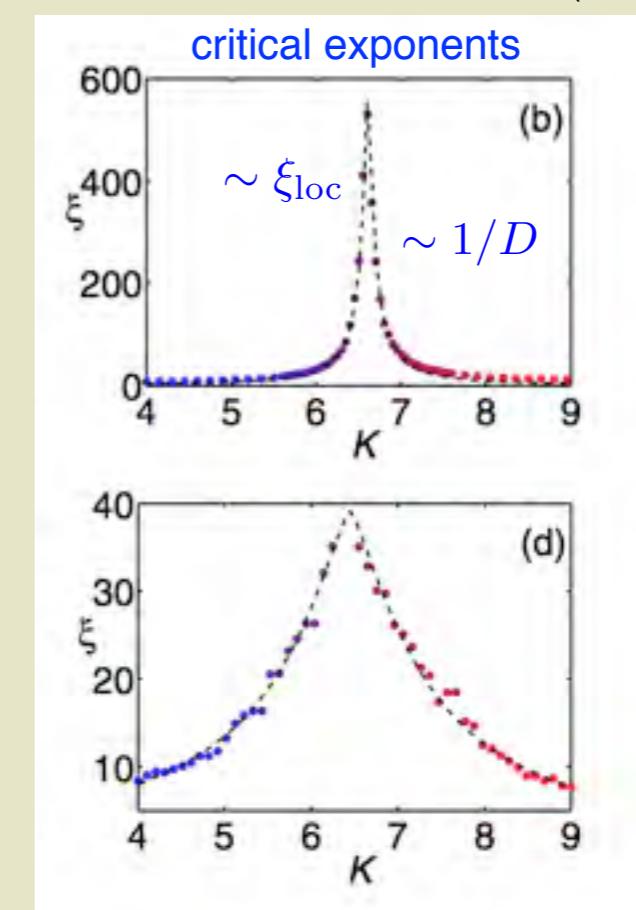
$\epsilon = 0$  equivalent to **1d disordered system**



$\epsilon \neq 0$  equivalent to **3d disordered system**



Chabé *et al.* (2008)

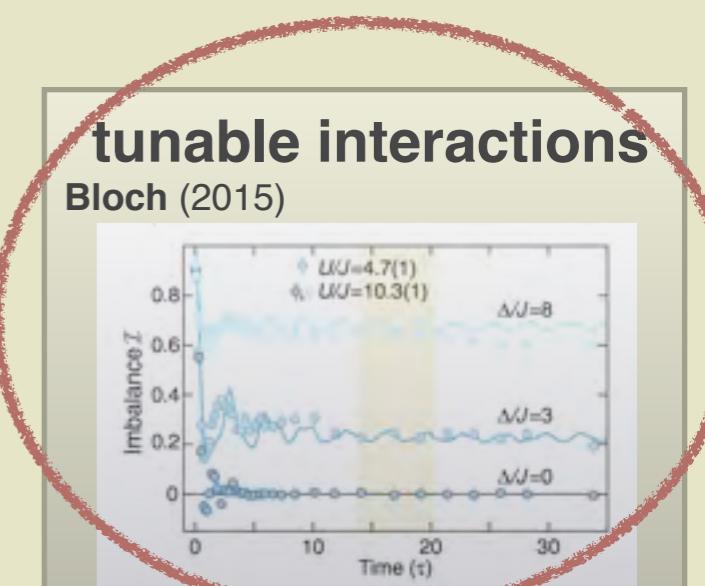
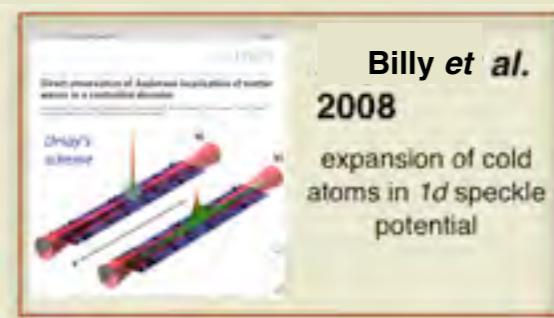
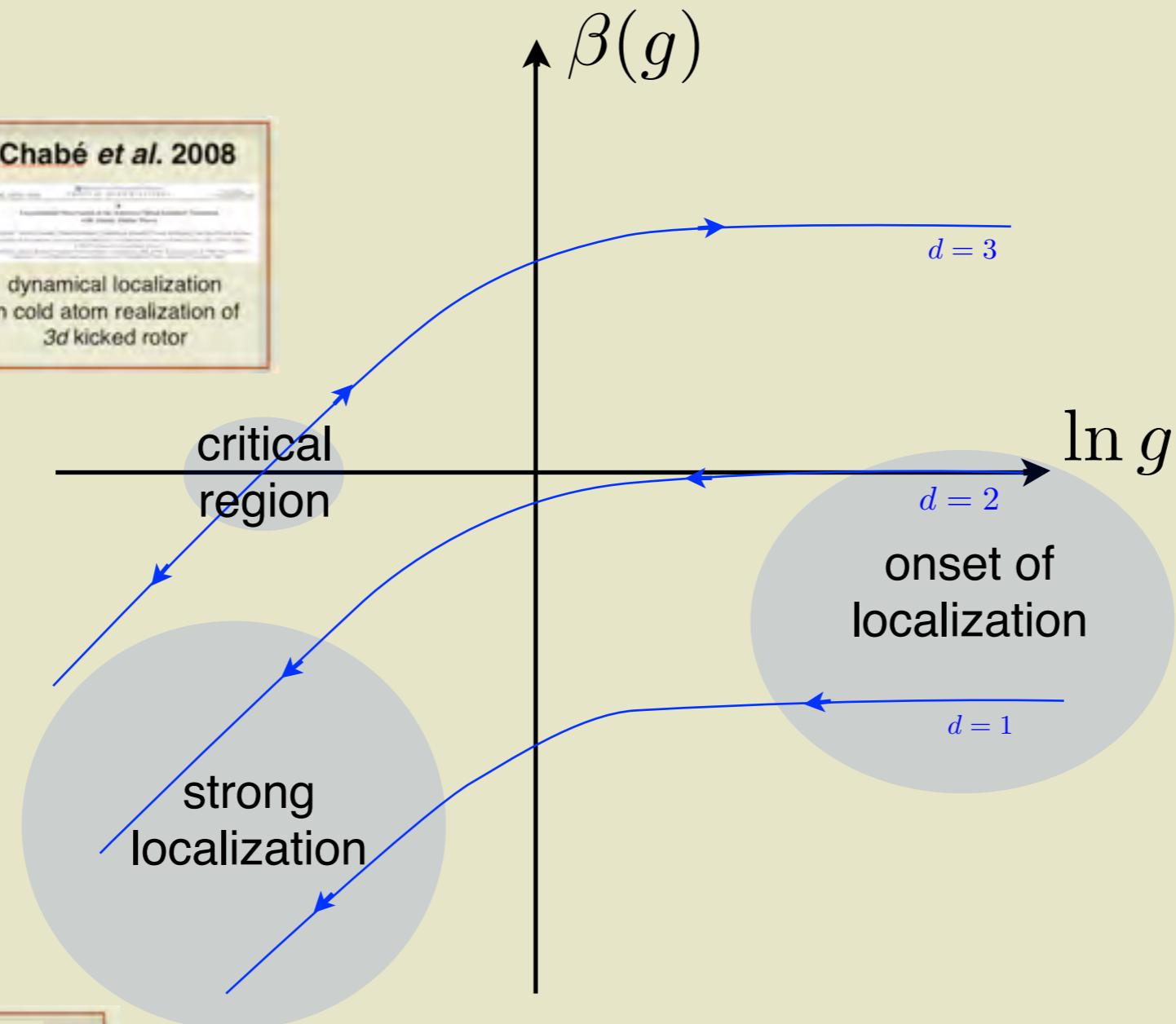


$$\xi \sim |K - K_c|^{-\nu}$$

$\nu = 1.60$  (theory)  
 $\nu = 1.4 \pm 0.3$  (experiment)

see also Lemar  e *et al.* (2010)

# some recent cold atom experiments



# many-body localization of interacting atoms in quasi-random potential?

## Observation of many-body localization of interacting fermions in a quasi-random optical lattice

Michael Schreiber<sup>1,2</sup>, Sean S. Hodgman<sup>1,2</sup>, Pranjal Bordia<sup>1,2</sup>, Henrik P. Lüschen<sup>1,2</sup>, Mark H. Fischer<sup>3</sup>, Ronen Vosk<sup>3</sup>, Ehud Altman<sup>3</sup>, Ulrich Schneider<sup>1,2</sup> and Immanuel Bloch<sup>1,2</sup>

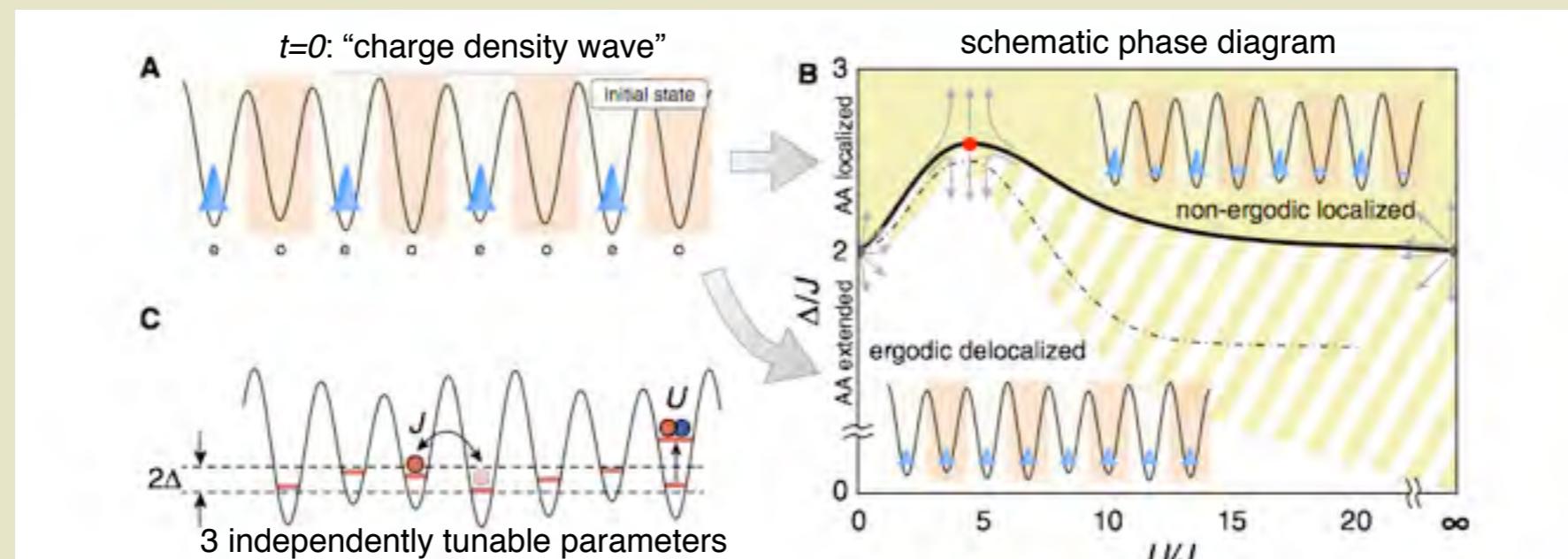
<sup>1</sup>Fakultät für Physik, Ludwig-Maximilians-Universität München, Schellingstr. 4, 80799 Munich, Germany

<sup>2</sup>Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1, 85748 Garching, Germany

<sup>3</sup>Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel

does localized phase withstand interactions in a **closed system**?

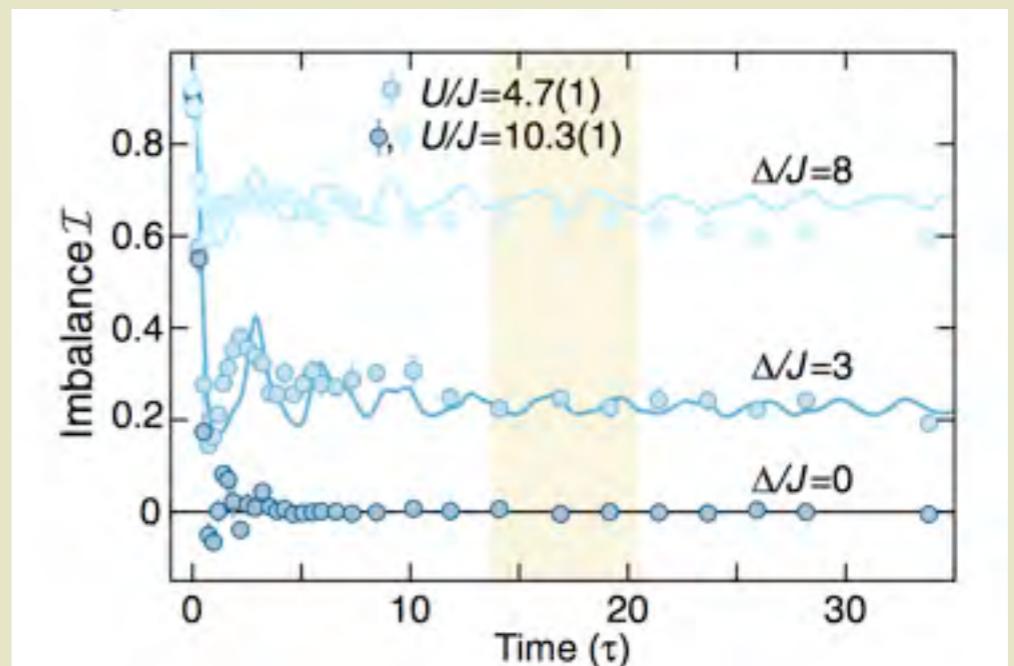
does an initial local perturbation (here density) decay to zero?



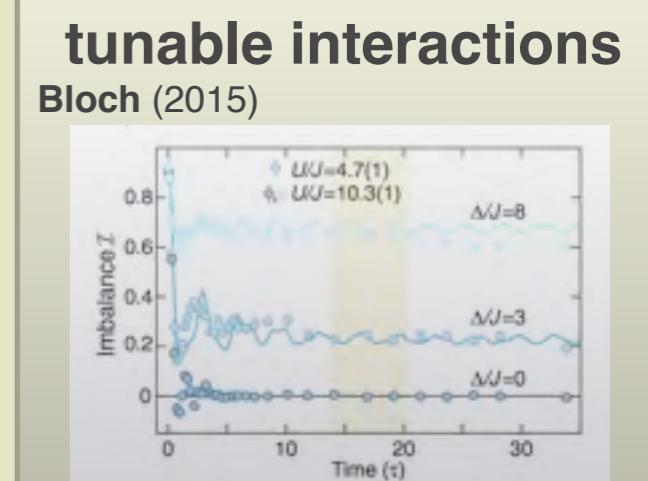
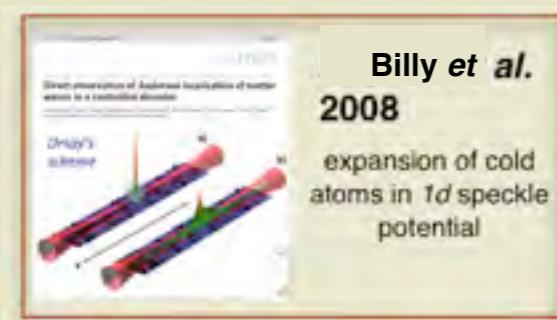
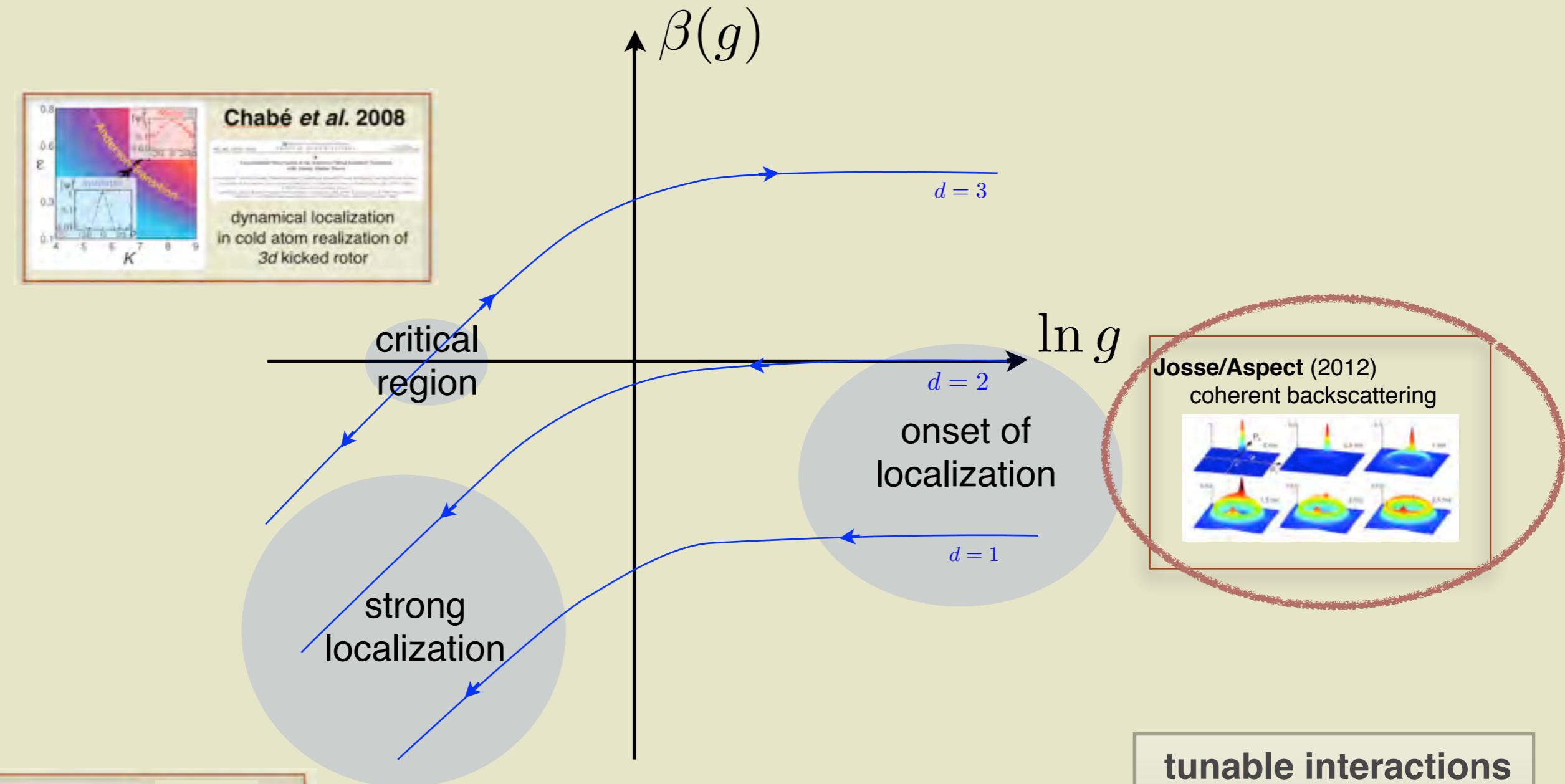
imbalance:

$$I(t) = \frac{N_e(t) - N_o(t)}{N_e(t) + N_o(t)} \neq 0 ?$$

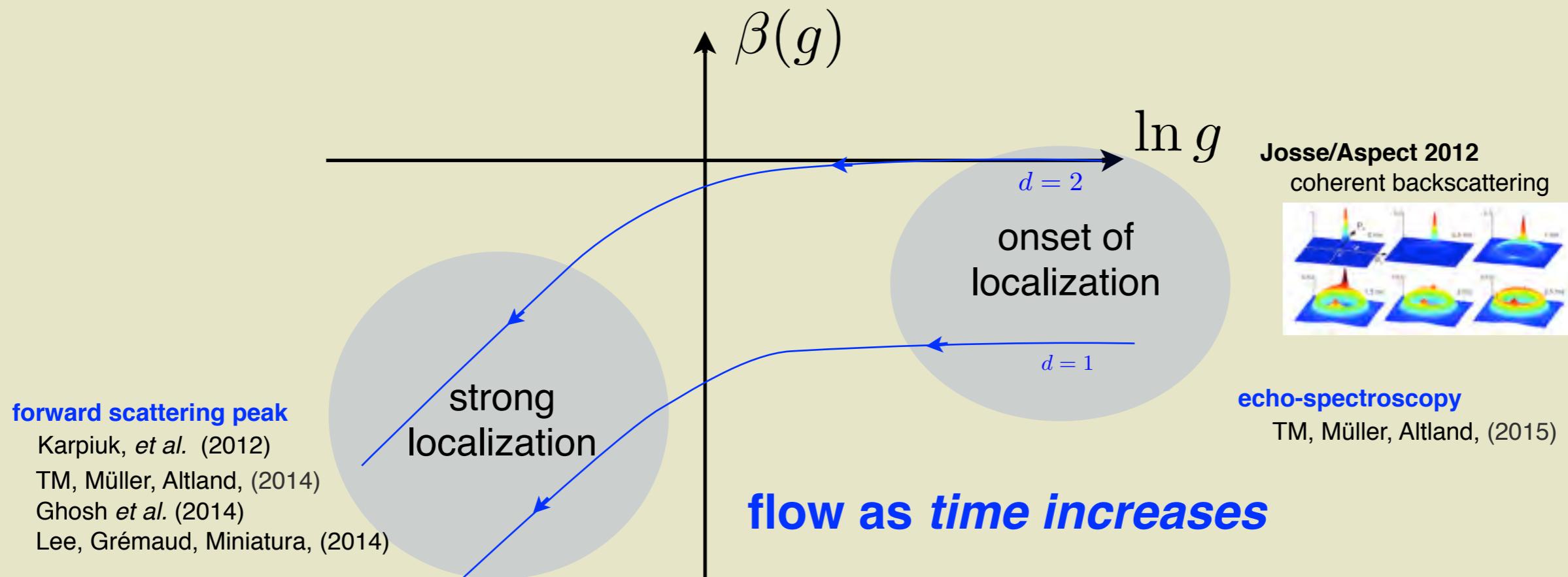
as  $t \rightarrow \infty$



# some recent cold atom experiments



# in this talk: cold atom “quench experiment”



## quantum quench experiment:

- controlled observation of strong Anderson localization
- control parameter: time

# Quantum Quench Experiment

PRL 109, 195302 (2012)

 Selected for a **Viewpoint** in *Physics*  
PHYSICAL REVIEW LETTERS

week ending  
9 NOVEMBER 2012



## Coherent Backscattering of Ultracold Atoms

F. Jendrzejewski,<sup>1</sup> K. Müller,<sup>1</sup> J. Richard,<sup>1</sup> A. Date,<sup>1</sup> T. Plisson,<sup>1</sup> P. Bouyer,<sup>2</sup> A. Aspect,<sup>1</sup> and V. Josse<sup>1,\*</sup>

<sup>1</sup>*Laboratoire Charles Fabry UMR 8501, Institut d'Optique, CNRS, Univ Paris Sud 11, 2 Avenue Augustin Fresnel, 91127 Palaiseau cedex, France*

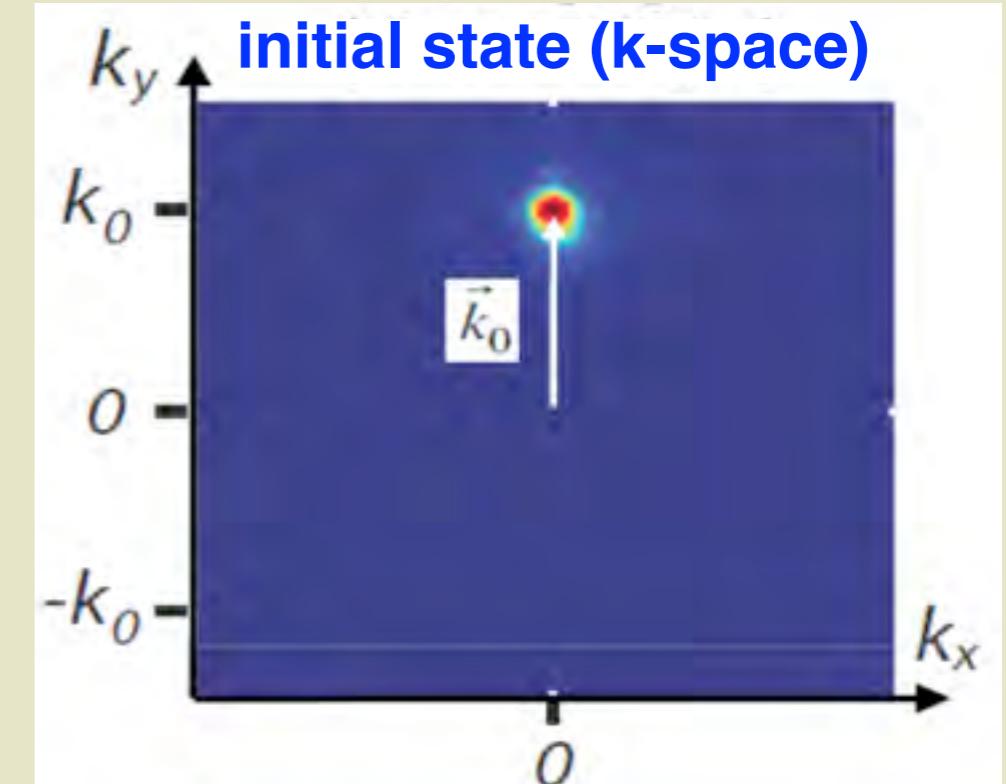
<sup>2</sup>*LP2N UMR 5298, Univ Bordeaux 1, Institut d'Optique and CNRS, 351 cours de la Libération, 33405 Talence, France*

(Received 19 July 2012; published 5 November 2012)

# Quantum Quench Experiment

## preparation of initial state:

1. cooling of atomic cloud in optical dipole trap, BEC of atoms in  $F = 2, m_F = -2$  ground sublevel
2. suppress interatomic interactions by releasing atomic cloud and letting it expand
3. freeze motion of atoms by switching on harmonic potential for well chosen amount of time almost
4. give atoms finite momentum without changing spread by applying additional magnetic gradient during 12 ms



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### Coherent Backscattering of Ultracold Atoms

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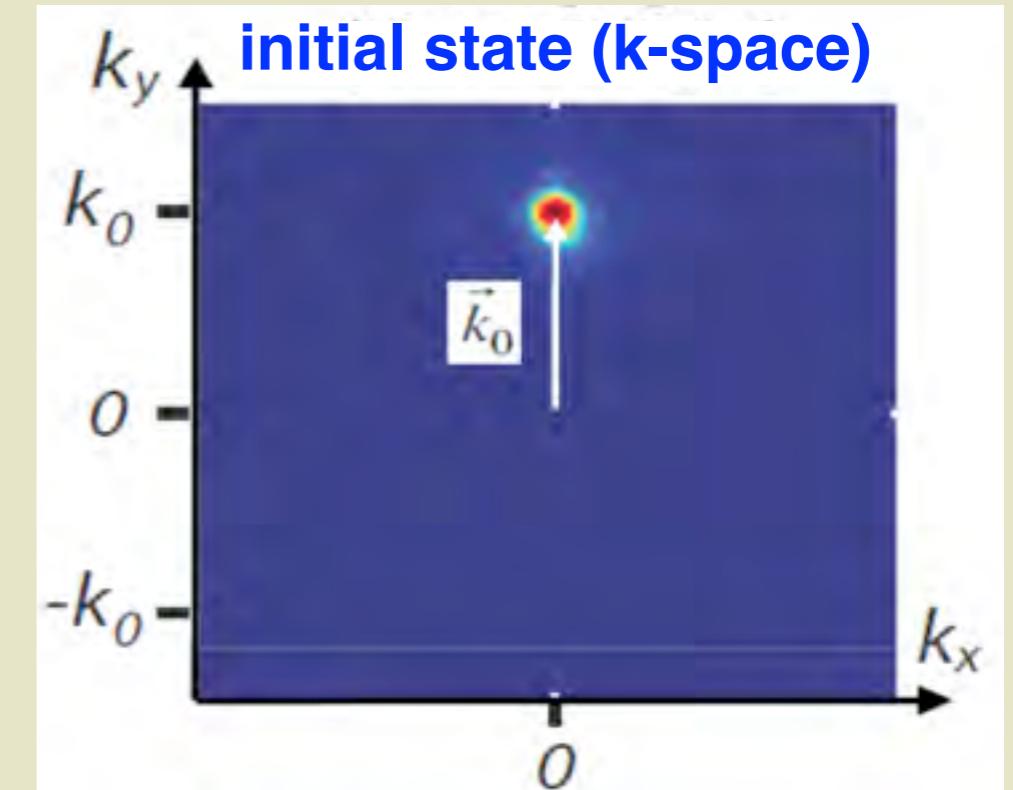
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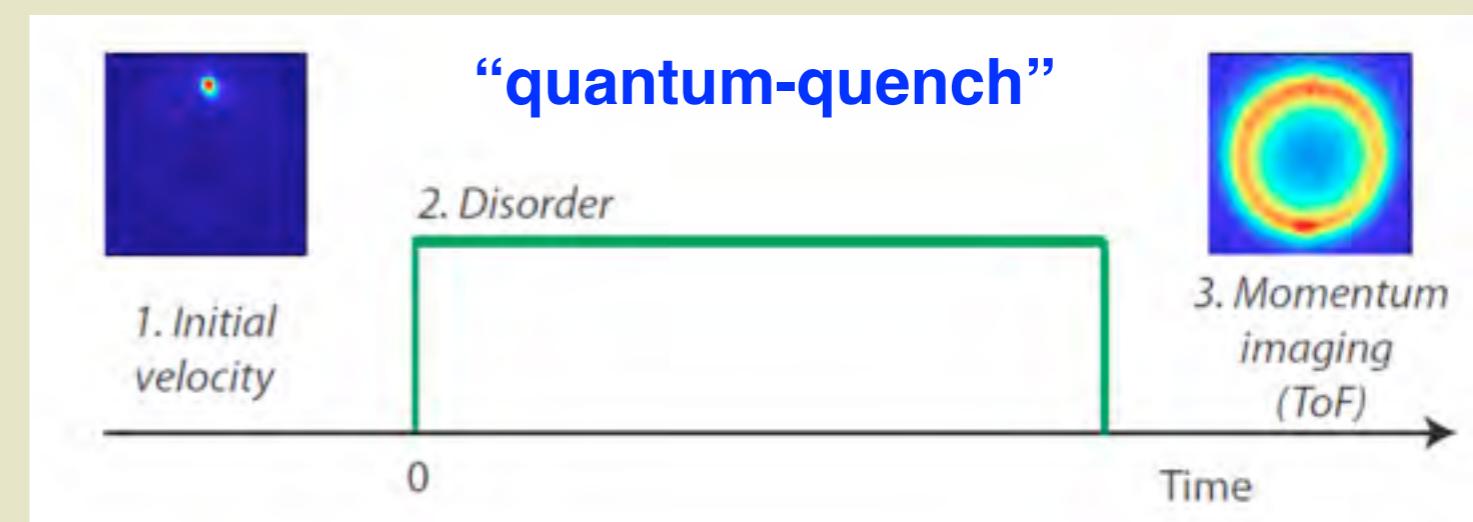
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4. give atoms finite momentum without changing spread by applying additional magnetic gradient during 12 ms



## experiment:

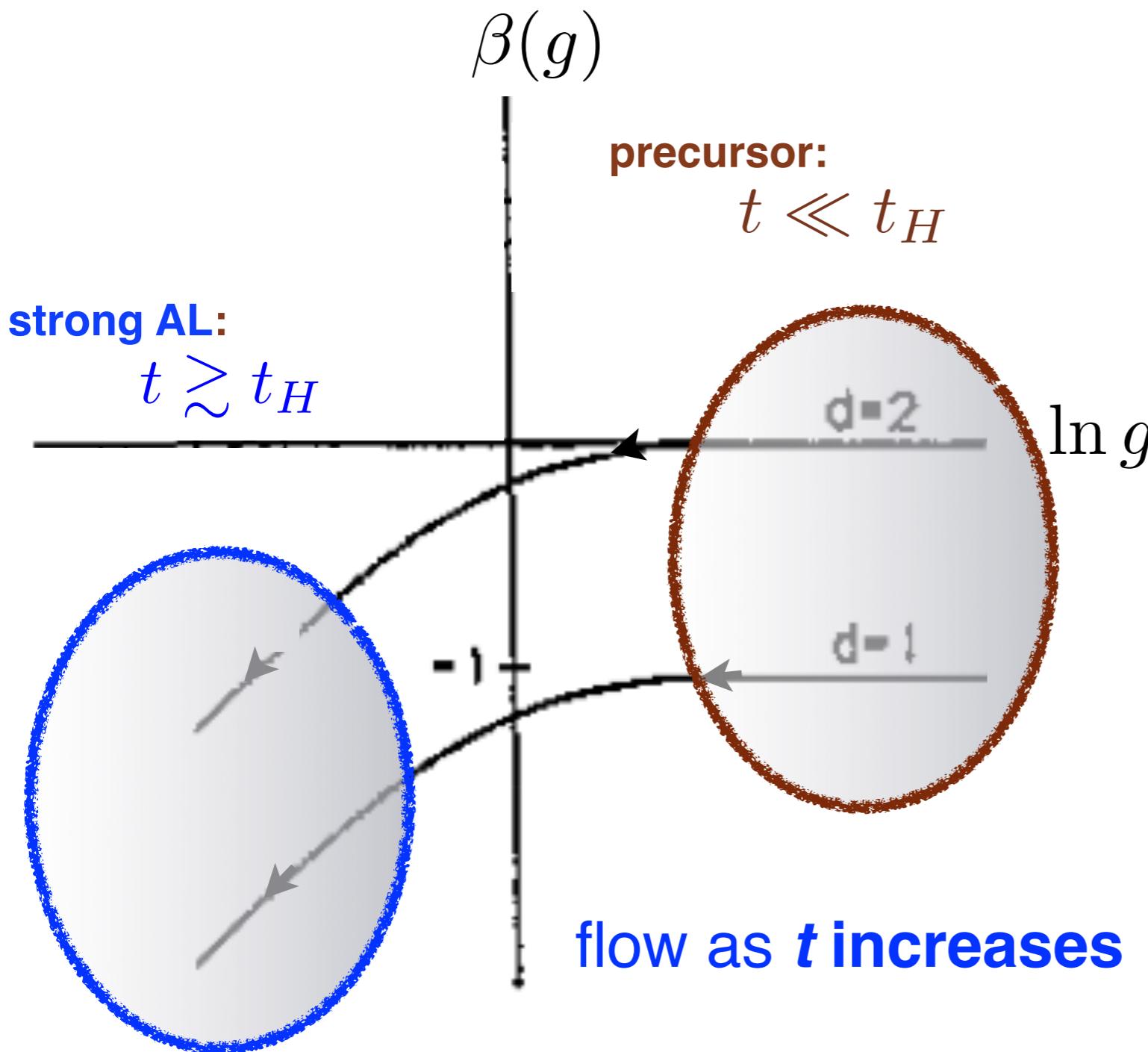
- release atoms from optical trap and suspend against gravity by magnetic levitation
- switch on anisotropic laser speckle disordered potential (2d: elongated along one axis)
- let atoms scatter for a time  $t$
- switch off the disorder and monitor momentum distribution at time  $t$  (time of flight imaging)



low densities... “it’s all single-particle physics”

# study momentum-correlations in time

observable:  $\mathcal{C}_{\mathbf{k}_i \mathbf{k}_f}(t) = |\langle \mathbf{k}_f | e^{-i \hat{H} t} | \mathbf{k}_i \rangle|^2$



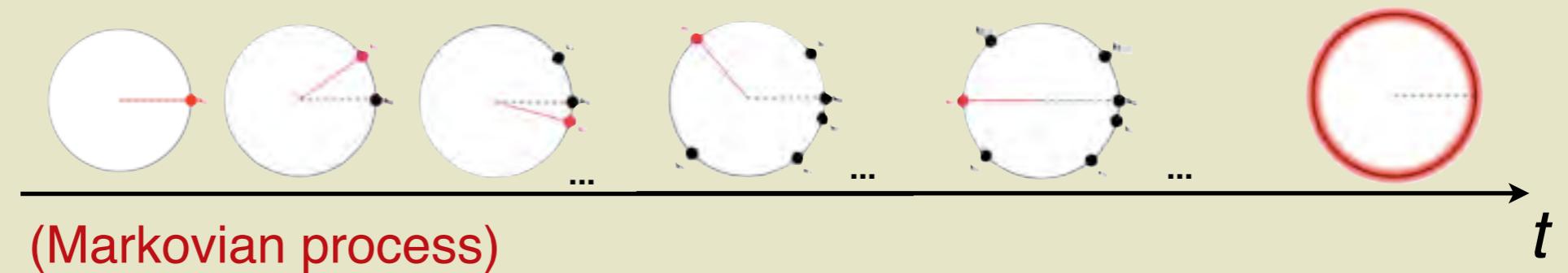
## quantum quench experiment:

- controlled observation of Anderson localization
- observable: momentum-distribution
- control parameter: time

# Classical vs. quantum diffusion



classical: isotropic redistribution of atoms over energy-shell



→ no momentum-correlations



quantum interference

(system has memory)

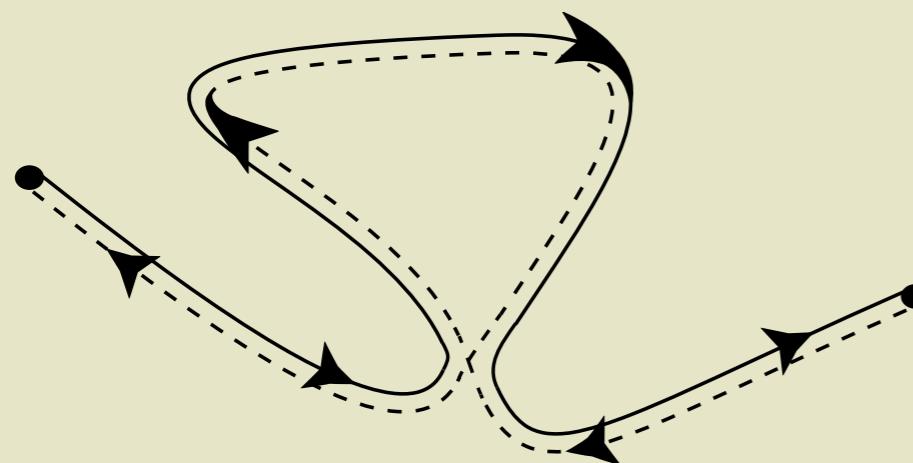
→ momentum-correlations

# Momentum-Correlations

## Precursor Effects 1

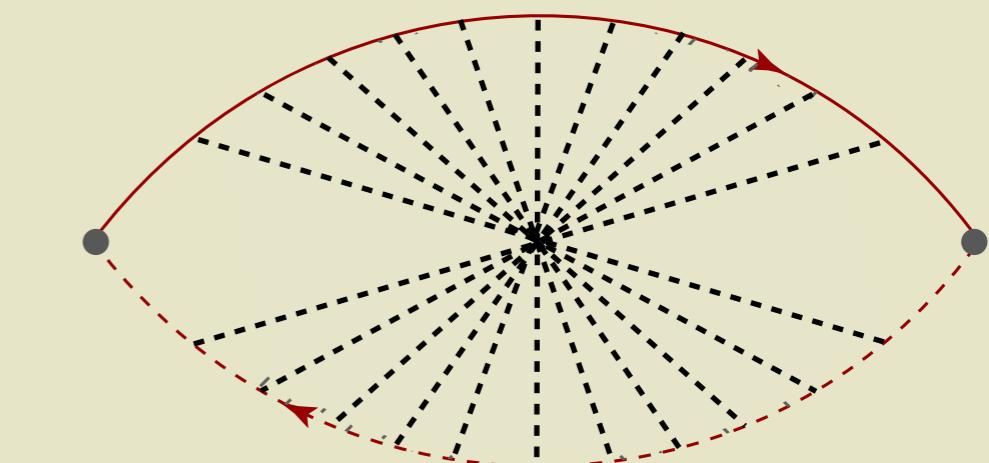
### leading contribution (orthogonal class)

space-correlations



$$P(\mathbf{r} \rightarrow \mathbf{r}, t)$$

momentum-correlations



$$P(\mathbf{k} \rightarrow -\mathbf{k}, t)$$

→ peak in **back-scattering** direction  
“coherent back-scattering”

# Coherent Backscattering peak Experiment 2012

## (orthogonal class)

PRL 109, 195302 (2012)

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PHYSICAL REVIEW LETTERS

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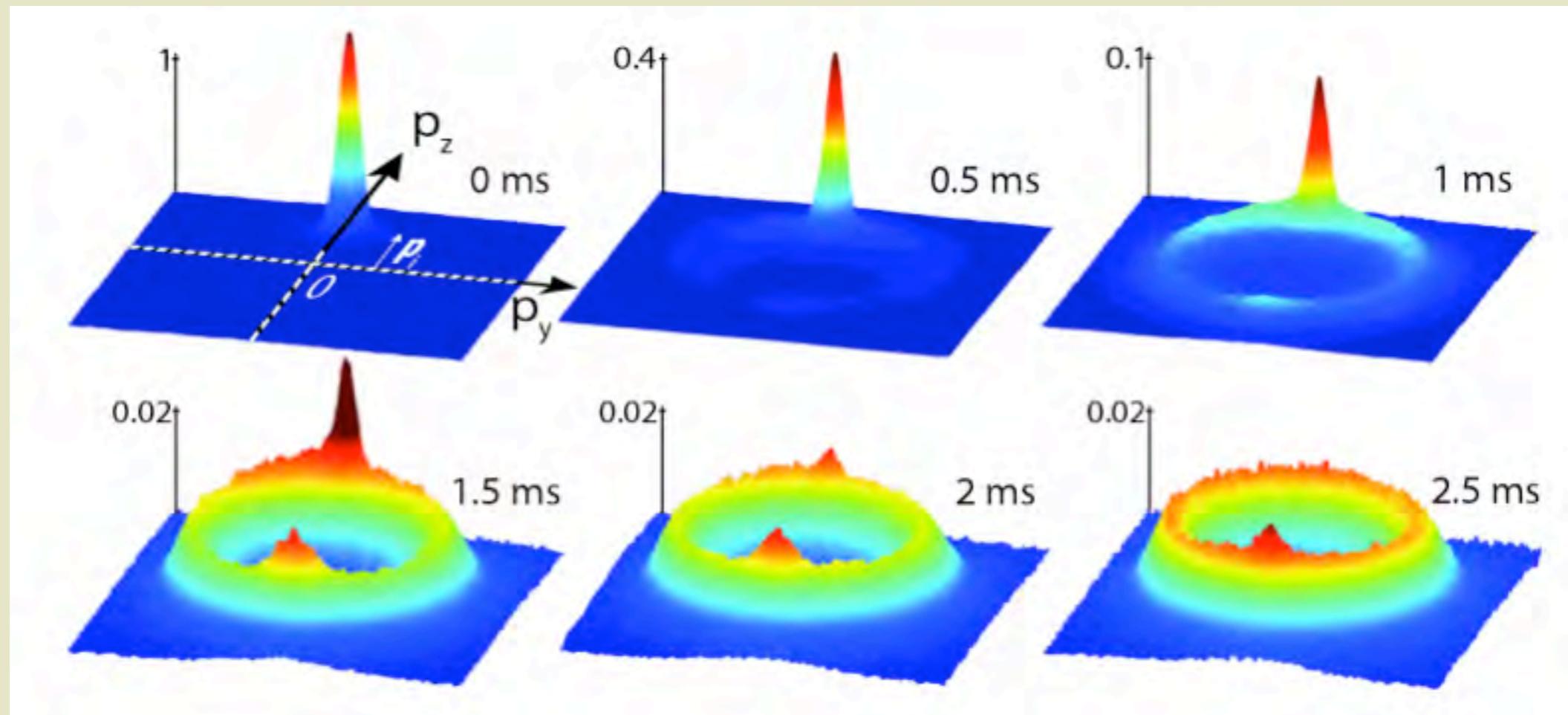
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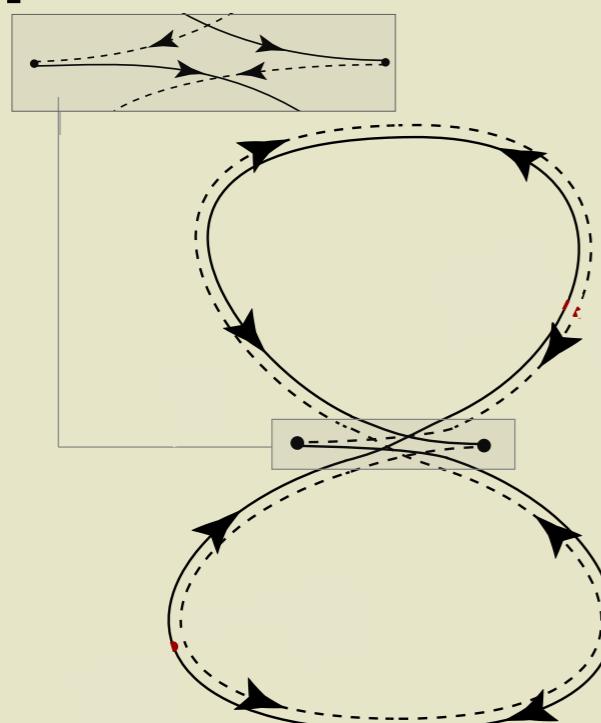


# Momentum-Correlations

## Precursor Effects 2

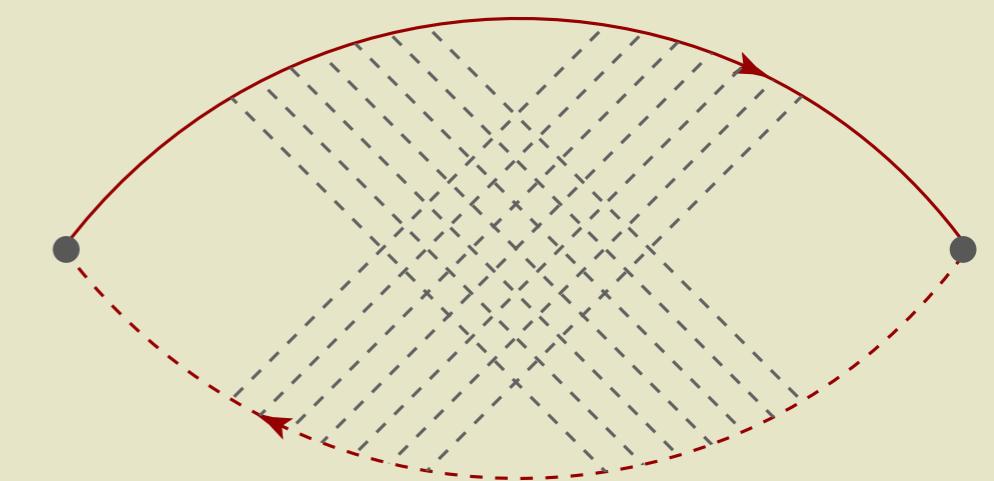
leading contribution (unitary class)

space-correlations



$$P(\mathbf{r} \rightarrow \mathbf{r}, t)$$

momentum-correlations



$$P(\mathbf{k} \rightarrow \mathbf{k}, t)$$

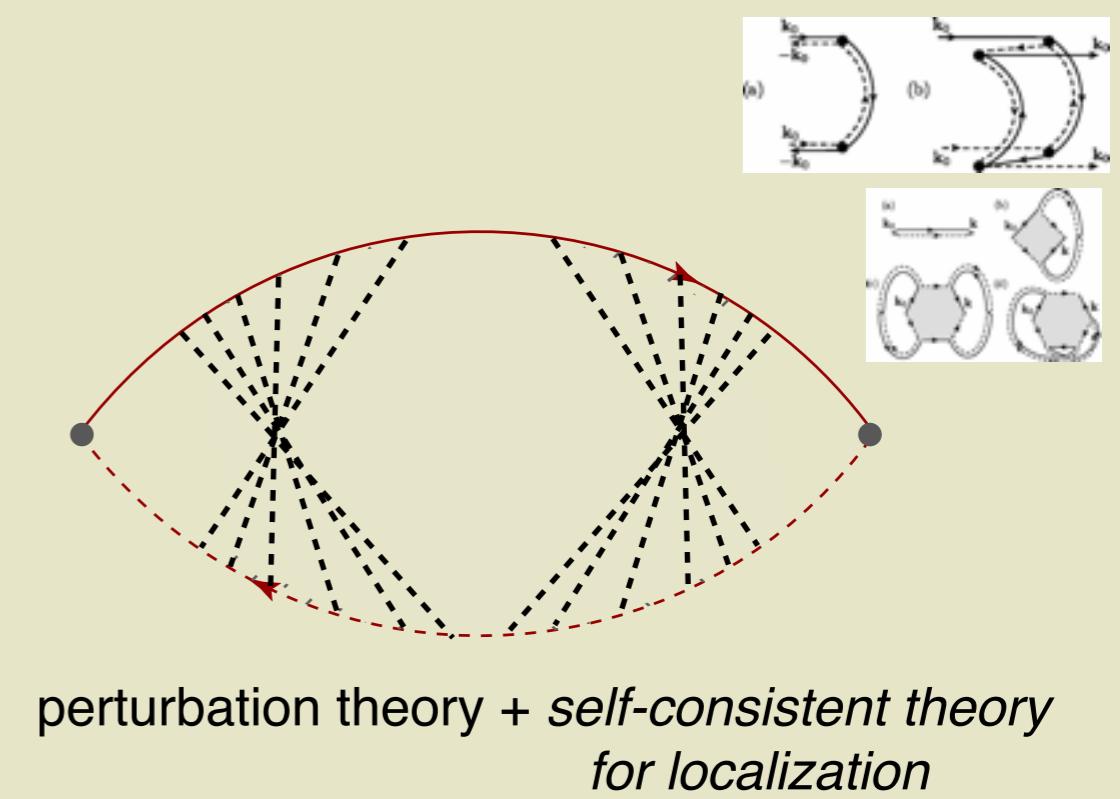
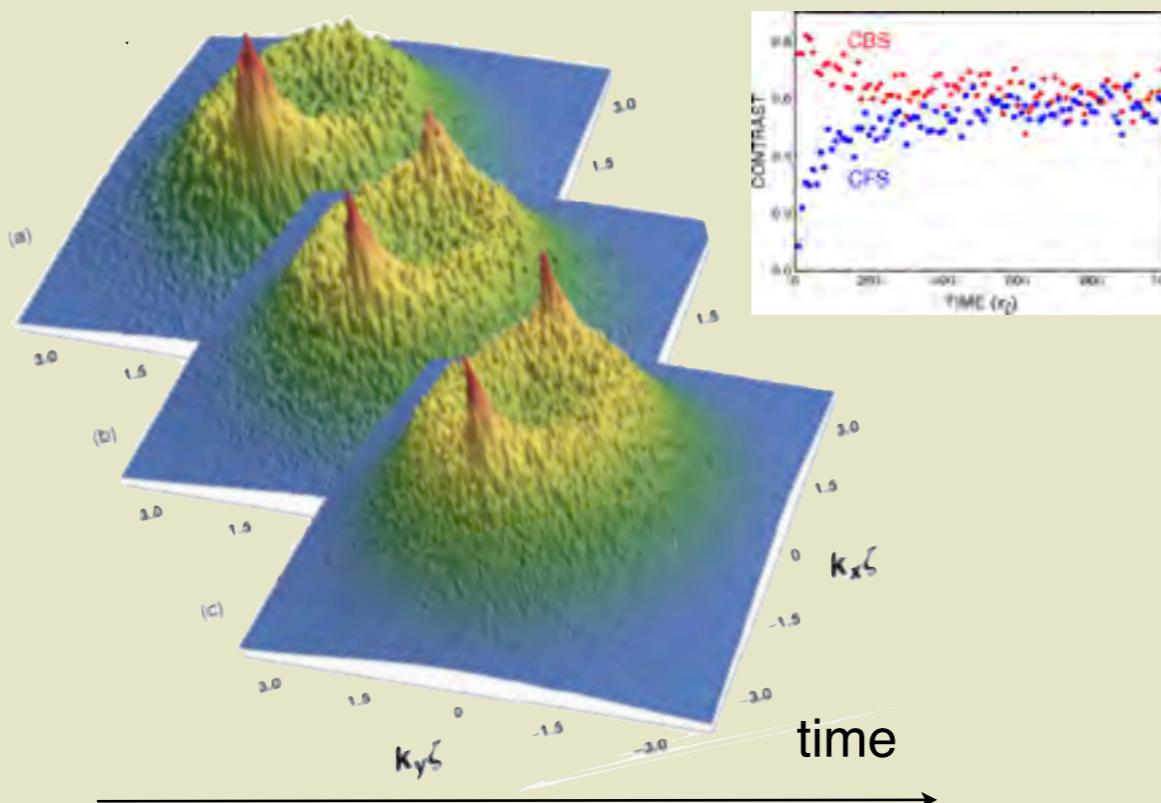


peak in **forward-scattering** direction  
“coherent forward-scattering”

# simulation + phenomenology

(2d, orthogonal class)

[T. Karpiuk, N. Cherroret, K.L. Lee, B. Grémaud,  
C.A. Müller, C. Miniatura, PRL 109, 190601 (2012)]



fwd-peak builds up on long time-scales ( $t \gtrsim t_H$ ),  
i.e. pronounced in strongly localized regime

a signature of strong localization...

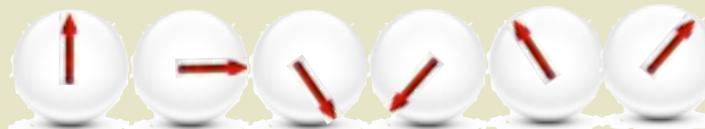
# fwd-peak: field theory

$$S[Q] = \pi\nu \int d\mathbf{x} \text{str} \left( \frac{D}{4} (\partial_{\mathbf{x}} Q)^2 - i\eta Q \Lambda \right)$$

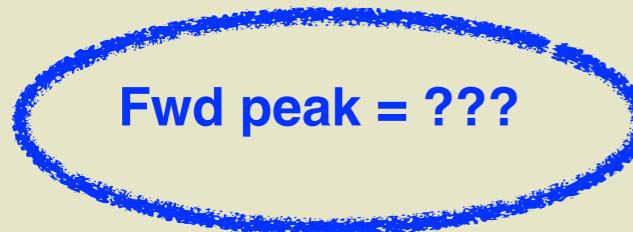
**symmetry-breaking**

**analogy:**  $H[\vec{S}] = \int d\mathbf{x} \left( J(\partial_{\mathbf{x}} \vec{S})^2 - \vec{S} \cdot \vec{h} \right) \quad \eta \sim t_H/t$

$Q(x) \in$   
 “sphere  $\times$  hyperboloid  
 dressed with Grassmann  
 variables”

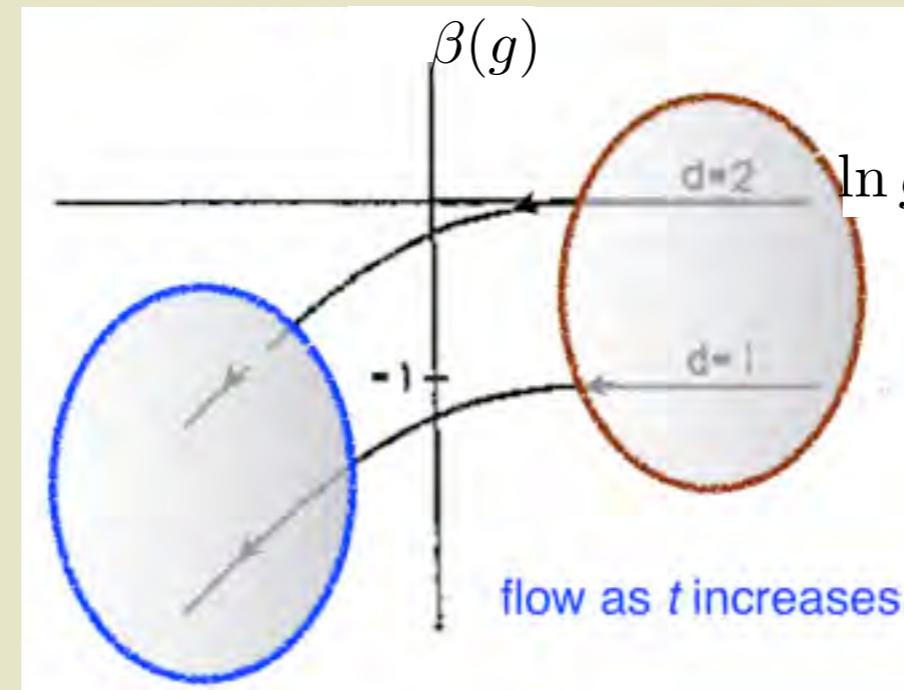


**strong AL:**  $t \gtrsim t_H$   
 (non-perturbative regime)



$$\mathbf{q} = \mathbf{k}_i - \mathbf{k}_f$$

$$\mathcal{C}(\mathbf{q}, \eta) = \delta_{\mathbf{0}, \mathbf{q}^\perp} \langle \text{tr} (\mathcal{P}_{-+} Q(q) \mathcal{P}_{+-} Q(-q)) \rangle_S$$



**precursor:**  $t \ll t_H$   
 (perturbative regime,  
 Goldstone-modes = diffusion)

Fwd peak = + ...

$$\propto \sqrt{t/t_H}$$

$$\langle \dots \rangle_S = \int \mathcal{D}Q e^{S[Q]}(\dots)$$

# solution strategy

quasi 1d geometry:

$$S[Q] = \int dx \left( \alpha \text{str}(\dot{Q}^2) + V(Q) \right)$$

$$\alpha \text{str}(\dot{Q}^2) = \pi \nu \text{str} \left( \frac{D}{4} (\partial_x Q)^2 \right)$$

$$V[Q] = -i\pi\nu\eta \text{str}(Q\Lambda)$$

■ Functional integral  $\mapsto$  “Schrödinger equation”

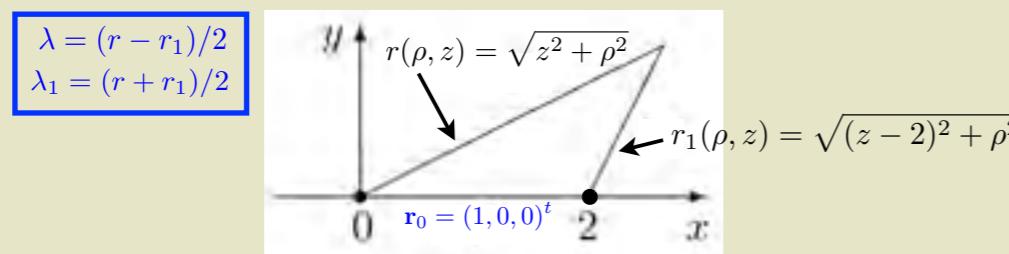
radial-symmetric  $V[Q]$ : few relevant variables  $\lambda_1, \lambda$

$$\mathcal{C}(q, \eta) \propto \int_1^\infty \int_{-1}^1 \frac{d\lambda_1 d\lambda}{\lambda_1 - \lambda} [\Psi_1^q(\lambda_1, \lambda) + \Psi_1^{-q}(\lambda_1, \lambda)] \Psi_0(\lambda_1, \lambda)$$

$$(\Delta_Q + V(\lambda_1, \lambda)) \Psi_0 = 0$$

$$(2\Delta_Q + 2V(\lambda_1, \lambda) - iq\xi_{\text{loc}}) \Psi_1^q = (\lambda_1 - \lambda) \Psi_0$$

■ elliptic coordinates  $\mapsto$  “3d Coulomb-problem”



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Local correlations of different eigenfunctions in a disordered wire

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Submitted 30 November 2006

$$\hat{H}_0 = -\frac{r_1^2 r}{2} \left[ \Delta_0 - \frac{2\kappa}{r} \right] \frac{1}{r_1}$$

$$\Delta_0 \equiv \partial_z^2 + \rho^{-1} \partial_\rho \rho \partial_\rho$$

$$\kappa \equiv -i\eta t_H/2$$

## 3d-Laplace + 1/r-potential

# fwd-peak from Coulomb-problem

using ***3d*-variables**:

$$\mathcal{C}(0, \omega) \propto \int \frac{d\mathbf{r}}{r} \Phi_0(\mathbf{r}) \Phi_1(\mathbf{r}) \quad \left( \partial_{\mathbf{r}}^2 + \frac{\kappa}{r} \right) \Phi_0(\mathbf{r}, t) = 0$$

$$\left( \partial_{\mathbf{r}}^2 + \frac{\kappa}{r} \right) \Phi_1(\mathbf{r}, t) = \frac{1}{r} \Phi_0(\mathbf{r}, t)$$



$$\begin{aligned} \mathcal{C}(0, \eta) &= 32\pi\xi \langle \mathbf{r}_0 | \hat{G}_0 \frac{1}{\hat{r}} \hat{G}_0 \frac{1}{\hat{r}} \hat{G}_0 | \mathbf{r}_0 \rangle \\ &= 8\pi\xi \partial_{\kappa}^2 G_0(\mathbf{r}_0, \mathbf{r}_0) \end{aligned}$$

Green's function for ***3d* non-relativistic Coulomb-problem**:

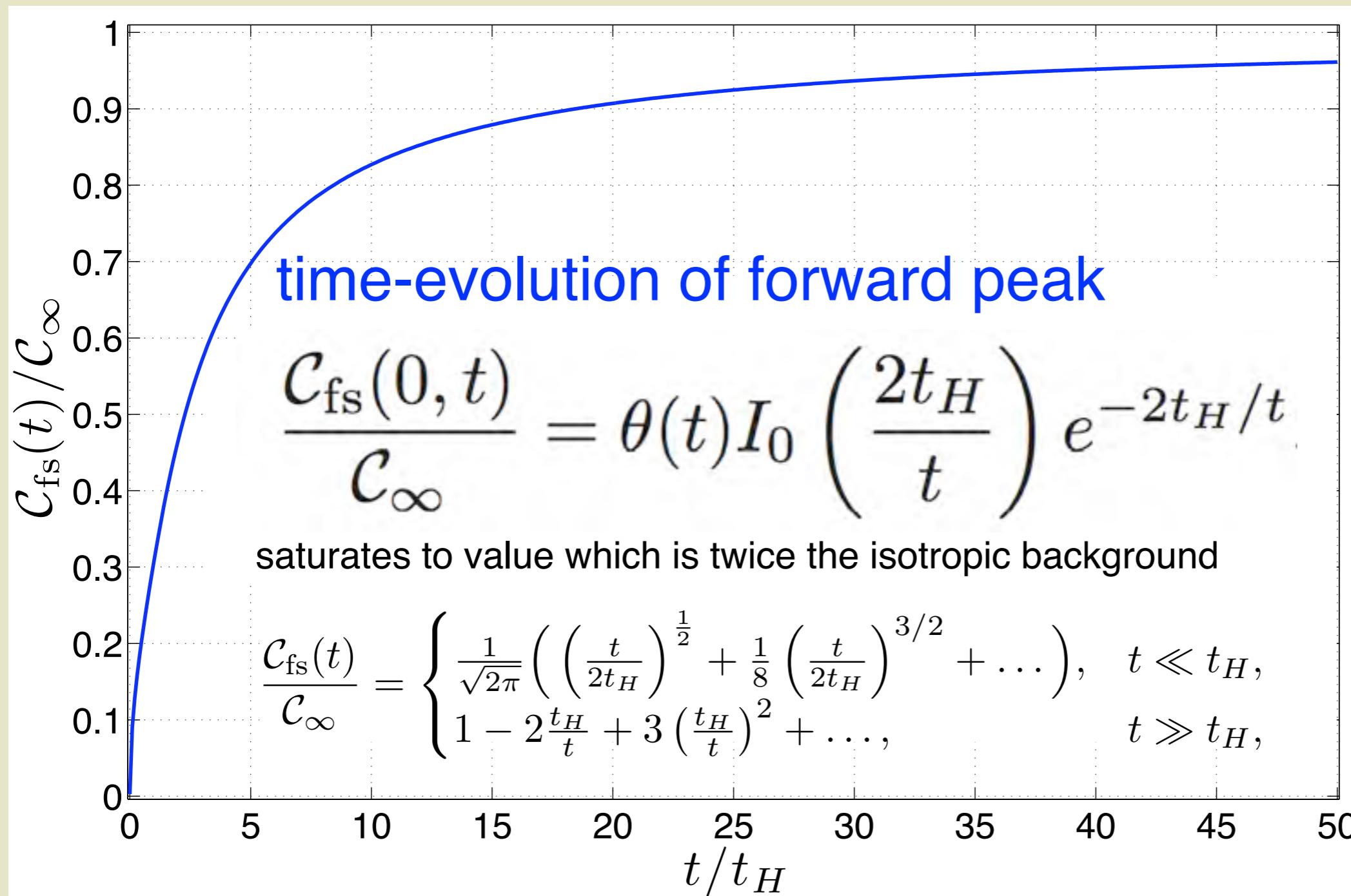
$$\left[ \Delta_0 - \frac{2\kappa}{r} \right] G_0(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}')$$

# analytical results

**L. Hostler (1964):**

$$G_0(\mathbf{r}, \mathbf{r}') = \frac{(\partial_u - \partial_v)\sqrt{u}K_1(2\sqrt{\kappa u})\sqrt{v}I_1(2\sqrt{\kappa v})}{2\pi|\mathbf{r} - \mathbf{r}'|}, \quad u = r + r' + |\mathbf{r} - \mathbf{r}'|$$

$$v = r + r' - |\mathbf{r} - \mathbf{r}'|$$

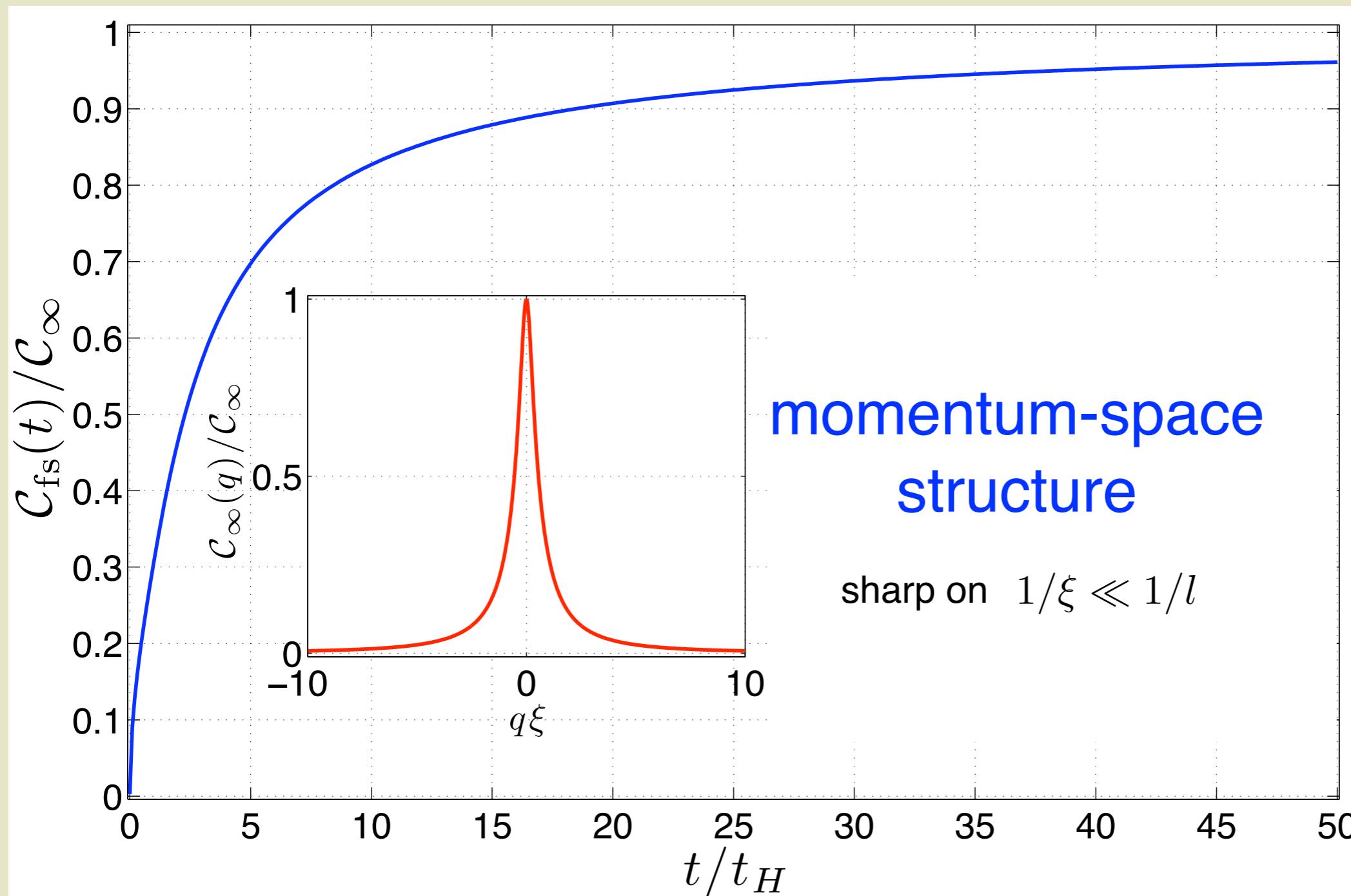


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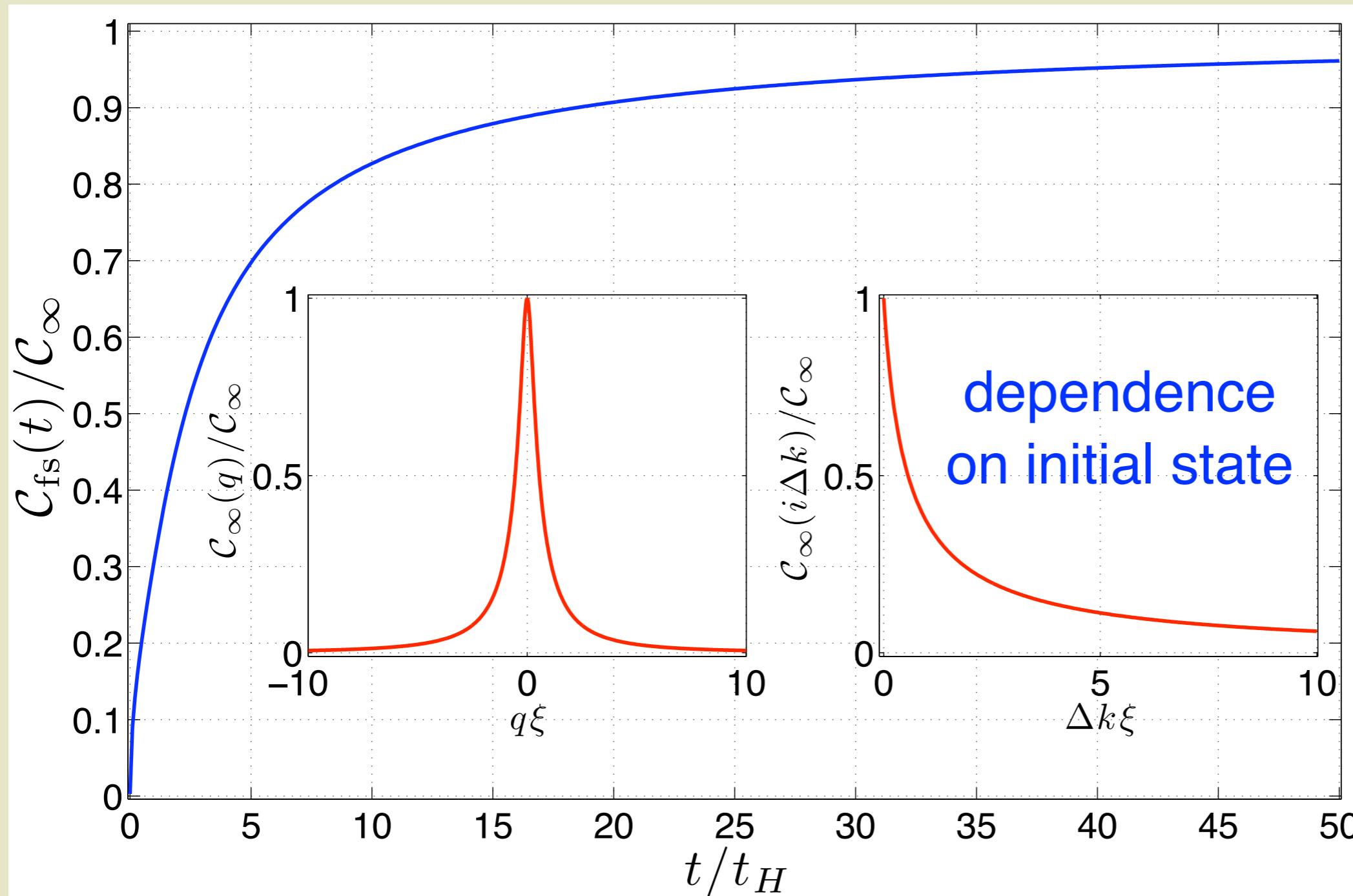
for details see:  
 T. M., C. A. Müller, A. Altland,  
 Phys. Rev. Lett. **112**, 110602  
 (2014)

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# fwd-peak and level statistics

$\mathcal{C}_{\text{fs}}(t)$  is the **form factor**, i.e. Fourier-transform of **level-level correlation function**  $K_2(\omega) = \frac{1}{\nu_0^2} \langle \nu(\epsilon)\nu(\epsilon + \omega) \rangle - 1$

# eigenstate-representation

$$\mathcal{C}_{\text{fs}}(t) \propto \int d\epsilon \int d\omega e^{i\omega t} \sum_{\alpha\beta} \langle |\alpha(\mathbf{k}_i)|^2 |\beta(\mathbf{k}_i)|^2 \delta(\epsilon_+ - E_\alpha) \delta(\epsilon_- - E_\beta) \rangle$$

$\epsilon_\pm = \epsilon \pm \omega/2$   
 $\alpha(\mathbf{k}) = \langle \alpha | \mathbf{k} \rangle$

## level-level correlations

$$\langle \nu(\epsilon_+) \nu(\epsilon_-) \rangle = \sum_{\alpha\beta} \langle \delta(\epsilon_+ - E_\alpha) \delta(\epsilon_- - E_\beta) \rangle$$

wave-function statistics in momentum-space is GUE independent of  $L/\xi_{\text{loc}}$

$$\frac{\mathcal{C}_{\text{fs}}}{\mathcal{C}_{\text{bgn}}} = \frac{\langle \alpha(\mathbf{k}_i) \alpha^*(\mathbf{k}_i) \alpha(\mathbf{k}_i) \alpha^*(\mathbf{k}_i) \rangle}{\langle \alpha(\mathbf{k}_i) \alpha^*(\mathbf{k}) \alpha(\mathbf{k}) \alpha^*(\mathbf{k}_i) \rangle} = 2$$

saturation value as  $t \gg t_H$

# fwd-peak and level statistics

$\mathcal{C}_{\text{fs}}(t)$  is the **form factor**, i.e. Fourier-transform of **level-level correlation function**  $K_2(\omega) = \frac{1}{\nu_0^2} \langle \nu(\epsilon)\nu(\epsilon + \omega) \rangle - 1$

$$\frac{\mathcal{C}_{\text{fs}}(t)}{\mathcal{C}_\infty} = \begin{cases} \frac{1}{\sqrt{2\pi}} \left( \left( \frac{t}{2t_H} \right)^{\frac{1}{2}} + \frac{1}{8} \left( \frac{t}{2t_H} \right)^{3/2} + \dots \right), & t \ll t_H, \\ 1 - 2\frac{t_H}{t} + 3 \left( \frac{t_H}{t} \right)^2 + \dots, & t \gg t_H, \end{cases}$$

$K_2(\omega) \propto \delta(\omega)$  ... self-correlations
 $K_2(\omega) \propto \omega^{-3/2}$ 
  
Mott scale (2 resonant levels)

spectral correlations of a *finite-size* Anderson insulator

$$K_L(\omega) = -\frac{\xi_{\text{loc}}}{L} \mathcal{K}(4\omega/\Delta_\xi),$$

$$\mathcal{K}(z) = \text{Re} \frac{8}{\sqrt{iz}} \left( K_1(\sqrt{iz}) I_0(\sqrt{iz}) - K_0(\sqrt{iz}) I_1(\sqrt{iz}) \right)$$

# fwd-peak: long-time asymptotics

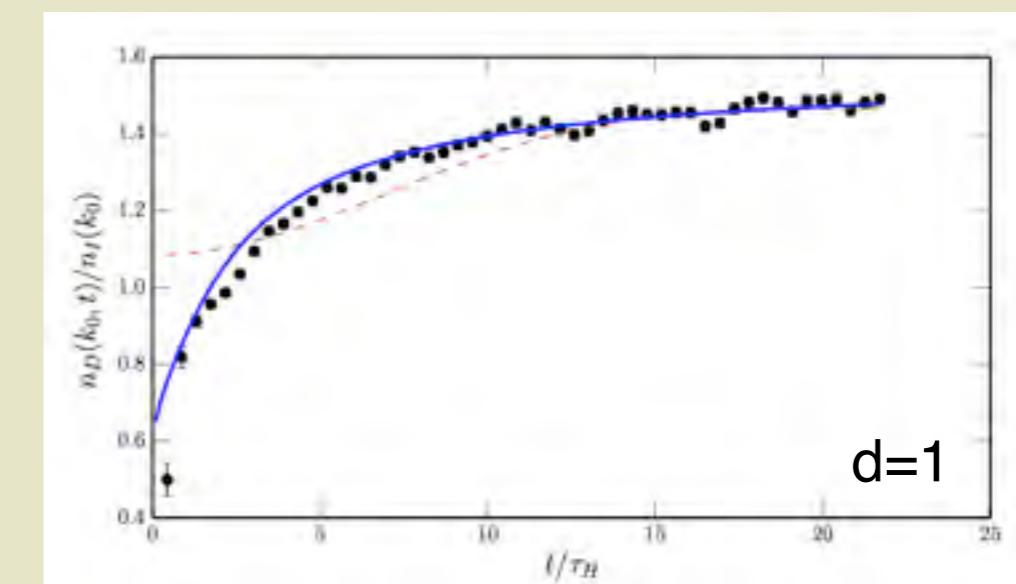
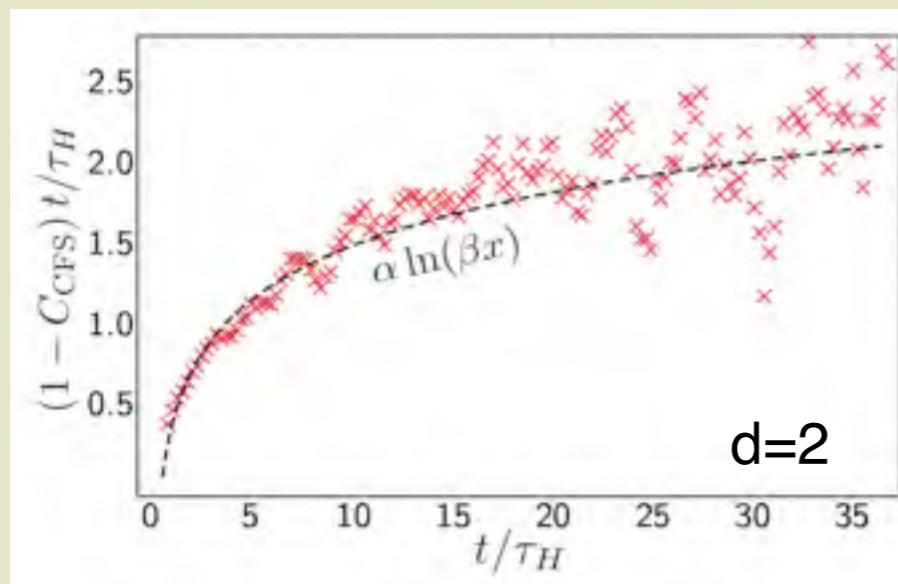
“two resonant levels”

$$H = \begin{pmatrix} \epsilon + \delta\epsilon & \Delta_\xi e^{-\frac{|\mathbf{x}-\mathbf{x}'|}{\xi_{\text{loc}}}} \\ \Delta_\xi e^{-\frac{|\mathbf{x}-\mathbf{x}'|}{\xi_{\text{loc}}}} & \epsilon - \delta\epsilon \end{pmatrix}$$

$\Rightarrow K(\omega) \propto -\left(\frac{\xi_{\text{loc}}}{L}\right)^d \ln^d (\omega/\Delta_\xi)$

$$\frac{\mathcal{C}_{\text{fs}}(t)}{\mathcal{C}_\infty} = 1 - \frac{\gamma_1 t_H}{t} \ln^{d-1}(\gamma_2 t/t_H)$$

long-time  
asymptotic  
general  $d$



[S. Ghosh, N. Cherroret, B. Grémaud, C. Miniatura, D. Delande, Phys. Rev. A 90, 063602 (2014)]

[K. L. Lee, B. Grémaud, C. Miniatura, Phys. Rev. A 90, 043605 (2014)]

**experiments: still challenging...**

# tunability of cold atoms

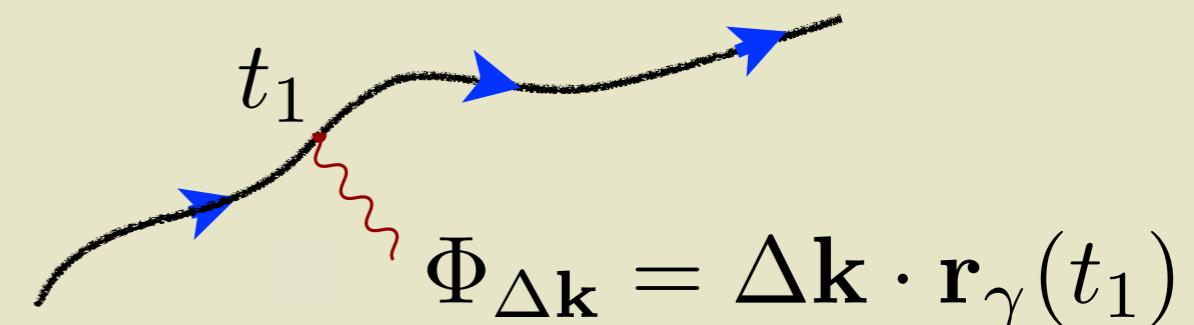
full control of system parameters opens new ways to study localization phenomena  
possible to **manipulate system on short time scales!** (compared to elastic scattering time)

## a proposal to study onset of localization

[TM, C. A. Müller, A. Altland, PRB. 91, 064203 (2015)]

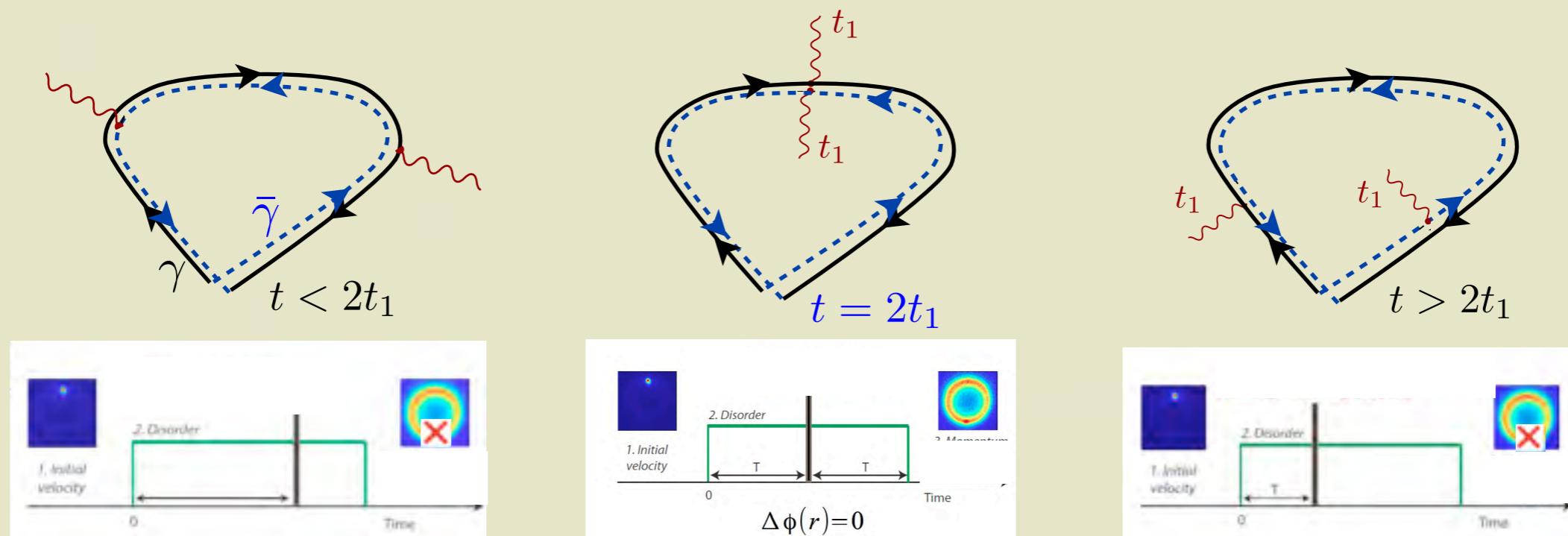
### in the quench experiment:

- release atoms from trap, **suspend against gravity by magnetic levitation**
- possible to change magnetic field on short time scales
- field pulse **weak enough** to not change the path of atom
- atoms pick up a **coordinate-dependent phase**

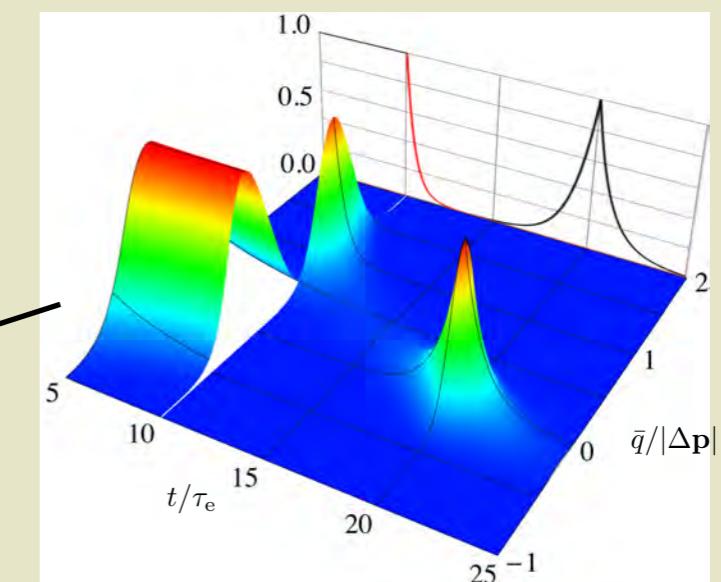
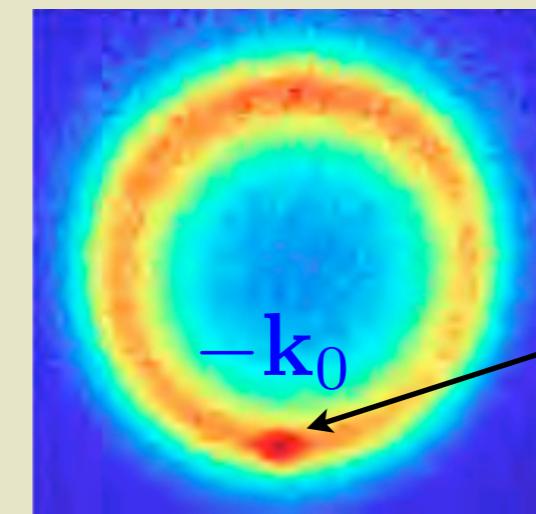


# Coherent backscattering echo (orthogonal class)

a single pulse at  $t = t_1$



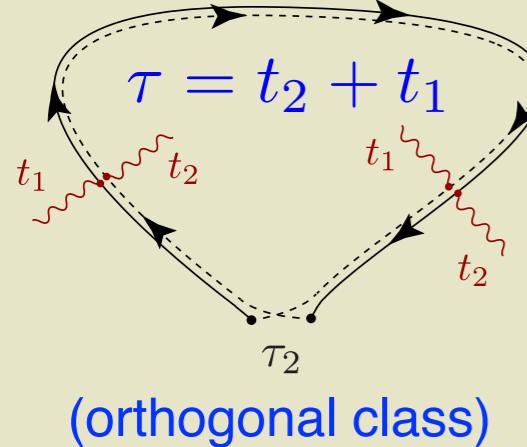
echo-  
structure in  
momentum-  
space



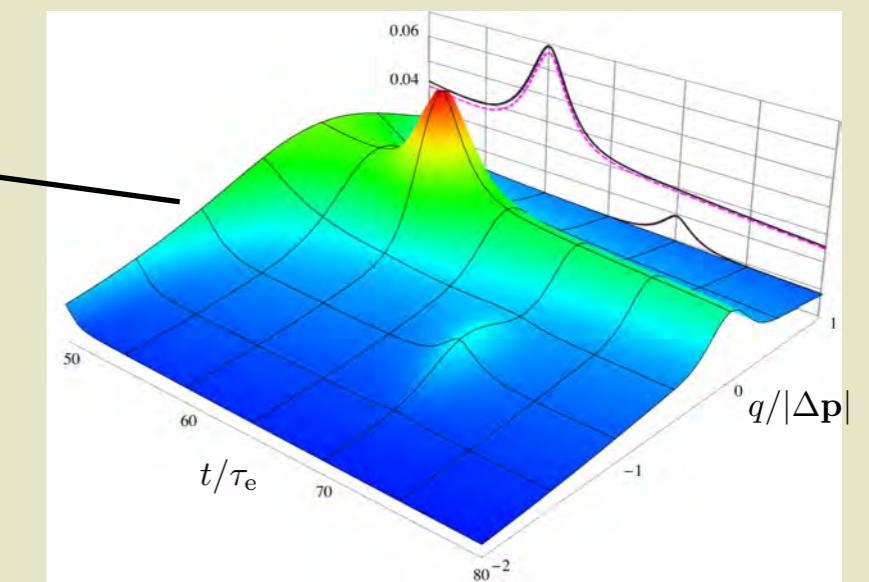
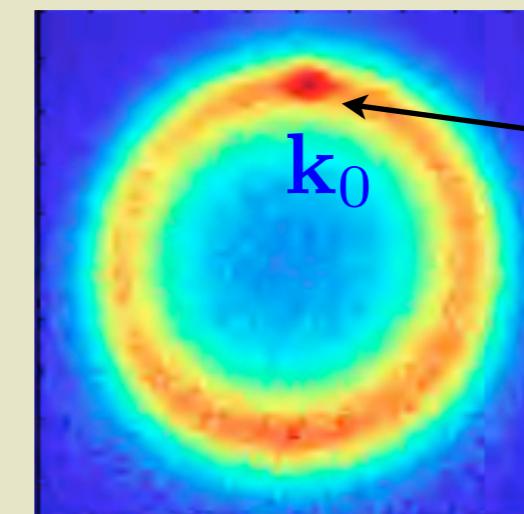
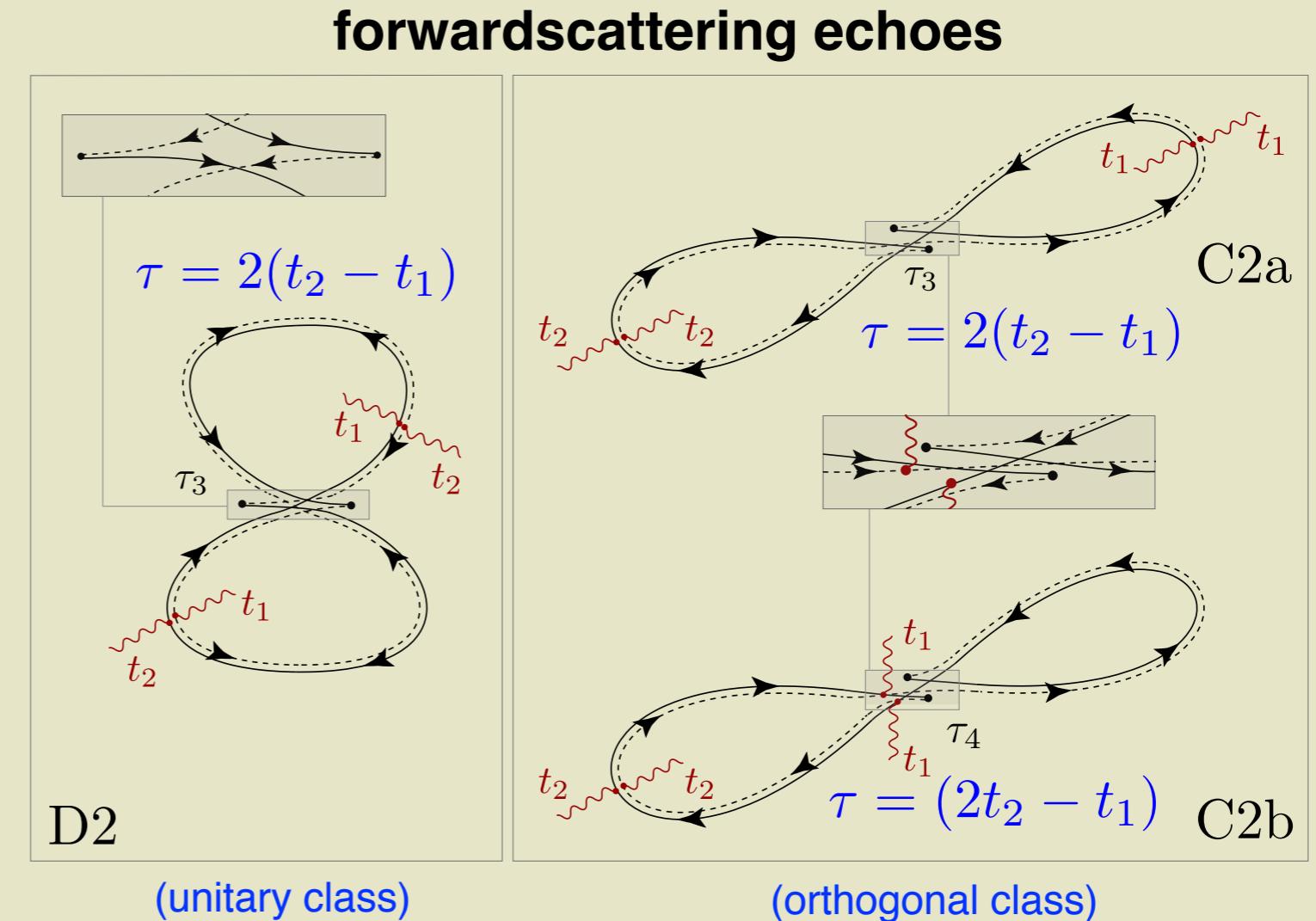
# Two-mode echoes

two pulses at  
 $t = t_1, t_2$

backscattering echo

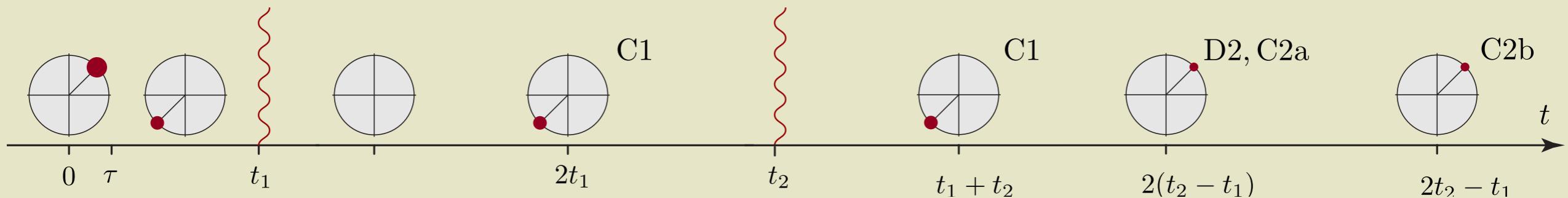


echo-  
structure in  
momentum-  
space



# Echo-spectroscopy at the onset of AL

[TM, C. A. Müller, A. Altland, PRB. 91, 064203 (2015)]

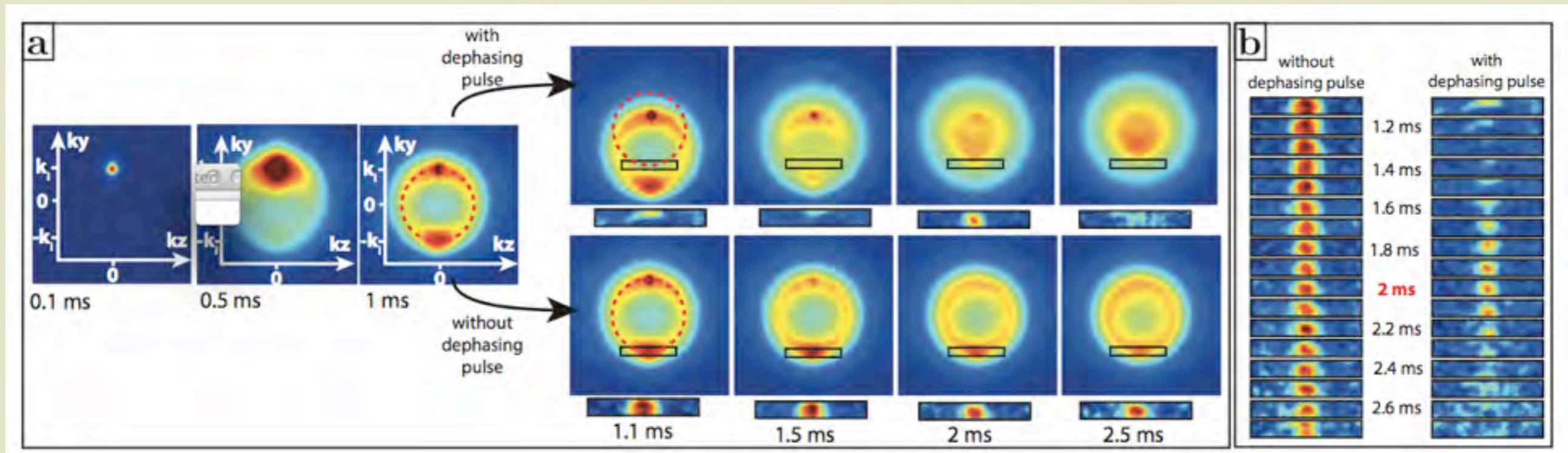


echo-signals in *forward-* and *backward-* scattering directions appear at moments which are in well-defined relations to applied pulses

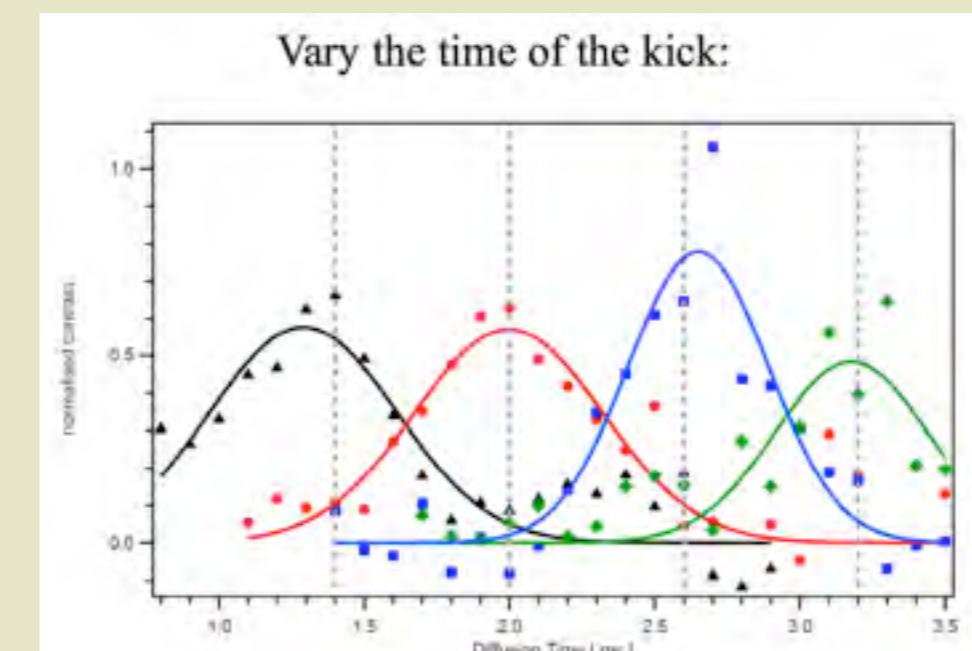
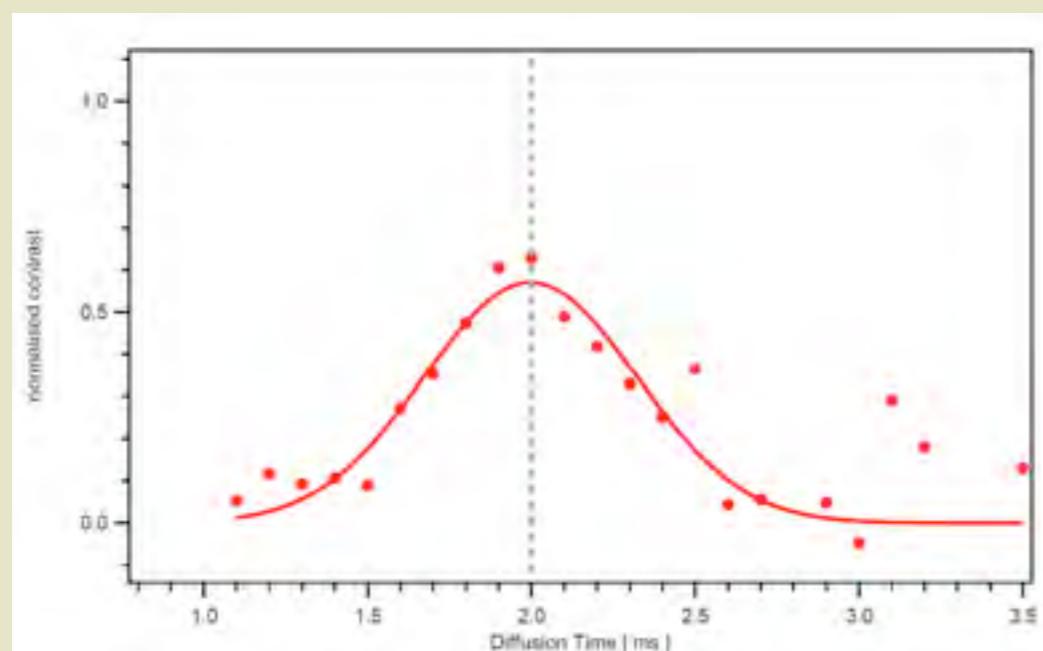
a systematic way to test elementary processes driving AL

# recent experiment: A. Aspect, V. Josse

[K. Müller, J. Richard, V.V. Volchkov, V. Denechaud, P. Bouyer, A. Aspect, V. Josse, PRL (2015)]



## coherent backscattering echo

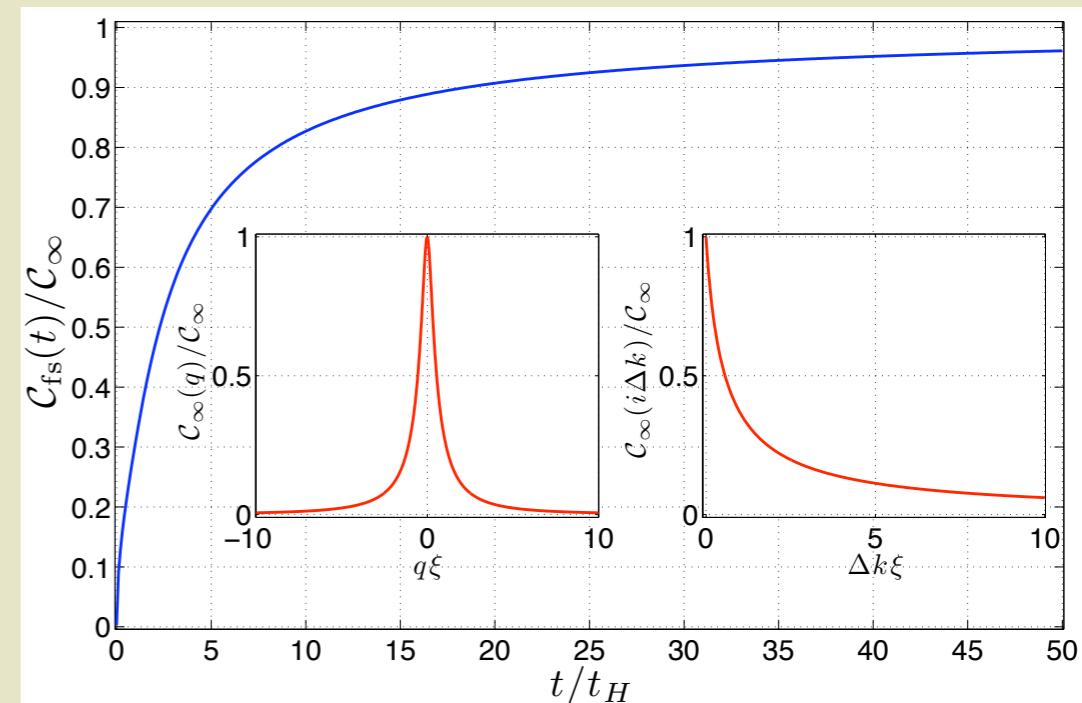


# Summary

## cold atom quantum quench experiment...

### forward peak

- time-resolved portrait of a strong localization phenomenon, with the perspective of observation using current device technology
- direct observation of level-level correlation function of finite size Anderson insulator
- full analytical description by mapping to 3d Coulomb problem



### echo-spectroscopy

- systematic way to study processes driving AL

