Physics of Weyl semimetals

Dima Pesin,
University of Utah, Salt Lake City, UT, USA

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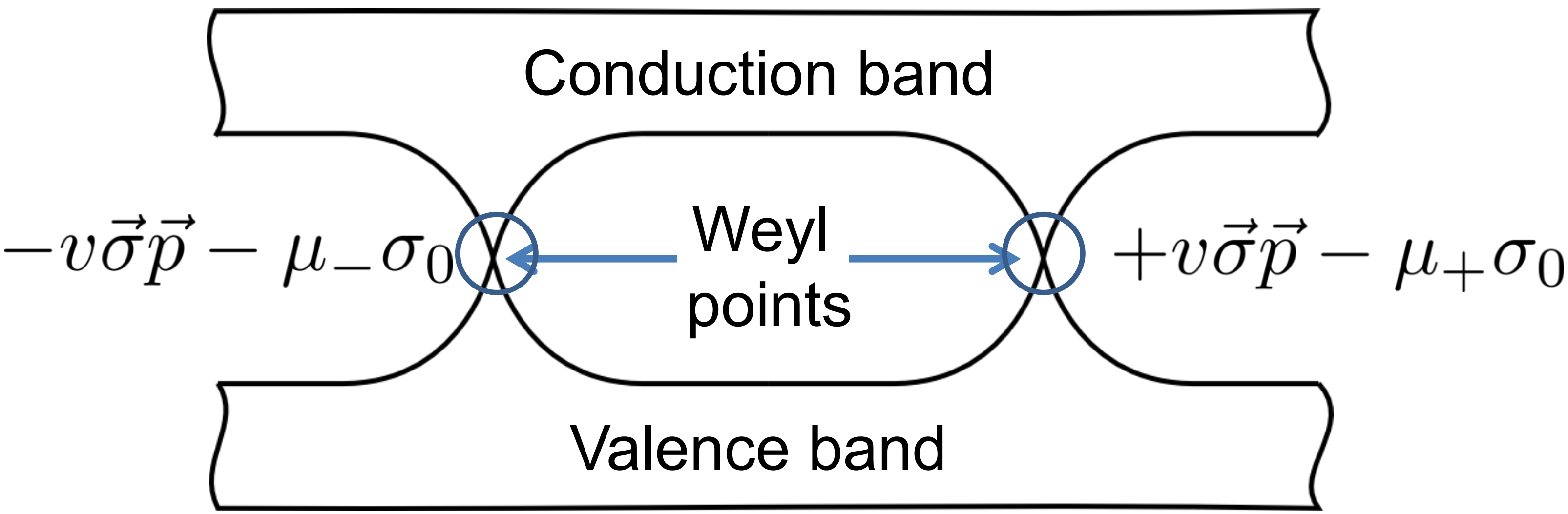
Weyl Semimetal (WS) is a collection of nondegenerate band touchings in 3D k-space

"3D Graphene"



(Herring'37, Abrikosov&Beneslavsky'71, Wan *et al'11,* Burkov&Balents'11)

Two bands => the spin degeneracy is lifted, either by T or I breaking

Where does one get a WS? (successful) theory

Classification: B.Yang, N.Nagaosa, Nat. Comm. 5, 4898 (2015)

Weyl semimetals with inversion breaking:

TaAs, NbAs:

H. Weng et. al, arxiv:1501.00060

Weyl semimetal with time-reversal breaking:

$A_2Ir_2O_7$:

Wan et al., PRB 83, 205101 (2011)

Dirac (with no gap) semimetals:

Cd_3As_2:

Wang et al., PRB 88, 125427 (2013)

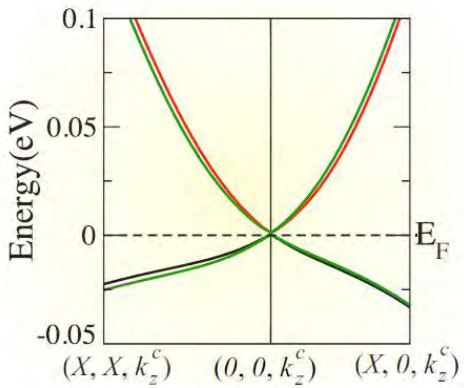


FIG. 2: (Color online) Band dispersions and band-splitting in the plane passing through Dirac point $(0,0,k_z^c)$ and perpendicular to Γ-Z for structure II. The k-points are indicated in cartesian coordinates. X and k_z^c are around 0.1 and 0.032 $Å^{-1}$, respectively.

Where does one get a WS? Experiment

Dirac (with no gap) semimetals:

Cd_3As_2:

T. Liang et al, Nat. Mater. (2014)

Borisenko et al., PRL 113, 027603 (2014)

Neupane et al. , Nat. Comm. 05, 3786 (2014)

$ZrTe_5$:

Q. Li et al., arXiv:1412.6543

Na_3Bi:

Z. K. Liu, Science 434, 864 (2014)

J. Xiong et al., arXiv: 1503.08179

Where does one get a WS? (II)

Weyl semimetals with inversion breaking:

Photonic crystal: Lu et al, arXiv:1502.03438.

TaAs:

S-Y. Xu et al., arXiv:1502.03807

B. Q. Lv et al., arXiv:1502.04684

NbAs:

Y. Luo et al. , arXiv: 1506.01751

Weyl semimetal with time-reversal breaking:

Er2Ir2O7:

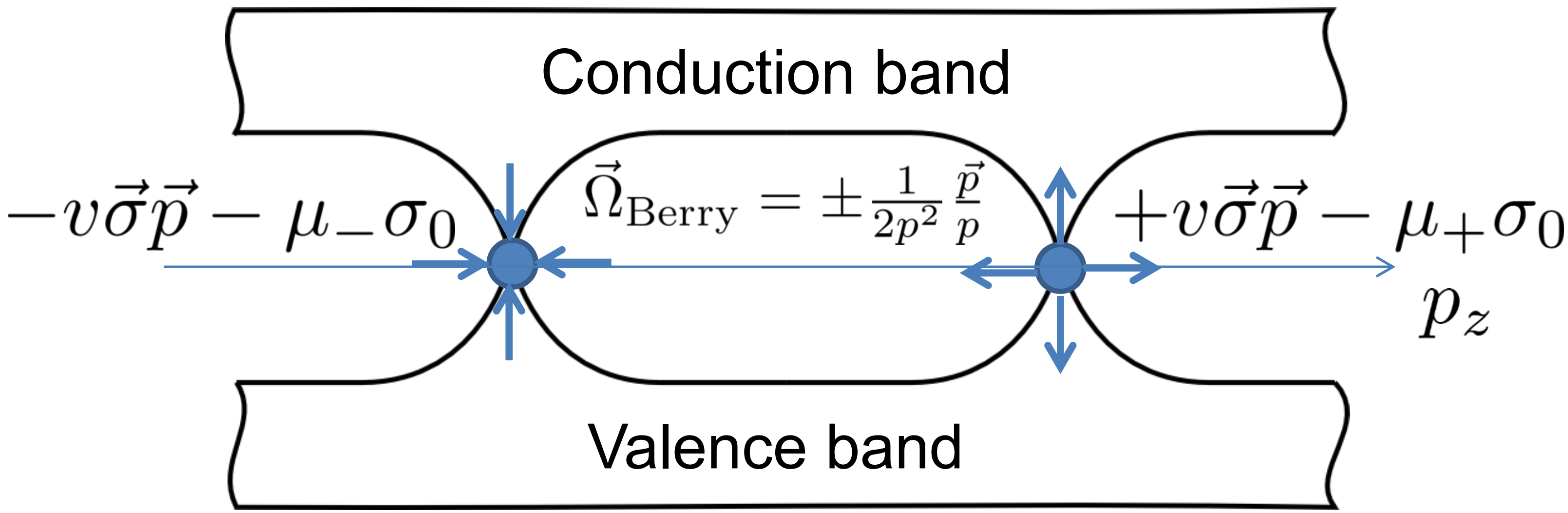
Sushkov et al., arxiv:1507.01038

$YbMnBi_2$:

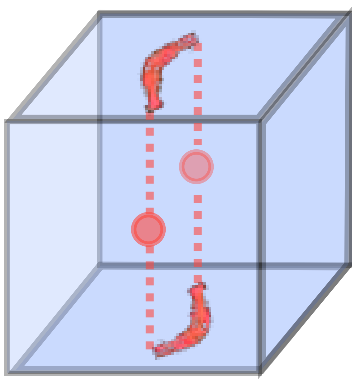
Borisenko et al., arxiv:1507.04847

Weyl semimetals (WS) are gapless phases with nontrivial topology

(review: Turner&Vishwanath, cond-mat: 1301.0330)



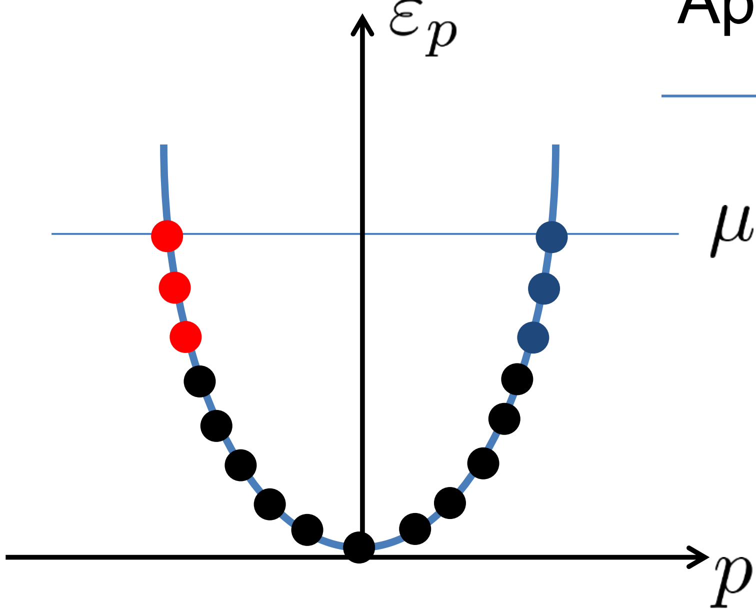
1. Nodes are stable due to topology, **not** symmetry (as, say, in graphene)
2. There are protected surface states('arcs'), which cannot be realized in a 2DEG.
3. Hall response is determined by the distance between nodes, and nothing else.
4. The chiral anomaly is also a topo. response

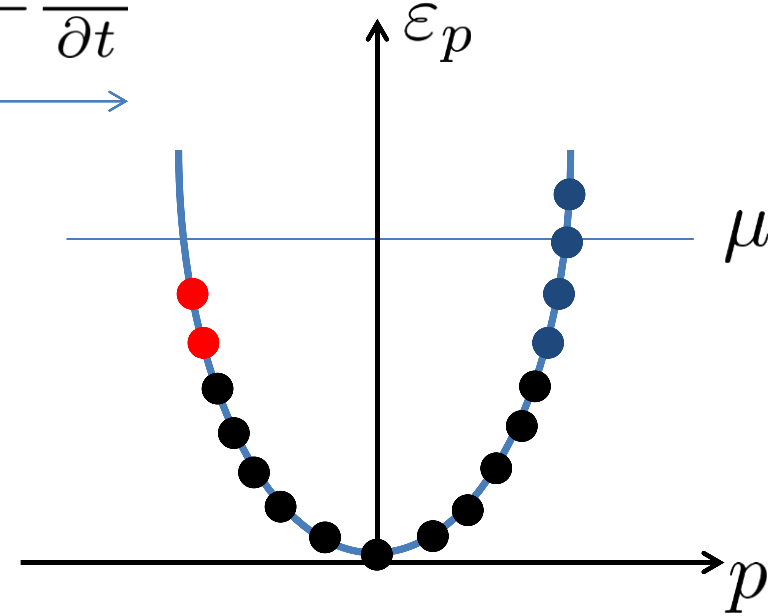


The chiral anomaly is the nonconservation of valley charges in the presence of EM fields

1D Example: 

Apply $E = -\frac{\partial A}{\partial t}$





$$(\partial_t \mp \partial_x)\,\Psi_{R,L} = 0$$

$$(\partial_t \mp [\partial_x - iA])\,\Psi_{R,L} = 0$$

yet

$$\dot{N}_R - \dot{N}_L = \frac{e}{\pi\hbar}E$$ **(!)**

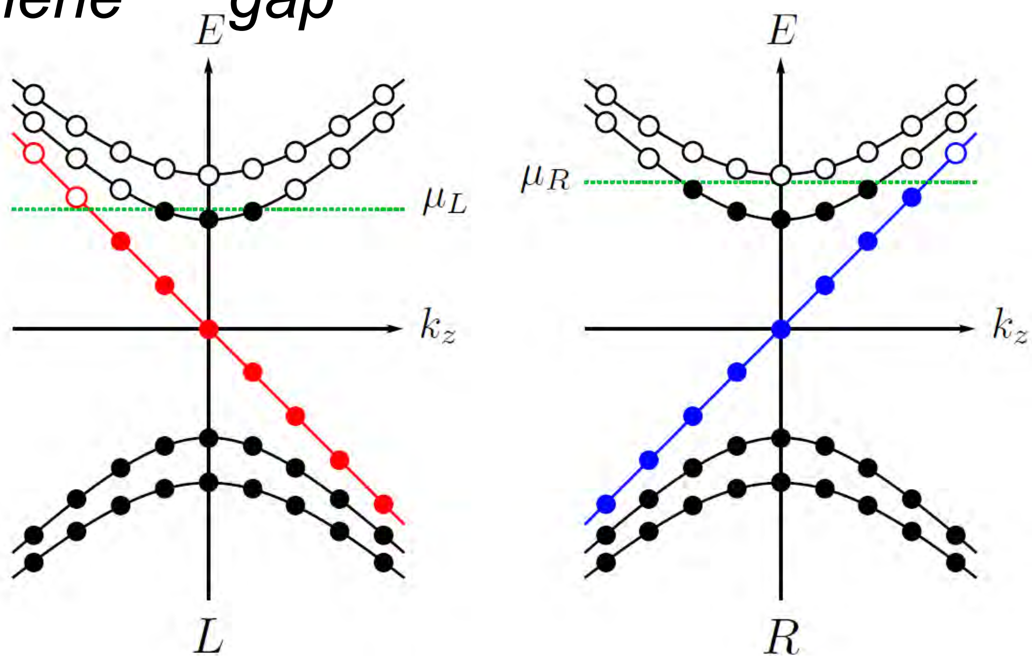
In the 3D case, the B-field reduces the problem to a collection of 1D ones

$$\vec{B} = (0,0,B),$$

$$H = \pm v \left[\underbrace{\vec{\sigma}_\perp (\vec{p}_\perp - e\vec{A})}_{\text{“graphene”}} + \underbrace{\sigma_z p_z}_{\text{“gap”}} \right]$$

$$\Rightarrow \quad E^{R,L}_{n\neq 0}(p_z) = \pm v \sqrt{2|n|eB\hbar/c + p_z^2}$$

$$E^{R,L}_{0}(p_z) = \pm v p_z, \quad n = 0$$



$$\boxed{\dot{N}_R - \dot{N}_L = \frac{e^2}{2\pi^2\hbar^2 c}\mathbf{E}\cdot\mathbf{B}}$$

“3D chiral anomaly”

(S. L. Adler, 1969 ; J. S. Bell and R. Jackiw, 1969; Nielsen&Ninomiya, 1983)

Practical problems:

1) symmetry-wise, WS are no different from more conventional phases, thus exhibit all the same responses, in principle.

2) Being gapless, these phases are sensitive to disorder, “fancy” properties masked by usual metallic transport

Example: "Unusual" electrodynamics in WS

$$S_{WS}[A] = \frac{e^2}{32\pi^2}\int d^4x\,\theta(\mathbf{r},t)\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

$$\text{``}(\mathbf{E}\cdot\mathbf{B})\text{''}$$

$$= -\frac{e^2}{8\pi^2}\int d^4x\,\epsilon^{\mu\nu\alpha\beta}\partial_\mu\theta A_\nu\partial_\alpha A_\beta$$

"3D Chern-Simons action", (Goswami&Tewari'12, Zyuzin&Burkov'12)

Current: $j^\rho = -\frac{e^2}{4\pi^2}\partial_\mu\theta\epsilon^{\mu\rho\alpha\beta}\partial_\alpha A_\beta$

$\mu \neq 0:\;\; j^\rho = -\frac{e^2}{4\pi^2}\partial_\mu\theta\epsilon^{\mu\rho\alpha\beta}\partial_\alpha A_\beta$ = Anomalous Hall effect

$\mu = 0:\;\; j^\rho = -\frac{e^2}{4\pi^2}\partial_0\theta\epsilon^{0\rho\alpha\beta}\partial_\alpha A_\beta$ or $\vec{j} \propto \lambda_{inv}\vec{B}$ **???**

“Chiral magnetic effect” = natural optical activity

$\vec{j} \propto \lambda_{inv} \vec{B}$ -- Chiral magnetic effect (Burkov et al., PRB’12,’13)

Proper relation: (Vazifeh, Franz, PRL’13, Chen, Wu, Burkov, PRB’13)

$$\vec{j} \propto \lambda_{inv} \vec{B}(\omega, q)\, f(\omega, q),$$

$$f(\omega, q \to 0) \neq 0, \;\; f(\omega \to 0, q) = 0$$

Because of the Faraday’s law $\mathbf{B} = \frac{1}{\omega}\mathbf{q} \times \mathbf{E}$, this is equivalent to

$\sigma_{ij},\ \varepsilon_{ij} \propto \epsilon_{ijk} q_k$ -- **natural optical activity**

Practical problem: symmetry-wise, WS are no different from more conventional phases, thus exhibit the same responses, in principle.

How does one detect them then? There are a few ways out of this complication:

1) Look for unusual magnitude of effect

2) Unusual sign (Son, Spivak, 2012 negative “classical” magneto-resistance)

3) Unusual parameter dependence. (Parameswaran et al. 2011, “quantum-critical” conductivity, $\sigma(\omega) \propto \max(\omega, T)$)

Q: How to distinguish a WS from a small-gap semiconductor? Both in principle, and in practice.

A: The chiral anomaly provides a nonlocal transport signature. The latter is absent in the usual small gap semiconductors.

We would like to have an easily observable effect that does not exist without the chiral anomaly.

PRX 4, 031035 (2014)

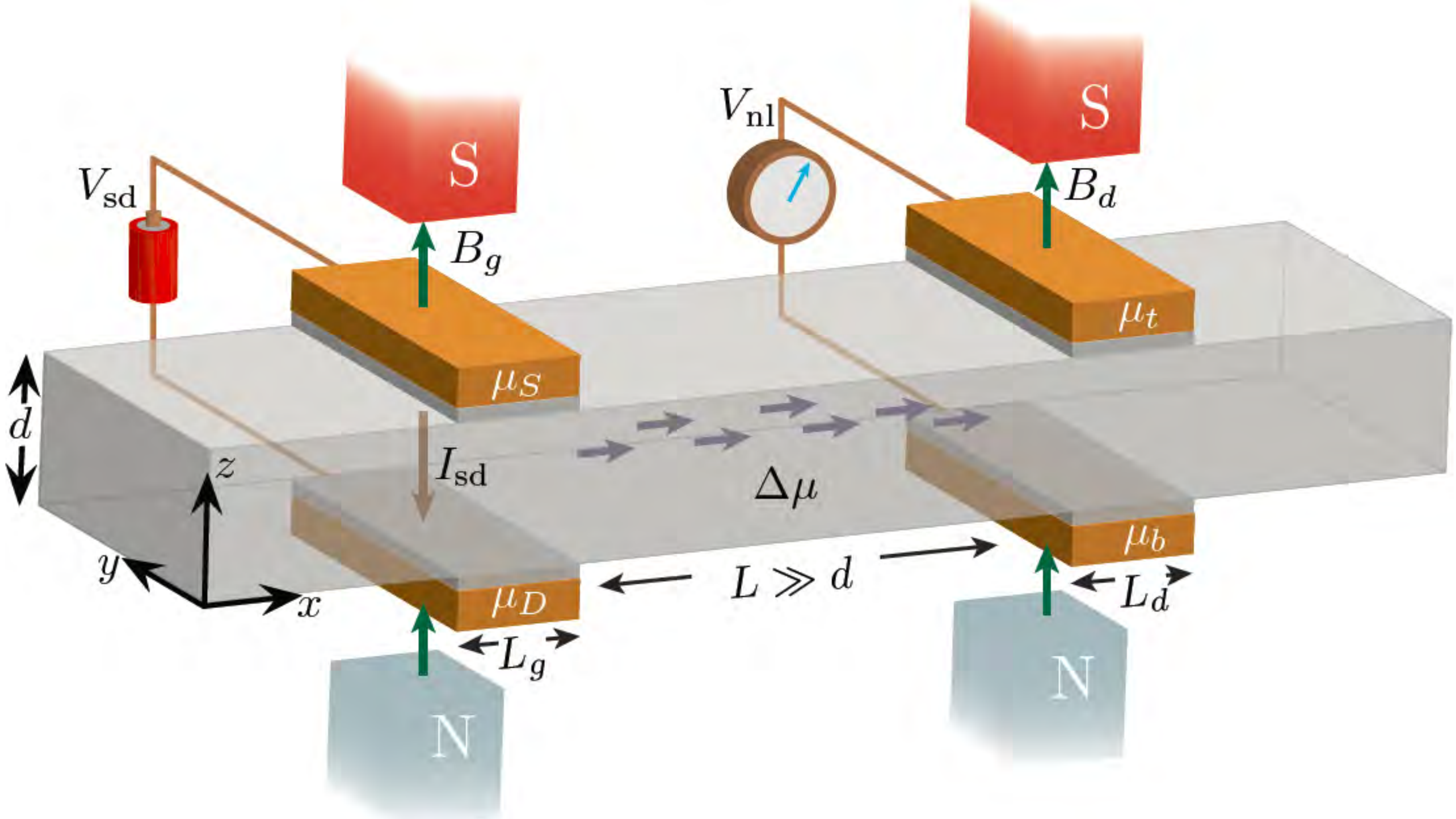
Sid Parameswaran
UC Irvine

Tarun Grover
UCSD

Dmitry Abanin
U of Geneva

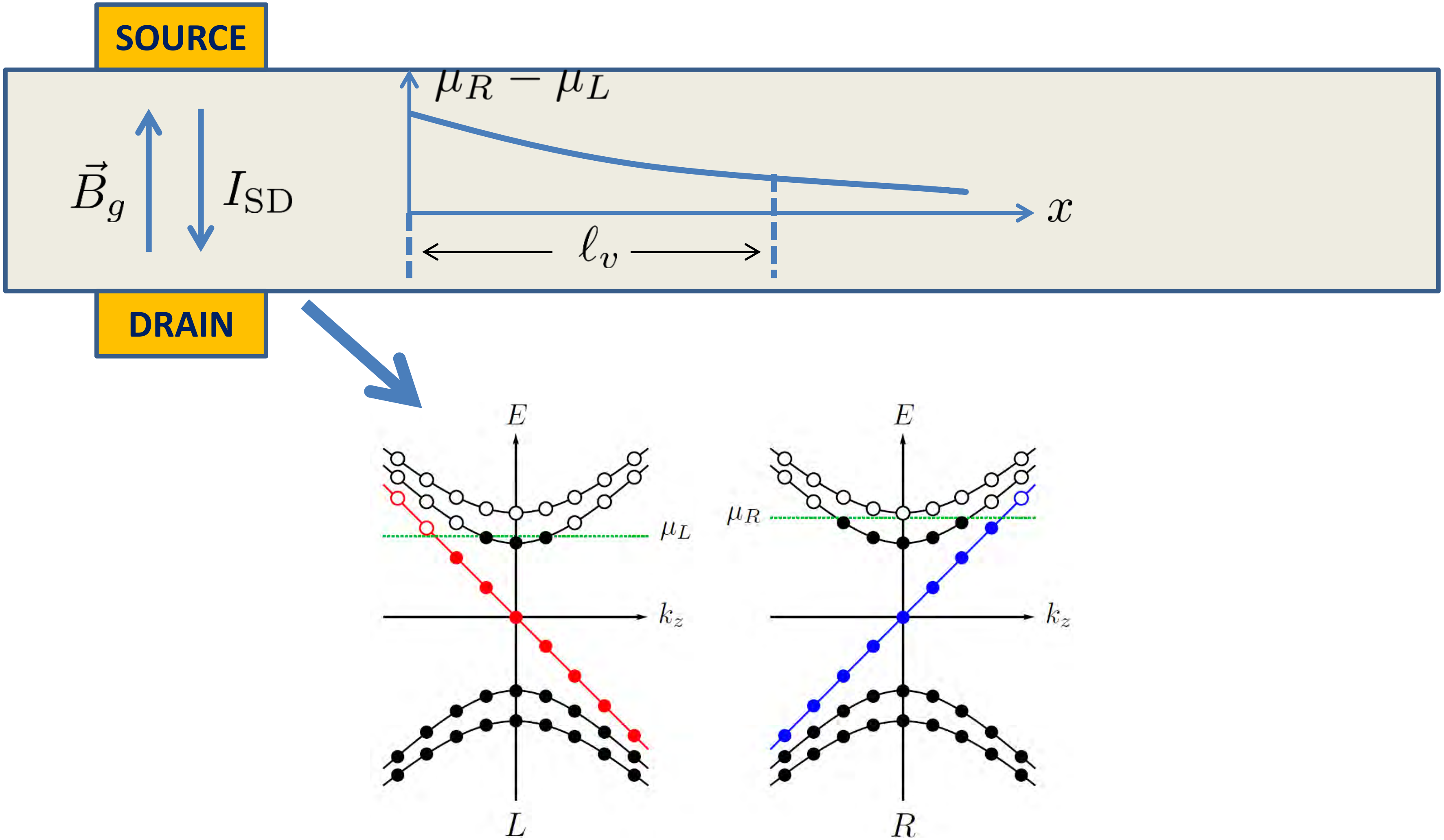
Ashvin Vishwanath
UC Berkeley

We will use the chiral anomaly to generate and detect nonlocal voltages, sensitive to magnetic field magnitude and direction.

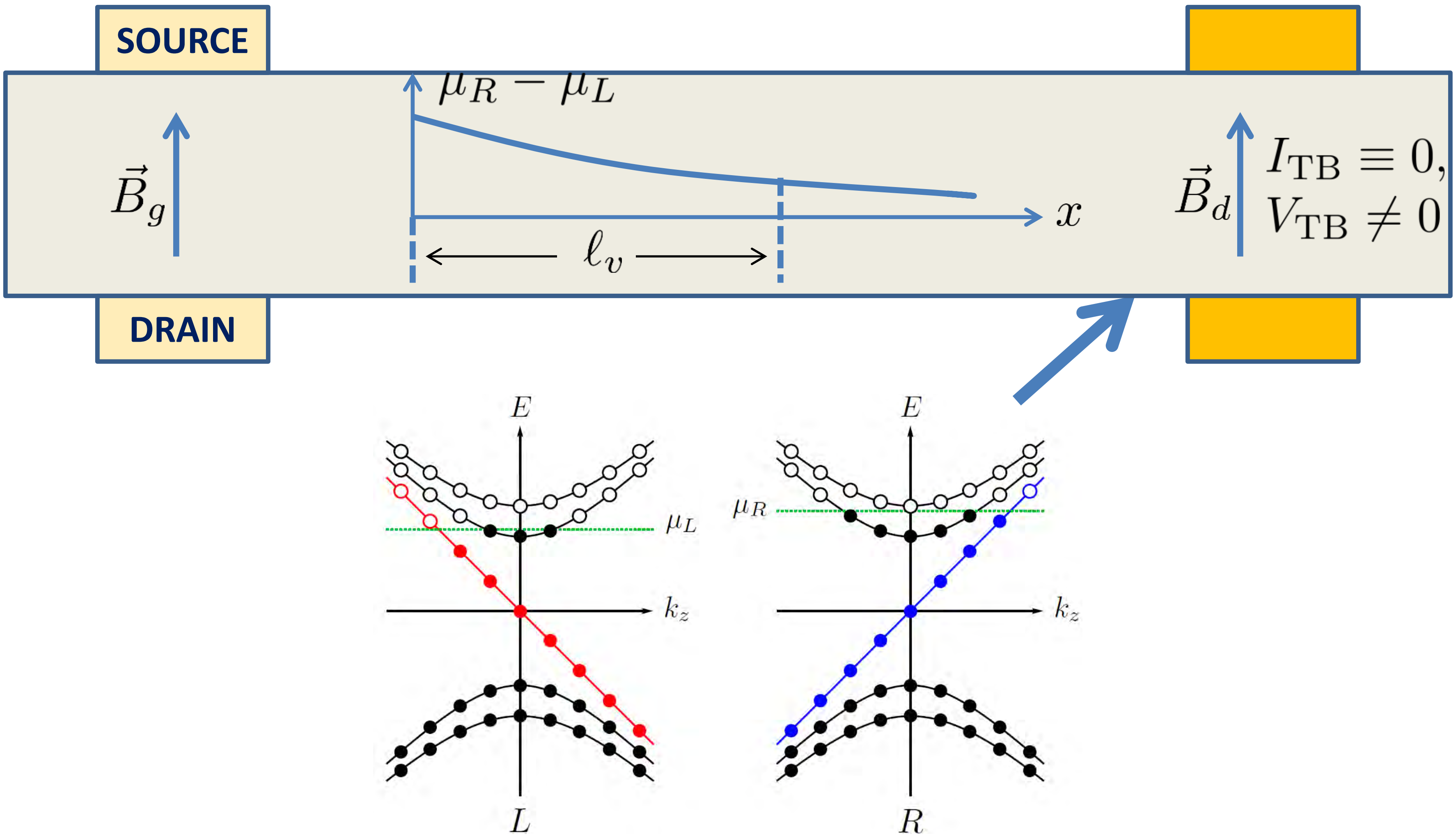


$$\frac{|V_{nl}(x)|}{V_{SD}} \propto e^{-x/\ell_v}, \quad \ell_v = \sqrt{D\tau_v} \gg d$$

In the presence of a magnetic field, transport current generates valley imbalance

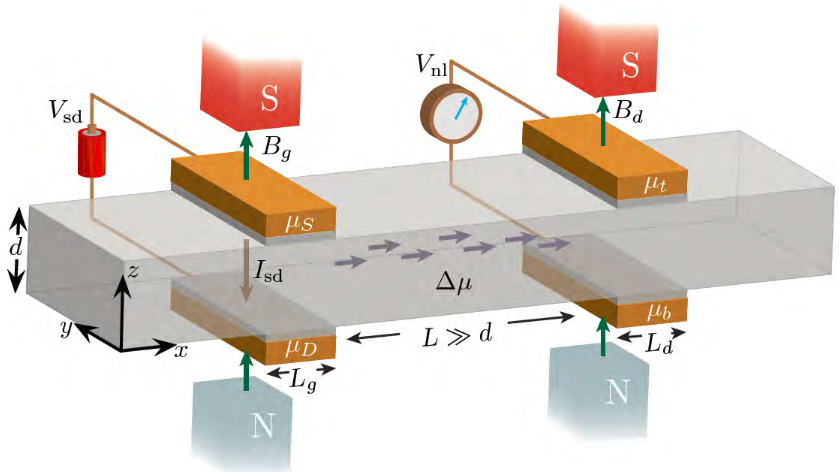


Inverse effect: In the presence of a magnetic field AND valley imbalance, there is a top-bottom voltage



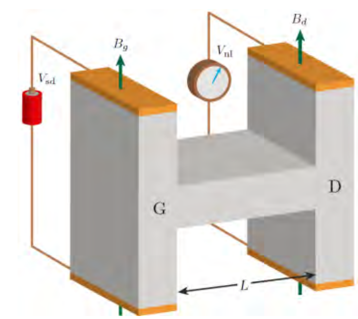
Summary so far:

1. Four-terminal nonlocal geometry can help detect electric signals due to the presence of the chiral anomaly:



$$\boxed{\frac{|V_{\mathrm{nl}}(x)|}{V_{\mathrm{SD}}} \propto e^{-x/\ell_v}, \qquad \ell_v = \sqrt{D\tau_v} \gg d}$$

2. Should employ either tunneling leads, or this:



3. Locally applied B-fields are preferred

4. Detectors can be made non-invasive, if they are not too long

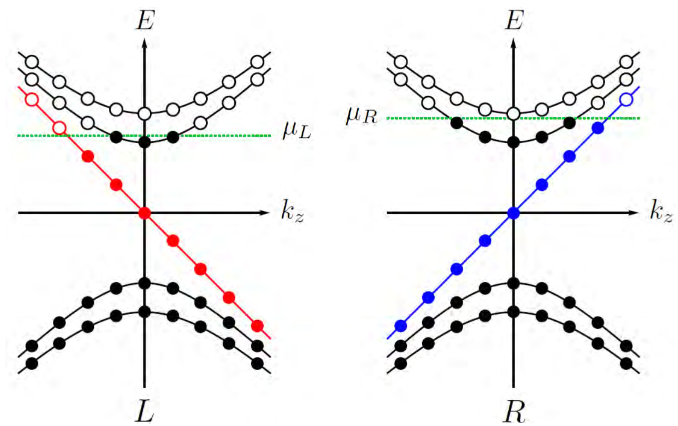
Transport theory = bulk transport equations + boundary conditions

The currents include the chiral modes contributions:

$$\mathbf{j}^{R,L} = -\frac{\sigma}{e}\boldsymbol{\nabla}\mu_{\rm ec}^{R,L} \pm \frac{e^2\mathbf{B}}{4\pi^2\hbar^2 c}\mu^{R,L}.$$

$$\mu_{\rm ec}^{R,L} = \mu^{R,L} + e\phi$$

(Vilenkin, 1980)



The continuity equations include the anomalous divergences:

$$\boldsymbol{\nabla}\cdot\mathbf{j}^{R,L} + \partial_t\rho^{R,L} = \pm\frac{e^3}{4\pi^2\hbar^2 c}\mathbf{E}\cdot\mathbf{B}$$

The final stationary transport equations contain only $\mu_{\rm ec}^{R,L}$

$$\boxed{-\frac{\sigma}{e}\nabla^2\mu_{\rm ec}^{R,L} \pm \frac{\beta}{e}\hat{n}\cdot\boldsymbol{\nabla}\mu_{\rm ec}^{R,L} = \mp\frac{e\nu_{\rm 3D}}{2\tau_v}(\mu_{\rm ec}^{R} - \mu_{\rm ec}^{L})} \quad \beta = \frac{1}{2\pi\ell_B^2}\frac{e^2}{h}$$

Boundary conditions: The simplest set of physically sound ones would suffice here

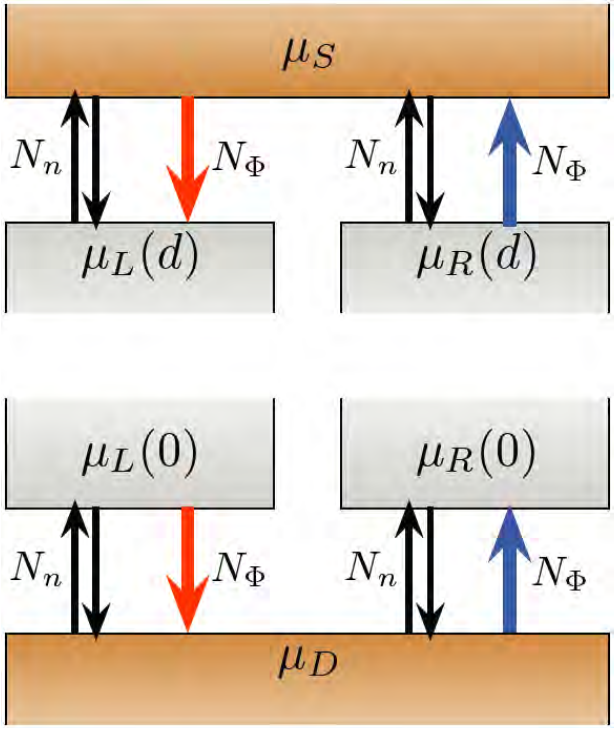
Top surface:

$$
\begin{aligned}
j_z^R(d) &= \frac{g}{e}(\mu_{\rm ec}^R(d) - \mu_S) + \frac{\beta}{e}\mu_{\rm ec}^R(d), \\
j_z^L(d) &= \frac{g}{e}(\mu_{\rm ec}^L(d) - \mu_S) - \frac{\beta}{e}\mu_S,
\end{aligned}
$$

Bottom surface:

$$
\begin{aligned}
j_z^R(0) &= \frac{g}{e}(\mu_D - \mu_{\rm ec}^R(0)) + \frac{\beta}{e}\mu_D, \\
j_z^L(0) &= \frac{g}{e}(\mu_D - \mu_{\rm ec}^L(0)) - \frac{\beta}{e}\mu_{\rm ec}^L(0).
\end{aligned}
$$

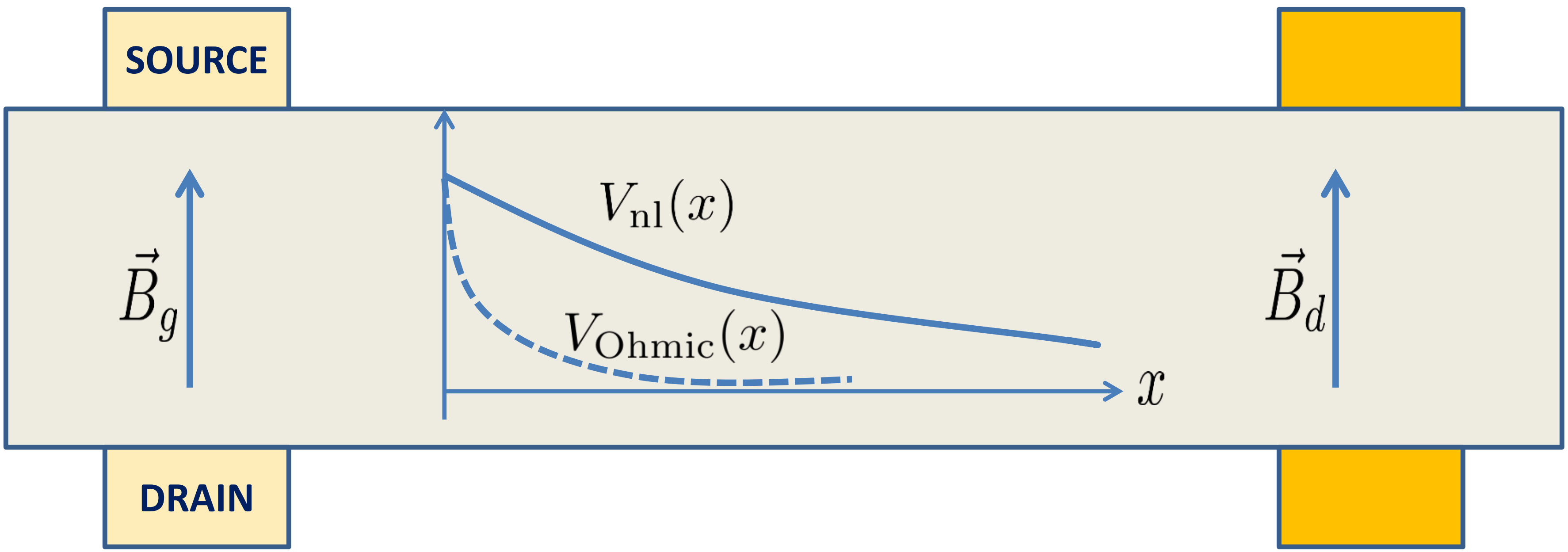
Assumption: no inter-valley scattering under a lead.



The nonlocal voltages reach afar, and depend on the orientation of the B-fields

$$\frac{V_{\rm nl}(x)}{V_{SD}} = -\mathrm{sign}(B_g)\mathrm{sign}(B_d)\frac{\beta_d}{2g_d+\beta_d}\frac{\beta_g}{2g_g+\beta_g}e^{-\frac{|x|}{\ell_v}}$$

$$\beta_{g,d} = \frac{1}{2\pi\ell^2_{B_{g,d}}}\frac{e^2}{h} \propto B_{g,d}, \qquad \ell_v = \sqrt{D\tau_v} \gg d$$



Physics near a node: WS with strong long-range disorder

http://arxiv.org/abs/1507.05349

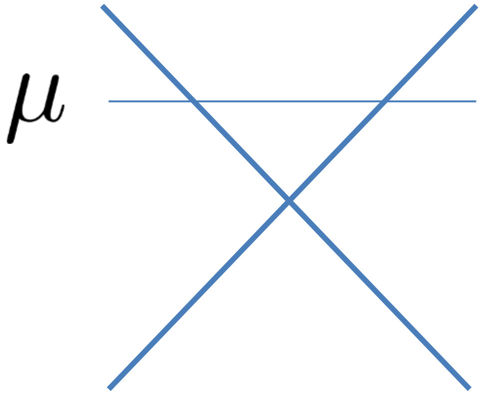
Alex Levchenko

U of Wisconsin, Madison

Eugene Mishchenko

U of Utah, SLC

We have a WS now. Anything interesting close to the nodal points?



- metallic behavior for substantial doping

What does disorder do to this picture?

No $k_f \ell$ parameter, hard to make quantitative statements

$$\frac{1}{\tau(\epsilon)} \propto \nu(\epsilon) \propto \epsilon^2 \ll \epsilon, \quad \epsilon \to 0$$

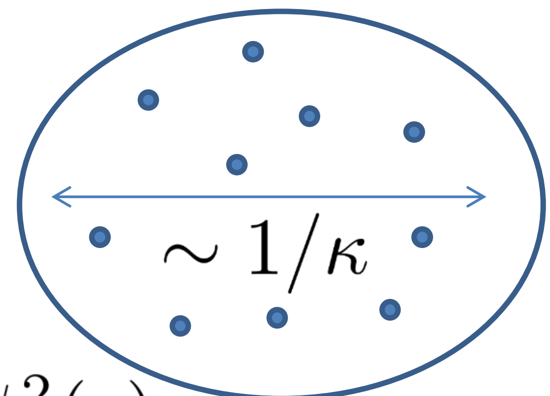
One needs a critical disorder strength to get a finite DoS at the node (unlike in 2D Dirac systems)

WS with long-range disorder

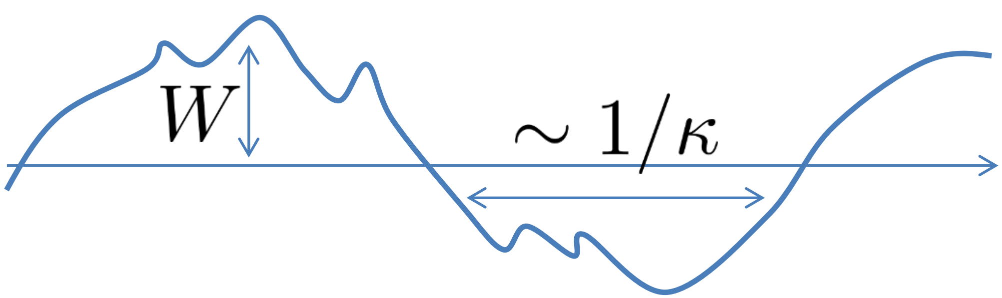
Many impurities with (self-consistently) screened potential:

$$u(r) = \sum_i \phi(r - r_i), \quad \phi(r) = \frac{e^2}{r} e^{-\kappa r}$$

If there are many impurities in the screeing volume, $n_i \kappa^{-3} \gg 1$, one can take the potential to be a Gaussian field:



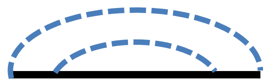
$$\langle u(r) u(0) \rangle = W^2 e^{-\kappa r}, \quad W^2 = \int d^3 r \phi^2(r)$$

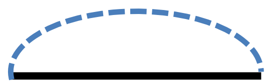


$$\boxed{\frac{W}{\kappa v} \propto \frac{1}{\sqrt{r_S}} \gg 1}$$

Born and SC-Born approximations are poorly controlled near Weyl point

Problems with (SC)BA close to a Weyl (or Dirac) point:

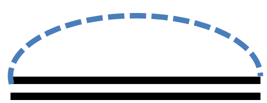
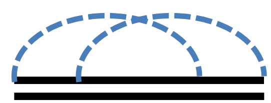
$\Sigma_{BA}^{(2)} =$ 

$\Sigma_{BA}^{(1)} =$ 

$$\frac{\Sigma_{BA}^{(2)}}{\Sigma_{BA}^{(1)}} \sim \frac{W^2}{\max(\kappa^2 v^2, \epsilon^2)}$$

(at least SCBA)

but then

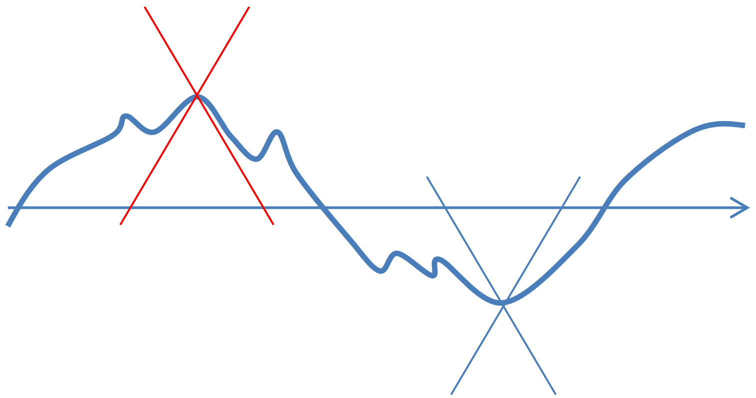
$\Sigma_{SCBA} =$  $\sim$  (B. Sbierski et al, PRL 2104)

SCBA does reproduce the main features of spectrum (e.g. a finite DoS above critical strength of disorder), but numerics is the only way to go

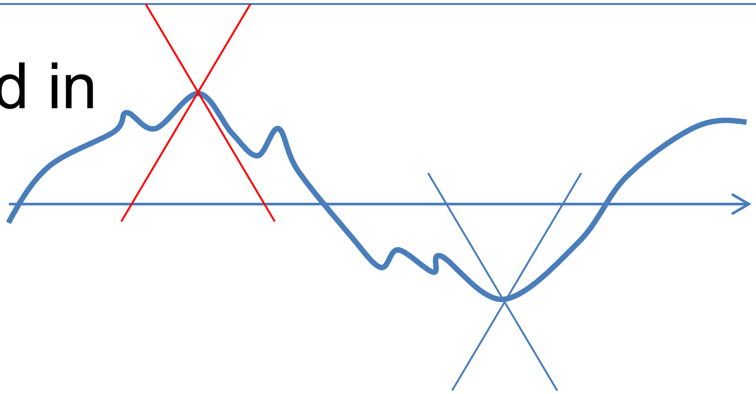
When BA fails, semiclassical approximation kicks in

$$\frac{d\lambda(x)}{dx} \sim \frac{v}{W}\frac{1}{1/\kappa} = \frac{\kappa v}{W} \ll 1$$

One should hope that Thomas-Fermi-kind of approximation will work well at that point: Disorder potential mosly modulates the local position of the Weyl point



Neglecting scattering yields classical effects



Single-particle properties are contained in

$$G_R[u(r)] = \frac{1}{\epsilon_+ - v\vec{\sigma}\vec{p} - u(r)}$$

If one keeps the smooth part of the potential only (no scattering):

$$\langle G_R[u(r)]\rangle = \int du F(u) G_R(\epsilon - u),$$

$$F(u) = \langle \delta(u - u(r))\rangle = \frac{1}{\sqrt{2\pi}W} e^{-u^2/2W^2}$$

In particular, the DoS is

$$\nu(\epsilon) = \langle \nu_0(\epsilon - u)\rangle_u = \frac{\epsilon^2 + W^2}{2\pi^2 v^3}$$

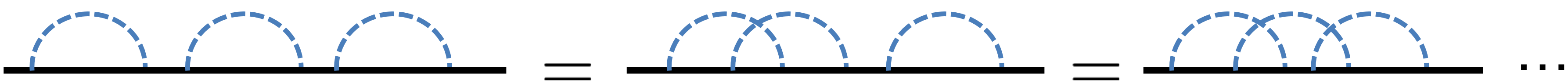
V.L. Bonch-Bruevich, 1962
E.O. Kane, 1963
L.V. Keldysh, 1964
A. L. Efros, 1970
B. Skinner, 2014

Keldysh model for WS with long-range disorder: controlled corrections to classics

Solution: rearrange the perturbation theory (Efros, 1970)

$$\langle u(r)u(0)\rangle \equiv D(r) = \underbrace{D(0)}_{\text{Treat exactly}} + \underbrace{(D(r) - D(0))}_{\text{perturbation}}$$

$D(0)$: Infinite correlation length, recovers the classical result



$$D(r) = W^2 e^{-\kappa r} :$$

$$\frac{D(r)-D(0)}{D(0)} \approx \kappa r \sim \kappa \lambda_F(\epsilon = W) \sim \sqrt{r_s} \ll 1$$

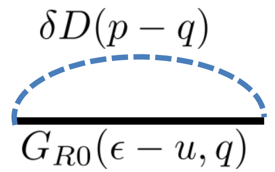
Keldysh model for WS: results

$$\delta D \equiv D(r) - D(0) :$$

$$\langle G_R[u(r)]\rangle = \int du \frac{1}{\sqrt{2\pi}W} e^{-u^2/2W^2} G_R(\epsilon - u),$$

$$G_R[\epsilon - u] = \frac{1}{\epsilon_+ - u - v\vec{\sigma}\vec{p} - \hat{\Sigma}_u},$$

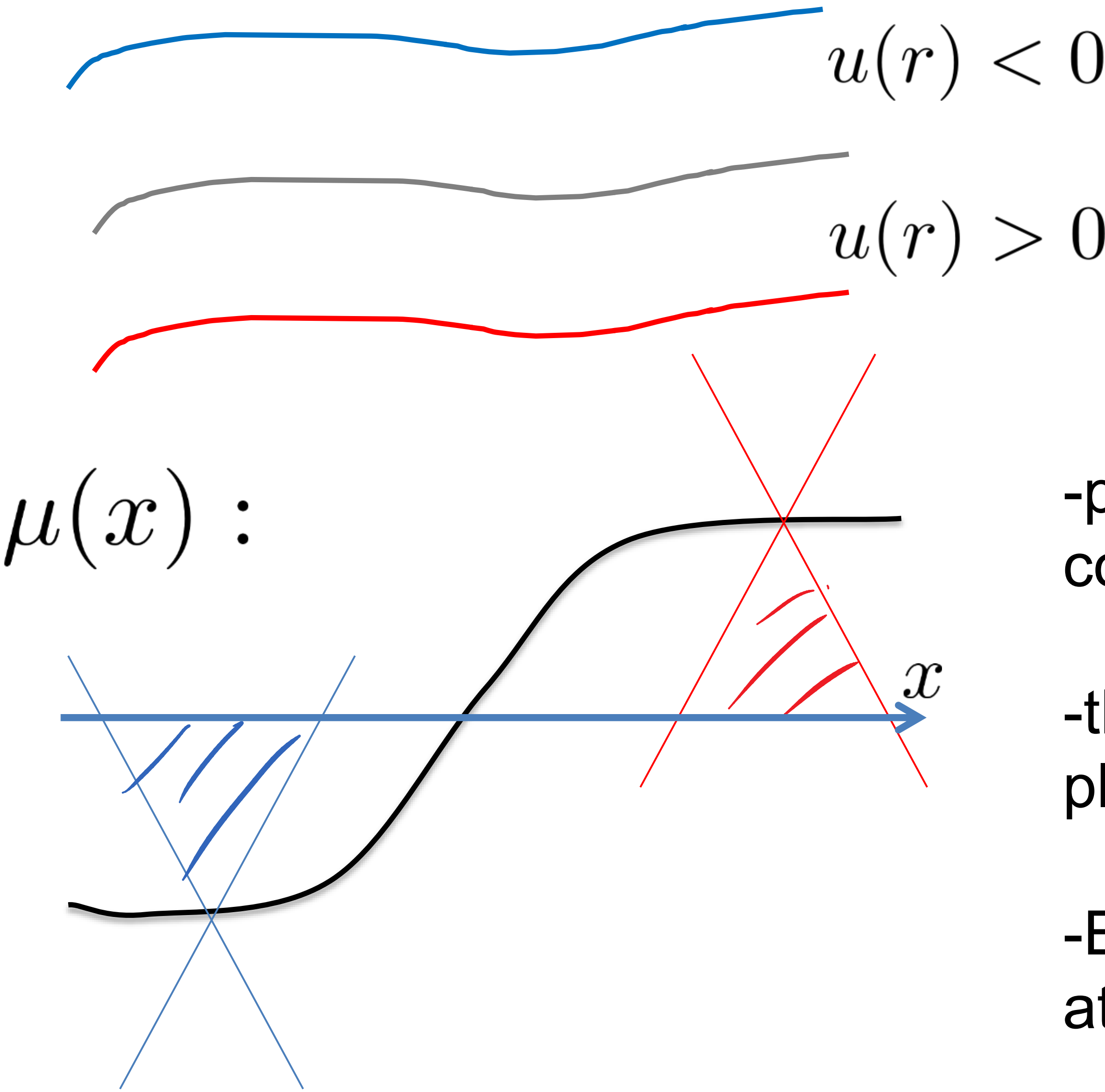
$$\hat{\Sigma}_u = \int \frac{d^3q}{(2\pi)^3} \delta D(p-q) \frac{1}{\epsilon_+ - u - v\vec{\sigma}\vec{q}}$$



Correction to the DoS:

$$\frac{\delta\nu(\epsilon)}{\nu(0)} = -\sqrt{\frac{\pi}{8}} \frac{\kappa v}{W} e^{-\epsilon^2/2W^2}, \qquad \frac{\delta\nu(0)}{\nu(0)} \sim \sqrt{r_S} \ll 1$$

Picture of transport: Electrons and holes flow in non-penetrating percolation channels



Both "colors" percolate in 3D

-p-n junctions have low conductance

-they must host low-energy plasmon modes

-EM absorption is localized at the junctions

Conclusions

1) The Adler-Bell-Jackiw (chiral) anomaly in Weyl semimetals is measurable via a simple nonlocal electric measurement.

2) "Keldysh model" provides a controlled way to calculate disorder-averaged quantities for WS with long-range disorder

3) Transport corresponds to flow of non-mixing electron and hole liquids, interband absorption is localized between the disorder-induced charge puddles