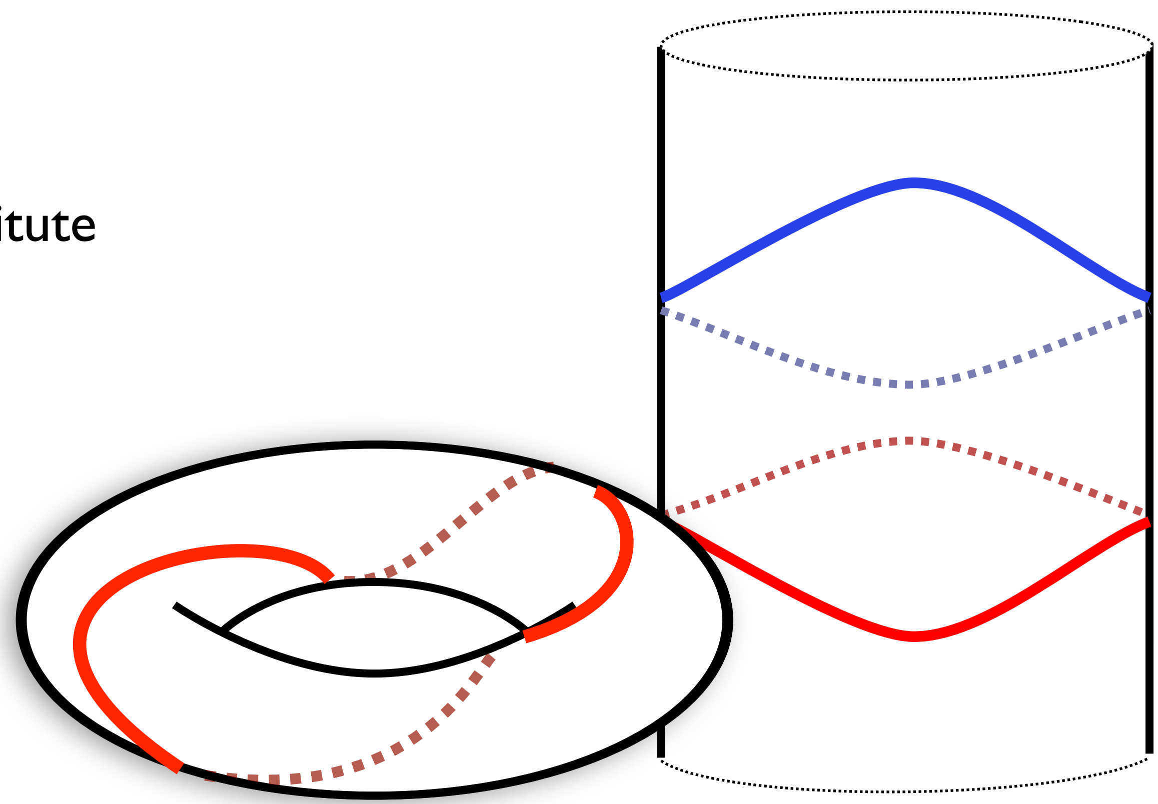
Topology in periodically driven systems

Mark Rudner

Niels Bohr Institute

6 August 2015



Collaborators (in order of appearance):

Takuya Kitagawa, Eugene Demler, Erez Berg, Michael Levin, Netanel Lindner, Paraj Titum, Gil Refael

Email: rudner@nbi.dk

What new types of robust quantum phenomena are possible in *driven* quantum systems?

~1200

$\mathrm{Pb} \rightarrow \mathrm{Au}$

2015

$\mathrm{Si} \rightarrow \mathrm{HgTe?}$

$\rightarrow \quad \ldots \quad ?$

The Plan

I. Review/introduce Floquet theory, quasienergy bands, etc.

II. Invariants of the evolution and topological classification

III. The AFAI and non-adiabatic quantized charge pumping

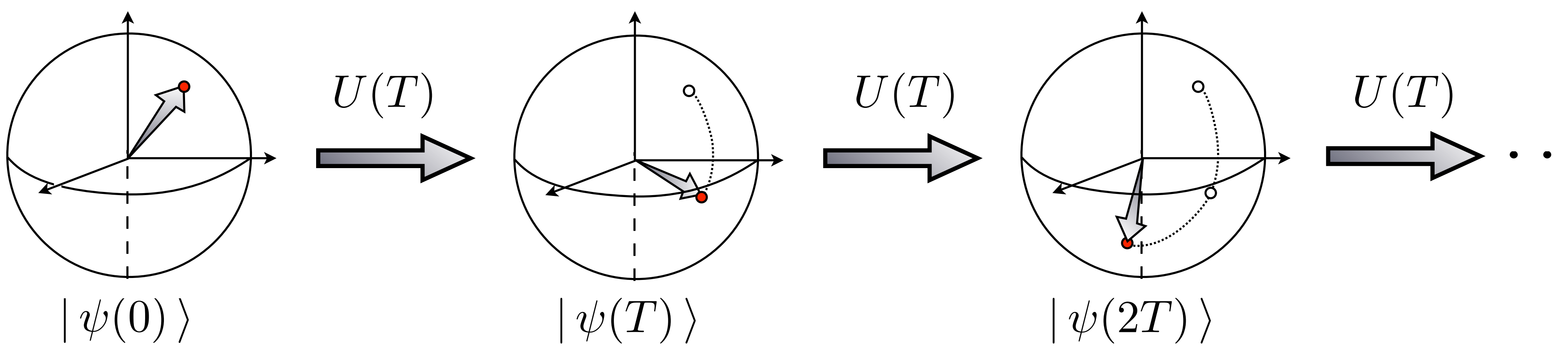
No ground state, energy conservation for driven system

$$i\frac{d}{dt}|\,\psi\,\rangle = H(t)|\,\psi\,\rangle; \quad H(t+T) = H(t)$$

periodic driving

<u>Quasi-energy</u> is conserved for system with discrete time translation symmetry

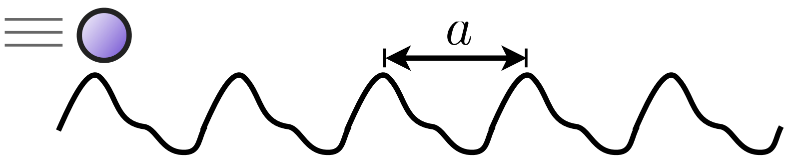
$$\boxed{U(T)|\,\psi_n\,\rangle = e^{-i\varepsilon_n T}|\,\psi_n\,\rangle}$$



$$\boxed{U(T) = \mathcal{T} e^{-i\int_0^T H(t)dt}}$$

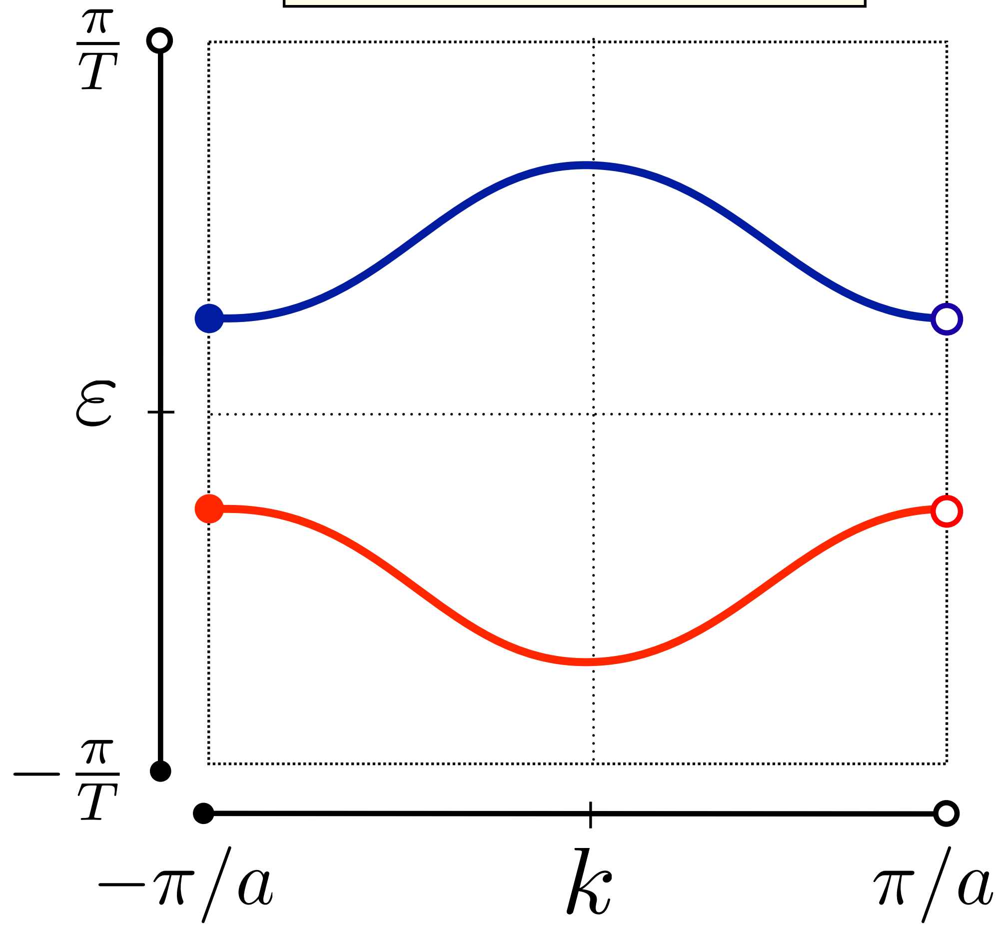
Eigenvalue invariant under $\varepsilon_n \to \varepsilon_n + 2\pi N/T$: quasi-energy lives on a <u>circle</u>

On a lattice find Floquet bands, similar to static system



$$V(x+a) = V(x)$$

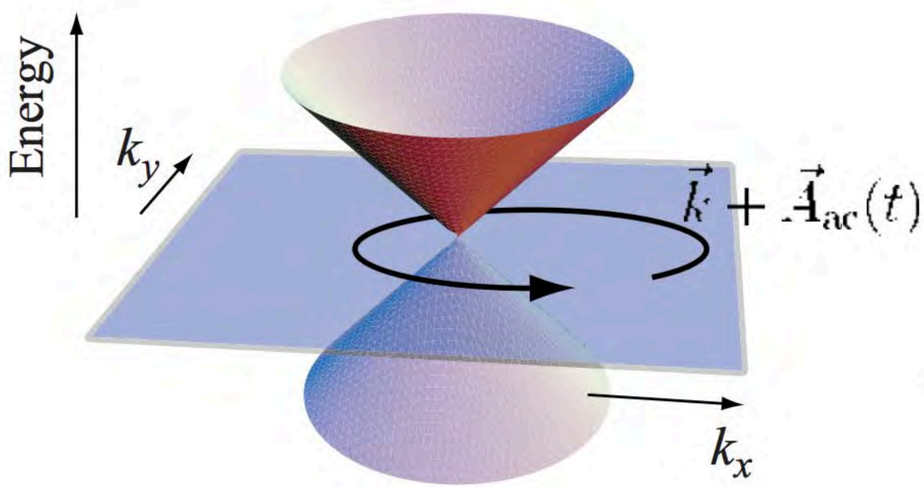
$$\boxed{U(T) \equiv e^{-iH_{\text{eff}}T}}$$



Suggests analogues of topological phenomena from static systems in driven systems

Optical control of band topology discussed for various setups

Circularly-polarized light opens Haldane gap in graphene



T. Oka and H. Aoki, Phys. Rev. B **79**, 081406 (2009).

ARPES experiment (3DTI surface):

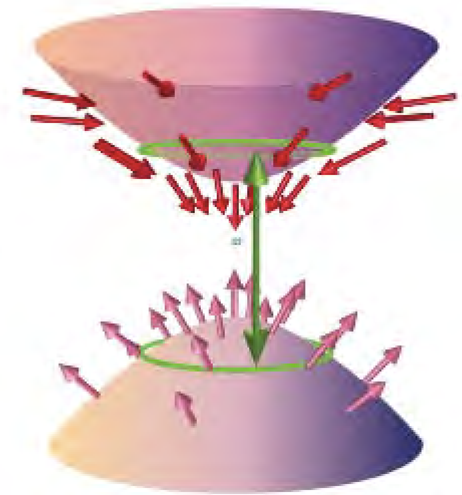
Y. H. Wang *et al.*, Science **342**, 453 (2013).

Resonant driving used to create band inversion

ARTICLES
PUBLISHED ONLINE: 13 MARCH 2011 | DOI: 10.1038/NPHYS1926
nature physics

Floquet topological insulator in semiconductor quantum wells

Netanel H. Lindner[1,2]★, Gil Refael[1,2] and Victor Galitski[3,4]

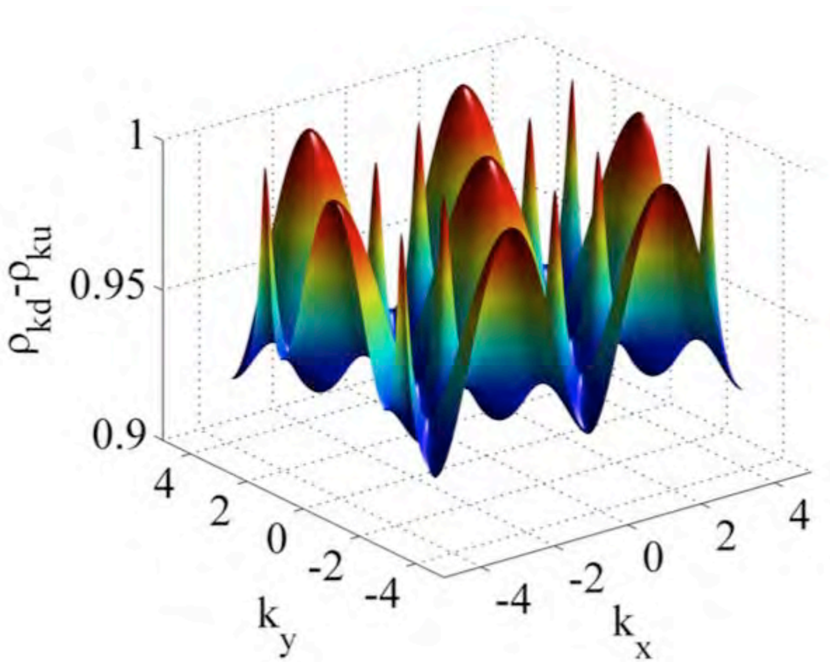


N. Lindner, G. Refael, and V. Galitski, Nature Physics **7**, 490 (2011).

+ driven optical lattices, photonic systems...

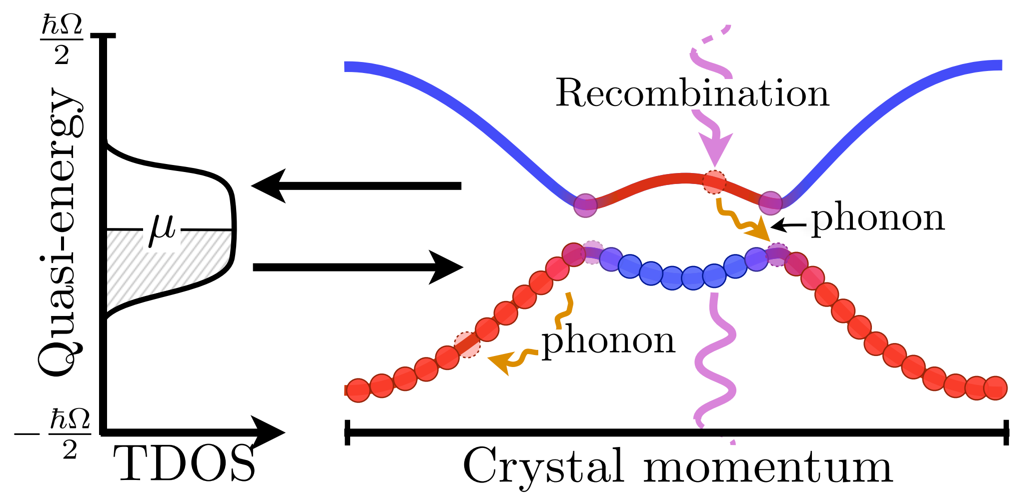
Insulator-like steady states can be reached via bath coupling

Driven graphene + phonons



H. Dehghani, T. Oka, and A. Mitra, Phys. Rev. B **91**, 155422 (2015).

Driven semiconductor + Bose, Fermi baths



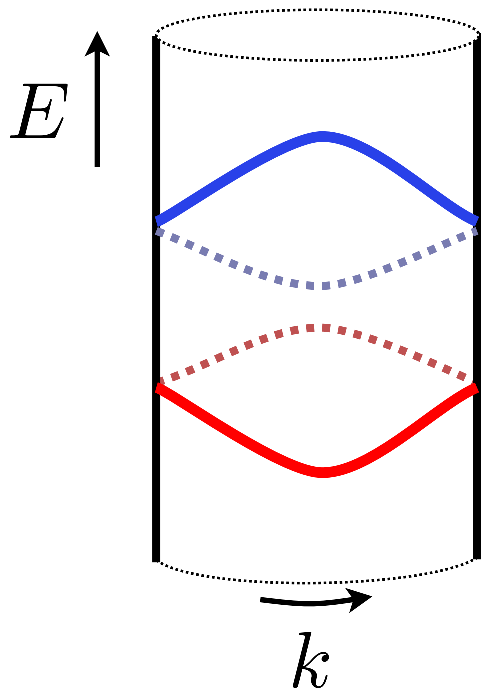
K. Seetharam, C.-E. Bardyn, N. Lindner, MR, and G. Refael, arXiv:1502.02664 (2015).

see also, for example:

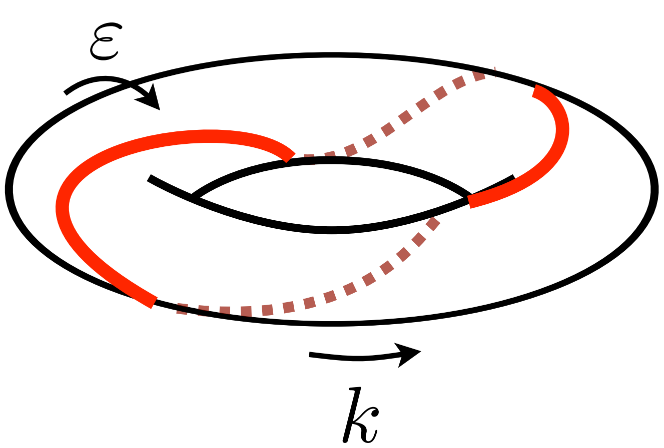
V. M. Galitskii, S. P. Goreslavskii, and V. F. Elesin, JETP **30**, 117 (1970).
T. Shirai, T. Mori, and S. Miyashita, Phys. Rev. E **91**, 030101(R) (2015).
D. E. Liu, arXiv:1410.2962 (2014).
T. Iadecola, T. Neupert, C. Chamon, Phys. Rev. B **91**, 235133 (2015).

New topological configurations possible in driven systems

Normal band structure: cylinder



Quasi-band structure: torus



Quasi-energy winding related to quantized adiabatic transport

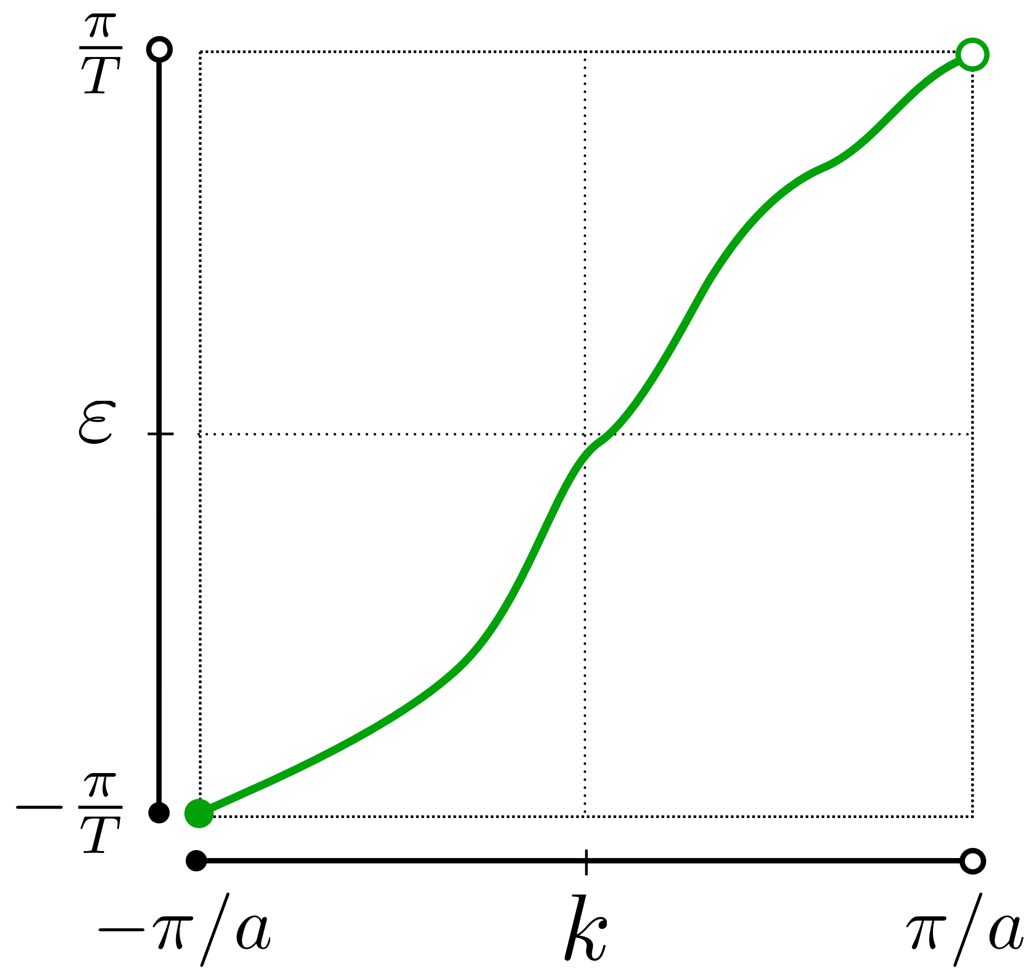
Average group velocity <u>quantized</u>

$$\bar{v}_g = \overline{\frac{d\varepsilon_k}{dk}} = a/T$$

Average displacement:

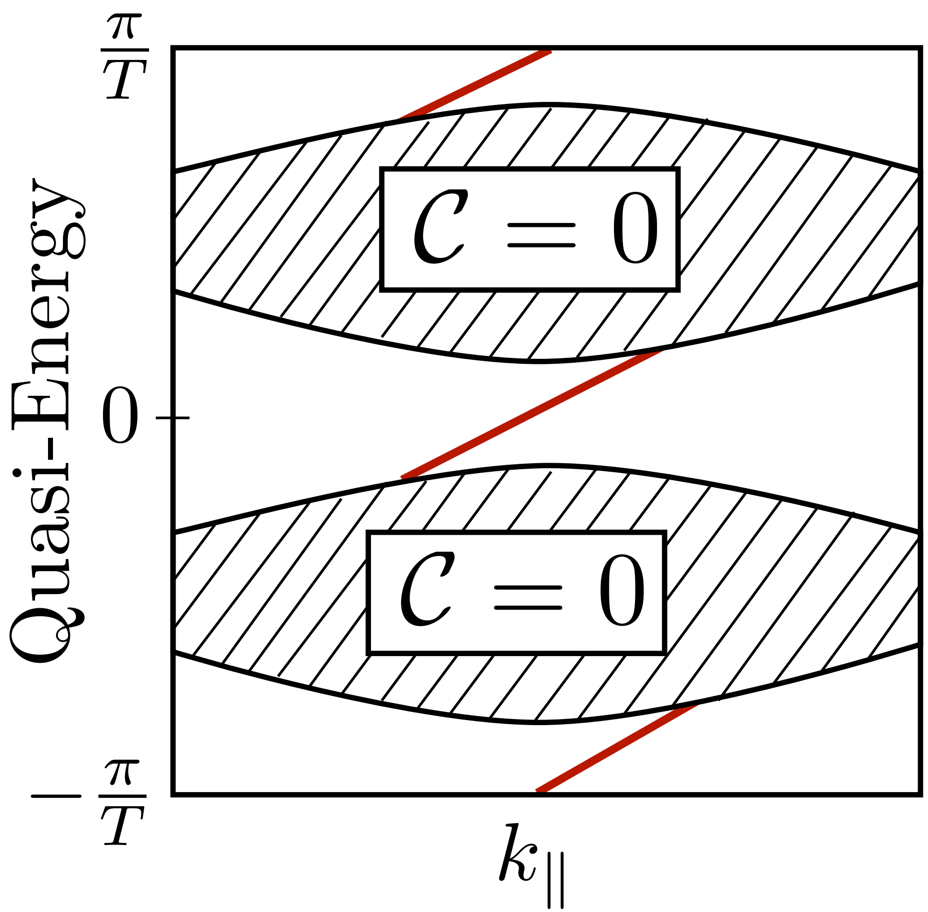
$$\overline{\Delta x} = \bar{v}_g \, T = a$$

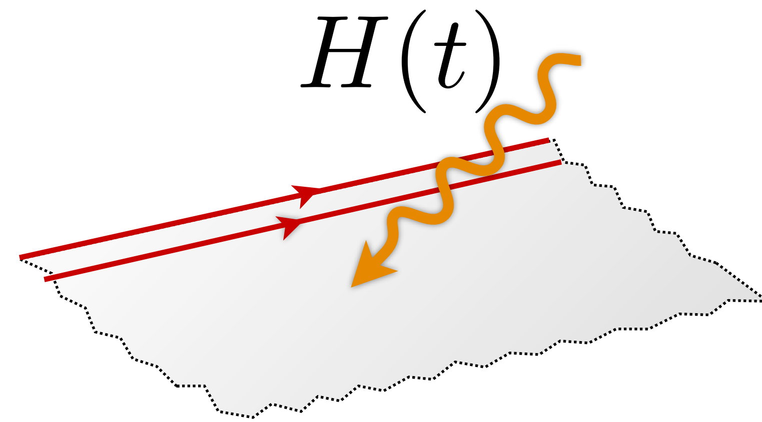
shift by one unit cell



D. J. Thouless, Phys. Rev. B **27**, 6083 (1983).
T. Kitagawa, E. Berg, MR, and E. A. Demler, Phys. Rev. B 82, 235114 (2010).

Driven 2D systems may support chiral edge modes even when all Chern numbers are <u>zero</u>





Chiral edge modes for $\mathcal{C} = 0$ bands

T. Kitagawa, E. Berg, MR, and E. A. Demler, Phys. Rev. B 82, 235114 (2010).

MR, N. Lindner, E. Berg, and M. Levin, Phys. Rev. X **3**, 031005 (2013).

Other examples (Floquet-Majorana, TRS, Chiral symmetry, ...):

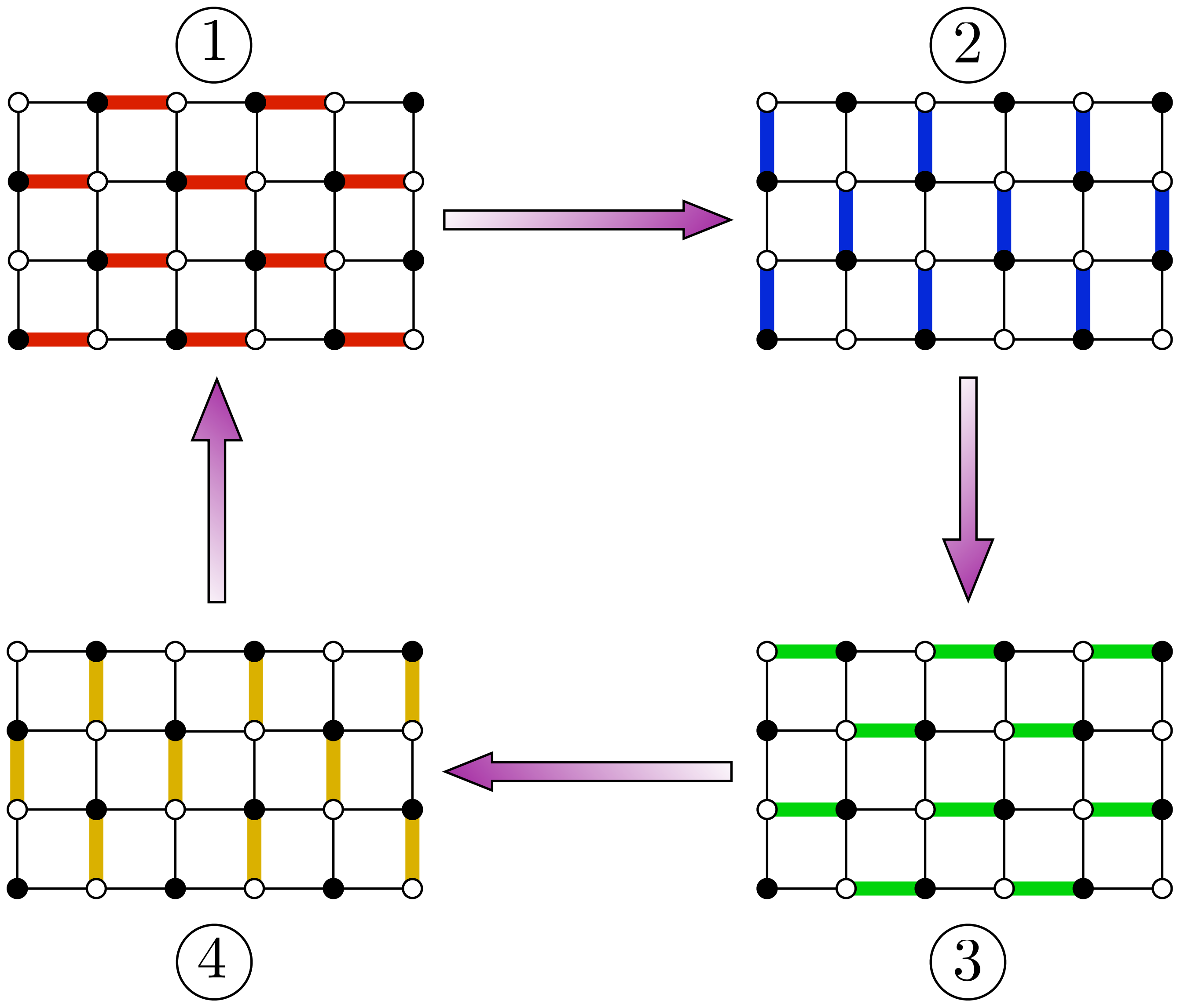
L. Jiang *et al.*, Phys. Rev. Lett. **106**, 220402 (2011).

D. Carptentier *et al.*, arXiv:1407.7747 (2014). J. K. Asboth *et al.*, Phys. Rev. B **90**, 125143 (2014).

Part II

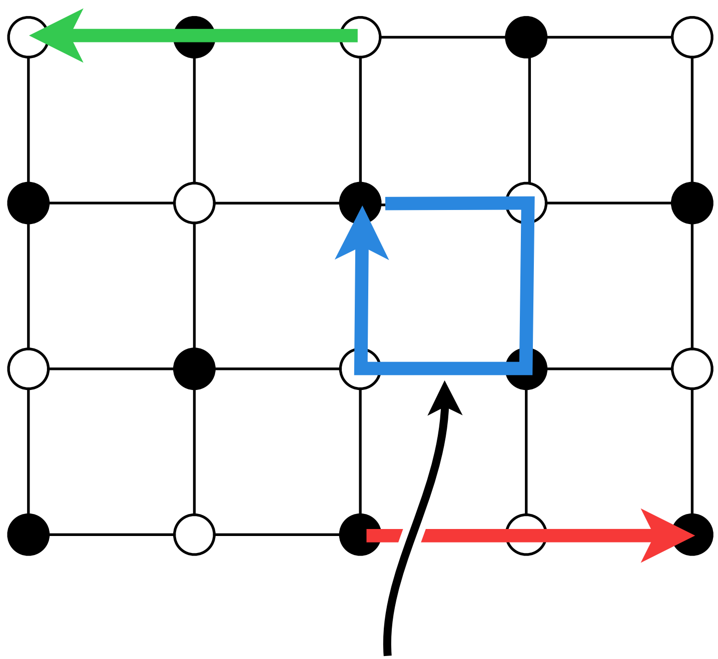
Invariants of the evolution and topological classification

New phase illustrated by model with modulated hoppings

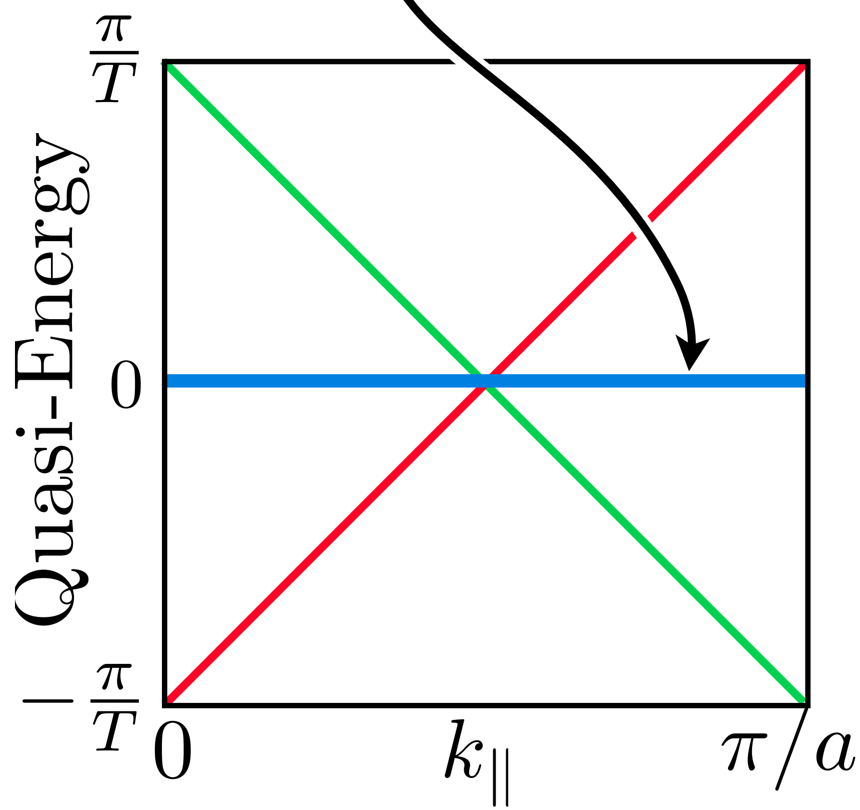


Bulk evolution trivial, chiral modes propagate along edges

Bulk Floquet operator is trivial: $U(T) = \mathbb{1}$



Particle returns to initial position after cycle



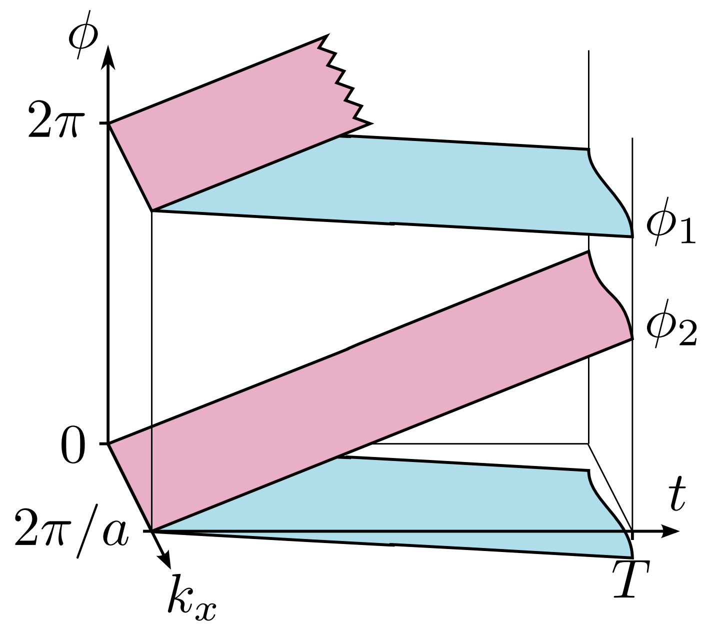
Winding number invariant:

MR, N. Lindner, E. Berg, and M. Levin, Phys. Rev. X **3**, 031005 (2013).

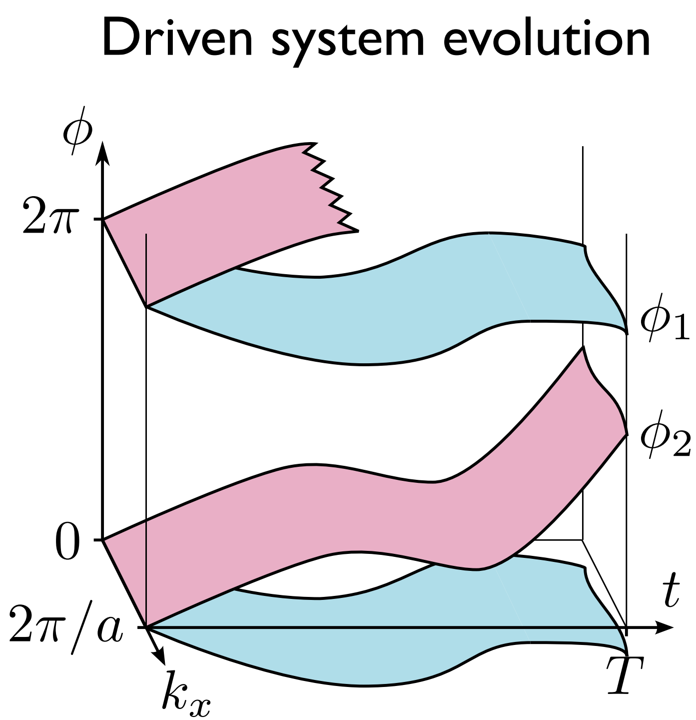
In non-driven system, phases wind *linearly* in time

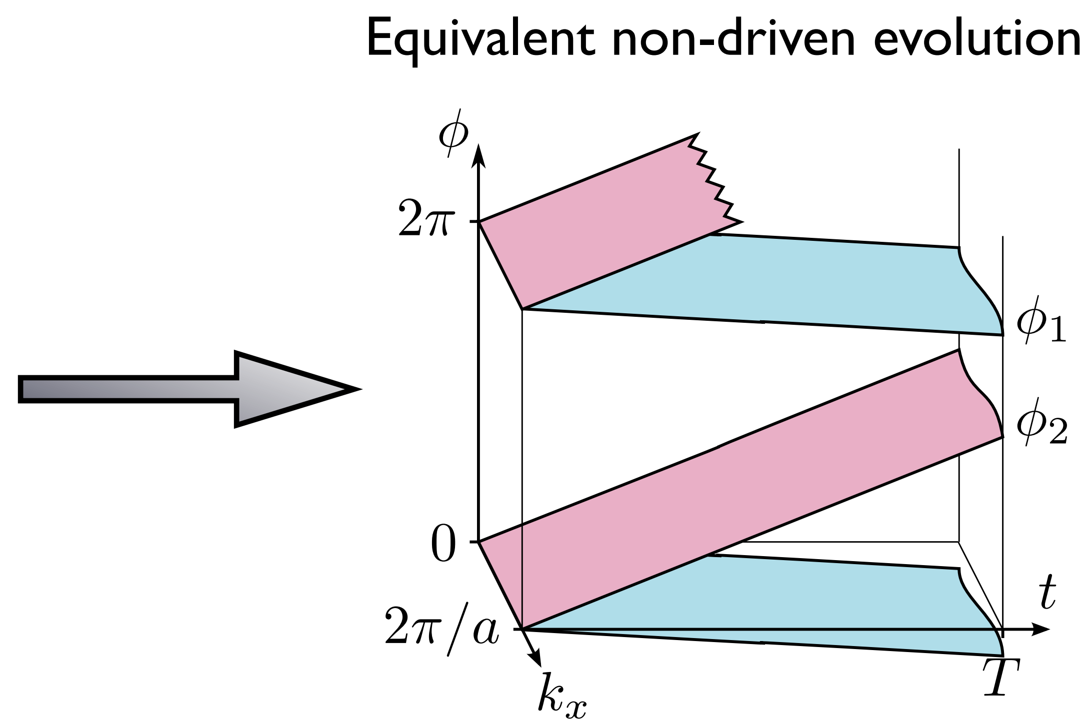
$$H|\,\psi_n\,\rangle = E_n|\,\psi_n\,\rangle, \quad U(t) = \sum_n e^{-iE_nt}|\,\psi_n\,\rangle\langle\,\psi_n\,|$$

<u>Example: two bands, one dimension</u>

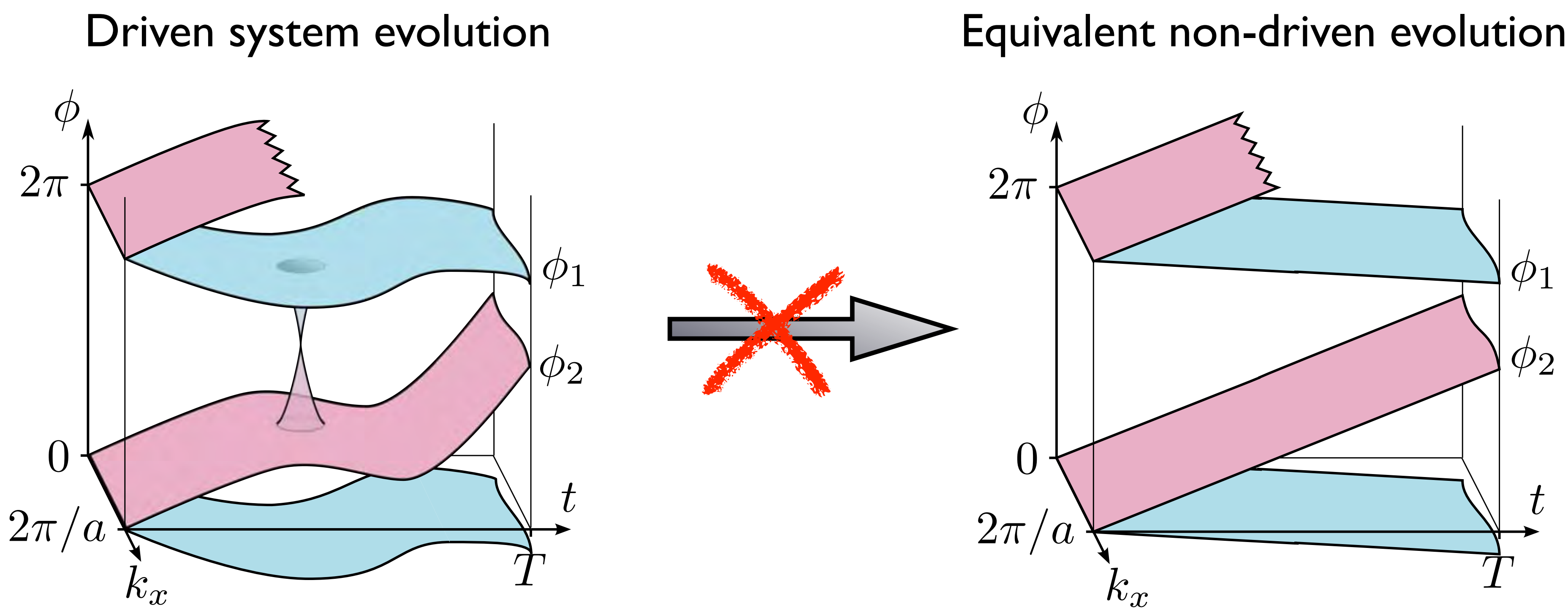


Topological equivalence: continuously adjust H(t) to "straighten" phase bands of driven system

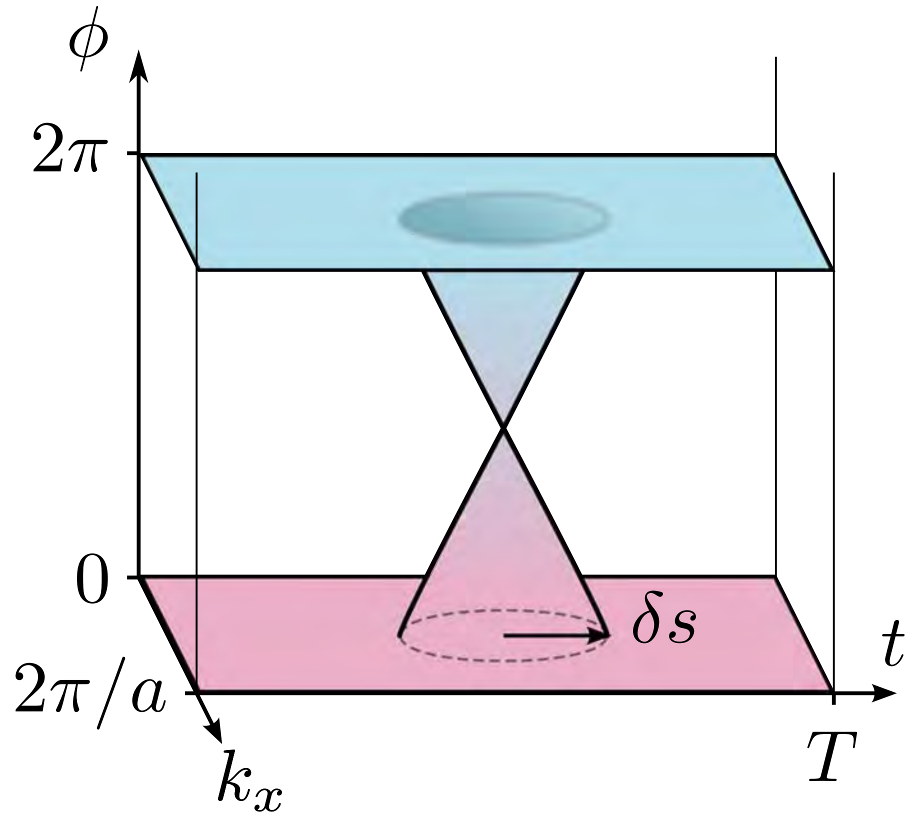




"Topological singularities" may break relation between driven and non-driven evolution



Similar to Weyl nodes, topological singularities can be shifted but not removed by local perturbations



$$\boxed{U(\mathbf{s}) \sim e^{-i(\mathbf{s}-\mathbf{s}_0)\cdot\boldsymbol{\sigma}}, \quad \mathbf{s} = (k_x, k_y, t)}$$

* Singularity-based classification captures topology of all FTIs and "anomalous" variants

F. Nathan and MR, arXiv:1506.07647 (2015).

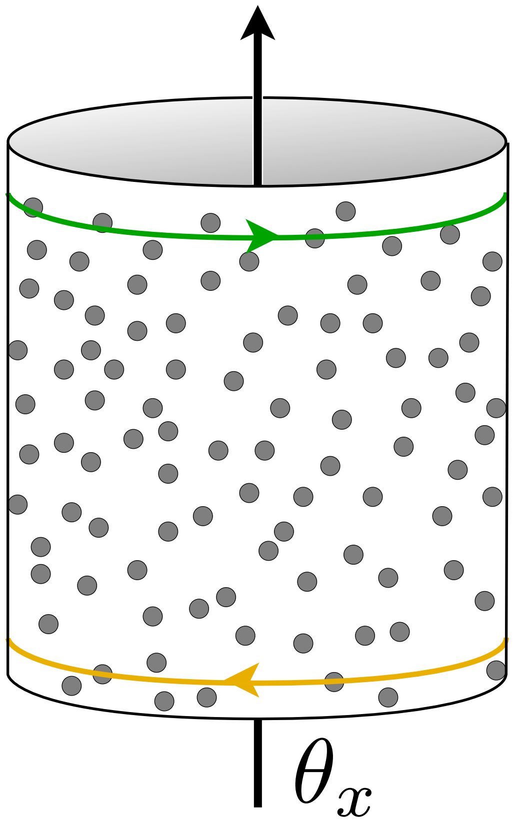
Part III

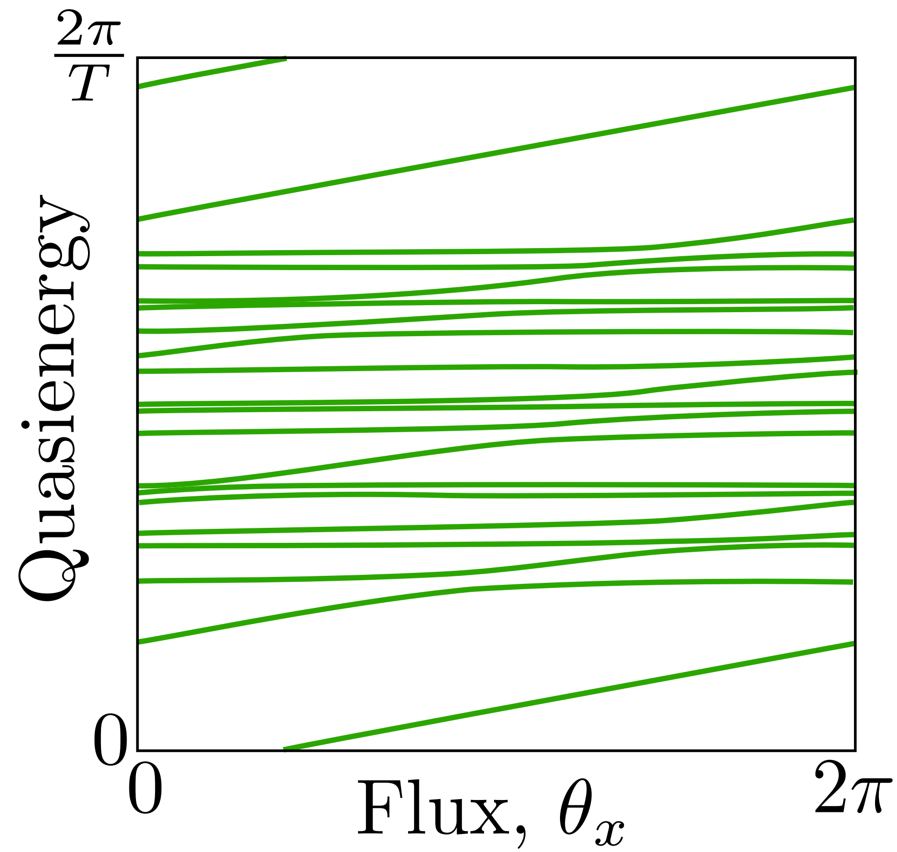
The AFAI and non-adiabatic quantized charge pumping

In collaboration with: Paraj Titum, Erez Berg, Gil Refael, and Netanel Lindner

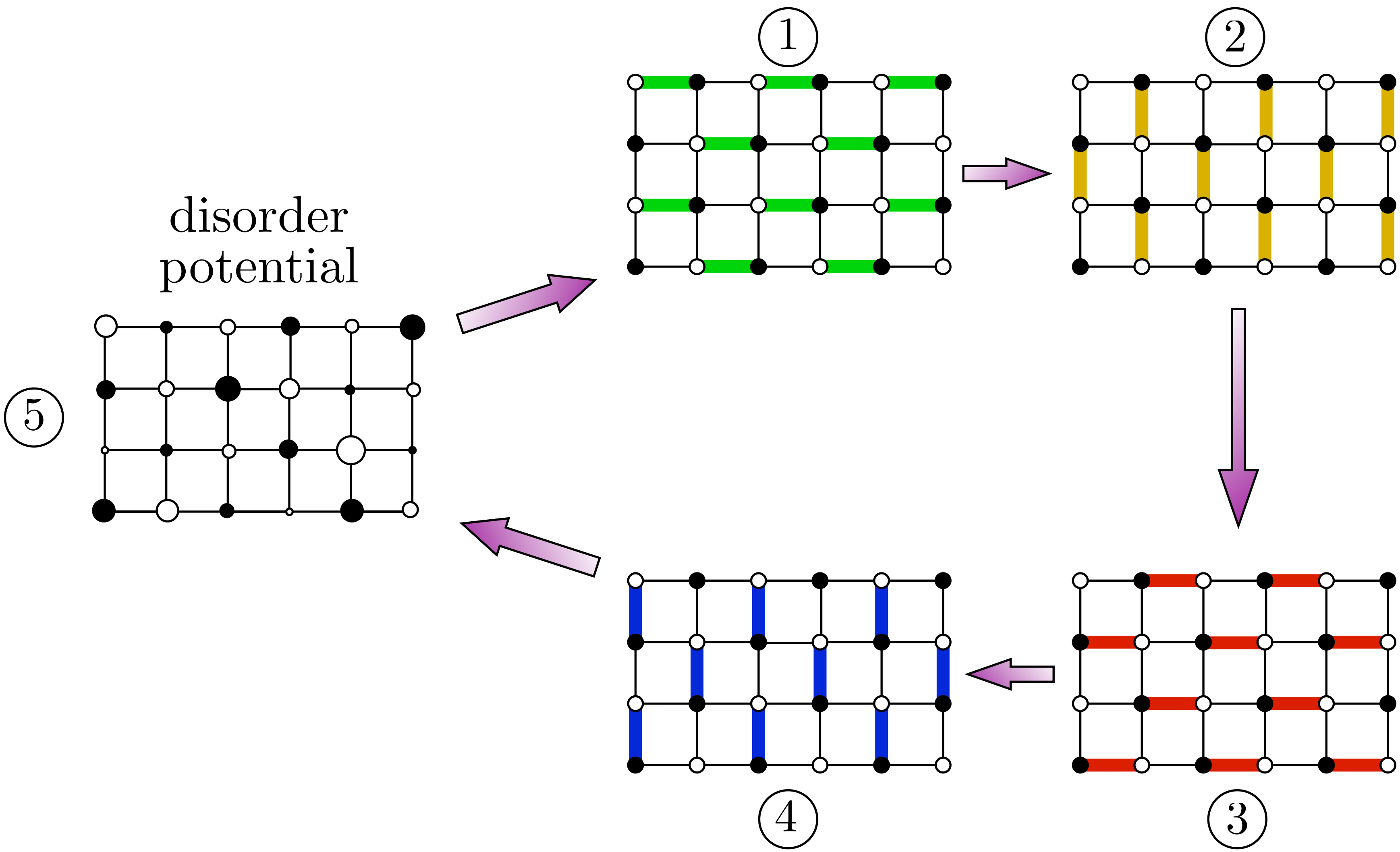
P. Titum, E. Berg, MR, G. Refael, and N. H. Lindner, arXiv:1506.00650 (2015).

Anomalous Floquet-Anderson Insulator: fully localized bulk with propagating chiral edge states at every quasienergy

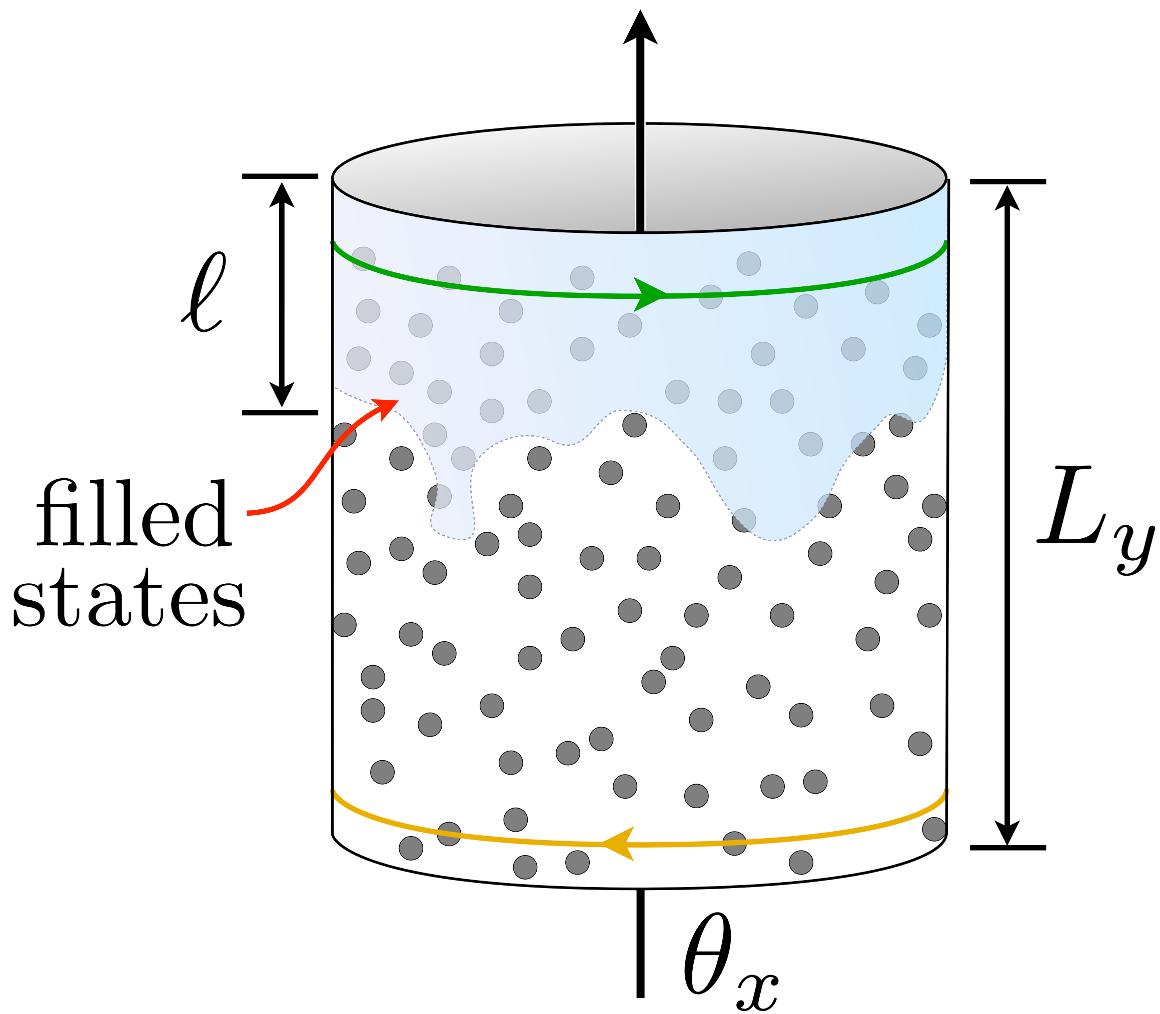




Disorder localizes all bulk states

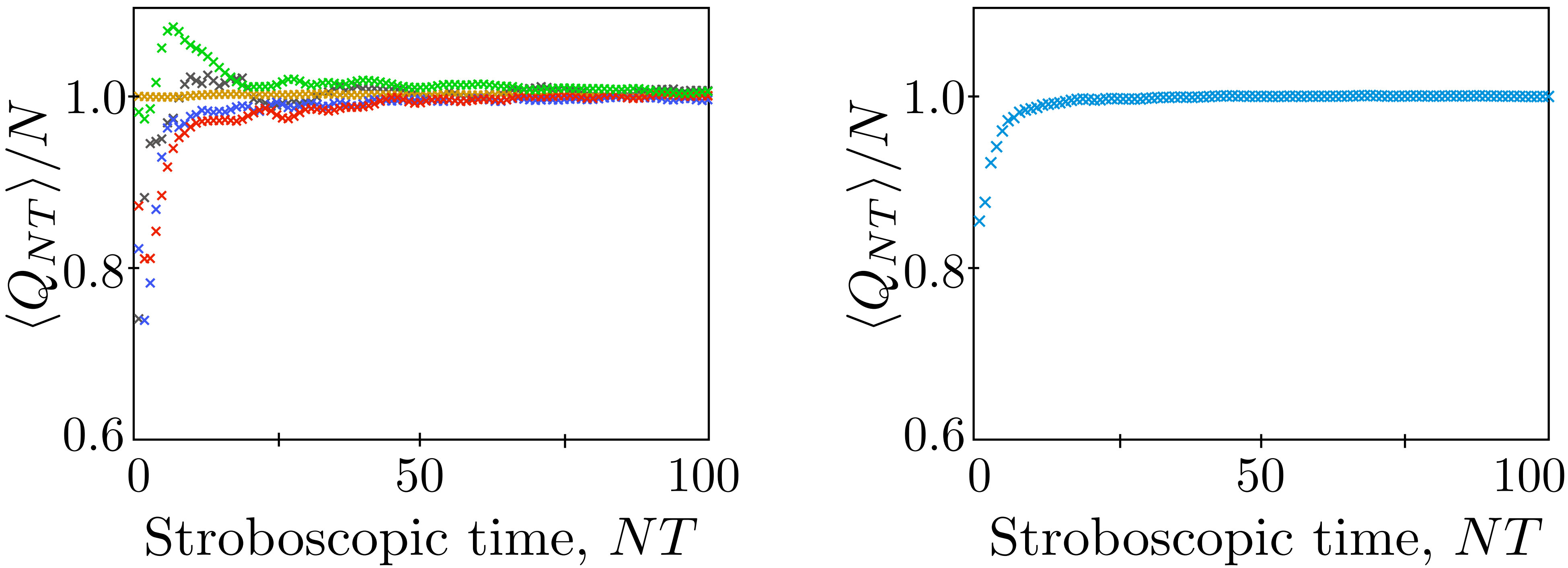


When fully filled, AFAI edges carry quantized current



* Counterpropagating channels spatially separated; no adiabaticity required!

Numerics confirm quantization of average pumped charge



Summary and open questions

Topology of driven systems is more rich than non-driven systems

Periodicity of quasi-energy due to discrete time plays key role

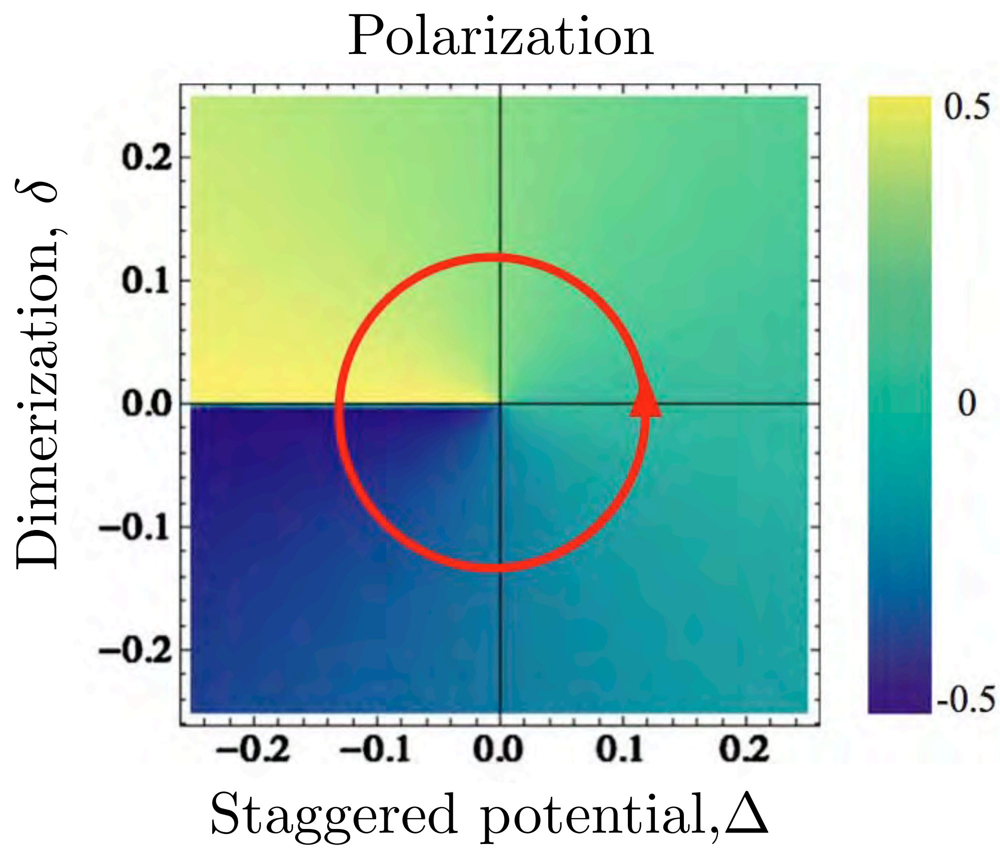
Interplay of driving, and disorder gives rise to a range of new and interesting phenomena

Generalizations to many-body order, MBL, ...

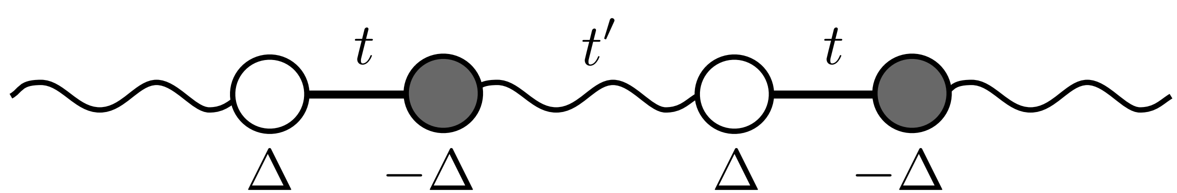
Email: rudner@nbi.dk

Support for this work provided by:

Gapped system: charge pumped via adiabatic cycle is quantized



Rev. Mod. Phys. **82**, 1959 (2010)



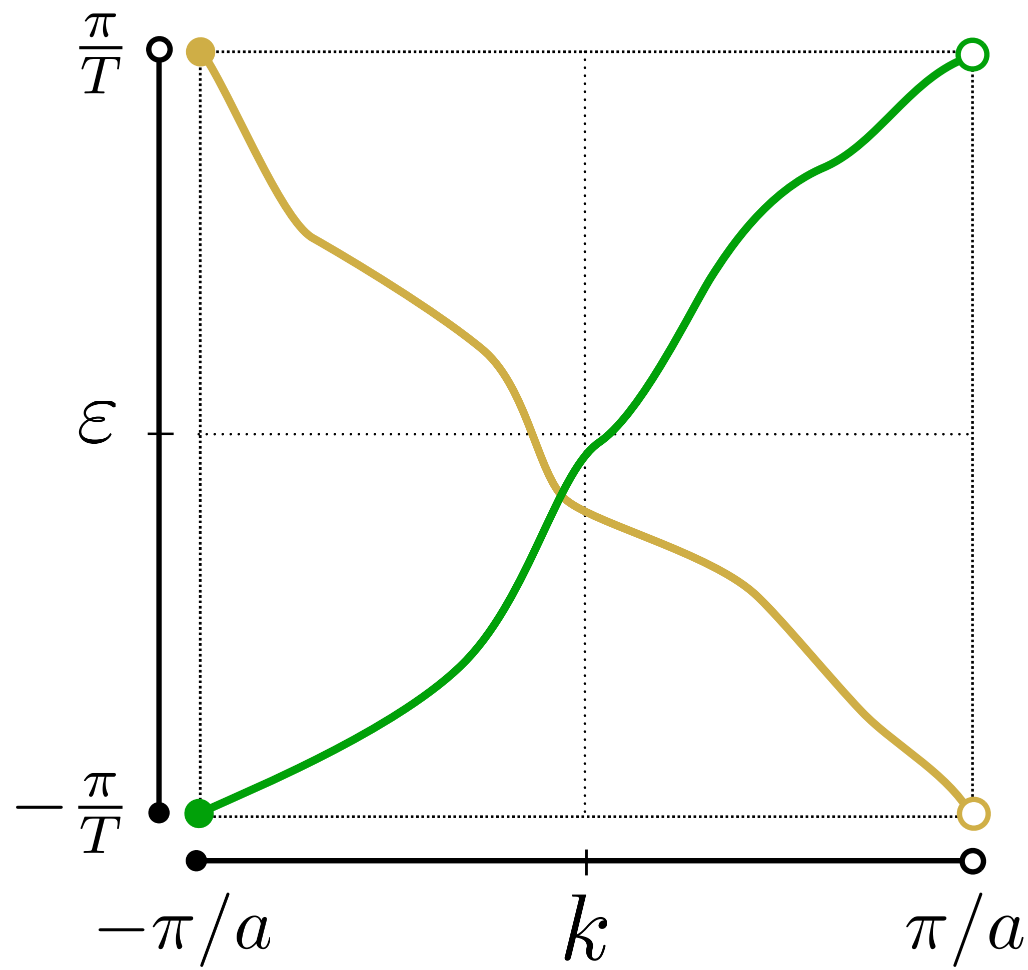
$$t = t_0 + \delta$$
$$t' = t_0 - \delta$$

D. J. Thouless, Phys. Rev. B **27**, 6083 (1983).

Counter-propagating modes decouple in adiabatic limit

Winding number

$$W = \oint \frac{dk}{2\pi} \, \mathrm{Tr}\left[U^{-1}\partial_k U\right]$$



Note: net winding number of all bands must be zero

“Phase band” picture reveals what distinguishes topology in driven and non-driven systems

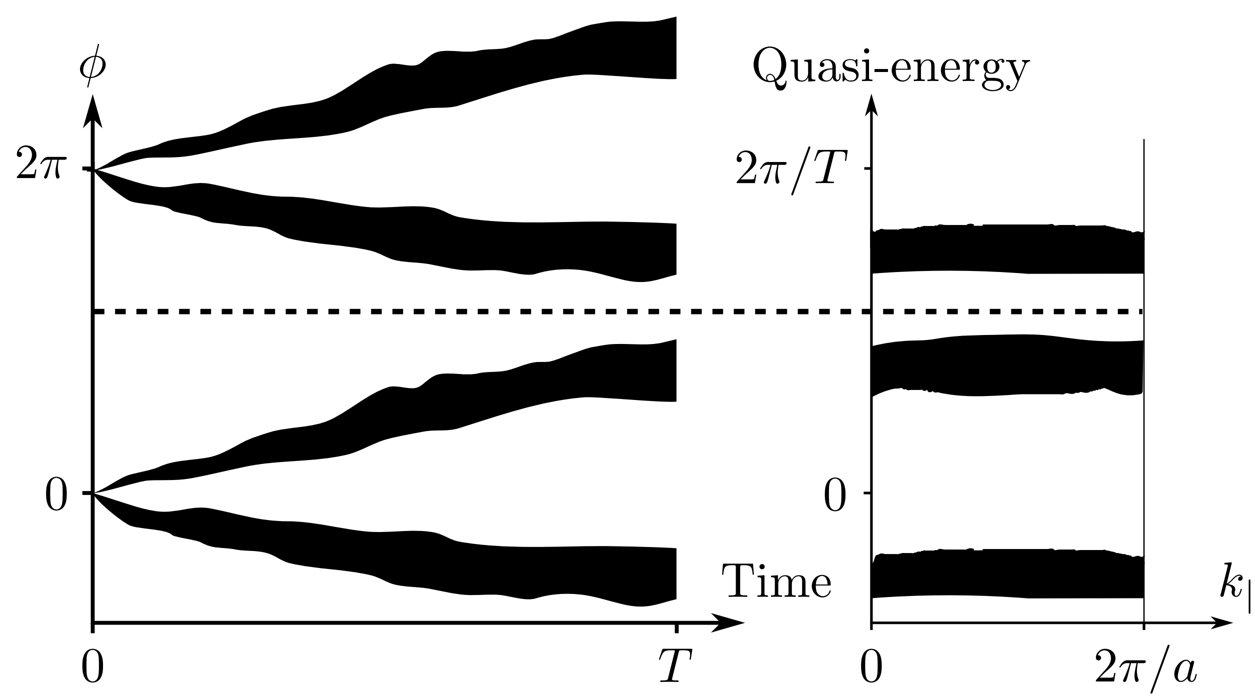
number of bands

$$U(\mathbf{k}, t) = \sum_{n=1}^{N} P_n(\mathbf{k}, t) e^{-i\phi_n(\mathbf{k}, t)}$$

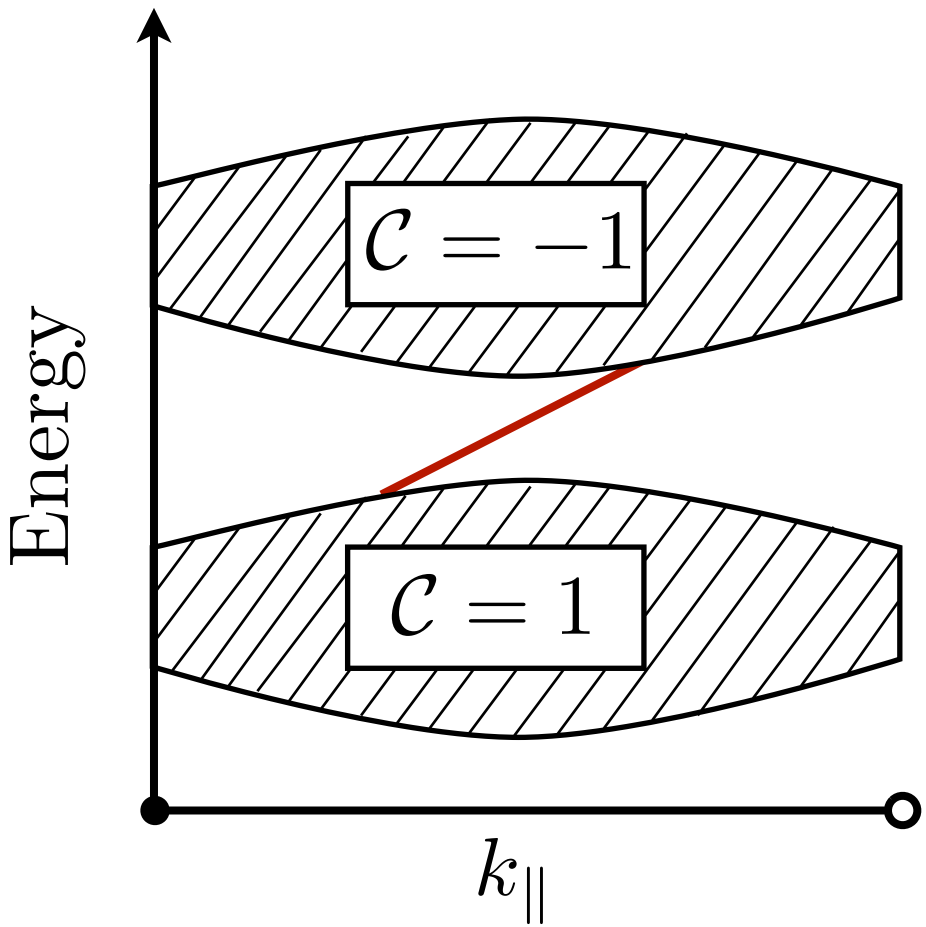
band projector

phase eigenvalue

<u>Example: two bands</u>

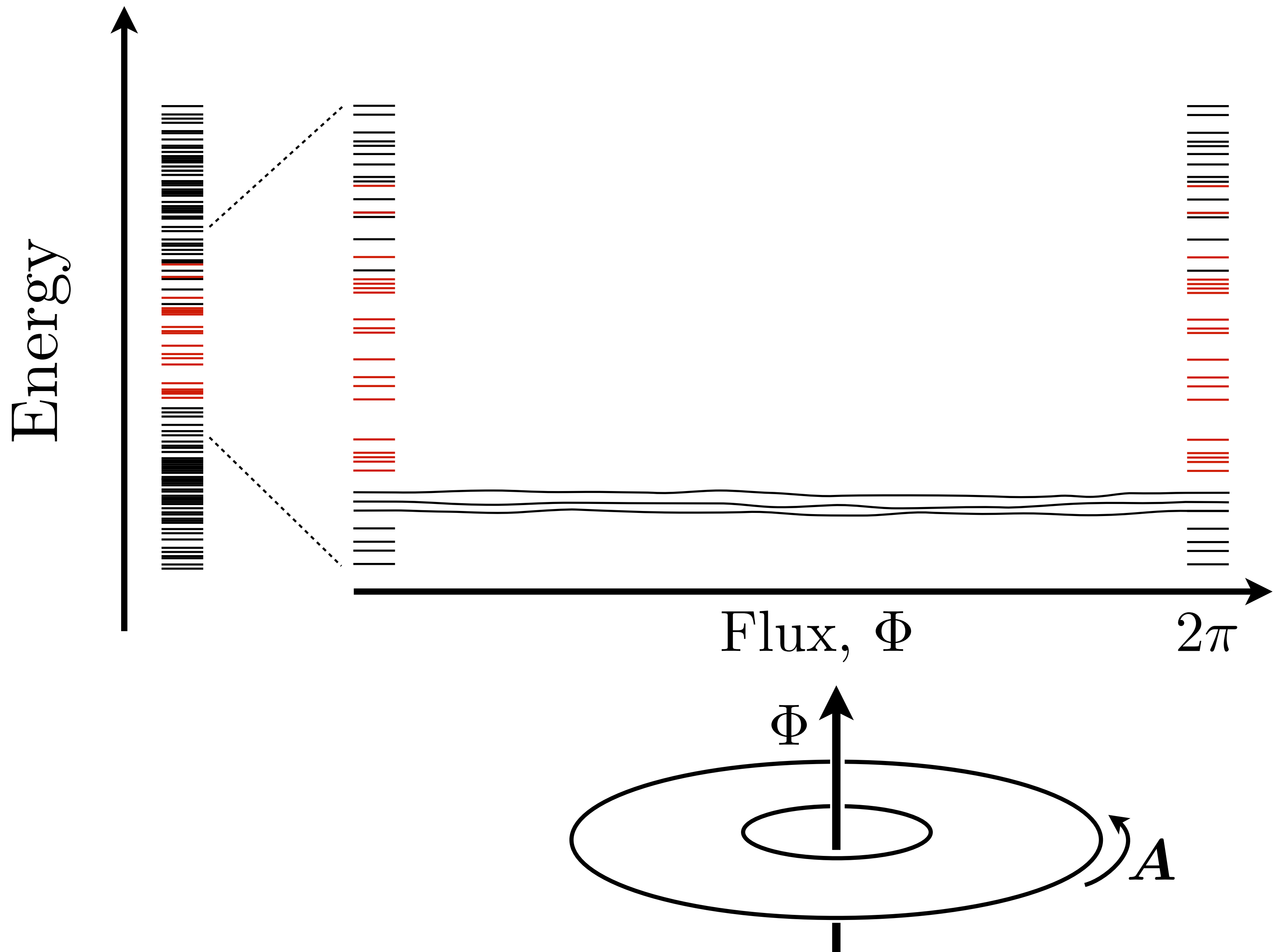


Impossible to localize *all* states in a Chern band

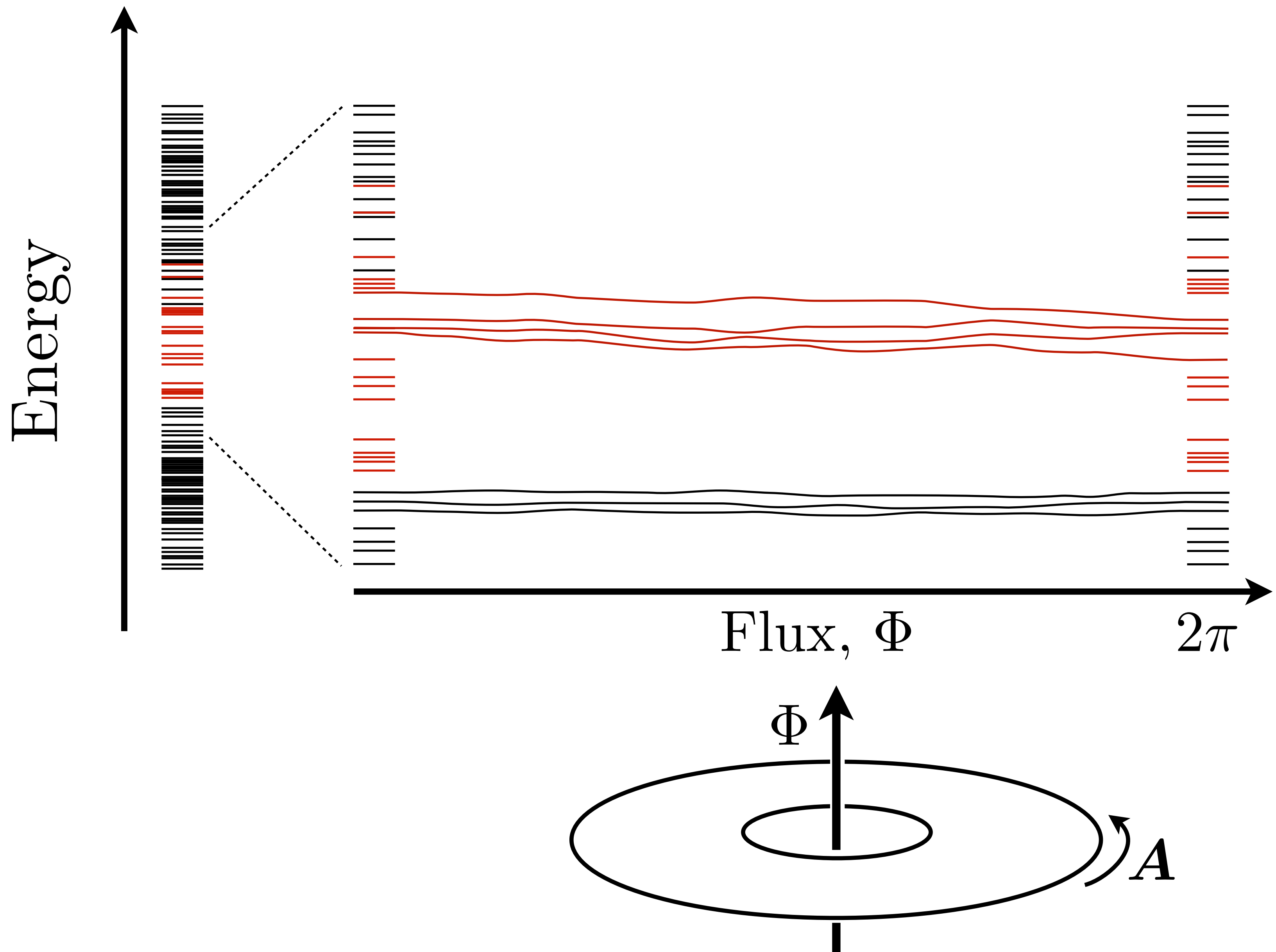


Impossible to localize *all* states in a Chern band

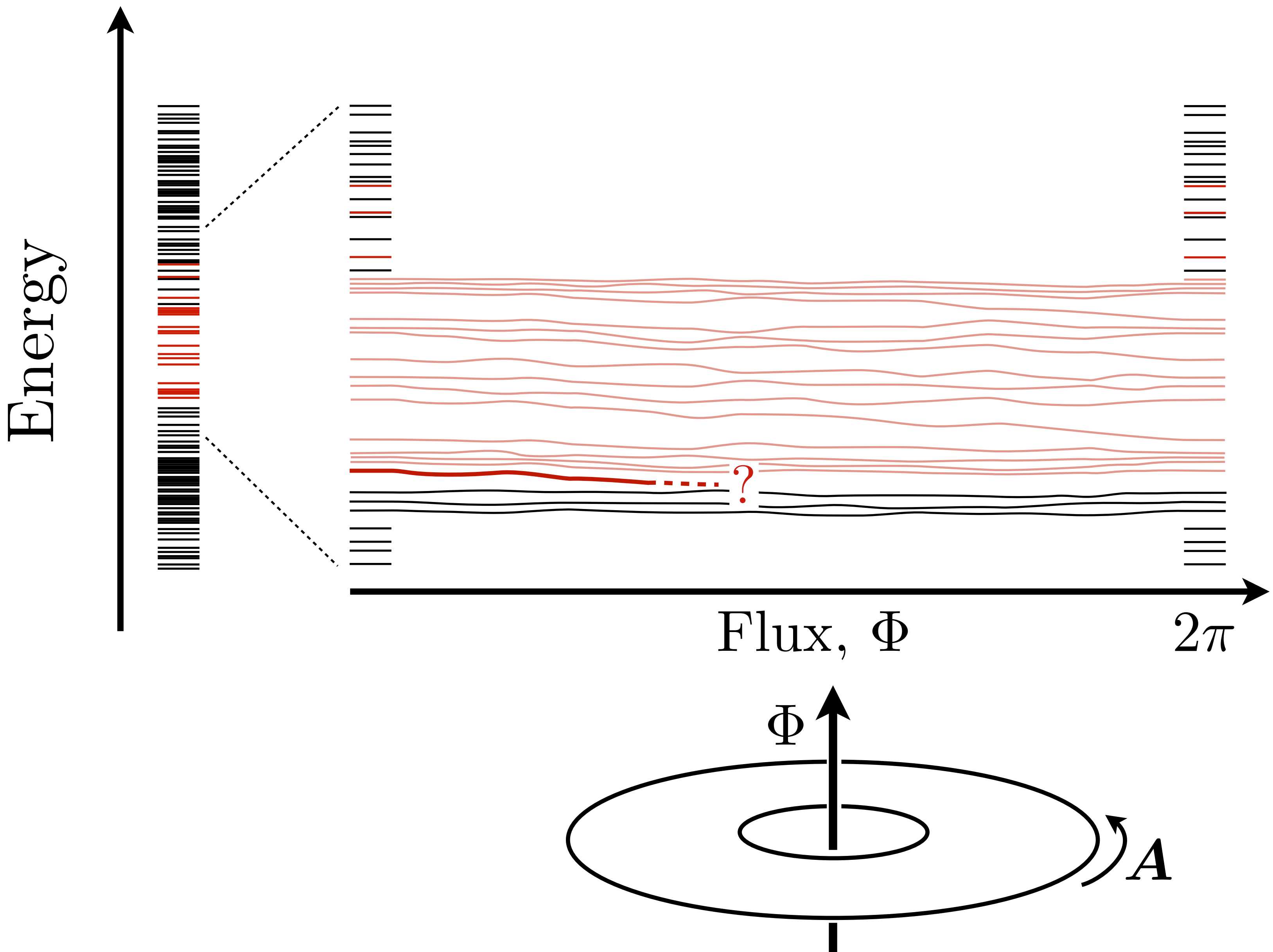
Localized states are exponentially insensitive to flux



Delocalized states feel flux, "flow" to neighbors



Problem: spectral flow cannot terminate



Anomalous Floquet-Anderson Insulator: fully localized bulk with propagating chiral edge states at every quasienergy