

Wire construction of topological phases

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Tel-Aviv University
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Topological order a definition ($T=0$)

- *Gap*
- *No local order*
- *edge states (not necessary)*



Topological order a definition ($T=0$)

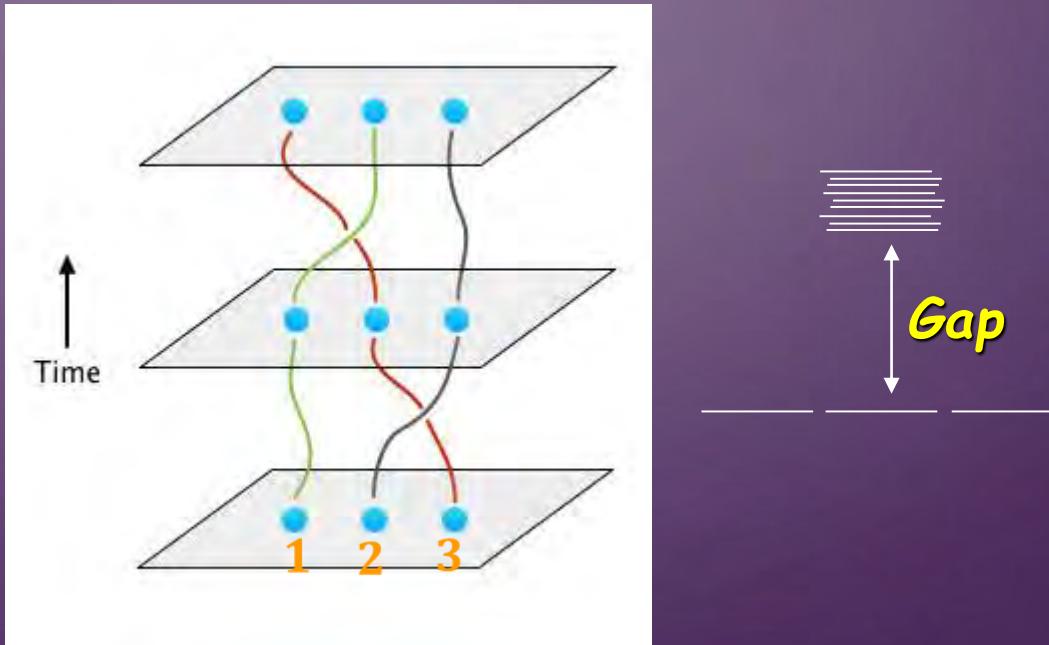
- *Gap*
- *No local order*
- *Edge states (not necessary)*
- *Degeneracy depends on genus*
- *Fractional statistics*



X.G. Wen



Non-Abelian statistics



- Degeneracy grows exponentially with number of QP
- Can operate on ground state manifold by braiding
- Perspectives for topological quantum computation
- Possibly realized in $\nu=5/2$ FQH plateau

Methods:

- Wave function /field theories
- numerics
- fine tuned models

Would like to have:
Simple and general
theoretical
approach for
topological states

Possibly lead to new
experiments

$$\psi(z_1, \dots, z_N) =$$

$$\prod_{i < j} (z_i - z_j)^3 \prod_i e^{-\frac{|z_i|^2}{2\ell_B}}$$

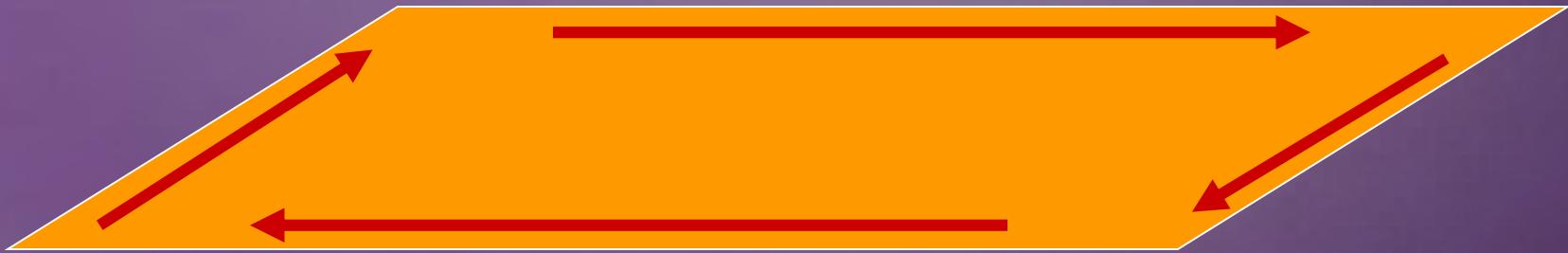


Outline

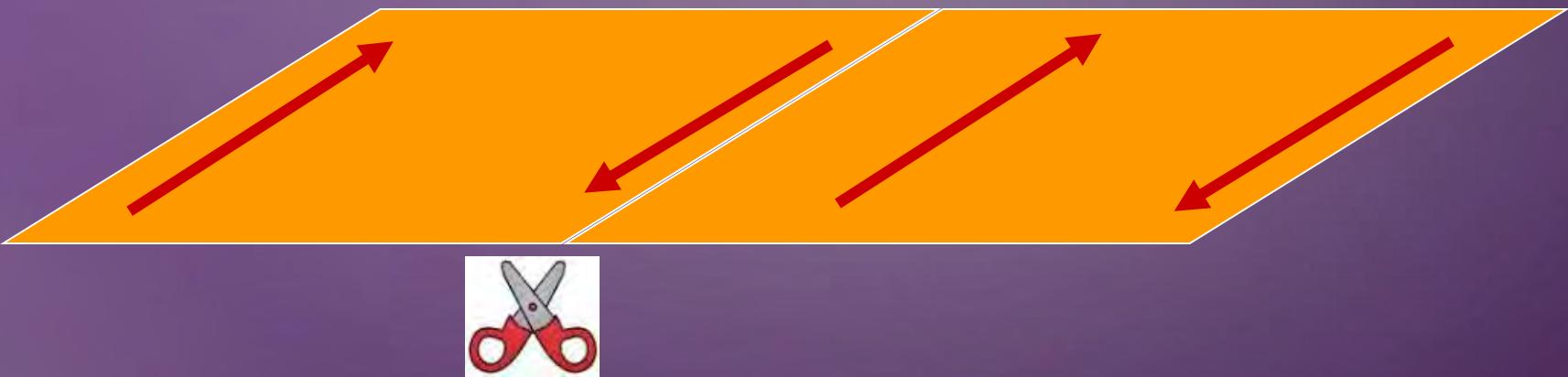
- *Introduction*
- *Wire construction*
- *Examples: Chiral Spin Liquid...*
- *Possible experimental realizations...*

Coupled wire
construction
of topological states

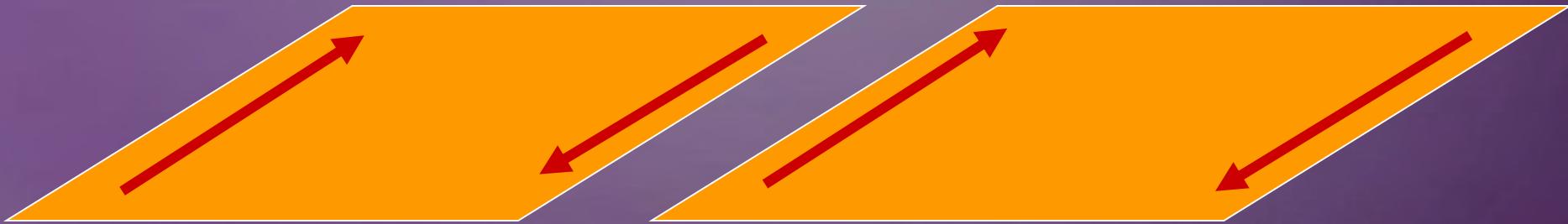
Coupled-wire construction of quantum Hall states



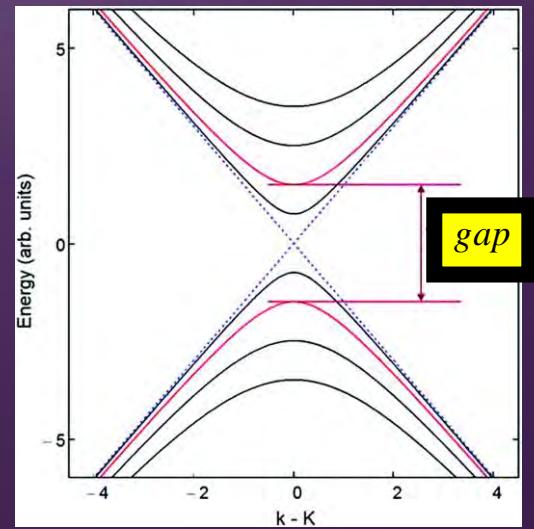
Coupled-wire construction of quantum Hall states



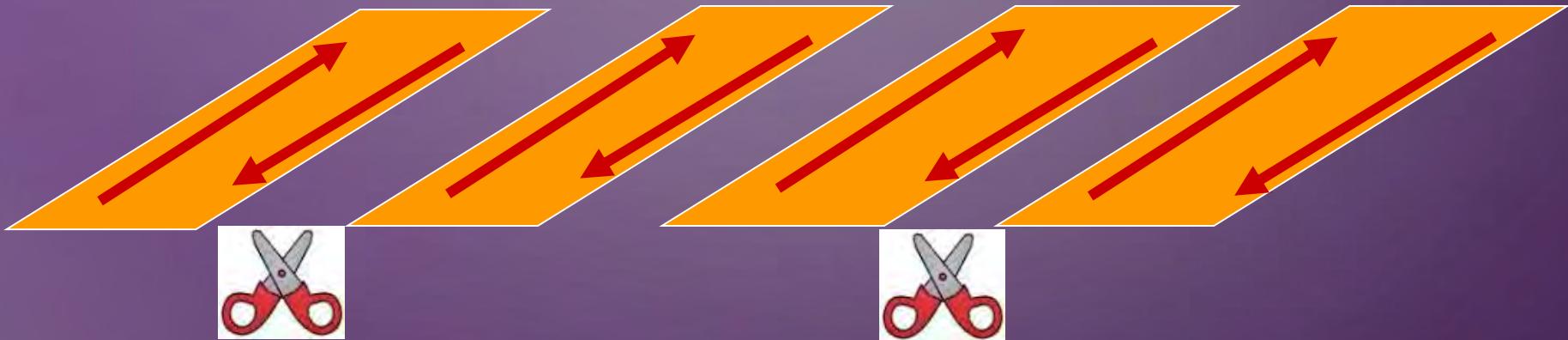
Coupled-wire construction of quantum Hall states



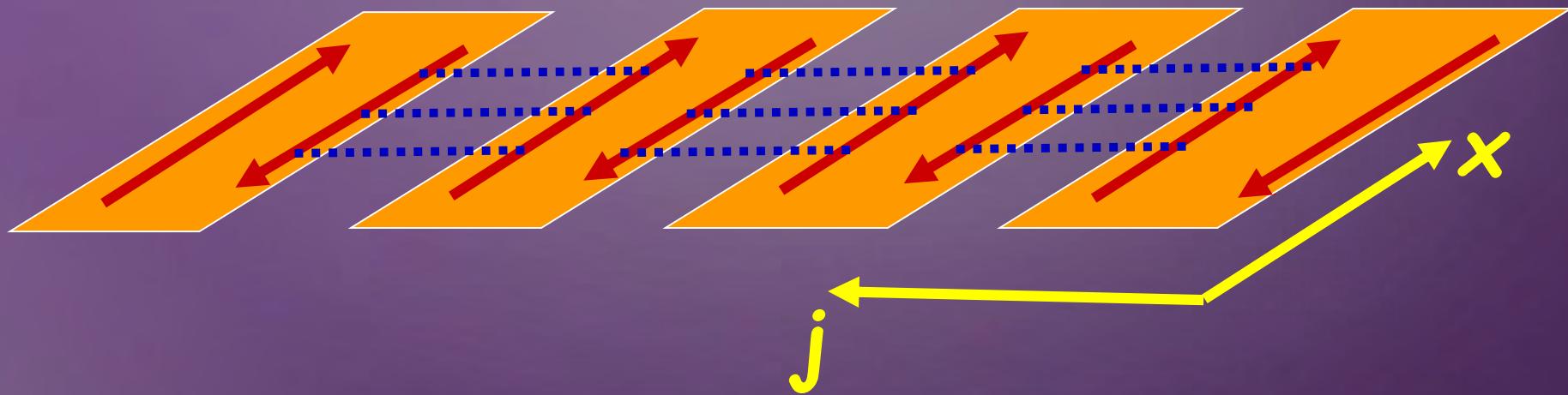
$$H = \int_{edge} dx \psi_{1,R}^+ \psi_{2,L} + (h.c.)$$



Coupled-wire construction of quantum Hall states



Coupled-wire construction of quantum Hall states



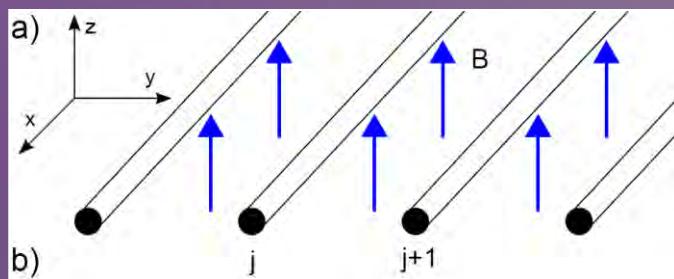
$$H_{topological \atop phase} = \sum_j \int dx \psi_{j,R}^+ \psi_{j+1,L} + (h.c.)$$



Fractional Quantum Hall Effect in an Array of Quantum Wires

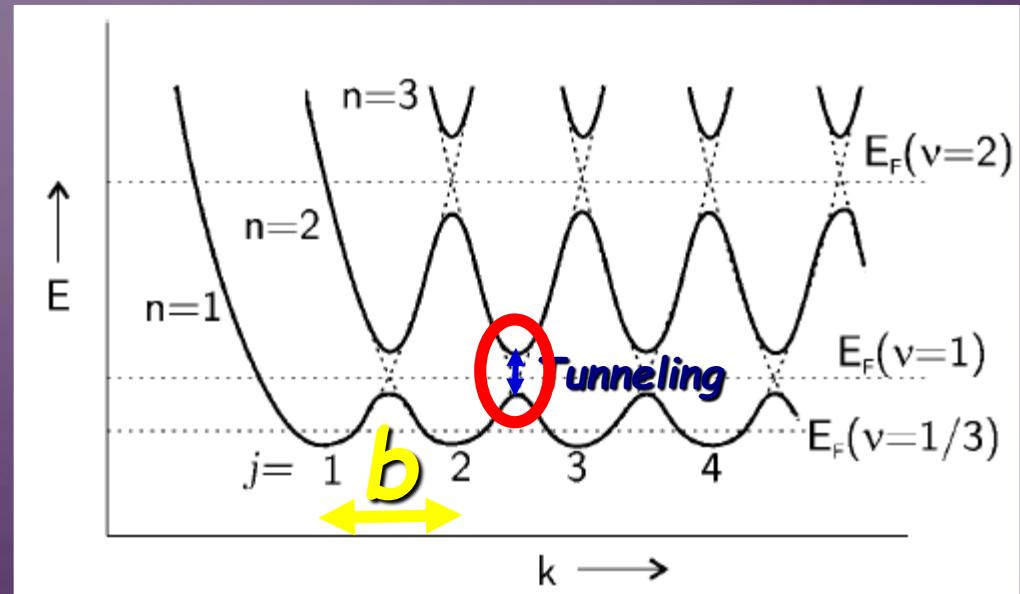
C. L. Kane, Ranjan Mukhopadhyay, and T. C. Lubensky

$$\mathcal{H} = \mathcal{H}_{1D} + \mathcal{H}_{tun}$$



$$b = eaB/\hbar c$$

$$E_j(k) = \frac{\hbar^2}{2m} (k - bj)^2$$

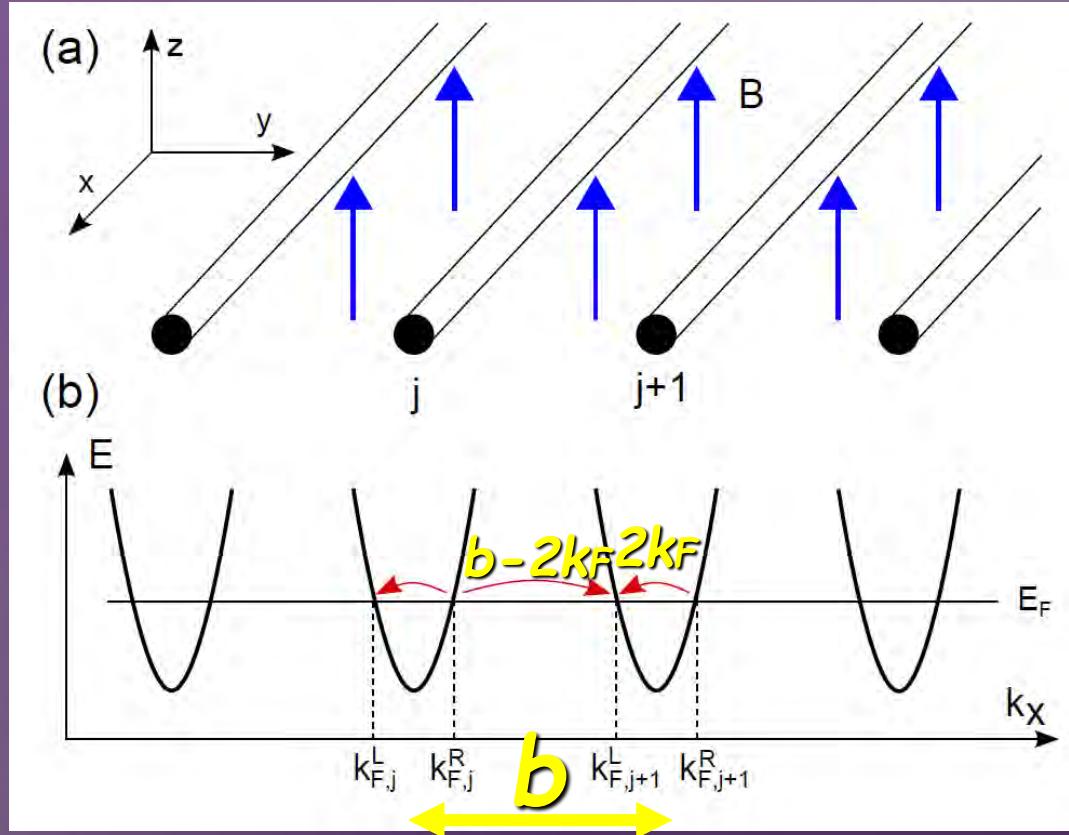


Filling factor: $v = hn_e/eB = 2k_F/b$

- $2k_F = b \leftrightarrow v = 1$; $2k_F = 2b \leftrightarrow v = 2\dots$
- Bulk gap, Edge states
- Near the IQH instability: Tunneling $\ll E_F \rightarrow$ Commuting 1D link problems

$$H = \psi_{j,R}^+ i v_F \partial_x \psi_{j,R} - \psi_{j+1,L}^+ i v_F \partial_x \psi_{j+1,L} + V_{tun} \sum_j \psi_{j,R}^+ \psi_{j+1,L}$$

FQHE



Add interactions

- Conserve momentum*
- $b - 2k_F = 4k_F \leftrightarrow v = 1/3$
- $b - 2k_F = (2k_F)2m \leftrightarrow v = 1/(2m+1)$

(Laughlin sequence)

Near FQH instability:
tunneling $\ll E_F$
 \rightarrow can linearize
 \rightarrow Bosonization valid

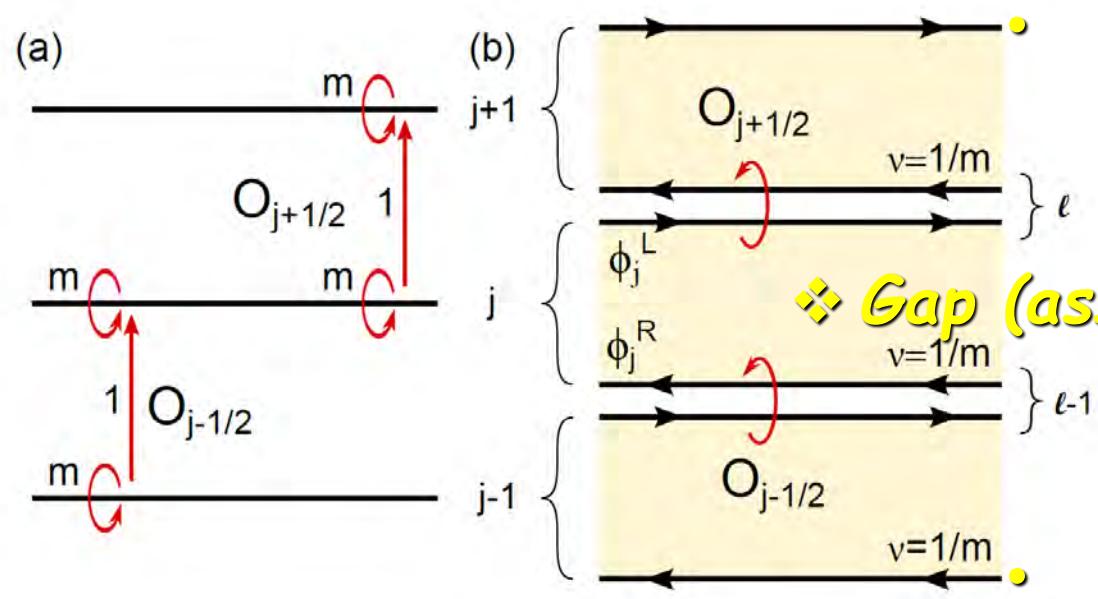
$$H = \psi_{j,R}^+ i v_F \partial_x \psi_{j,R} - \psi_{j+1,L}^+ i v_F \partial_x \psi_{j+1,L} + V_{tun}^{eff} \sum_j (\psi_{j,L}^m \psi_{j,R}^{\dagger 1+m}) (\psi_{j+1,L}^{1+m} \psi_{j+1,R}^{\dagger m})$$

$$\begin{aligned}
O_{j+1/2} &= \sum_j (\psi_{j,L}^m \psi_{j,R}^{\dagger 1+m})(\psi_{j+1,L}^{1+m} \psi_{j+1,R}^{\dagger m}) + h.c. \\
&= \sum_j e^{-i(-m\phi_{j,L} + (m+1)\phi_{j,R})} e^{i((1+m)\phi_{j+1,L} - m\phi_{j+1,R})} + h.c. \\
&= \sum_j e^{-i\tilde{\phi}_{j,R}} e^{i\tilde{\phi}_{j+1,L}} + h.c. = \cos(\tilde{\phi}_{j,R} - \tilde{\phi}_{j+1,L})
\end{aligned}$$

□ **Bosonize**

$$\psi_{j,L/R} \propto e^{i\phi_{j,L/R}}$$

□ **Redefine bosons**
➤ Still commuting 1D problems!

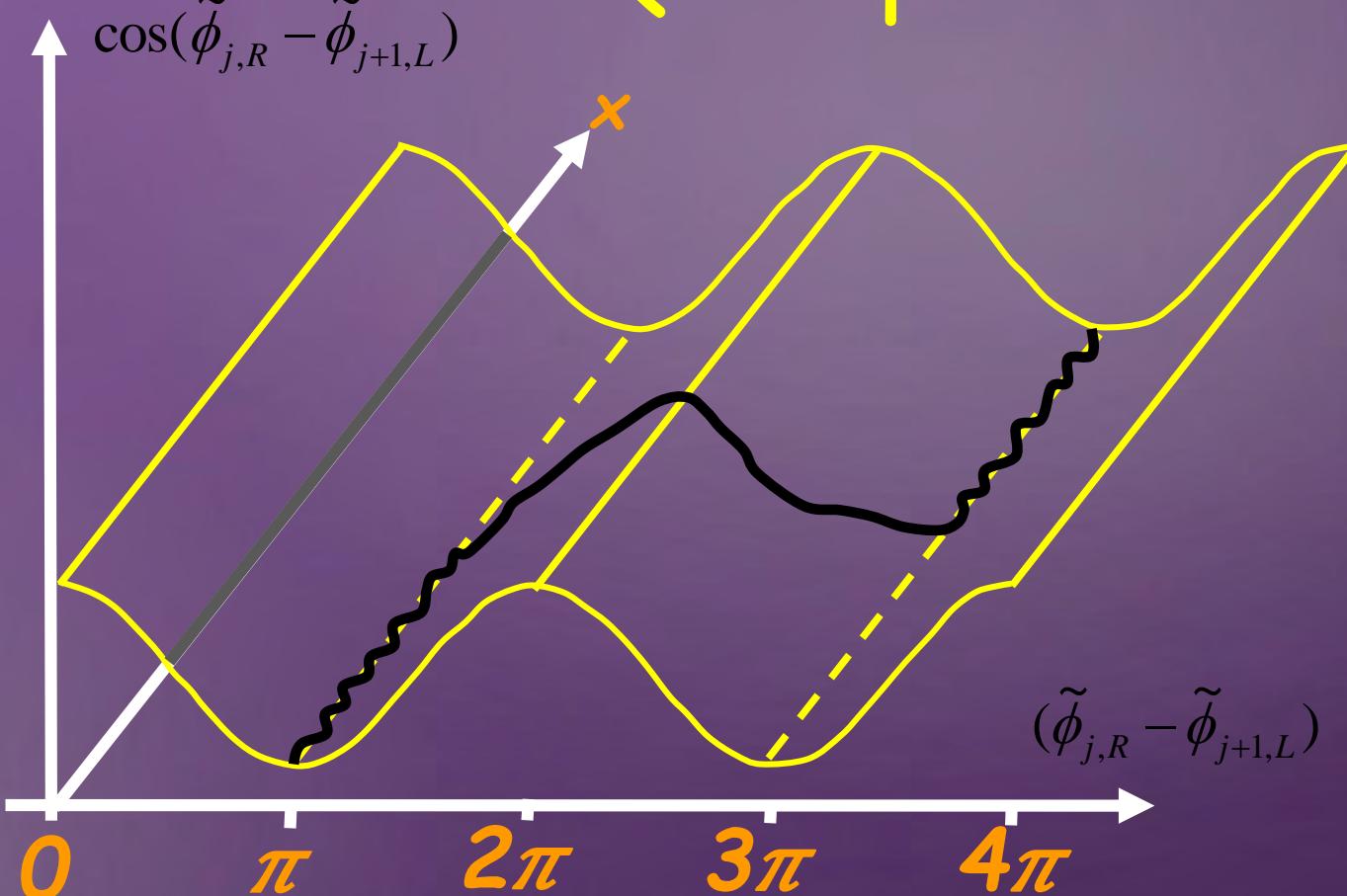


Gapless edge states

❖ **Gap (assume cosine relevant)**

Gapless edge states

Quasiparticles

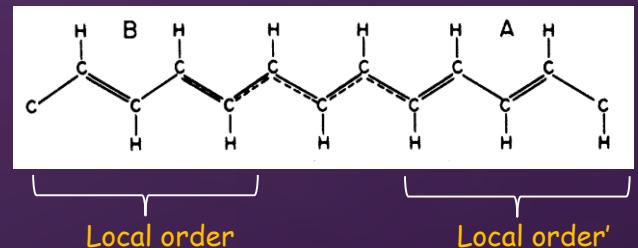


□ QP's are 1D sine-Gordon solitons

$$\begin{aligned}
 q &= \sum_j dx \rho_j = \\
 &= \sum_j \frac{1}{(2m+1)2\pi} \int dx \partial_x (\Delta \phi_{j+1/2}) \\
 &= \frac{1}{2m+1}
 \end{aligned}$$

- c.f. fractional Su-Schrieffer-Hegger solitons:
- In FQH different minimas are equivalent
- QPs can hop between wires via local operators

$$\psi_{jL}^+ \psi_{jR}$$



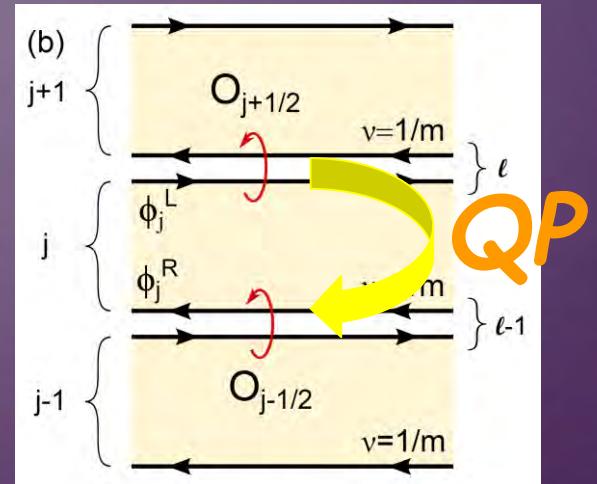
Moving QPs

□ Local operator:

$$\begin{aligned}\psi_{jL}^+ \psi_{jR}(x) &= e^{i(\phi_{jL} - \phi_{jR})} = e^{\frac{i}{2m+1}(\tilde{\phi}_{jL} - \tilde{\phi}_{jR})} \\ &= e^{\frac{i}{2(2m+1)}((\tilde{\phi}_{jL} - \tilde{\phi}_{j-1R}) + (\tilde{\phi}_{jL} + \tilde{\phi}_{j-1R}))} e^{\frac{i}{2(2m+1)}((\tilde{\phi}_{jR} - \tilde{\phi}_{j+1L}) - (\tilde{\phi}_{jR} + \tilde{\phi}_{j+1L}))}\end{aligned}$$

pinned conjugate

$$[(\tilde{\phi}_{jL} + \tilde{\phi}_{j-1R})(x), (\tilde{\phi}_{jL} - \tilde{\phi}_{j-1R})(y)] = i\pi(2m+1)\theta(x-y)$$

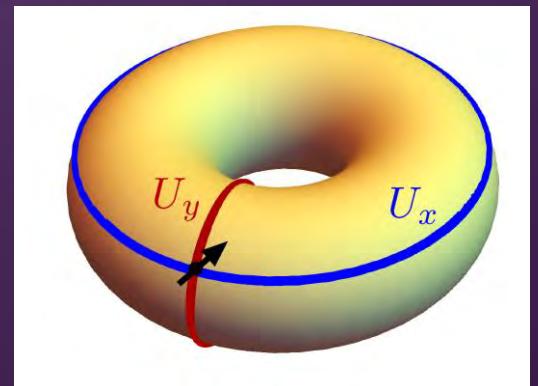


□ Non-Local string operators:

$$U_y = \prod_j \psi_{jL}^+ \psi_{jR}, \quad U_x = e^{i2\pi l_{ij}} = e^{i2\pi \oint dx \partial_x \phi_{Lj}}$$

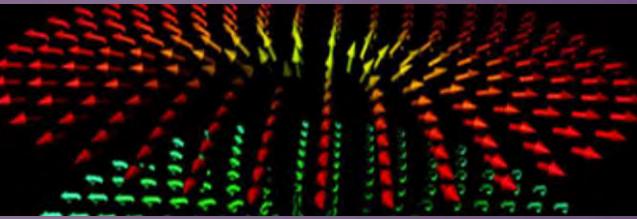
State changes each time one takes a QP around the torus

$$U_x U_y = e^{\frac{2\pi i}{2m+1}} U_y U_x \rightarrow 2m+1\text{-fold degeneracy}$$



Journal Club for Condensed Matter Physics

A Monthly Selection of Interesting Papers by Distinguished Correspondents



Topological quantum Lego

Posted in [Journal Articles](#) on March 30, 2015 at 8:31 am by JCCMP

Imprint of topological degeneracy in quasi-one-dimensional fractional quantum Hall states.

Authors: Eran Sagi, Yuval Oreg, Ady Stern, and Bertrand Halperin.

[arXiv:1502.01665](#)

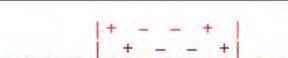
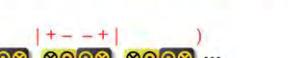
Recommended with a commentary by [Anton Akhmerov](#), Delft University of Technology.

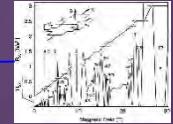
[View Commentary](#)

Add symmetry

Wire deconstructionism of two-dimensional topological phases

Titus Neupert,¹ Claudio Chamon,² Christopher Mudry,³ and Ronny Thomale⁴

	$\Theta^2 \Pi^2 C^2$	Short-range entangled (SRE) topological phase	Long-range entangled (LRE) topological phase
A	0 0 0 \mathbb{Z}	$T=(- +)$ 	$T=(- 2-1 1-2)$ 
AIII	0 0 +	NONE	
AII	- 0 0 \mathbb{Z}_2	$T=(- +)$  $T=(- +)$ 	$T=(1 -2 2 +1)$ 
DIII	- + + \mathbb{Z}_2	$T=(+ - - +)$  $T=(+ - - +)$  $T=(+ - - +)$  $T=(+ - - +)$ 	$T=(+ - - +)$  $T=(-1 2 -2 1)$  $T=(-2 1 2 -1)$ 
D	0 + 0 \mathbb{Z}	$T=(+ - -)$  $T=(+ - -)$ 	$T=(+ - - +)$ 
BDI	+ + +	NONE	



FTI?



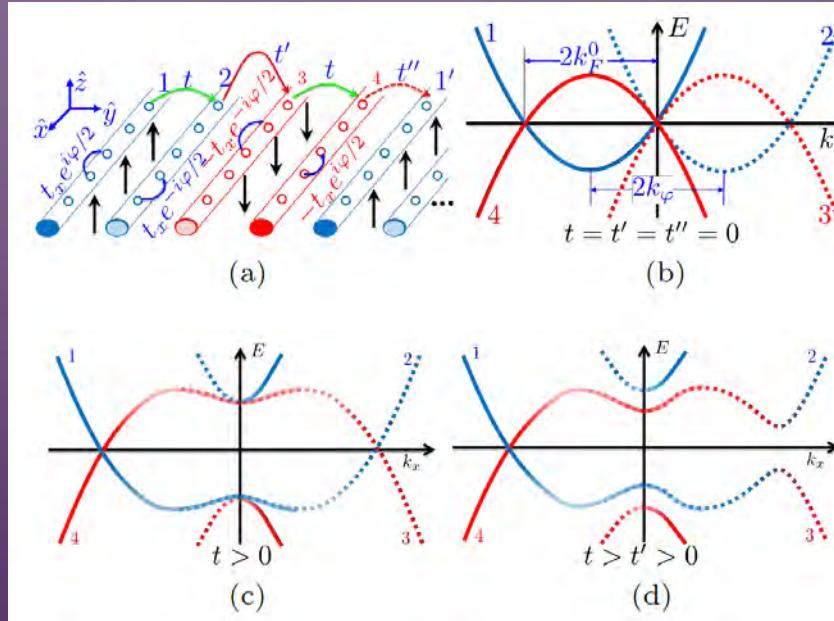
Zoo of fractional topological phases

- Easy to find a wire-construction realization for each symmetry class
- Easy to predict new fractionalized symmetric phases

Fractional Chern insulator

Non-Abelian topological insulators from an array of quantum wires

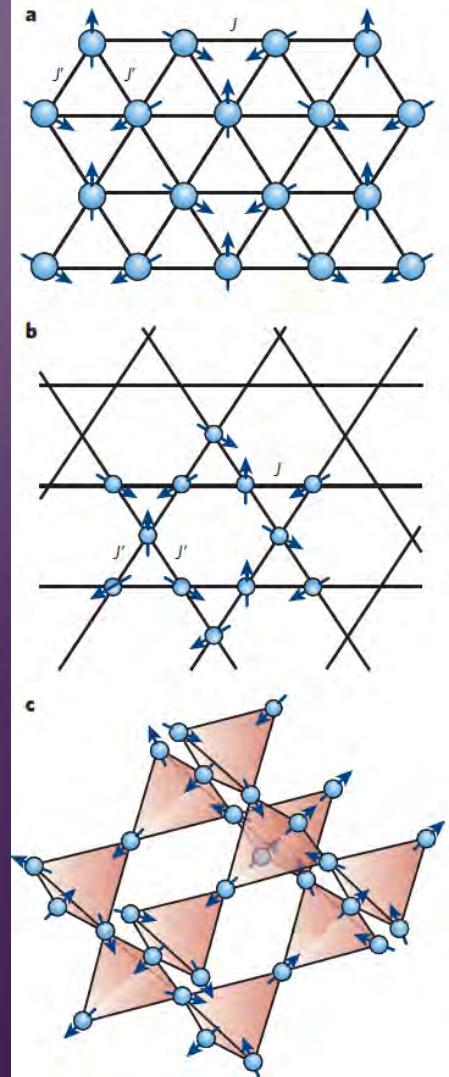
Eran Sagi and Yuval Oreg



- Haldane model: QHE without (net) magnetic field
- How to fractionalize Haldane model?
- Idea: combine electron- and hole- wires
- Generalizations with SO coupling to time reversal symmetry

Spin liquids

- Phases of quantum spins that don't order
- Typically happens due to frustration
- Anderson (1973): First proposal of "spin liquids"
- Kalmeyer and Laughlin (1987): suggestion (not correct): ground state of $s=1/2$ Heisenberg quantum antiferromagnet on triangular lattice breaks time reversal, producing the Bosonic Laughlin quantum Hall state, a '*Chiral Spin Liquid*'

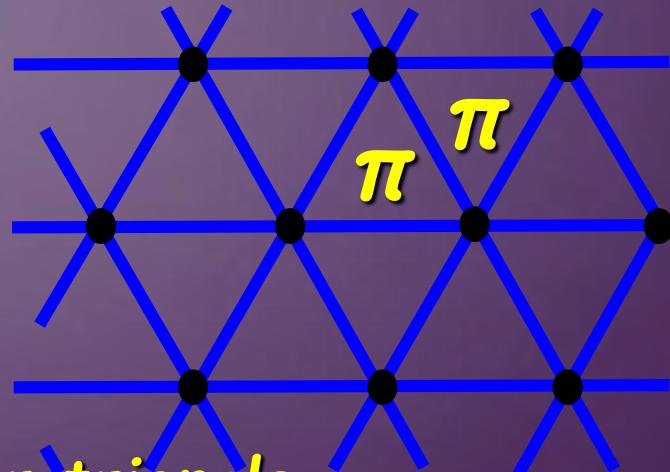


From: Balents, Nature (2010)

Frustrated spins and FQHE

- Spins-1/2=hard core bosons

$$\mathcal{H} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j \quad T = \frac{J}{2} \sum_{\langle ij \rangle} (a_j^\dagger a_i + a_i^\dagger a_j)$$



- Gauge transformation:
- $J \rightarrow -J$ & 1/2 flux quantum per triangle
- Interacting bosons in a magnetic field at $v=1/2$
- Candidate ground state:

$$\begin{aligned} \psi_{\text{gs}}(z_1, \dots, z_{N_b}) \\ = \prod_{j < k} (z_j - z_k)^2 \exp \left\{ \frac{-1}{4l_0^2} \sum_{i=1}^{N_b} |z_i|^2 \right\} \end{aligned}$$

- ❖ Our Goal: controllable approach to tell when this ground state is actually stabilized

A chiral spin liquid has appeared in the past

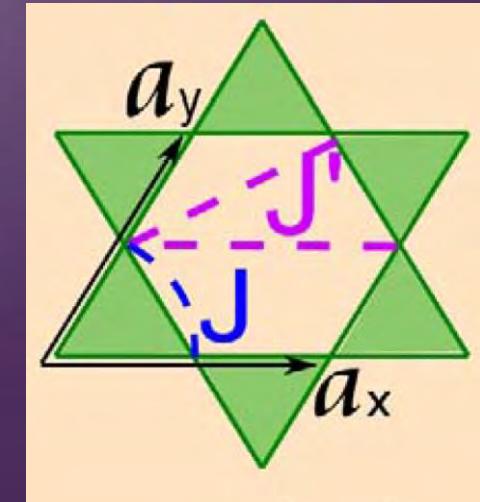
(i): in models with somewhat artificial Hamiltonians:

- Ch. Mudry (1989) , Schroeter, Thomale, Kapit, Greiter (2007); long-range interactions
- Yao+Kivelson (2007 - 2012); certain decorations of Kitaev's honeycomb model

(ii): particles with topological bandstructure plus interactions:

- Tang et al. (2011), Sun et al. (2011), Neupert et al. (2011); "flat bands"
- Nielsen, Sierra, Cirac (2013)

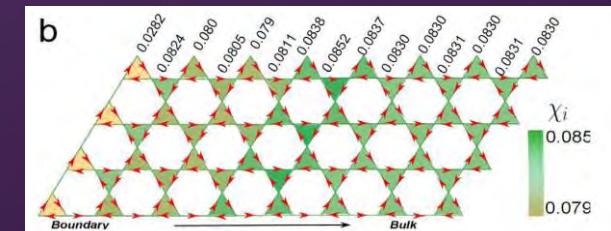
(iii): $SU(N)$ cases, cold atom systems: Hermele, Gurarie, Rey (2009).



(iv): Numerics on Kagome lattice:

Bauer et al. (2014),
He, Sheng, Chen (2014),
Gong, Zhu, Shen (2014)

Wietek, Sterdyniak, Lauchli (2015)...



Chiral Spin Liquids in Arrays of Spin Chains

Gregory Gorohovsky,¹ Rodrigo G. Pereira,² and Eran Sela¹

Also: T. Meng, T. Neupert, M. Greiter, and R. Thomale

Coupled wire
construction
of chiral spin liquid

Coupled spin chains

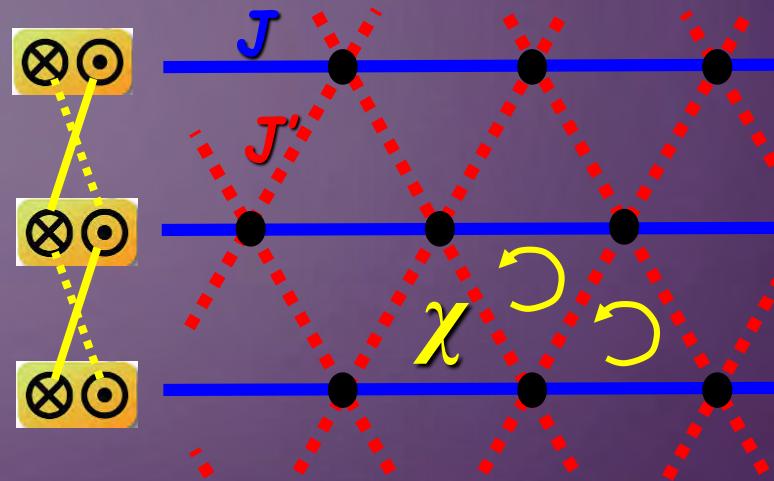
- Each spin chain has L- and R-moving spinons

$$H_0 = \sum_{l=1}^N \frac{2\pi v_s}{3} \int dx (\mathbf{J}_{l,R}^2 + \mathbf{J}_{l,L}^2)$$

- ## Field theory of AF spin chain

$$\mathbf{S}_j \rightarrow \mathbf{S}(x) \sim \mathbf{J}_R(x) + \mathbf{J}_L(x) + (-1)^x \mathbf{n}(x)$$

spin currents Staggered spin field



Interchain couplings:

Heisenberg J'

$$\mathbf{S}_i \cdot \mathbf{S}_j \quad \text{TP even}$$

T.P even

$$\delta H_g = 2\pi v_s g \sum_l \int dx (\mathbf{J}_{l,R} \cdot \mathbf{J}_{l+1,L} + R \leftrightarrow L)$$

spin chirality χ

Wen, Zee, Wilczek; Barkaran (1989)

$$\mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_k) \quad \text{T,P odd}$$

T,P odd

$$\delta H_\chi = 2\pi v_s \tilde{\chi} \sum_l \int dx (\mathbf{J}_{l,L} \cdot \mathbf{J}_{l+1,R} - R \leftrightarrow L)$$

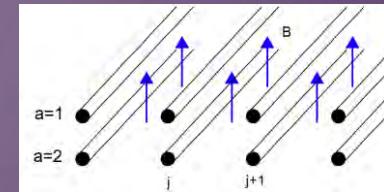
- $\chi < g$: $J_{j,R} \cdot J_{j+1,L}$ and $J_{j,L} \cdot J_{j+1,R}$ flow to strong coupling
 - $\chi > g$: $J_{j,R} \cdot J_{j+1,L}$ flows to weak coupling \rightarrow Edge states

Competing Relevant operator

- *Exists relevant coupling driving AF ordered state
(Starykh and Balents PRL 2007)*
- *Small coupling constant in kagome lattice*
- *In Kagome lattice CSL instability could have sufficiently larger coupling constant and flow to strong coupling first*

Non-Abelian States

- Add copies (Teo and Kane)
- Chiral Spin liquids (see Meng.):
Coupled spin- S chains
Effective 1D theory: $SU(2)_k$, $k=S/2$



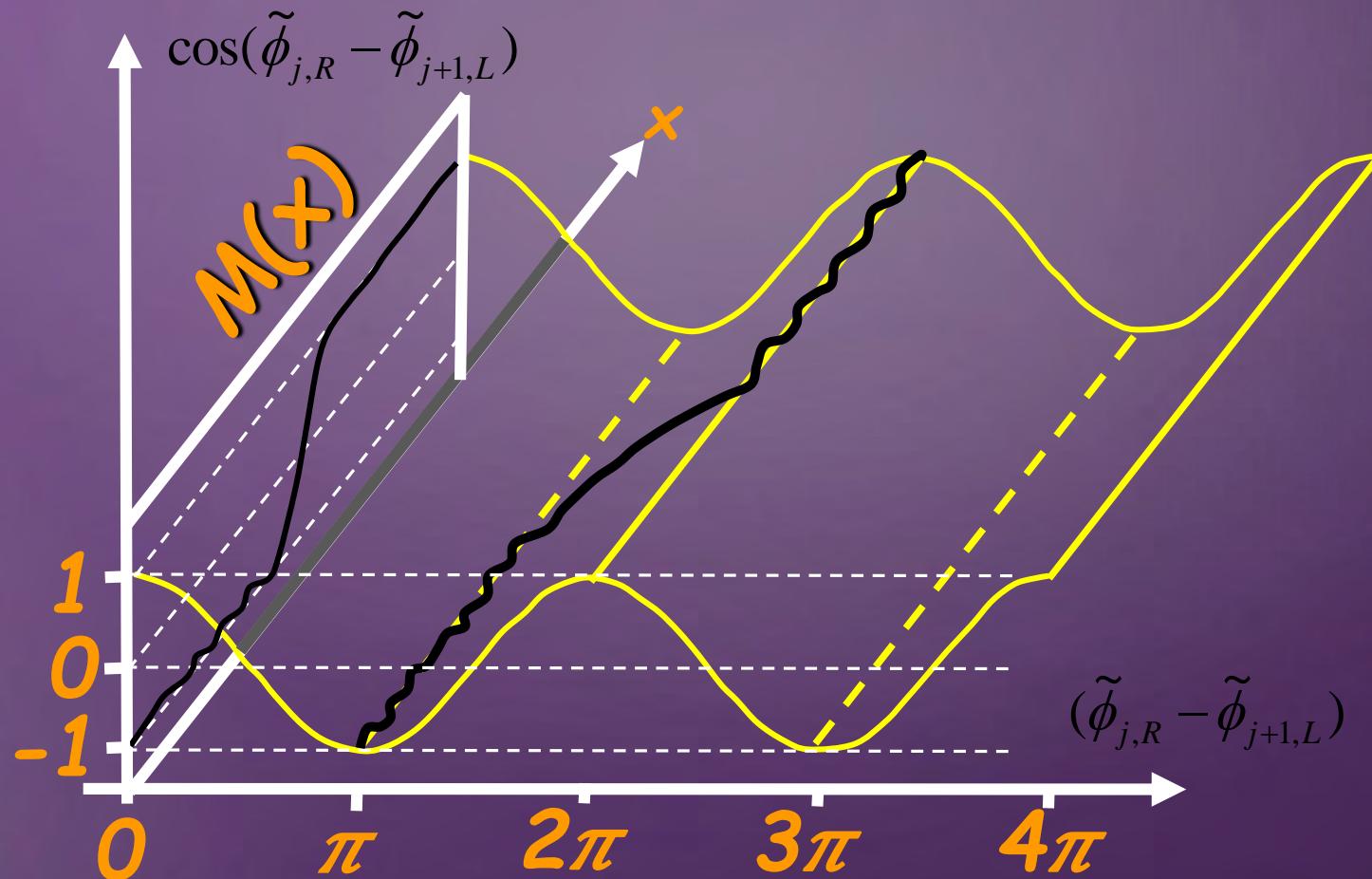
$$\cos(\tilde{\phi}_{j,\rho}^R - \tilde{\phi}_{j+1,\rho}^L)) \cos \tilde{\phi}_{j,\sigma}^R \cos \tilde{\phi}_{j+1,\sigma}^L$$

$\underbrace{\hspace{10em}}$ $\underbrace{\hspace{10em}}$

$M(x) \quad i\xi_{j,\sigma}^R \xi_{j+1,\sigma}^L$

Mass profile changes sign →

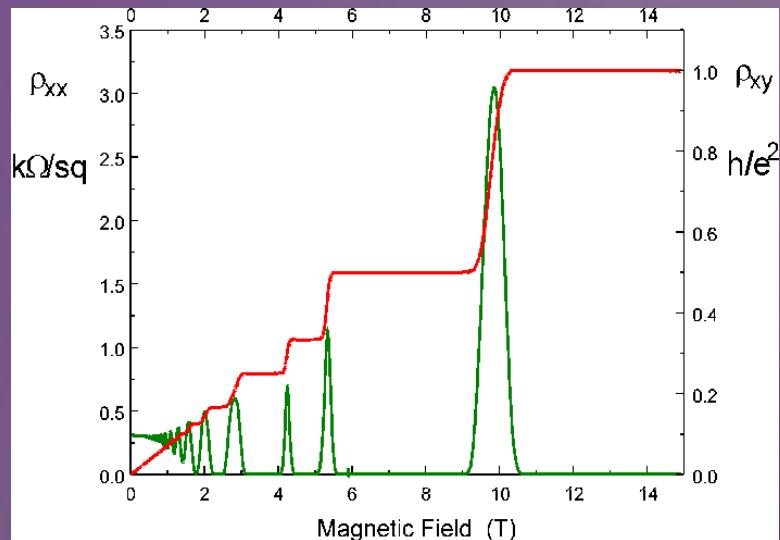
- Majorana zero mode attached to each quasiparticle
- Half QP charge (Moore Read $e^*=e/4$ at $\nu=1/2$)



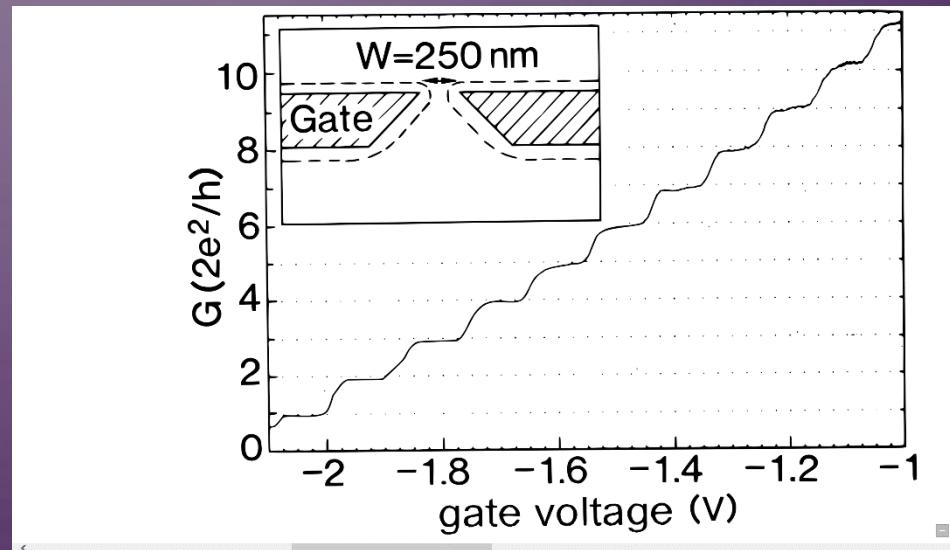
Can we have
similar states in
1D?

Quantization of the conductance in...

The quantum Hall effect



Short 1D constrictions



Protected by distance



Protected by cleanliness
and adiabaticity



Rashba wires

Semiconductor wires & strong SO

Quantum wire Hamiltonian

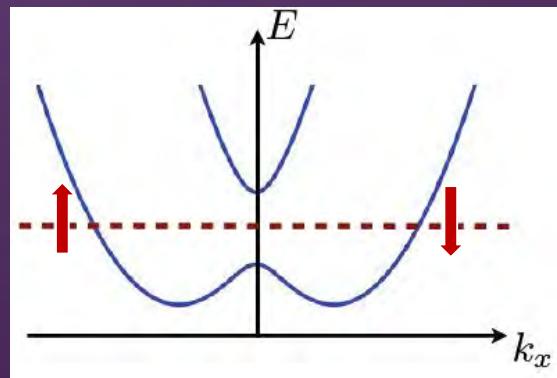
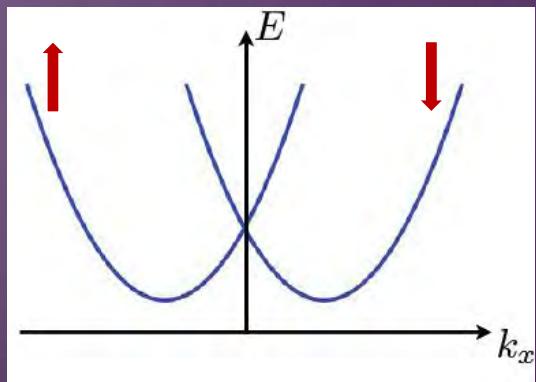
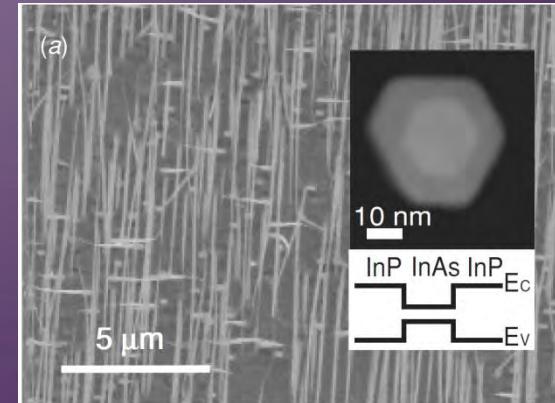
e.g. InAs wires:
(Delft group)

$$H = \frac{p^2}{2m} + up\sigma_x - B\sigma_z$$

Rashba spin-orbit coupling

$$(p \times E) \cdot \sigma$$

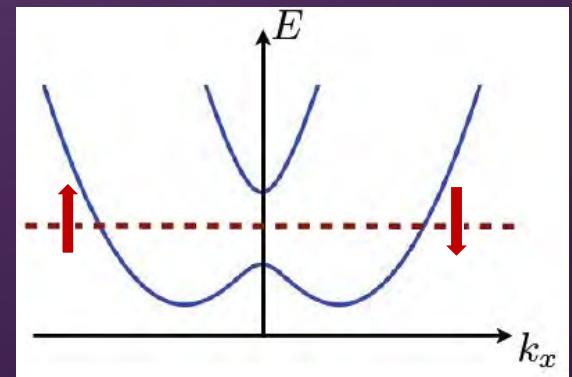
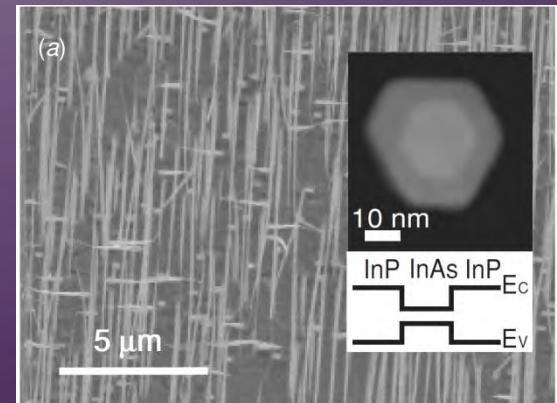
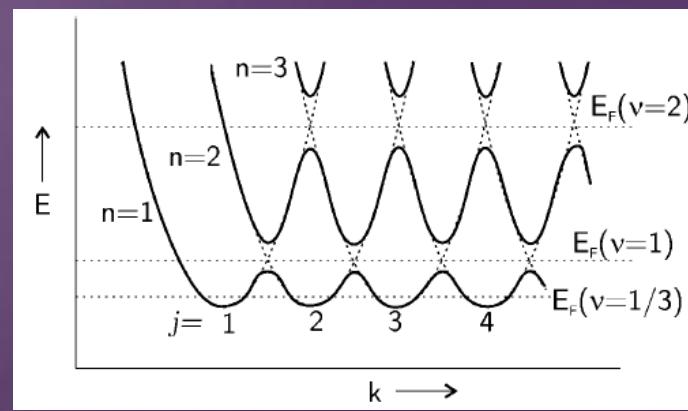
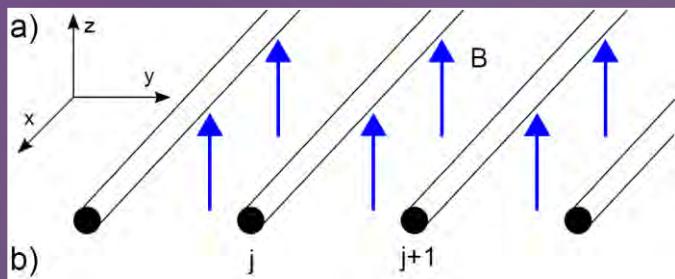
Zeeman field



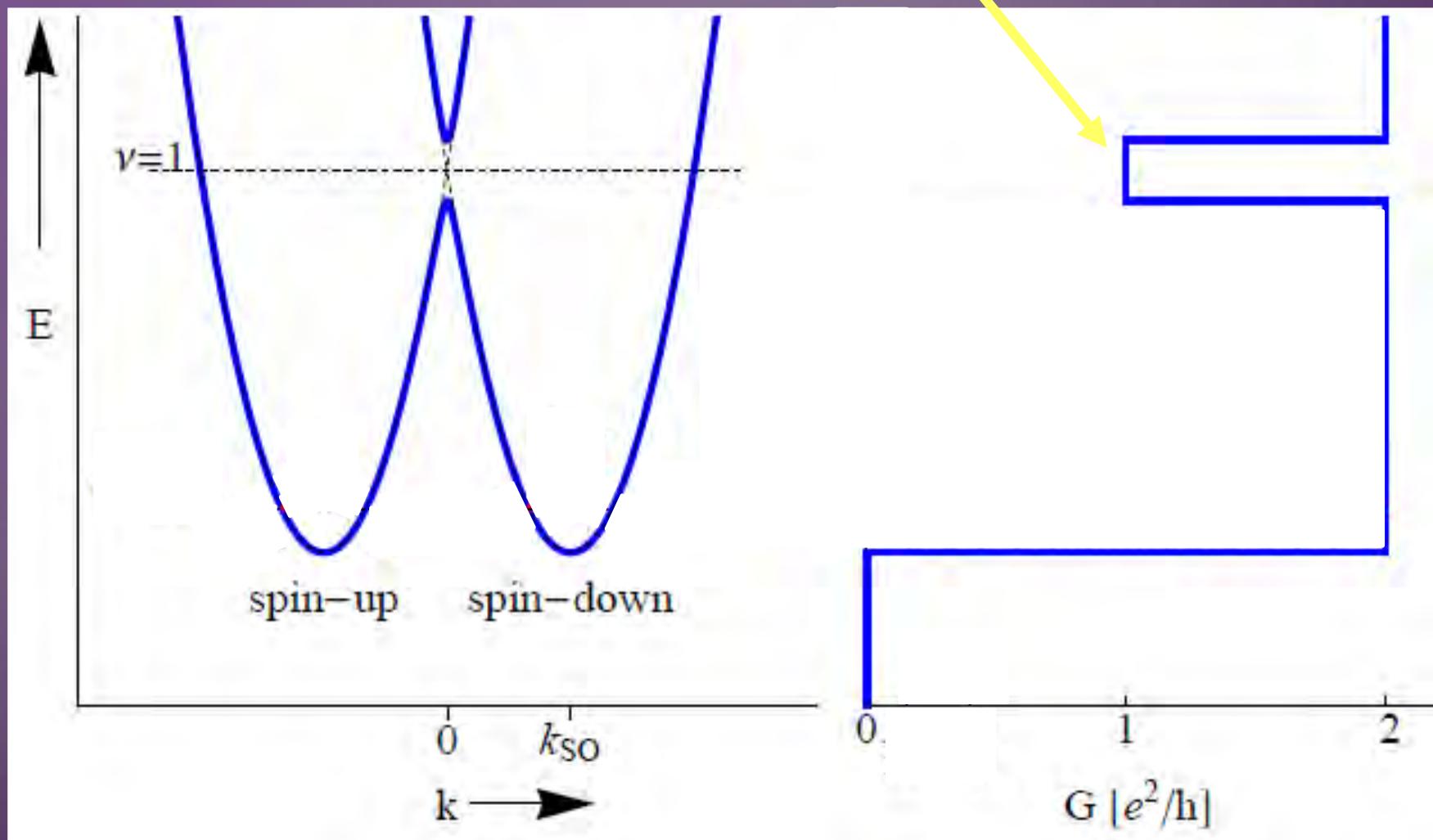
Number of wires = 2 spin species

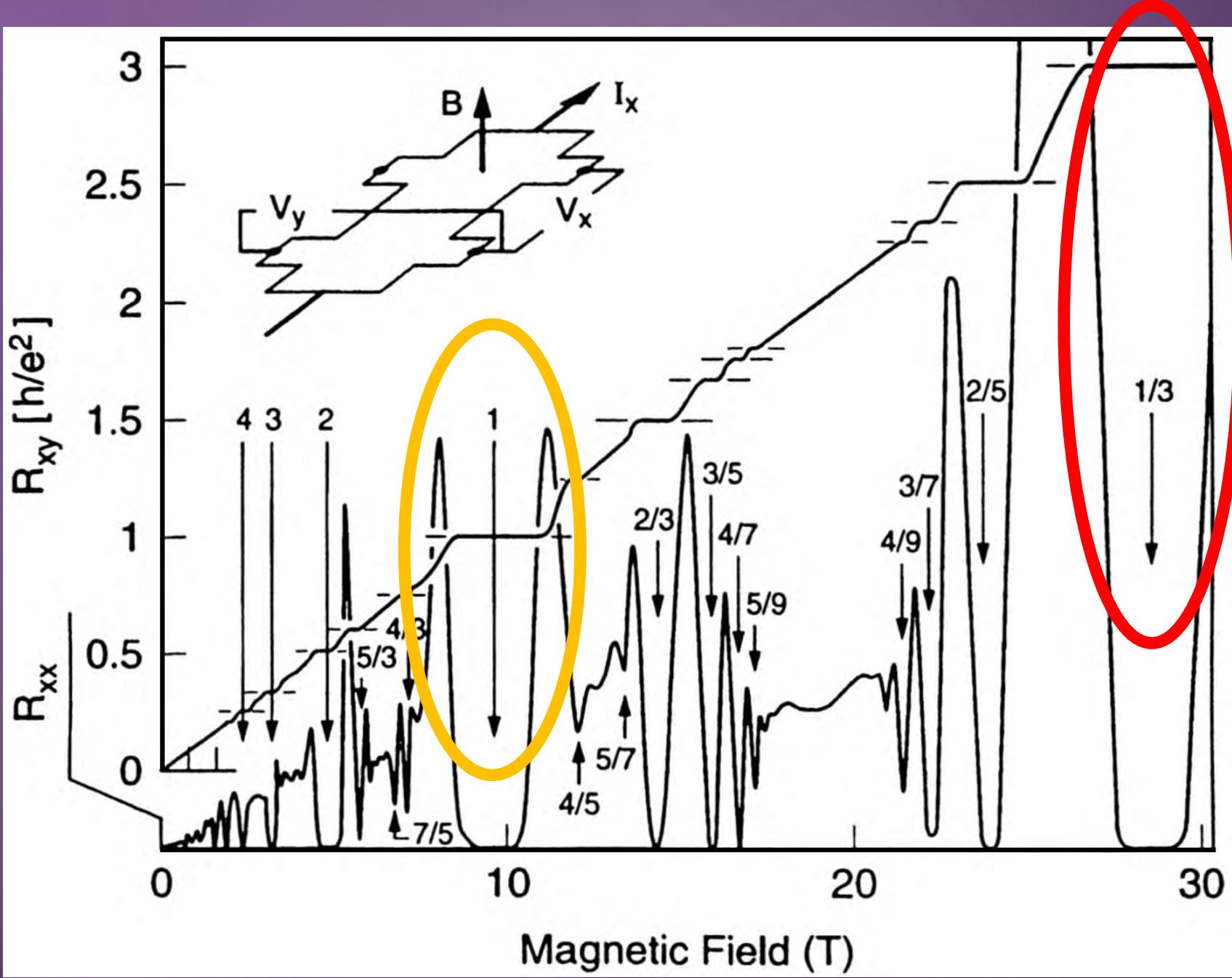
Orbital B-field = Rashba SO coupling

Tunneling = Zeeman field

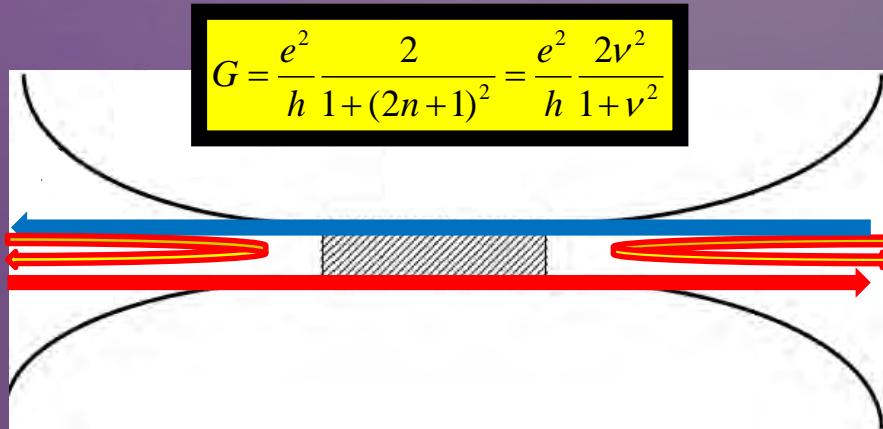


Conductance dip

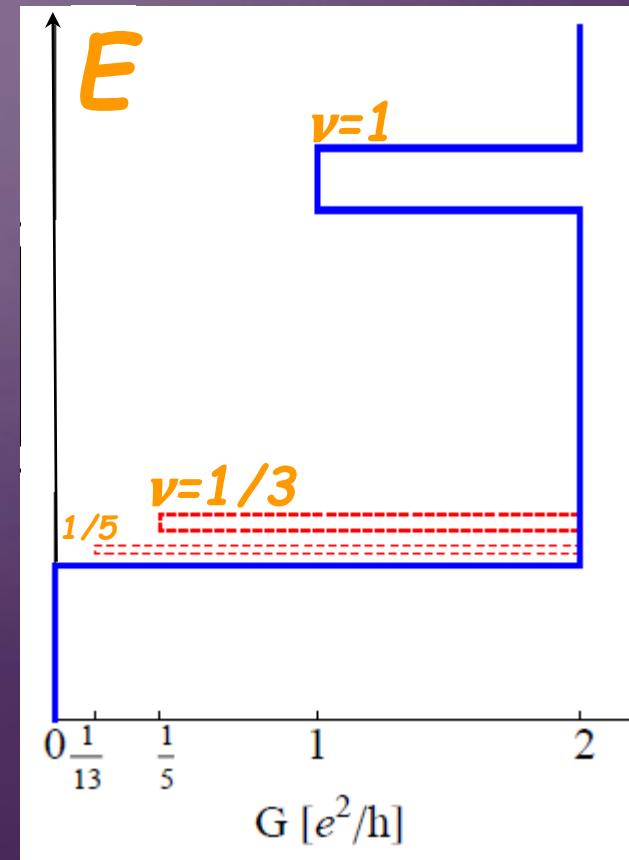




Need: Strongly interacting quantum wire Luttinger parameter $K < 1/3$; Clean, Adiabatic contacts



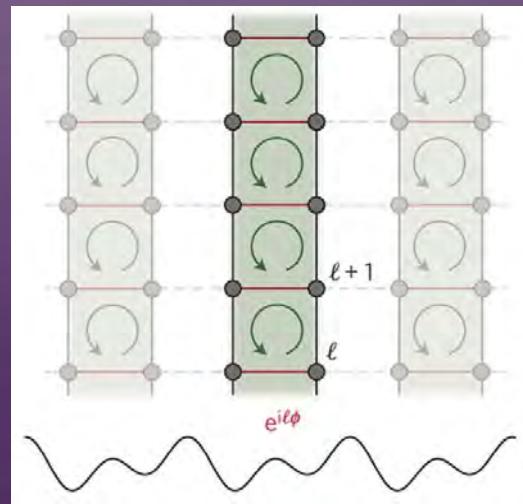
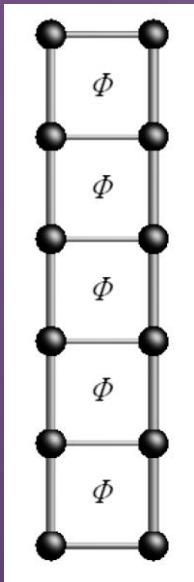
- Fractional mode is transmitted



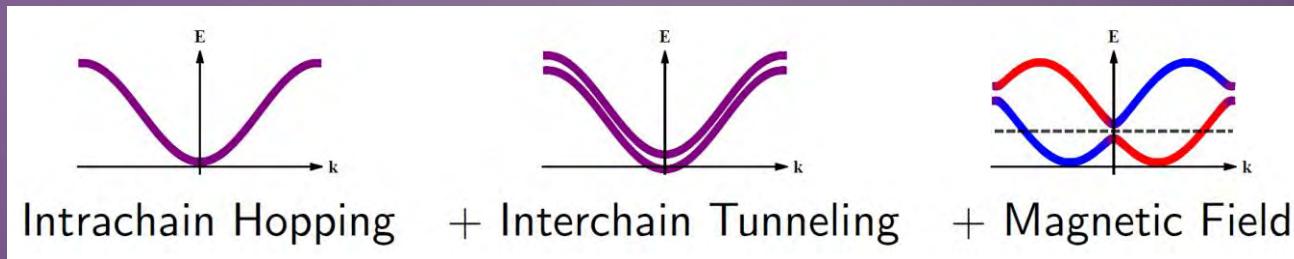
Two leg ladders

Why two leg ladders?

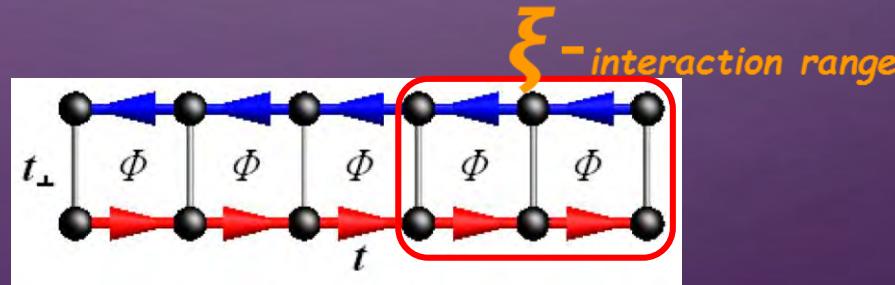
- Minimal 1D model affected by orbital field
- Realized using cold atoms with synthetic field



I. Bloch et. al



- *A non-interacting model equivalent to $\nu=1$ IQHE*
- *Would interactions stabilize new phases?*

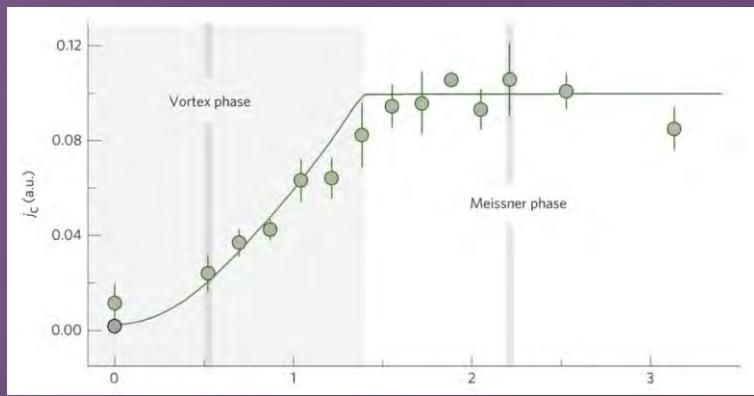


$$\mathcal{H} = -t \sum_j \left(a_{1,j}^\dagger a_{1,j+1} + a_{2,j}^\dagger a_{2,j+1} \right) + t_\perp \sum_j e^{i\Phi_j} a_{1,j}^\dagger a_{2,j} + \text{h.c.} + \sum_{ij} V_{ij} \rho_i \rho_j$$

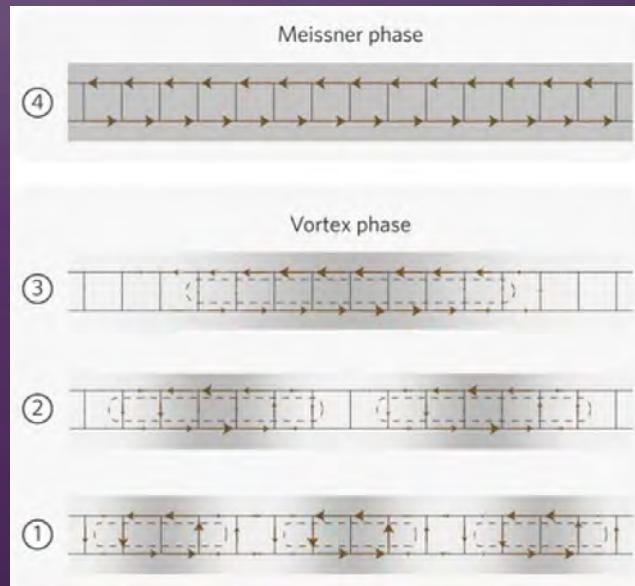
	$\nu = \frac{1}{2}$	$\nu = \frac{1}{3}$	$\nu = \frac{1}{4}$	$\nu = \frac{1}{5}$
$\xi = 0$	✓	-	-	-
$\xi = 1$	✓	✓	✓	-
$\xi = 2$	✓	✓	✓	✓

Some recent experimental results

- Cold atoms ladders under synthetic magnetic field display a phase transition between Meissner & vortex lattice state
- Can measure circulating current
- Are there other accessible phases?

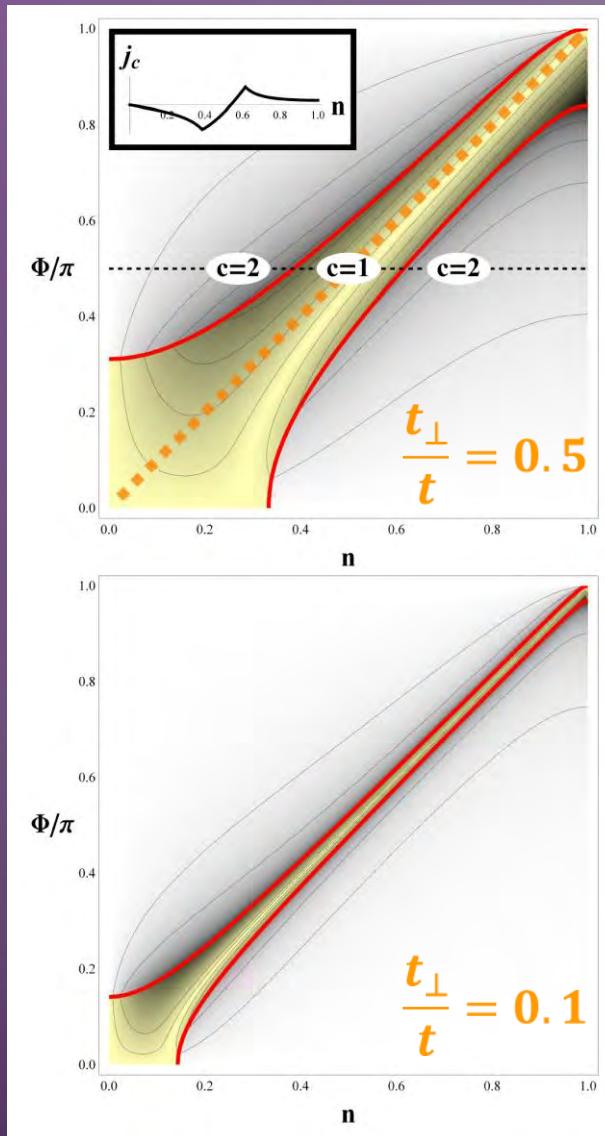


M. Atala, M. Aidelsburger, M. Lohse,
J. T. Barreiro, B. Paredes, & I.
Bloch (2014)



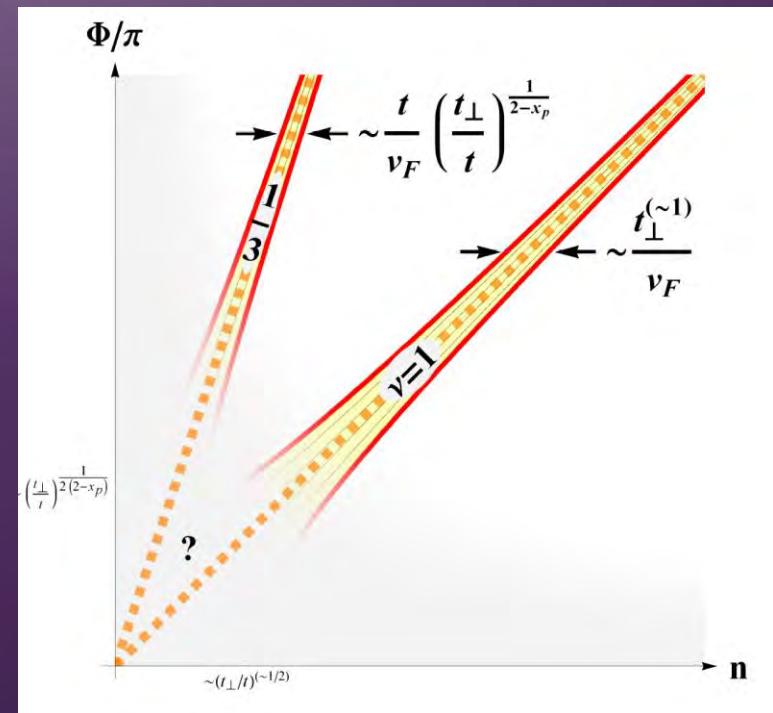
Chiral currents in one-dimensional fractional quantum Hall states

Eyal Cornfeld¹ and Eran Sela¹



$$j_c \propto \left(n - \frac{\nu}{\pi} \Phi \right)$$

for $t_{\perp} \ll t$



Summary

- *Wire construction gives a simple approach to complicated topological phases*
- *Easy to fractionalize phases*
- *Possible experimental signatures of 2D topological states in 1D systems*