Electron-Phonon Interaction in Nanoelectronic Circuits
toward the control of single phonons

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Phonon meets Electron

- electron-phonon interaction in 2D, one electron at a time...

- influence of confinement in nanostructures

- coherent electron-phonon interaction

- control of single phonons?
introduction – samples

Werner Wegscheider
wafers
@ ETH Zürich
A GaAs/AlGaAs heterostructure is shown with a 2DES layer approximately 100 nm thick, δ-doped with Si. Metal gates are used to control the device. A QPC and tunnel barriers are also present. The energy level diagram shows the confinement of electrons in the 2DES layer and QDs.
degenerate Fermi sea

\[ E_F \sim 10 \text{ meV} \]

\[ T < 100 \text{ mK} \quad [k_B T < 9 \mu \text{eV}] \]

electron energy in real space & in k-space
E degenerate Fermi sea

\[ T < 100 \text{ mK} \]

energy transfer: \( \epsilon = \frac{\hbar^2}{2m} \left( \vec{k}_i^2 - \vec{k}_f^2 \right) \)

momentum transfer: \( \vec{k}_f - \vec{k}_i = \vec{q} \)

phonon dispersion: \( \epsilon = \hbar |\vec{q}| v_{\text{ph}} \)
degenerate Fermi sea

$T < 100 \text{ mK}$

$E_F \sim 10 \text{ meV}$

phonon
length scales (for $E_{\text{kin}} - E_F \sim 1 \text{ meV}$)

- momentum mean-free-path$^\dagger$: $l_m \sim 10$-$100 \mu\text{m}$
- electron-electron scattering*: $l_{e-e} \sim 10 \mu\text{m}$
- electron-phonon scattering (phonon emission)$^\dagger$: $l_{e-ph} \sim 100 \mu\text{m}$
- phonon-electron scattering (phonon absorption)$^\dagger$: $l_{\text{ph-e}} \sim 1 \mu\text{m}$

J. Appl. Phys. 109, 102412 (2011)

$^\dagger$ Schinner et al.:
Phys. Rev. Lett. 102, 186801 (2009)
design of an electron-phonon scattering (in 2D) experiment

emitter circuit

detector circuit

\[ E_F \]

\[ eV_{SD} \]

\[ \sim 2 \mu m \]
electron back scattering $\rightarrow$ maximum momentum & energy transfer

$$E_{\text{max}} \approx \hbar 2k_F v_{s\text{max}}$$

$$\Delta k_{\text{max}} = 2k_F$$

$\rightarrow$ phonon mediated current only for detector barrier height $< E_F + E_{\text{max}}$
$V_{SD} = -55 \text{ mV}$

$V_{SD} = -15 \text{ mV}$
\[ E_{\text{max}} \approx \hbar 2k_F v_{s_{\text{max}}} \]

\[ \Delta k_{\text{max}} = 2k_F \]
\[ E_{\text{max}} \approx 1.3 \text{ meV} \]

\[ v_{s_{\text{max}}} \approx 6000 \frac{\text{m}}{\text{s}} \]
vertical onset of detector current ↔ upper bound of transferred momentum corresponds to electrons back-scattered by phonon absorption

electron-phonon interaction matters in driven mesoscopic circuits

Schinner et al., PRL 102, 186801 (2009)
QD tunnel barriers

~100 nm

\[ \alpha \]

\[ \mathbf{V} \]

\[ \mathbf{L} \]

\[ \mathbf{V} \]

\[ \mathbf{R} \]

500 nm

400 nm

GaAs

Source

Drain

1 \mu m

400 nm

500 nm

... and in nanostructures?

... and in nanostructures?
how is the electron-phonon interaction affected, if the phonon wavelength is comparable to the structure size?
**size effects**

phonon energies: 50 μeV – 1 meV \[\leftrightarrow\] phonon wavelength: 20 nm – 500 nm

**electron confinement**

… enhances overlap of electron and phonon wavefunctions

… relaxes momentum conservation law \[\Delta k \sim \frac{2\pi}{\Delta x}\]

(1) enhanced electron-phonon interaction
(2) relative phase between electron and phonon wave functions matters
phonon emission in a double quantum dot (DQD)

phonon dispersion: $\epsilon = \hbar |q| v_{ph}$
Interaction with the environment can change the quantum mechanical phase of a system.

- dephasing / decoherence
- coherent dynamics

Early proposals related to coherent electron-phonon interaction in solids:

A. Miller and E. Abrahams, Phys. Rev. 120, 745 (1960) [phonons induced electron hopping between impurities]

determine electron-phonon coupling from decoherence

DQD charge qubit

\[ \epsilon = \bar{\epsilon} + Af(t) \]

\[ T = 18 \text{ mK} \]

\[ 2.75 \text{ GHz} \]
coherence time of our two-electron [undriven] charge qubit

\[ T_2^{-1} = \frac{\pi \alpha_Z}{\hbar} \left( \frac{2 \varepsilon^2}{E^2} + \frac{\Delta^2}{2E} \coth \left( \frac{E}{2k_B T} \right) \right); \quad E = \sqrt{\Delta^2 + \varepsilon^2} \]

\[ \alpha_Z = 1.5 \times 10^{-4} \]

\[ T_2 \text{ (ns)} \]

\[ \Delta/k_B = 150 \text{ mK} \]

Interaction with the environment can change the quantum mechanical phase of a system.

dephasing / decoherence

coherent dynamics

early proposals related to coherent electron-phonon interaction in solids:

A. Miller and E. Abrahams, Phys. Rev. 120, 745 (1960)
[phonons induced electron hopping between impurities]

[very general, tunneling involving defects in solids]
coherent phonon emission in a DQD

Spontaneous emission spectrum in double quantum dot devices.

theory: T. Brandes, Physics Reports 408, 315 – 474 (2005):
Coherent and collective quantum optical effects in mesoscopic systems.
Fermi's golden rule:

\[
\hat{H}_{\text{int}} = \frac{t_c}{\epsilon} \sum_{\vec{q},\mu} \lambda_{\vec{q},\mu} e^{i\vec{q} \cdot \vec{d}} \left( \hat{a}_{\mu,\vec{q}} + \hat{a}_{\mu,-\vec{q}}^\dagger \right) \langle g \rangle \langle e | + \text{h.c.}
\]

- Electron-phonon interaction matrix element
- Phonon annihilation & creation operators
- Electron wave function
- Phonon branches
- Phase factor

Period of interference pattern:

\[
\vec{d} \cdot \vec{q} \equiv N2\pi \Rightarrow \\
\Delta \epsilon = 2\pi \hbar \nu_{\text{ph}} \frac{|\vec{q}|}{\vec{d} \cdot \vec{q}}
\]

Using energy conservation:

\[
\epsilon = \hbar |\vec{q}| \nu_{\text{ph}}
\]
• phonon wavelengths are typically in the order or smaller than the distance of QDs

• **photon wavelengths are much longer** ⇒ $\Delta \varphi_{\text{photon}} = 0$

• **here:** phonon mediated interaction
**coherent phonon emission in a DQD**


**observation:** the non-equilibrium current through a double QD oscillates as a function of energy detuning $\varepsilon$, i.e. the energy of the emitted phonons.

can we do the same for phonon absorption?

emission ↔ absorption

phonon source

phonon detector
a driven QPC can act as a broad band phonon emitter ($E_{\text{max}} = eV_{\text{sd}}$)

Khaprai et al.:  
quantum point contact (QPC) as phonon source
quantum point contact (QPC) as phonon source

transmission: \( 0 < T \ll 1 \Rightarrow \text{local charge fluctuations} \)

\[ E_{ph} \leq eV_{QPC} \]

our model (Aash Clerk):

- **standard scattering theory**
  \( \Rightarrow \) charge noise spectrum of the QPC.

- link the QPCs charge noise to its phonon emission spectrum (using **Keldysh Green functions** of the acoustic phonons to first order in the electron-phonon coupling to the QPC).

_Nature Phys. 8, 522 (2012)_
coherent phonon absorption in a DQD
Fermi's golden rule:

\[ \hat{H}_{\text{int}} = \frac{t_c}{\epsilon} \sum_{\vec{q},\mu} \lambda_{\vec{q},\mu} e^{i \vec{q} \cdot \vec{d}} (\hat{a}_{\mu,\vec{q}} + \hat{a}_{\mu,-\vec{q}}^\dagger) (|g\rangle\langle e| + h.c.) \]

our model (Aash Clerk):

- **Golden rule rates** for electron-phonon interaction in the double QD (only piezoelectric coupling) [as in *].
- **standard elasticity theory** [as in **] but in addition account for **anisotropy** of sound velocities and polarizations, include **screening effects**

sensitive measurement of phonon absorption: QPC as charge detector ...

\[ I_{QPC}(V_R) \]

\[ V_{QPC} = 0.1 \text{ mV} \]

measured is the transconductance: \[ \frac{dI_{QPC}}{dV_L} \text{ (a.u.)} \]

proportional to changes of the steady state occupation of the DQD

Daniela Taubert
... to measure the steady state occupation of the DQD

**QPC charge detector:**
- is a voltage biased 1D-tunnel barrier
- acts as a **broad band phonon emitter**
- re-absorption of phonons at the DQD cause **detector backaction**

**Literature:**
- Khrapay et al.: PRL 97, 176803 (2006);
- Schinner et al.: PRL 102, 186801 (2009);
- Harbusch et al.: PRL 104, 196801 (2010);
- Prokudina et al.: PRB 82, 201310(R) (2010)
same effect observed in a triple quantum dot

data from Andy Sachrajda's group, NRC Canada

Nature Phys. 8, 522 (2012)
The figure illustrates a quantum point contact (QPC) with a closed barrier between source and drain regions. The QPC width is 500 nm. The QPC is shown in three different states:

- **Ground state**: The particle is localized in the lowest energy level.
- **Intermediate state**: The particle is in a metastable state, sandwiched between the ground state and the excited state.
- **Excited state (metastable state)**: The particle has a higher energy level.

The figure also shows the change in density of states with applied voltage, indicating transitions between these states. The states are labeled as:

- **Ground state metastable state**
- **Intermediate state**
- **Excited state (metastable state)**

The energy levels are denoted as \( E \) and \( E' \), with the transition energy \( h\omega \) between states.
our model (Aash Clerk):

- master equation approach considering the three relevant double QD states.
- **constructive interference:** considerable occupation of excited configuration (0,0)
- **destructive interference:** ground state configuration (0,1) is always occupied
- the intermediate state (1,0) is short living and does not contribute to the detector signal
reproducible beating patterns

repeated measurements:

contributions of different phonon modes including deformation potential and piezoelectric coupling...
most relevant acoustic phonon modes

acoustic phonons in GaAs:

phonon focusing
(radius $\propto$ emission strength)

model calculations for different geometries
geome try of back action region
maximum triangle size as a function of $V_{SD}$

$\epsilon_{\text{max}} = |eV_{\text{QPC}}|$
coupled quantum dots as single-phonon detector

- we detect one phonon at a time

- we can tune $\varepsilon$ and, hence, measure the phonon spectrum

- we are sensitive to different phonon modes
• electron-phonon interaction is relevant in non-equilibrium mesoscopic circuits

• coherent electron-phonon coupling is accessible

• can we control single phonons?