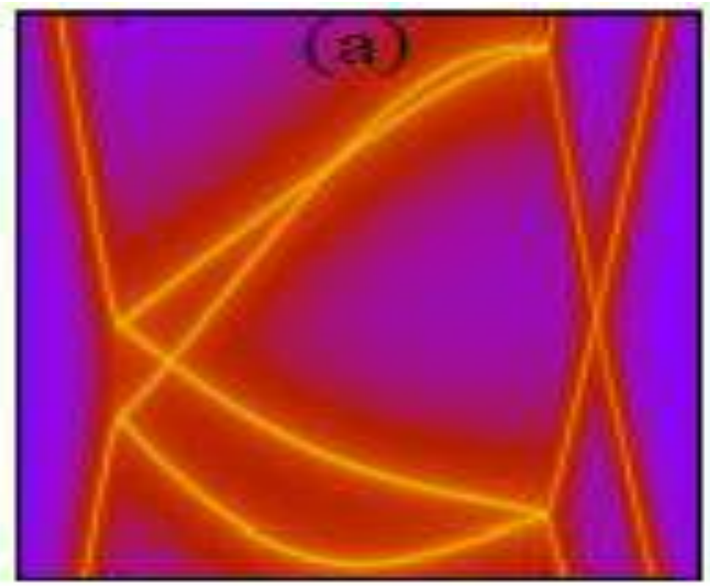
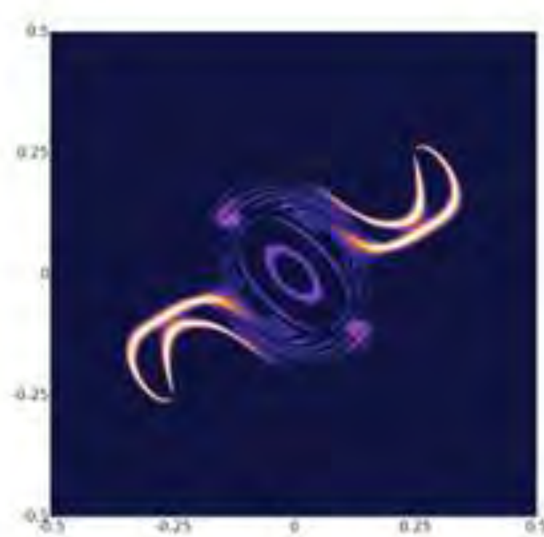
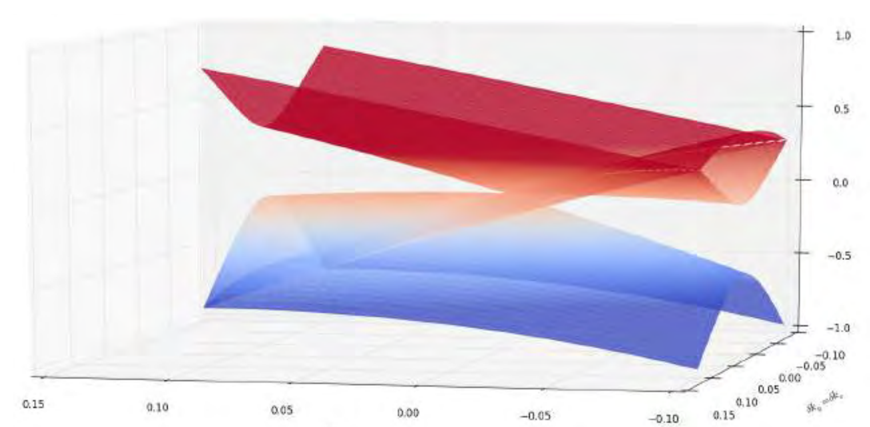
Cohomological Insulators



Also, Non-Symmorphic Fermions





(b) SG 220

Barry
Bradlyn

Jennifer
Cano

Wang
Zhijun

Alexey
Soluyanov

Aris
Alexandradinata

Maia G
Vergniory

Dominik Gresch
Quansheng Wu
S. Kushwaha
Max Hirschberger
Weiwei Xie

Bob Cava
Xi Dai
C. Felser
N.P. Ong
A Yazdani

Hiroyuki Inoue,
András Gyenis,
Jian Li,
Stefanos Kourtis
A Yazdani

How Many Types Of Energy Fermions Are There in Nature?

HourGlass Fermions

Group extensions by Wilson loops

Generalizes spatial nonsymmorphism to the Brillouin zone

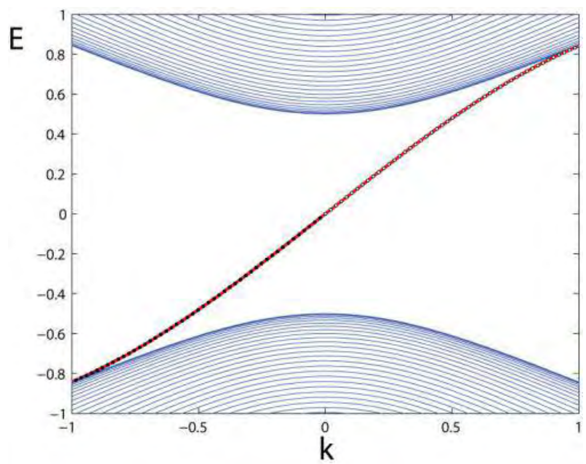
Bulk fermions with 3,6,8 -fold degeneracies

Degeneracies on planes and surfaces

Exotic Transport Response

Chiral edge states

Do not rely on any symmetry.

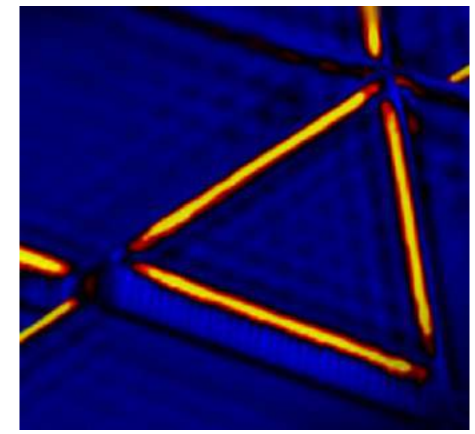


C can be defined within
a mirror subspace (Teo,Fu,Kane)
e.g., SnTe, BiSb,CeBi
(Hsieh, Fu, Suyang, Nasser, Hasan, .)

Symmetry-protected edge states

Time-reversal symmetry, spin-orbit coupling
(Kane,Mele)

e.g., HgTe, Bismuth



2D+ 3D Mollenkamp, Hassan and many others

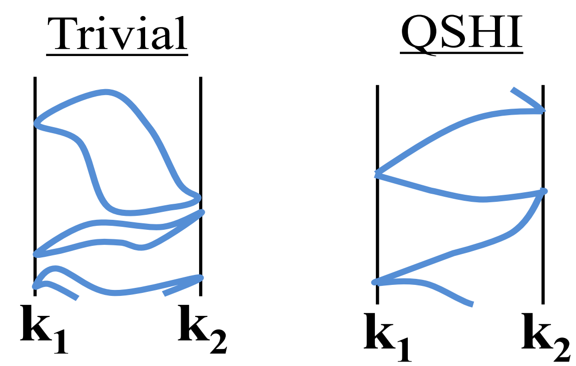
Or no edge states at all: Hughes Prodan BAB, Turner and Vishwanath 2011; Teo, Ryu, Turner, others

Initial criterion for symmetry-protected edge states

Degeneracies at isolated points

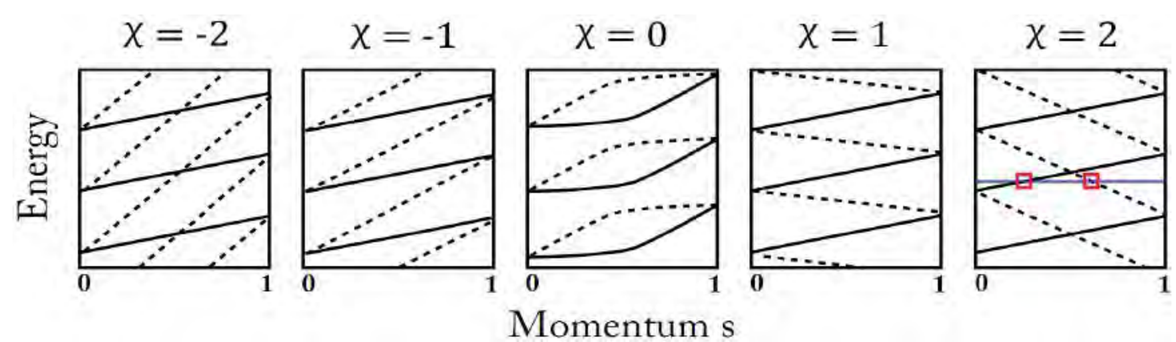
1) Two points ($\mathbf{k_1}$,$\mathbf{k_2}$) with enhanced degeneracy
2) Trivial degeneracy on the line bridging $\mathbf{k_1}$-$\mathbf{k_2}$

Is This All?



Z_2 topology

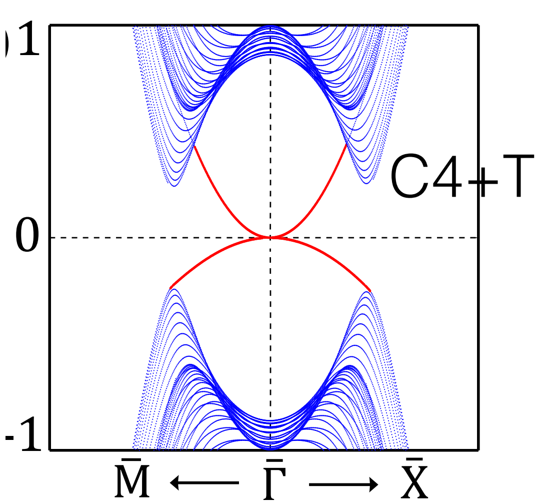
(**Fu,** Kane,Mele)

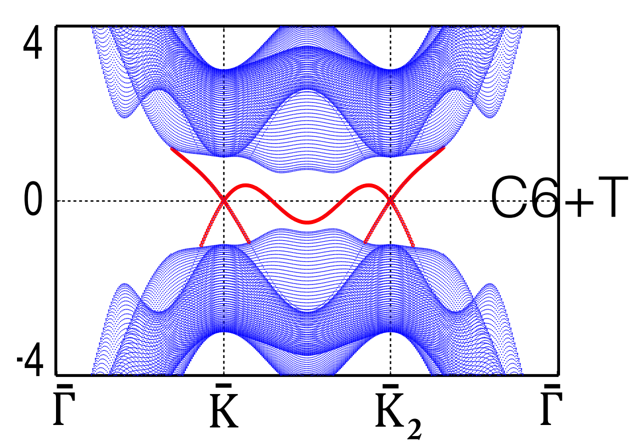


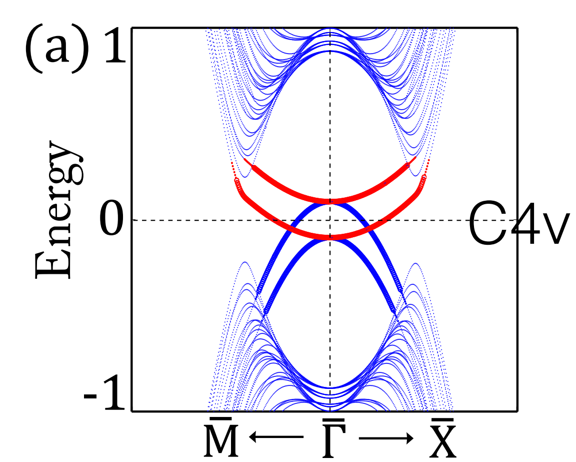
Z topology

(AA,Chen,Gilbert,BAB)

Many others







Better criterion for symmetry-protected edge states

Connectivity of submanifold ($\boldsymbol{S}$)
Degree of connectivity = D

1) Two submanifolds ($\boldsymbol{S_1}$,$\boldsymbol{S_2}$) with equal and enhanced connectivity ($D_1 = D_2 > 1$).

2) $\boldsymbol{S_3}$ bridges $\boldsymbol{S_1}$ and $\boldsymbol{S_2}$, and has a connectivity (D_3) that nontrivially divides D_1,

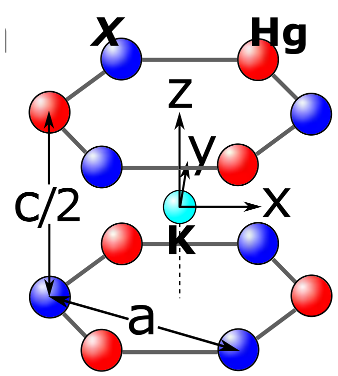
i.e., D_1/D_3 = an integer greater than one.

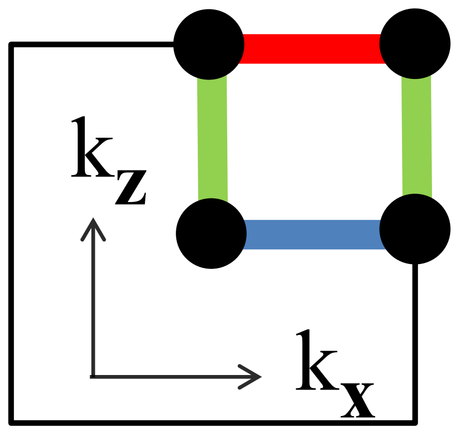
Which symmetry groups have nontrivial connectivity?

More complete answer: *all* nonsymmorphic space groups.

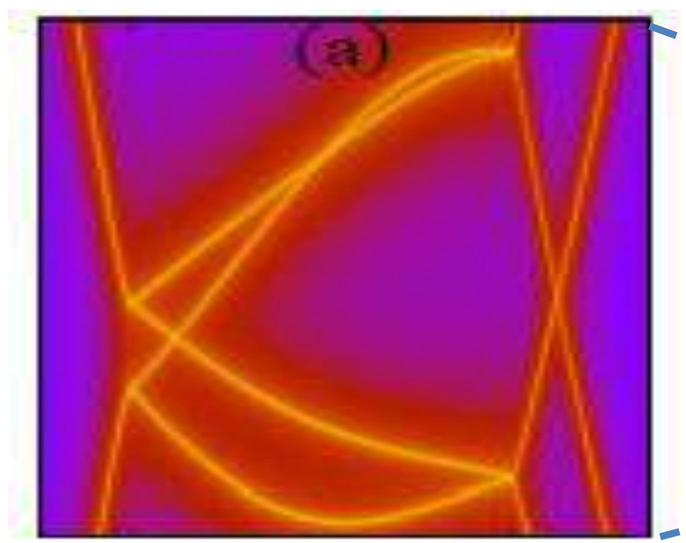
Concept of connectivity: Zak

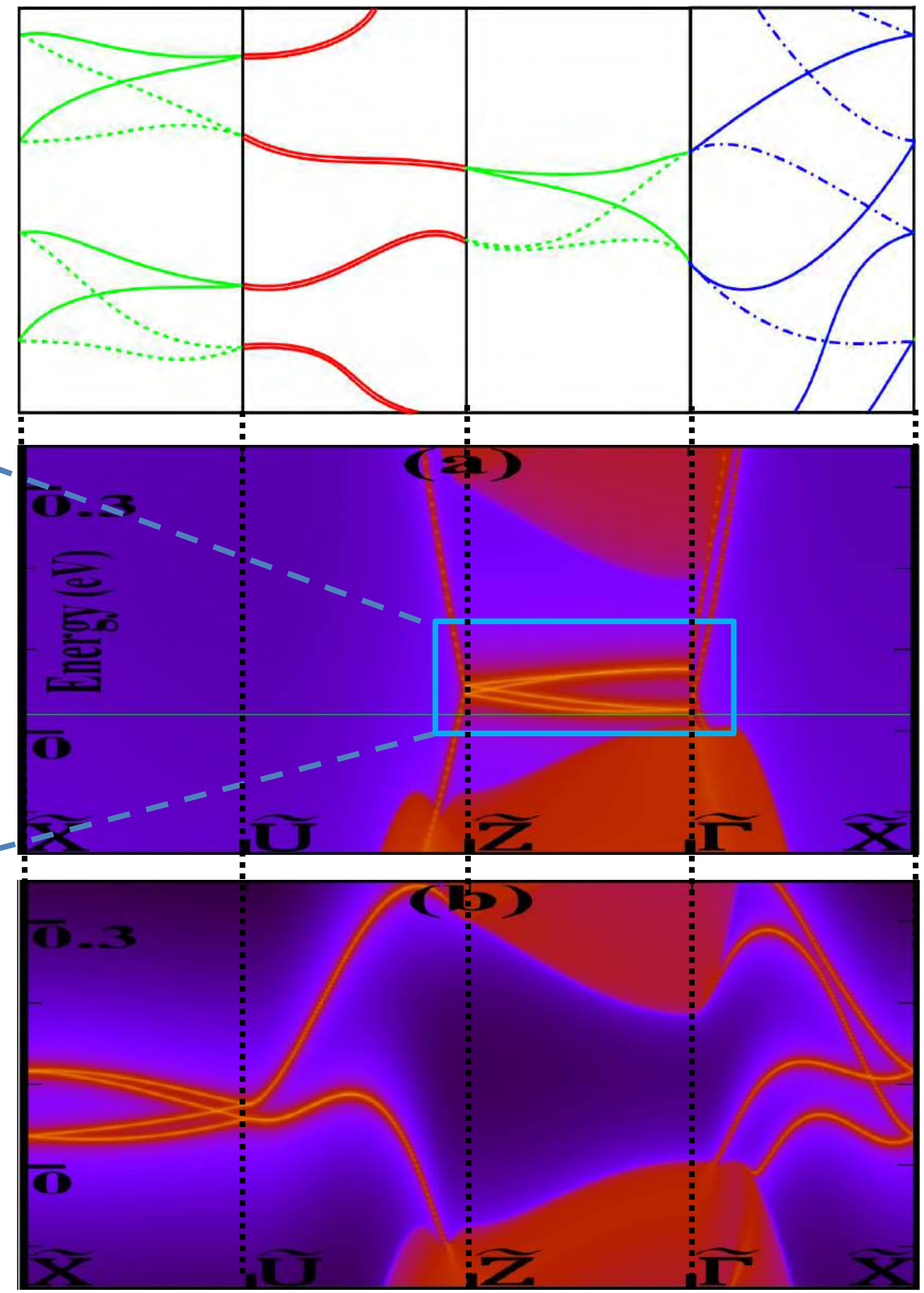
Example of Connectivity in Our New Top Ins





Weak TI: B. Yan, L. Mu¨chler, and C. Felser, Phys. Rev. Lett. 109, 116406 (2012).





Perfect termination

Modified surface potential

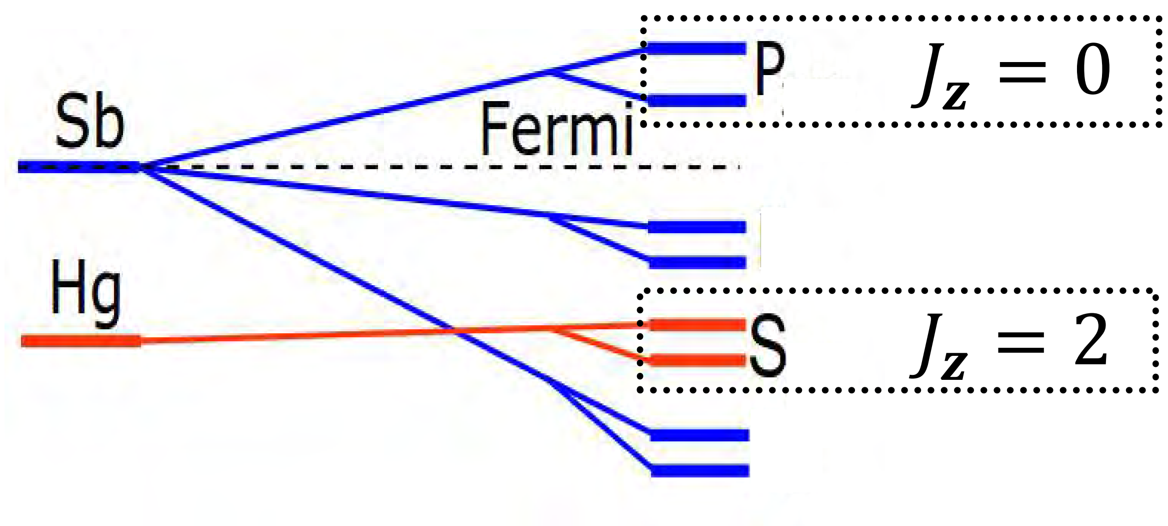
HourGlass Fermion.
3 Realistic Materials Found and Synthesized

KHgX; X= As,Bi, Sb

KZnP: trivial phase | KHgSb: topological phase

Different quantum numbers under spatial transformations.

- Inversion of (screw) rotational eigenvalues: $\exp[-iJ_z\pi/3]$



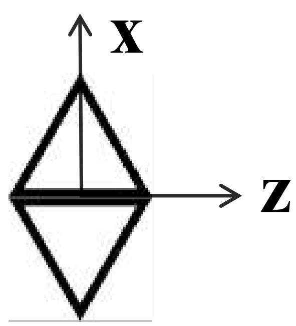
$\Delta J_z =$ Mirror Chern number mod 6

Criterion for nontrivial topology in ***any*** space group with screw/rotational symmetry.

- generalizes previous criterion for symmorphic rotation (Chen Fang et al)

Cohomology and Crystals

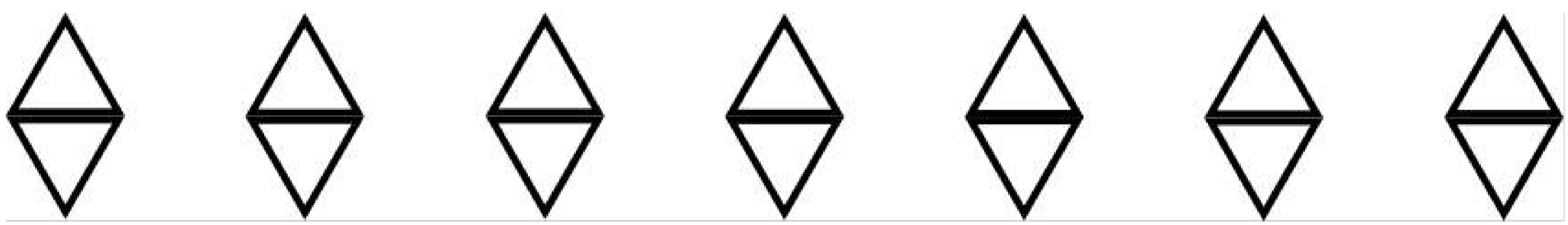
Cohomology in crystallography



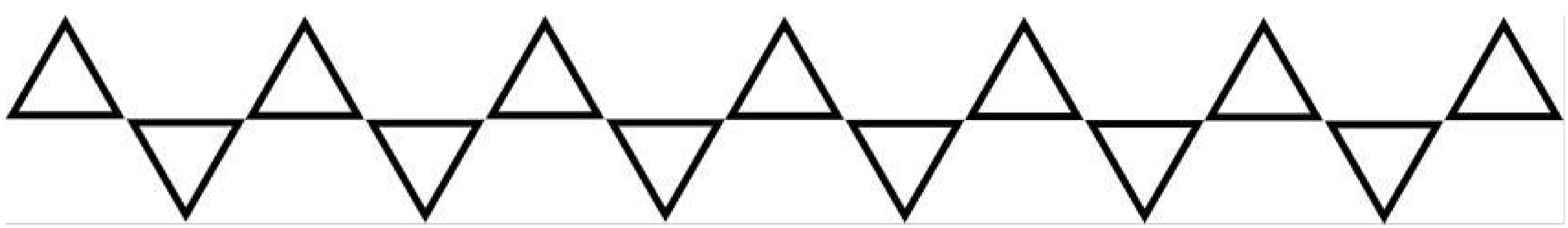
$$M_x^2 = I$$

How many ways can I make a 1D crystal that extends in z, and has also this reflection symmetry?

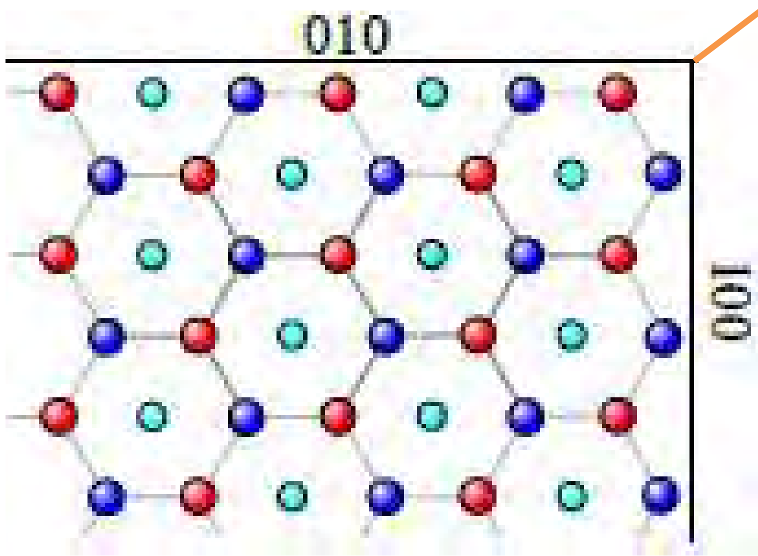
Obvious (for cohomologists, *trivial/split*; for crystallographers: **symmorphic**) $M_x^2 = I$



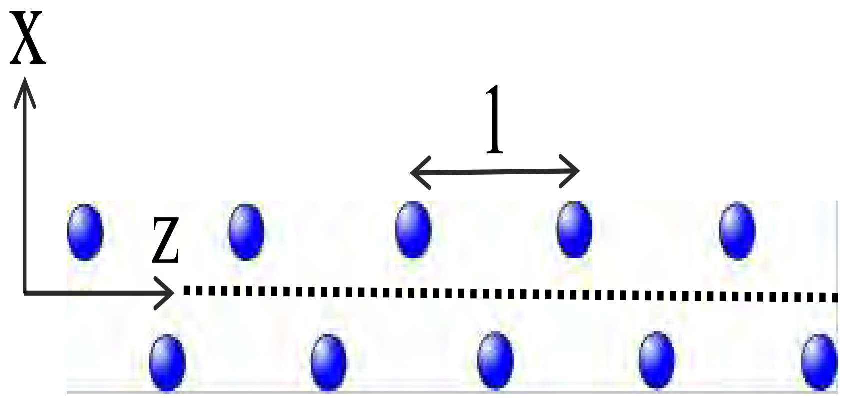
Less obvious (for crystallographers: **nonsymmorphic**) $\bar{M}_x^2 = t(\hat{z})$

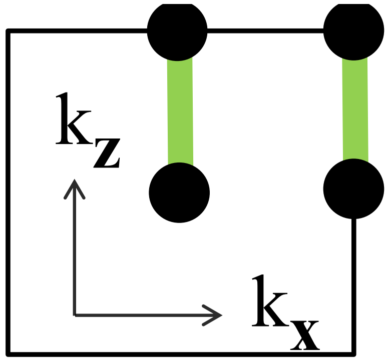


The two algebras differ only by insertion of spatial translations.



HourGlass Fermion



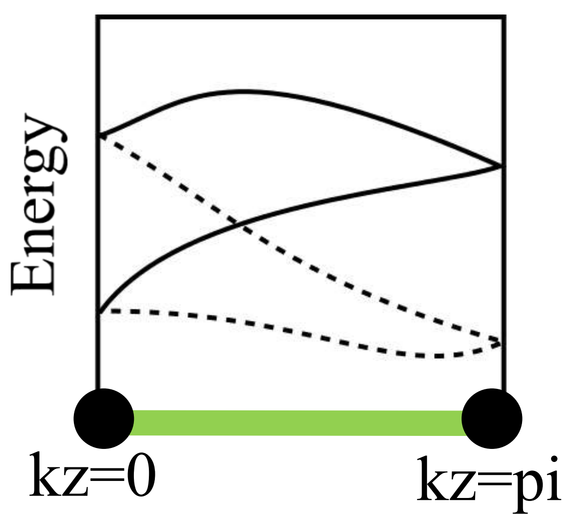


Symmetries:
Mx, Mz

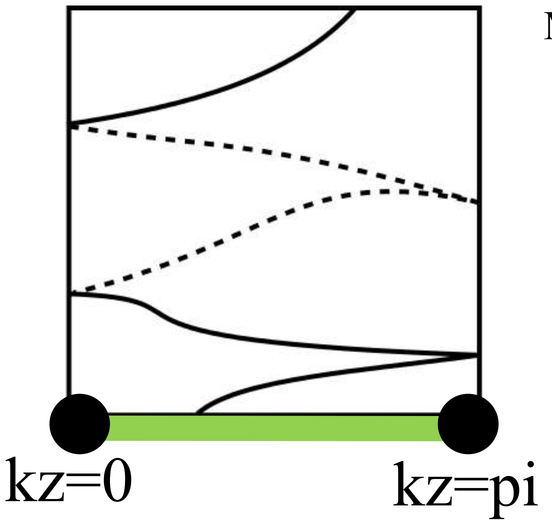
Group of the wavevector

$\underline{Mx}^2$ = - exp (i k_z) → 2 branches of eigenvalues = +/- i * exp(i k_z/2)

4-fold-connected
Hourglass



Glide spin Hall effect



Monodromy: $k_z \to k_z + 2\pi$,
$eig(\bar{M}_x) \to -eig(\bar{M}_x)$.

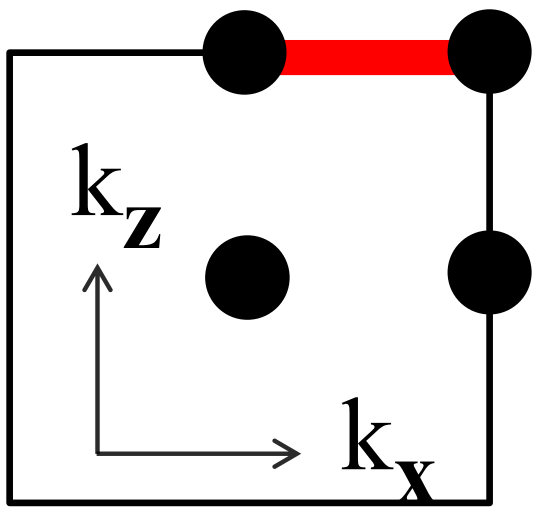
A degeneracy that is movable but unremovable.

Connectivity

Symmetries:

T,<u>Mx,</u>

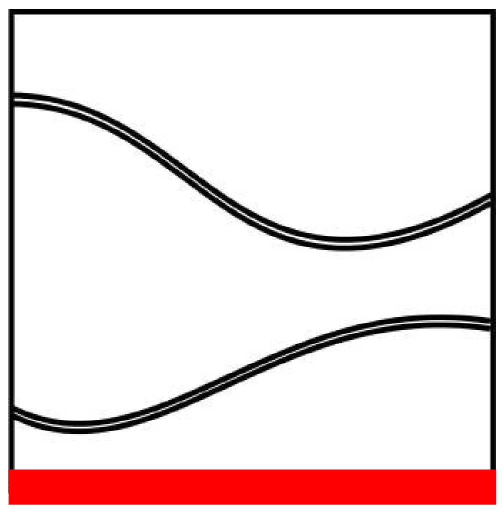
$$(\bar{M}_x)^2 = t(\hat{z}) * (2\pi\, rotation) = -e^{-ik_z}$$



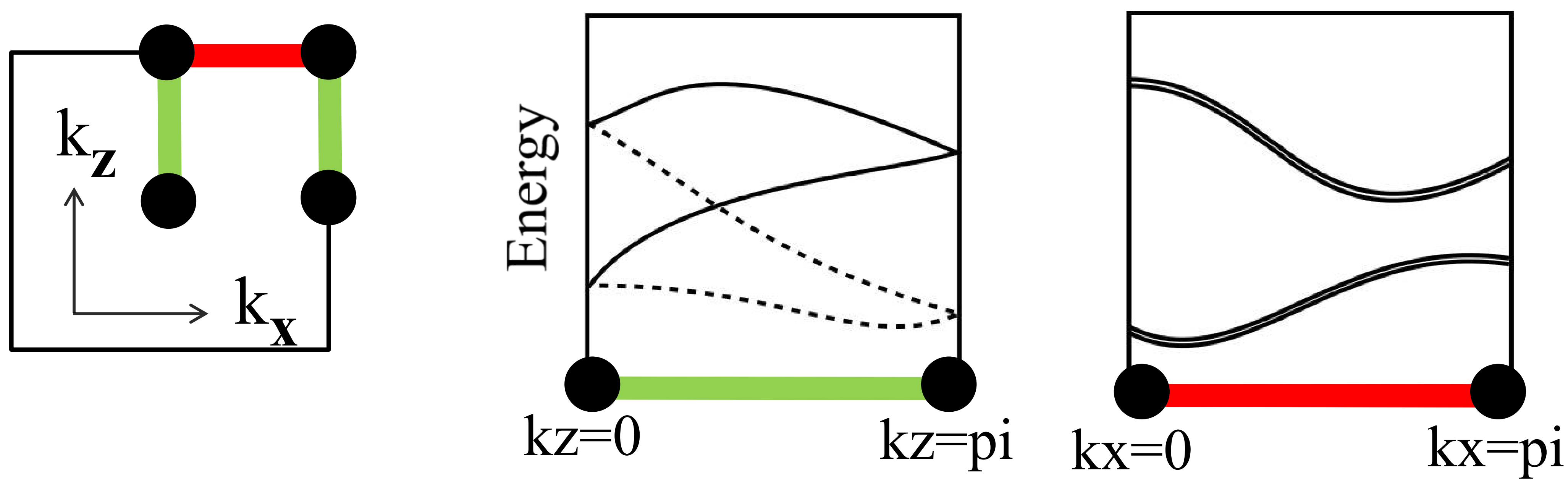
<u>Group of the wavevector</u>

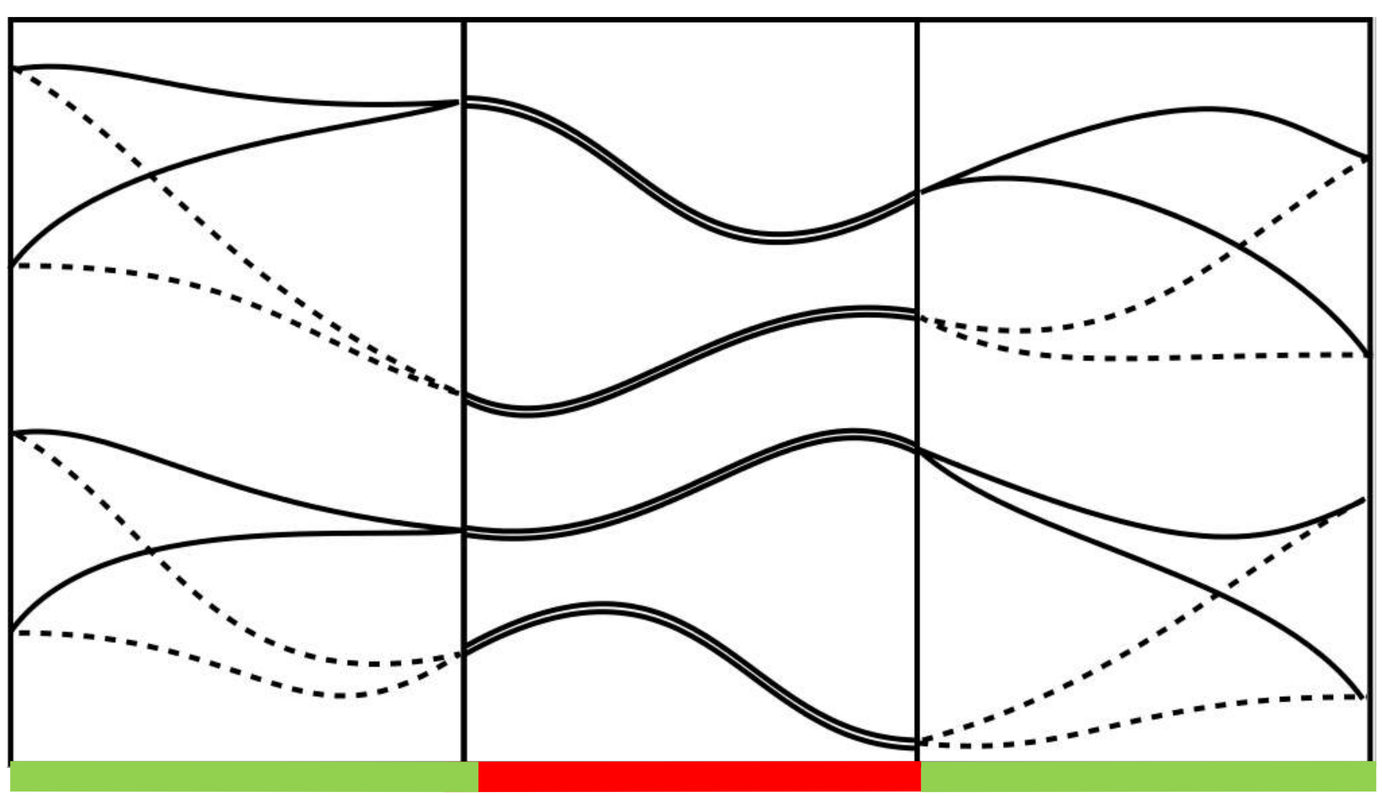
$$(\, T^*\underline{Mx}\,)^{\mathbf{2}} = -1$$

Kramers-like degeneracy at every wavevector,
i.e., ▬ is two-fold connected.

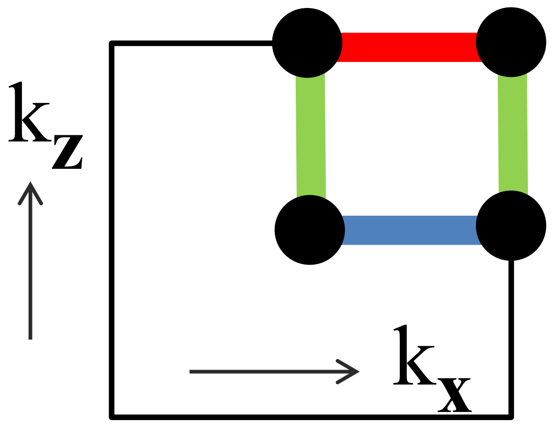


Connectivity





Putting Everything Together

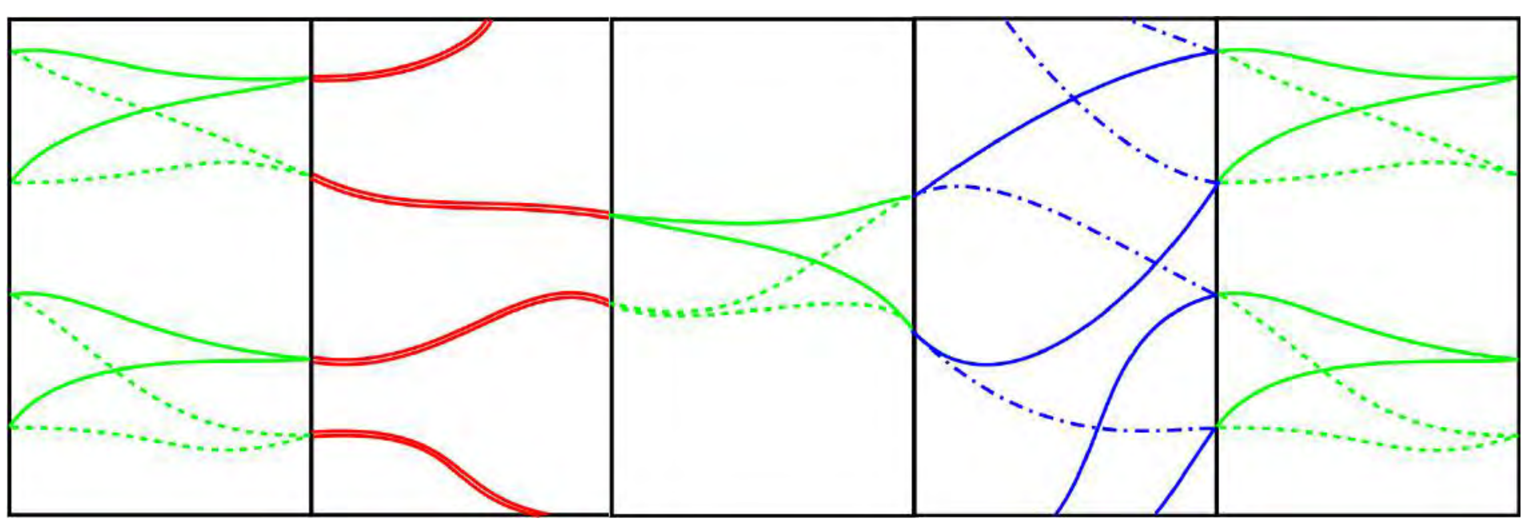


Mz : z → -z

Mirror Chern number: (Teo, Fu, Kane)

$\int \mathcal{F}(k)d^2k$ in the even subspace.

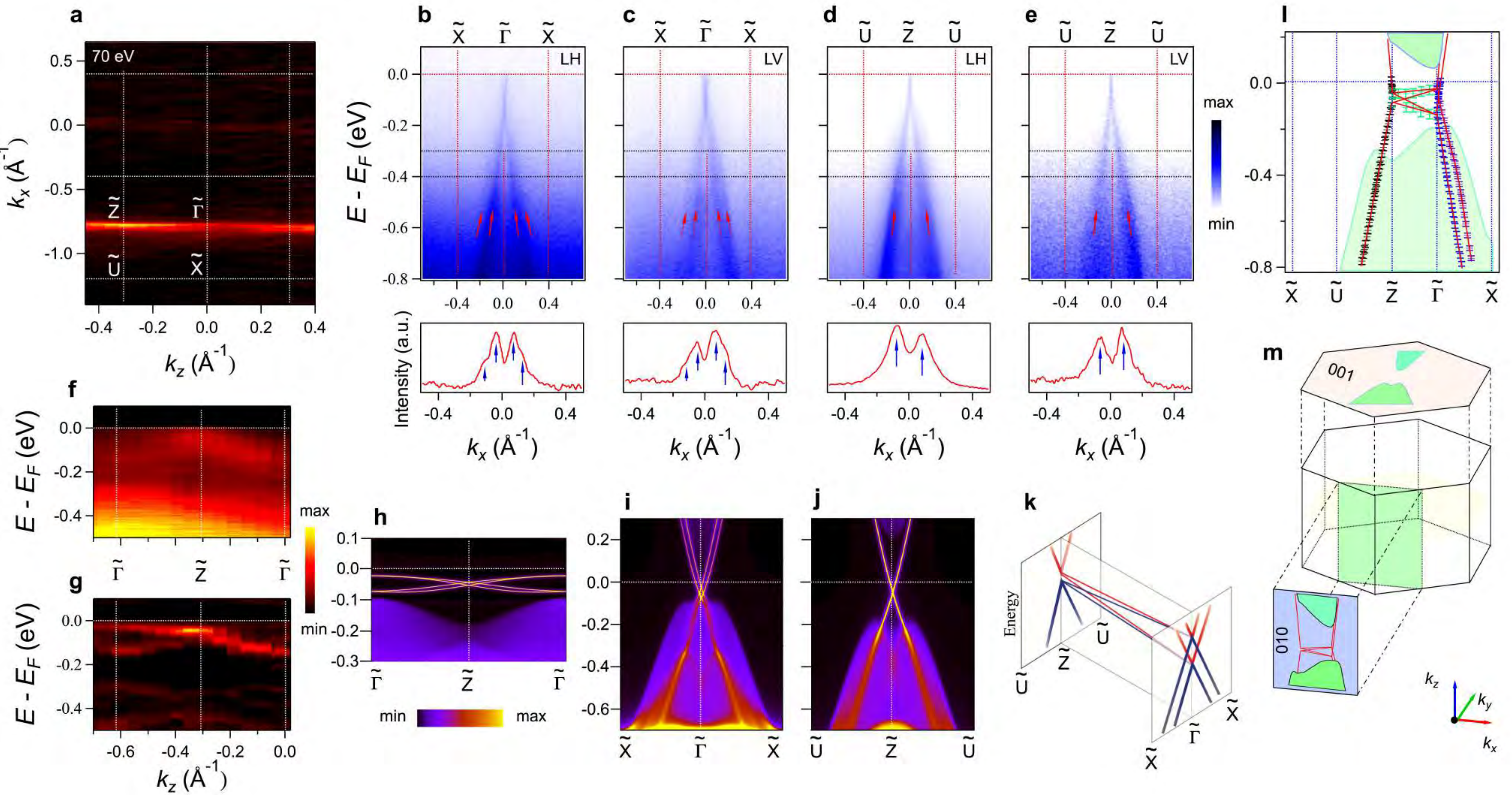
Hourglass-flow topology



Rest of this talk: how do we diagnose this topology in the bulk wavefunction?

A non-abelian generalization of the theory of polarization.

ARPES on KHgSb

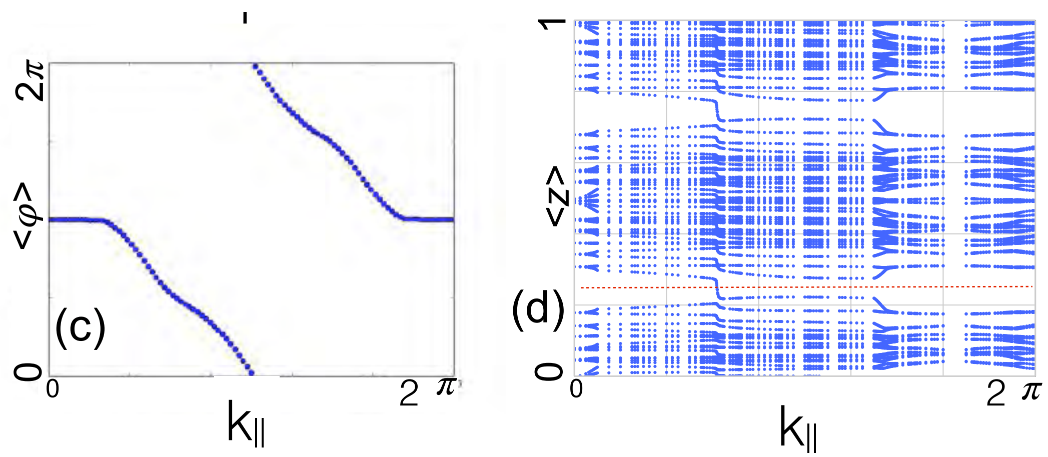


Bulk Indices

And Wilson Group Extensions

Wilson Band Structures Have ALWAYS Provided Faithful Representations of Any Topological Insulator We Thought Of

Wilson loops: matrix representation of parallel transport around momentum loops.

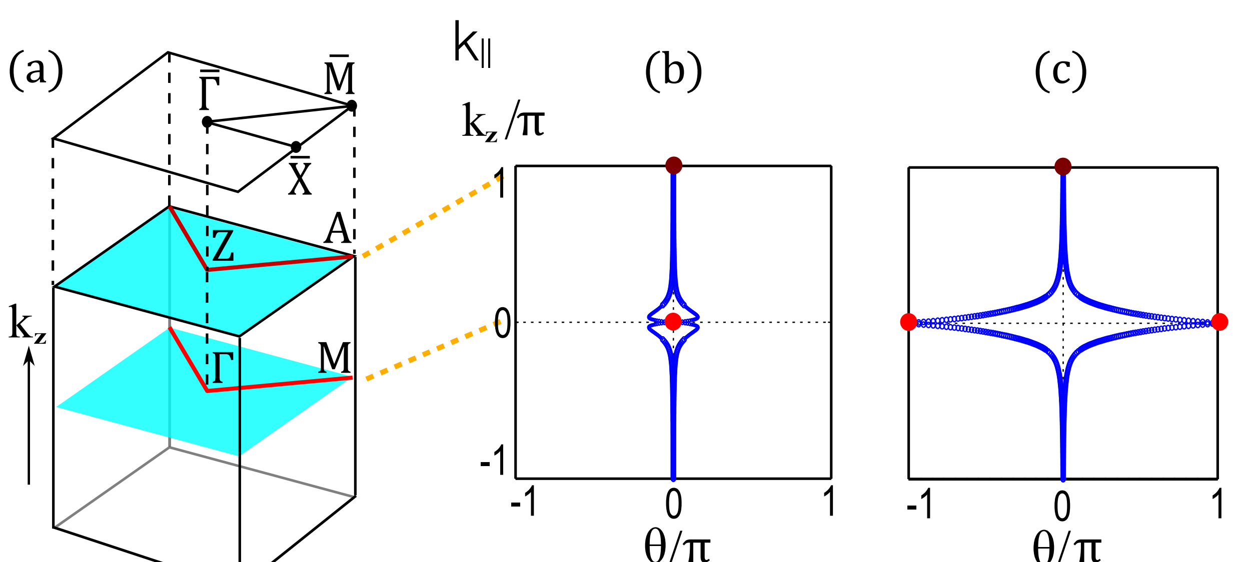


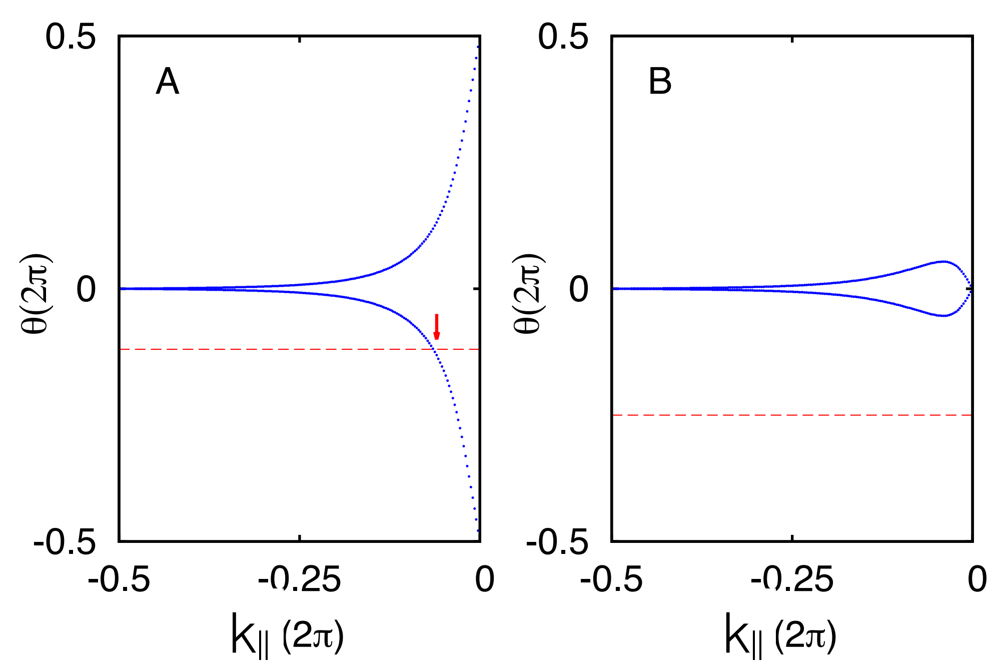
Vanderbilt, Resta

$$\mathcal{W} = \exp\left(-\int A\, dk_y\right);$$

$$A_{mn} = \left\langle U_{m,k_y} \middle| \partial_{k_y} U_{n,k_y} \right\rangle$$

$$\exp[i\theta_{n,\boldsymbol{k}_\parallel}].$$





Dai Xi, Z. Fang, BAB, X. Qi, Rui Yu, A Alexandradinata
Also Soluyanov and Vanderbilt

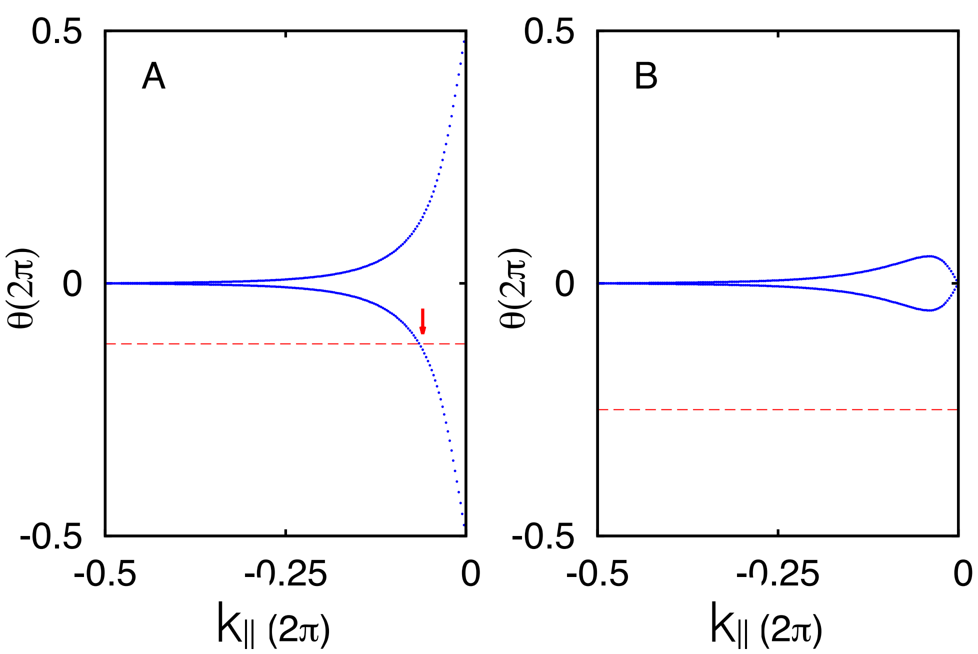
We Define A Wilson Group

Since Wilson loops winding gives correct surface symmetries, we define a wilson group

All elements g (spatial or time-reversal symmetries) such that

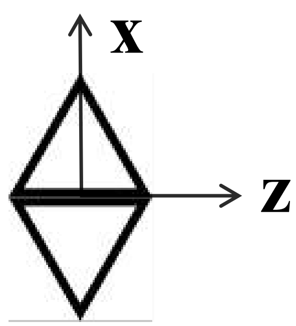
$$\hat{g}\,\mathcal{W}\,\hat{g}^{-1} = \mathcal{W}^{\pm 1}$$

$$\breve{T}(-\pi, \boldsymbol{k}_{\shortparallel})\,\mathcal{W}(\boldsymbol{k}_{\shortparallel})\,\breve{T}(-\pi, \boldsymbol{k}_{\shortparallel})^{-1} = \mathcal{W}(-\boldsymbol{k}_{\shortparallel})^{-1}$$



Cohomology and Crystals

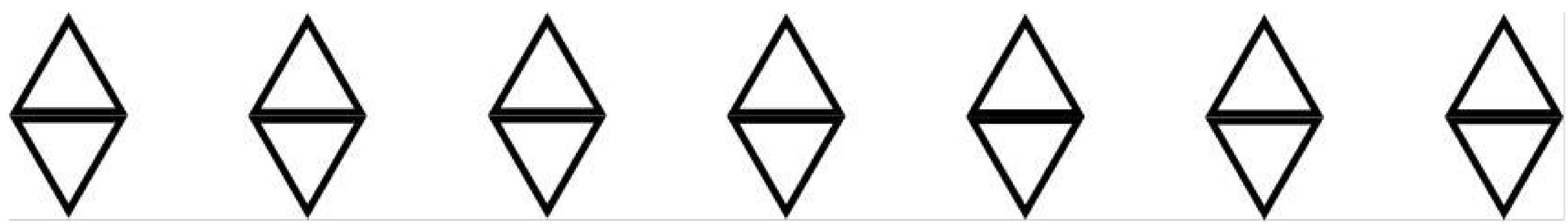
<u>Cohomology in crystallography</u>



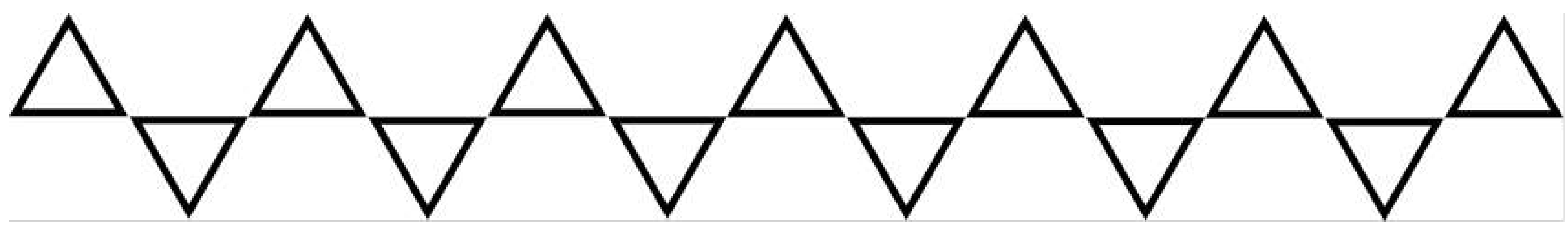
$$M_x^2 = I$$

How many ways can I make a 1D crystal that extends in z, and has also this reflection symmetry?

Obvious (for cohomologists, *trivial/split*; for crystallographers: **symmorphic**) $M_x^2 = I$



Less obvious (for crystallographers: **nonsymmorphic**) $\bar{M}_x^2 = t(\hat{z})$



The two algebras differ only by insertion of spatial translations.

Group Extensions

Cohomology in crystallography

Different extensions are classified by the second cohomology group:

$$H^2\left(G_p, \mathcal{T}\right) = \mathbb{Z}_2$$

$$G_p = \{1, M_x\} \qquad \mathcal{T} = \{\, t(\hat{z})^n \mid n \in \mathbb{Z} \,\}$$

– each extension corresponds to a different crystal order / space group.

In essence, this is how one determines there are

1) 17 space groups in two spatial dimensions,
2) 219 in 3D,
3) 4783 in 4D, and
4) a finite number in any finite spatial dimension.

(Hilbert's 18'th problem, 1900)

Most groups nonsymmorphic

Nonsymmorphicity exists only in real space, why not in momentum space?

Group Extensions

Topological Insulators exist because of a nontrivial extension of the time - reversal symmetry group with that of spin.

With Spin, We can have two situations:

$T^2=1$ (spinful, but no spin-orbit coupling}

$T^2=-1$ (spinful, with spin-orbit coupling, nontrivial TI possible)

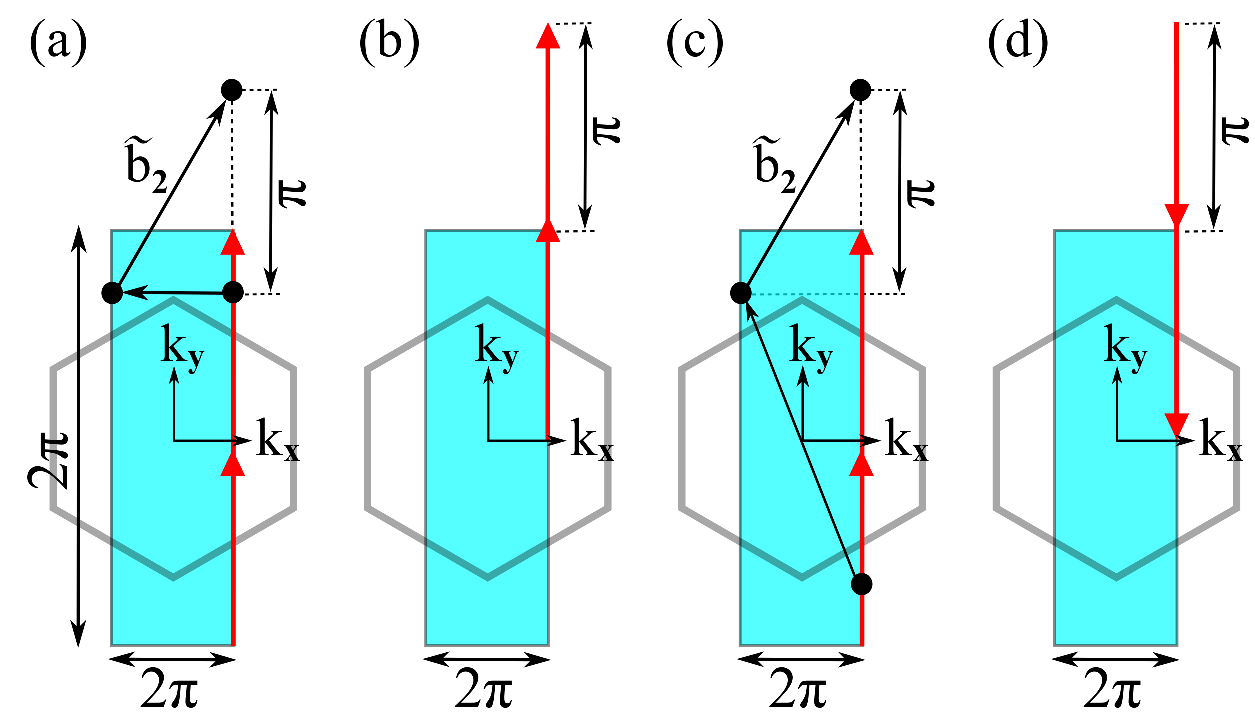
New Topological Classes
Wilson Group Can Be Extended Nontrivially

$$\hat{g}\,\mathcal{W}\,\hat{g}^{-1} = \mathcal{W}^{\pm 1}$$

$$\bar{M}_x\,\mathcal{W}_{-\pi}(\pi, k_z)\,\bar{M}_x^{-1} = \mathcal{W}_0(\pi, k_z)$$

For kx=Pi, the Mirror is NOT part of the Wilson Group - translates the origin of the Wilson line.

Can Build a Nontrivial Mirror:



$$\bar{\mathcal{M}}_x\,\mathcal{W}_{-\pi}\,\bar{\mathcal{M}}_x^{-1} = \mathcal{W}_{-\pi}, \text{ with } \bar{\mathcal{M}}_x = \mathcal{W}_{-\pi\leftarrow 0}\,\bar{M}_x.$$

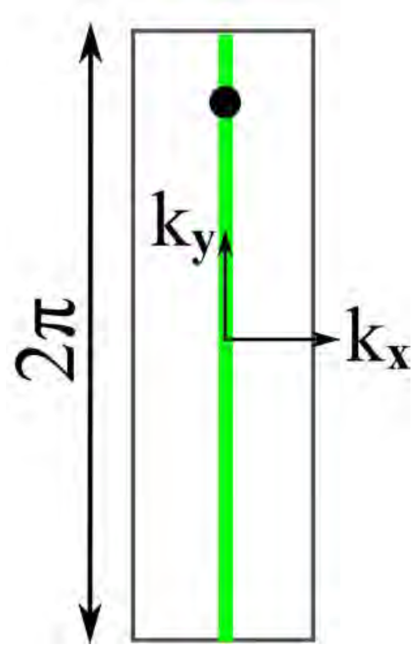
$$\bar{\mathcal{M}}_x^2 = \bar{E}\,t(\vec{z})\,\mathcal{W}_{-\pi}^{-1}$$

There exists an extension by the Wilson loop!
Gives the correct prediction of the possible surface modes, Gives the equivalent of spatial nonsymmorphic symmetries to the Brillouin zone

The two glide-invariant planes in our case study correspond to different representations of the same symmetries – by deducing the possible Wilson 'bandstructures' in both planes, we demonstrated that one plane allows for a glide-spin-Hall subtopology, and the other does not.

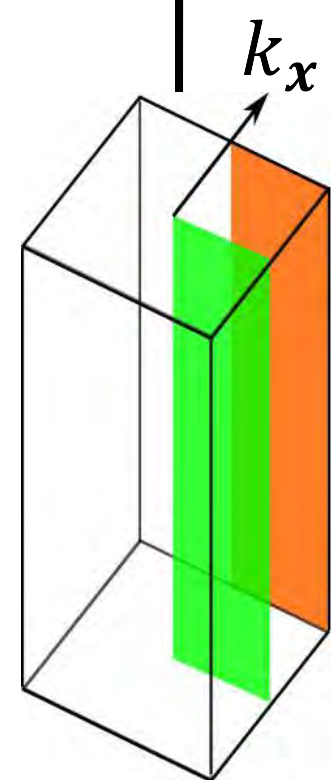
Ordinary glide plane: $k_x = 0$

$$\bar{M}_x : k_x \rightarrow -k_x$$

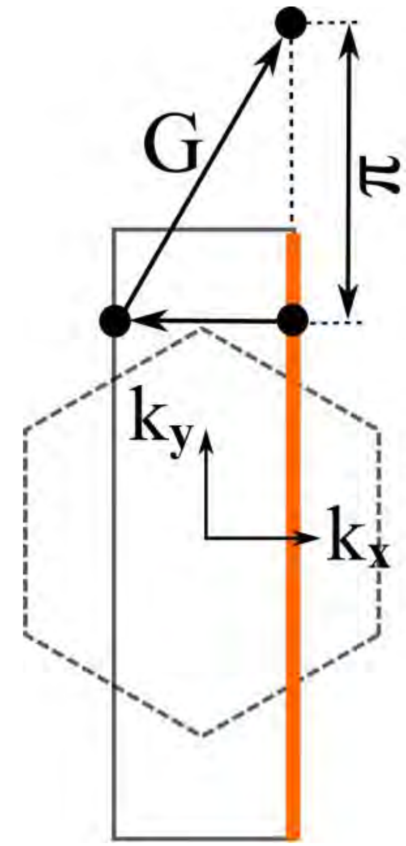


$$\bar{M}_x^2 = -\,\boldsymbol{t}(\hat{z})$$

Trivial extension by quasimomentum translation.



Projective glide plane: $k_x = \pi$



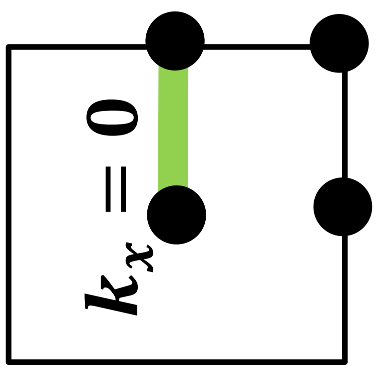
$$\bar{\mathcal{M}}_x^2 = -\,\boldsymbol{t}(\hat{z})\mathcal{W}^{-1}$$

Nontrivial extension by quasimomentum translation.

Different extensions lead to different 2D topologies.

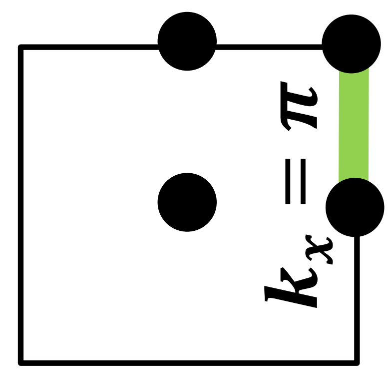
Cohomological Insulators

$$\bar{\mathcal{M}}_x = \mathcal{W}_{-\pi\leftarrow 0}\, \bar{M}_x$$



$$\bar{M}_x \mathcal{W} \bar{M}_x^{-1} = \mathcal{W}$$

$$T\mathcal{W}T^{-1} = \mathcal{W}^{-1}$$



$$\bar{\mathcal{M}}_x \mathcal{W} \bar{\mathcal{M}}_x^{-1} = \mathcal{W}$$

$$\mathcal{T}\mathcal{W}\mathcal{T}^{-1} = \mathcal{W}^{-1}$$

<u>Ordinary representation</u>

$$\bar{M}_x^2 = (2\pi\, rot.)(\text{spatial trans.})$$
$$= -\boldsymbol{t}(\hat{z})$$

$$T^2 = -I$$

$$\bar{M}_x T = T\bar{M}_x$$

Identical to the algebra of the surface symmetries

→ surface prediction is valid.

<u>Projective representation</u>

$$\bar{\mathcal{M}}_x^2 = -\boldsymbol{t}(\hat{z})\mathcal{W}^{-1}$$

$$\mathcal{T}^2 = -I$$

$$\bar{\mathcal{M}}_x \mathcal{T} = \mathcal{T}\bar{\mathcal{M}}_x \mathcal{W}$$

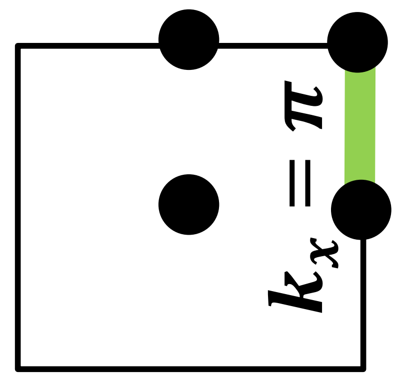
$\bar{\mathcal{M}}_x, \mathcal{T}$ are generalized symmetries that encode parallel transport.

Different from the algebra of the surface symmetries.

The Wilsonian algebra determines the possible topologies of the Wilson bands along $\tilde{\mathrm{X}}\,\tilde{\mathrm{U}}$.

$$H_2(G_\circ, N) \qquad \mathcal{N} = \{\, \bar{E}^a\, t(\vec{z})^b\, \mathcal{W}^c \mid a \in \mathbb{Z}_2, b, c \in \mathbb{Z} \,\}$$

Cohomology determines the band sub-topology



$$\bar{\mathcal{M}}_x \mathcal{W} \bar{\mathcal{M}}_x^{-1} = \mathcal{W}$$

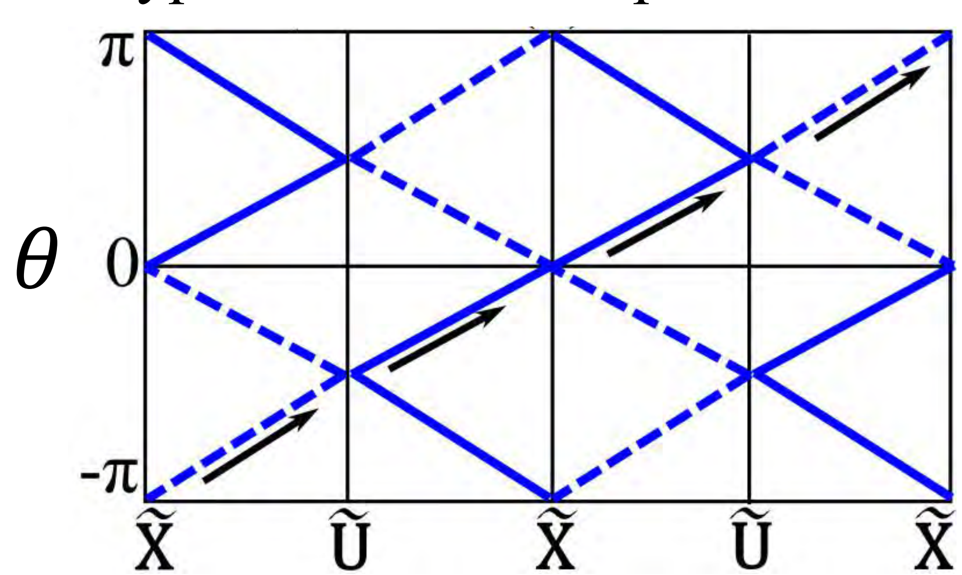
$$\bar{\mathcal{M}}_x^2 = -\,\boldsymbol{t}(\hat{z})\mathcal{W}^{-1}$$

$$\mathcal{W}|\theta, k_z\rangle = e^{i\theta}|\theta, k_z\rangle, \qquad \boldsymbol{t}(\hat{z})|\theta, k_z\rangle = e^{-i\boldsymbol{k_z}}|\theta, k_z\rangle$$

Energy- and momentum-dependent quantum numbers:

$$\mathrm{eig}[\bar{\mathcal{M}}_x] = \pm i\, e^{-i(\boldsymbol{k_z}+\theta)/2}$$

Hypothetical Glide Spin Hall Effect



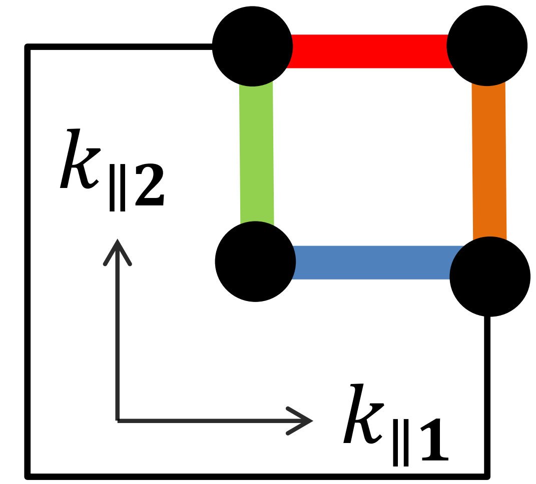
$$k_z \to k_z + 4\pi$$

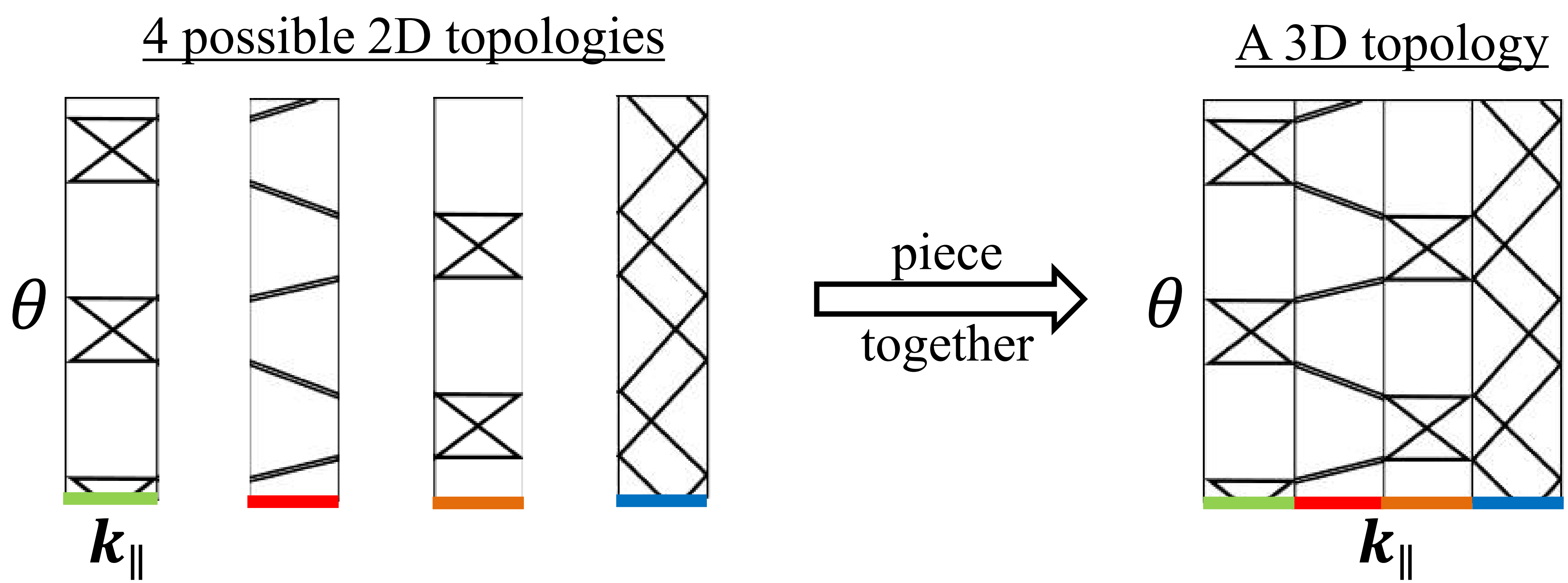
$$\theta \to \theta + 2\pi$$

$$\mathrm{eig}[\bar{\mathcal{M}}_x] \;\to -\mathrm{eig}[\bar{\mathcal{M}}_x]$$

Assumption: Wilson bands come in multiples of four.

Multiplication rules in the group extension,
e.g., $\bar{\mathcal{M}}_x^2 = -\boldsymbol{t}(\hat{z})\mathcal{W}^{-1}$,
constrain the Wilson energies.





Distinct connectivities of the Wilson energies
correspond to topologically inequivalent groundstates.

Summary and outlook

- We introduced a criterion for symmetry-protected band topologies. The idea is to generalize symmetry-protected degeneracy by connectivity.

- This motivates nonsymmorphic crystals which have nontrivial connectivity.

- While the surface analysis gets many things right, the sure-fire method to classify band topology is through the Wilson loop. The connectivity criterion should be applied to the Wilson 'bands'.

Spin-off: There exists further of the 230 Space Group topologies!

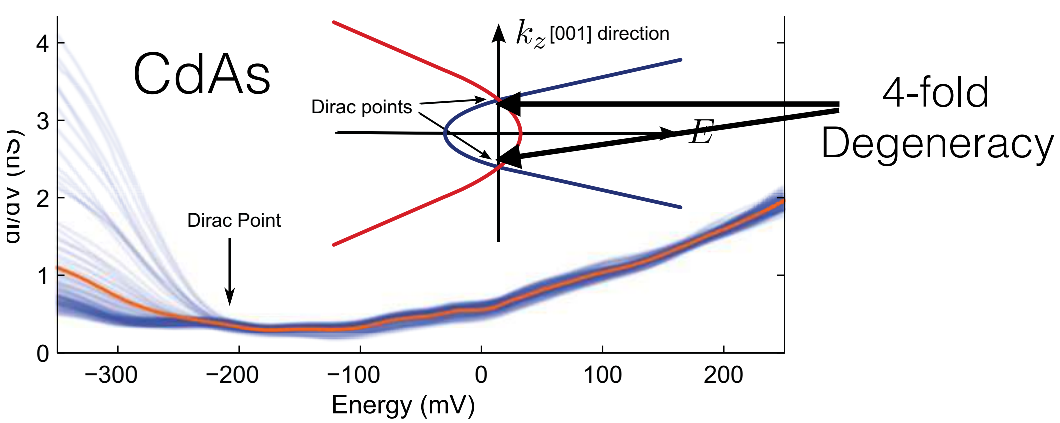
Wang, Alexandradinata, Cava, BAB, Nature
Alexandradinata, Wang, BAB, PRX

Semimetals: Why Are They Interesting?

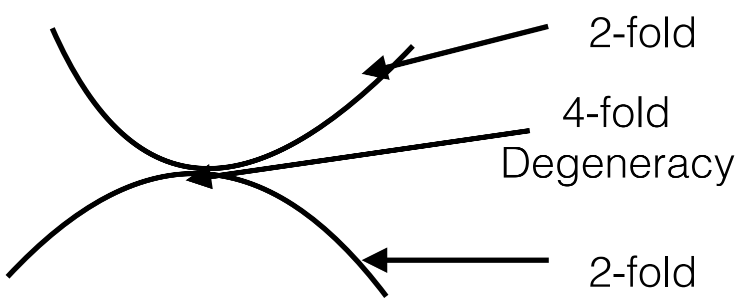
See Ali's Talk Tomorrow

Weyl, Dirac: Experimentally Discovered materials firs theoretically predicted**.**

Two - types of "Dirac" Semimetal:
1. Two 2-fold irreps crossing linearly (CdAs):

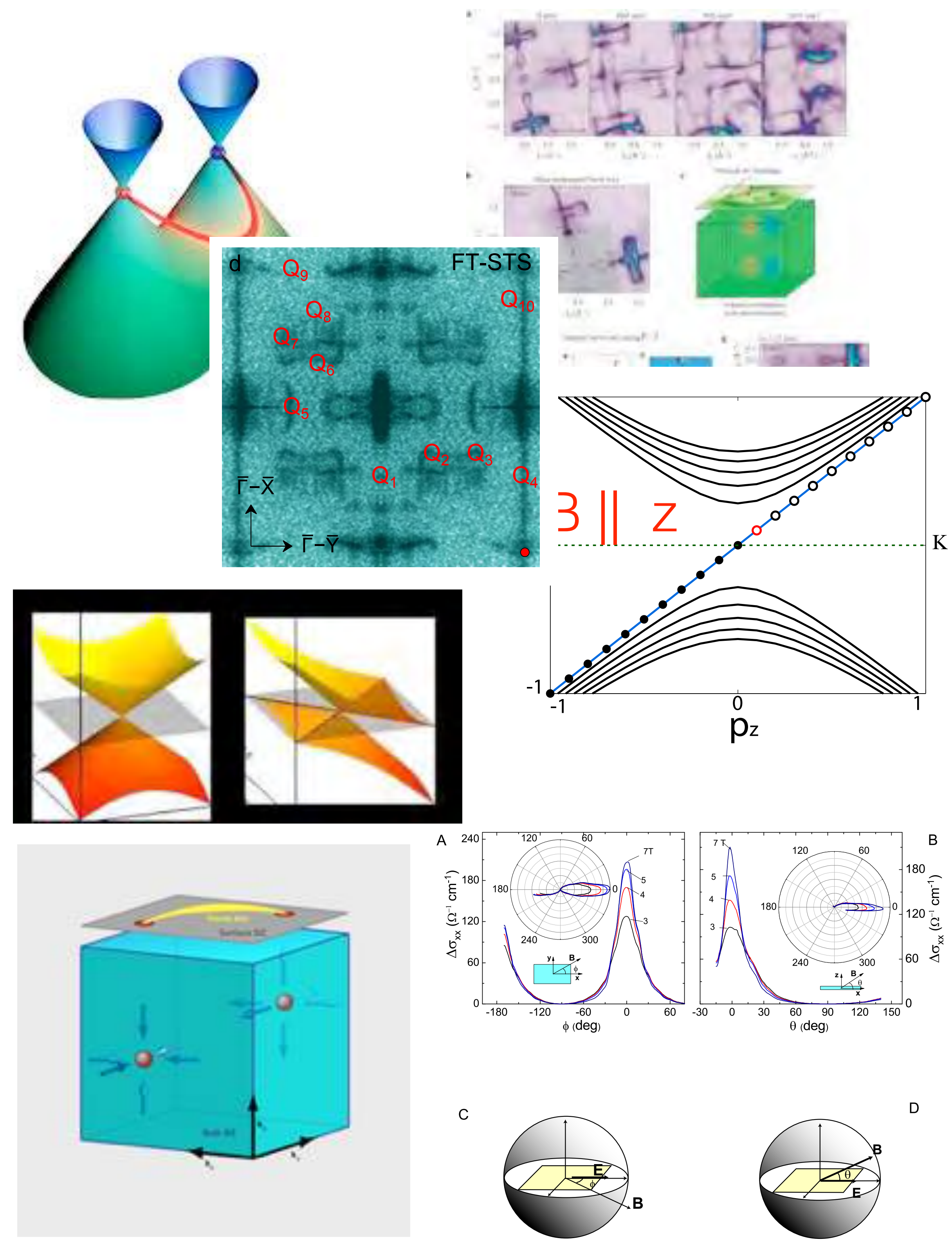


2. One 4-fold irrep (HgTe):



Topological Semimetals: weird transport

Dirac Semimetals generically break into Weyl (due to symmetry) and can have negative magneto-resistance.



Ong Felser Cava Hasan Ding Chen Yazdani Mollenkamp

Volovik, Murakami, Burkov, Balents, Moore, Vishwanath, Savrasov, Kane, Fu, Grushin, Zhang, Qi, Hosur, Bardarson, Weng, Dai, Soluyanov, Bergholtz, Fang, Yao, Nagaosa, Vanderbilt, Sachdev and others

New Types of Fermions

Barry Bradlyn, Jennifer Cano, Zhijun Wang, Maia Vergniory, Claudia Felser, Bob Cava, BAB, submitted

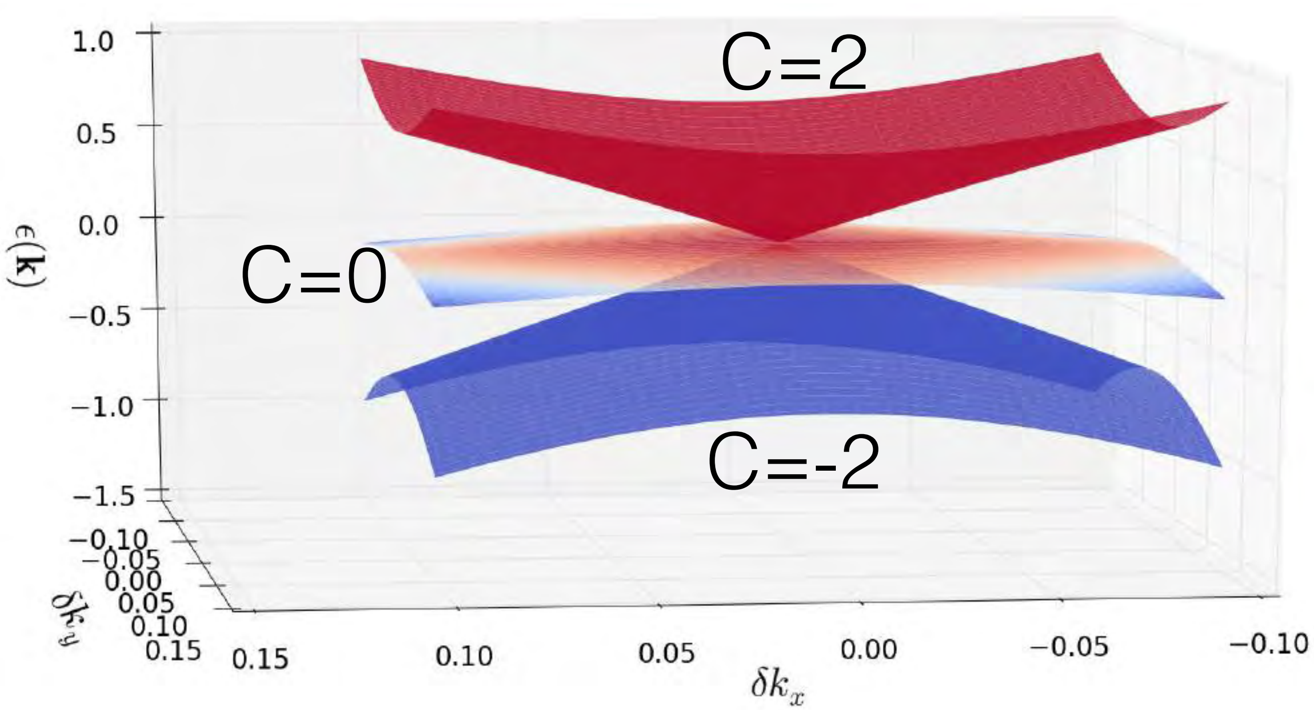
3-fold Degeneracy (in several symmetry groups)

Violates spin 2S+1 degeneracy (always even)

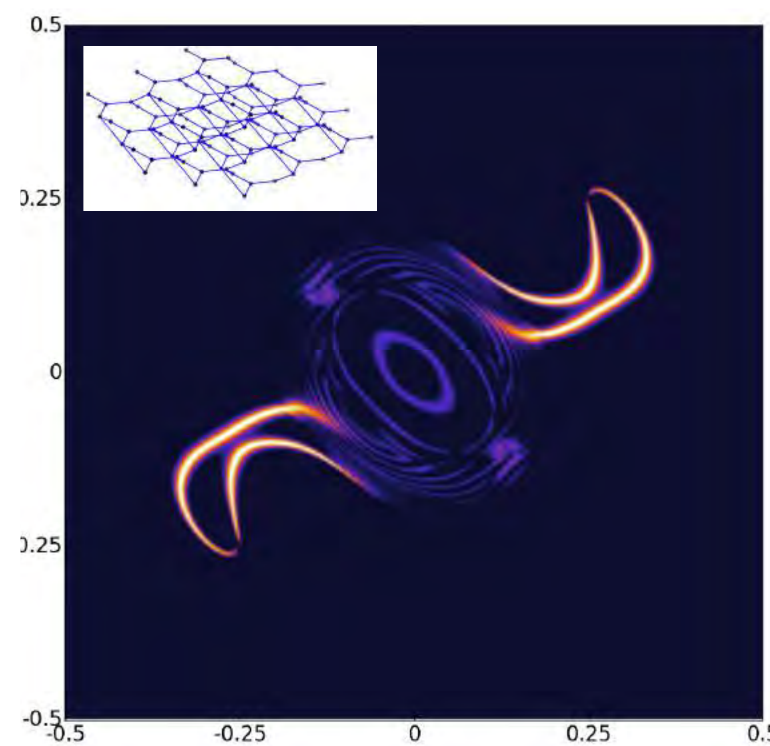
Linear crossing, but **Chern number 2** bands.

$$\vec{k} \cdot \vec{S}$$

Spin 1, 3/2



(a) SGs 199 and 214

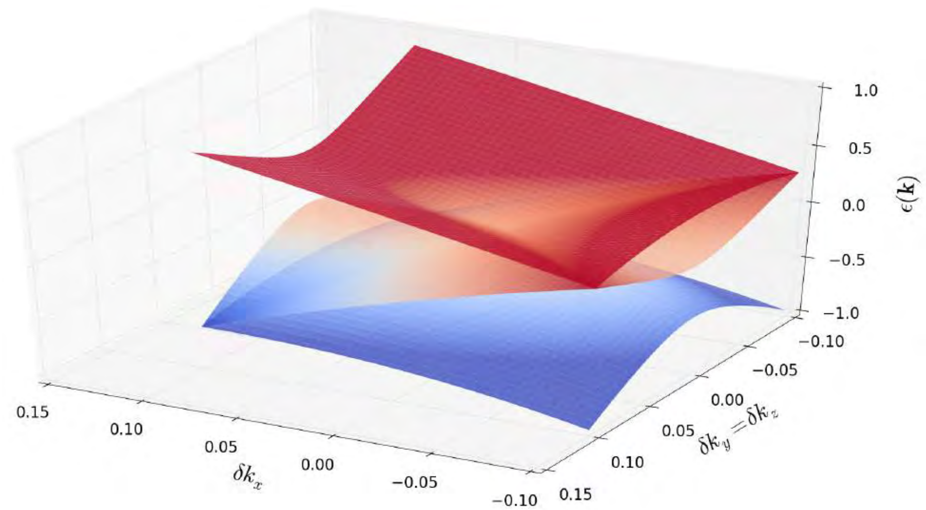


arXiv:1509.00861
Dirac Cone Protected by **Non-Symmorphic** Symmetry: 4 fold nonsymmorphic
Leslie M. Schoop, Mazhar N. Ali, Carola Straßer, Viola Duppel, Stuart S. P. Parkin, Bettina V. Lotsch, Christian R. Ast

3-fold, 6-fold, 8-fold Crossings: All Different Fermions

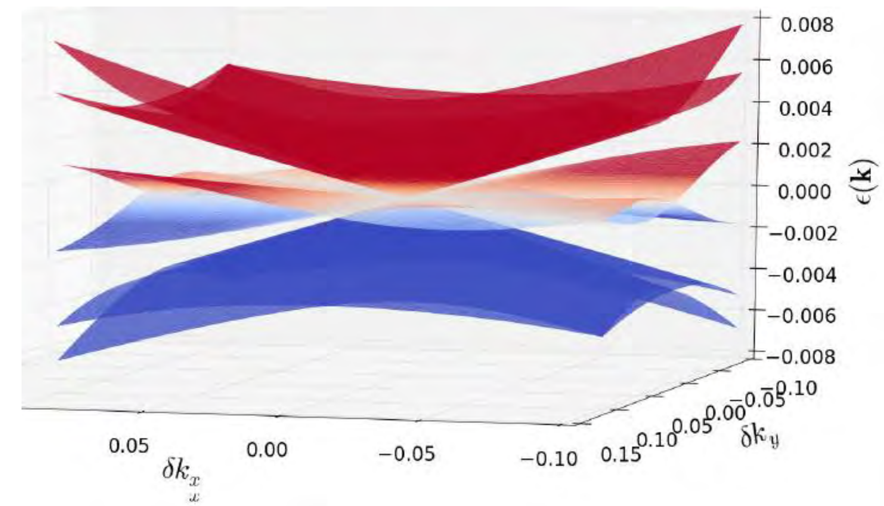
For 8-fold see also Benjamin J. Wieder, Youngkuk Kim, A. M. Rappe, C. L. Kane, arXiv:1512.00074

k dot p models



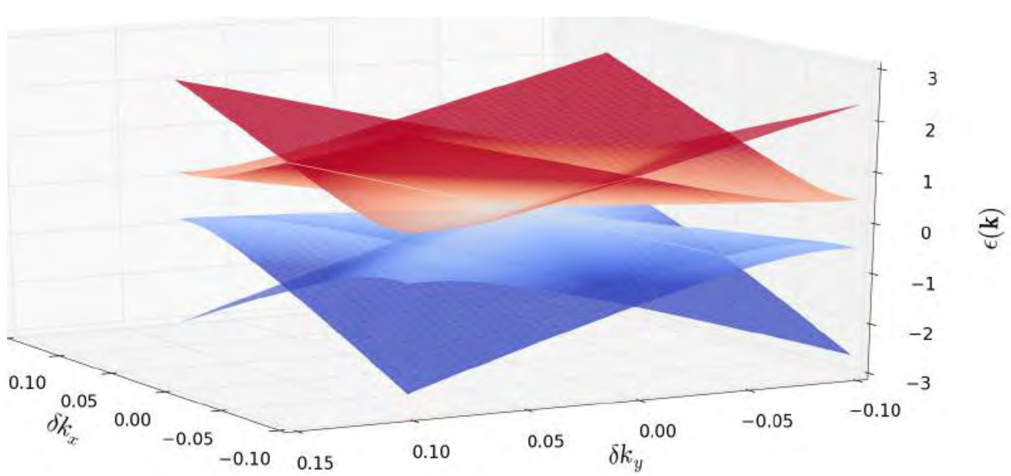
(b) SG 220

3-fold degeneracy,
Line-nodes on $|\delta k_x| = |\delta k_y| = |\delta k_z$



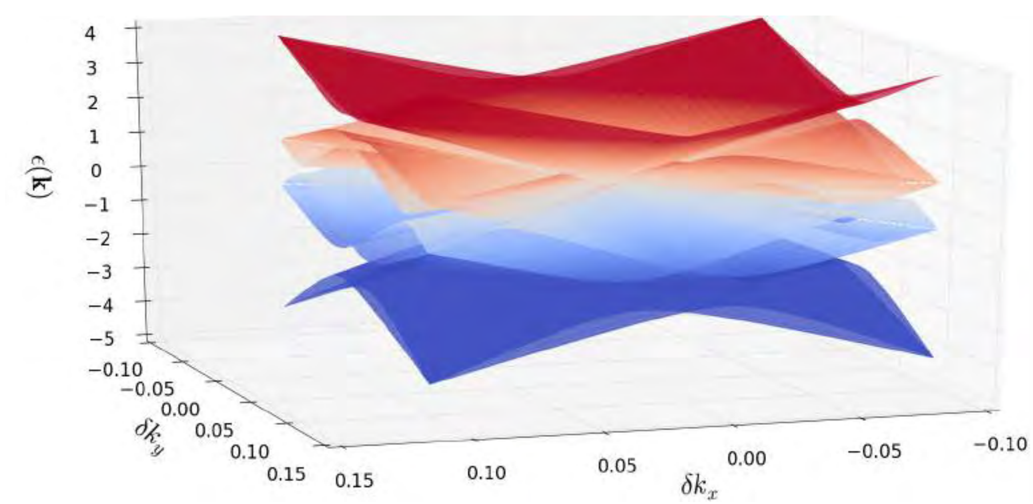
(c) SGs 198, 212 and 213

6-fold degeneracy,
Surface-nodes on $\delta k_i = 0$ of the BZ



(a) SGs 130 and 135

4-fold degenerate at corner of BZ:
Dirac Line Nodes



(c) SGs 218 and 220

Four non-degenerate and two doubly degenerate pairs of bands on $|\delta k_x| = |\delta k_y| = |\delta k_z|$

NonSymmorphicity, Degeneracies, and Group Cohomology

(8-fold crossings example, everything else is similar)

Start With Non-Symmorphic 4-Fold

$$T, R_1, R_2 \longrightarrow T_1, T_2, T_3$$

3 antiunitaries

$$T_1 = T$$

$$T_2 = e^{i\frac{\theta_{12}}{3}} R_1 \cdot T$$

$$T_3 = e^{i\frac{\theta_{13}}{3}} R_2 \cdot T$$

$$T^2 = -1$$

$$R_1^2 = R_2^2 = 1$$

Non Symmorphic

$$[T, R_1] = [T, R_2] = 0$$

$$R_1 R_2 = e^{i(\theta_{12}+\theta_{13}+\theta_{31})} R_2 R_1$$

If $\{R_1, R_2\} = 0$

only 4-fold degeneracy available

$$T_i^2 = -1$$

$$T_i T_j = e^{i\theta_{ij}} T_j T_i$$

$$e^{i(\theta_{12}+\theta_{23}+\theta_{31})} \neq 1$$

4-dim irrep, projective

NonSymmorphicity, Degeneracies, and Group Cohomology

SG 130

$$T_1 = \mathcal{T};\ T_2 = \breve{\mathcal{T}}\bar{I}\bar{C}_{2x};\ T_3 = \mathcal{T}\bar{I}\bar{C}_{2y}$$

To obtain an 8-fold, we need another unitary

$$C_{4z}|\varphi\rangle = \lambda \cdot |\varphi\rangle$$

$\{\lambda; \lambda^*\}$ $\xleftrightarrow{\text{Inversion}}$ $\{-\lambda; -\lambda^*\}$

4-fold, 3 projective antiunitaries

4-fold, 3 projective anti-unitaries

All other Fermions 3, 6, 8 fold, here also have projective representations

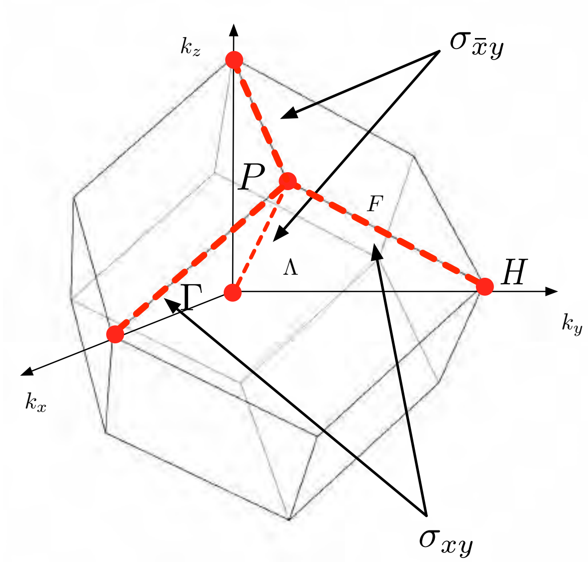
All distinct, all different responses

New Fermions: Classification And Surface/Line Degeneracies

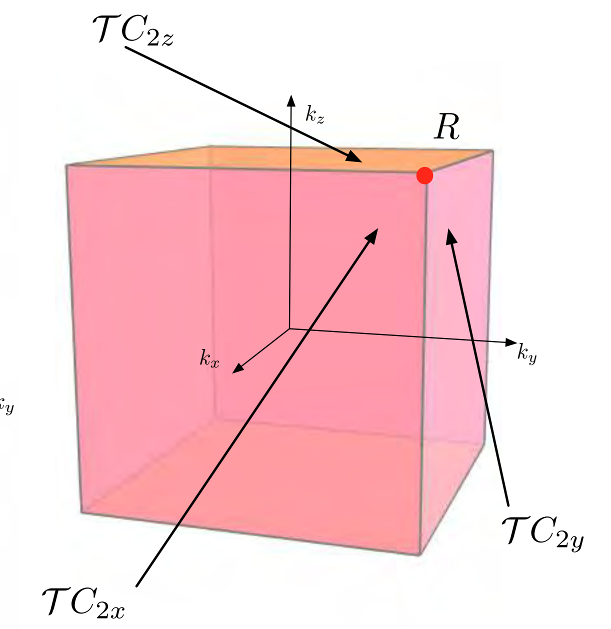
Barry Bradlyn, Jennifer Cano, Zhijun Wang, Maia Vergniory, Claudia Felser, Bob Cava, BAB

Bravais lattice	Lattice vectors	Reciprocal lattice vectors
Primitive cubic	$(a,0,0),(0,a,0),(0,0,a)$	$\frac{2\pi}{a}(1,0,0),\frac{2\pi}{a}(0,1,0),\frac{2\pi}{a}(0,0,1)$
Body-centered cubic	$\frac{a}{2}(-1,1,1),\frac{a}{2}(1,-1,1),\frac{a}{2}(1,1,-1)$	$\frac{2\pi}{a}(0,1,1),\frac{2\pi}{a}(1,0,1),\frac{2\pi}{a}(1,1,0)$
Primitive tetragonal	$(a,0,0),(0,a,0),(0,0,c)$	$\frac{2\pi}{a}(1,0,0),\frac{2\pi}{a}(0,1,0),\frac{2\pi}{c}(0,0,1)$

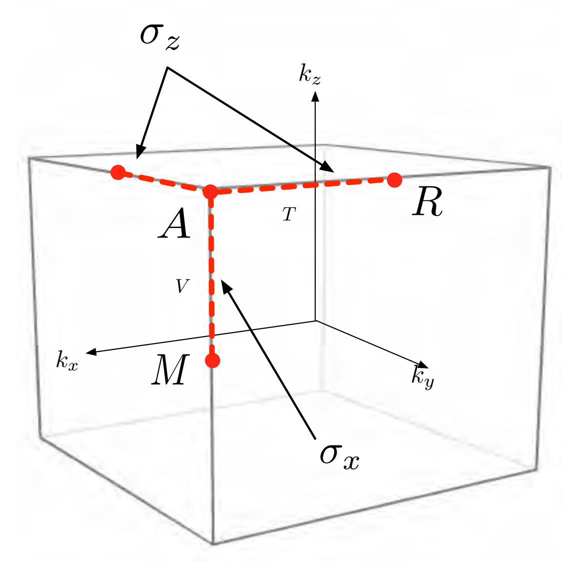
SG	La	k	d	Generators
198	cP	R	6	$\{C^{-}_{3,111}\vert 010\},\{C_{2x}\vert \frac{1}{2}\frac{3}{2}0\},\{C_{2y}\vert 0\frac{3}{2}\frac{1}{2}\}$
199	cI	P	3	$\{C^{-}_{3,111}\vert 101\},\{C_{2x}\vert \frac{1}{2}\frac{1}{2}0\},\{C_{2y}\vert 0\frac{1}{2}\frac{1}{2}\}$
205	cP	R	6	$\{C^{-}_{3,111}\vert 010\},\{C_{2x}\vert \frac{1}{2}\frac{3}{2}0\},\{C_{2y}\vert 0\frac{3}{2}\frac{1}{2}\},\{I\vert 000\}$
206	cI	P	6	$\{C^{-}_{3,111}\vert 101\},\{C_{2x}\vert \frac{1}{2}\frac{1}{2}0\},\{C_{2y}\vert 0\frac{1}{2}\frac{1}{2}\}$
212	cP	R	6	$\{C_{2x}\vert \frac{1}{2}\frac{1}{2}0\},\{C_{2y}\vert 0\frac{1}{2}\frac{1}{2}\},\{C^{-}_{3,111}\vert 000\},\{C_{2,1\bar{1}0}\vert \frac{1}{4}\frac{1}{4}\frac{1}{4}\}$
213	cP	R	6	$\{C_{2x}\vert \frac{1}{2}\frac{1}{2}0\},\{C_{2y}\vert 0\frac{1}{2}\frac{1}{2}\},\{C^{-}_{3,111}\vert 000\},\{C_{2,1\bar{1}0}\vert \frac{3}{4}\frac{3}{4}\frac{3}{4}\}$
214	cI	P	3	$\{C^{-}_{3,111}\vert 101\},\{C_{2x}\vert \frac{1}{2}\frac{1}{2}0\},\{C_{2y}\vert 0\frac{1}{2}\frac{1}{2}\}$
220	cI	P	3	$\{C_{3,\bar{1}\bar{1}1}\vert 0\frac{1}{2}\frac{1}{2}\},\{C_{2y}\vert 0\frac{1}{2}\frac{1}{2}\},\{C_{2x}\vert \frac{3}{2}\frac{3}{2}0\},\{IC^{-}_{4x}\vert \frac{1}{2}11\}$
230	cI	P	6	$\{C_{3,\bar{1}\bar{1}1}\vert 0\frac{1}{2}\frac{1}{2}\},\{C_{2y}\vert 0\frac{1}{2}\frac{1}{2}\},\{C_{2x}\vert \frac{3}{2}\frac{3}{2}0\},\{IC^{-}_{4x}\vert \frac{1}{2}11\}$
130	tP	A	8	$\{C_{4z}\vert 000\},\{\sigma_{\bar{x}y}\vert 00\frac{1}{2}\},\{I\vert \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$
135	tP	A	8	$\{C_{4z}\vert \frac{1}{2}\frac{1}{2}\frac{1}{2}\},\{\sigma_{\bar{x}y}\vert 00\frac{1}{2}\},\{I\vert 000\}$
218	cP	R	8	$\{C_{2x}\vert 001\},\{C_{2y}\vert 000\},\{C^{-}_{3,111}\vert 001\},\{\sigma_{\bar{x}y}\vert \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$
220	cI	H	8	$\{C_{2x}\vert \frac{1}{2}\frac{1}{2}0\},\{C_{2y}\vert 0\frac{1}{2}\frac{3}{2}\},\{C^{-}_{3,111}\vert 001\},\{\sigma_{\bar{x}y}\vert \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$
222	cP	R	8	$\{C^{-}_{4z}\vert 000\},\{C_{2x}\vert 000\},\{C^{-}_{3,111}\vert 010\},\{I\vert \frac{1}{2}\frac{1}{2}\frac{1}{2}\}$
223	cP	R	8	$\{C^{-}_{4z}\vert \frac{1}{2}\frac{1}{2}\frac{1}{2}\},\{C_{2x}\vert 000\},\{C^{-}_{3,111}\vert 010\},\{I\vert 000\}$
230	cI	H	8	$\{C_{4z}\vert 0\frac{1}{2}0\},\{C_{2y}\vert 1\frac{1}{2}\frac{1}{2}\},\{C_{3,111}\vert 111\},\{I\vert 000\}$



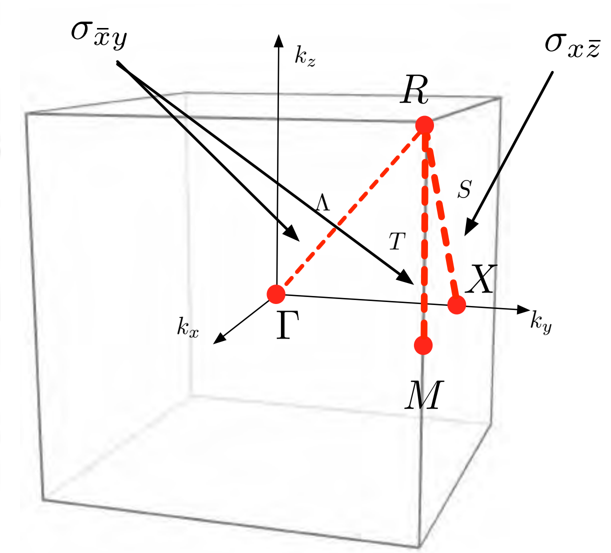
(a) Line Nodes in SG 220



(b) Surface Nodes in SGs 198,212, and 213



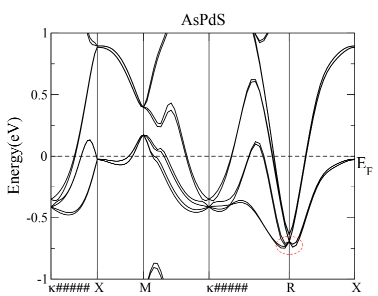
(c) Dirac line nodes in SGs 130 and 135

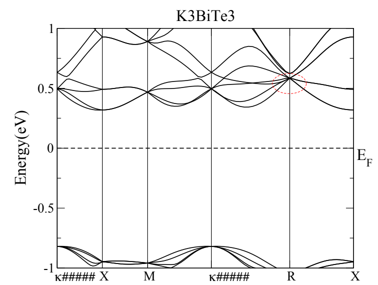


(d) Line nodes in SG 218

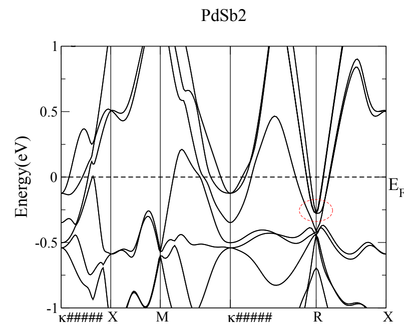
For 8-fold see also Benjamin J. Wieder, Youngkuk Kim, A. M. Rappe, C. L. Kane

The Different Classes of New Fermions: 3,6,8 Fold

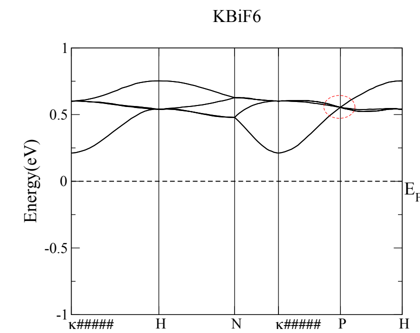




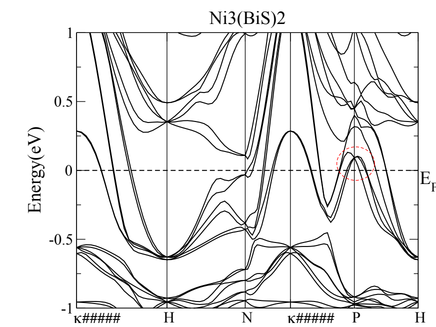
A 6-fold fermion can occur at the R point in the dashed circle. *SG 198*

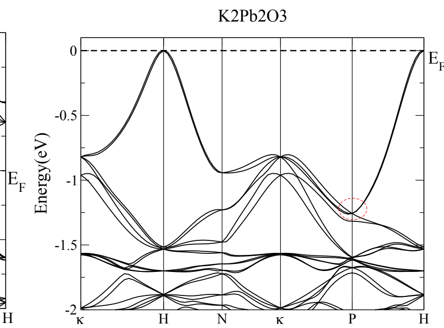


6-fold fermion occur below E_F at the R point *SG 205*

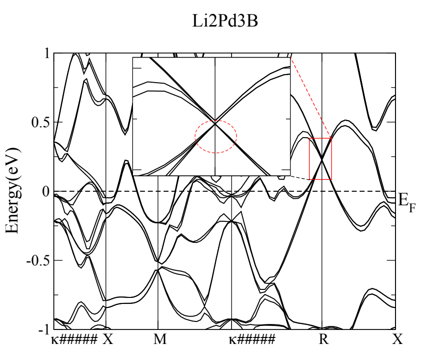


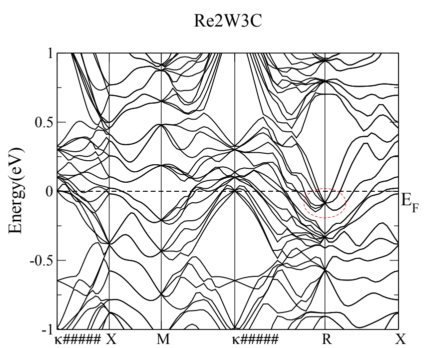
A 6-fold fermion occur above E_F at the P point *SG 206*



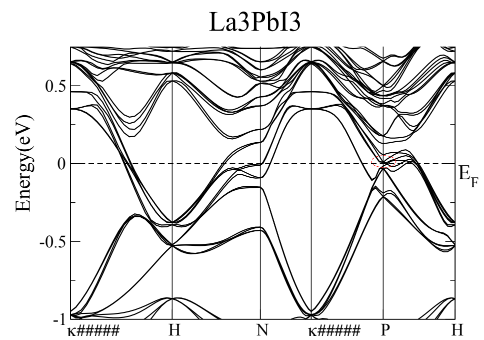


A 3-fold fermion can occur at the P point. *SG 199*





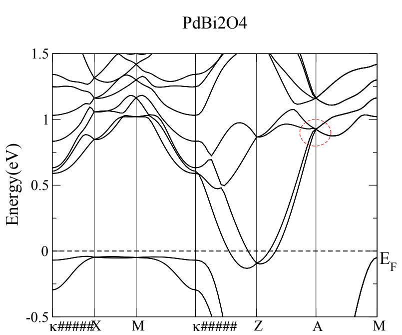
A 6-fold fermion can occur at the R point *SGs 212 and 213*

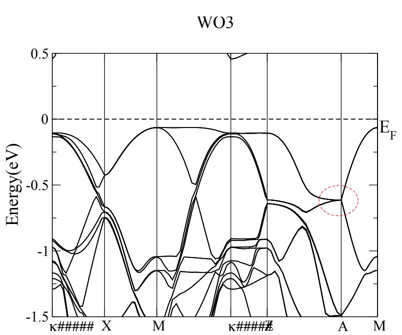


unique 3-fold fermion can occur at the P point *SG 214*

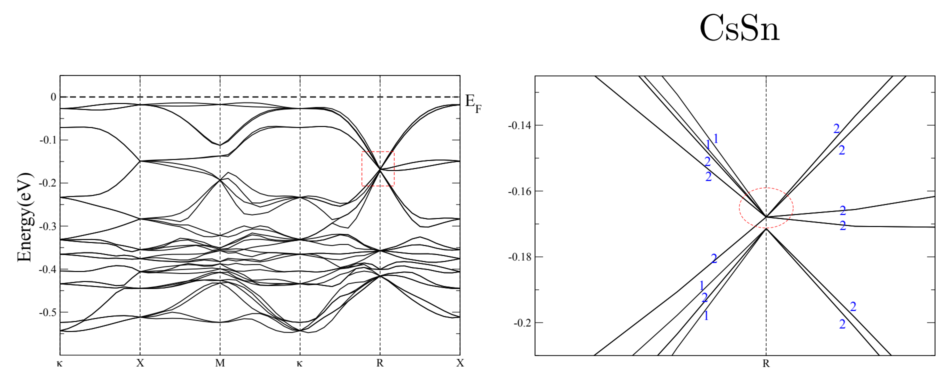
AsPdS,
K3BiTe3
M3(XS)2 where M= Ni,Pd, X= Pb, Bi
A2 B2 O3 (A=K, Rb, B= Ge, Sn, Pb)
PdSb2

FeS2
PtP2
KBiF6
Li2(Pd/Pt)3B
Re2(W/Mo)3C
La3PbI3

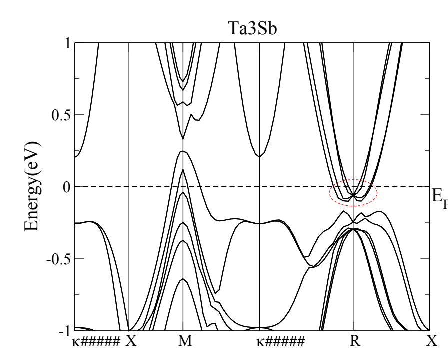


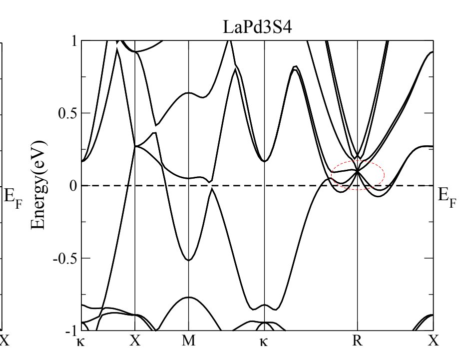


8-Fold Fermions at the A-point *SGs 130 and 135*

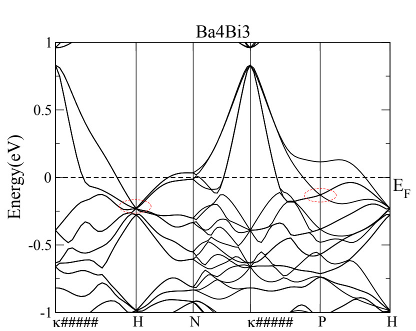


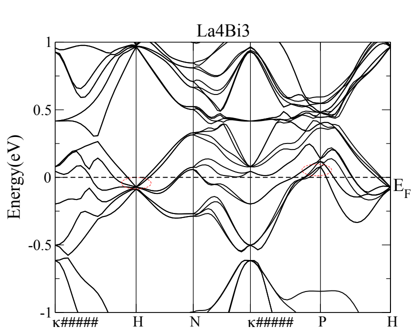
An 8-fold fermion can occur at the R point *SGs 218*





An 8-fold) fermion can occur at the R point. *SGs 222 and 223*





A 3-fold (resp. 8-fold) fermion can occur at the P (resp. H) point. *SG 220*

A4Pn3 where A= Ca, Sr, Ba, Eu and R= La, Ce and Pn = pnictogen (As, Sb, Bi)
MPd3S4 where M= rare earth (see La very close to the Fermi level)
X3Y where X=Nb,Ta and Y= A-IV; A-V (Sb for ex)
Th3P4, PdBi2O4, AuBi2O5, WO3, CsSn, CsSi

Cleaner Materials: M. Vergniory, C Felser, B. Cava Bradlyn, Cano, in preparation

Results

New Types of Fermions in Nature!

Large Degeneracy Fermions, but degeneracy point not their exotic only property

Degeneracies on surfaces

Transport, localization, superconductors, interaction behaviors when large degeneracies are involved

New Types of Fermi Arcs

New Types of Dirac Lines

Measuring Projective Representations - physical consequences