Two-Fluid Theory for Spin Superfluidity in Magnetic Insulators

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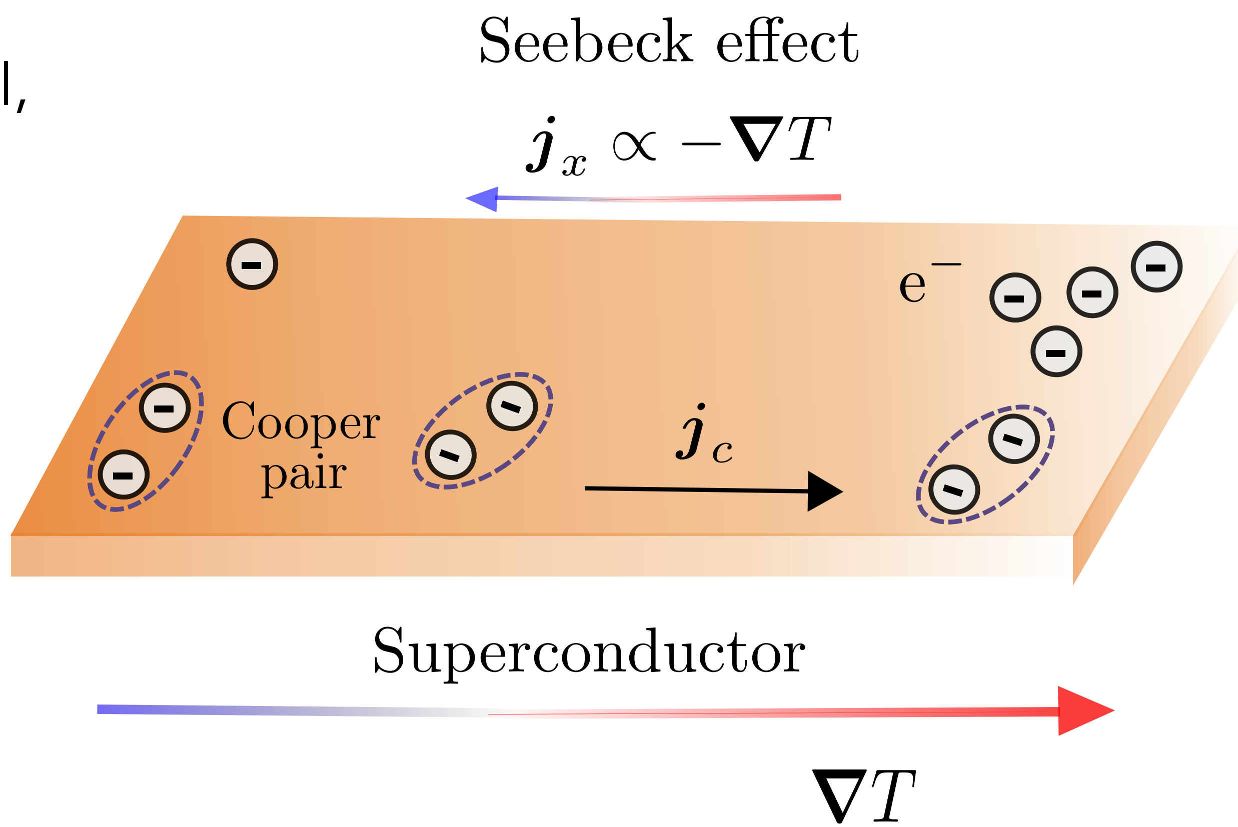
Two-Fluid Model

Below T_c system as mixture of a normal fluid, viscous and carrying entropy and a superfluid one that propagates collectively and carries no thermal energy.

Experimentally verified model, e.g. fountain effect in Helium and thermoelectric effects in superconductors.

Origin of superfluidity resides in BEC.

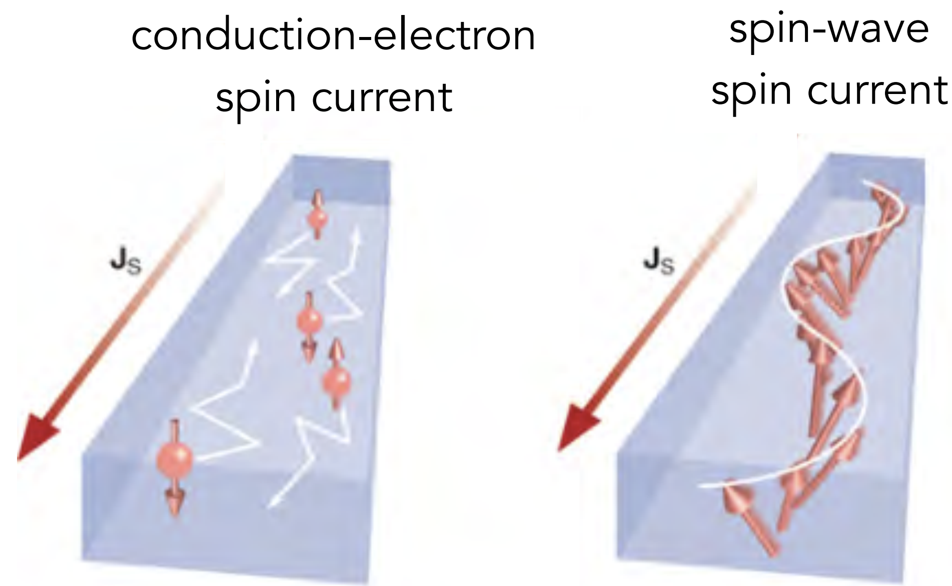


Magnetic insulators

Spin-wave currents persist over longer distances

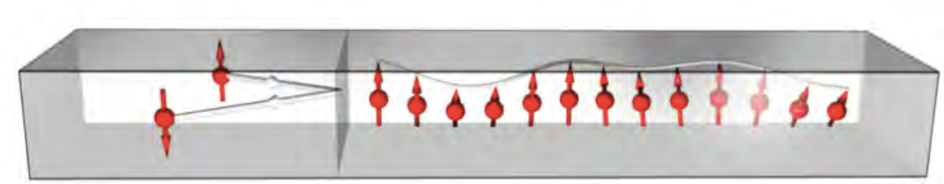
Free of Joule heating

Interconversion of magnetic and electric signal



K. Uchida *et al.*, J. Appl. Phys. **111**, 103903 (2012).

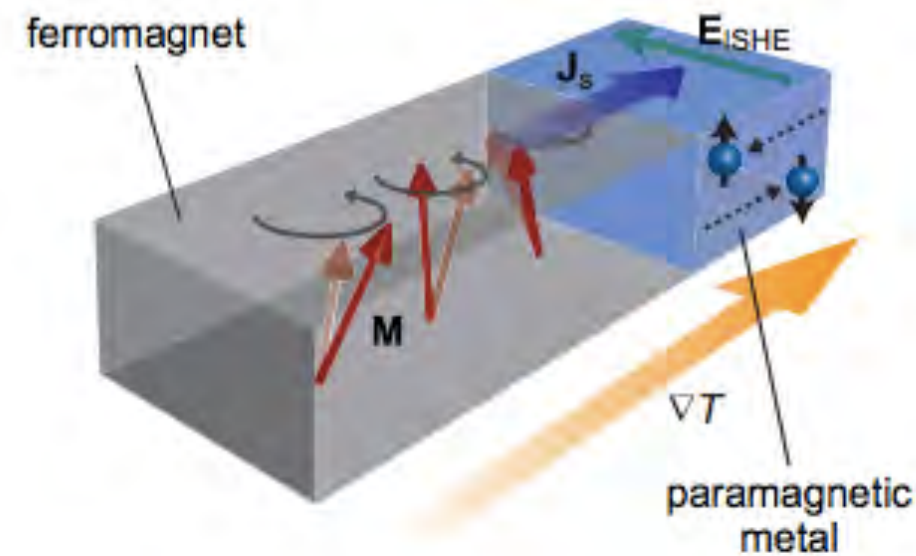
Macroscopic quantum phenomena



L. J. Cornelissen *et al.*, Nature Physics **11**, 1022–1026 (2015).

Spin Seebeck effect

The Spin Seebeck effect is the direct conversion of a temperature gradient into a spin current.



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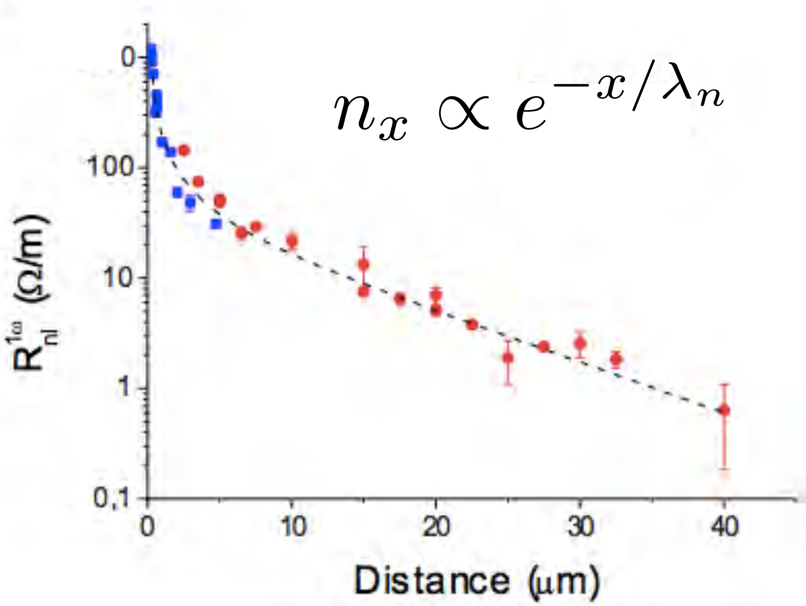
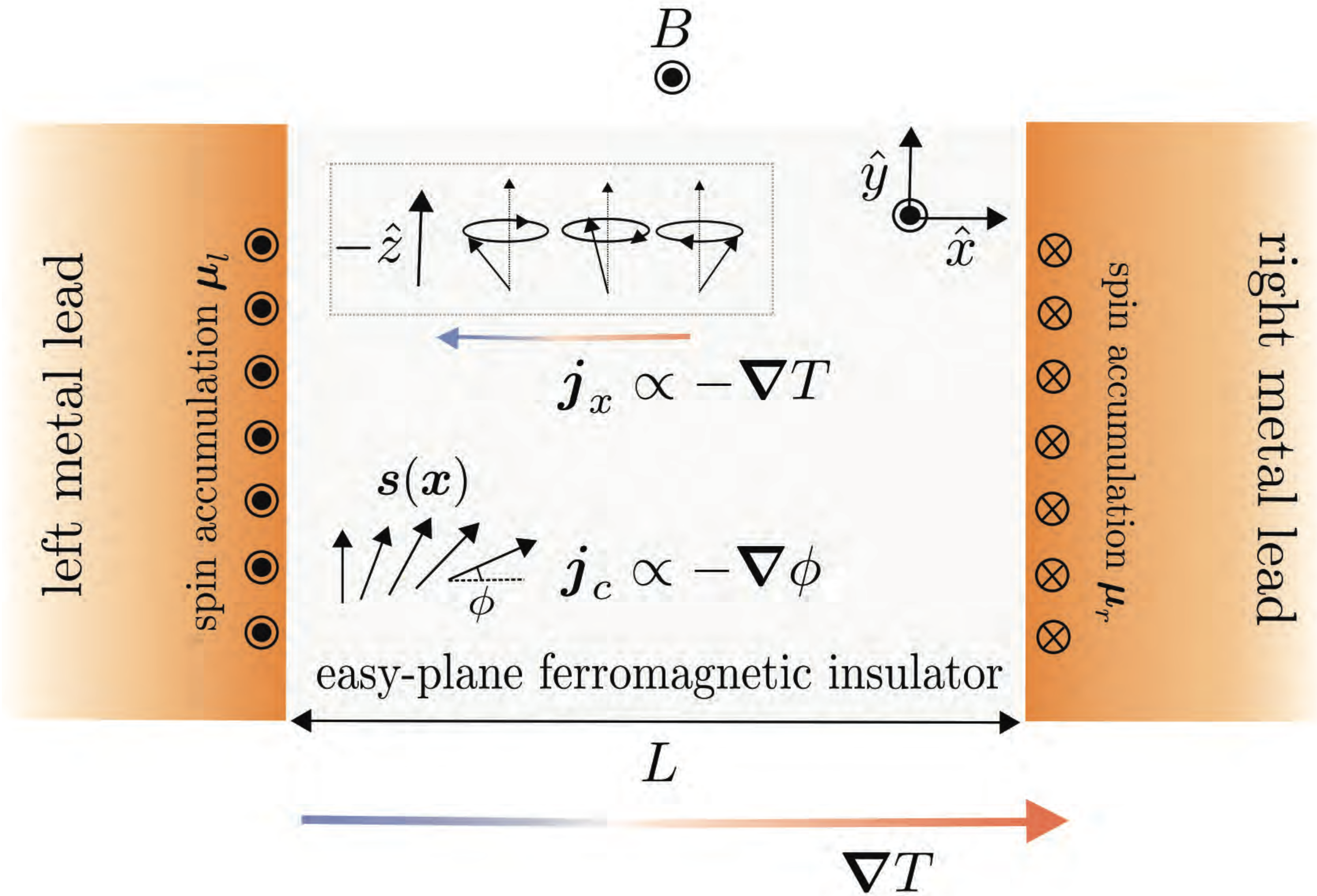
$$n_x \propto e^{-x/\lambda_n}$$



L. J. Cornelissen *et al.*, Nature Physics **11**, 1022–1026 (2015).

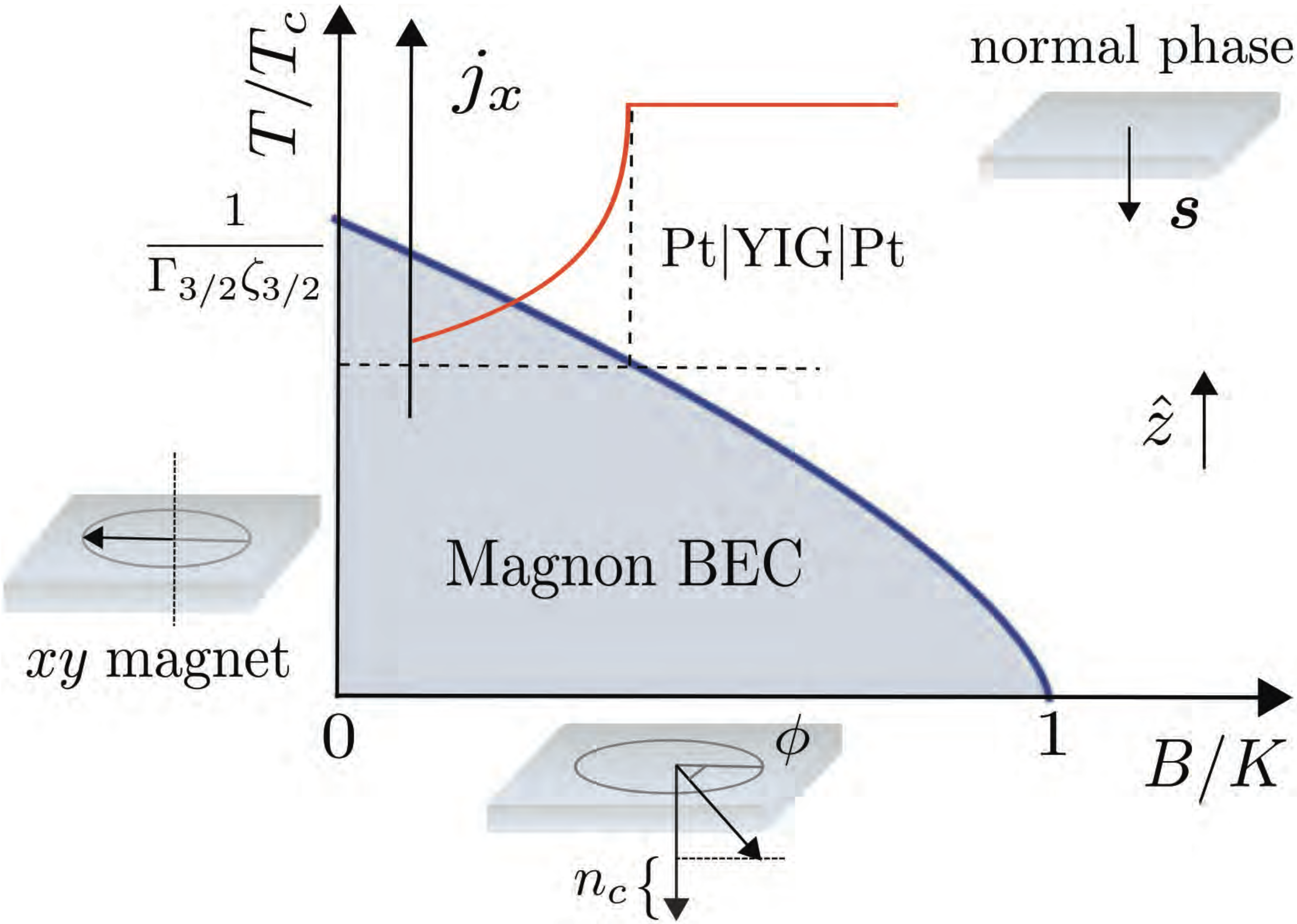
transport of out-of-equilibrium magnons characterized by the magnon diffusion length λ_n

Set up



B
left metal lead
spin accumulation $\boldsymbol{\mu}_l$
$-\hat{z}$
$\hat{y}$
$\hat{x}$
$\boldsymbol{j}_x \propto -\boldsymbol{\nabla} T$
$\boldsymbol{s}(\boldsymbol{x})$
ϕ
$\boldsymbol{j}_c \propto -\boldsymbol{\nabla} \phi$
spin accumulation $\boldsymbol{\mu}_r$
right metal lead
easy-plane ferromagnetic insulator
L
$\boldsymbol{\nabla} T$

Results



T/T_c
j_x
normal phase
$\boldsymbol{s}$
$\frac{1}{\Gamma_{3/2}\zeta_{3/2}}$
Pt|YIG|Pt
$\hat{z}$
Magnon BEC
xy magnet
0
ϕ
1
B/K
n_c

Spin superfluid

$$\mathcal{H} = \int d^3r \left(-\frac{A}{2s}\hat{\mathbf{s}} \cdot \nabla^2 \hat{\mathbf{s}} + B\hat{s}_z + \frac{K}{2s}\hat{s}_z^2 \right)$$

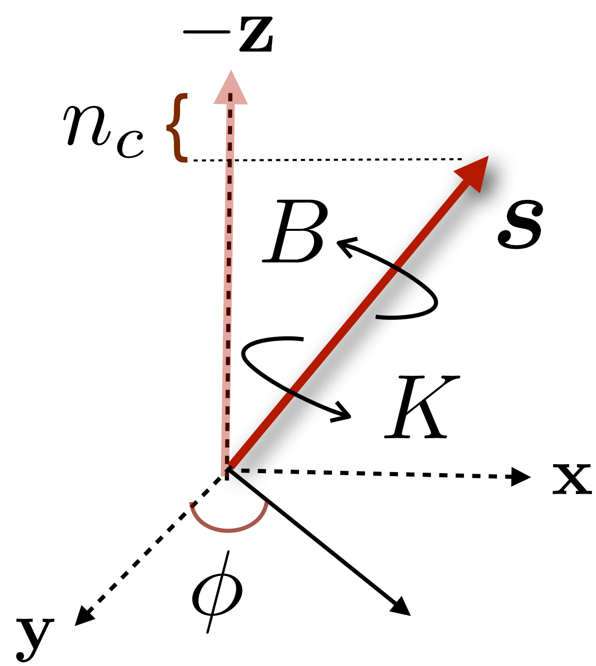
(Exchange) (Zeeman) (Easy-plane anisotropy)

$K > 0$

At zero temperature

$$\langle \hat{s}_z \rangle = -B/K = n_c - s$$

$$\langle \hat{s}_- \rangle = \langle \hat{s}_x - i\hat{s}_y \rangle = \sqrt{2sn_c}e^{-i\phi}$$



Holstein-Primakoff (HP) transformation

$$\hat{s}_z = \hat{\Psi}^\dagger \hat{\Psi} - s$$

$$\hat{s}_- \simeq \sqrt{2s}\hat{\Psi}$$

$\hat{\Psi}^\dagger, \hat{\Psi}$ creates/annihilates a magnon

$\langle \hat{\Psi}^\dagger \hat{\Psi} \rangle$ magnon density

At T=0 macroscopical occupation of the GS by bosonic quasi-particles

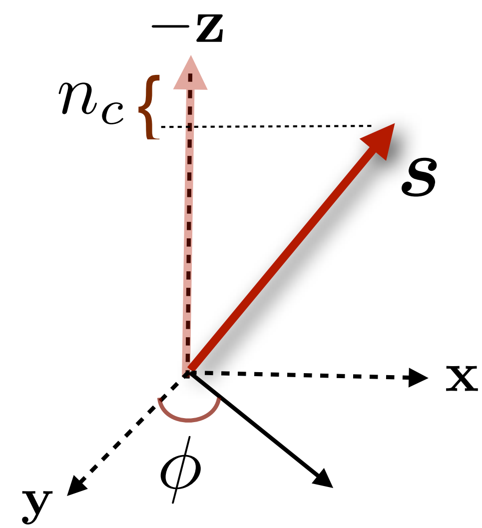
$\boxed{\textbf{Magnon BEC at equilibrium}}$

Gross-Pitaevskii theory at T=0

$$\hat{s}_z = \hat{\Phi}^\dagger \hat{\Phi} - s \to n_c - s \quad \text{and} \quad \hat{s}_- \approx \sqrt{2s}\hat{\Phi}$$

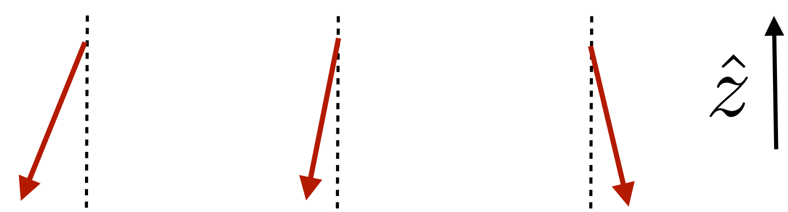
$$\hat{\Phi} = \langle \hat{\Phi} \rangle = \sqrt{n_c} e^{-i\phi}$$

Bose macroscopic wavefunction



$$\mathbf{v}_c = -i \frac{\hbar}{2m|\Phi|^2} (\Phi^* \boldsymbol{\nabla} \Phi - \Phi \boldsymbol{\nabla} \Phi^*) = -\frac{\hbar}{m} \boldsymbol{\nabla} \phi$$

Superfluid velocity



$$\mathbf{j}_c \propto -\boldsymbol{\nabla}\phi$$

$$m = \hbar^2/2A, \;\; \mathbf{j}_c = n_c \mathbf{v}_c$$

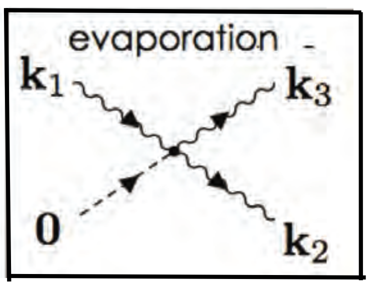
Superfluid dynamics at finite temperature

$$\mathcal{H} = \int d^3r \left(-\frac{A}{2s}\hat{\mathbf{s}} \cdot \nabla^2 \hat{\mathbf{s}} + B\hat{s}_z + \frac{K}{2s}\hat{s}_z^2 \right)$$

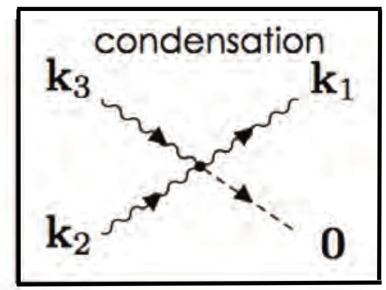
HP transformation

$$\longrightarrow \hat{s}_z = \hat{\Psi}^\dagger \hat{\Psi} - s \xrightarrow{\text{GP theory}} \hat{\Psi} = \Phi$$

+ coupling to the lattice $\propto \alpha$



+ coupling to the thermal cloud $\propto R$



Gross-Pitaevskii equation

$$(i - \alpha)\hbar\partial_t \Phi = (\hbar\Omega + Kn_c/s - iR)\,\Phi - A\nabla^2\Phi$$

$\hbar\Omega \equiv B - K(1 - 2n_x/s)$

Superfluid hydrodynamic equations

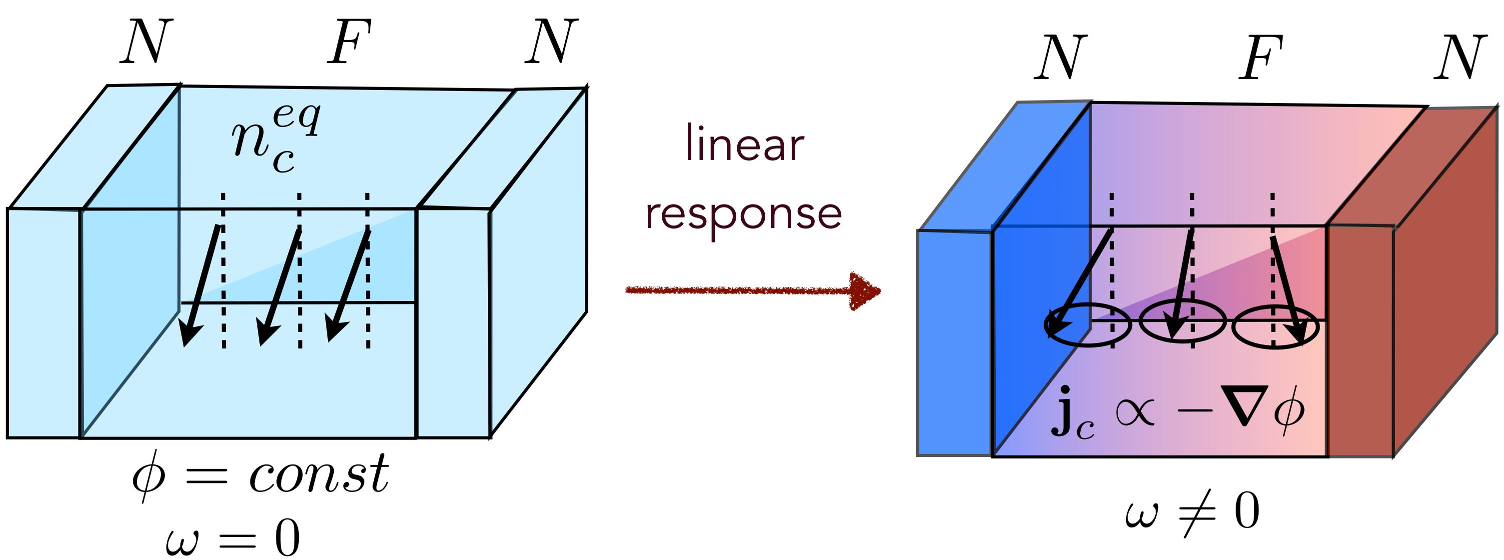
$$\Phi = \sqrt{n_c}e^{-i\phi}$$

$$\omega = \dot{\phi}$$

$$\mathbf{j}_c = \nabla(n_c\mathbf{v}_c)$$

$$\hbar\omega = \hbar\Omega + K\frac{n_c}{s} + A\left[(\boldsymbol{\nabla}\phi)^2 - \frac{\nabla^2\sqrt{n_c}}{\sqrt{n_c}}\right]$$

$$\dot{n}_c + \boldsymbol{\nabla}\cdot\mathbf{j}_c = -\Gamma_{cx} - 2\alpha\omega n_c$$



linear response variables:

$$\delta n_c = n_c - n_c^{eq},$$
$$\mathbf{v}_c,\ \omega,\ \mu,\ \delta T$$

Superfluid linearized hydrodynamic equations

$$\delta\dot{n}_c + n_c^{(eq)}\nabla\cdot\mathbf{v}_c = 2\eta(\mu/\hbar - \omega)n_c^{(eq)} - 2\alpha\omega n_c^{(eq)}$$

$$\hbar\omega = K\frac{\delta n_c + 2\delta n_x}{s} - A\frac{\nabla^2\delta n_c}{2n_c^{(eq)}}$$

Thermal cloud hydrodynamics

$\hat{\Psi} = \langle \hat{\Psi} \rangle + \hat{\phi}$

bosonic field

$\Psi = \langle \hat{\Psi} \rangle = \sqrt{n_c} e^{-i\phi}$

condensate component

$n_x = \langle \hat{\phi}^\dagger \phi \rangle$

thermal cloud

Kinetic theory for the thermal cloud

+ relaxation of magnon T to the phonon T_p $\propto g_{nT}, g_{uT}$

+ relaxation of μ into the lattice $\propto g_{n\mu}, g_{u\mu}$

+ interactions with the condensate $\propto \eta$

$$\delta \dot{n}_x + \nabla \cdot \mathbf{j}_x = 2\eta(\omega - \mu/\hbar) n_c^{(eq)} - g_{n\mu}\mu - g_{nT}(T - T_p)$$

$$\delta \dot{u} + \nabla \cdot \mathbf{j}_q = -g_{uT}(T - T_p) - g_{u\mu}\mu$$

Linear response currents

$$\begin{pmatrix} \mathbf{j}_x \\ \mathbf{j}_q \end{pmatrix} = - \overbrace{\begin{pmatrix} \sigma & \zeta \\ \rho & \kappa \end{pmatrix}}^{\text{bulk coefficients}} \begin{pmatrix} \nabla \mu \\ \nabla T \end{pmatrix}$$

General coupled two-fluid model

zero entropy, subjected to Gilbert damping

coherent magnetic dynamics

hydrodynamic equations for $n_c, \ \phi$

η

kinetic theory for thermal magnons

hydrodynamic equations for $n_x(\mu, T), \ u(\mu, T)$

carries heat, subjected to disorder and Gilbert damping

Boundary conditions

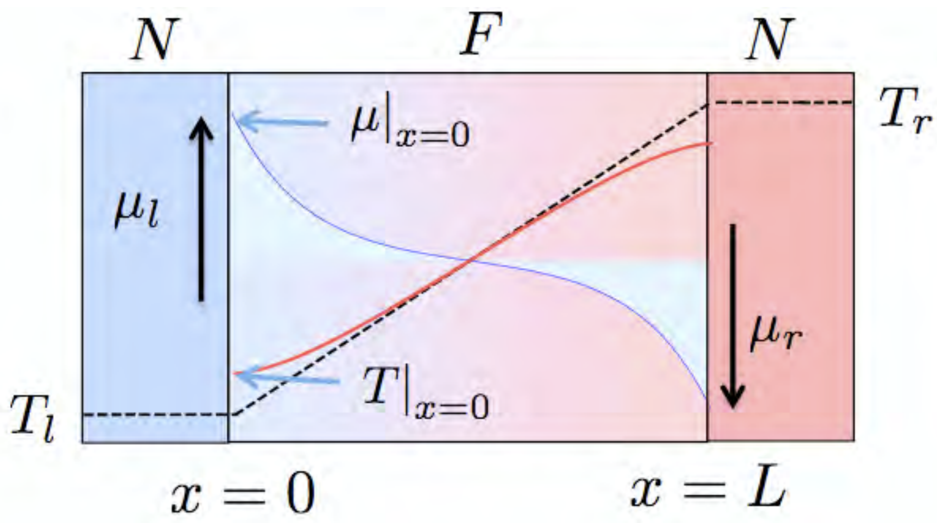
Superfluid component

$$\mathbf{j} = \frac{g_r^{\uparrow\downarrow}}{4\pi}\mathbf{n} \times (\mu_l \mathbf{z} \times \mathbf{n} - \hbar\dot{\mathbf{n}})|_{x=0}$$

Spin transfer torque

Spin pumping

$$j_c|_{x=0} = n_c g^{\uparrow\downarrow}(\mu_l - \hbar\omega)/2\pi\hbar s$$



Normal component

$$j_x|_{x=0} = G(\mu_l - \mu)|_{x=0} + S(T_l - T)|_{x=0},$$

$$j_q|_{x=0} = K(T_l - T)|_{x=0} + \Pi(\mu_l - \mu)|_{x=0}$$

Separation of length scales

$$\lambda_u \equiv \sqrt{k/g_{uT}}$$

magnon-phonon
scattering length

$$\lambda_n \equiv \sqrt{\sigma/g_{n\mu}}$$

thermal magnon
diffusion length

$$\lambda_{cx} = \sqrt{\hbar\sigma/2\eta n_c}$$

condensate-cloud
equilibration length

fast inelastic spin-preserving
magnon-phonon scattering $\longrightarrow$ $\lambda_u \ll \lambda_n, \lambda_{cx}$ $\longrightarrow$ DECOUPLED
spin transport
and heat dynamics

$$T \to T_p$$

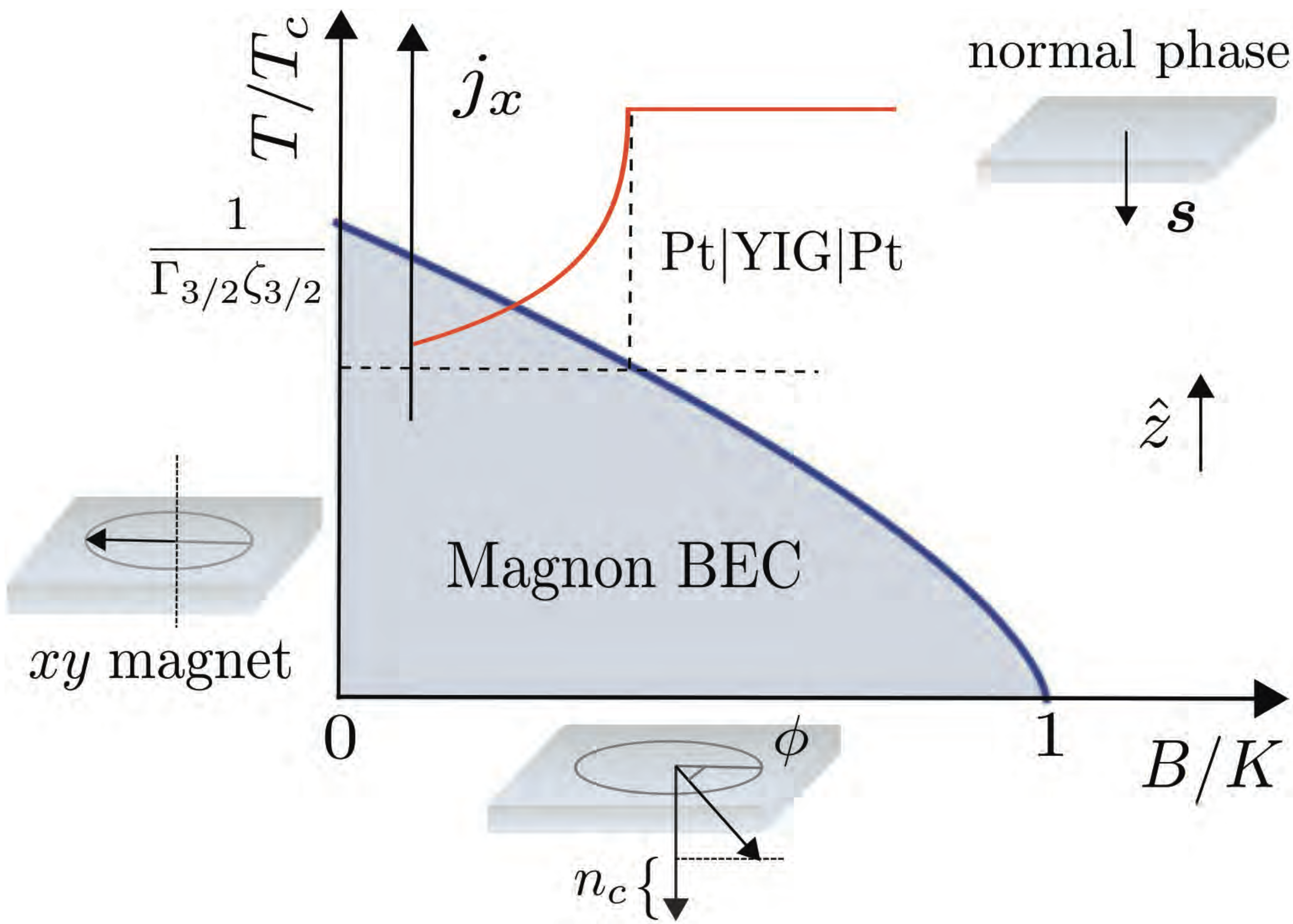
Solving hydrodynamic equations:

$$\boxed{\mu \sim e^{\pm x/\lambda_m}, \quad \lambda_m^{-2} = \lambda_n^{-2} + \lambda_{cx}^{-2}}$$

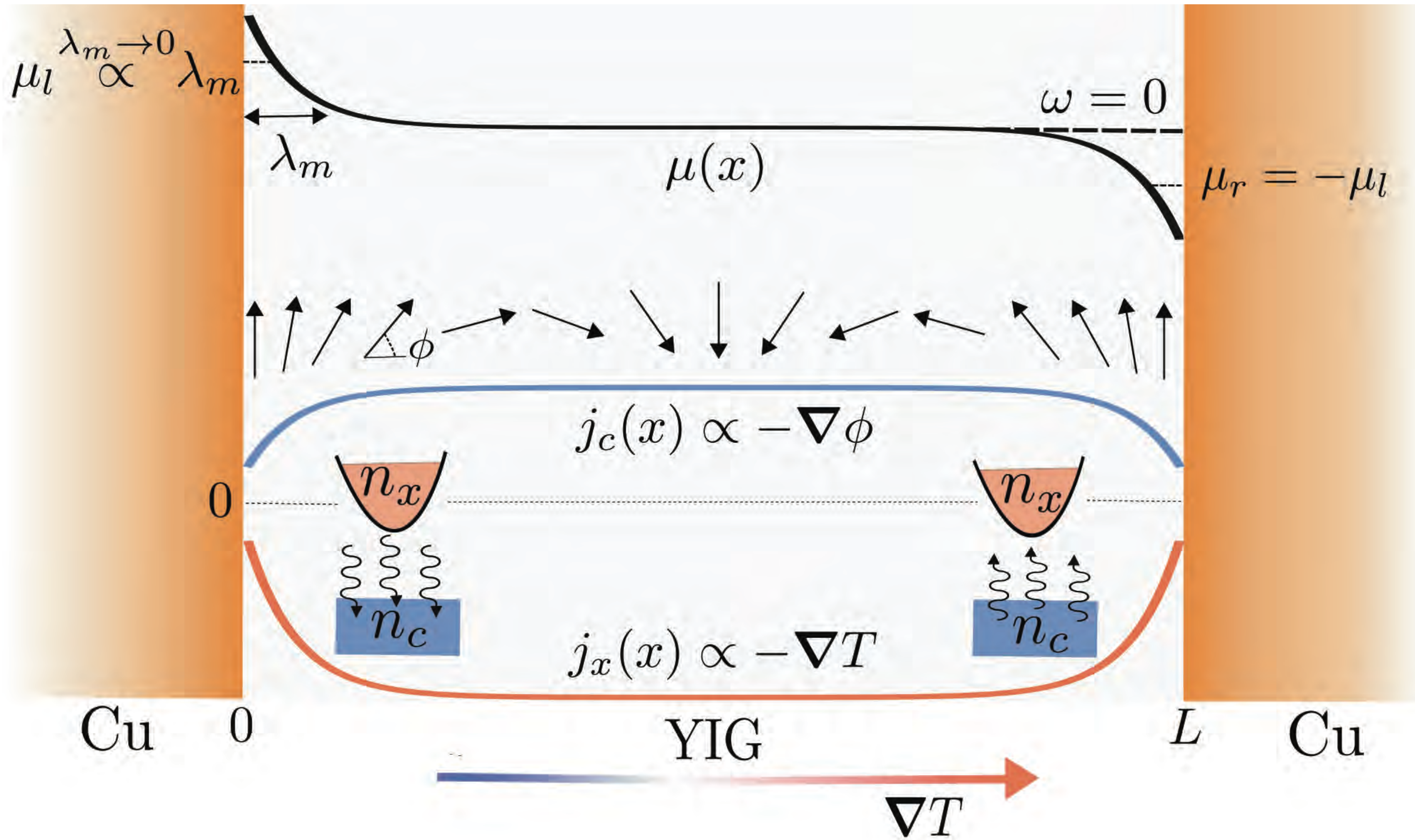
Results

$$j_x \propto \frac{1}{1+\sqrt{1+(\lambda_n/\lambda_{cx})^2}}$$

$$n_c/s \propto 1 - B/K$$
$$\lambda_{cx} \propto 1/\sqrt{n_c}$$



Conveyer-belt physics



Conclusions and outlooks

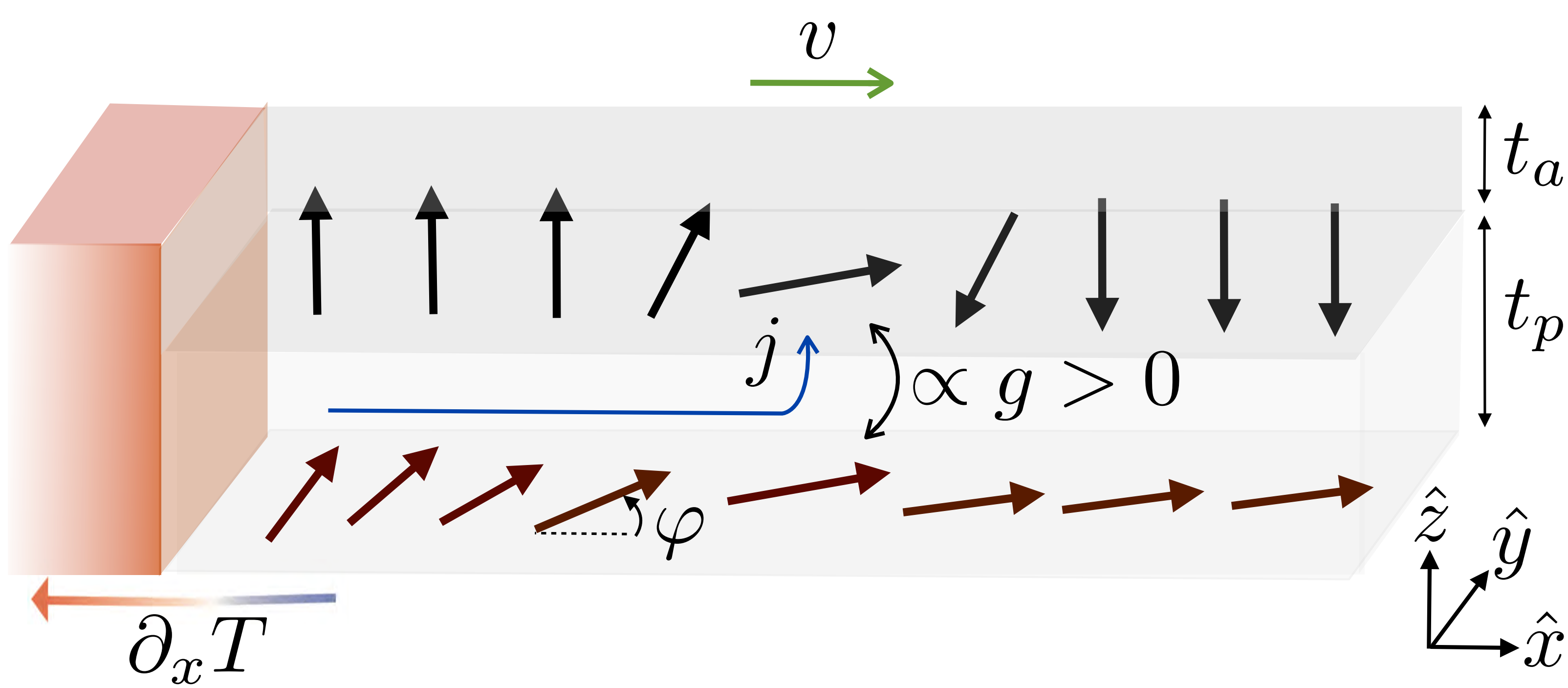
We built a hydrodynamic theory which describes the interactions between thermal and condensed magnons in an easy-plane magnetic insulator in the presence of a thermal gradient

We predicted that spin superfluidity can be induced by sweeping the external magnetic field and experimentally probed via spin Seebeck effect.

Future works should more systematically address the magnon-phonon relaxation mechanisms and study the role of magnons in the net heat transport

Nonlinear response still needs to be addressed.

What for?



Thanks for your attention!