SPICE - Quantum Spintronics

Are Topological Materials Useful in Spintronics ?

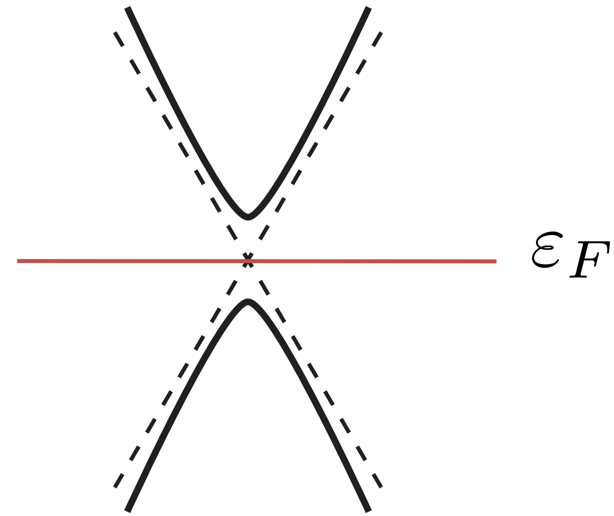
Allan MacDonald
University of Texas at Austin

Classes of Spintronic Materials

- Ferromagnetic Metals
- Light Paramagnetic Metals
- Antiferromagnetic Metals
- Antiferromagnetic Insulators
- Ferromagnetic Insulators
- Heavy Paramagnetic Metals
- ….
- Topological Insulators ?

TIs have extreme SO-coupling

$$H = v\boldsymbol{\tau}\cdot\boldsymbol{k} + \Delta\tau_z$$

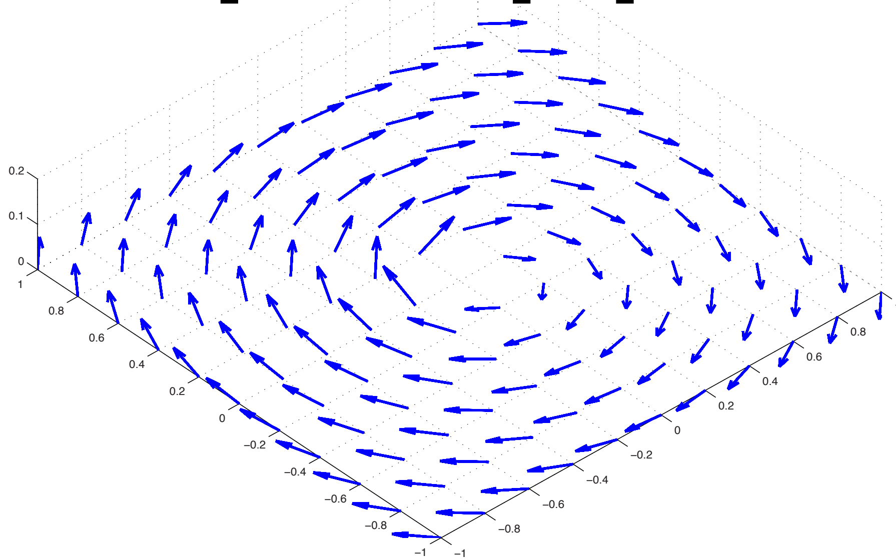


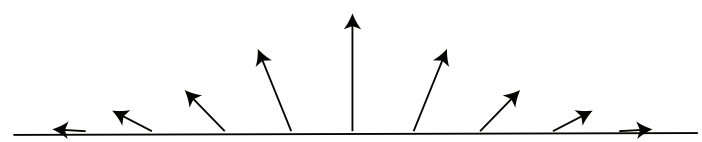
Eigenstates and eigenenergies:

$$\boldsymbol{d} = (vk\cos\phi_k, vk\sin\phi_k, \Delta)$$

$$|\pm\rangle = \begin{bmatrix}\cos\theta_k \\ \sin\theta_k\, e^{i\phi_k}\end{bmatrix}, \begin{bmatrix}\sin\theta_k \\ -\cos\theta_k\, e^{i\phi_k}\end{bmatrix}$$

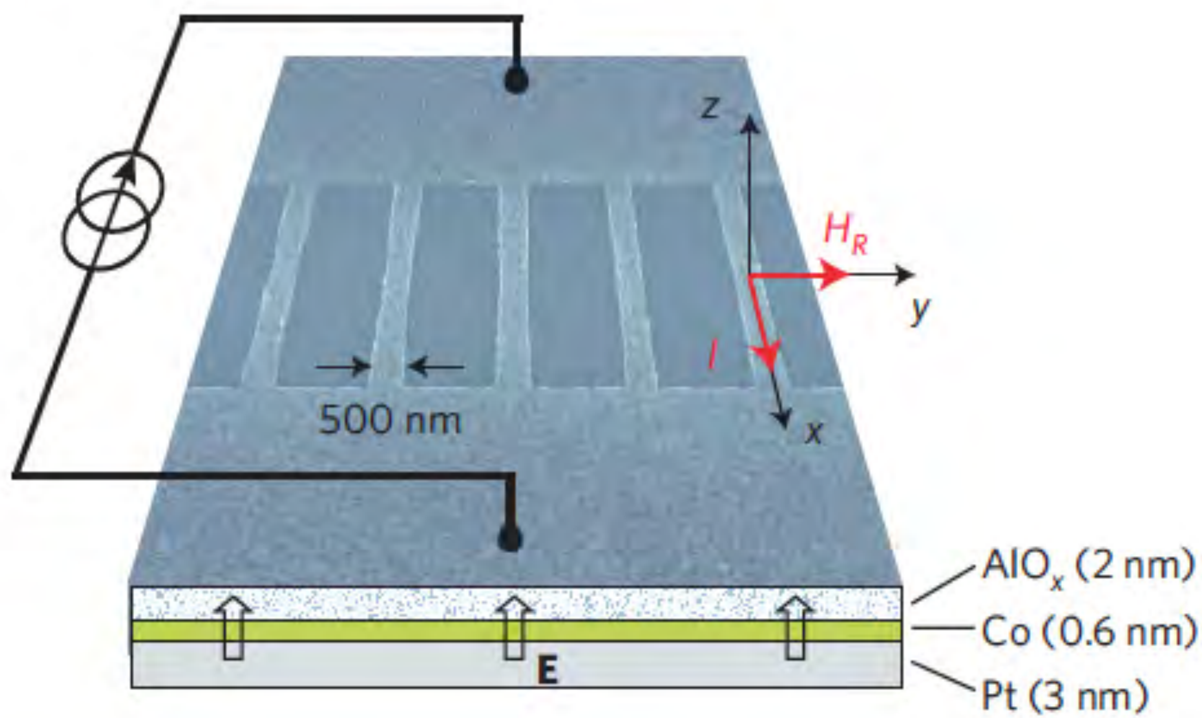
$$\varepsilon = \pm\sqrt{(vk)^2 + \Delta^2}$$





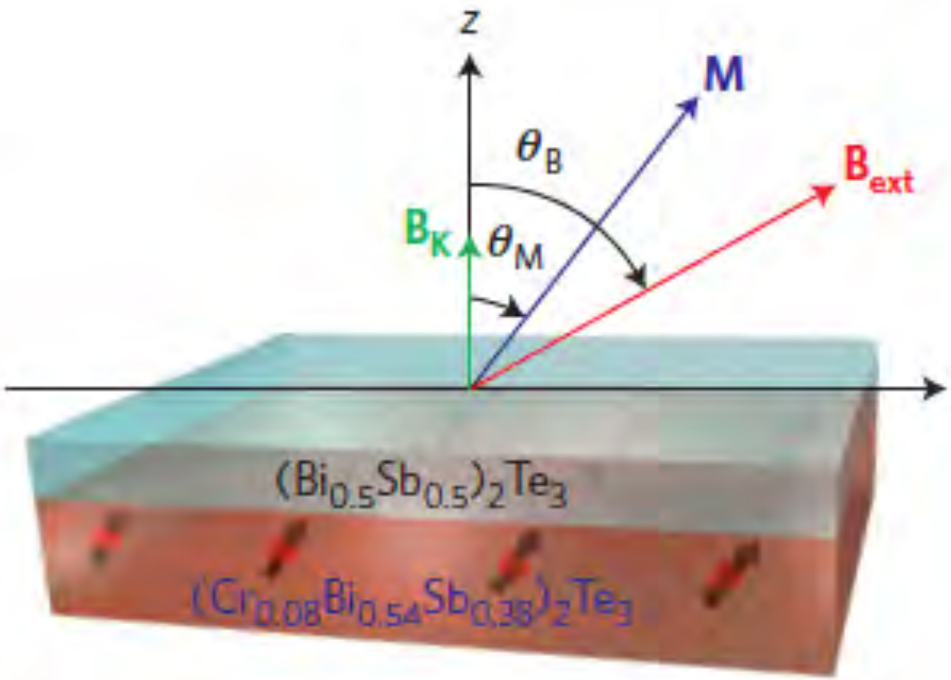
momentum space merons
- Half Skyrmions

Spin-Orbit Torques



Miron *et al.* Nature Mat. 9 (2010) Nature 476 (2011)
Liu *et al.* PRL 109 (2012) Science 336 (2012)

Spin-Orbit Torques in TI DMSs

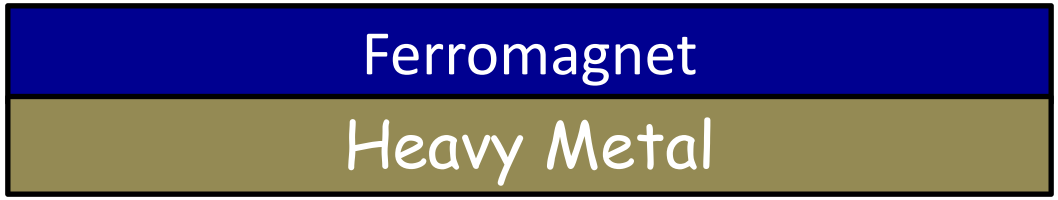


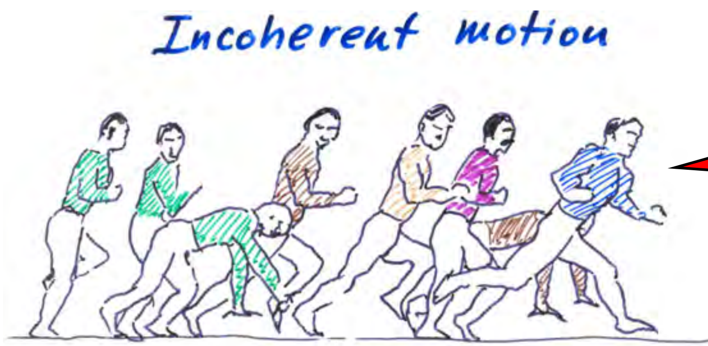
Colossal

$\theta_{SHE} = (I_s/I)\,(e/\hbar)$

Fan et al., Nature Mat. (2014)
Ndiaye et al. arXiv:1509.06929
Franz and Garate, PRL (2010)

Emergent Collective DOF





Fermi Liquid Theory

LLG Equation

Courtesy Catherine Kallin

Landau-Liftshitz Gilbert Equation

Magnetization Direction

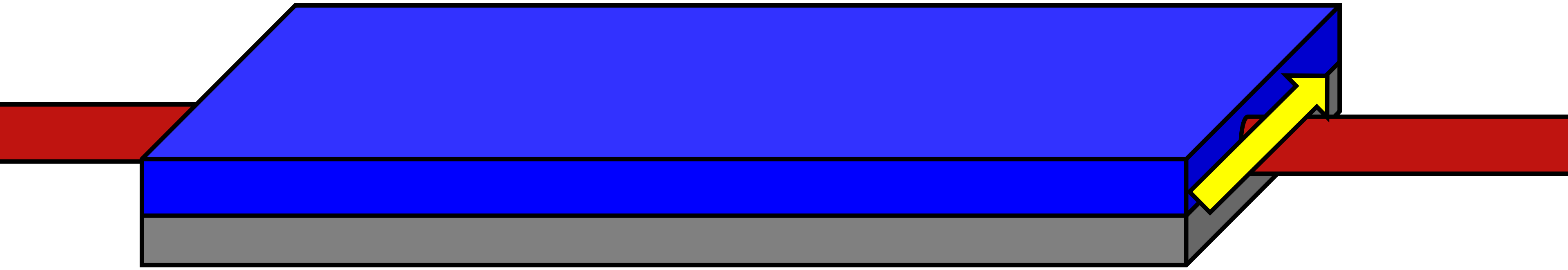
$$\frac{\partial \hat{m}}{\partial t} = \hat{m} \times \left(-\frac{\delta E[\hat{m}]}{\hbar \delta \hat{m}} \right) - \alpha \hat{m} \times \frac{\partial \hat{m}}{\partial t}$$

Anisotropy
External Fields
Exchange

Damping

SOT 100

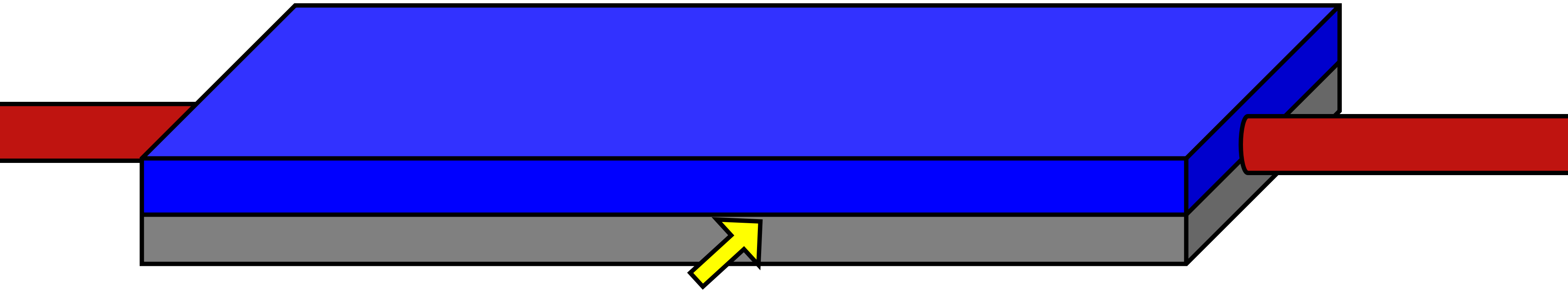
(Current Parallel to Magnetization)



i) Rashba Interactions and Spin Momentum Locking

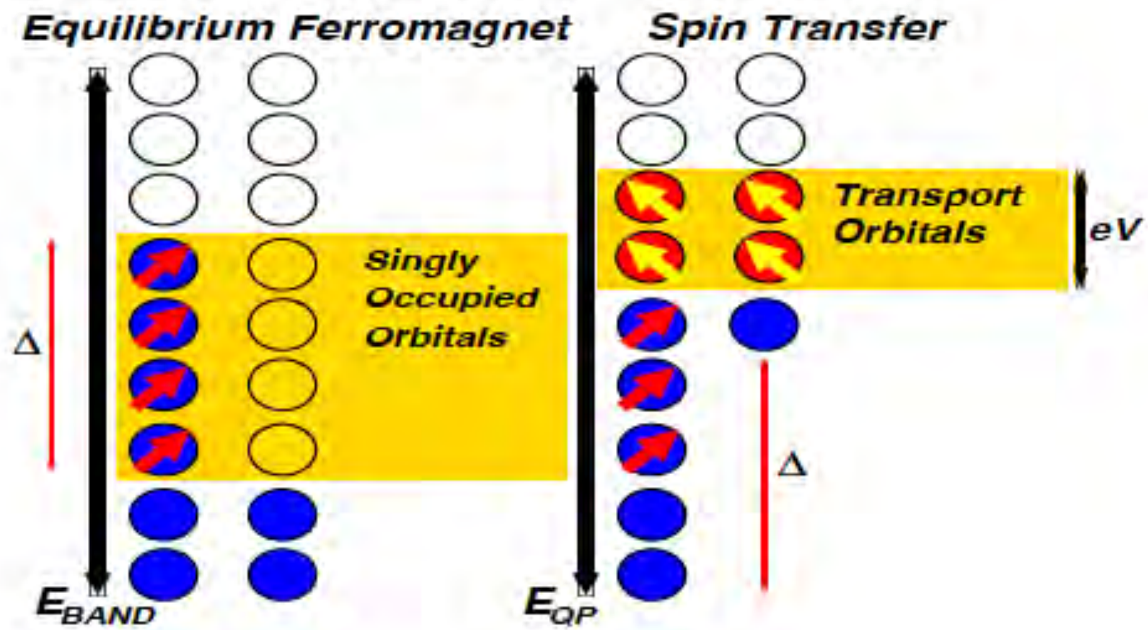
SOT 100

(Current Parallel to Magnetization)



ii) Spin Hall Effect

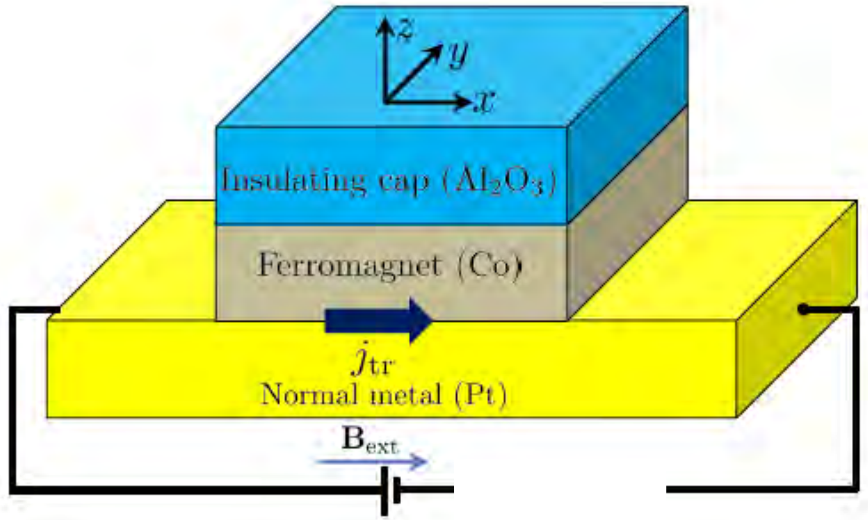
Spin-Torques Beyond Spin-Transfer



Alvaro Nunez – Ph.D. Thesis (2004)
SSC (2006)

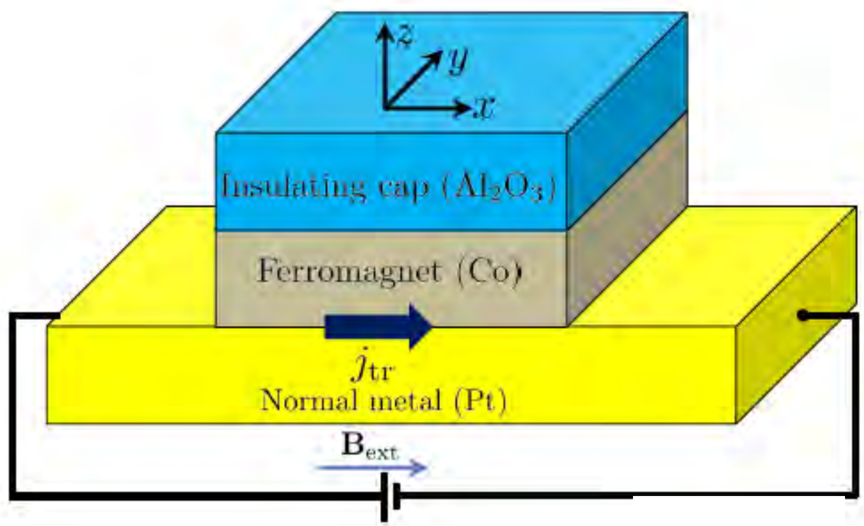
Rashba Torque

$M_y \approx p_x z \,.\, p_x$ = allowed



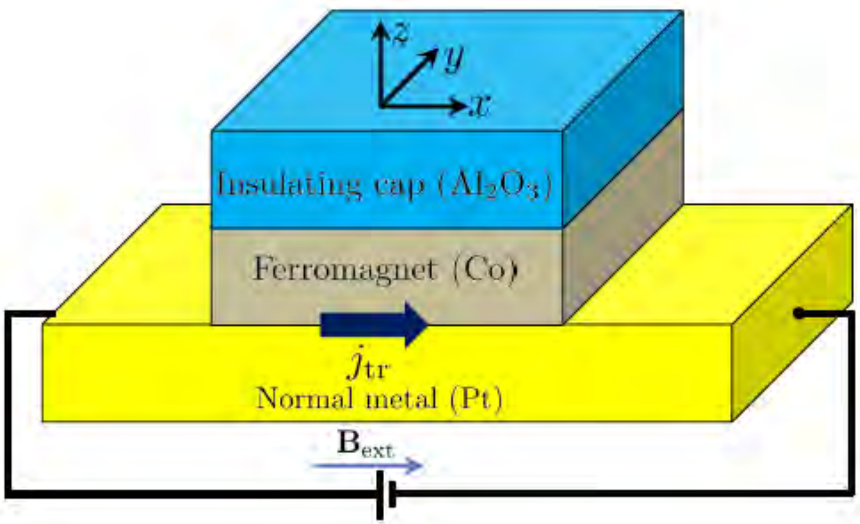
Spin Hall Torque

$M_z \approx p_y z \; . \; p_x$ = not allowed



Spin Hall Torque

$M_y \approx p_y z \;.\; (p_y x) \;.\; p_x$ = allowed



Bulk Transport Theory
Relaxation Time Approx

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[H, \rho] + \frac{1}{\hbar}\frac{\partial \rho}{\partial \mathbf{k}} \cdot e\mathbf{E} - \frac{\rho - \rho_0}{\tau}$$

Relaxation Time Approximation

Driving Term

Relaxation Time Approximation

Bulk Transport Theory

$$\frac{\partial \rho^{(0)}}{\partial \mathrm{k}} = \sum_m \left\{ \frac{\partial f_{m\mathrm{k}}}{\partial \mathrm{k}} |mk\rangle\langle mk| + \right.$$

$$\left. + f_{m\mathrm{k}} \left| \frac{\partial}{\partial \mathrm{k}} mk \right\rangle \langle mk| + f_{m\mathrm{k}} |mk\rangle \left\langle \frac{\partial}{\partial \mathrm{k}} mk \right| \right\}$$

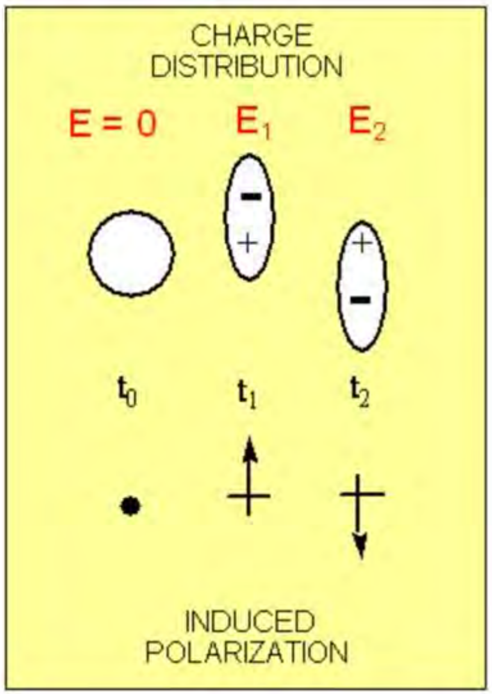
$$[H, \rho]_{nn'} = (\varepsilon_{nk} - \varepsilon_{n'k})\, \rho_{nn'}$$

Off diagonal response is intrinsic when bands well defined

$$\left| \frac{\partial}{\partial \mathrm{k}} nk \right\rangle = \sum_{m \neq n} \left(\frac{\langle mk | \frac{\partial H}{\partial \mathrm{k}} | nk \rangle}{\varepsilon_{nk} - \varepsilon_{mk}} \right) |mk\rangle$$

Response of Atom to Static Electric Field

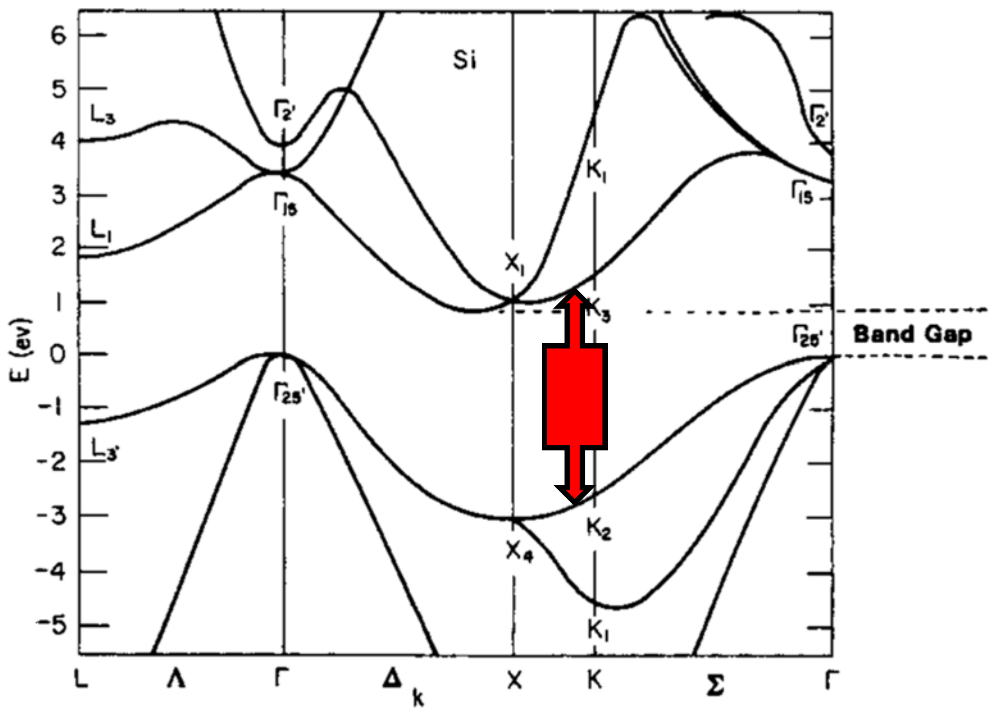
F = qE



$$\Psi_n^{(1)} = \sum_{k \neq n} \Psi_k^{(0)} \frac{V_{kn}}{E_n^{(0)} - E_k^{(0)}}$$

$$\alpha = e^2 \sum_{k \neq n} \frac{\mathbf{r}_{nk}\mathbf{r}_{kn} + \mathbf{r}_{kn}\mathbf{r}_{nk}}{E_k^{(0)} - E_n^{(0)}}$$

Response of Insulator to static Electric Field



Interband Transitions

$$\rho^{(1)}_{n'n}(\vec{k}) = ieE\,\frac{f_{n',\vec{k}} - f_{n,\vec{k}}}{(E_{n',\vec{k}} - E_{n,\vec{k}})^2}\,\langle \Psi_{n',\vec{k}}|\frac{\partial H}{\partial k_x}|\Psi_{n,\vec{k}}\rangle$$

Response of Metal to static Electric Field

$$\rho^{(1)}_{n,n}(\vec{k}) = f_{n,\vec{k}+\vec{\delta}} - f_{n,\vec{k}}$$

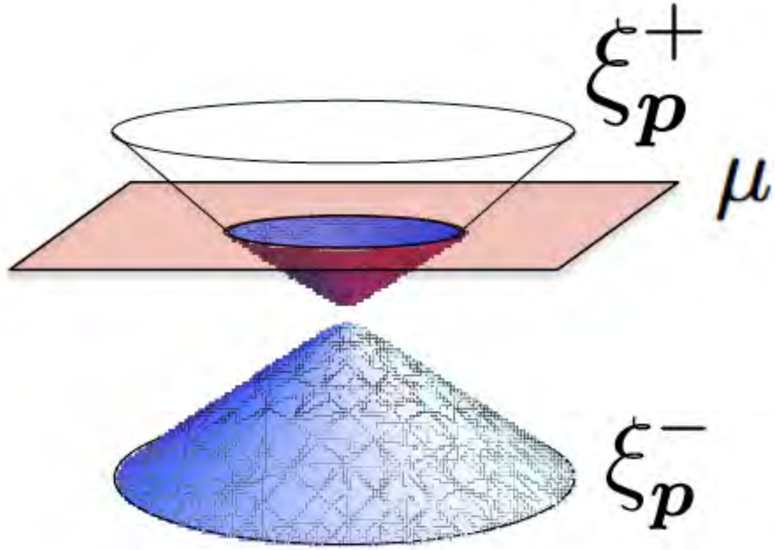
$$\frac{\delta}{K} \sim \frac{eE\tau_{tr}}{K\hbar} \sim \frac{eEa}{\hbar/\tau_{tr}}$$

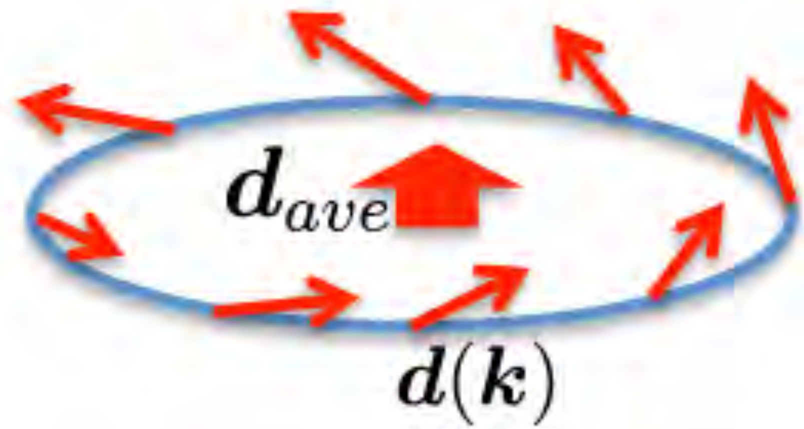
$E = \rho \times j \approx 10^{-4}\ \Omega\text{cm} \times 10^{6}\ \text{A/cm}^2$

$eE = \rho \times j \approx 100\ \text{eV/cm}$

$$eEa \ll h/\tau_{tr} \ll W$$

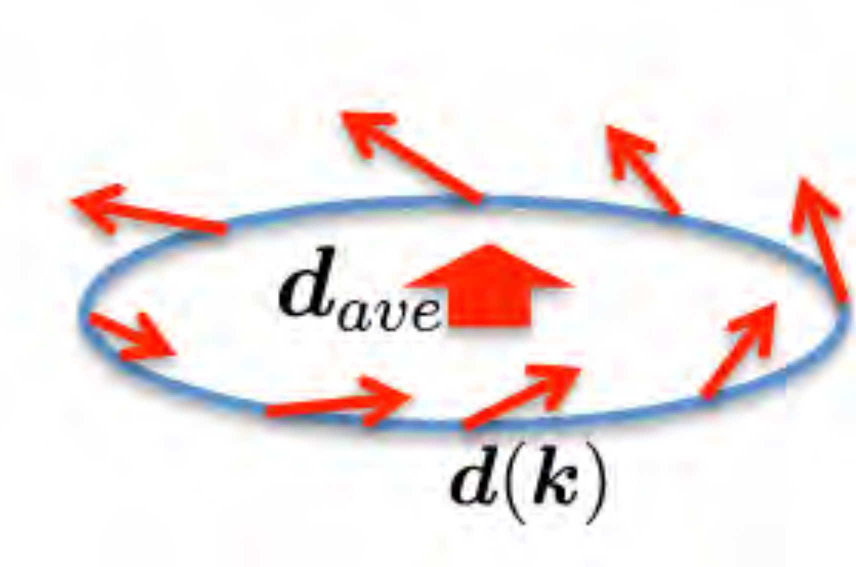
Topological Insulators
A Model Spin-Orbit-Torque System

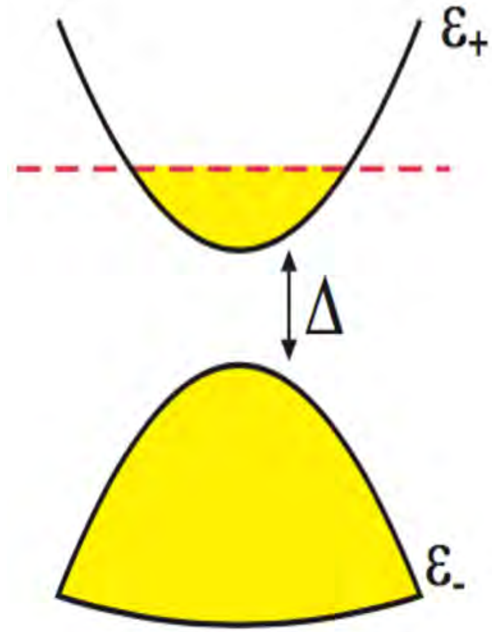




$$\hat{\mathcal{H}} = v(-p_x\hat{\sigma}_2 + p_y\hat{\sigma}_1) + \Delta\hat{\sigma}_3$$

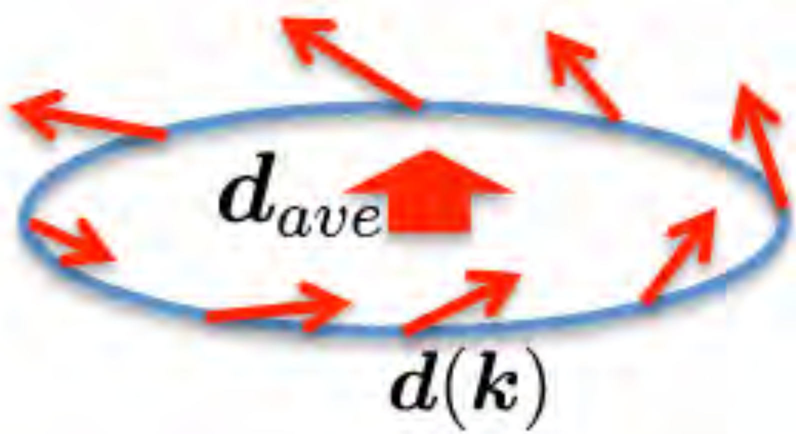
Theory-free Rashba SO Torque Coefficient

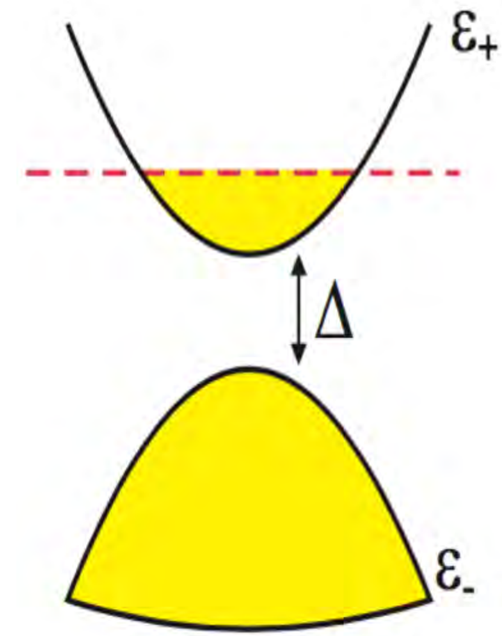




$$\tau_R/(I/e) \approx (WLs_{tr} \times \Delta/\hbar)/(Wv_D s_{tr})$$
$$= \Delta/(\hbar v_D/L)$$

Massive Dirac Model for Current-Induced Spin Density





$$2v_D \delta \mathbf{s_k} \times \mathbf{b_k} + \frac{\delta \mathbf{s_k}}{\tau} = -\mathbf{F_k}$$

$$\mathbf{F_k} = \left(\sum_n \langle n, \mathbf{k} | \mathbf{s} | n, \mathbf{k} \rangle \frac{\partial f_{n,\mathbf{k}}}{\partial \mathbf{k}} + \sum_n \sum_{m \neq n} \langle n, \mathbf{k} | \mathbf{s} | m, \mathbf{k} \rangle \left(\frac{f_{n,\mathbf{k}} - f_{m,\mathbf{k}}}{\varepsilon_{n,\mathbf{k}} - \varepsilon_{m,\mathbf{k}}} \right) \langle m, \mathbf{k} | \frac{\partial H}{\partial \mathbf{k}} | n, \mathbf{k} \rangle \right) \cdot \frac{e\mathbf{E}}{\hbar}$$

Magneto-resistance of Massive Dirac Model

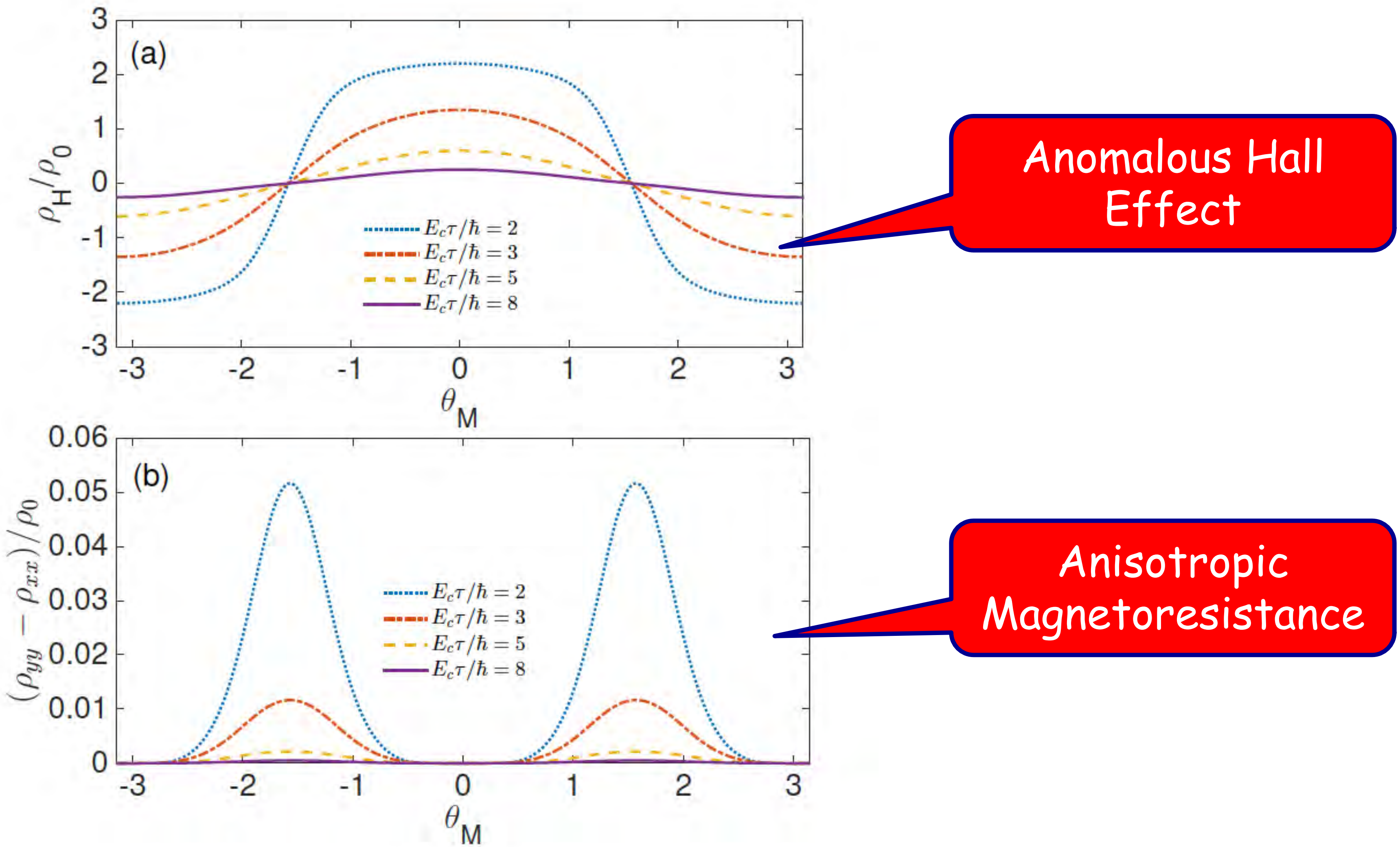
Anomalous Hall Effect

$$\rho = \begin{bmatrix} \rho_\perp \cos^2\varphi_M + \rho_{||}\sin^2\varphi_M & \rho_P \cos\varphi_M \sin\varphi_M + \rho_H \\ \rho_P \cos\varphi \sin\varphi_M - \rho_H & \rho_\perp \sin^2\varphi_M + \rho_{||}\cos^2\varphi_M \end{bmatrix}$$

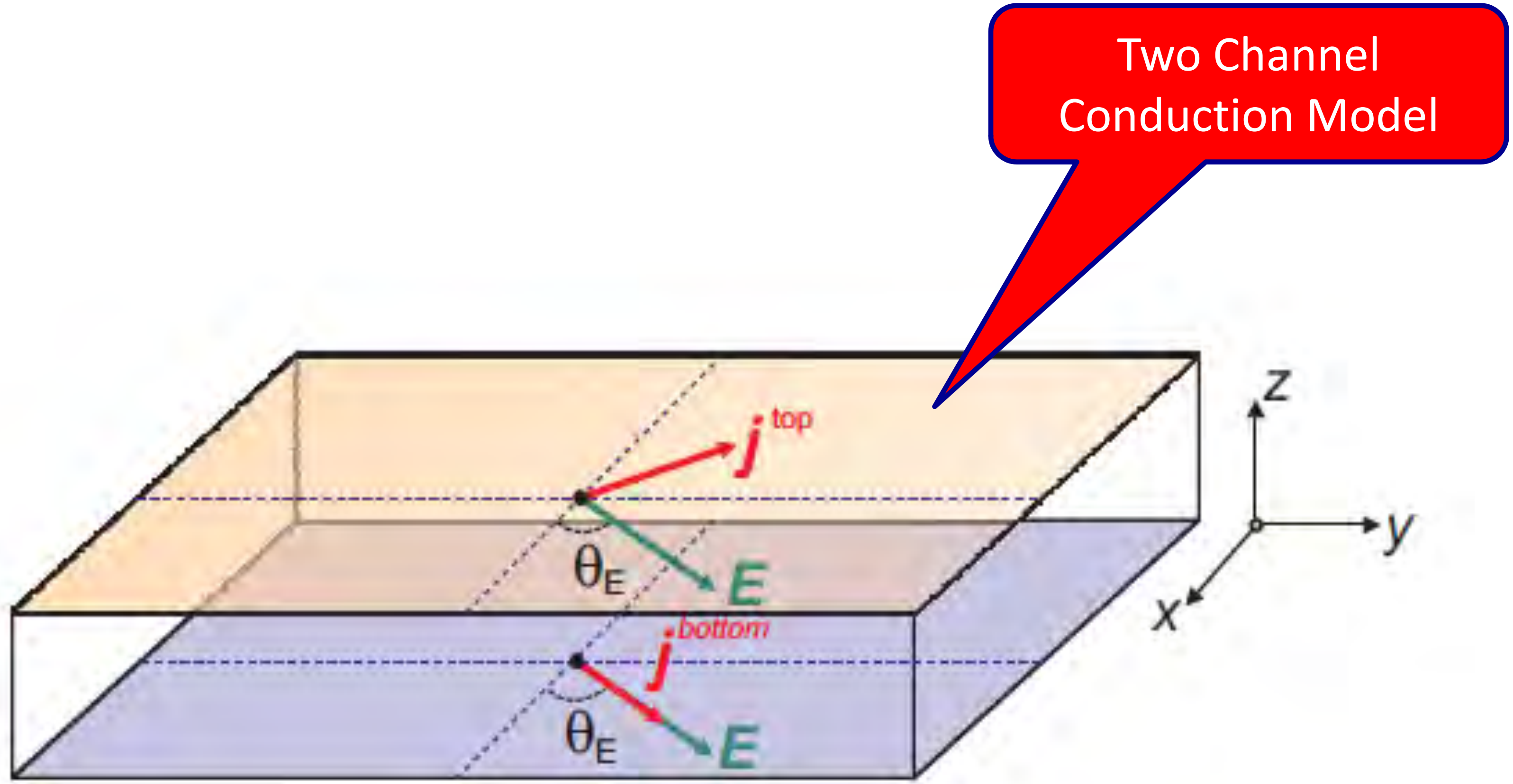
Anisotropic Magneto-resistance

$$\rho_P = \rho_{||} - \rho_\perp$$

Magneto-resistance of Massive Dirac Model



Resistively Detected SOT



Electric Field Induced Spin Densities

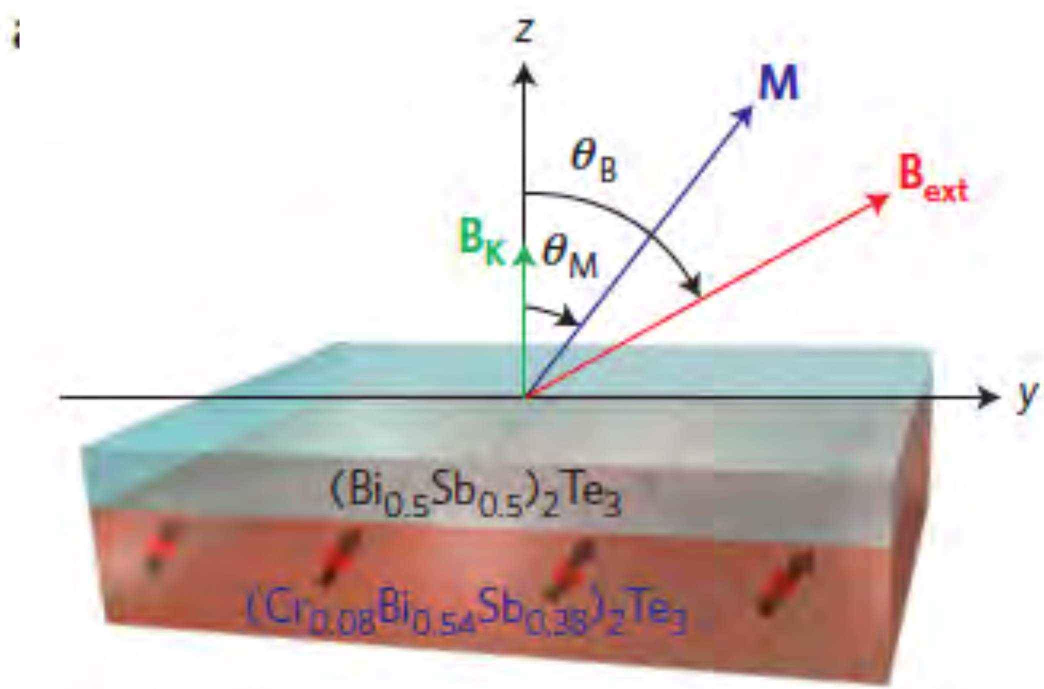
$$S_y/n \approx (e E a)/(\hbar/\tau) \times (1/k_F a)$$

Rashba

$$S_z/n \approx (e E a)/(E_g) \times (1/k_F a) \times (\Delta/E_F)$$

` Spin Hall '

Spin-Orbit Torques in TI DMSs



Experiment:

$\Delta\,(s^z_{tr}/S) \approx 10^{-6}$ eV

$\gg \Delta\,(s^y_{tr}/S)$

Fan et al., Nature Mat. (2014)
Ndiaye et al. arXiv:1509.06929
Franz and Garate, PRL (2010)

Numbers

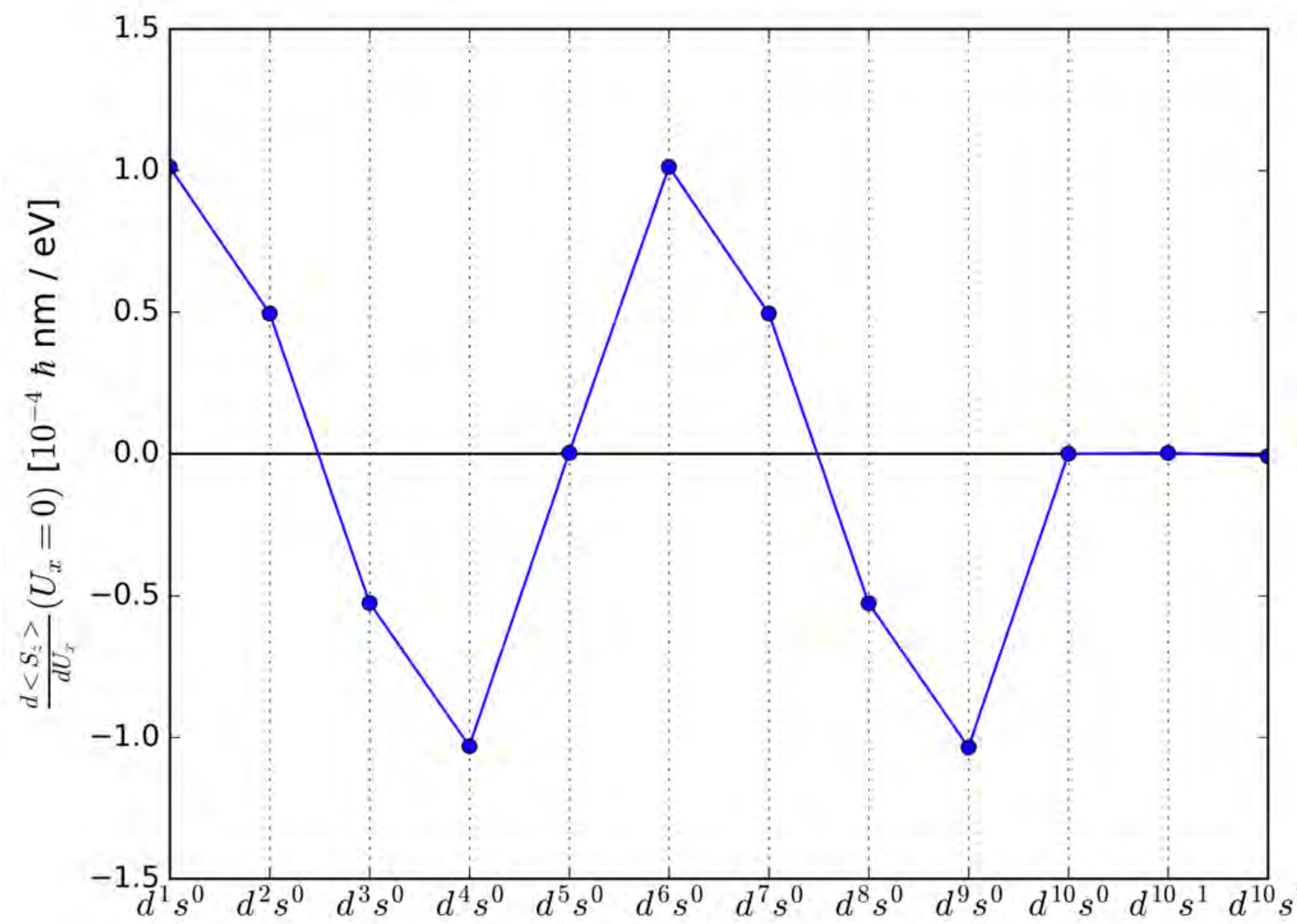
- $k_c = 10^7\ cm^{-1}$
- $10\ k\Omega$
- $eE = 10\ eV/cm$
- $B_{so} = 1.5 \times 10^{-6}\ eV$ (expt)
- $B_{so}/\Delta = s_{ci}/S_{bulk} = 10^{-5}$
- $S_{bulk} = 10^{14}\ QL^{-1}\ cm^{-2}$
- $s_{ci} = k_c^2\ (eE/k_c)/E_{gap}$

$= 10^{14}\ cm^{-2}\ (10^{-6}eV/10^{-1}eV)$

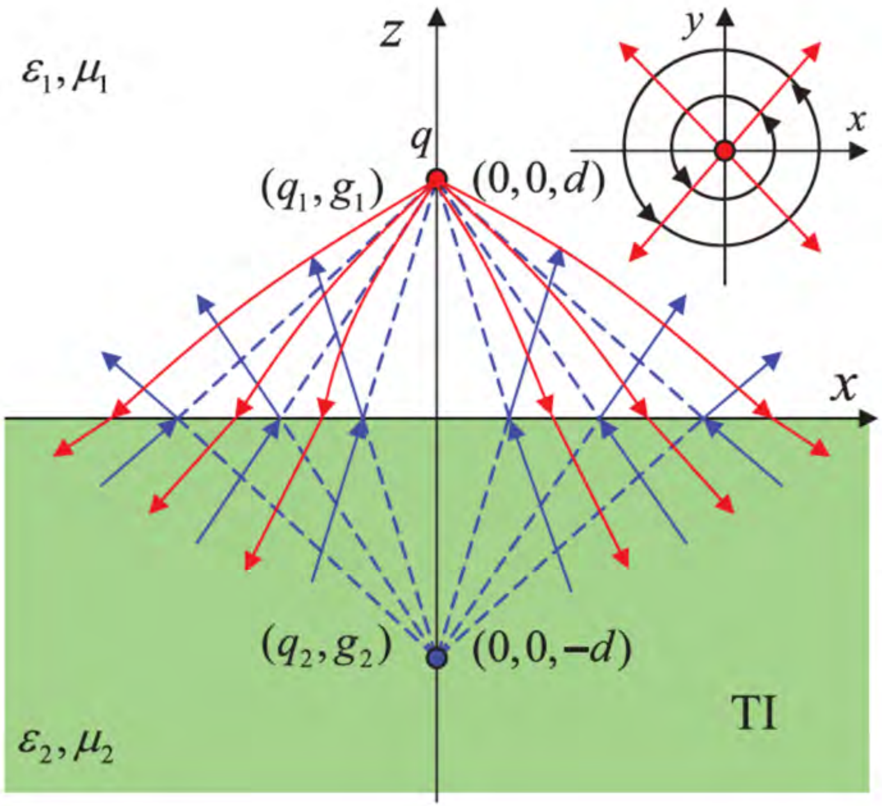
- Diagonal (Normal - Rashba) vs. Off-Diagonal (Anomalous - `Spin Hall') Response

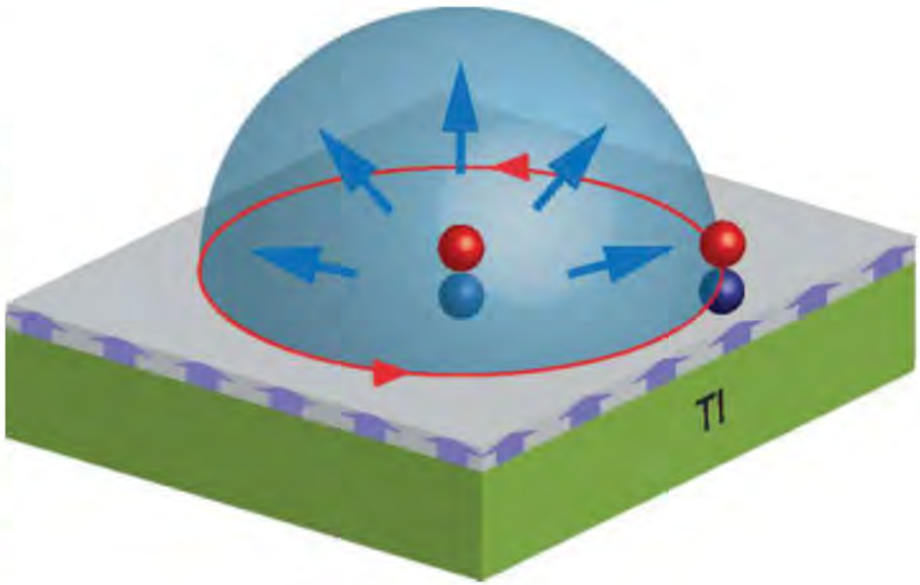
- Experimental Findings `Make Sense'

Atomic Limit
Electric Field Induced Torques



TI Thin Films as Multiferroic Materials





Qi and Zhang – RMP (2011)
Zang and Nagaosa – PRB (2010)
Pesin and AHM – PRL (2013)

Maxwell's Equations

$$\nabla \cdot \mathbf{E} = 4\pi \rho_{\text{tot}}$$

$$\nabla \cdot \mathbf{B} = 0$$

Still True

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

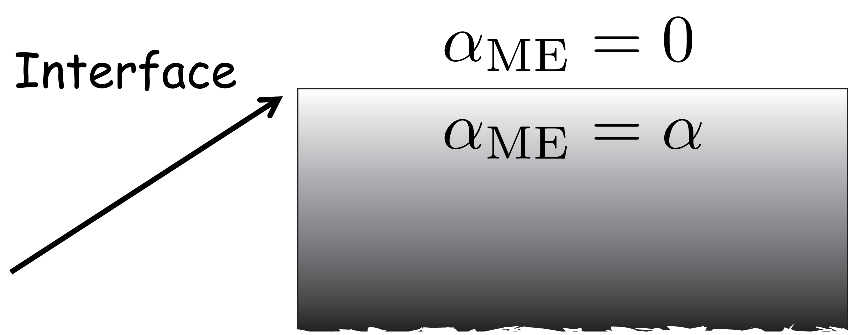
$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{J}_{\text{tot}}$$

Axion Maxwell's Equations

Magneto-Electric Coupling Coefficent

$$\nabla \cdot \boldsymbol{E} = 4\pi\rho - \nabla\alpha_{\mathrm{ME}} \cdot \boldsymbol{B},$$

$$\nabla \times \boldsymbol{B} - (1/c)\,\partial \boldsymbol{E}/\partial t = (4\pi/c)\boldsymbol{J} + \nabla\alpha_{\mathrm{ME}} \times \boldsymbol{E}$$



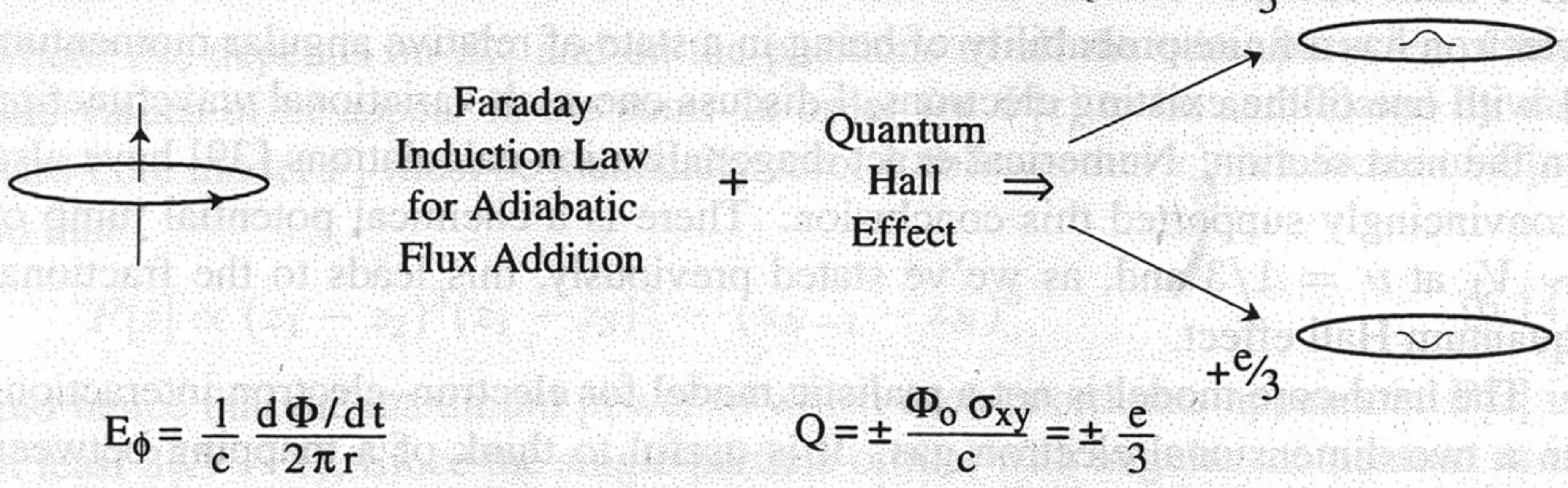
$$\nabla\alpha_{\mathrm{ME}} \times \boldsymbol{E}$$

with

$$\nabla\alpha_{\mathrm{ME}} = \alpha\delta(z) = \frac{4\pi}{c}\sigma_{xy}\delta(z)$$

can represent surface Hall currents

Adiabatic Flux Addition

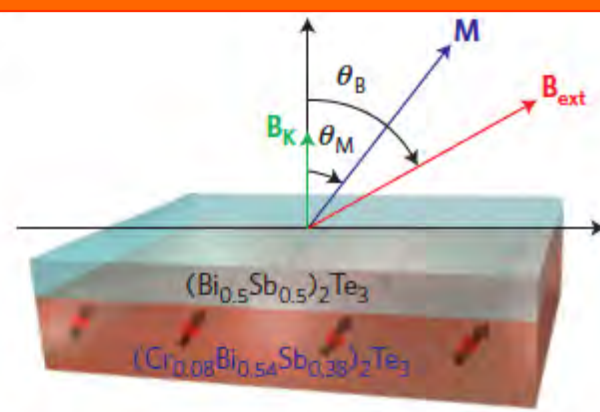


K. Everschor-Sitte, M. Sitte, and AHM, PRB (2015)
Wu and Armitage, arXiv:1603.04317
Dziom and Molenkamp, arXiv:1603.05482
Beenakker – CM Journal Club

$P = e\,(n_T - n_B)/2$

$dP/dB = e/\Phi_0$

$= dM/dE$



Spice – Quantum Spintronics

Are Topological Materials Useful in Spintronics ?

Allan MacDonald
University of Texas at Austin