Non-equilibrium transport properties of spin-dependent nanostructures

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with

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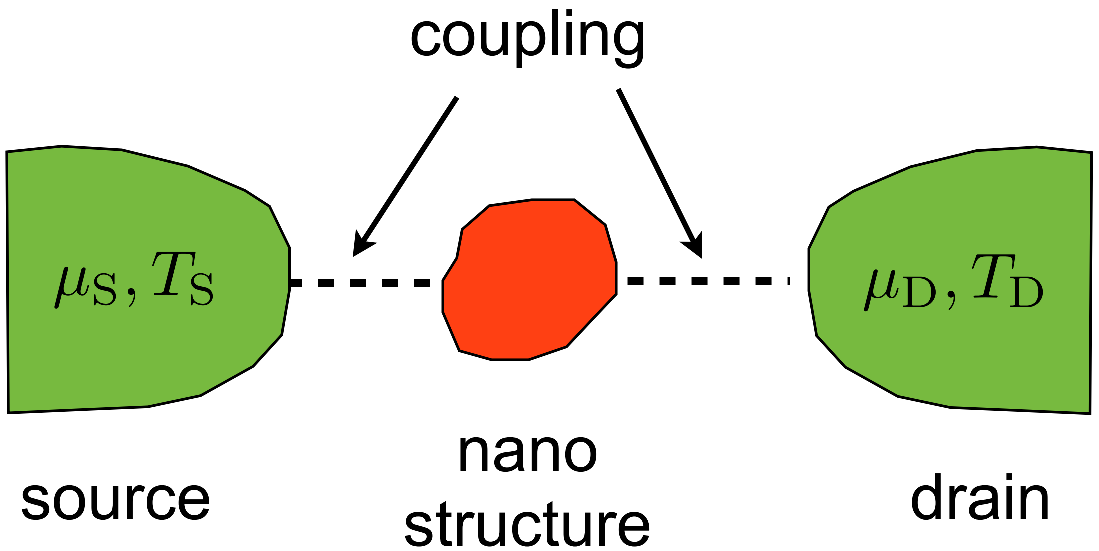
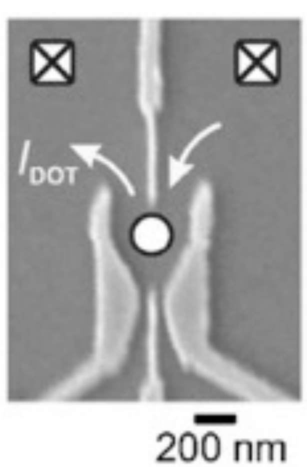
Basic transport setup



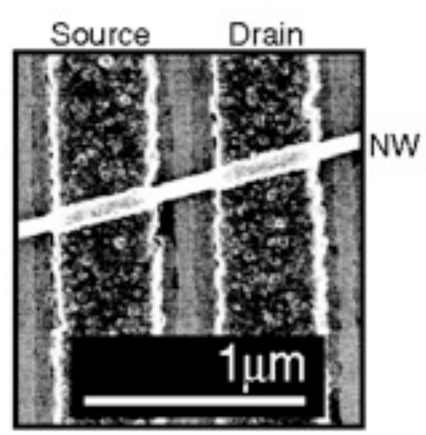
discrete quantum levels

spatial restriction → interactions important

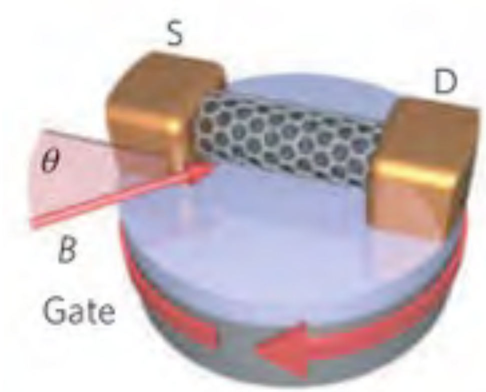
experimental realisations: heterostructures, nano wires, carbon nanotubes, molecules, ...



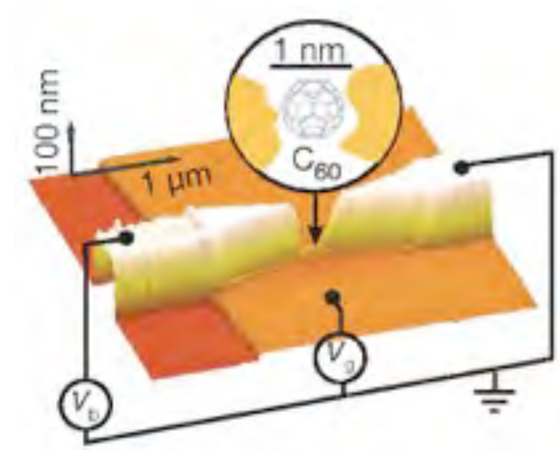
Hanson et al.,
RMP 2007



Sand-Jespersen et al.,
PRL 2007

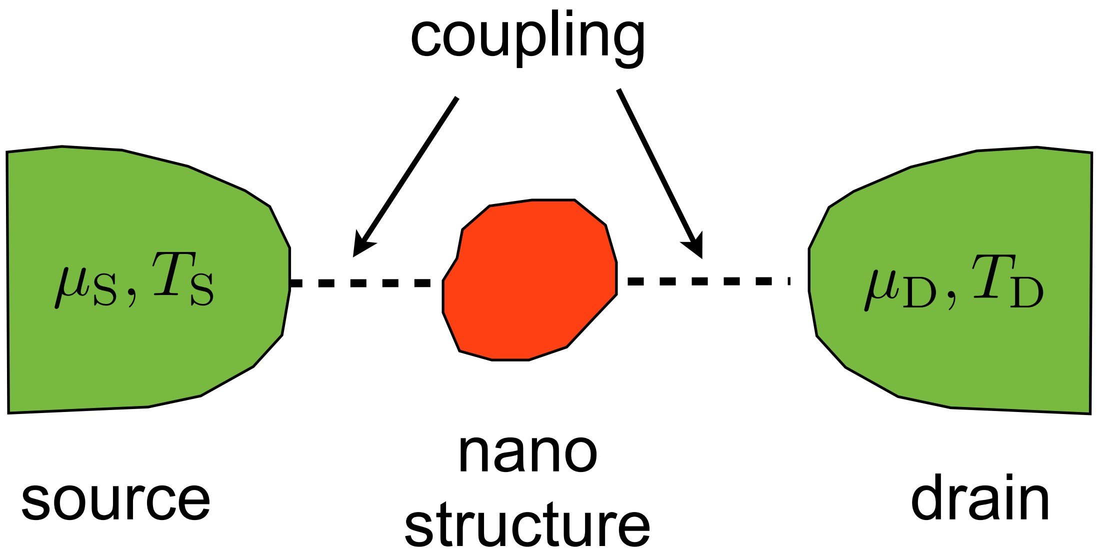


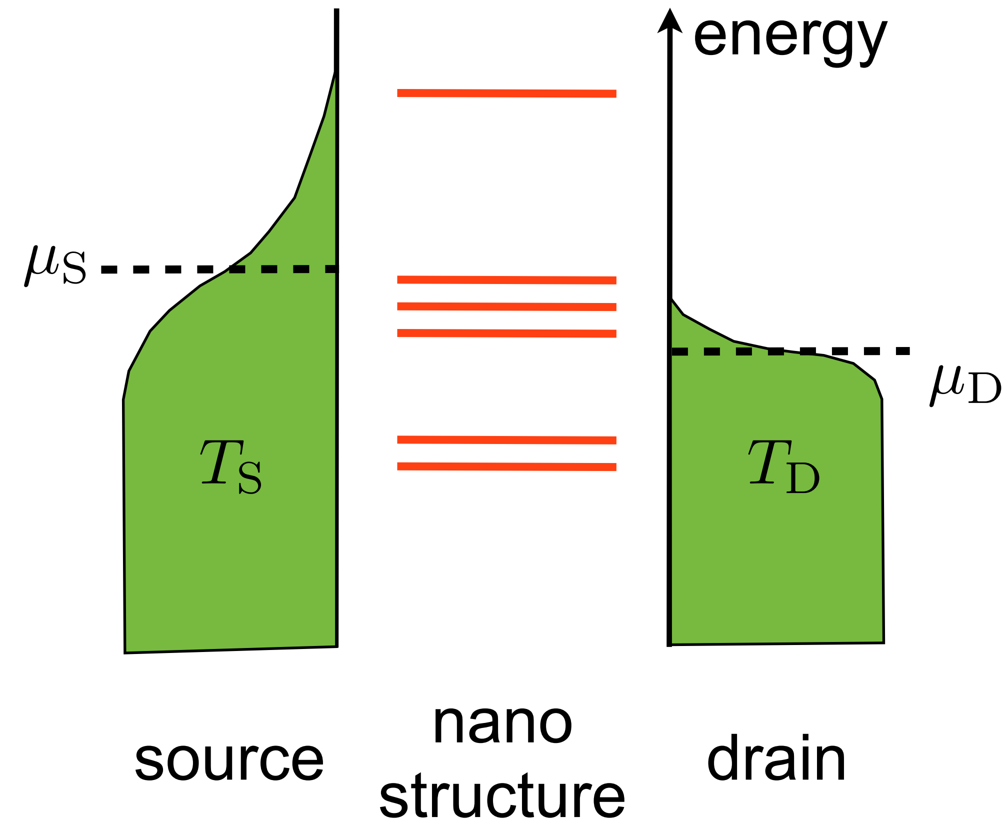
Jespersen et al.,
Nat. Phys. 2011



Roch et al.,
Nature 2008

Basic transport setup





finite bias voltage $V = \mu_{\mathrm{S}} - \mu_{\mathrm{D}}$

temperature bias $\Delta T = T_{\mathrm{S}} - T_{\mathrm{D}}$

⇒ drive currents through nanostructure $I_{\mathrm{C}}, \quad I_{\mathrm{E}}, \quad I_{\mathrm{S}}$

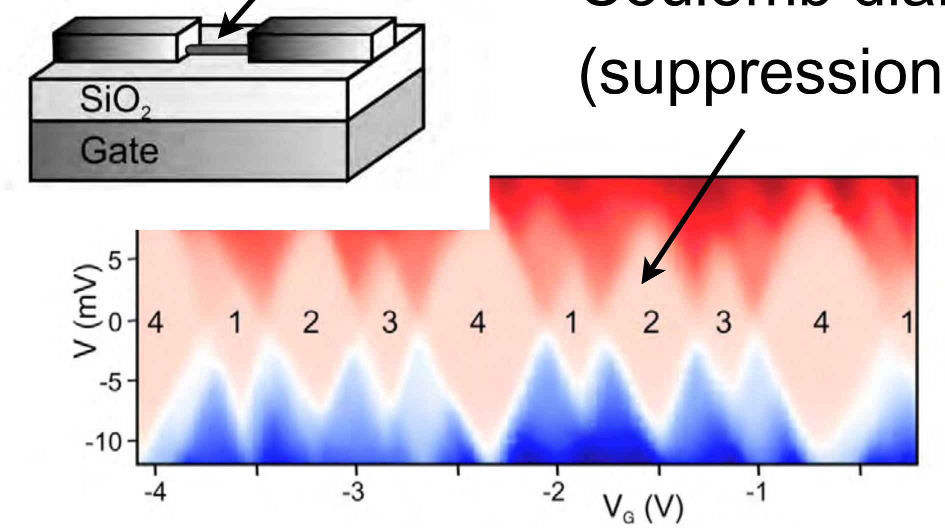
charge energy spin

Goal: understand transport phenomena

Experimental examples

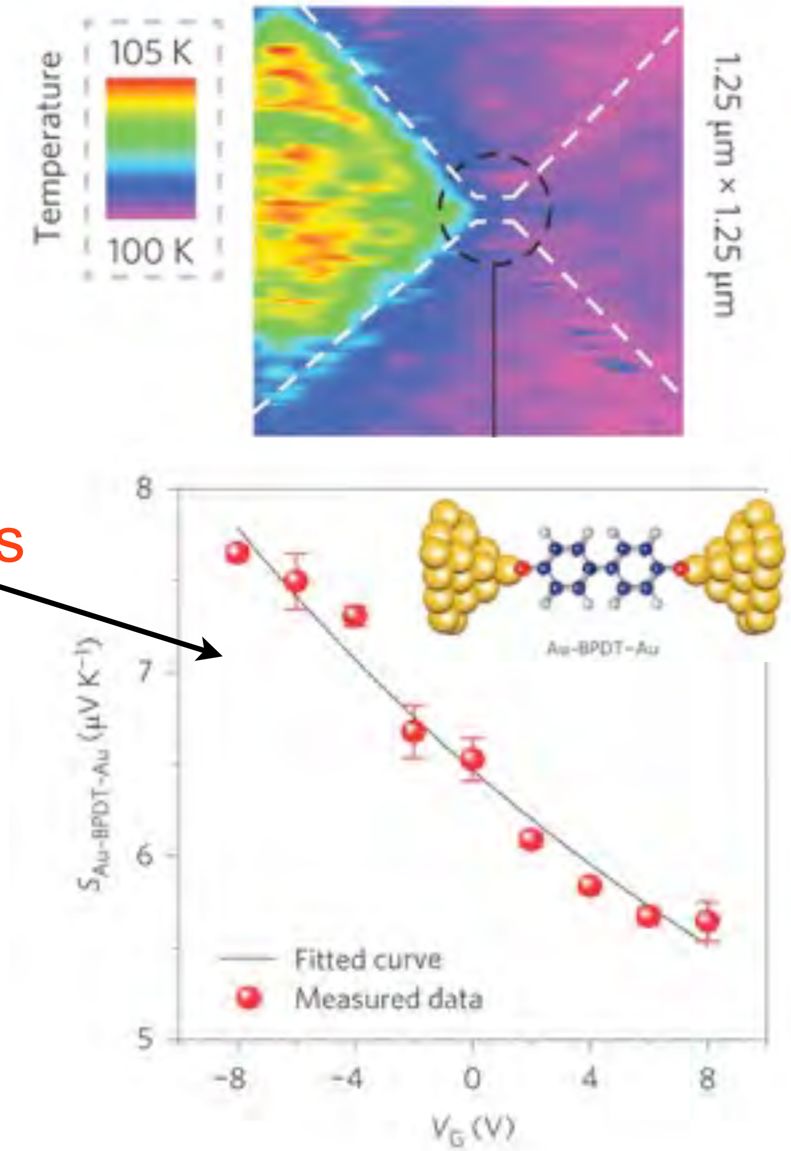
transport through
nanotube quantum dot

Coulomb diamonds
(suppression of transport)



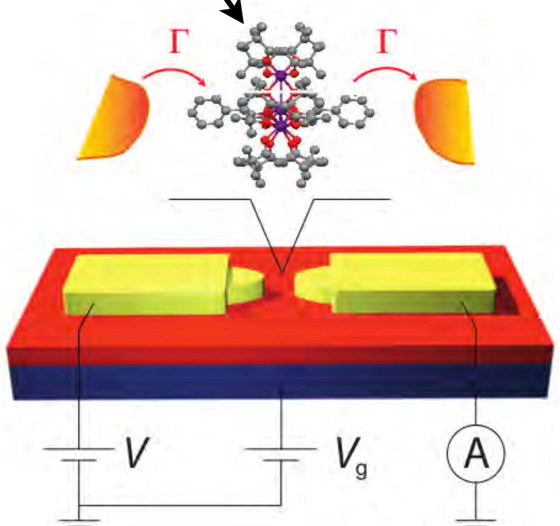
Sapmaz et al, PRB 2005

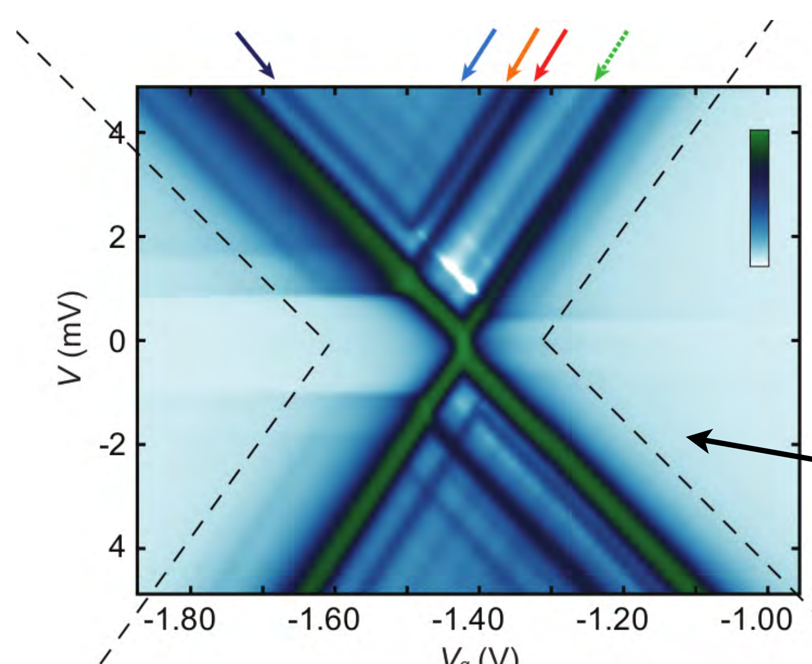
thermotransport
through molecules



Kim et al, Nat Nano 2014

transport through
gated molecule



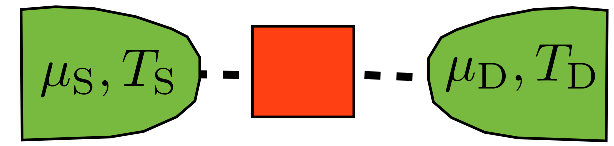


several resonant and
off-resonant transport
processes visible

Gaudenzi, Misiorny, Burzuri, van der Zant & Wegewijs, to appear in J Chem Phys 2016

Outline

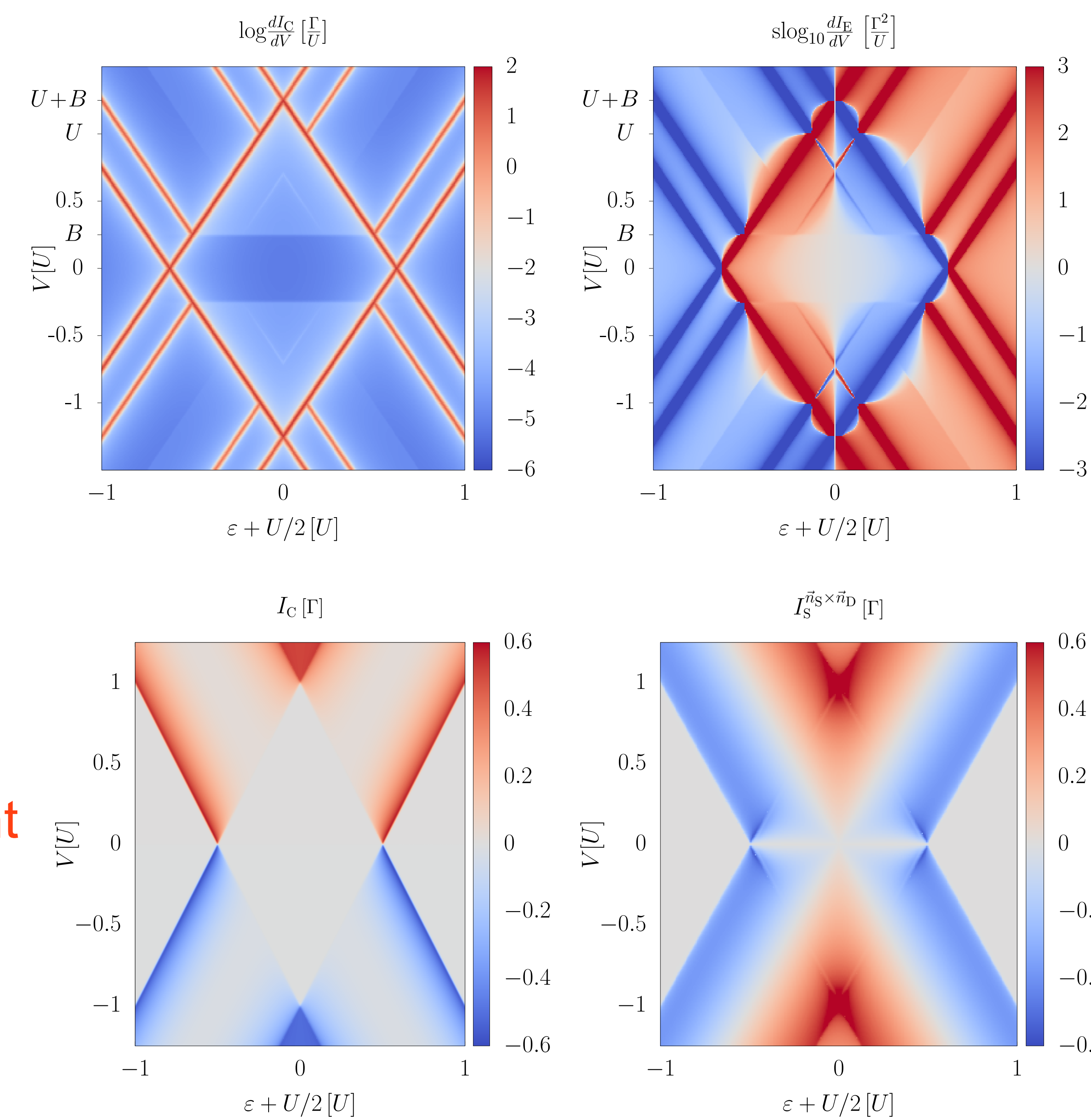
- paradigmatic system: Anderson model



- basic transport processes
  (well known in literature)

- energy and heat transport
  energy transport spectroscopy

- multi-level systems



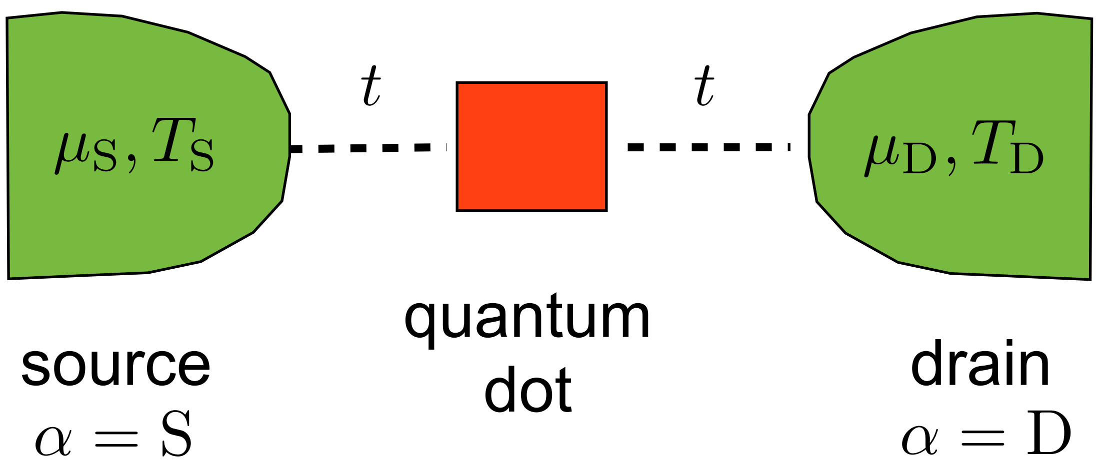
- spin-dependent transport
  spin current without charge current

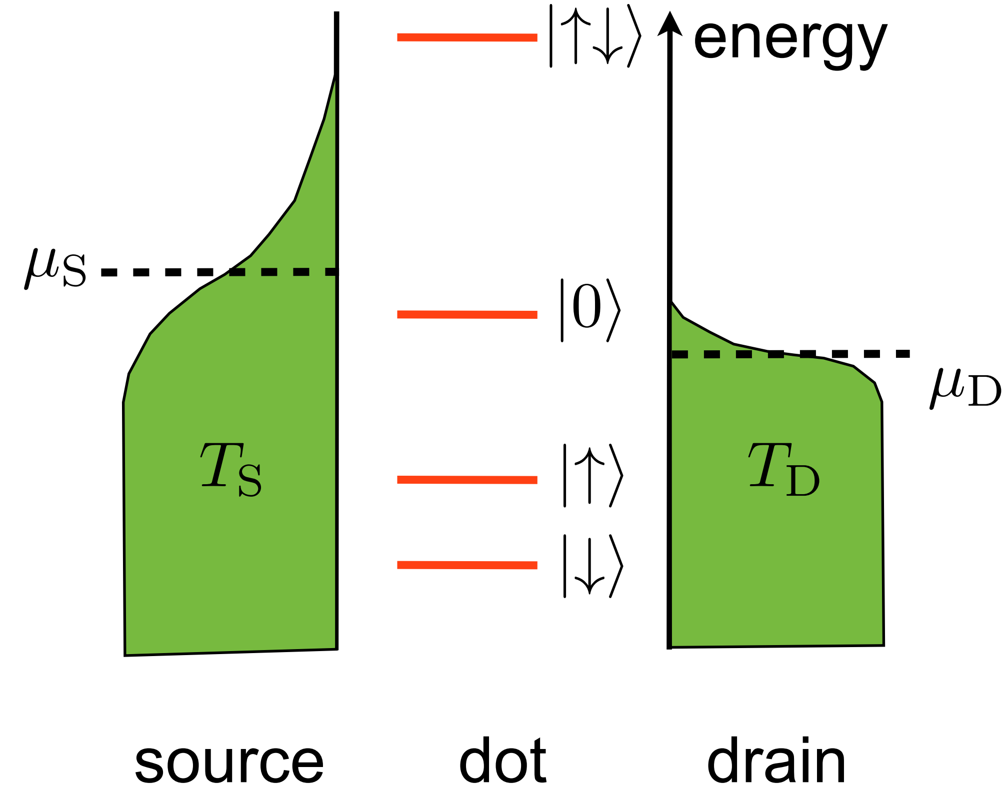
- spin-valve setups

Basic transport processes

$\hbar = 1$

Single-level Anderson model





$$H = H_\mathrm{res} + H_\mathrm{d} + H_\mathrm{tun}$$

$$H_\mathrm{res} = \sum_{\alpha k \sigma} \epsilon_k c^\dagger_{\alpha k \sigma} c_{\alpha k \sigma}$$

electrodes, density of states ρ_0

$$H_\mathrm{d} = \sum_\sigma \left(\varepsilon + \frac{B\sigma}{2} \right) d^\dagger_\sigma d_\sigma + U d^\dagger_\uparrow d_\uparrow d^\dagger_\downarrow d_\downarrow$$

quantum dot, gate voltage ε,
Coulomb repulsion U, magnetic field B

$$H_\mathrm{tun} = t \sum_{\alpha k \sigma} \left(c^\dagger_{\alpha k \sigma} d_\sigma + d^\dagger_\sigma c_{\alpha k \sigma} \right)$$

tunneling, rate $\Gamma = 2\pi \rho_0 t^2$

Real-time technique

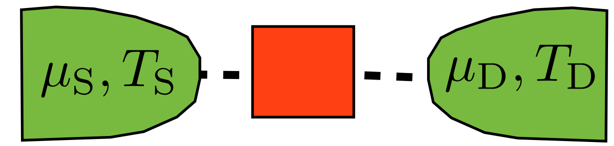
cf. Saptsov & Wegewijs, PRB 2012

von-Neumann equation for full system

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = [H, \rho(t)] = L\rho(t)$$

Liouville operator $L = L_{\mathrm{res}} + L_{\mathrm{d}} + L_{\mathrm{tun}}$

$$\Rightarrow \quad \rho(t) = e^{-\mathrm{i}Lt}\rho(0)$$



contain bias voltage and temperatures

initially $\rho(0) = \rho_{\mathrm{d}}(0)\rho_{\mathrm{S}}^{\mathrm{eq}}\,\rho_{\mathrm{D}}^{\mathrm{eq}}$

reduced density matrix of dot

$$\rho_{\mathrm{d}}(t) = \mathrm{tr}_{\mathrm{res}}\,\rho(t)$$

integrate out source and drain degrees of freedom perturbatively in L_{tun} $\Rightarrow$ rate equation for occupations on dot applicable for weak tunneling $\Gamma \ll T, U$

stationary density matrix satisfies

$$L_{\mathrm{d}}^{\mathrm{eff}}\,\rho_{\mathrm{d}}^{\mathrm{st}} = 0$$

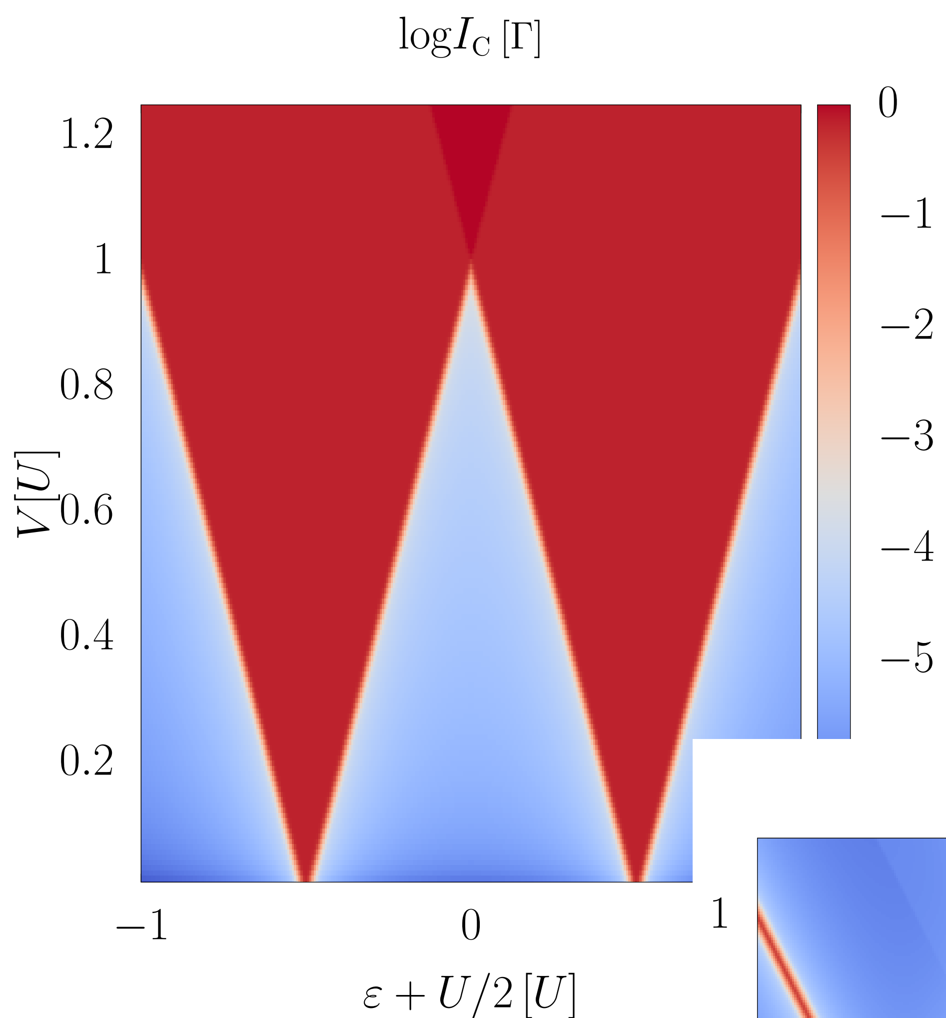
contains relaxation rates

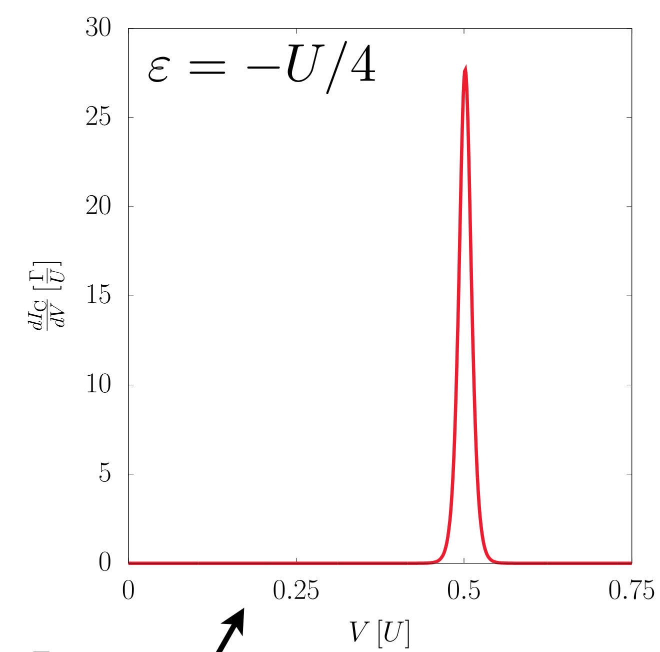
$\Rightarrow$ stationary observables, eg, currents possible to describe time evolution

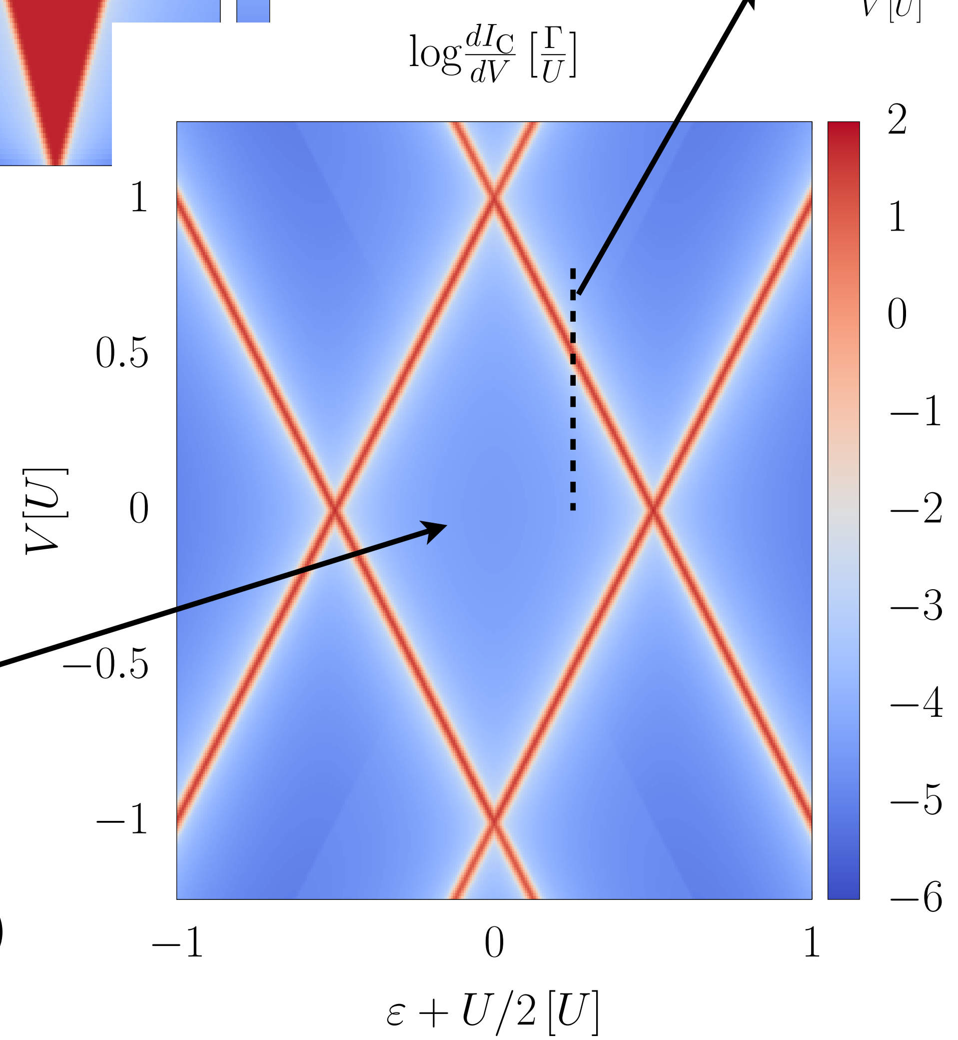
remark: many other methods on the market

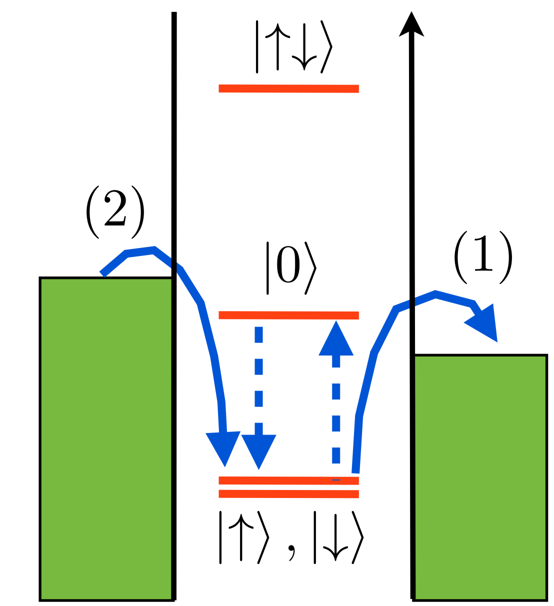
Single-electron tunneling (SET)

$U = 10^5\,\Gamma$
$T = 300\,\Gamma$
$B = 0$









Coulomb blockade regime (Coulomb diamond)

process in $O(\Gamma)$:
when energy differences on dot equal $V/2$, eg, electron
(1) hops off dot to drain
(2) hops from source to dot

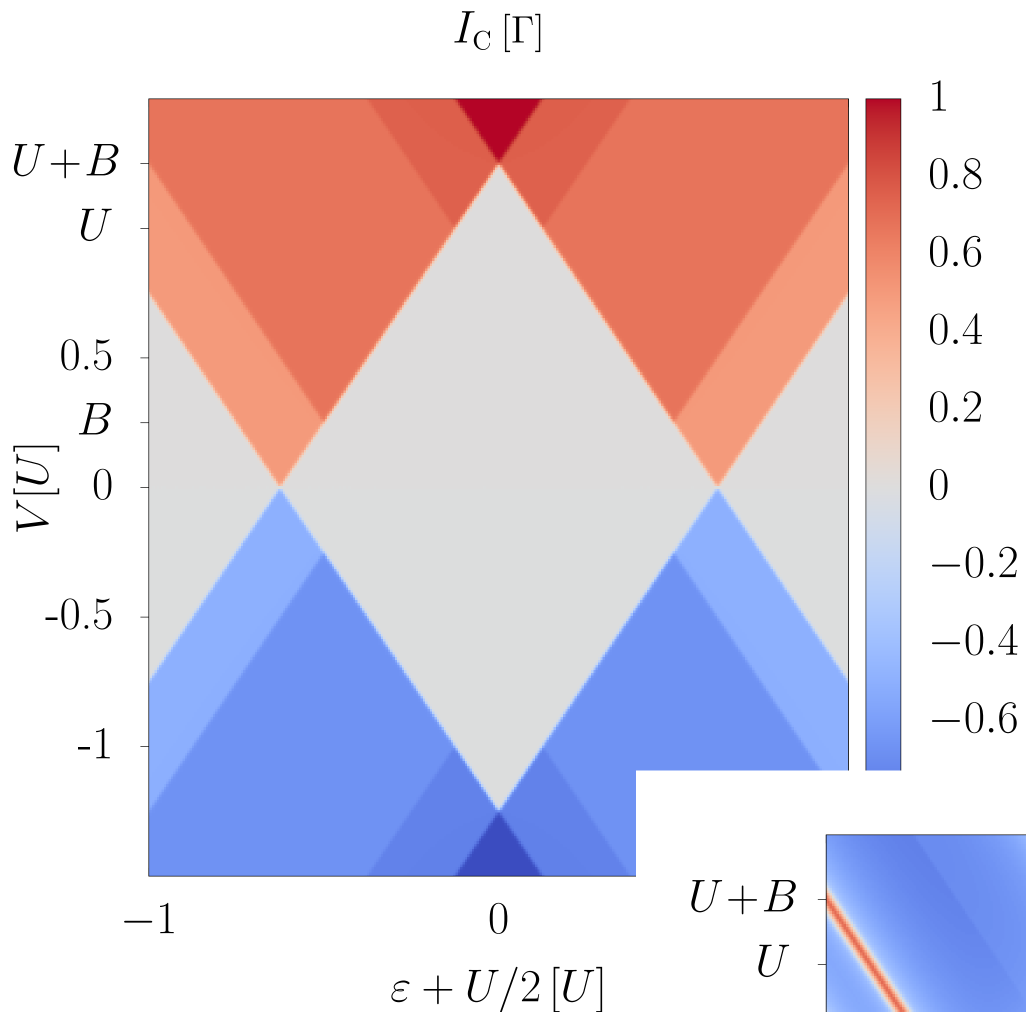
broadening of SET resonance by temperature

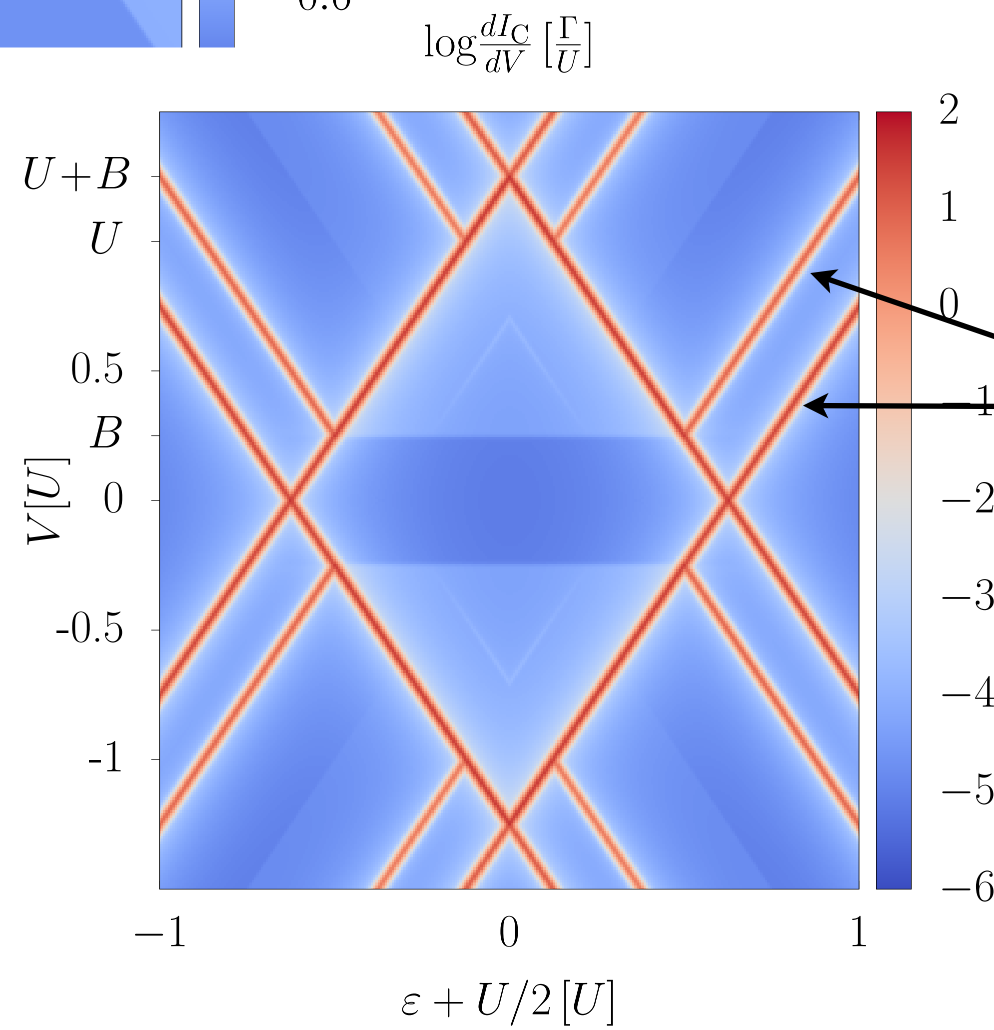
Single-electron tunneling (SET)

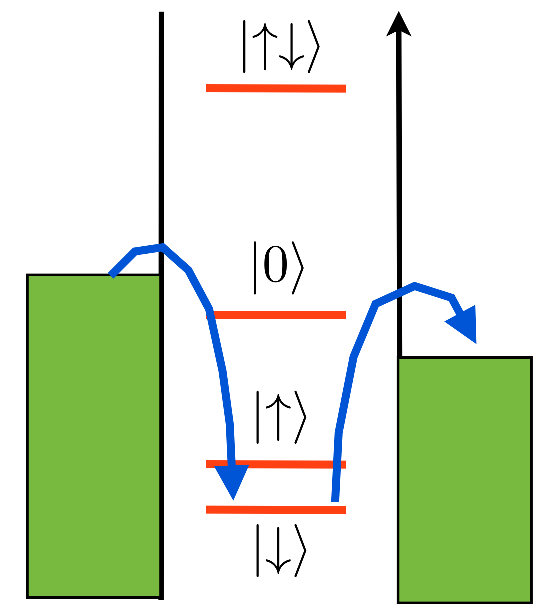
$U = 10^5\,\Gamma$

$T = 300\,\Gamma$

$B = U/4$



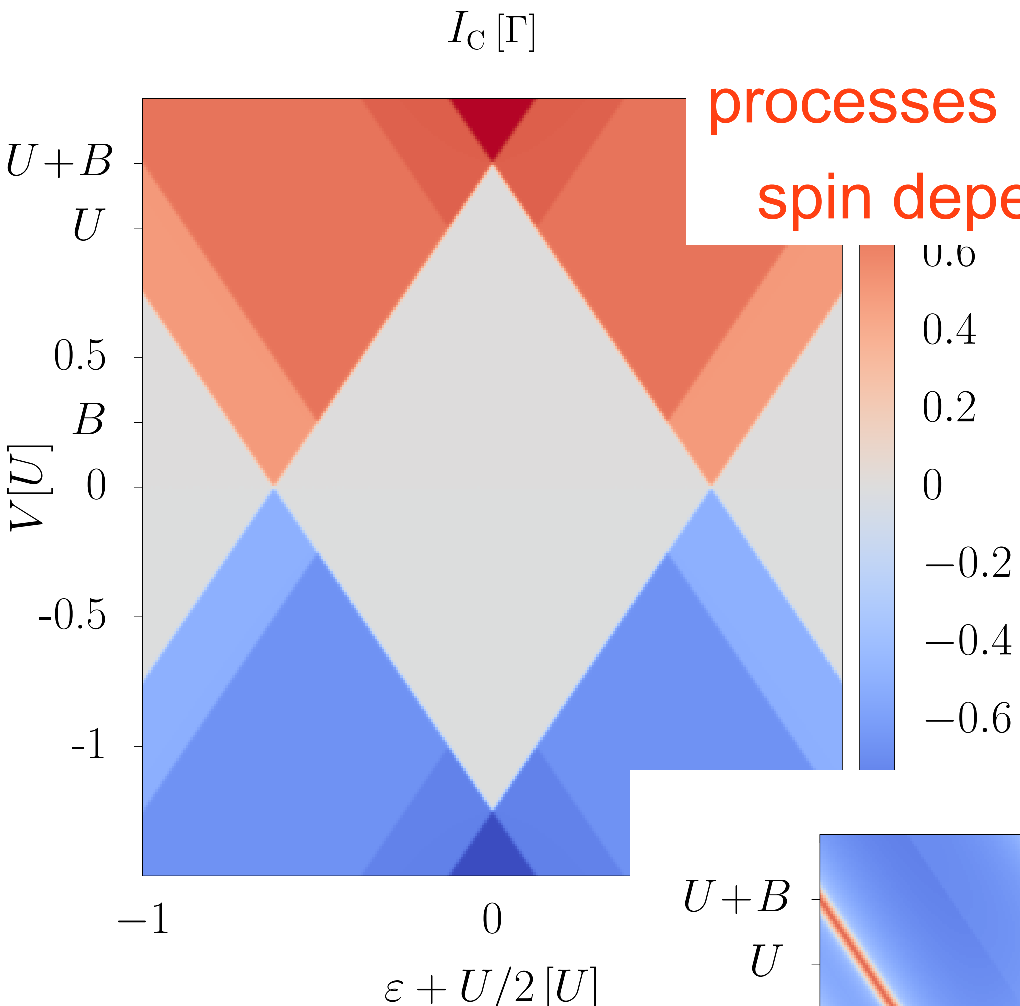




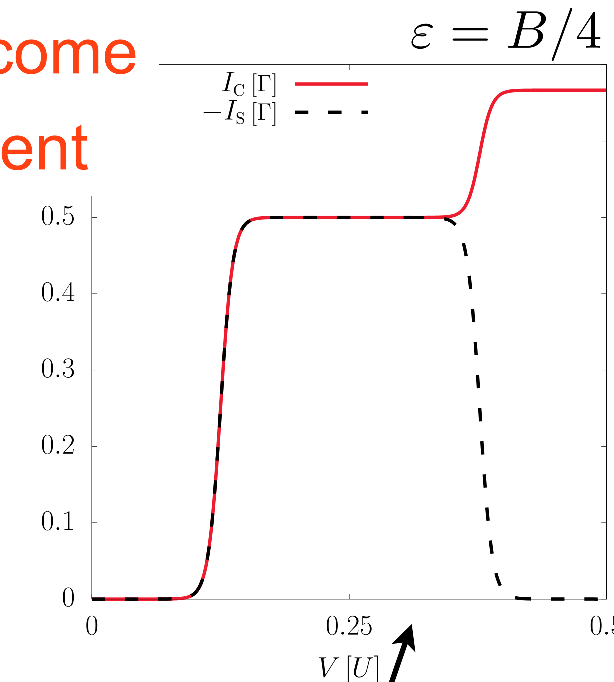
finite magnetic field leads to splitting of SET lines

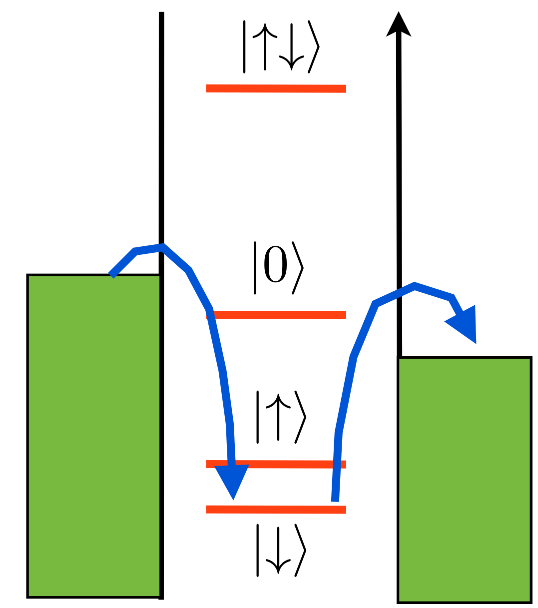
Single-electron tunneling (SET)

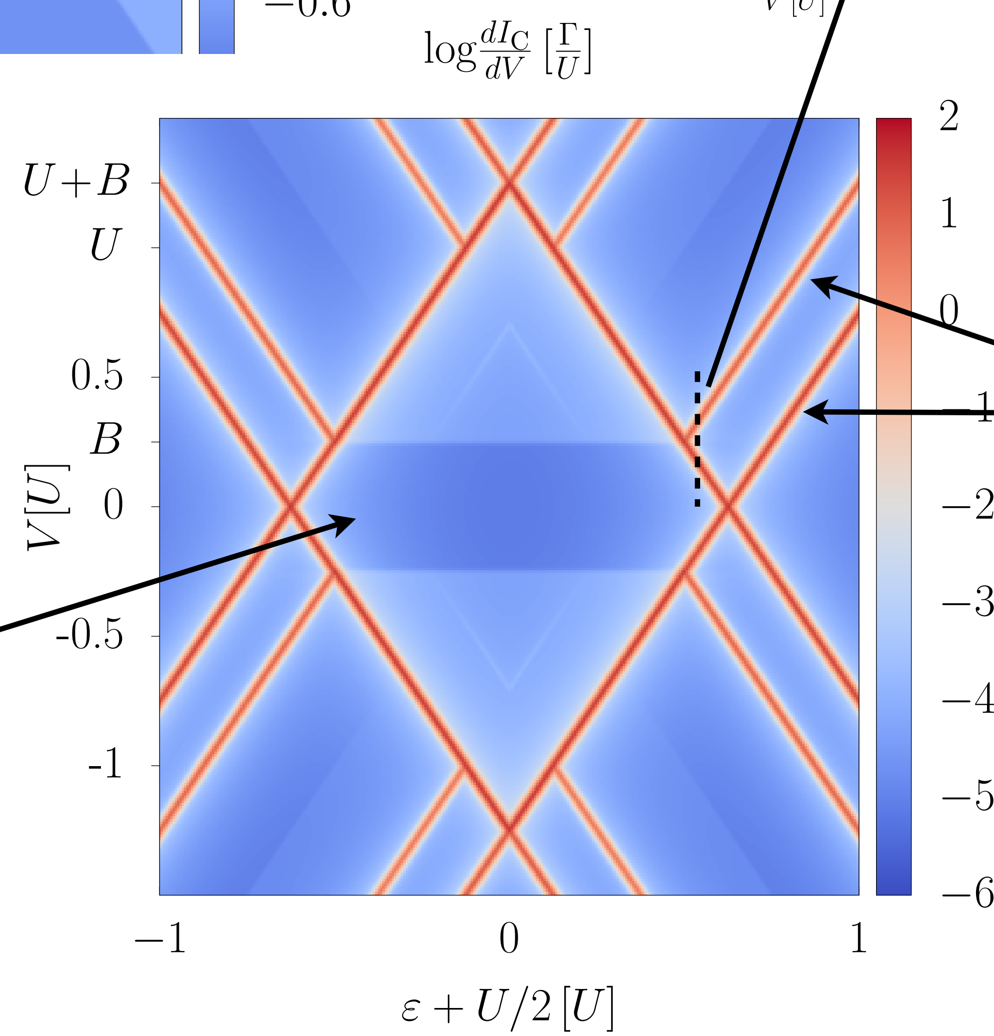
$U = 10^5\,\Gamma$
$T = 300\,\Gamma$
$B = U/4$



processes become spin dependent







finite magnetic field leads to splitting of SET lines

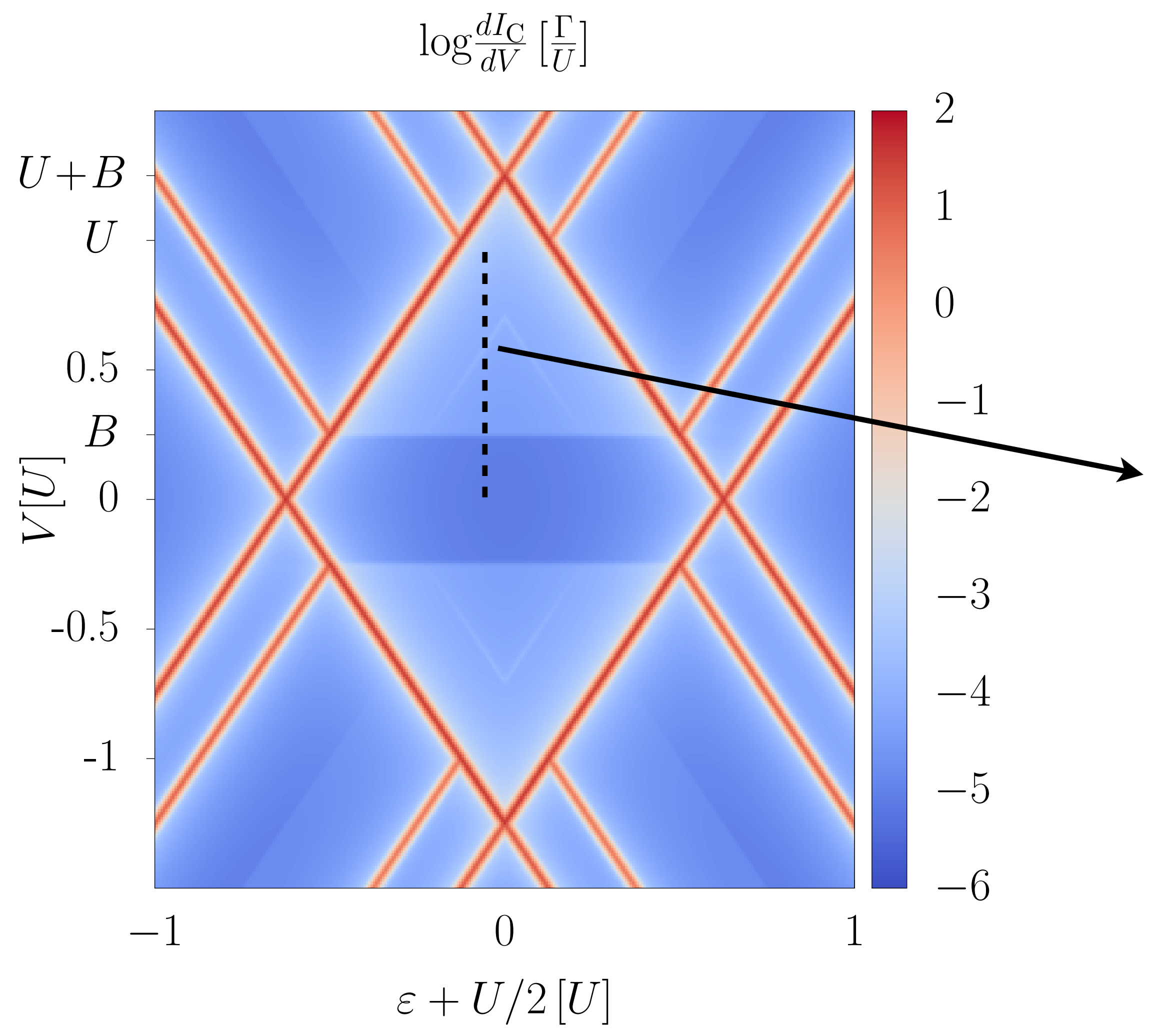
weak features in $O(\Gamma^2)$ in Coulomb diamond

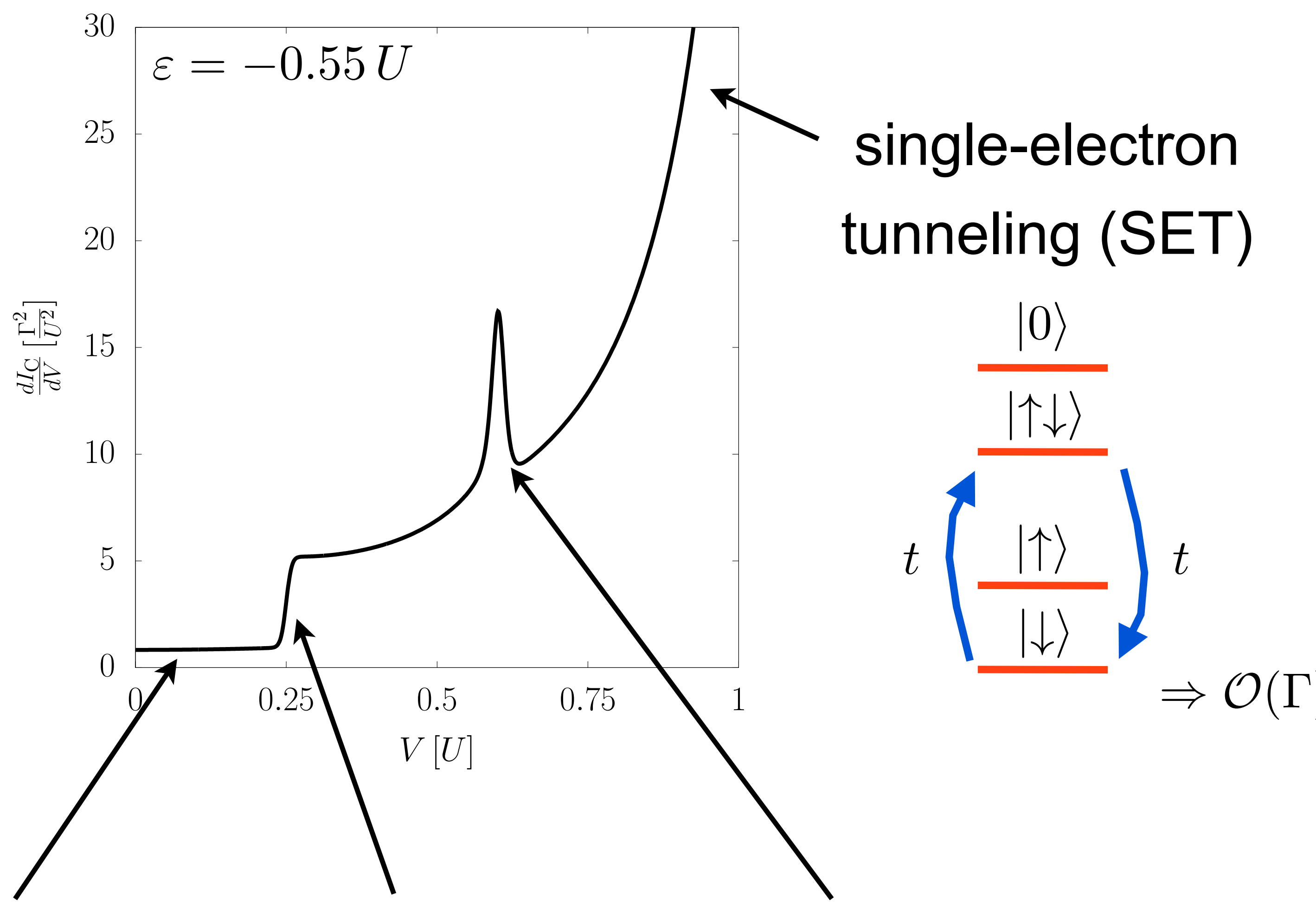
Cotunneling (COT)

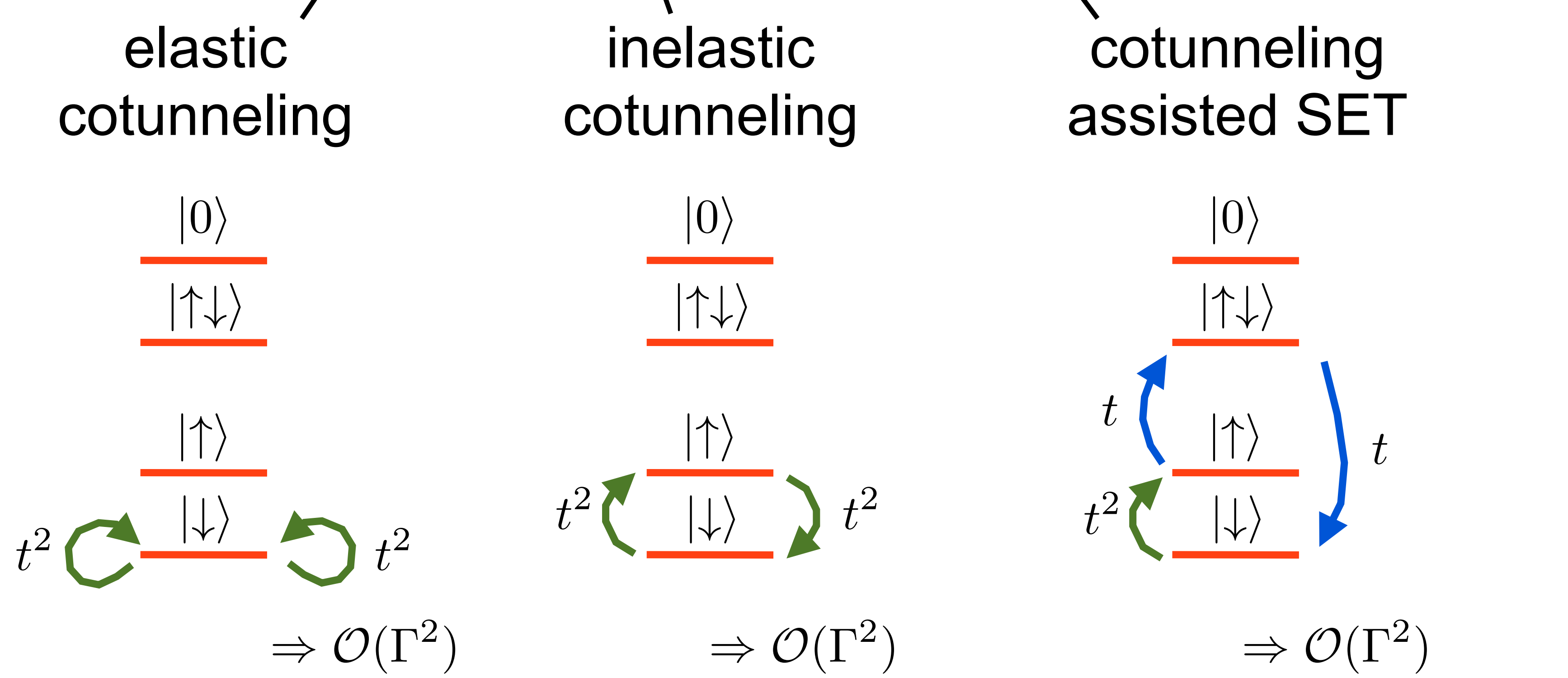
$U = 10^5\,\Gamma$

$T = 300\,\Gamma$

$B = U/4$







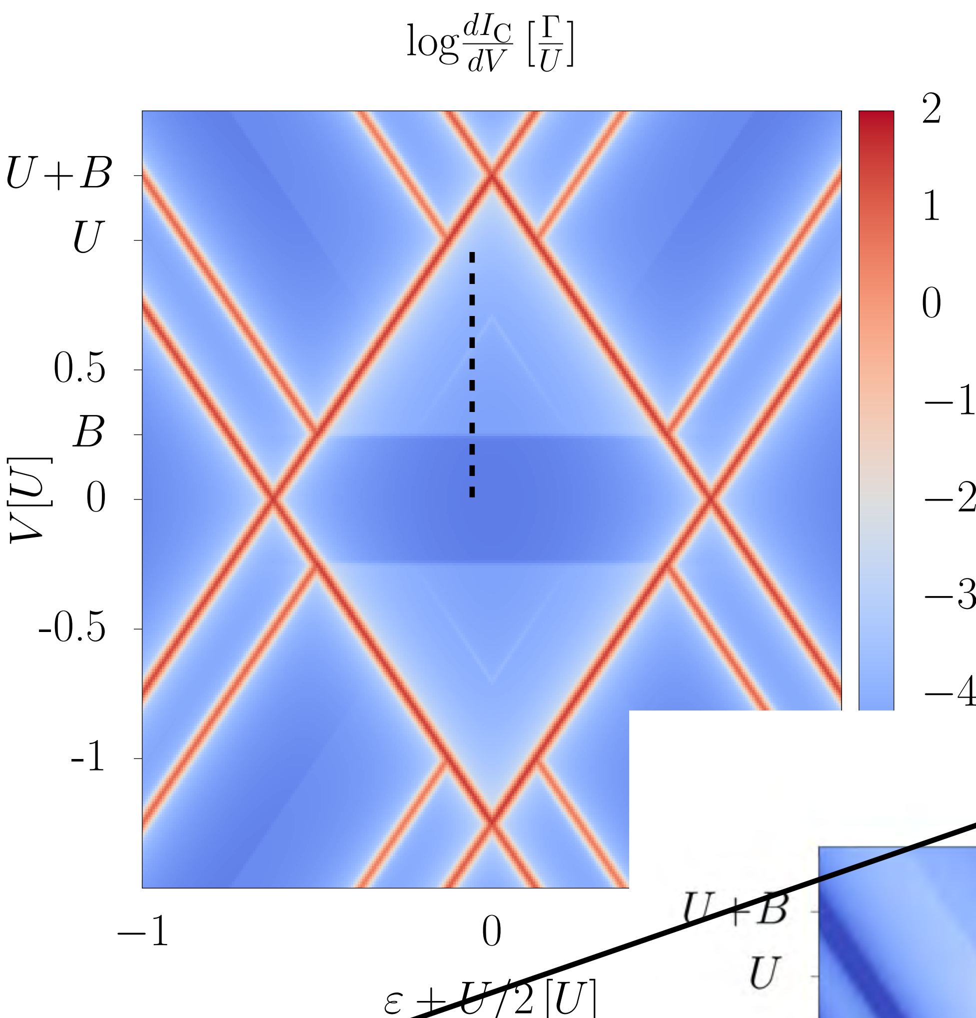
cotunneling processes starting from $|\downarrow\rangle$, $|\uparrow\rangle$ dot states $|0\rangle$, $|\uparrow\downarrow\rangle$ only visited virtually

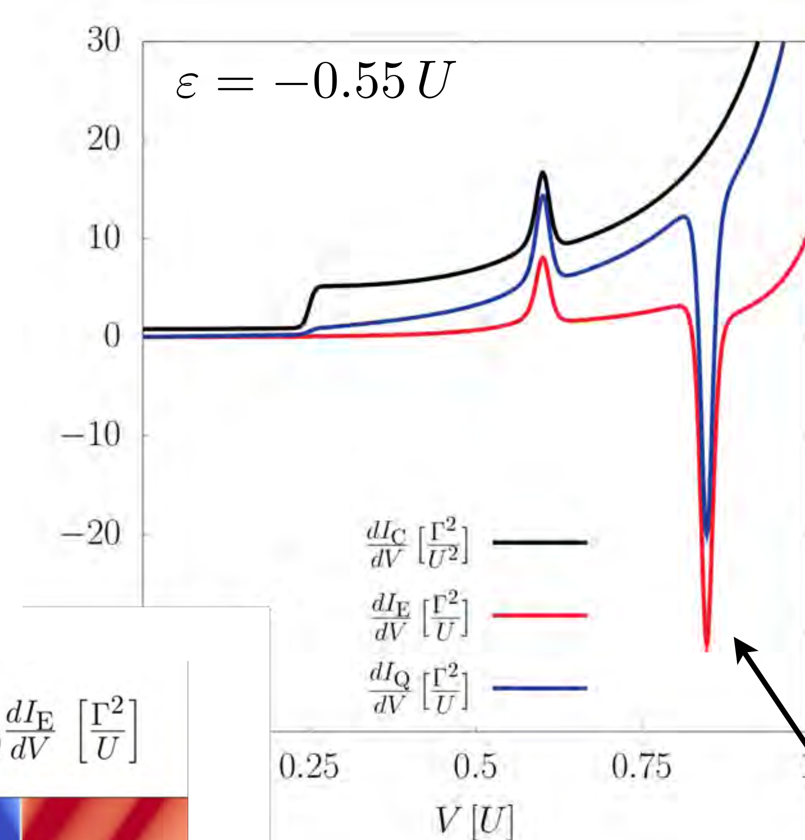
Energy transport

$U = 10^5\,\Gamma$

$T = 300\,\Gamma$

$B = U/4$



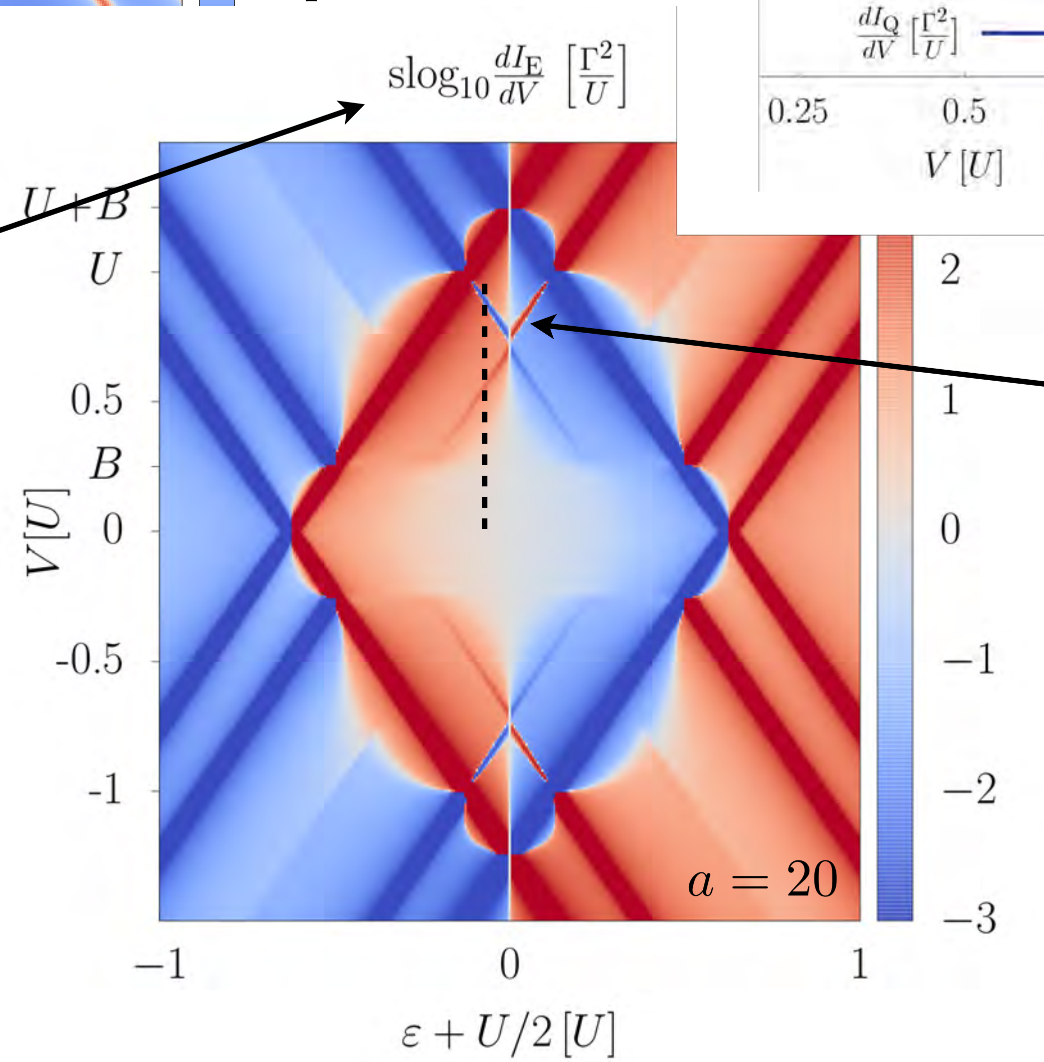


energy current

$$I_\mathrm{E} = \left\langle \frac{\mathrm{d}}{\mathrm{d}t} H^\mathrm{D}_\mathrm{res} \right\rangle$$

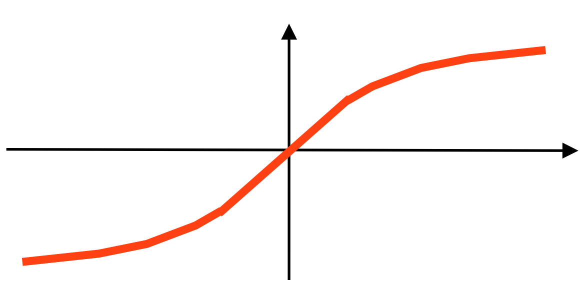
heat current

$$I_\mathrm{Q} = I_\mathrm{E} - \mu_\mathrm{D} I_\mathrm{C}$$



signed log

$$\mathrm{slog}_{10} x = \begin{cases} \frac{ax}{10}, & a|x| \leq 10, \\ \mathrm{sgn}(x)\log_{10}|ax|, & a|x| > 10 \end{cases}$$



cotunneling

assisted SET

much better visible

(no feature in charge

conductance)

see: Gergs et al, PRB 2015

Thermal bias

$U = 10^5\,\Gamma$

$T_\mathrm{D} = 3T_\mathrm{S} = 900\,\Gamma$

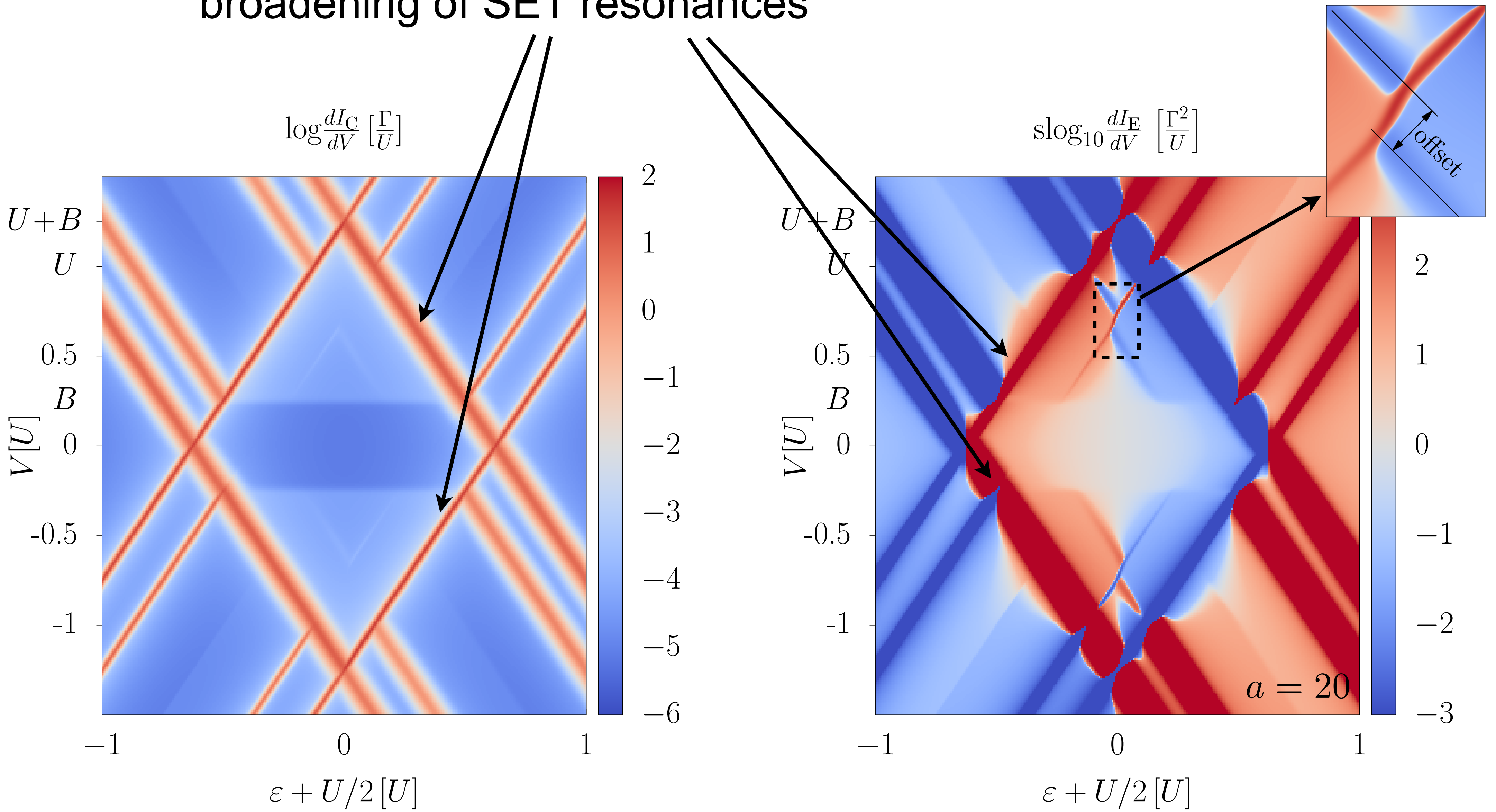
$B = U/4$

one can also add a finite thermal bias $\Delta T = T_\mathrm{S} - T_\mathrm{D}$

temperature bias results in different broadening of SET resonances

COSET lines offset $\propto T_\mathrm{D}$

COSET-thermometry?



see: Gergs et al, PRB 2015

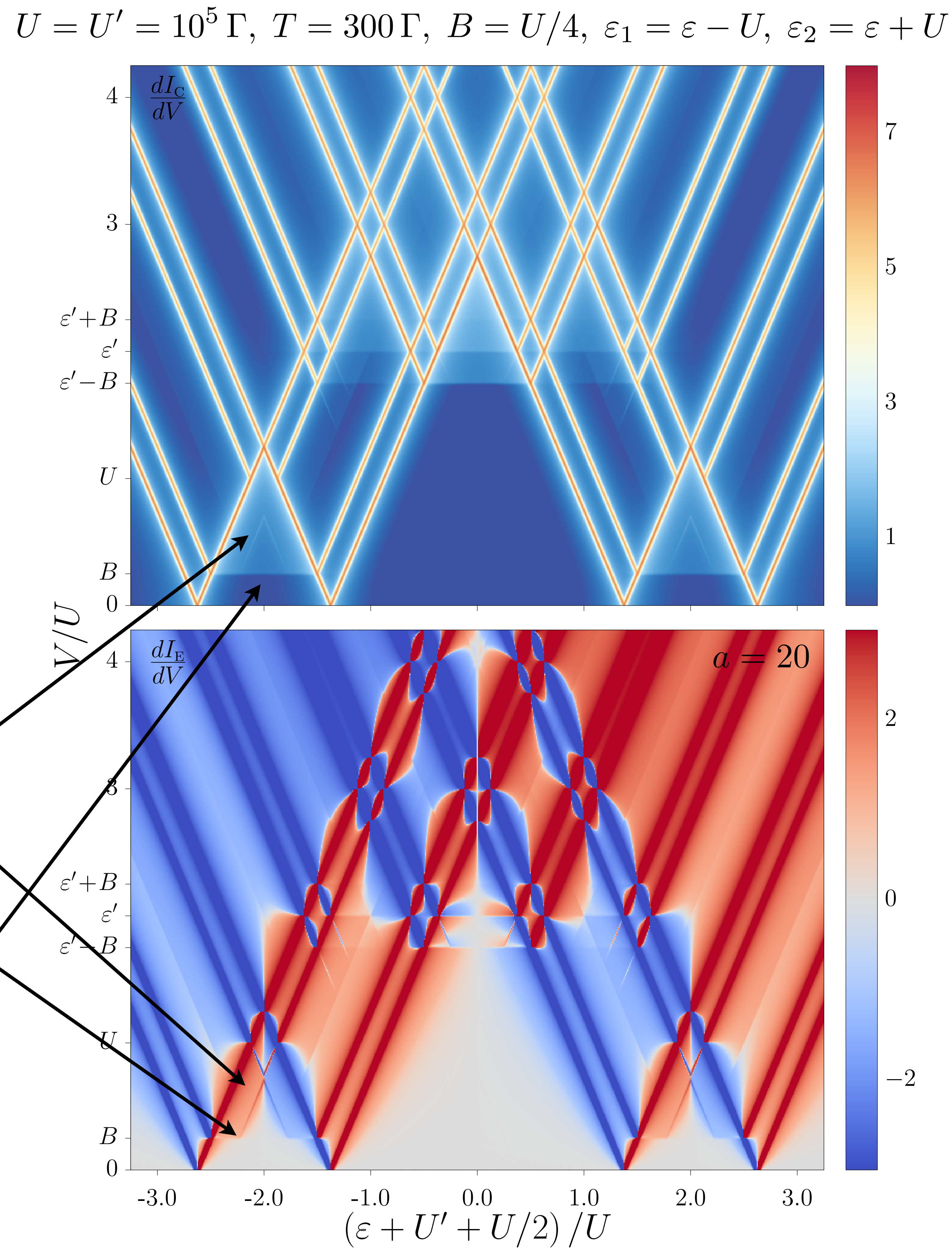
Multi-level dots

eg, two levels

$$H_{\mathrm{d}} = \sum_{i\sigma} \left(\varepsilon_i + \frac{B\sigma}{2} \right) d^\dagger_{i\sigma} d_{i\sigma}$$
$$+U \sum_i d^\dagger_{i\uparrow} d_{i\uparrow} d^\dagger_{i\downarrow} d_{i\downarrow}$$
$$+U' \left(d^\dagger_{1\uparrow} d_{1\uparrow} + d^\dagger_{1\downarrow} d_{1\downarrow} \right)$$
$$\times \left(d^\dagger_{2\uparrow} d_{2\uparrow} + d^\dagger_{2\downarrow} d_{2\downarrow} \right)$$

COSET

ICOT



→ basic transport processes appear in multi-level systems

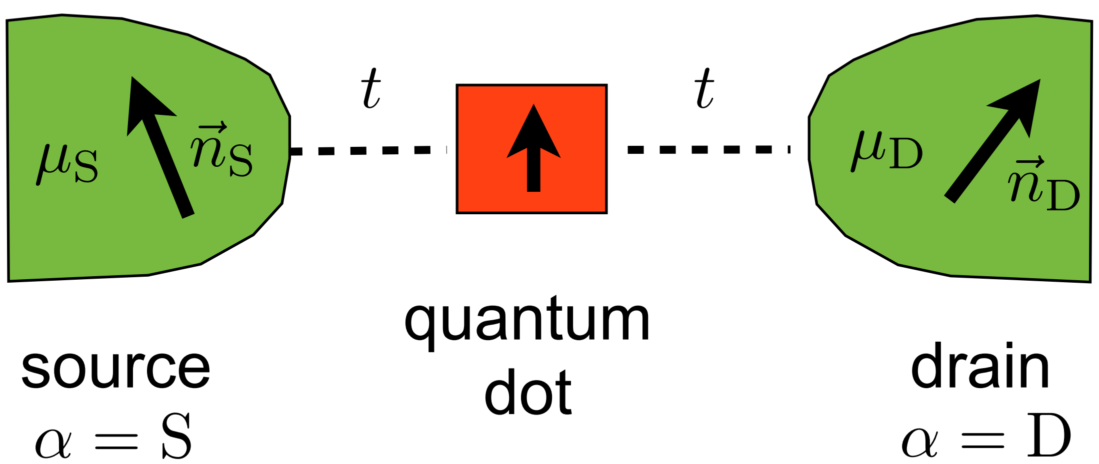
see: Gergs et al, PRB 2015

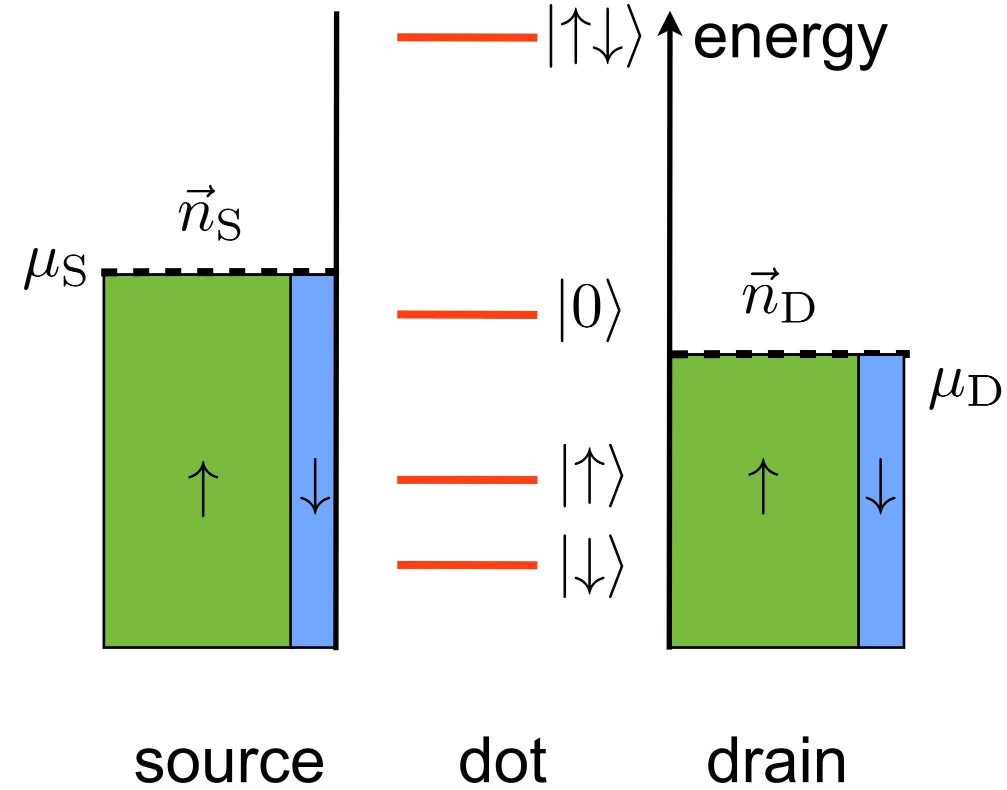
Spin-valve setup

Spin-valve Anderson model

cf. Braun et al, PRB 2004

spin quantisation in reservoirs and on dot may differ





$$H = H_{\mathrm{res}} + H_{\mathrm{d}} + H_{\mathrm{tun}}$$

$$p = \frac{\rho_+ - \rho_-}{\rho_+ + \rho_-}$$

$$H_{\mathrm{res}} = \sum_{\alpha k \sigma} \epsilon_k c^\dagger_{\alpha k \sigma} c_{\alpha k \sigma}$$

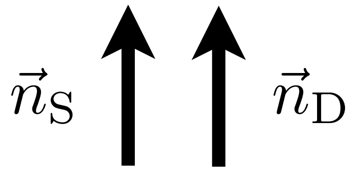
electrodes, density of states $\rho_\pm = (1 \pm p)\rho_0$, polarisation *p*

$$H_{\mathrm{d}} = \sum_{\sigma} \varepsilon\, d^\dagger_\sigma d_\sigma + U d^\dagger_\uparrow d_\uparrow d^\dagger_\downarrow d_\downarrow$$

quantum dot, gate voltage ε, Coulomb repulsion *U*

$$H_{\mathrm{tun}} = t \sum_{\alpha k \sigma} \left(c^\dagger_{\alpha k \sigma} d_\sigma + d^\dagger_\sigma c_{\alpha k \sigma} \right)$$

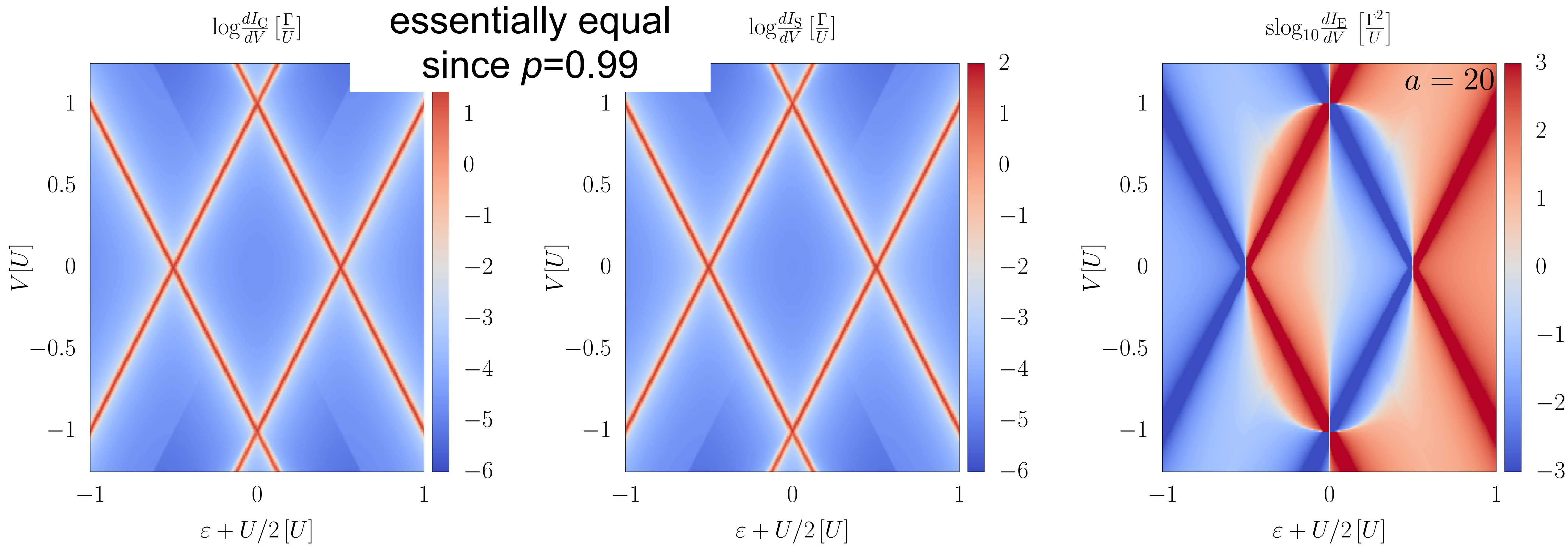
tunneling, rate $\Gamma = 2\pi \rho_0 t^2$



Parallel setup

$U = 10^5\,\Gamma$
$T = 300\,\Gamma$
$p = 0.99$

essentially equal since *p*=0.99



induced magnetic field on dot (Braun et al, PRB 2004)

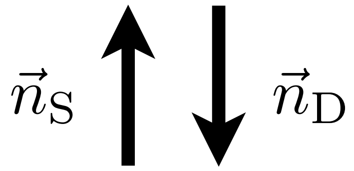
interaction essential

$$\vec{B}_\mathrm{ind} = \frac{\pi \rho_0 t^2 p}{2} \sum_\alpha \mathfrak{Re}\left[\psi\left(\frac{1}{2} + \mathrm{i}\frac{\varepsilon - \mu_\alpha}{2\pi T}\right) - \psi\left(\frac{1}{2} + \mathrm{i}\frac{\varepsilon + U - \mu_\alpha}{2\pi T}\right)\right]\vec{n}_\alpha$$

polarisation essential

but no interesting effect since $\vec{n}_\mathrm{S} \parallel \vec{B}_\mathrm{ind} \parallel \vec{n}_\mathrm{D}$

generalisation to different polarisations, thermal bias, ... straightforward

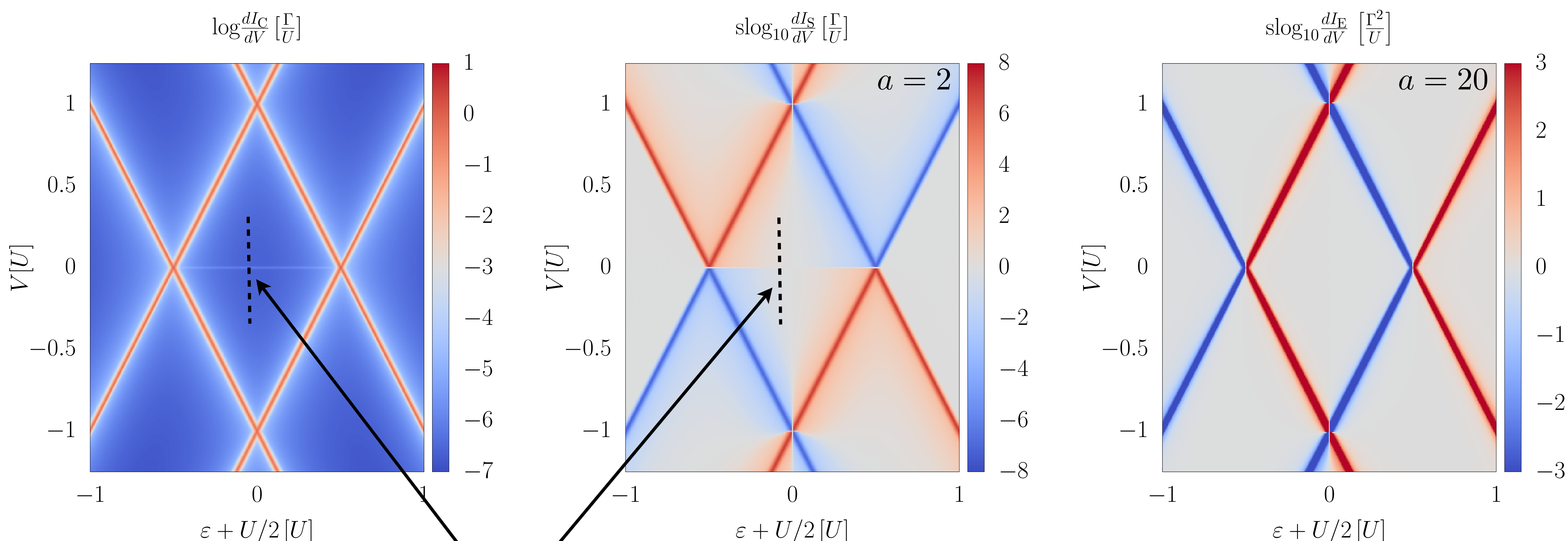


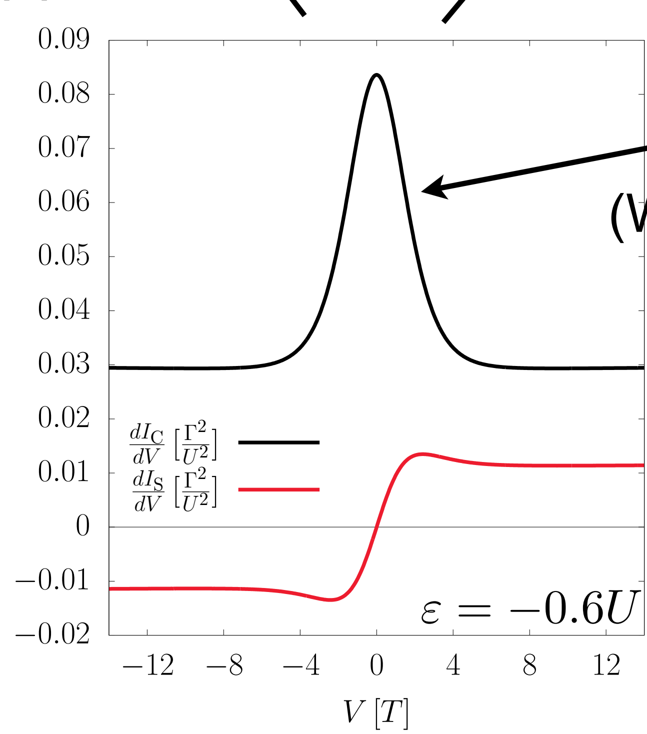
Antiparallel setup

$U = 10^5\,\Gamma$

$T = 300\,\Gamma$

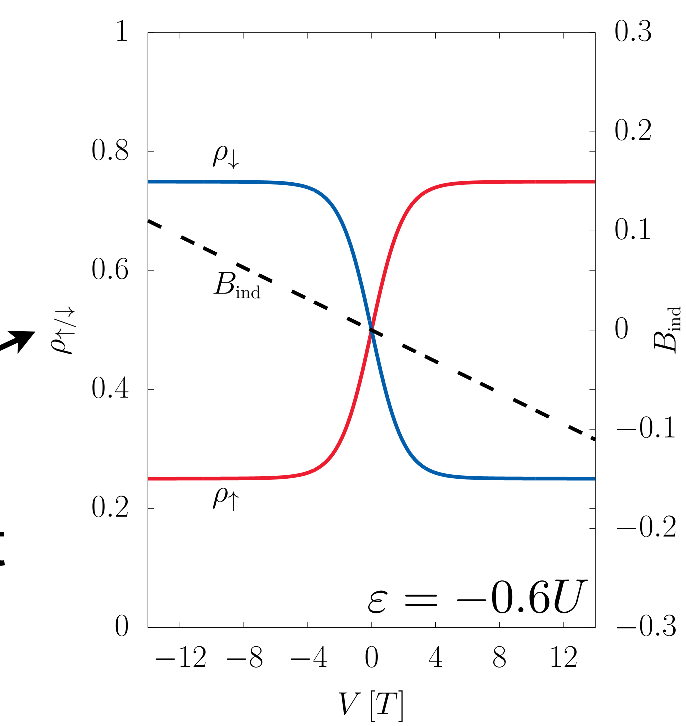
$p = 0.99$

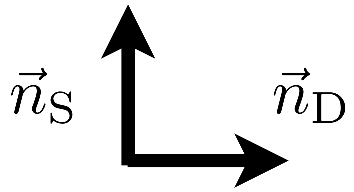




zero-bias anomaly
(Weymann et al, PRB 2005)

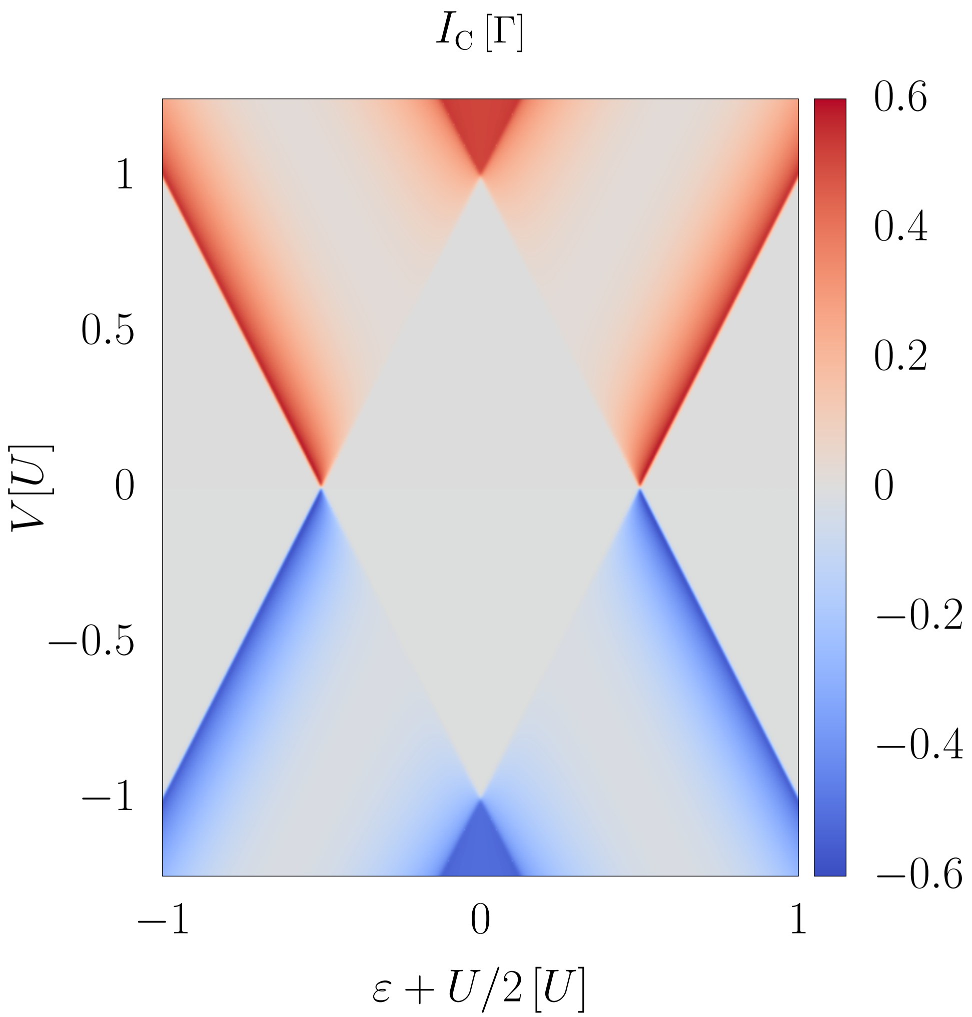
due to non-equilibrium
spin accumulation on dot





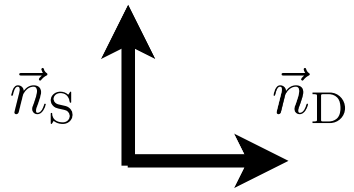
Orthogonal setup

$U = 10^5\,\Gamma$
$T = 300\,\Gamma$
$p = 0.99$



non-trivial transport due to precession of electron spin on dot in induced magnetic field

(Hell et al, PRB 2015)



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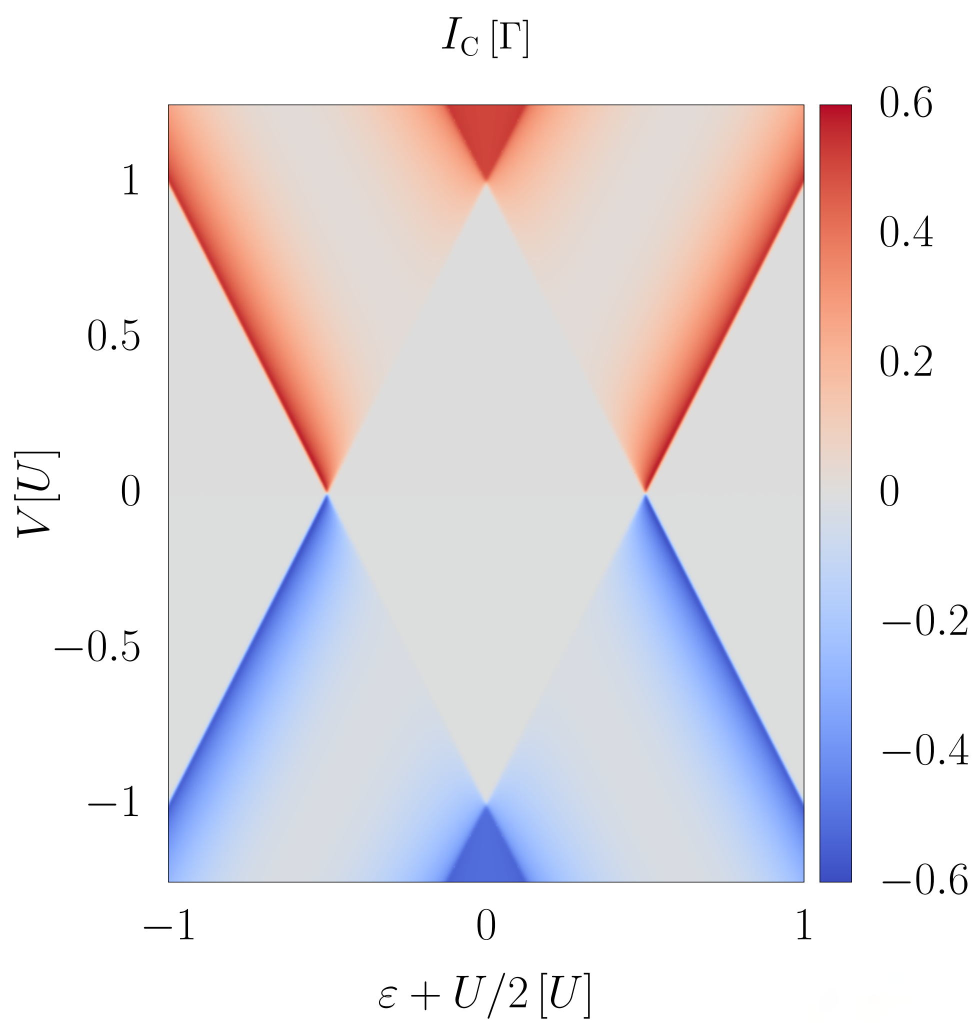

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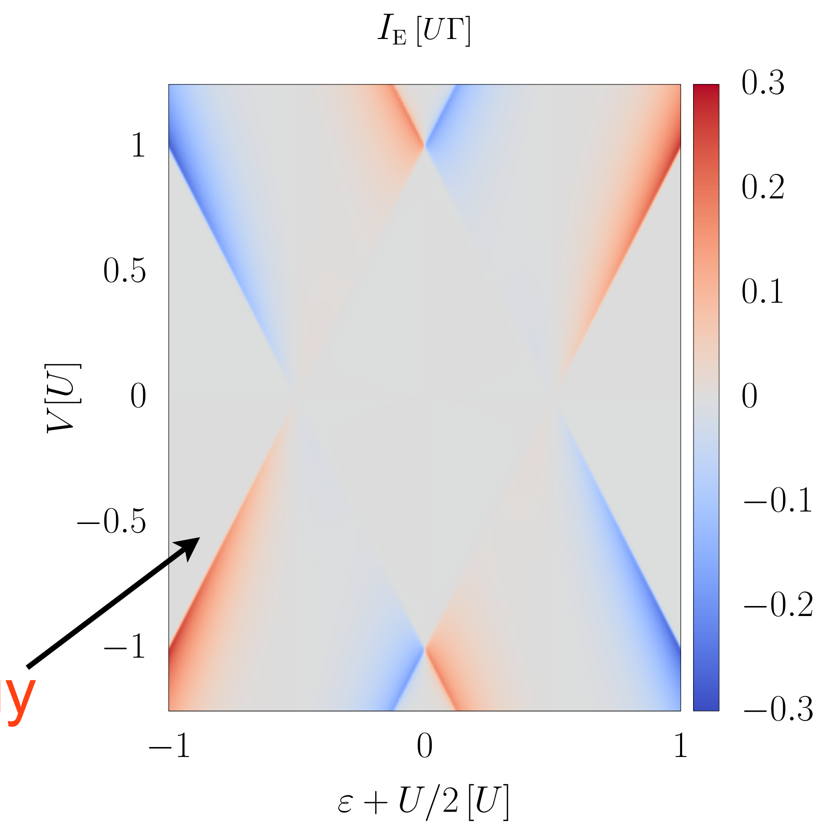
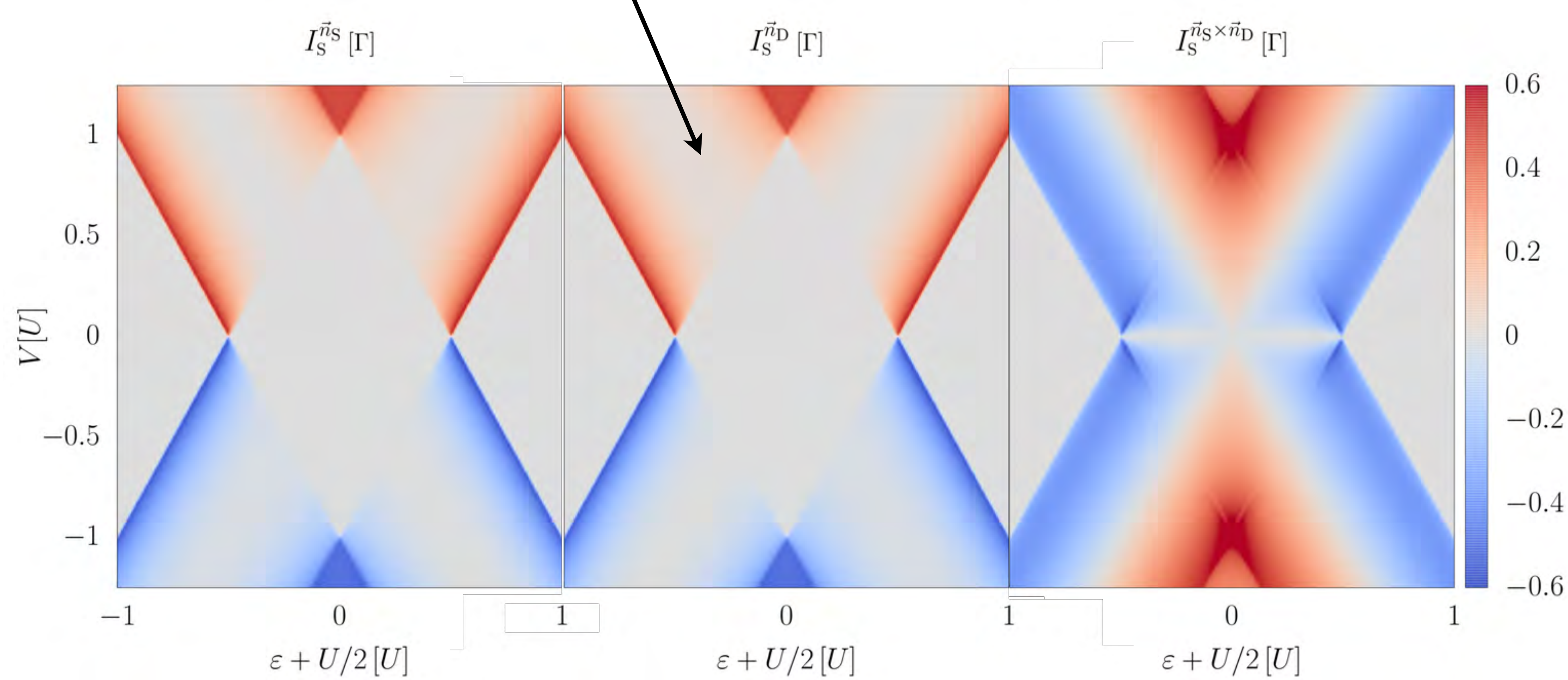
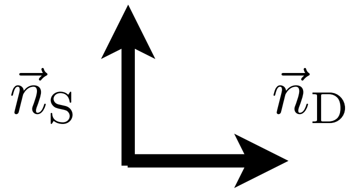


also visible in energy and spin transport





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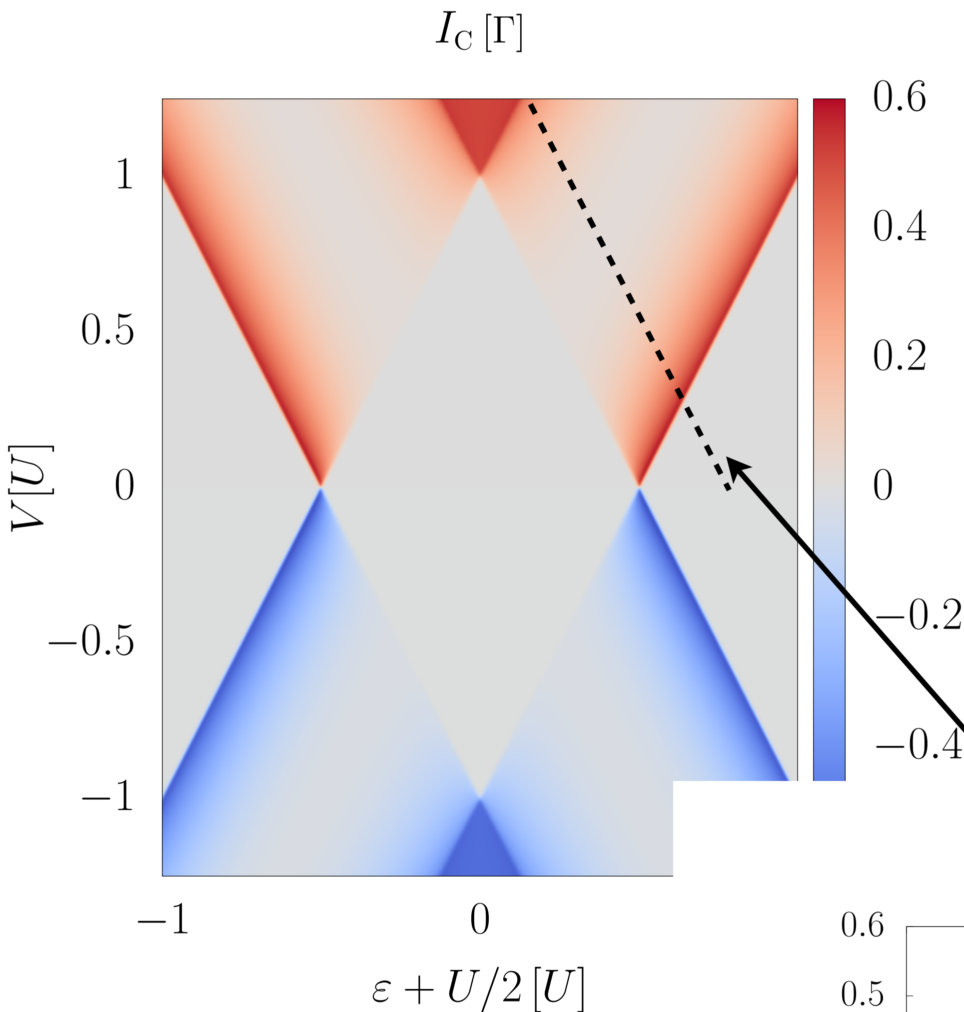
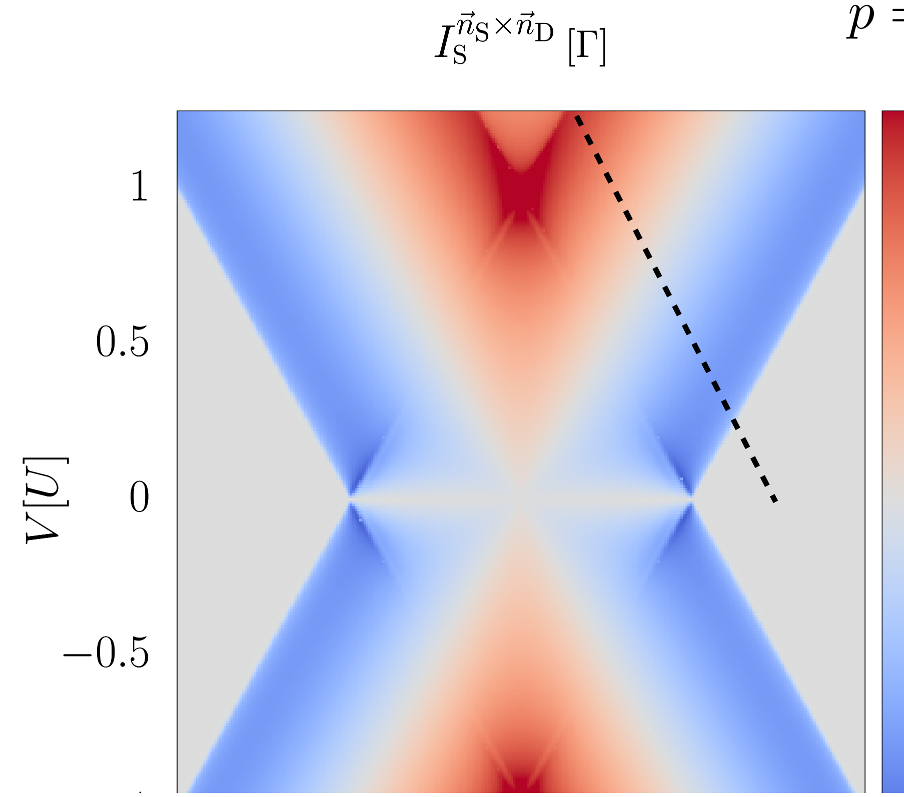
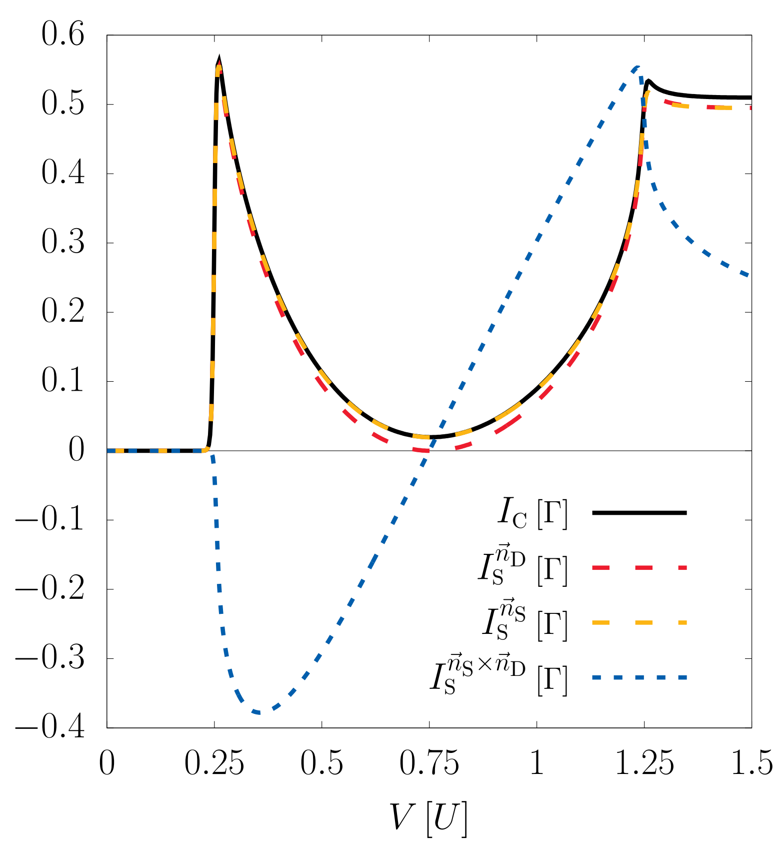
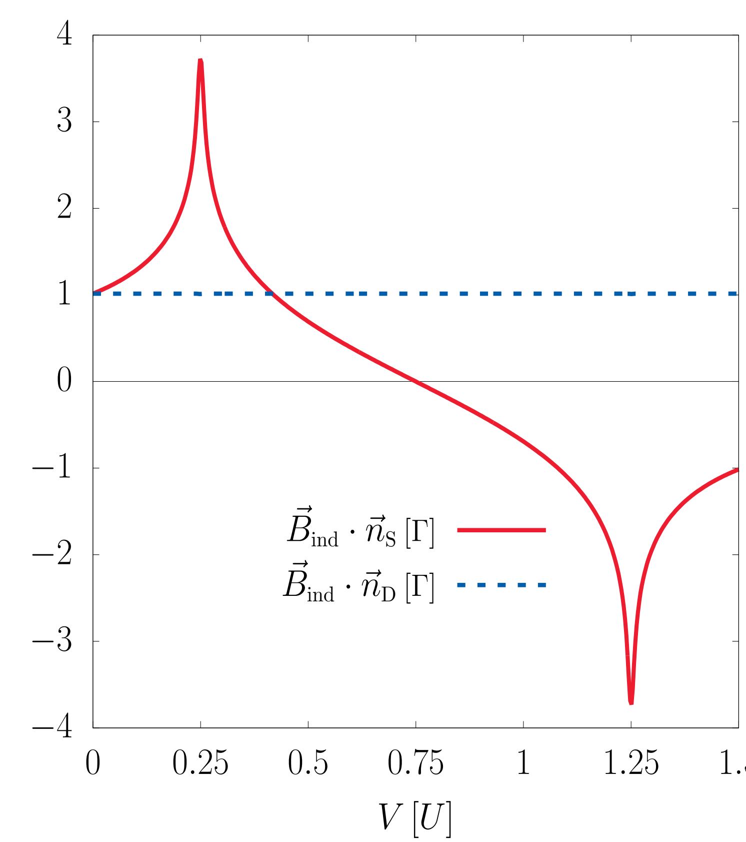




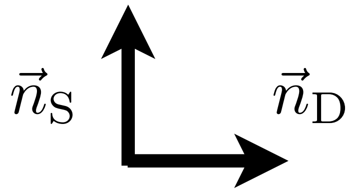
non-trivial
spin currents
and induced
magnetic field

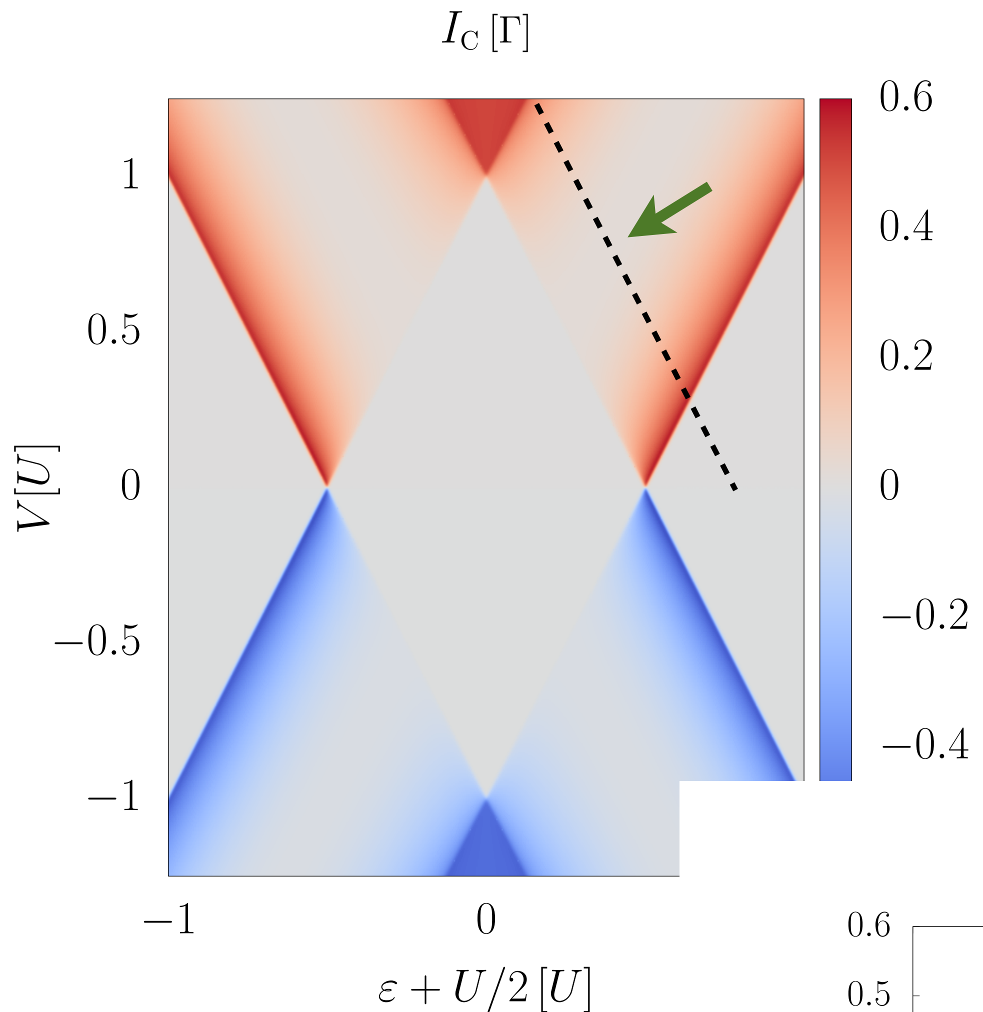


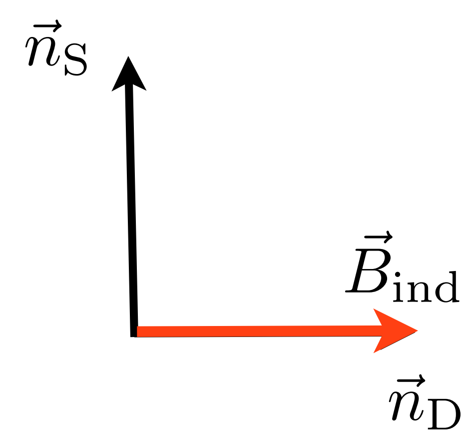


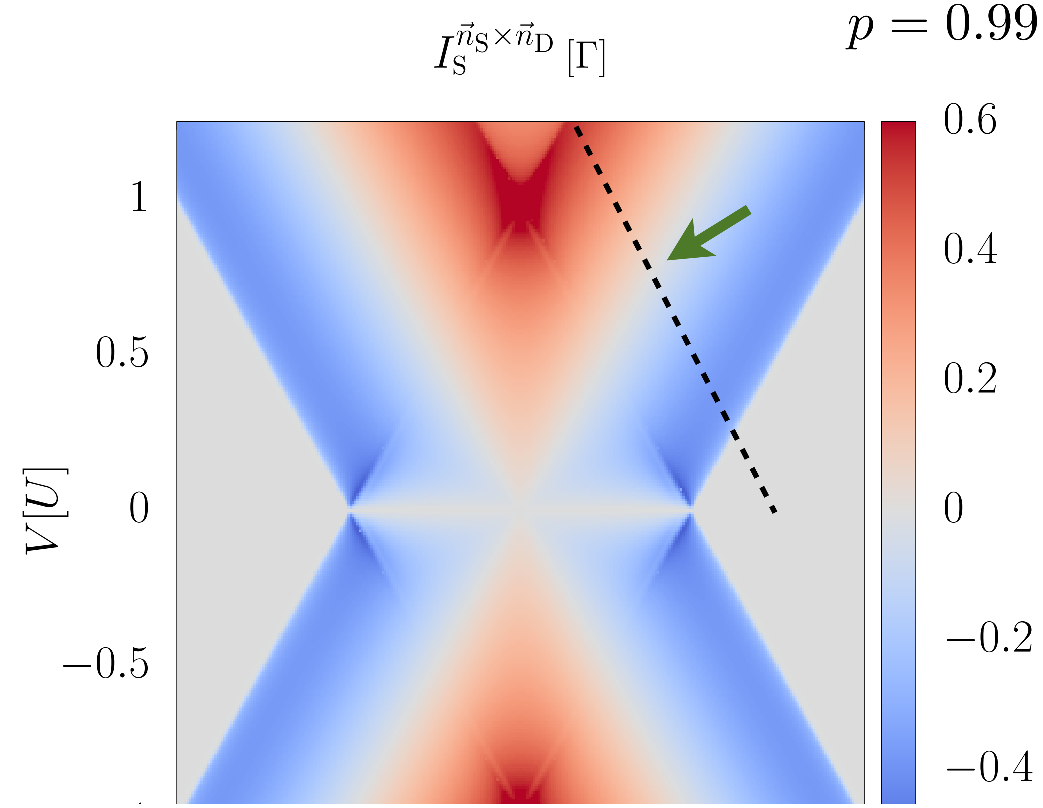
Orthogonal setup

$U = 10^5\,\Gamma$
$T = 300\,\Gamma$
$p = 0.99$





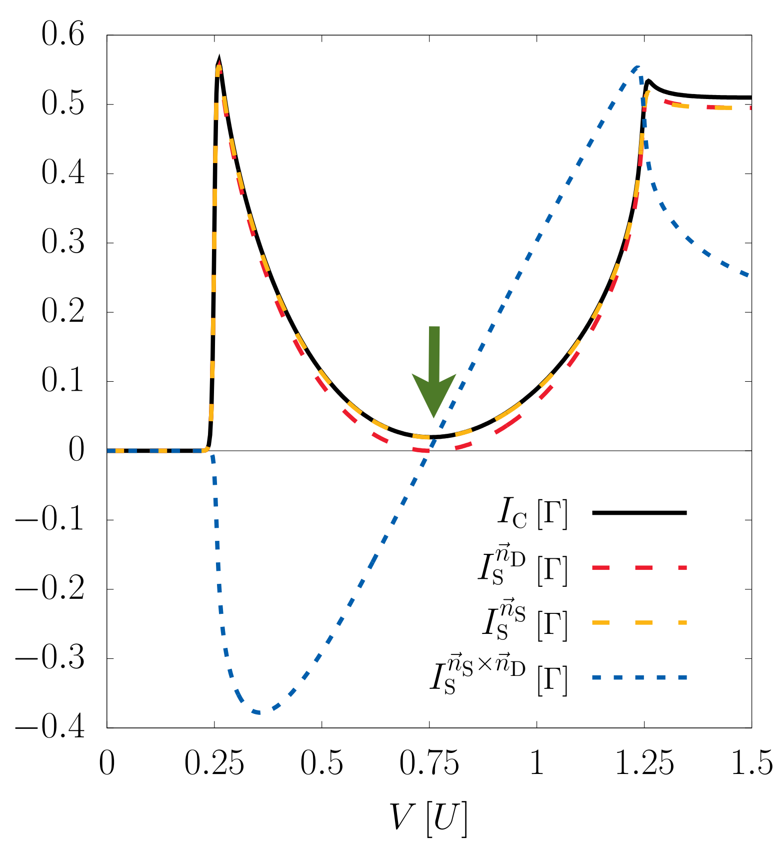


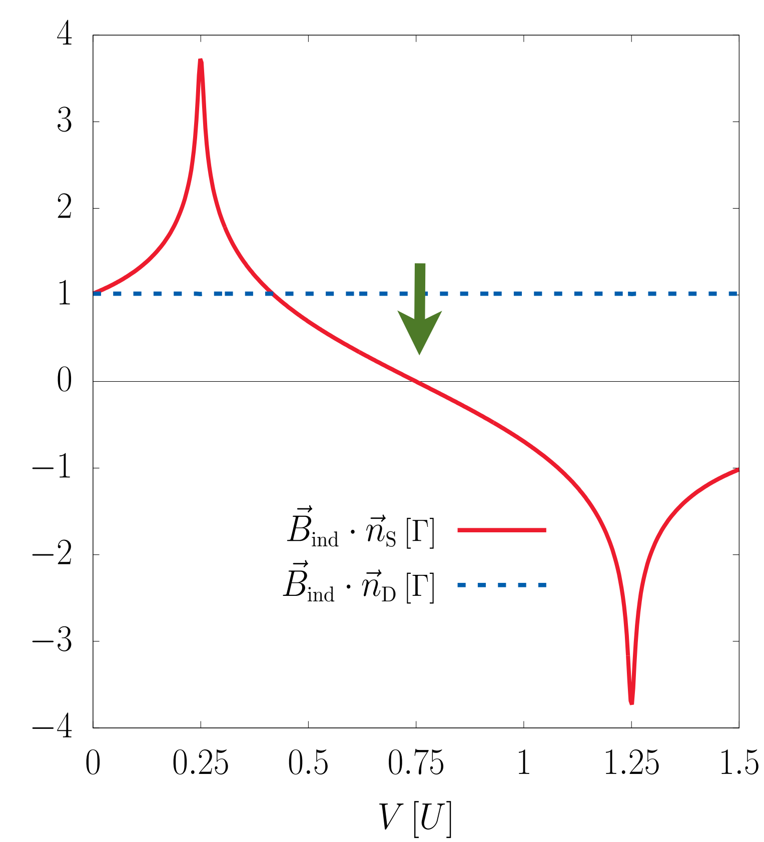


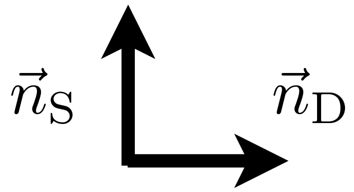
$\vec{B}_\mathrm{ind} \parallel \vec{n}_\mathrm{D}$

no precession of electron spin in $\vec{n}_\mathrm{D}$-direction

$I_\mathrm{S}^{\vec{n}_\mathrm{D}} = 0$





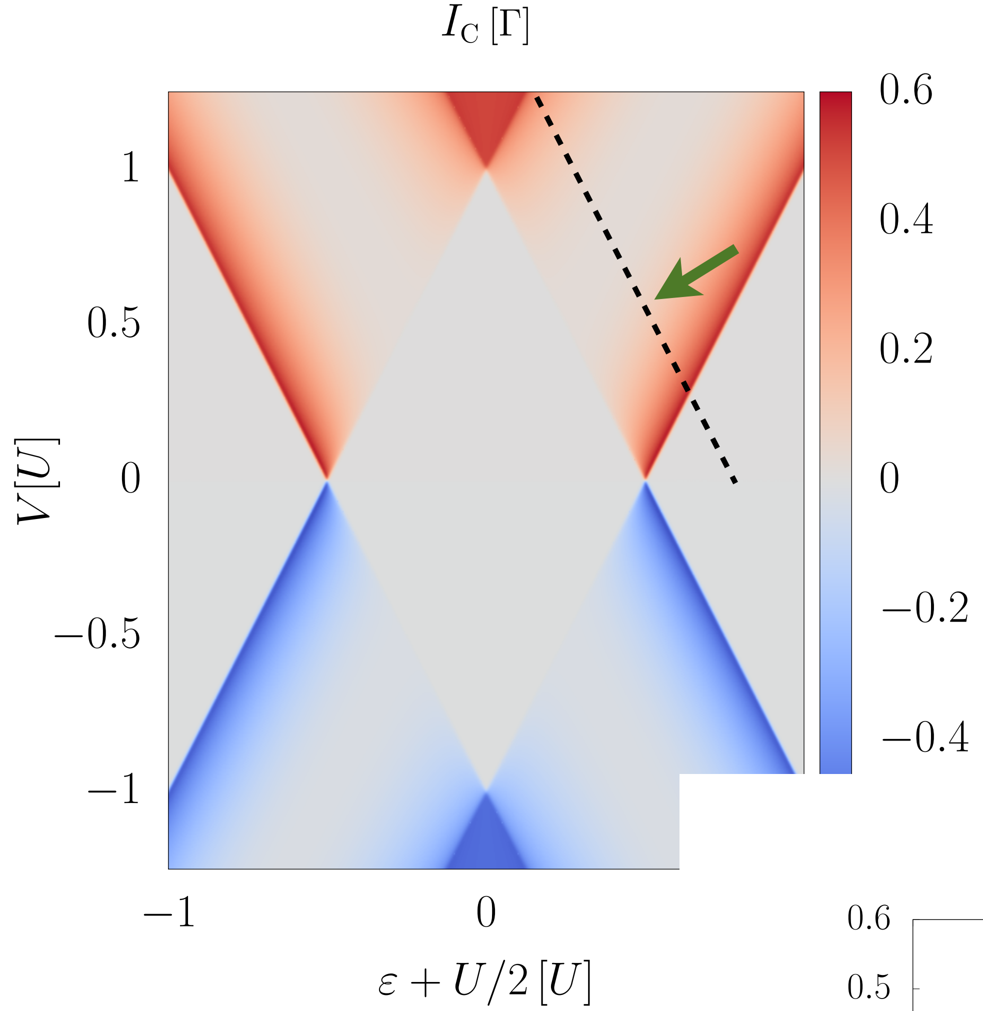


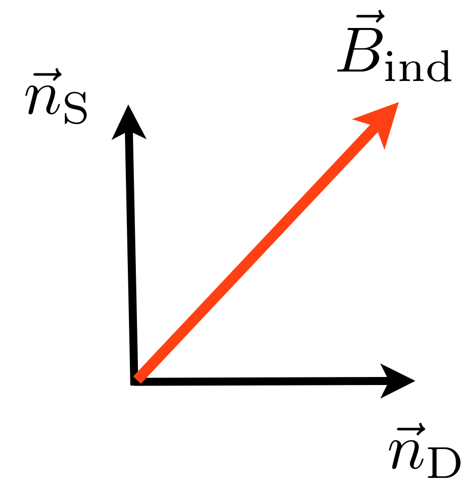
Orthogonal setup

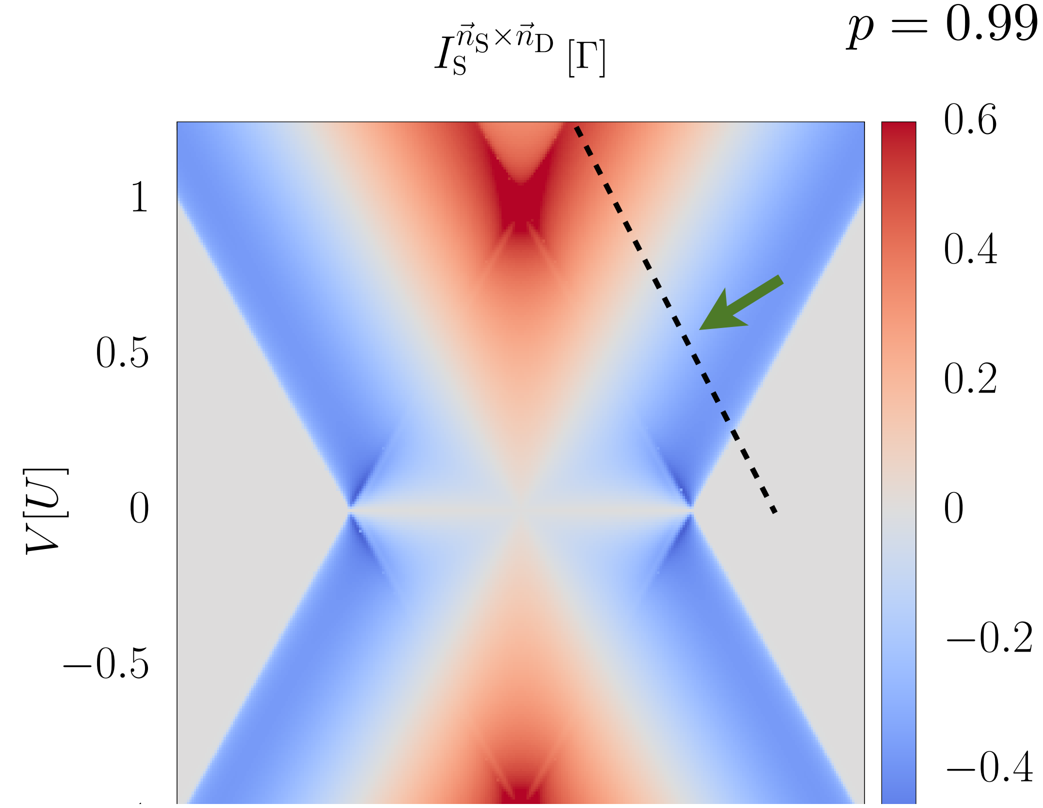
$U = 10^5\,\Gamma$

$T = 300\,\Gamma$

$p = 0.99$





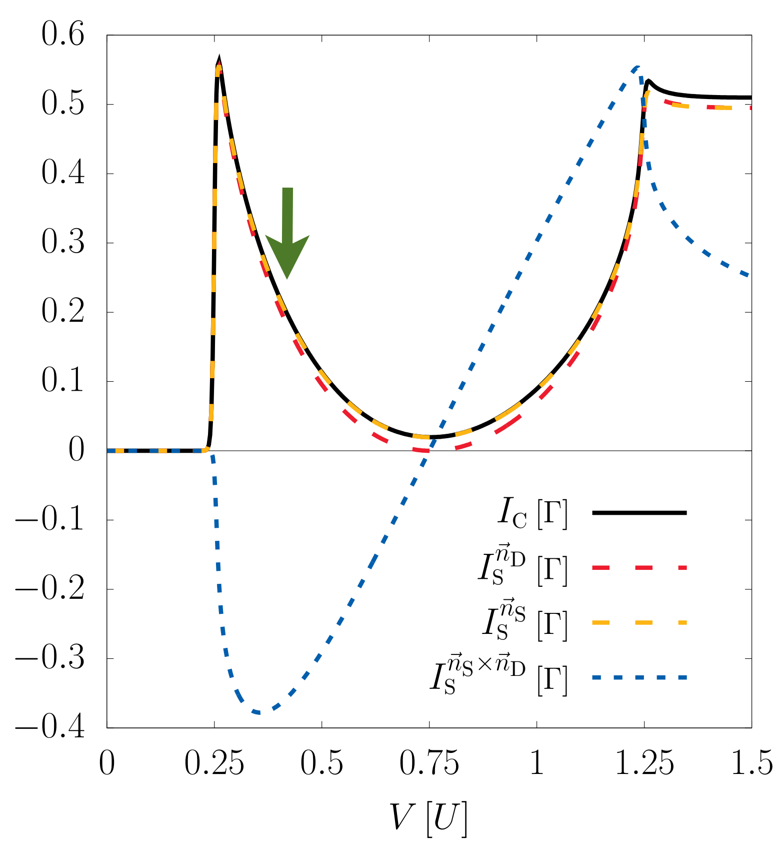


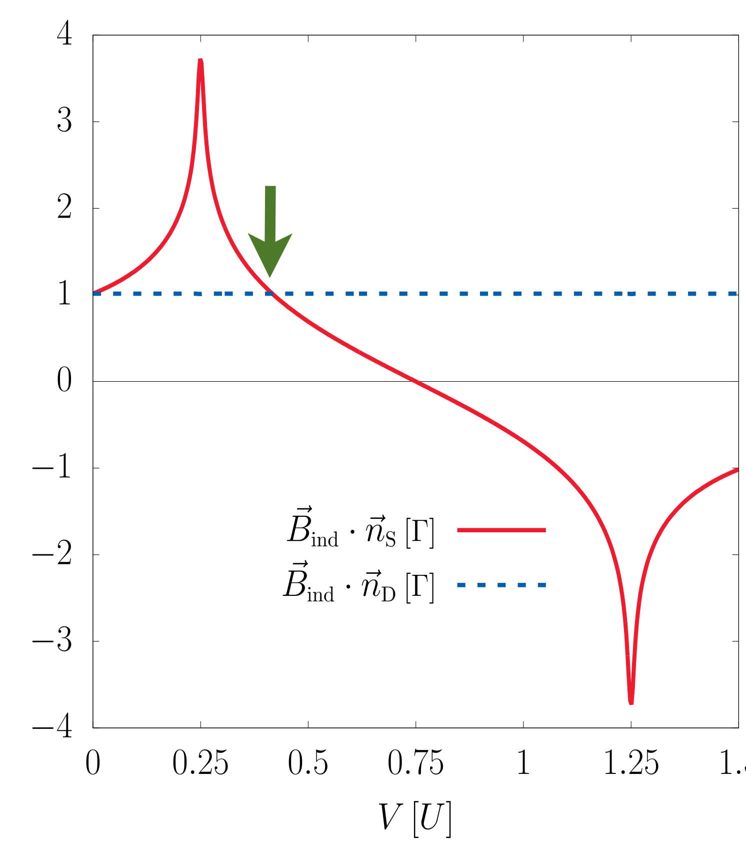
$\vec{B}_\mathrm{ind} \parallel (\vec{n}_\mathrm{S} + \vec{n}_\mathrm{D})$

precession of electron spin

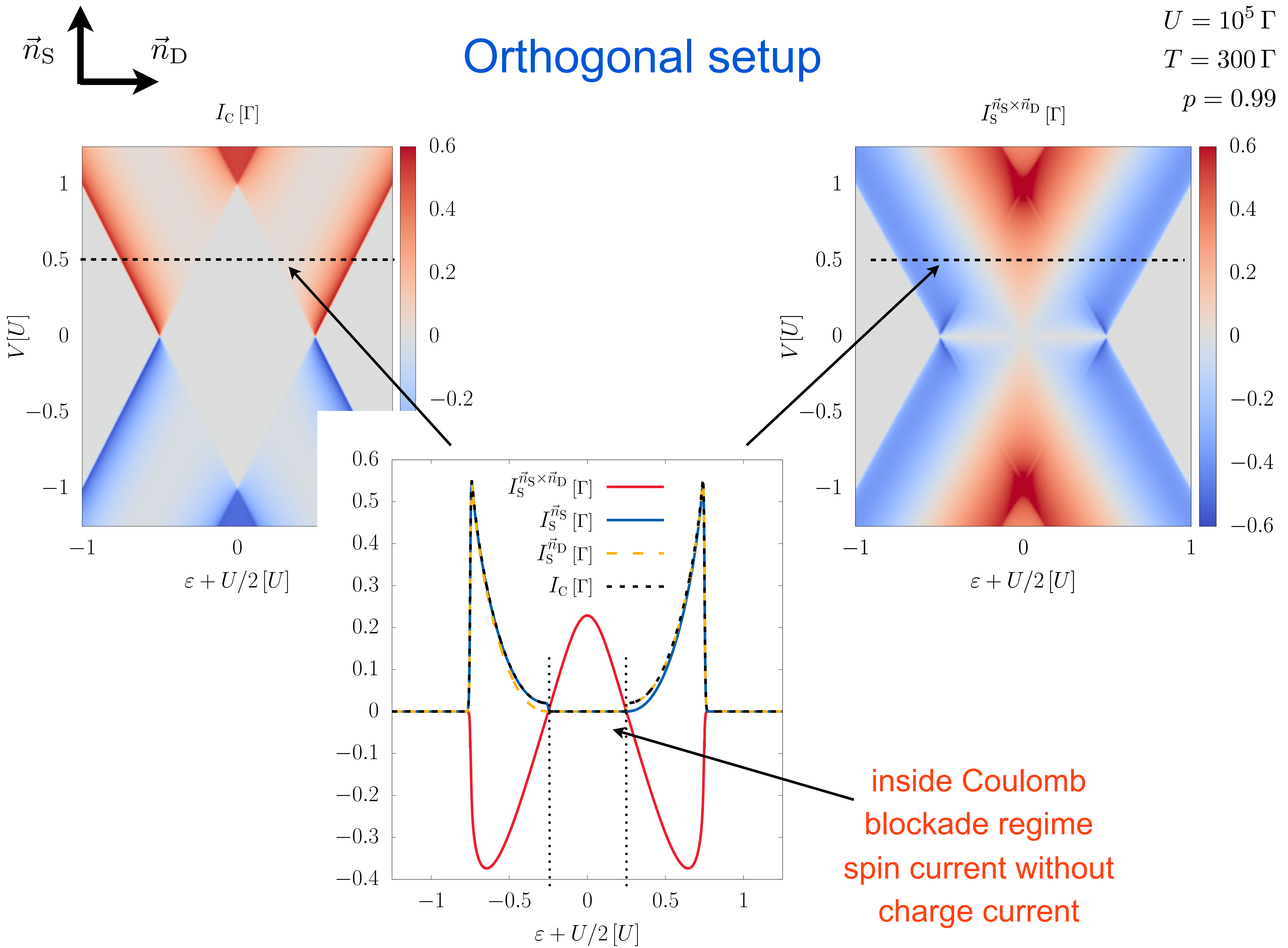
$I_\mathrm{S}^{\vec{n}_\mathrm{D}} \neq 0$

$I_\mathrm{S}^{\vec{n}_\mathrm{D}\times\vec{n}_\mathrm{S}} \neq 0$





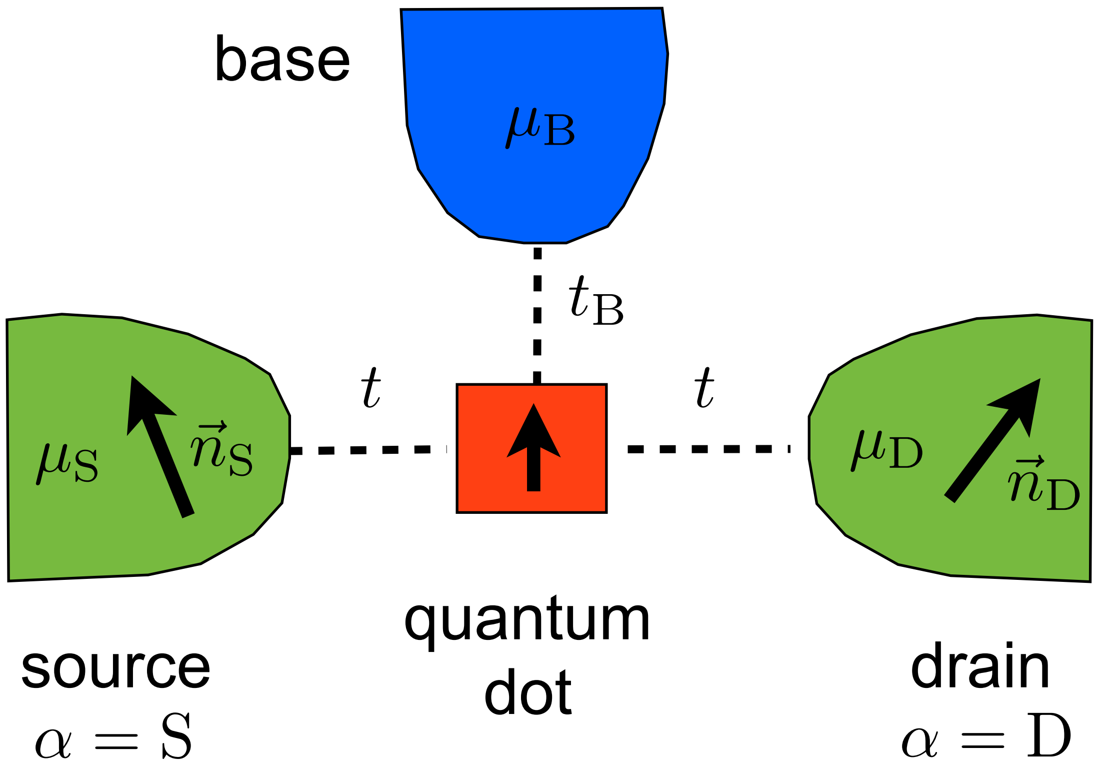
Orthogonal setup



Controllable spin valve

(in progress, see also poster by Niklas Gergs)

Controllable spin valve



add base reservoir

chemical potential $\mu_{\rm B}$

tunneling rate $\Gamma_{\rm B} = 2\pi\rho_0 t_{\rm B}^2 \gg \Gamma$

tunneling to base possible

$$H = H_{\rm res} + H_{\rm d} + H_{\rm tun} + H_{\rm B}$$

$$H_{\rm res} = \sum_{\alpha k\sigma} \epsilon_k c^\dagger_{\alpha k\sigma} c_{\alpha k\sigma}$$

electrodes, density of states $\rho_\pm = (1 \pm p)\rho_0$, polarisation *p*

$$H_{\rm d} = \sum_\sigma \varepsilon\, d^\dagger_\sigma d_\sigma + U d^\dagger_\uparrow d_\uparrow d^\dagger_\downarrow d_\downarrow$$

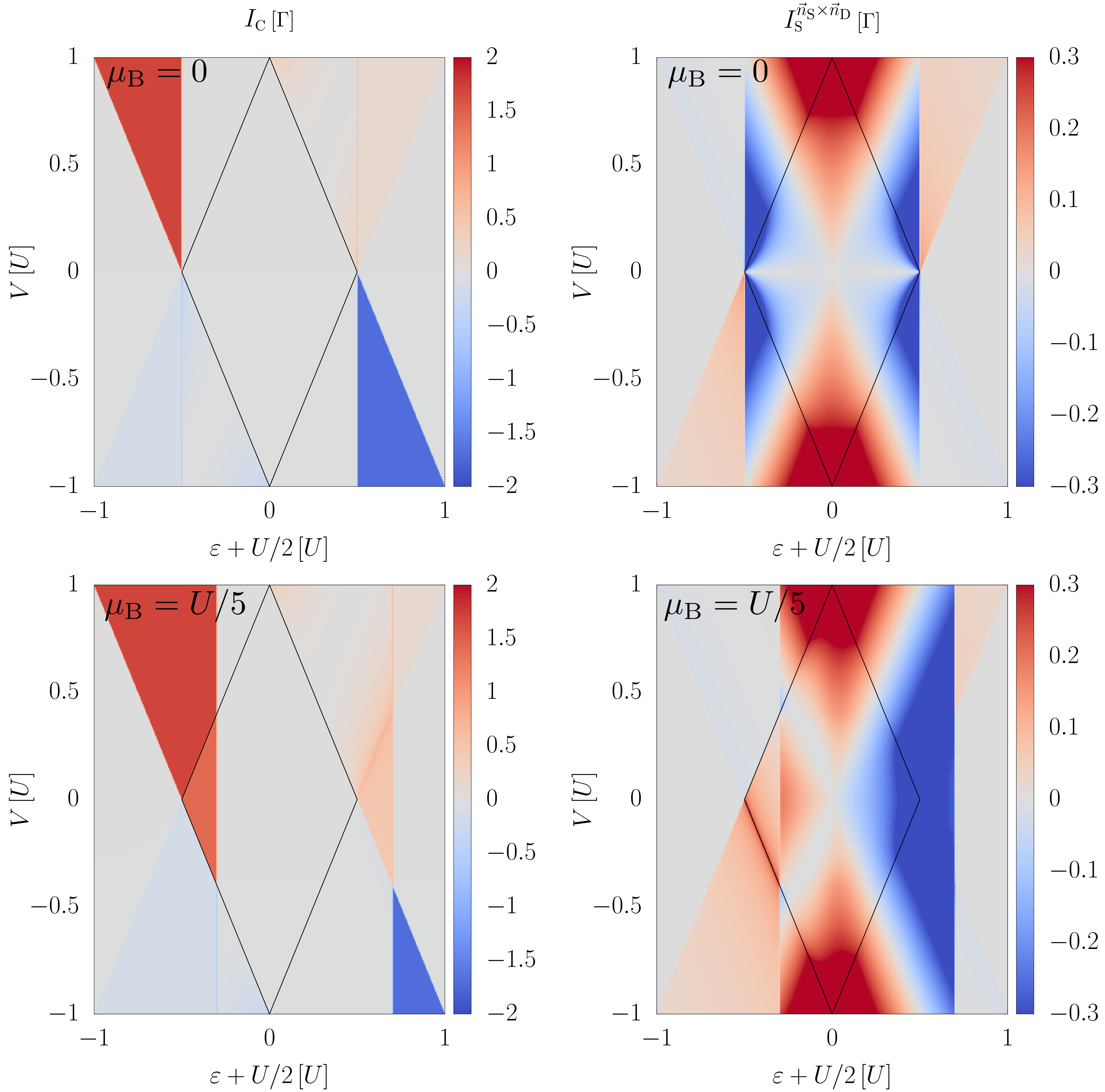
quantum dot, gate voltage ε, Coulomb repulsion *U*

$$H_{\rm tun} = t \sum_{\alpha k\sigma} \left(c^\dagger_{\alpha k\sigma} d_\sigma + d^\dagger_\sigma c_{\alpha k\sigma} \right)$$

tunneling, rate $\Gamma = 2\pi\rho_0 t^2$

Controllable spin valve

$U = 10^6\,\Gamma$
$T = 300\,\Gamma$
$p = 0.99$
$\Gamma_B = 10\,\Gamma$

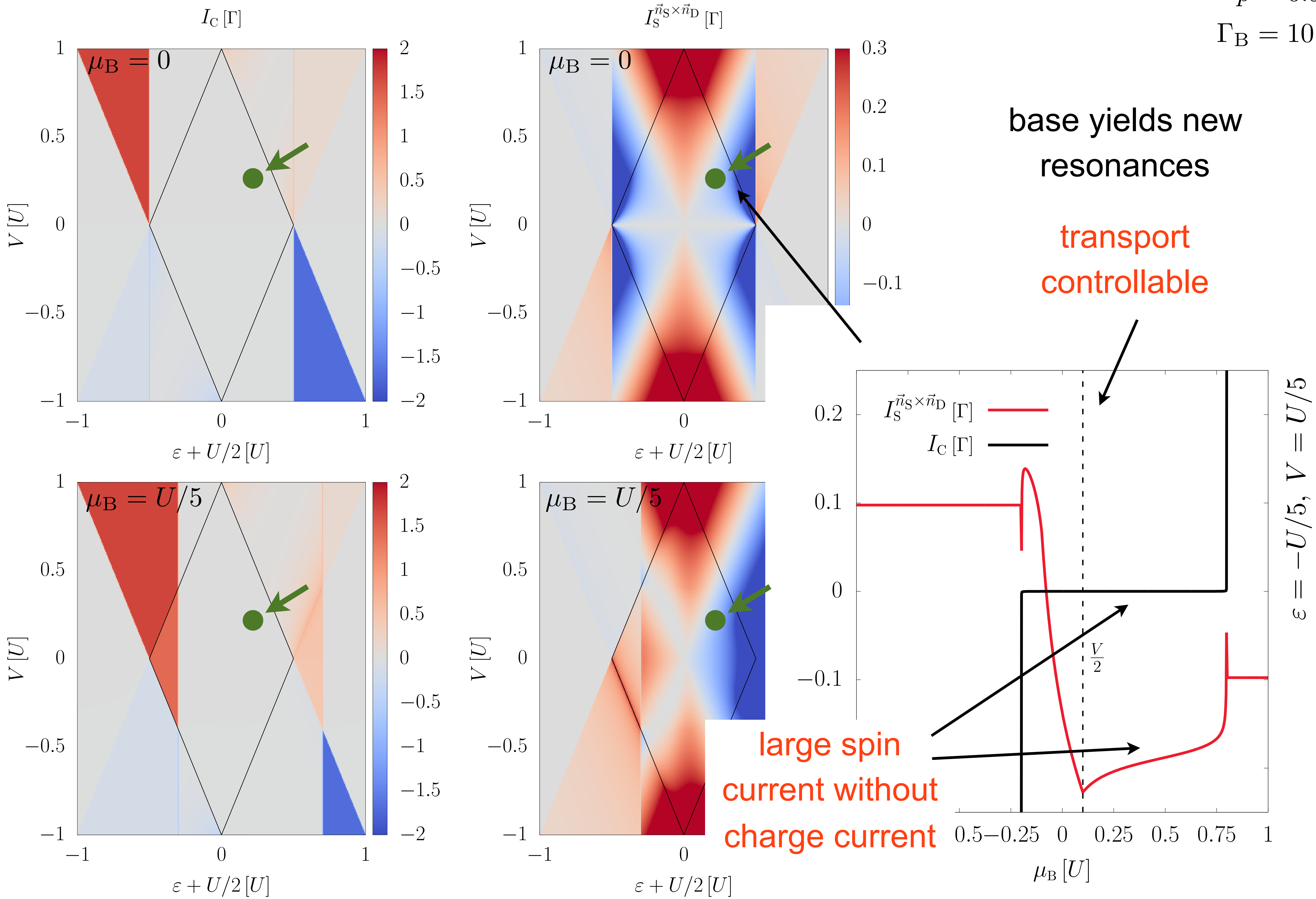


base yields new resonances

transport controllable

Controllable spin valve

$U = 10^6\,\Gamma$
$T = 300\,\Gamma$
$p = 0.99$
$\Gamma_B = 10\,\Gamma$



Conclusions

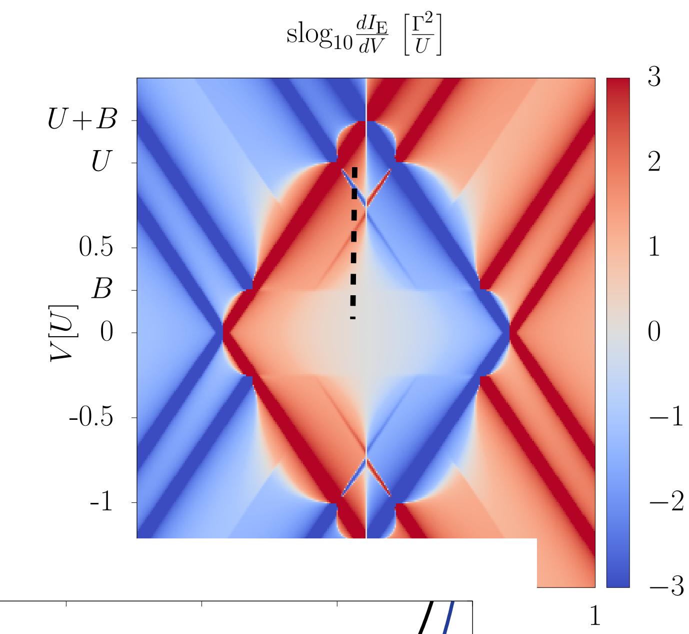
- showed a lot of plots
- discussed basic transport processes in quantum dots

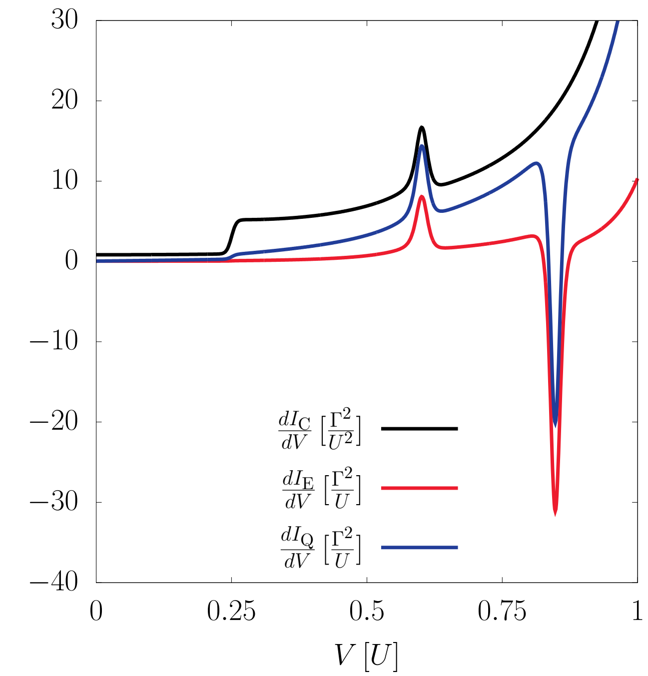
  single-electron tunneling, elastic and inelastic cotunneling, cotunneling assisted single-electron tunneling
- processes are well visible in energy/heat transport
- spin precession in induced magnetic field in spin-valve setup

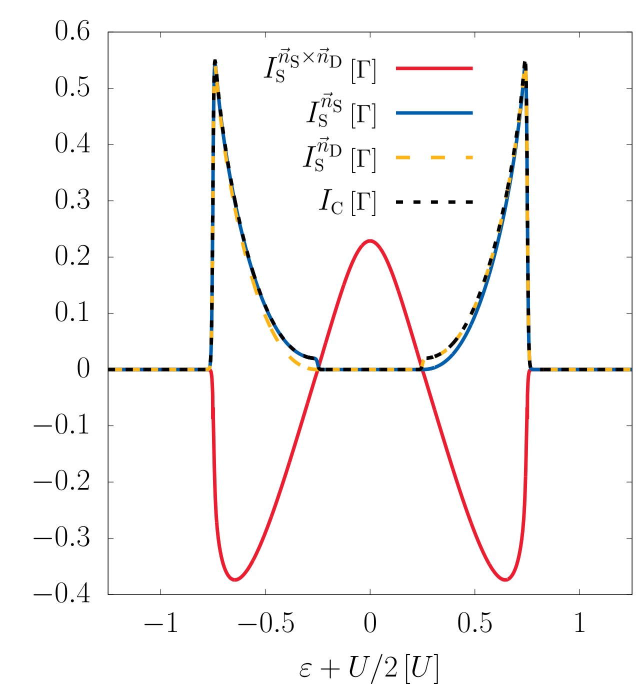
  spin current without charge current in Coulomb blockade regime
- controllable spin transport in three-terminal setup

  see also poster by Niklas Gergs
- reference for energy transport:

  Phys. Rev. B 91, 201107(R) (2015)





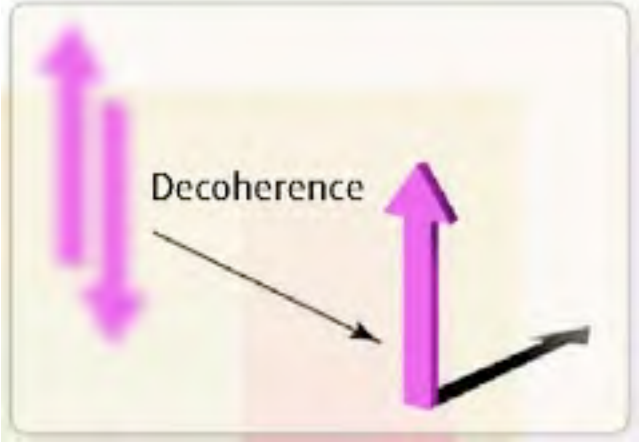


Relaxation dynamics in Kondo quantum dots

Pletyukhov, Schuricht & Schoeller, PRL 104, 106801 (2010)

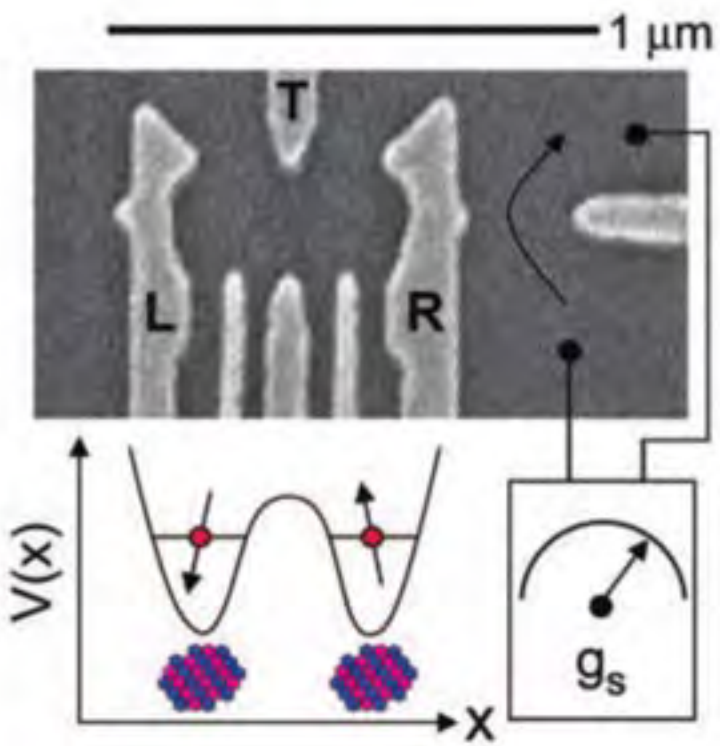
Time evolution in quantum dots

studies e.g. in the context of quantum information theory, q-bits, error correction, ...

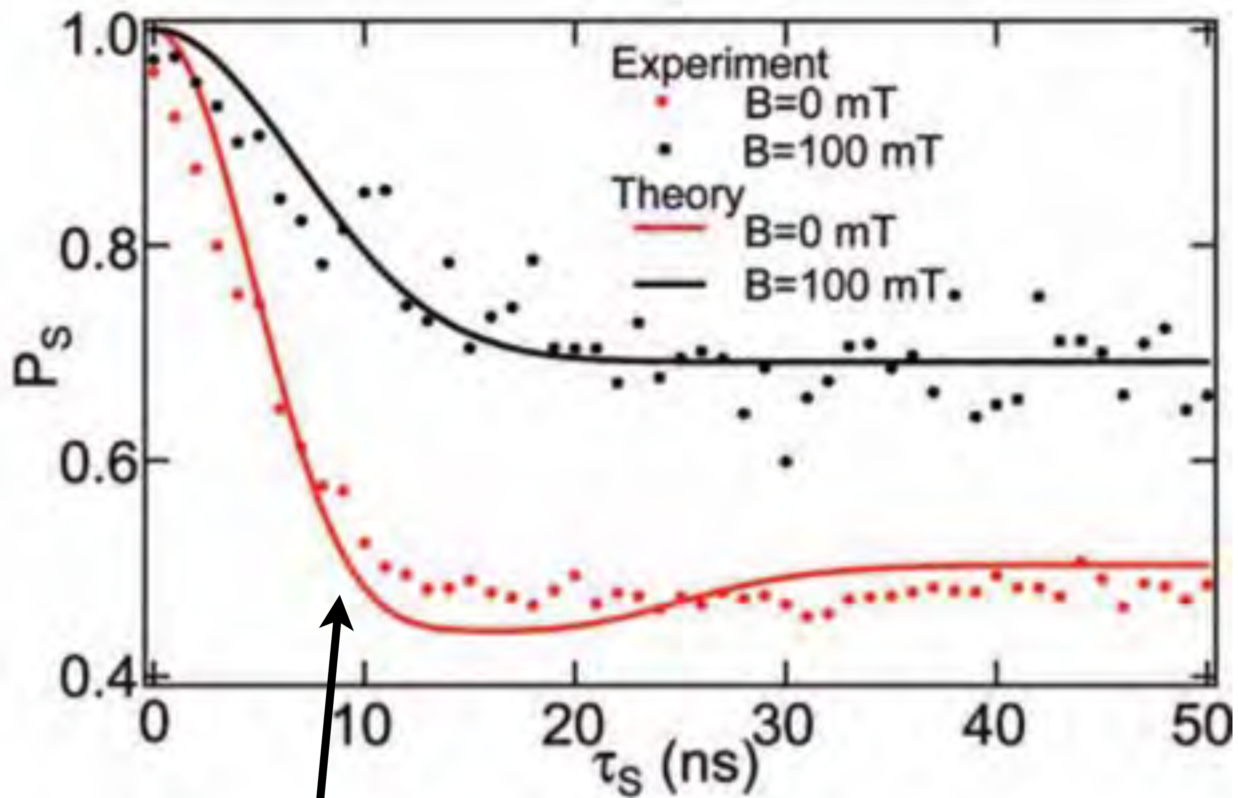


Fisher & Loss, Nature 2007

GaAs/AlGaAs double dot
(Petta et al., Science 2005)



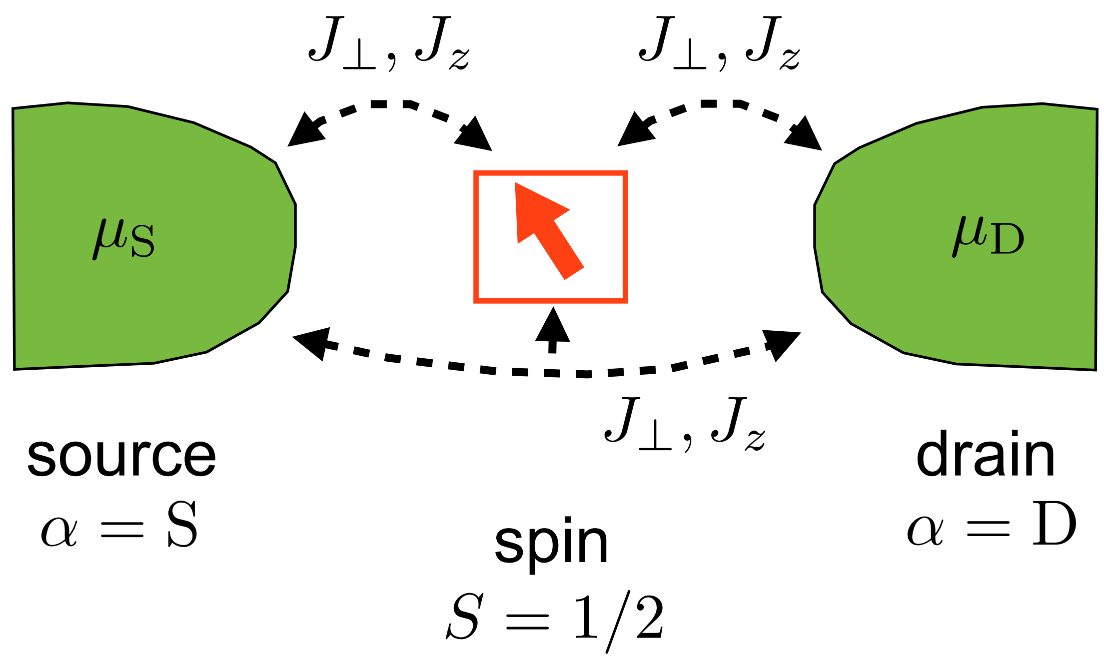
two-state system from singlet and *m*=0 triplet



dephasing time
$T_2^* \sim 10\,\mathrm{ns}$

limited by hyperfine interactions with substrate nuclei

Anisotropic Kondo model



obtained from Anderson model by Schrieffer-Wolff transformation

$$H = H_{\mathrm{res}} + H_{\mathrm{d}} + H_{\mathrm{tun}}$$

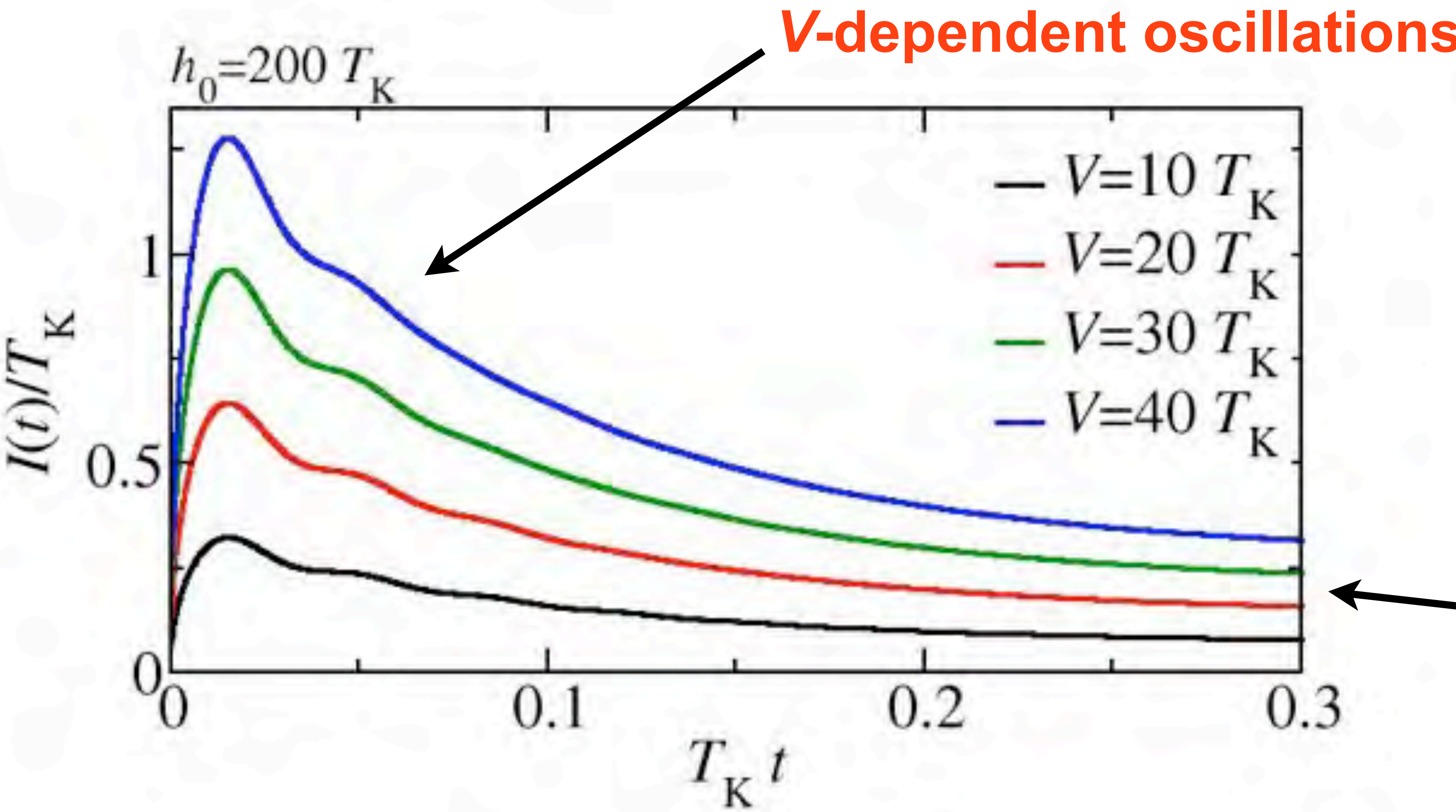
$$H_{\mathrm{res}} = \sum_{\alpha k \sigma} \epsilon_k c^\dagger_{\alpha k \sigma} c_{\alpha k \sigma}$$ electrodes, density of states ρ_0

$$H_{\mathrm{d}} = h S^z$$ quantum dot, spin-1/2, external magnetic field h

$$H_{\mathrm{tun}} = J_\perp \left(S^x s^x + S^y s^y \right) + J_z S^z s^z$$ exchange interaction J

local reservoir spin $\vec{s} = \frac{1}{2} \sum_{\alpha\alpha' k k' \sigma\sigma'} c^\dagger_{\alpha k \sigma} \vec{\sigma}_{\alpha\alpha'} c_{\alpha' k' \sigma'}$

Relaxation in anisotropic Kondo model



decay rates
$$\Gamma_i \sim J_c^2\ \max\{V, h, T\}$$

relaxation to stationary value

all decay rates appear

oscillations and algebraic decay

off resonance: $I(t > 1/\Lambda_c) = I_{\rm st} - \pi\, J_\perp^2\, M_{\rm st}\ \min\{V, h\}\, e^{-\Gamma_1 t}$

$$- \frac{J_\perp^2}{2}\, e^{-\Gamma_2 t} \left[\frac{\sin\big((h+V)t\big)}{(h+V)t^2} - \frac{\sin\big((h-V)t\big)}{(h-V)t^2} \right]$$

on resonance: divergencies cut-off by decay rates $\dfrac{1}{(\Gamma_1 - \Gamma_2)t^2}$

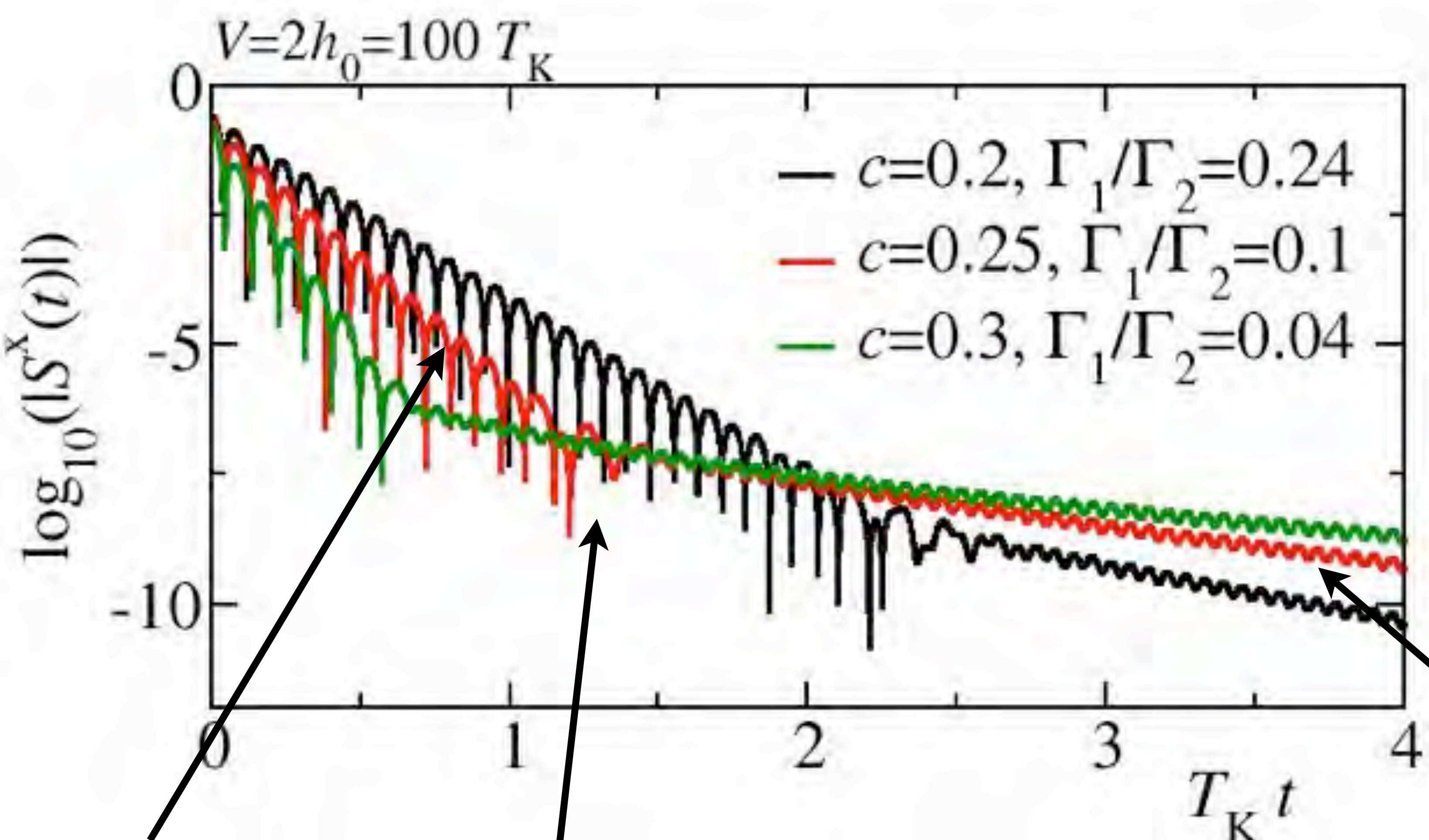
Strongly anisotropic model

e.g. in single molecule magnets $J_z \gg J_\perp$

(Romeike et al., PRL 2006)

at finite bias voltage: $$\frac{\Gamma_2}{\Gamma_1} = \frac{1}{2} + \frac{J_z^2}{J_\perp^2}\frac{V}{2(h + \max\{V, h\})} \gg 1$$

$\Rightarrow$ **separation of time scales**



anisotropy

$$c^2 = J_z^2 - J_\perp^2$$

oscillation with h

$$t \sim \frac{2|\ln J_\perp|}{\Gamma_2 - \Gamma_1}$$

oscillation with V

Universal short-time behaviour

short times: $t < \dfrac{1}{\max\{V,h\}}$ $\Longrightarrow$ $\Lambda_t = \dfrac{1}{t}$

$\Longrightarrow$ $J_t = J(\Lambda_t) = -\dfrac{1}{2\ln(T_{\mathrm{K}}t)}$ (isotropic model)

magnetization $\langle\vec{S}(t)\rangle = \left[1 + \dfrac{1}{\ln(T_{\mathrm{K}}t)}\right]\langle\vec{S}(0)\rangle$

conductance $G(t) = \dfrac{I(t)}{V} = \dfrac{e^2}{h}\dfrac{3\pi^2}{8\ln^2(T_{\mathrm{K}}t)}$ both for AFM and FM

anisotropic model: power laws with exponents $\propto \sqrt{J_z^2 - J_\perp^2}$

Hackl et al., PRL 2009: FM model

$\langle S^z(t)\rangle, V = h_0 = 0$