Panoply of orders near quantum Lifshitz point of a frustrated ferromagnet

Oleg Starykh
University of Utah

Quantum Spintronics: Spin Transport Through Quantum Magnetic Materials,
SPICE, Johannes Gutenberg Universitat Mainz, Sept. 21-23, 2016

Life at (and near) University of Utah

Collaborators

Leon Balents, KITP, UCSB

Andrey Chubukov, FTPI, U Minnesota

Jason Alicea, Caltech

General motivation: **Exotic** *but* ordered phases

composite order parameter

$$\mathcal{O}^{\alpha\beta}(\mathbf{r}_i, \mathbf{r}_j) = \tfrac{1}{2}(S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha) - \tfrac{1}{3}\delta^{\alpha\beta}\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle$$

LE JOURNAL DE PHYSIQUE TOME 38, AVRIL 1977, PAGE 385

Classification
Physics Abstracts
7.480 — 8.514

A MAGNETIC ANALOGUE OF STEREOISOMERISM : APPLICATION TO HELIMAGNETISM IN TWO DIMENSIONS

J. VILLAIN

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Centre d'Etudes Nucléaires de Grenoble, 85 X, 38041 Grenoble Cedex, France

(Reçu le 13 juillet 1976, révisé le 8 novembre 1976, accepté le 4 janvier 1977)

Spin nematics

A. F. Andreev and I. A. Grishchuk

Institute of Physics Problems, USSR Academy of Sciences

(Submitted 6 March 1984)
Zh. Eksp. Teor. Fiz. **87**, 467–475 (August 1984)

We investigate possible properties of exchange magnets in which the onset of magnetic order leads to spontaneous violation of the isotropy of the spin space, but invariance to time reversal is preserved. These magnets do not differ from antiferromagnets in their macroscopic magnetic properties and can be identified only by neutron scattering or NMR investigations. The possibility of similar ordering in the nuclear system of solid ^{3}He is discussed.

Quantum magnetism *vs* Spintronics

Spin is almost conserved

- No dipolar coupling (small magnetic moments)
  Notable exception —> spin ice

- No coupling to phonons (basically isolated system of spins)
  Notable exception -> hybridization of magnons and phonons in **non-collinear** spin structures

- Spin transport —> mostly **thermal**

1) heat transport in chains/ladders
2) thermal transport in organic spin liquid candidate materials (spinon Fermi surface?)
3) magnon Hall effect (due to DM interactions)

Magnetic Coulomb Phase in the Spin Ice $Ho_2Ti_2O_7$

Science 2009

T. Fennell[1,*], P. P. Deen[1], A. R. Wildes[1], K. Schmalzl[2], D. Prabhakaran[3], A. T. Boothroyd[3], R. J. Aldus[4], D. F. McMorrow[4], S. T. Bramwell[4]

Spontaneous decays of magneto-elastic excitations in noncollinear antiferromagnet (Y,Lu)MnO_3

Joosung Oh[1,2], Manh Duc Le[1,2], Ho-Hyun Nahm[1,2], Hasung Sim[1,2], Jaehong Jeong[1,2], T. G. Perring[3], Hyungje Woo[3,4], Kenji Nakajima[5], Seiko Ohira-Kawamura[5], Zahra Yamani[6], Y. Yoshida[7], H. Eisaki[7], S.-W. Cheong[8], A. L. Chernyshev[9], and Je-Geun Park[1,2*]

arxiv:1609.03262

Nature Physics **5**, 44 - 47 (2009)
Published online: 23 November 2008 | doi:10.1038/nphys1134

Subject Category: Condensed-matter physics

Thermal-transport measurements in a quantum spin-liquid state of the frustrated triangular magnet κ-(BEDT-TTF)$_2Cu_2(CN)_3$

Minoru Yamashita[1], Norihito Nakata[1], Yuichi Kasahara[1,2], Takahiko Sasaki[2], Naoki Yoneyama[2], Norio Kobayashi[2], Satoshi Fujimoto[1], Takasada Shibauchi[1] & Yuji Matsuda[1]

Observation of the Magnon Hall Effect

Y. Onose[1,2,*], T. Ideue[1], H. Katsura[3], Y. Shiomi[1,4], N. Nagaosa[1,4], Y. Tokura[1,2,4]

- Author Affiliations

*To whom correspondence should be addressed. E-mail: onose@ap.t.u-tokyo.ac.jp

Science 16 Jul 2010
Vol. 329, Issue 5989, pp. 297-299
DOI: 10.1126/science.1188260

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arxiv:1609.03262

Nature Physics **5**, 44 - 47 (2009)
Published online: 23 November 2008 | doi:10.1038/nphys1134

arXiv.org > cond-mat > arXiv:1609.06410

Condensed Matter > Strongly Correlated Electrons

Observation of spin current in quantum spin liquid

Daichi Hirobe, Masahiro Sato, Takayuki Kawamata, Yuki Shiomi, Ken-ichi Uchida, Ryo Iguchi, Yoji Koike, Sadamichi Maekawa, Eiji Saitoh

(Submitted on 21 Sep 2016)

Spin liquid is a state of electron spins in which quantum fluctuation breaks magnetic ordering while maintaining spin correlation. It has been a central topic in magnetism because of its relevance to high-Tc superconductivity and topological states. However, utilizing spin liquid has been quite difficult. Typical spin liquid states are realized in one-dimensional spin systems, called quantum spin chains. Here, we show that a spin liquid in a spin-1/2 quantum chain generates and carries spin current via its long-range spin fluctuation. This is demonstrated by observing an anisotropic negative spin Seebeck effect along the spin chains in Sr2CuO3. The results show that spin current can flow even in an atomic channel owing the spin liquid state, which can be used for atomic spin-current wiring.

Outline

- Magnon BEC
- Materials
- Basic theory and some numerics
- *Field theory of the Lifshitz point*

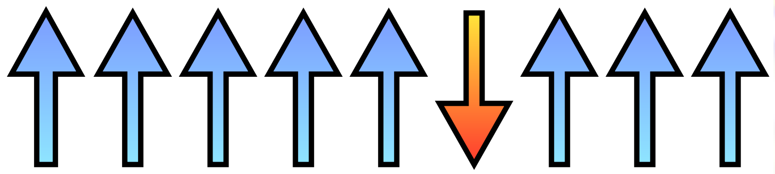
  magnon "BCS" superconductor
- *Spin-current state* near the end-point of 1/3 magnetization plateau

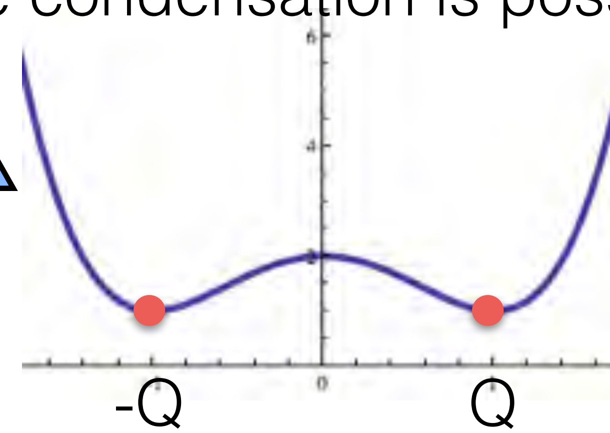
  exotic superconductor of magnons

Magnon BEC and superfluidity

frustration shows up via presence of two or more degenerate minima where condensation is possible

1-magnon



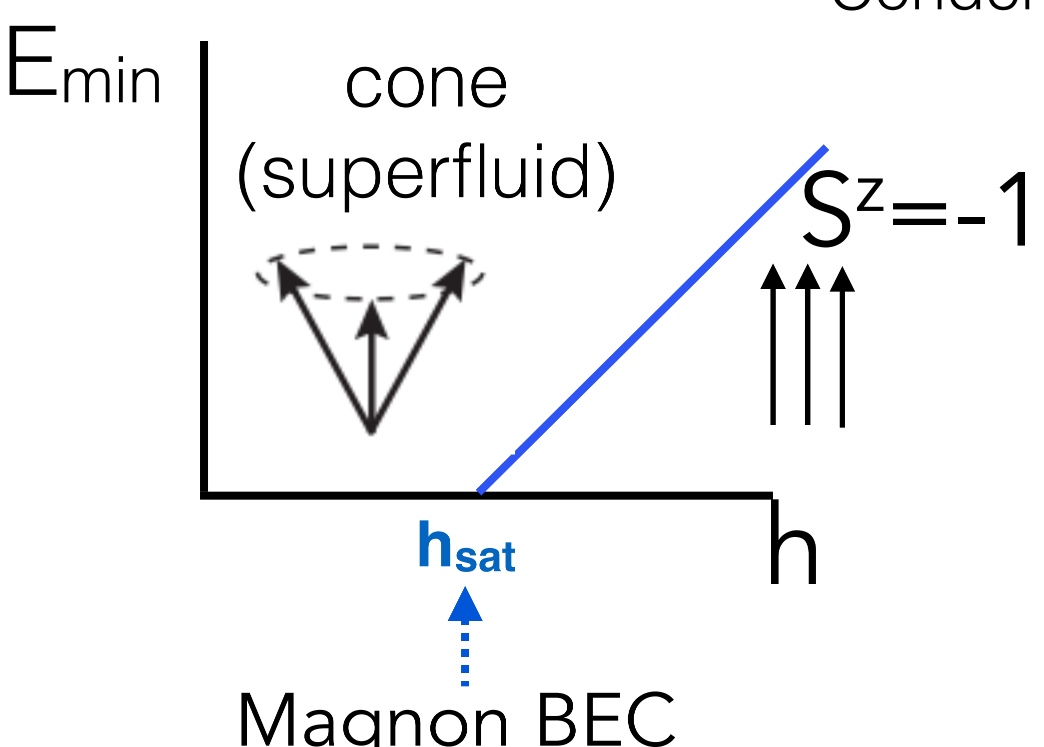


$$\omega \sim (k^2 - Q^2)^2 - (h_{\rm sat} - h)$$

$$h_{\rm sat} = \frac{S(4J_2 - |J_1|)^2}{4J_2}$$

chirality!

Condensation at one of the two minima —> $U(1) \times Z_2$



$$\langle a_k^+ \rangle = \sqrt{N}\ \Psi_Q\ \delta_{k,Q}$$

$$\langle S_n^- \rangle \sim \Psi_Q\ e^{iQx_n}$$

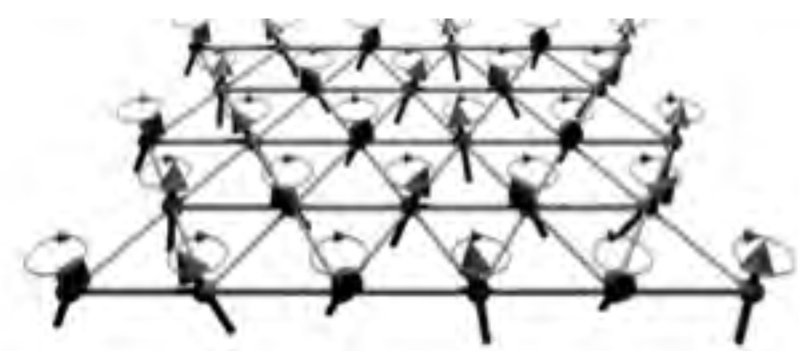
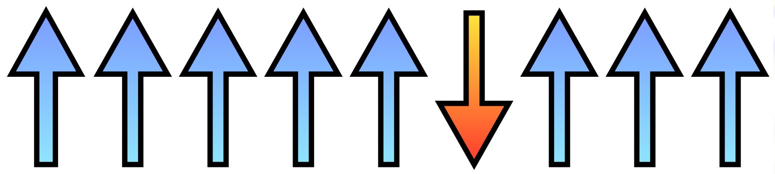


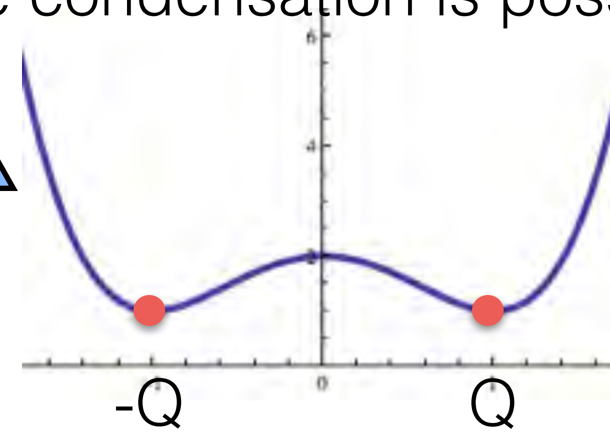
FIG. 3. A single triangular layer of the cone state, illustrated for a field along the a axis. Circles with arrows indicate the sense of precession of the spins, as one moves along the x axis. This is most

Magnon BEC and superfluidity

frustration shows up via presence of two or more degenerate minima where condensation is possible

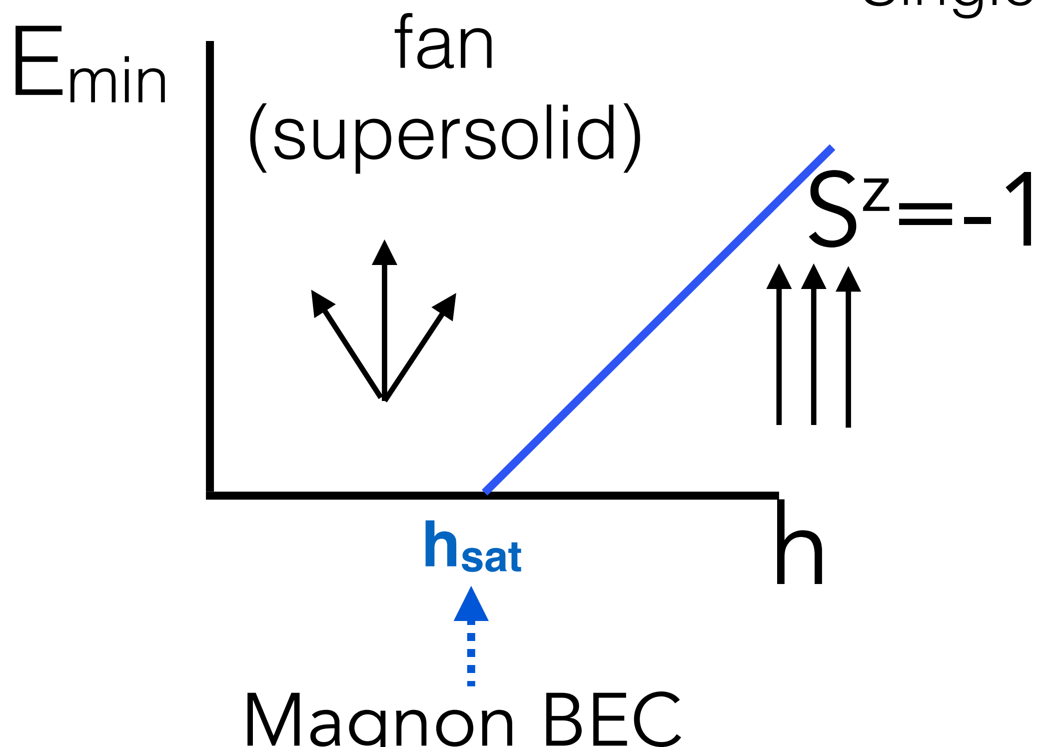
1-magnon





$$\omega \sim (k^2 - Q^2)^2 - (h_{\mathrm{sat}} - h)$$

$$h_{\mathrm{sat}} = \frac{S(4J_2 - |J_1|)^2}{4J_2}$$



Single magnon condensation at both minima —> $U(1)$

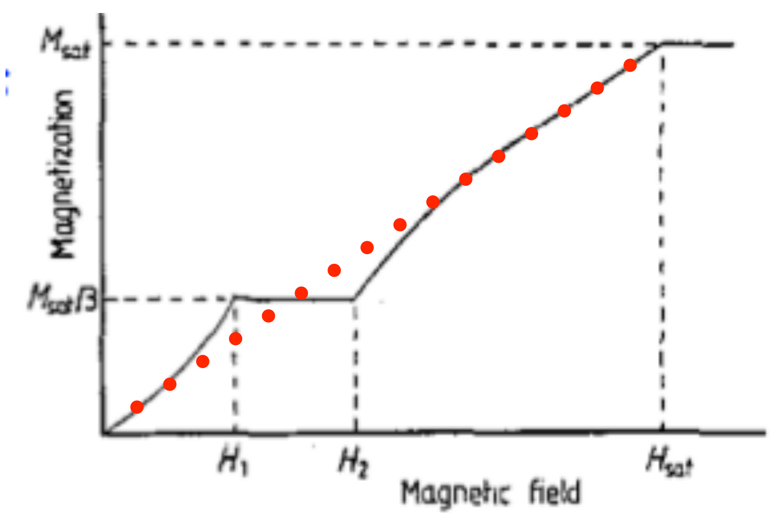
$$\langle a_{\mathbf{k}} \rangle = \sqrt{N}\psi_{+\mathbf{Q}}\delta_{\mathbf{k},\mathbf{Q}} + \sqrt{N}\psi_{-\mathbf{Q}}\delta_{\mathbf{k},-\mathbf{Q}}$$

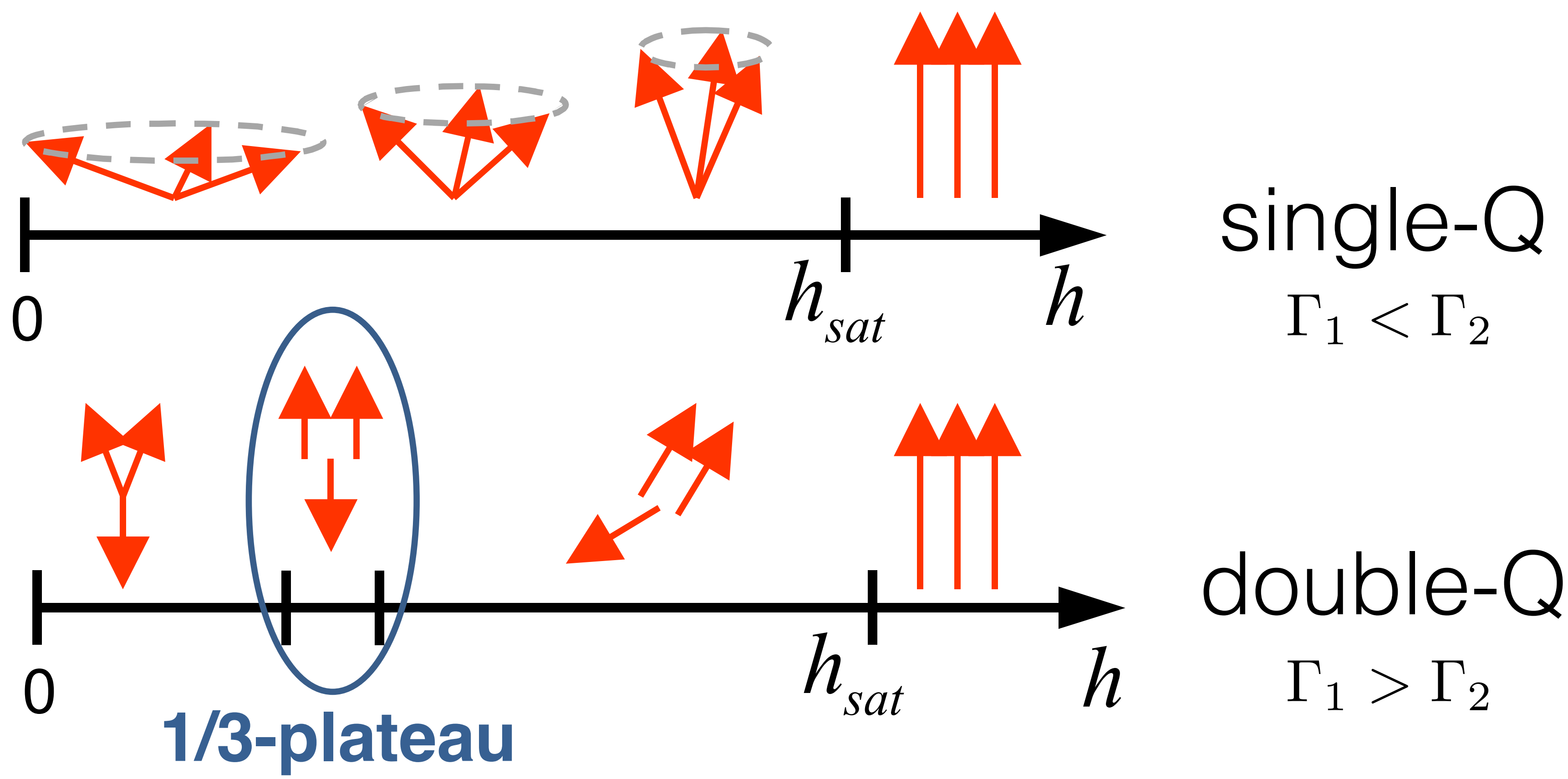
$$\langle S^-(x) \rangle = |\psi| e^{i\phi^+} \cos[\mathbf{Q}\cdot\mathbf{r} + \phi^-]$$

$$\langle S^z(x) \rangle = S - |\psi|^2 \cos^2[\mathbf{Q}\cdot\mathbf{r} + \phi^-]$$

Single-Q *vs* double-Q condensation is decided by interaction between magnons which is strongly renormalized by quantum fluctuations.

The difference is not small —
the entire magnetization M(h) of the triangular lattice antiferromagnet is determined by quantum fluctuations





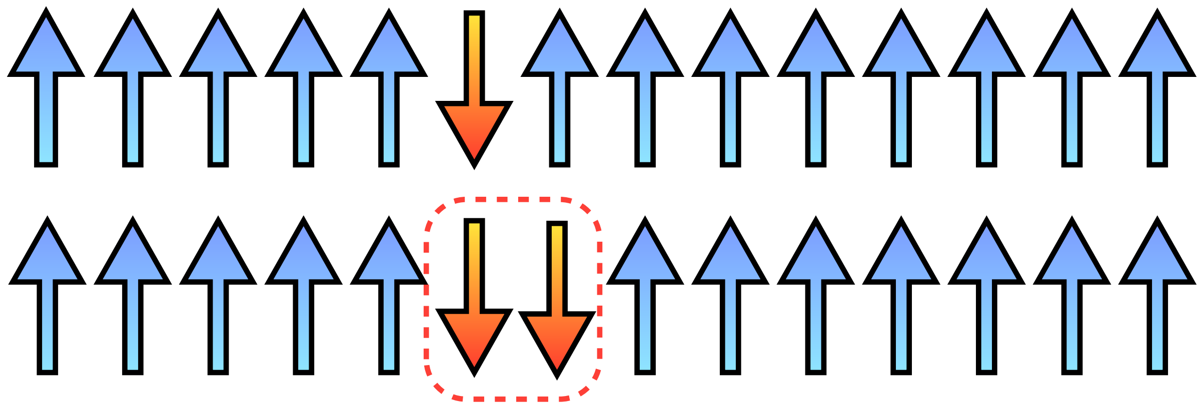
$$\frac{E}{N} = -S\mu(\rho_1 + \rho_2) + \tfrac{1}{2}\Gamma_1\left(\rho_1^2 + \rho_2^2\right) + \Gamma_2\rho_1\rho_2.$$

magnon superconductor

Today: condensation of *magnon pairs*

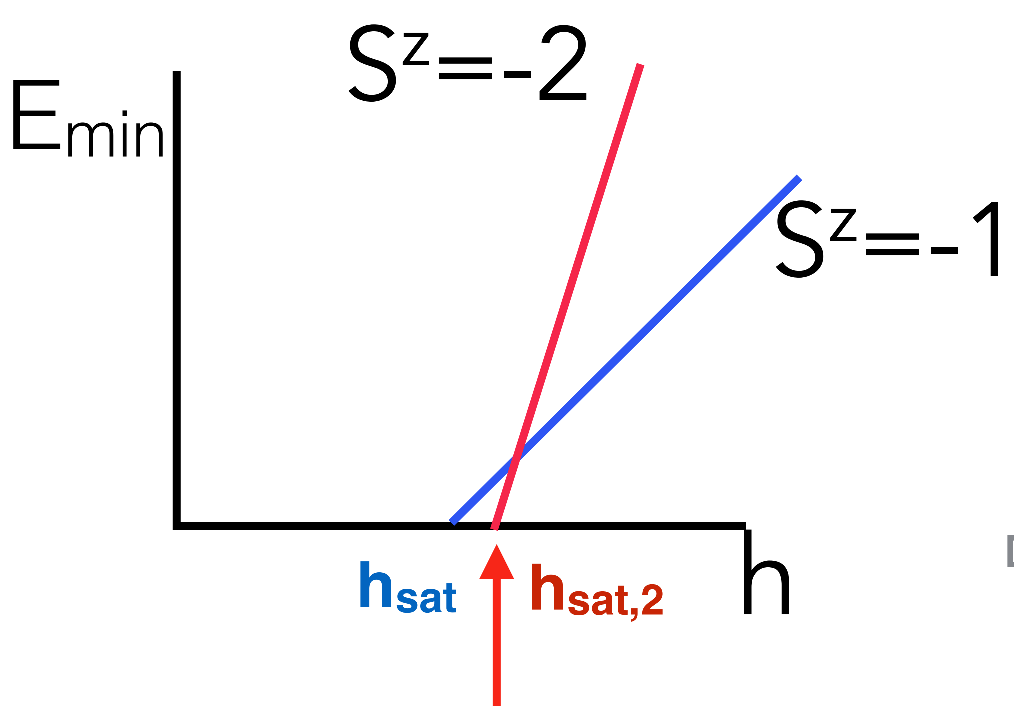
1-magnon

2-magnon bound state



$$E_{\rm min} = \epsilon_1(k) + h$$

$$E_{\rm min} = \Omega_2(k) + 2h$$



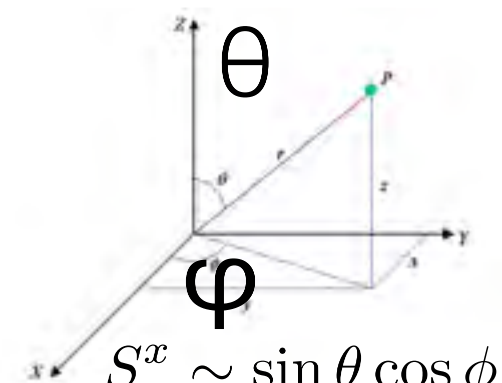
"molecular" bound state

$$\mathrm{Min}\{\Omega_2\} = 2\epsilon_1(Q) - \Delta_{\rm bound}$$

$$\Rightarrow h_{\rm sat,2} = h_{\rm sat} + \Delta_{\rm bound}/2$$

Formation of molecular fluid: for d>1 at T=0 this is a molecular BEC = true spin nematic (magnon superconductor)

Hidden order



$S^x \sim \sin\theta\cos\phi$

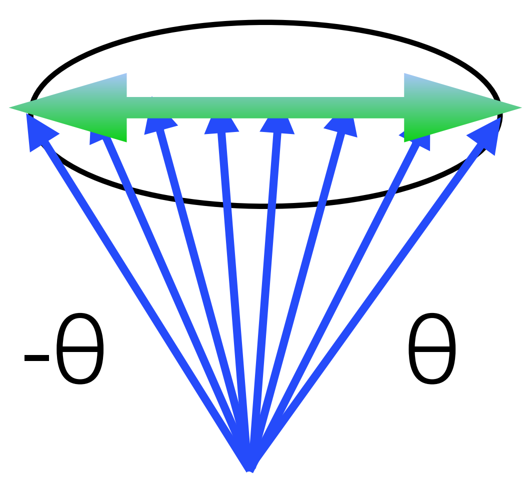
No dipolar order $\qquad \langle S_n^- \rangle = 0$

S^z=1 gap $\qquad \langle S_i^+ S_j^- \rangle \sim e^{-|i-j|/\xi}$

Nematic order $\qquad \langle S_n^- S_{n+a}^- \rangle = \Phi \neq 0$

$$\langle S_n^- S_m^- \rangle = \langle S_n^x S_m^x - S_n^y S_m^y - i(S_n^x S_m^y + S_n^y S_m^x) \rangle \sim \sin^2\theta(\cos 2\varphi - i\sin 2\varphi)$$

nematic director



Magnetic quadrupole moment

think of a fluctuating fan state:
φ is constant, while θ fluctuates (in time)
in the interval (θ_0, - θ_0)

Outline

- Magnon BEC
- Materials
- Basic theory and some numerics
- *Field theory of the Lifshitz point*
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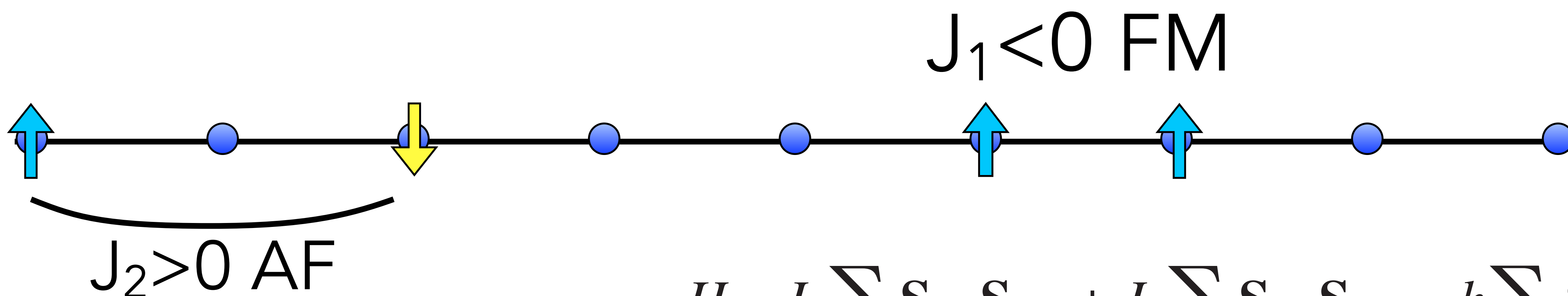
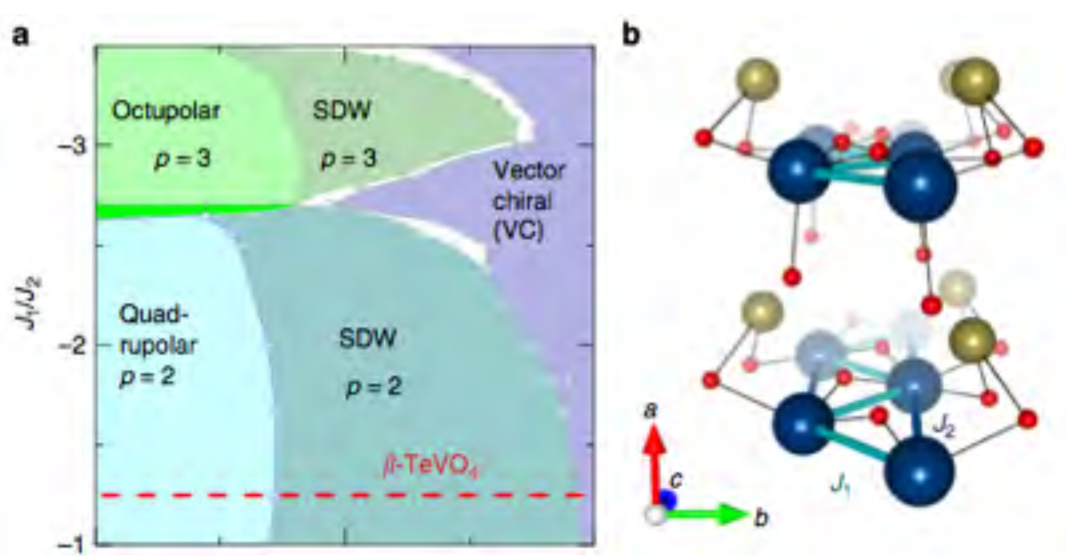
New system: Frustrated ferromagnet

1d S=1/2 chain



$$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$



$\beta - \mathrm{TeVO_4}$

Pregelj *et al.*, Nat.Comm.2015

Compound	J_1, J_2 (K)	∠ Cu-O-Cu (deg)	T_N (K)	H_s (T)
Li_2ZrCuO_4[12, 13]	−151, 35	94.1	6.4	-
$Rb_2Cu_2Mo_3O_{12}$[14, 15]	−138, 51	89.9, 101.8 91.9, 101.1	< 2	14
$PbCuSO_4(OH)_2$[16–18]	−100, 36	91.2, 94.3	2.8	5.4
$LiCuSbO_4$[19]	−75, 34	89.8, 95.0 92.0, 96.8	< 0.1	12
$LiCu_2O_2$[20–22]	−69, 43	92.2, 92.5	22.3	110
$LiCuVO_4$[23–31]	−19, 44	95.0	2.1	44.4
$NaCuMoO_4(OH)$	−51, 36	92.0, 103.6	0.59	26

K. Nawa *et al*, arXiv:1409.1310

Complex field-induced states in Linarite $PbCuSO_4(OH)_2$ with a variety of high-order exotic SDW_p states

PRL 2016

B. Willenberg[1,2], M. Schäpers[3], A.U.B. Wolter[3], S.-L. Drechsler[3], M. Reehuis[2], J.-U. Hoffmann[2], B. Büchner[3,4], A.J. Studer[5], K.C. Rule[5], B. Ouladdiaf[6], S. Süllow[1], and S. Nishimoto[3,4]

multi-polar states with $9 \geq \mathbf{p} \geq 2$

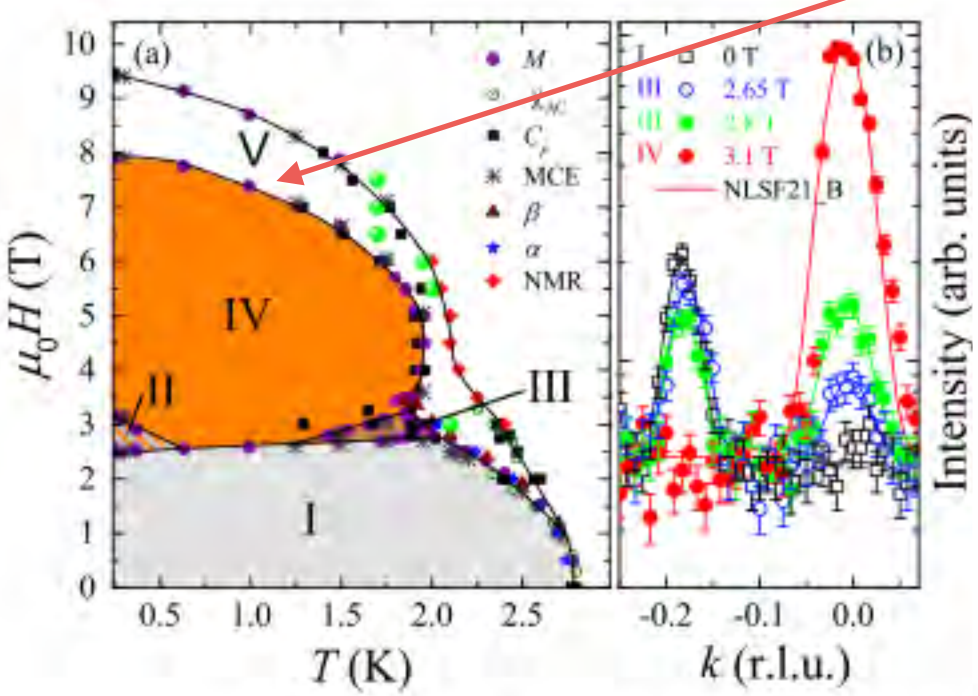


FIG. 1. (color online) (a) Phase diagram of linarite with $H \parallel b$. For a description of the phases I to V see text. Transition temperatures into phase V are depicted as red diamonds for the first time. Green balls indicate (H,T) points of Fig. 3. (b) Neutron scattering scans along k at 1.7 K as function of magnetic field, crossing from phase I via III into IV. Solid lines: Gaussians fits for peak position determination.

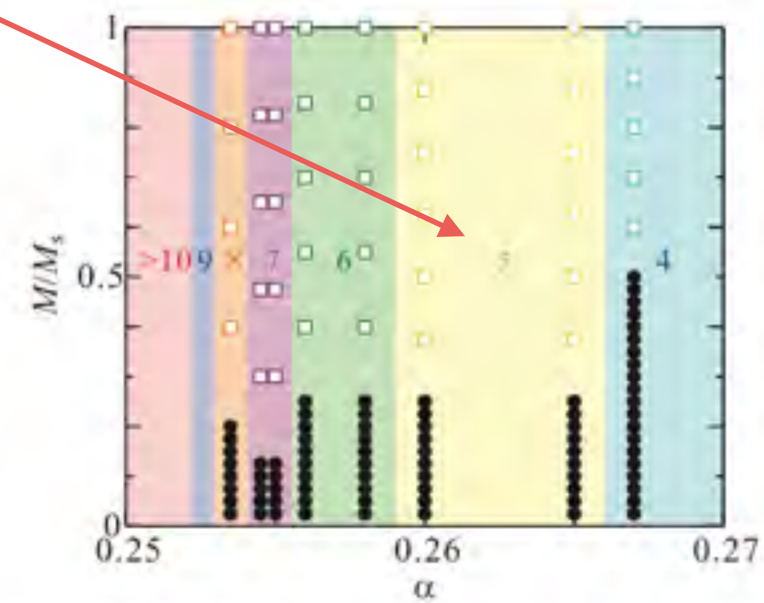
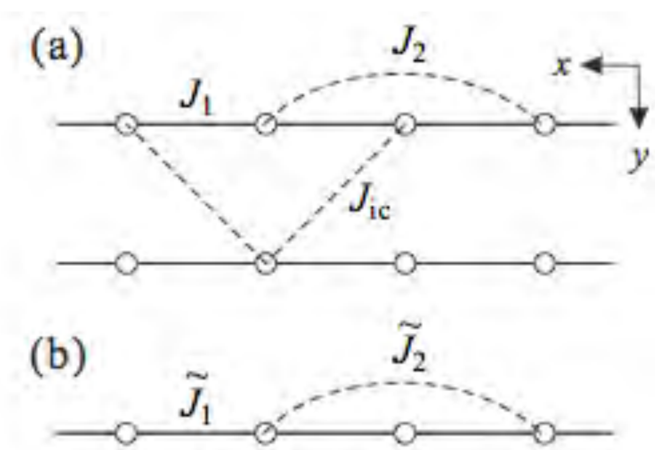
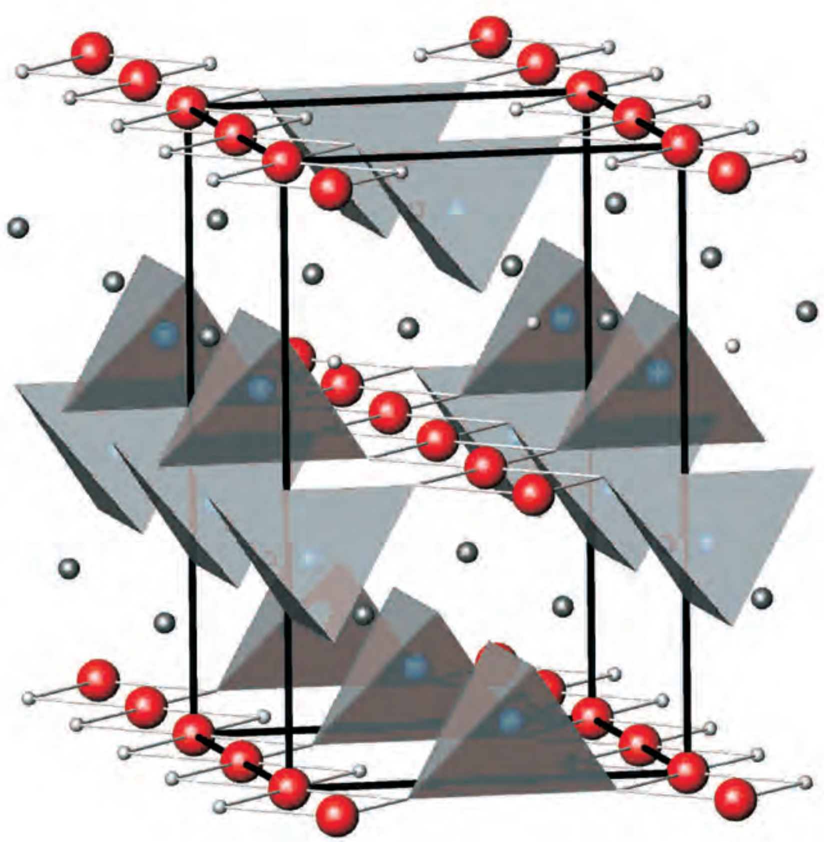


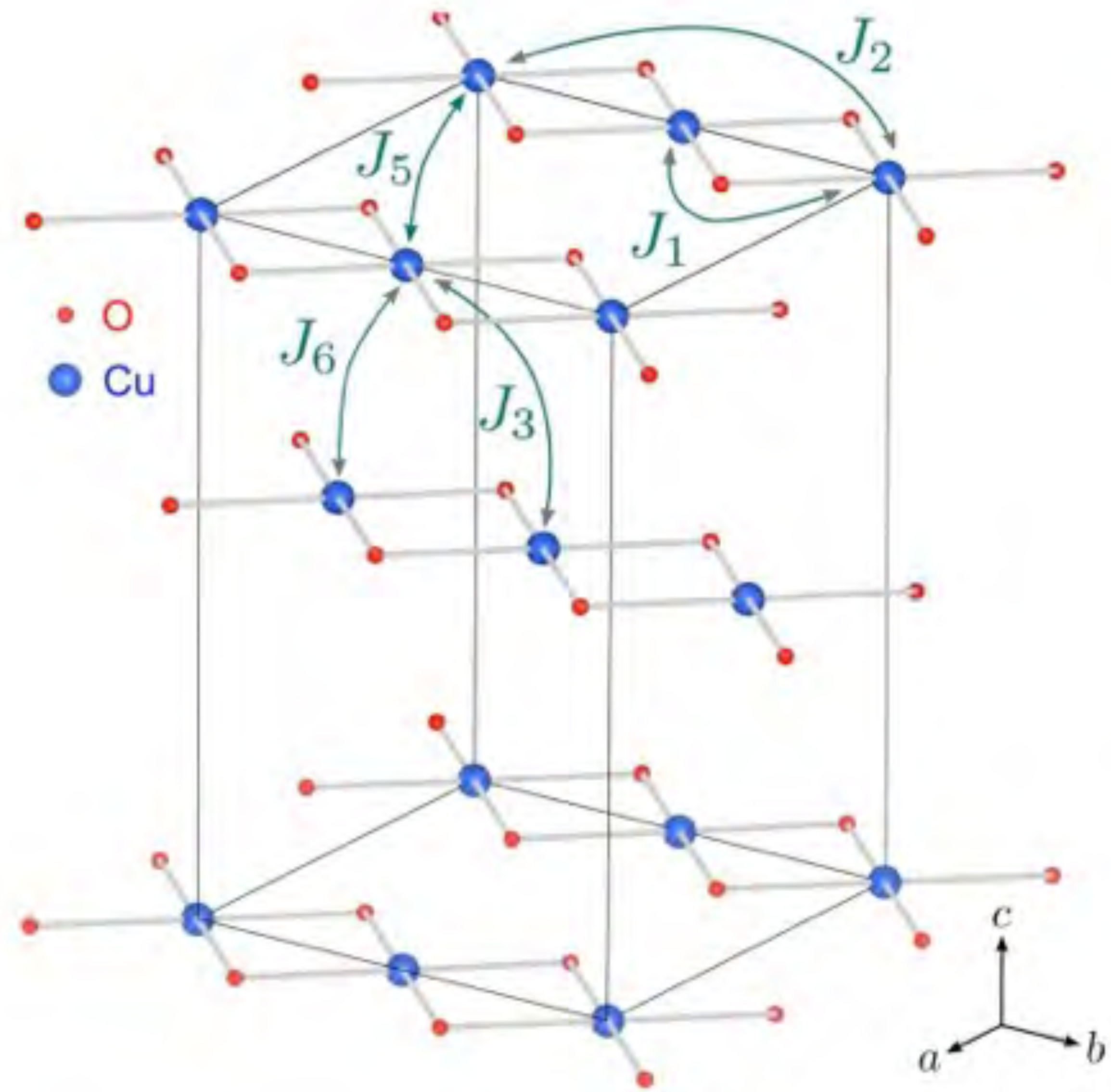
FIG. S5. (color online) Phase diagram showing dominant p-magnon bound states near the ferromagnetic critical point $\alpha_{eff} = 1/4$. The numbers from 4 to 10 are estimated $p(\equiv p_{full})$ values in the fully polarized state. The largest binding energy is given by $p = p_{full}$ ($p = 1$) by open squares (filled circles).

$$\alpha = J_{2,\mathrm{eff}}/|J_{1,\mathrm{eff}}|$$



$LiCuVO_4$: spin nematic?





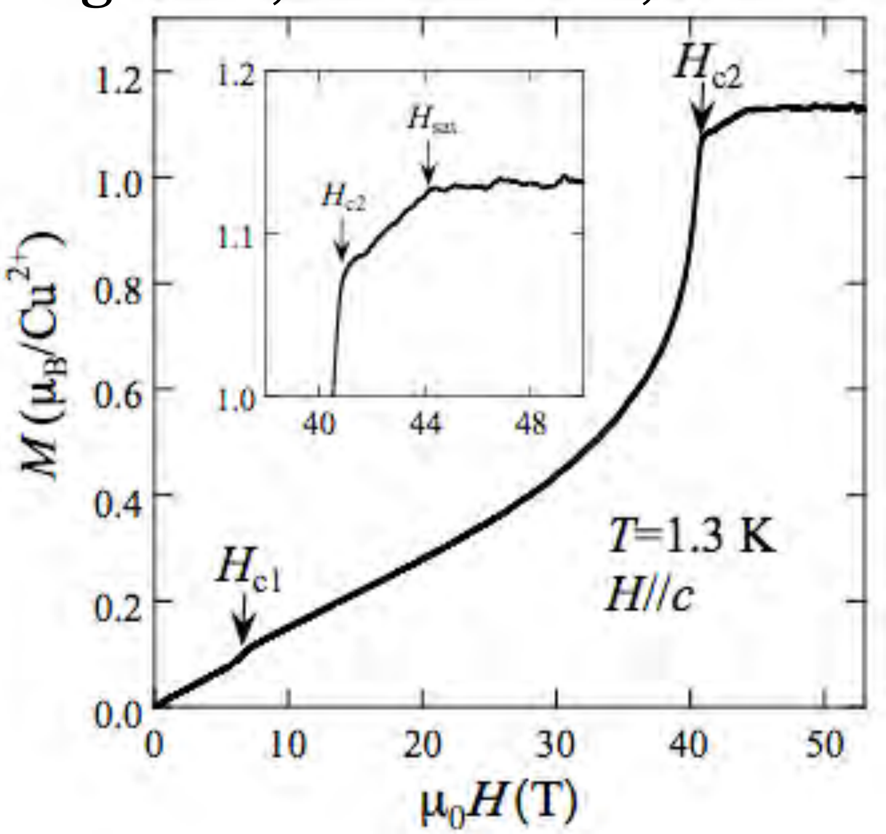
$$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$

+ weak interchain coupling J_5…

estimates:
J_1 = - 1.6 meV
J_2 = 3.9 meV
J_5 = -0.4 meV

$LiCuVO_4$ experiment: collinear SDW along **B**

Hagiwara, Svistov et al, 2011



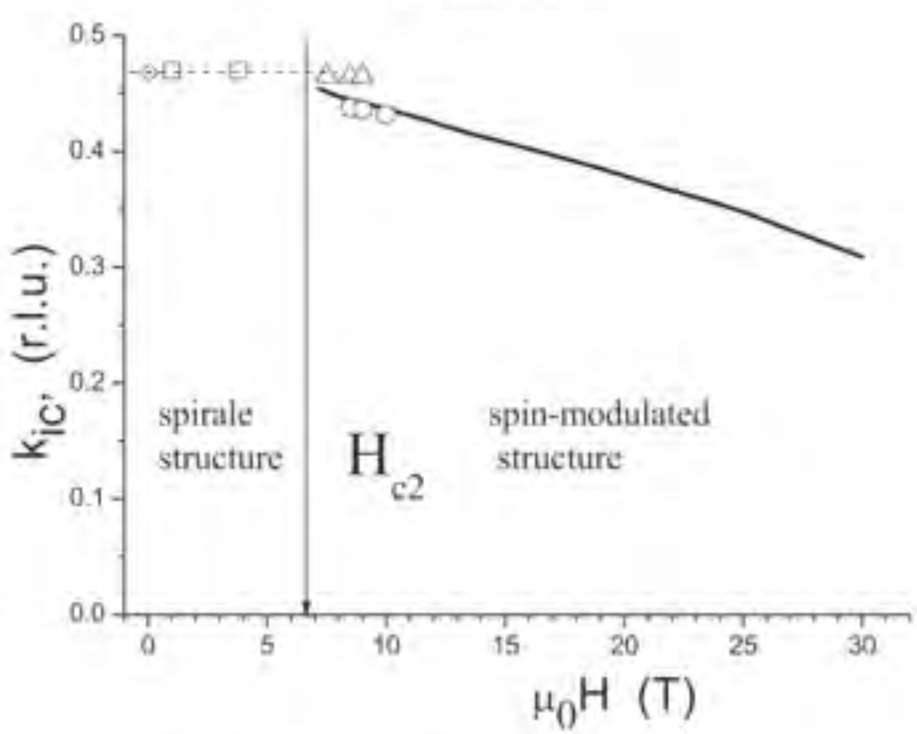
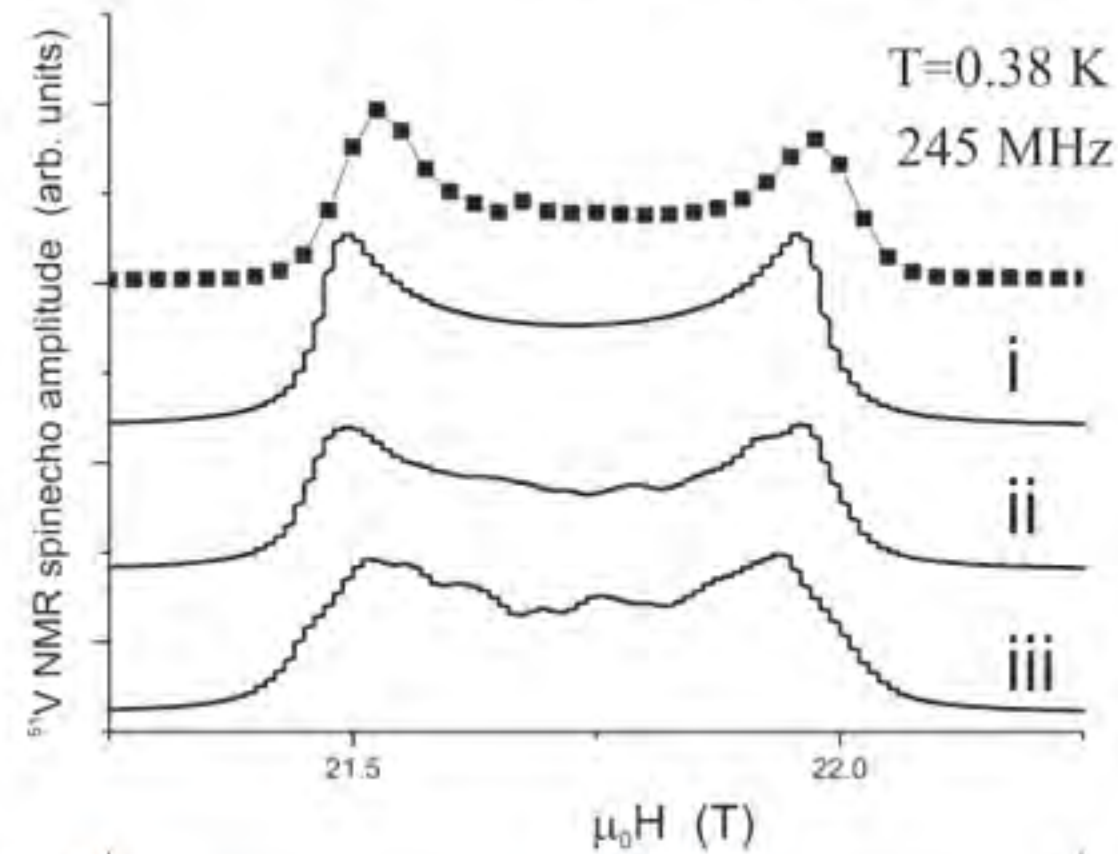
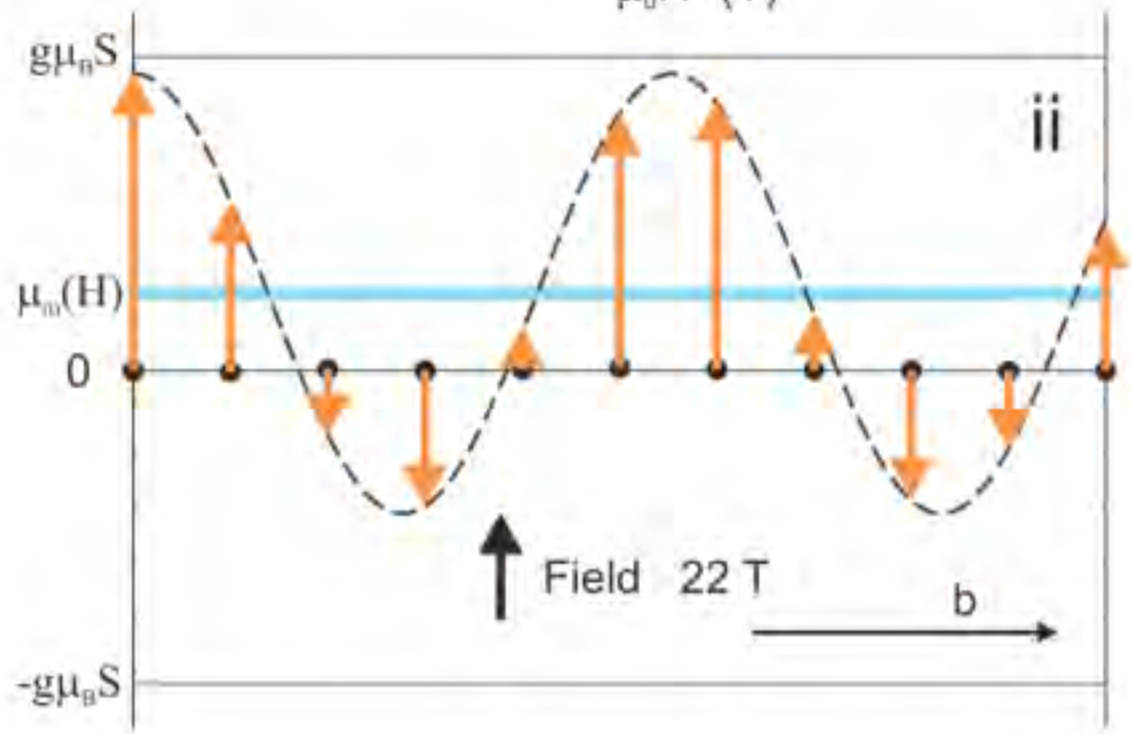


FIG. 2. Field dependence of the incommensurate wave vector k_{ic} for applied magnetic fields **H** ∥ **c** in $LiCuVO_4$. The open symbols

Buttgen et al 2012, 2014

PHYSICAL REVIEW B **85**, 214421 (2012)





PRL 109, 027203 (2012) PHYSICAL REVIEW LETTERS week ending 13 JULY 2012

longitudinal SDW

Evidence of a ~~Bond-Nematic~~ Phase in $LiCuVO_4$

M. Mourigal,[1,2] M. Enderle,[1] B. Fåk,[3] R. K. Kremer,[4] J. M. Law,[4,*] A. Schneidewind,[5] A. Hiess,[1,†] and A. Prokofiev[6,7]

No spin-flip scattering above ~ 9 Tesla: **longitudinal SDW state**

SF = spin flip, ΔS = 1
NSF = no spin flip, ΔS = 0

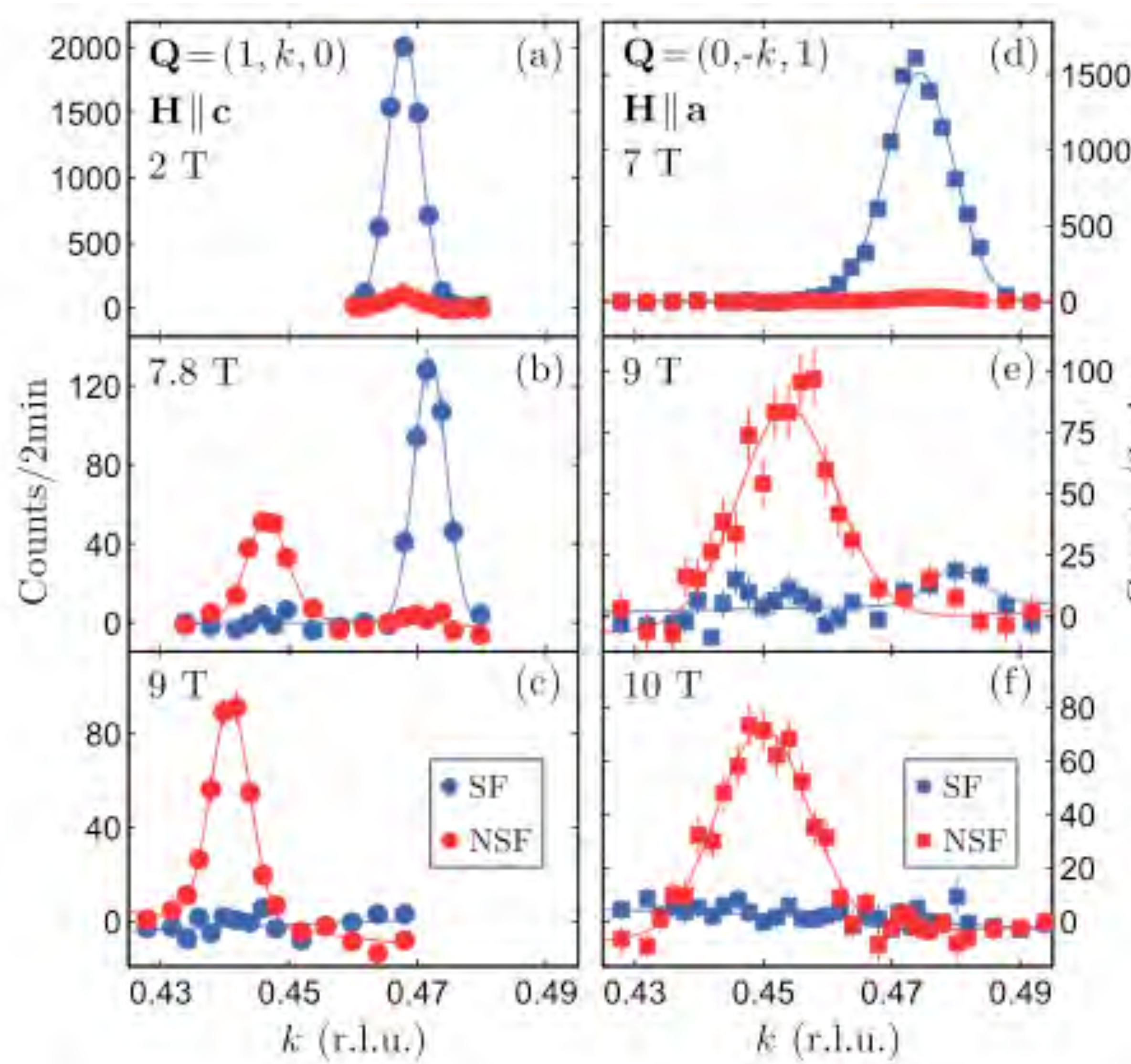


FIG. 3 (color online). Polarized cross sections measured at $T = 70$ mK for the magnetic reflections $\mathbf{Q} = (1, k_{IC}, 0)$ with $\mathbf{H}\|\mathbf{c}$ [left panels, (a)–(c)] and $\mathbf{Q} = (0, -k_{IC}, 1)$ with $\mathbf{H}\|\mathbf{a}$ [right panels, (d)–(f)].

Cold reality

PHYSICAL REVIEW B **90**, 134401 (2014)

Search for a spin-nematic phase in the quasi-one-dimensional frustrated magnet $LiCuVO_4$

N. Büttgen,[1,*] K. Nawa,[2,3] T. Fujita,[4] M. Hagiwara,[4] P. Kuhns,[5] A. Prokofiev,[6] A. P. Reyes,[5] L. E. Svistov,[7,†] K. Yoshimura,[2] and M. Takigawa[3,‡]

"Our results suggest that the theoretically predicted spin-nematic phase, if it exists in LiCuVO4, can be established only within the narrow field range 40.5<H<41.4 T."

- so far, extensive experimental evidence for longitudinal SDW
- Spin Nematic phase is constrained to field interval < 1 T right below the saturation field (of the order 40 T)

One-Third Magnetization Plateau with a Preceding Novel Phase in Volborthite

H. Ishikawa,[1] M. Yoshida,[1] K. Nawa,[1] M. Jeong,[2] S. Krämer,[2] M. Horvatić,[2] C. Berthier,[2] M. Takigawa,[1] M. Akaki,[1] A. Miyake,[1] M. Tokunaga,[1] K. Kindo,[1] J. Yamaura,[3] Y. Okamoto,[1,4] and Z. Hiroi[1]
[1]*Institute for Solid State Physics, The University of Tokyo, Kashiwa, Chiba 277-8581, Japan*
[2]*Laboratoire National des Champs Magnétiques Intenses, LNCMI-CNRS (UPR3228), UJF, UPS and INSA, B.P. 166, 38042 Grenoble Cedex 9, France*
[3]*Materials Research Center for Element Strategy, Tokyo Institute of Technology, Yokohama 226-8503, Japan*
[4]*Department of Applied Physics, Nagoya University, Nagoya 464-8603, Japan*
(Received 23 November 2014; published 1 June 2015)

Huge 1/3 magnetization plateau !

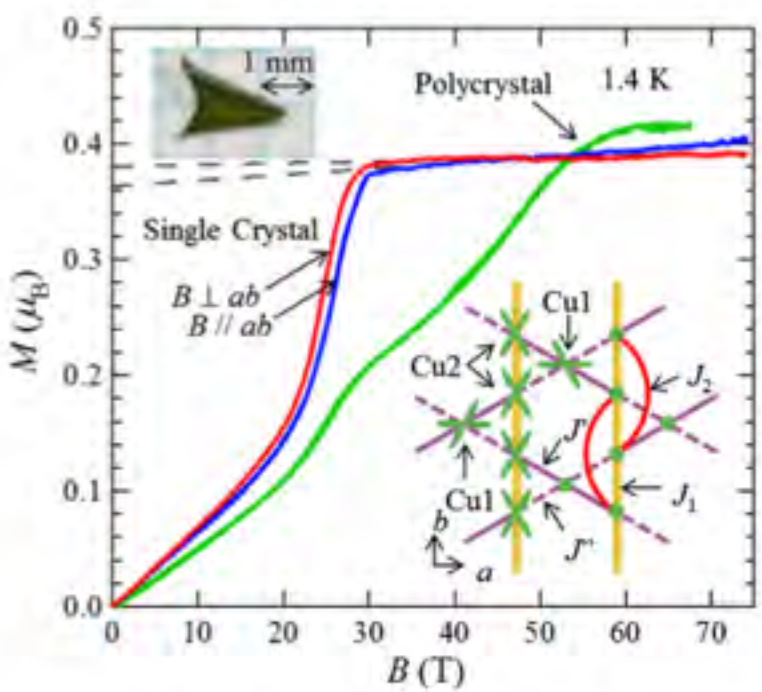


FIG. 1 (color online). Magnetization curves of volborthite measured at 1.4 K on two piles of single crystals in magnetic fields perpendicular (red line) and parallel (blue line) to the *ab* plane, and on a polycrystalline sample (green line) [32]. Shown also are a typical single crystal of volborthite (upper left) and the arrangement of Cu d_{x2-y2} orbitals projected onto the *ab* plane in the low-temperature $P2_1/a$ structure (lower right). J_1 and J_2 represent the NN and NNN interactions in the Cu2 spin chains, respectively. J' and J'' represent the NN interactions between Cu1 and Cu2 spins.

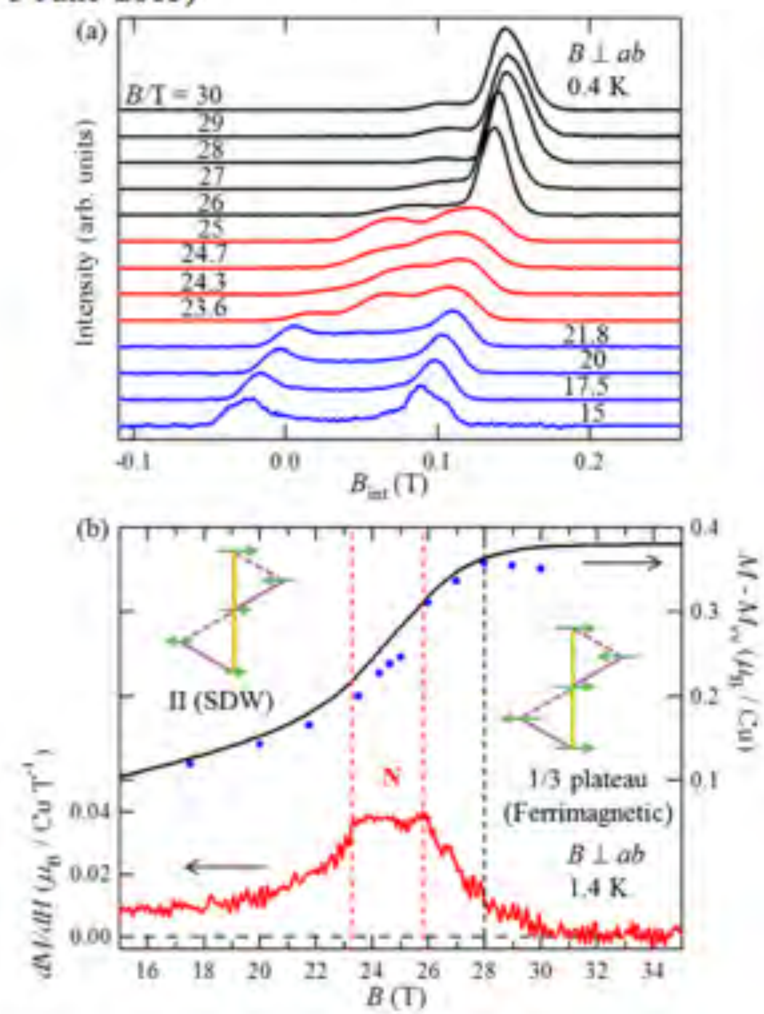
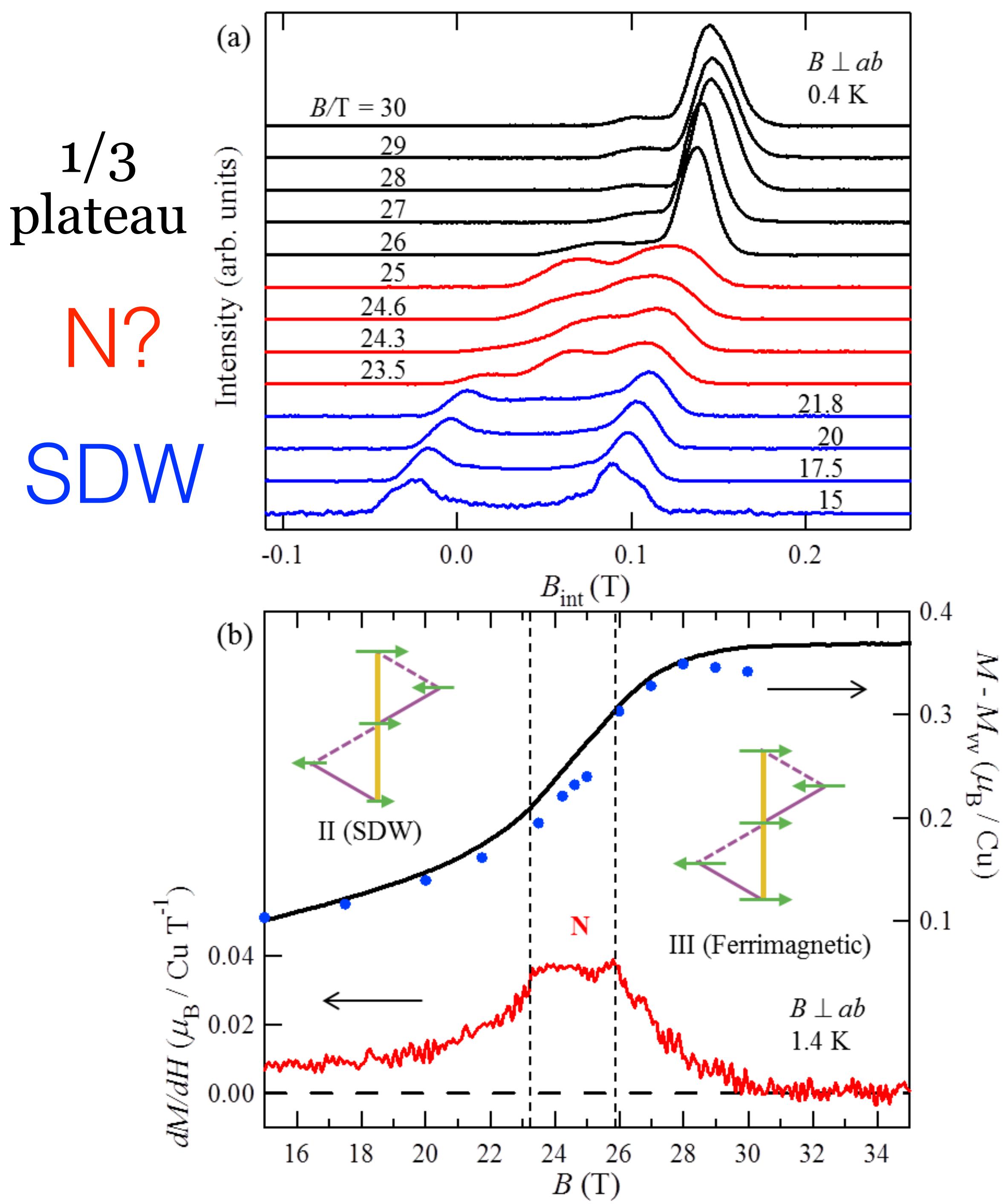


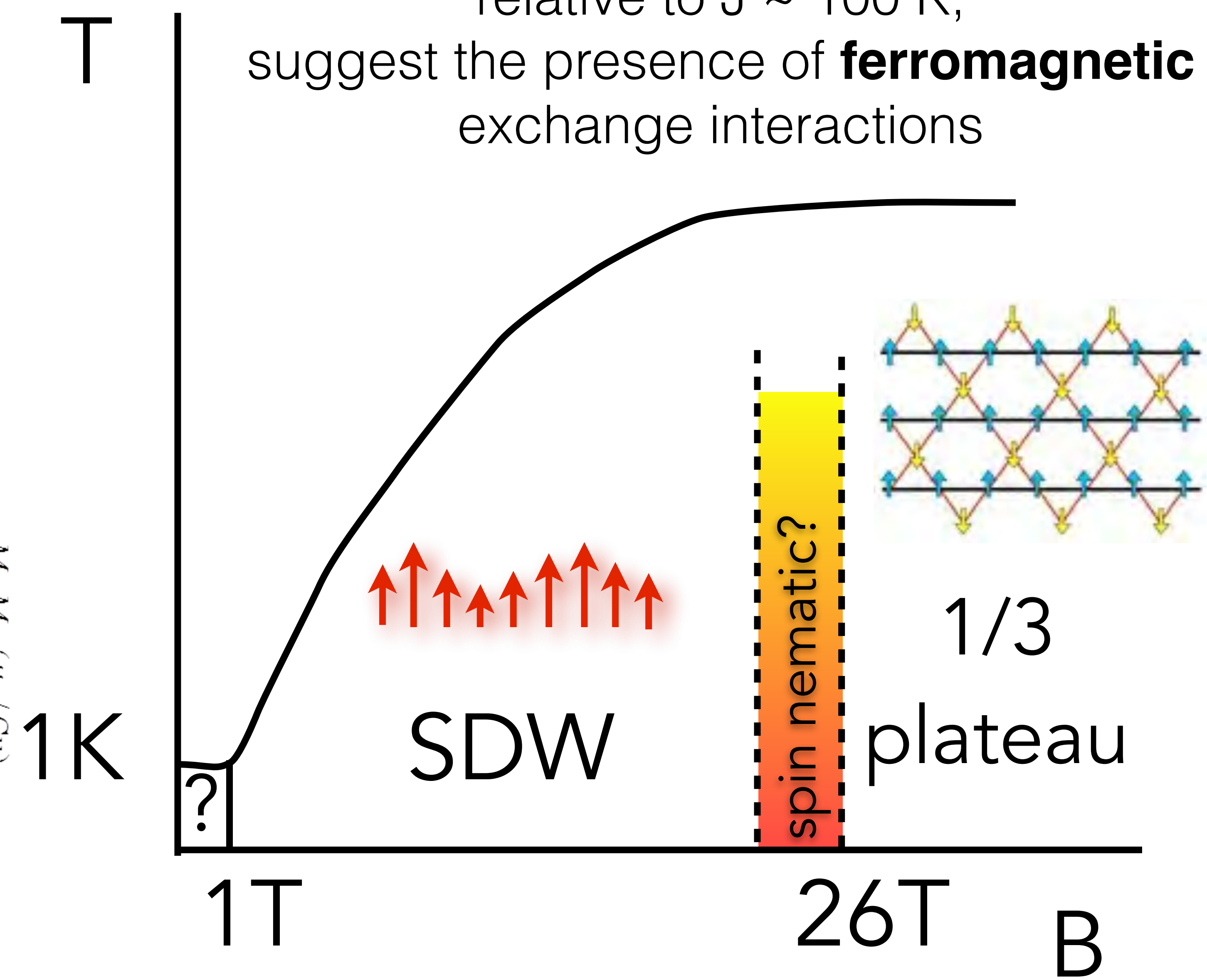
FIG. 2 (color online). (a) ^{51}V NMR spectra measured on a single-domain piece of a crystal in magnetic fields applied perpendicular to the *ab* plane at $T = 0.4$ K. The labeled fields correspond to $B = \nu_0/\gamma(B_{int} = 0)$. (b) Magnetization curve of single crystals (top, black line) and its field derivative (bottom) in $B \perp ab$ at 1.4 K after the subtraction of the Van Vleck paramagnetic magnetization (M_{VV}). Magnetization deduced from the center of the gravity of the NMR spectra is also plotted (top, blue circles).

Phase diagram



H. Ishikawa *et al*, PRL 2015

small plateau's onset field of 27 Tesla,
relative to J ~ 100 K,
suggest the presence of **ferromagnetic**
exchange interactions



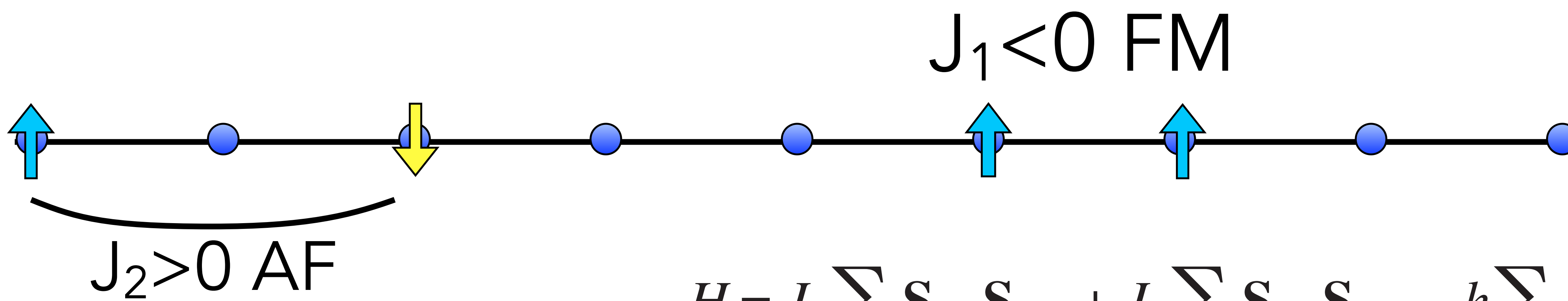
may be a spin nematic??

Outline

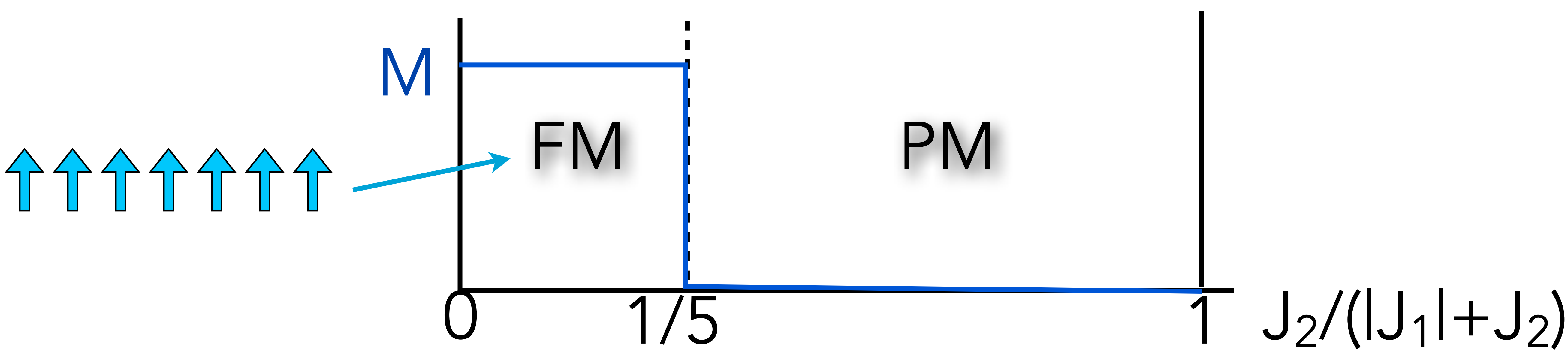
- Magnon BEC
- Materials
- Basic theory and some numerics
- *Field theory of the Lifshitz point*
- *Spin-current state* near the end-point of 1/3 magnetization plateau

Frustrated ferromagnet

1d S=1/2 chain



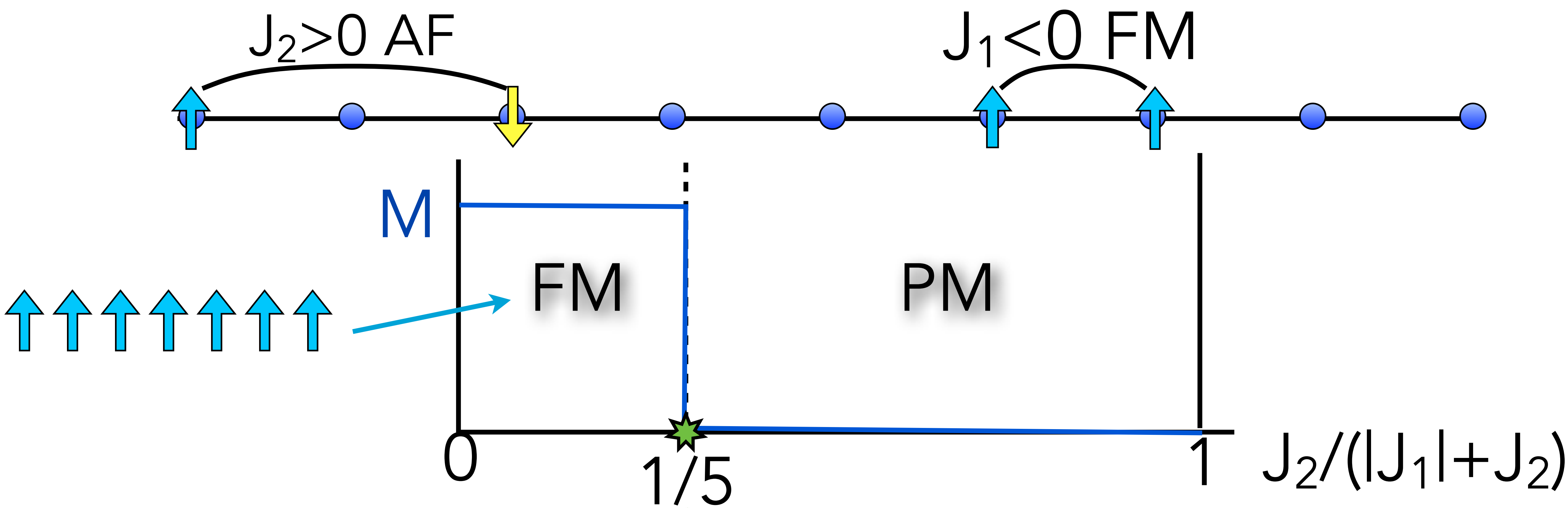
$$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$



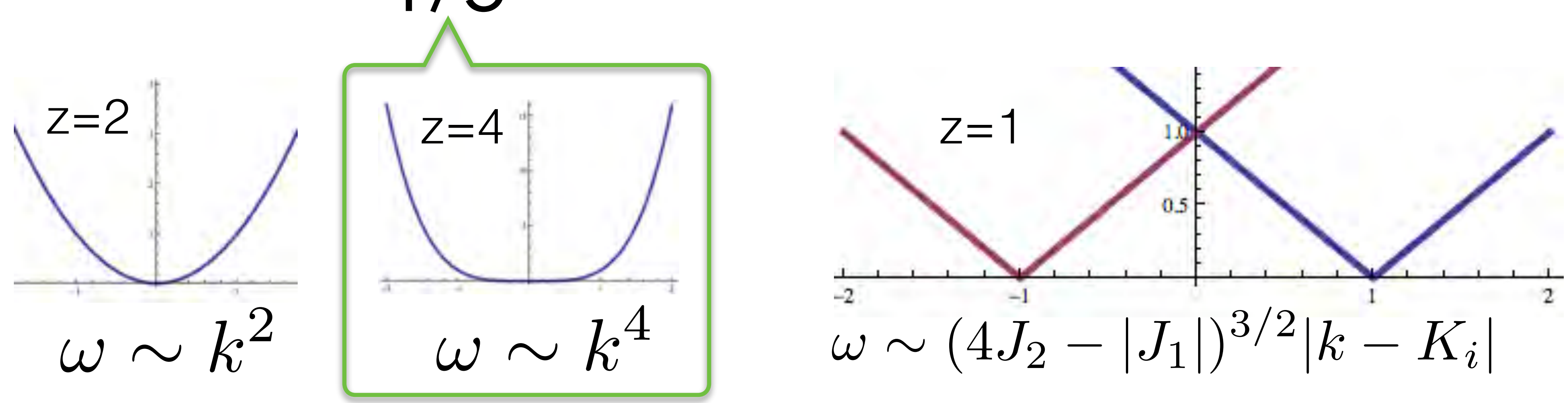
Frustrated ferromagnet

1d S=1/2 chain

$$H = J_1 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} - h \sum_i S_i^z$$

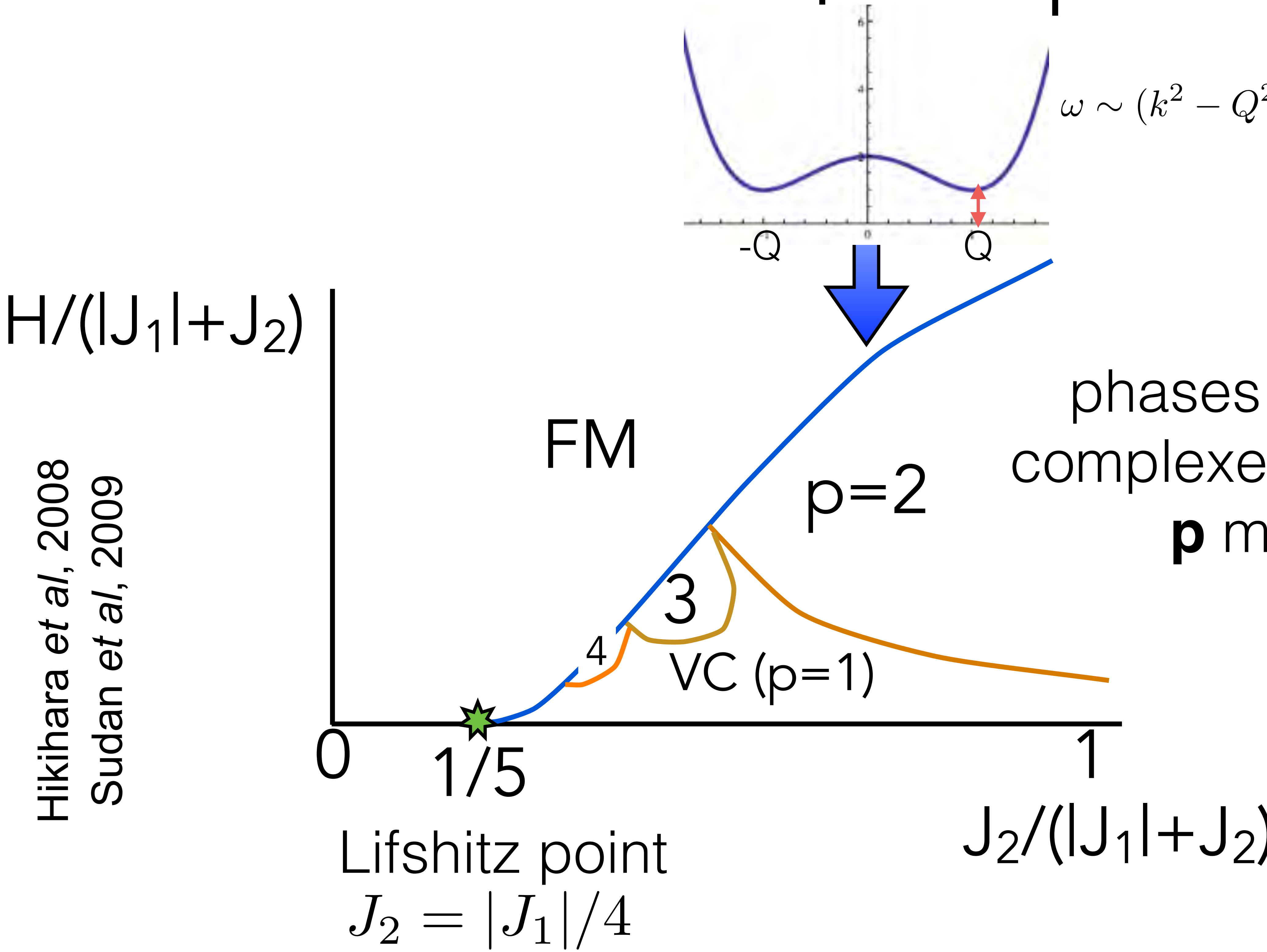


(spin-wave) dispersion



Approaching from fully polarized state — Multipolar phases

$\omega \sim (k^2 - Q^2)^2 - (h_{\mathrm{sat}} - h)$



phases with bound complexes made out of **p** magnons

Hikihara *et al*, 2008
Sudan *et al*, 2009

Spin chain numerics

PHYSICAL REVIEW B **74**, 020403(R) (2006)

Frustrated ferromagnetic spin-$\frac{1}{2}$ chain in a magnetic field: The phase diagram and thermodynamic properties

F. Heidrich-Meisner,[1,2] A. Honecker,[3,4] and T. Vekua[5,6]

We compute the ground-state energies $E_0(S^z_{\mathrm{total}}, h=0)$ in subspaces labeled by S^z_{total} on chains with periodic boundary conditions (PBC) using the Lanczos algorithm. The ground-state energies of substantially larger chains with open boundary conditions (OBC) are calculated with DMRG. Typically, we keep up to $m=400$ states in our DMRG calculations.

Then, we include the Zeeman term and obtain the field-dependent ground-state energies

$$E_0(S^z_{\mathrm{total}}, h) = E_0(S^z_{\mathrm{total}}, h=0) - hS^z_{\mathrm{total}}. \qquad (4)$$

The magnetization curves are constructed by solving the equations $E_0(S^z_{\mathrm{total}}, h_{\mathrm{step}}) = E_0(S^z_{\mathrm{total}}+s, h_{\mathrm{step}})$, which define those magnetic fields at which the magnetization increases from $M = S^z_{\mathrm{total}}/(NS)$ to $M' = (S^z_{\mathrm{total}}+s)/(NS)$. Steps larger than $s=1$ may occur.

s=1/2

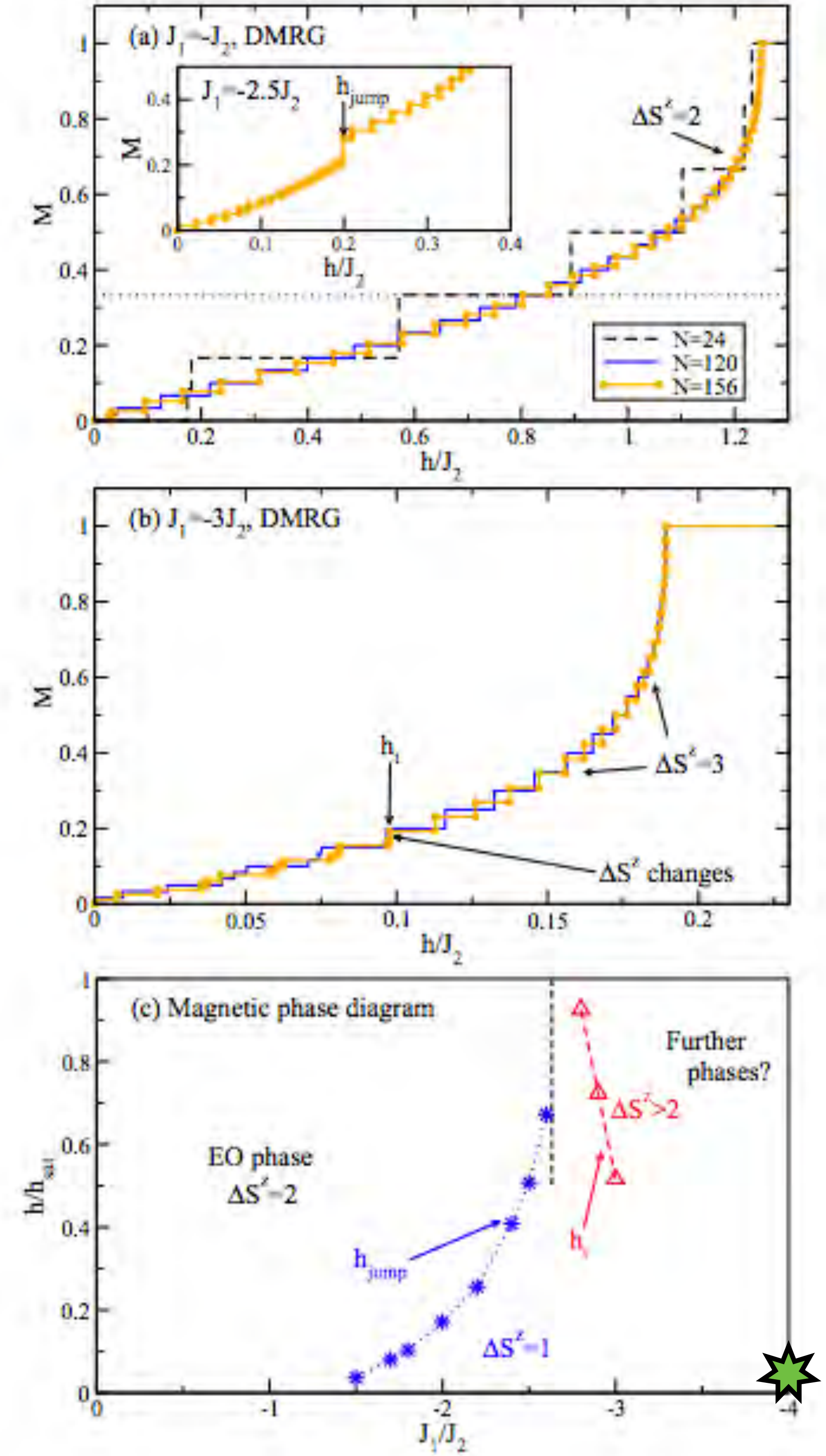


FIG. 1: (Color online) (a), main panel (inset): Magnetization curve $M(h)$ for $J_1 = -J_2$ ($J_1 = -2.5\,J_2$). The horizontal dotted line marks $M = 1/3$. (b): $M(h)$ for $J_1 = -3\,J_2$. (c): Magnetic phase diagram of the frustrated FM chain. The dotted line (with stars) marks the first-order transition between the EO phase and the $\Delta S^z = 1$ region, while the line $h = h_1$ (dashed, triangles) separates the $\Delta S^z = 1$ region from the $\Delta S^z \geq 3$ part. Uncertainties of the transition lines, e.g. due

PHYSICAL REVIEW B **78**, 144404 (2008)

Vector chiral and multipolar orders in the spin-$\frac{1}{2}$ frustrated ferromagnetic chain in magnetic field

Toshiya Hikihara,[1] Lars Kecke,[2,3] Tsutomu Momoi,[2] and Akira Furusaki[2]

s=1/2

TABLE I. Number of magnons p and total momentum k of the multimagnon bound states which become gapless at the saturation field.

Parameter range	p	k
$-2.669 < J_1/J_2 < 0$	2	π
$-2.720 < J_1/J_2 < -2.669$	2	$\pi \pm \delta\,(\delta > 0)$
$-3.514 < J_1/J_2 < -2.720$	3	π
$-3.764 < J_1/J_2 < -3.514$	4	π
$-3.888 < J_1/J_2 < -3.764$	5	π
$-3.917 < J_1/J_2 < -3.888$	6	π
$-4 < J_1/J_2 < -3.917$	7	π

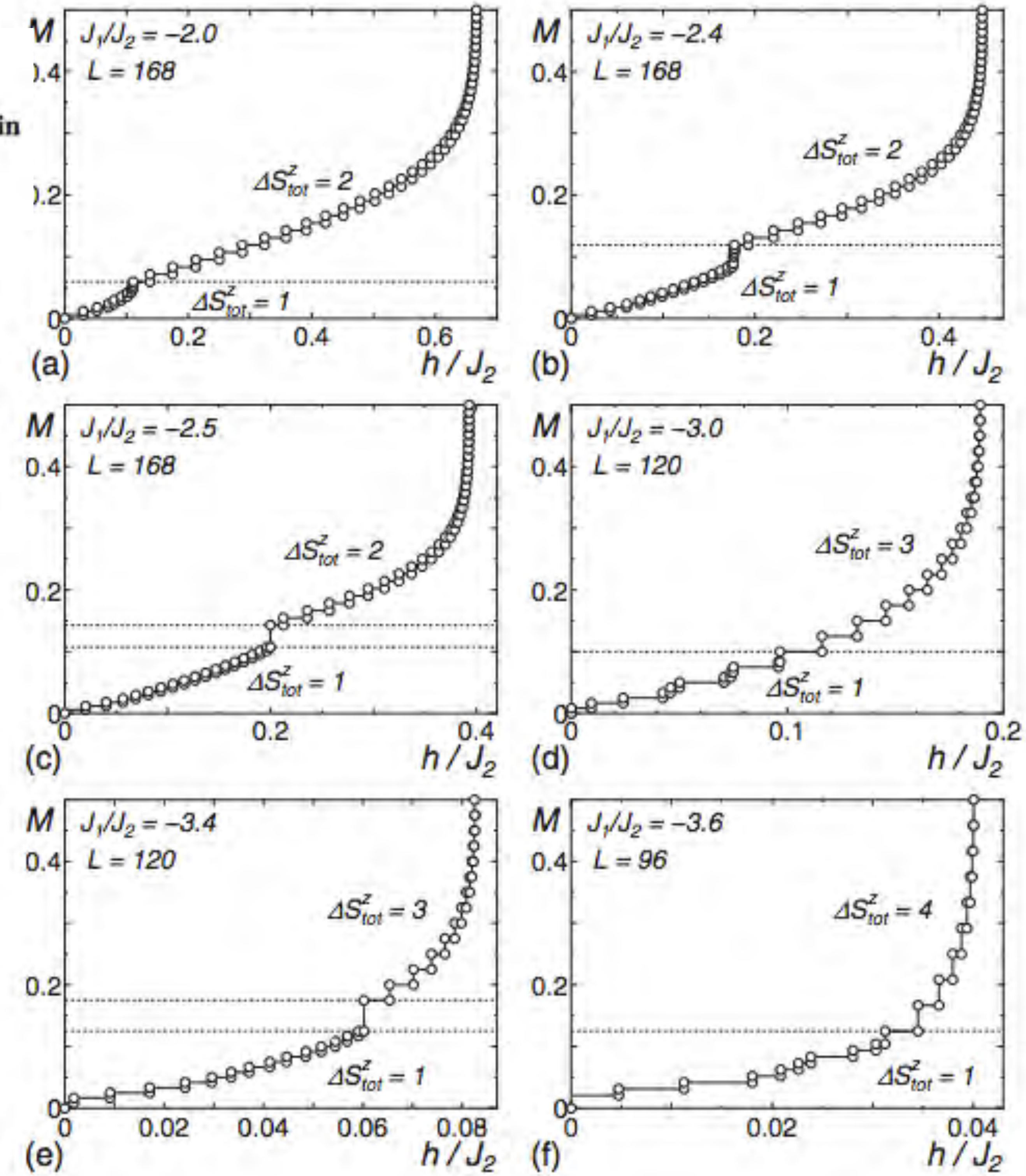
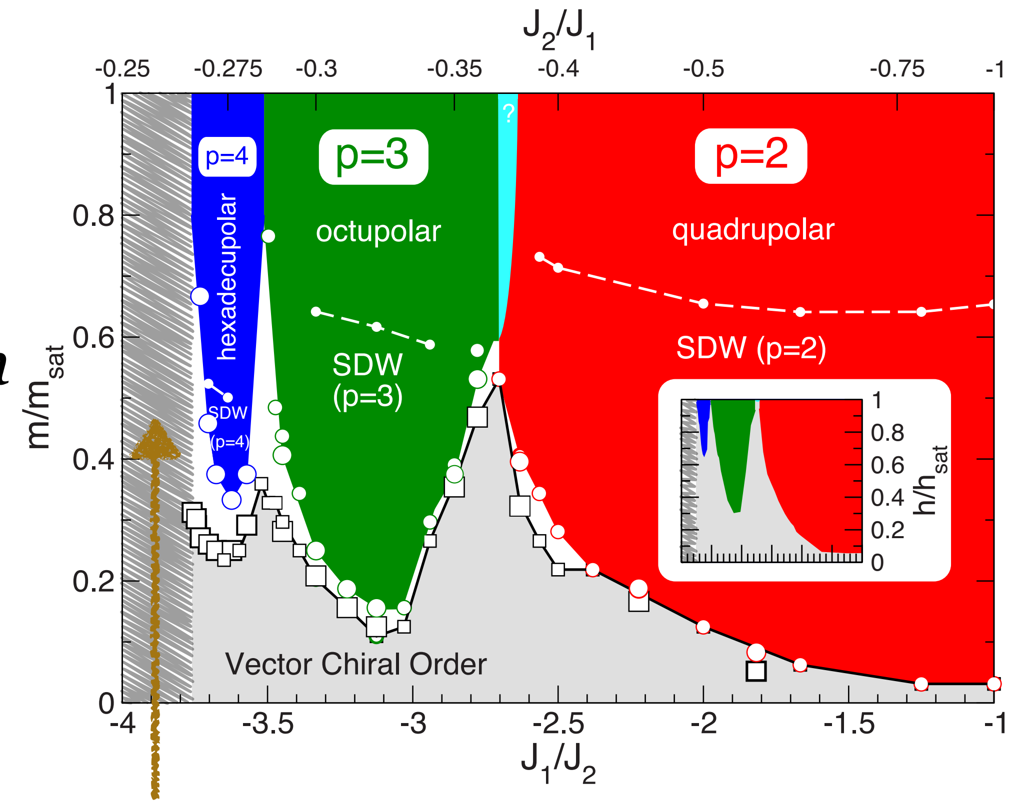


FIG. 9. Magnetization curves for (a) $J_1/J_2 = -2.0$, (b) $J_1/J_2 = -2.4$, (c) $J_1/J_2 = -2.5$, (d) $J_1/J_2 = -3.0$, (e) $J_1/J_2 = -3.4$, and (f) $J_1/J_2 = -3.6$. The dotted lines represent the boundaries of the regions of $\Delta S^z_{tot} = 1$ and $\Delta S^z_{tot} \geq 2$.

Numerics: nematicity and 1^{st} order seem connected?

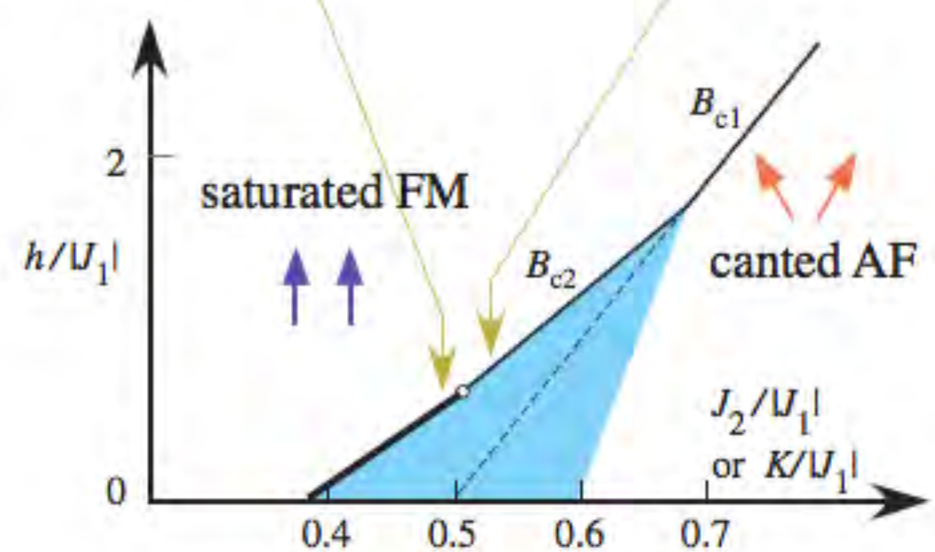
1d frustrated chain



Sudan et al, 2009

1st order 2nd order

2d frustrated square lattice



Shannon, Momoi, Sindzingre PRL 2006

Outline

- Magnon BEC
- Materials
- Basic theory and some numerics
- *Field theory of the Lifshitz point* magnon “BCS” superconductor
- *Spin-current state* near the end-point of 1/3 magnetization plateau exotic superconductor of magnons

Lifshitz Point

Balents, Starykh PRL 2016

- Unusual QCP: order-to-order transition
- Effective action - NLσM for unit vector **m**

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

$$\mathcal{A}_B = \frac{\hat{m}_1 \partial_\tau \hat{m}_2 - \hat{m}_2 \partial_\tau \hat{m}_1}{1 + \hat{m}_3}$$

Berry phase term

tunes QCP $\delta \propto |J_1| - 4J_2$

two symmetry allowed interactions at O(q^4)

All properties near Lifshitz point obey "one parameter universality" dependent upon u/K ratio

Lifshitz Point

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

- Intuition: behavior near the Lifshitz point should be semi-classical, since “close” to FM state which is classical

$$x \to \sqrt{\frac{K}{|\delta|}} x \qquad \tau \to \frac{K}{\delta^2}\tau$$

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

Large parameter: saddle point!

$$v = \frac{u}{K} \qquad \overline{h} = \frac{hK}{\delta^2}$$

Lifshitz point

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

v derives from quantum fluctuations

Need it be positive?

$$\hat{m} \cdot \hat{m} = 1 \quad \Rightarrow \quad \partial_x \hat{m} \cdot \partial_x \hat{m} = -\hat{m} \cdot \partial_x^2 \hat{m} \leq |\partial_x^2 \hat{m}|$$

Theory is stable for v>-1

In fact, v<0

- Semiclassical large s limit: $v = -\dfrac{3}{2s}$
- s=1/2 estimate: $v_{s=1/2} = -\dfrac{9}{(2+\sqrt{7})^2} \approx -0.42$

Saddle point

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

Solution:

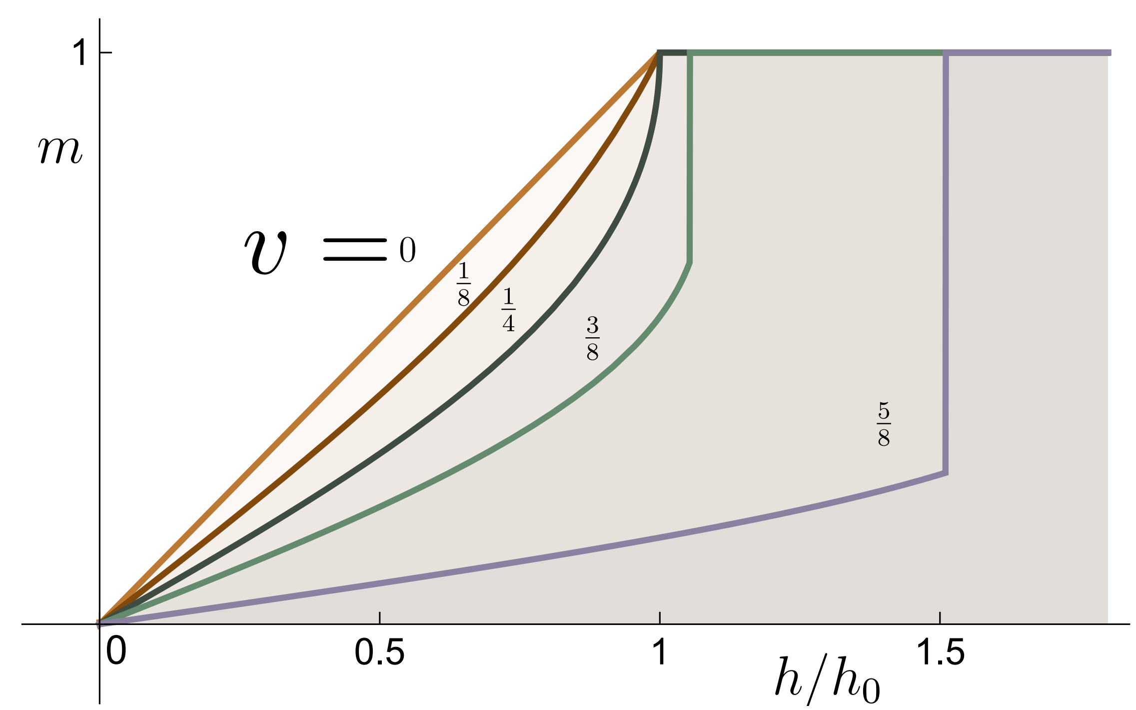
$$\hat{m} = \begin{pmatrix} |\Psi|\cos(qx+\phi) \\ \pm|\Psi|\sin(qx+\phi) \\ \sqrt{1-|\Psi|^2} \end{pmatrix}$$

cone state

Obtain

q, Ψ versus h, v, δ

→ many physical quantities



Saddle point

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

Solution:

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cone state

Obtain

q, Ψ versus h, v, δ

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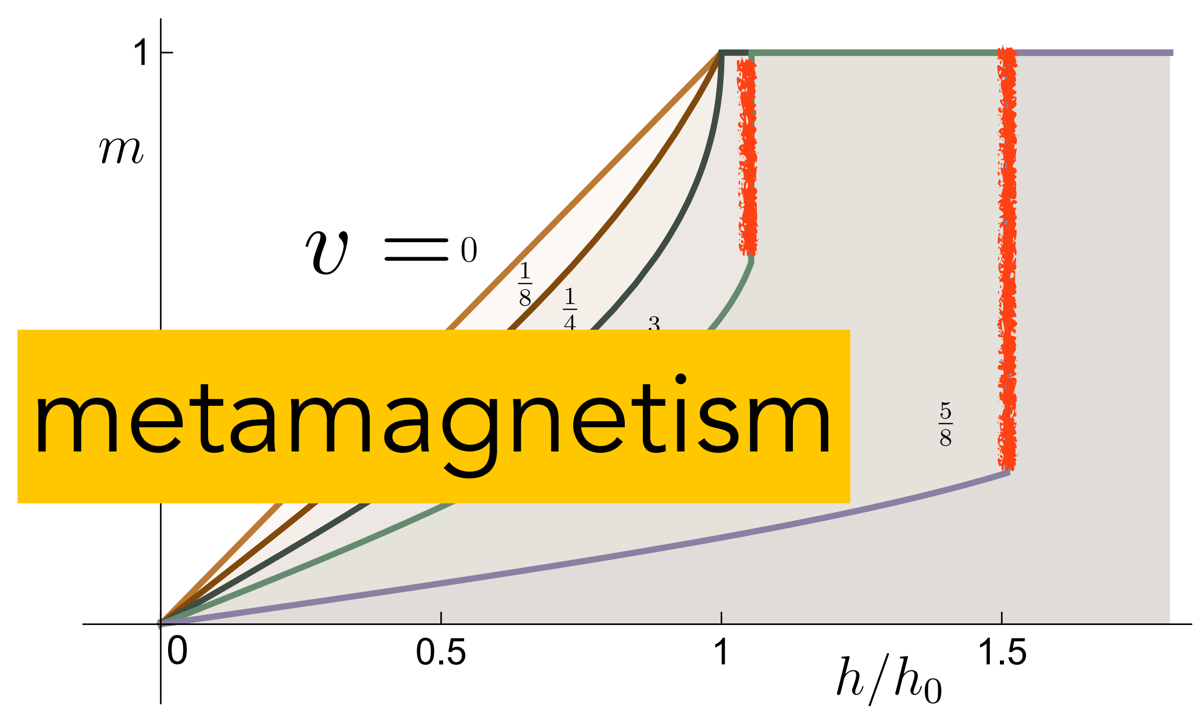
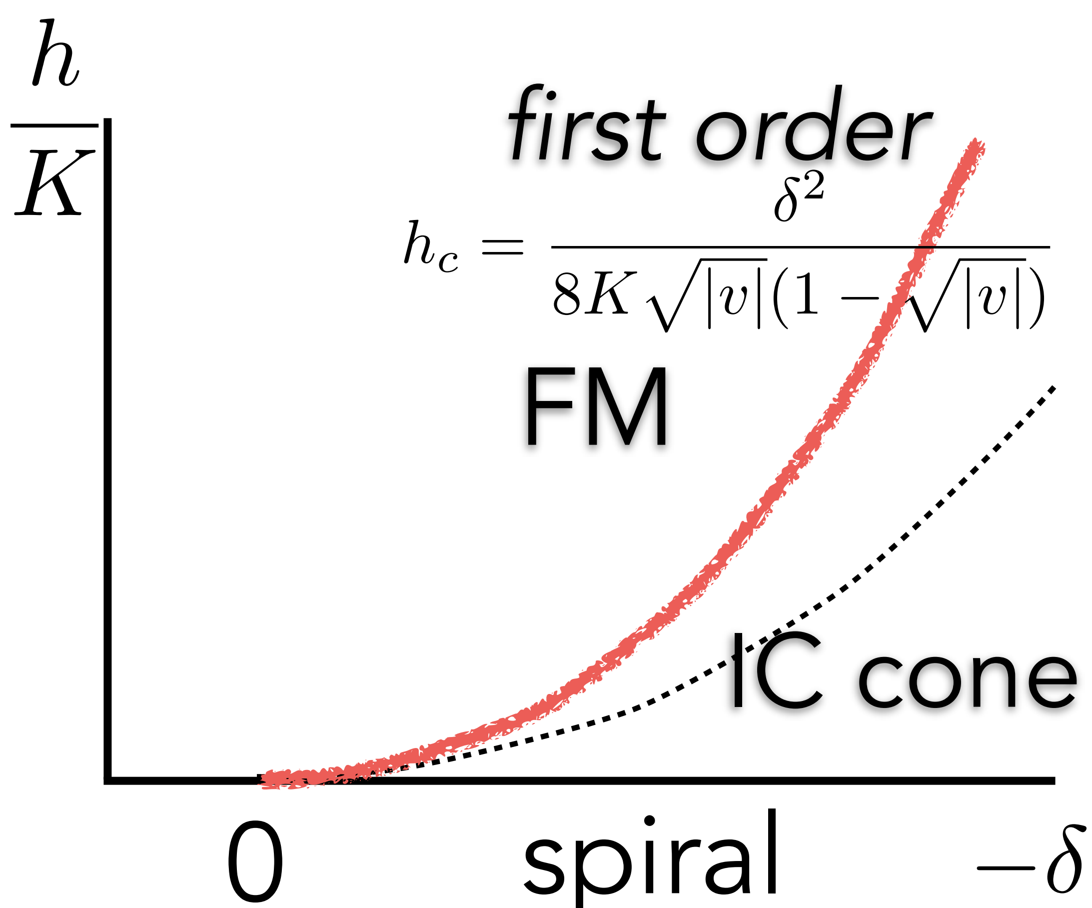
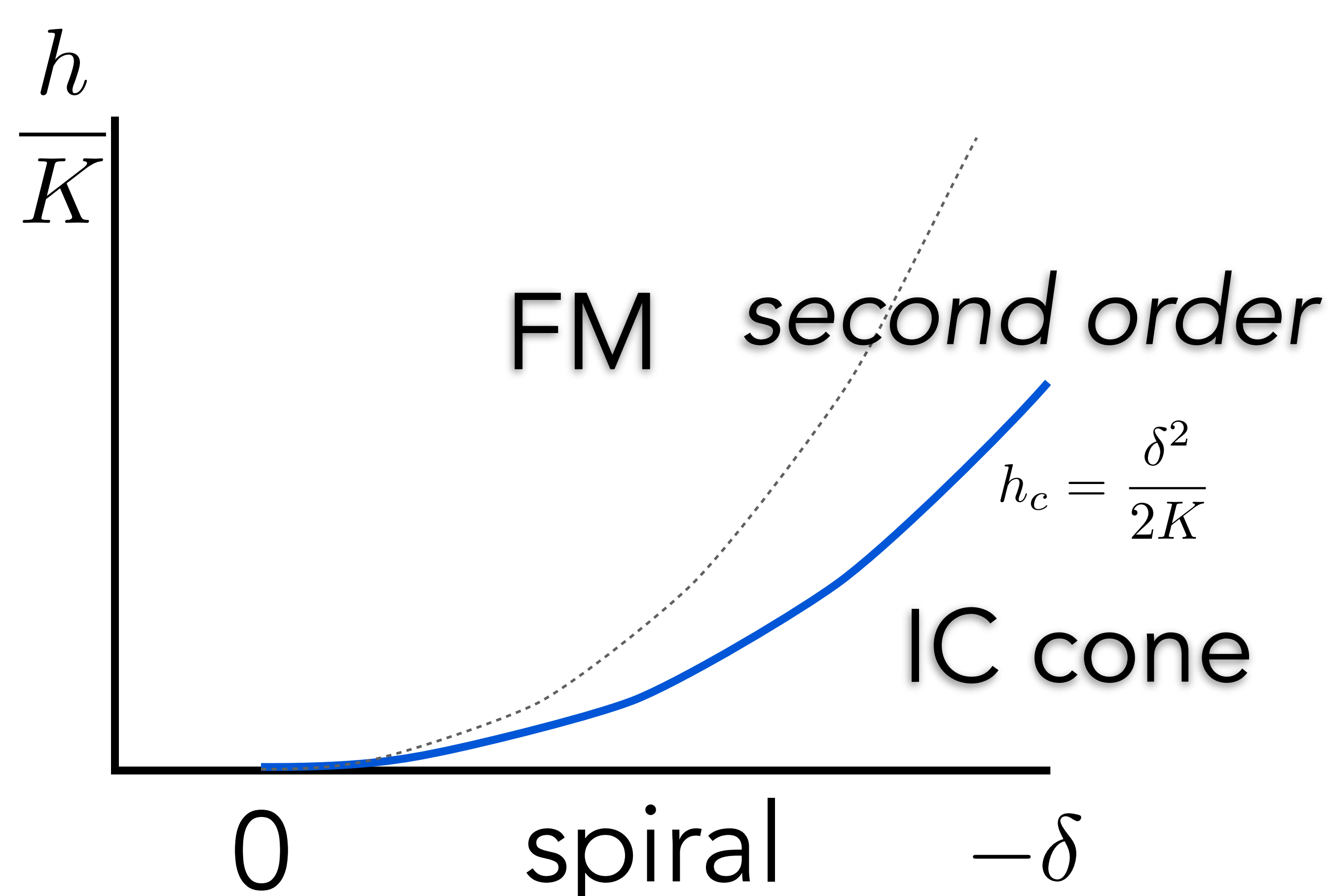


Saddle point

$$S = \sqrt{\frac{K}{\delta}} \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$



$-1 < v < -1/4$



$v > -1/4$

Note: at saddle point level there is no scale for δ

Saddle point predicts 1st order transition for S < 6 !

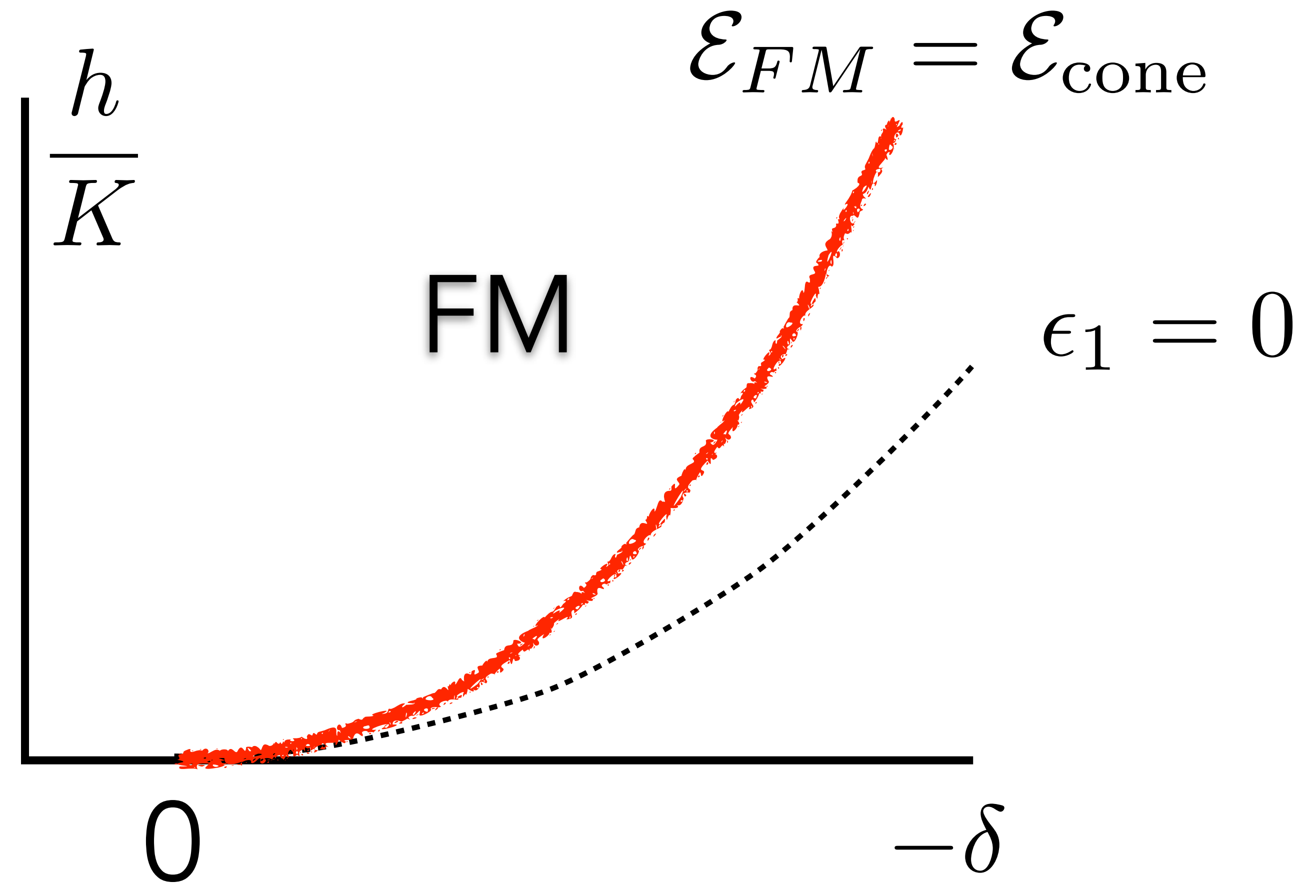
PHYSICAL REVIEW B **84**, 224409 (2011)

Resonances in a dilute gas of magnons and metamagnetism of isotropic frustrated ferromagnetic spin chains

M. Arlego,[1] F. Heidrich-Meisner,[2] A. Honecker,[3] G. Rossini,[1] and T. Vekua[4]
[1]*Departamento de Física, Universidad Nacional de La Plata, C.C. 67, 1900 La Plata, Argentina*
[2]*Physics Department and Arnold Sommerfeld Center for Theoretical Physics, Ludwig-Maximilians-Universität München, D-80333 München, Germany*
[3]*Institut für Theoretische Physik, Georg-August-Universität Göttingen, D-37077 Göttingen, Germany*
[4]*Institut für Theoretische Physik, Leibniz Universität Hannover, D-30167 Hannover, Germany*
(Received 22 November 2011; published 14 December 2011)

We show that spin-S chains with SU(2)-symmetric, ferromagnetic nearest-neighbor and frustrating antiferromagnetic next-nearest-neighbor exchange interactions exhibit metamagnetic behavior under the influence of an external magnetic field for small S, in the form of a first-order transition to the fully polarized state. The corresponding magnetization jump increases gradually starting from an S-dependent critical value of exchange couplings and takes a maximum in the vicinity of a ferromagnetic Lifshitz point. The metamagnetism results from resonances in the dilute magnon gas caused by an interplay between quantum fluctuations and frustration.

$$v = -\frac{3}{2s}$$ ⇨ find S_{cr}=6



PHYSICAL REVIEW B **85**, 064420 (2012)

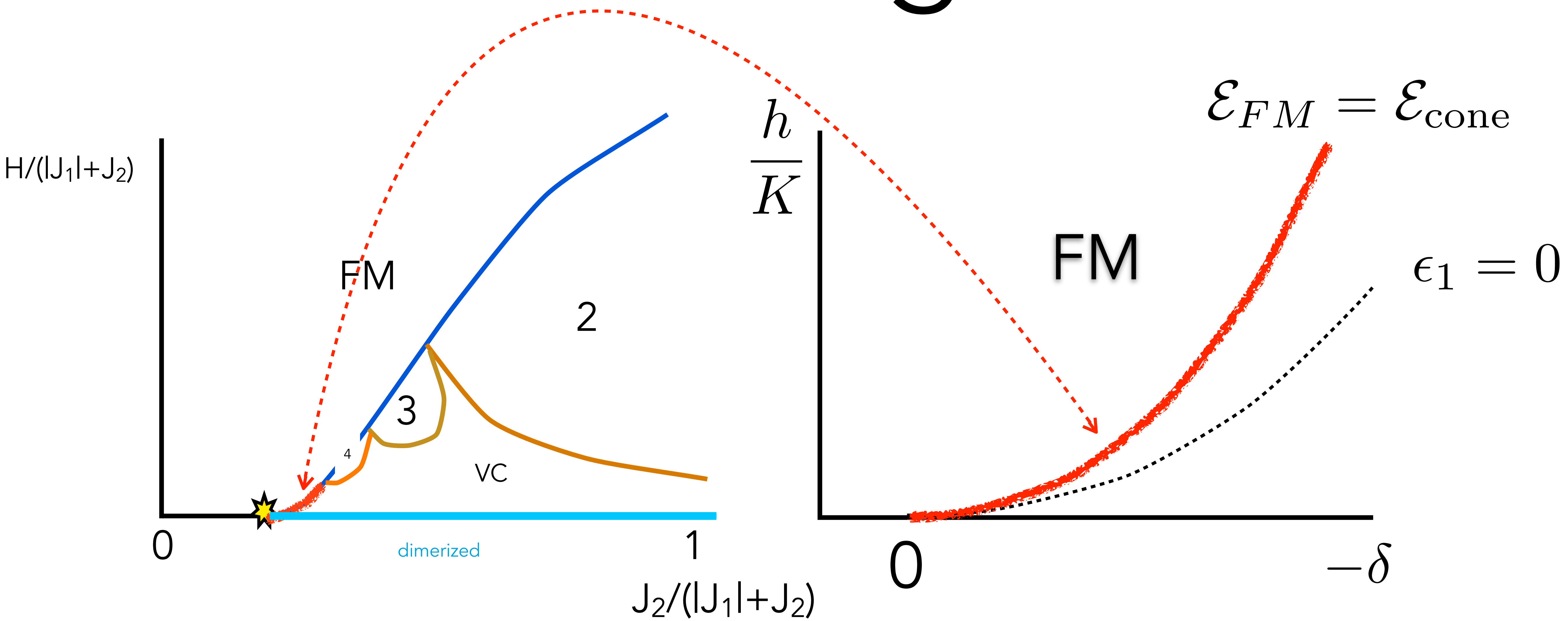
Frustrated spin chains in strong magnetic field: Dilute two-component Bose gas regime

A. K. Kolezhuk,[1,2] F. Heidrich-Meisner,[3] S. Greschner,[4] and T. Vekua[4]
[1]*Institute of Magnetism, National Academy of Sciences and Ministry of Education, 36-b Vernadskii av., 03142 Kiev, Ukraine*
[2]*Institute of High Technologies, T. Shevchenko Kiev National University, 64 Volodymyrska str., 01601 Kiev, Ukraine*
[3]*Department of Physics and Arnold Sommerfeld Center for Theoretical Physics, Ludwig-Maximilians-Universität München, D-80333 München, Germany*
[4]*Institut für Theoretische Physik, Leibniz Universität Hannover, Appelstr. 2, 30167 Hannover, Germany*
(Received 24 December 2011; published 28 February 2012)

We study the ground state of frustrated spin-S chains in a strong magnetic field in the immediate vicinity of saturation. In strongly frustrated chains, the magnon dispersion has two degenerate minima at inequivalent momenta $\pm Q$, and just below the saturation field the system can be effectively represented as a dilute one-dimensional lattice gas of two species of bosons that correspond to magnons with momenta around $\pm Q$. We present a theory of effective interactions in such a dilute magnon gas that allows us to make quantitative predictions for arbitrary values of the spin. With the help of this method, we are able to establish the magnetic phase diagram of frustrated chains close to saturation and study phase transitions between several nontrivial states, including a two-component Luttinger liquid, a vector chiral phase, and phases with bound magnons. We study those phase transitions numerically and find a good agreement with our analytical predictions.

Thus, $S_{cr} = 5$ is the critical value of spin where the metamagnetic behavior vanishes in isotropic chains (for $S > 5$, it exists only in the presence of an easy-axis anisotropy). We would like to note that Ref. 22 reported a slightly different value of $S_{cr} = 6$; this discrepancy is due to the fact that Ref. 22 used just the leading term in the large-S expansion, while our present approach is exact to all orders in $1/S$. Figure 6 illustrates the behavior of S_{cr} as a function of the order of the $1/S$ expansion.

Missing?

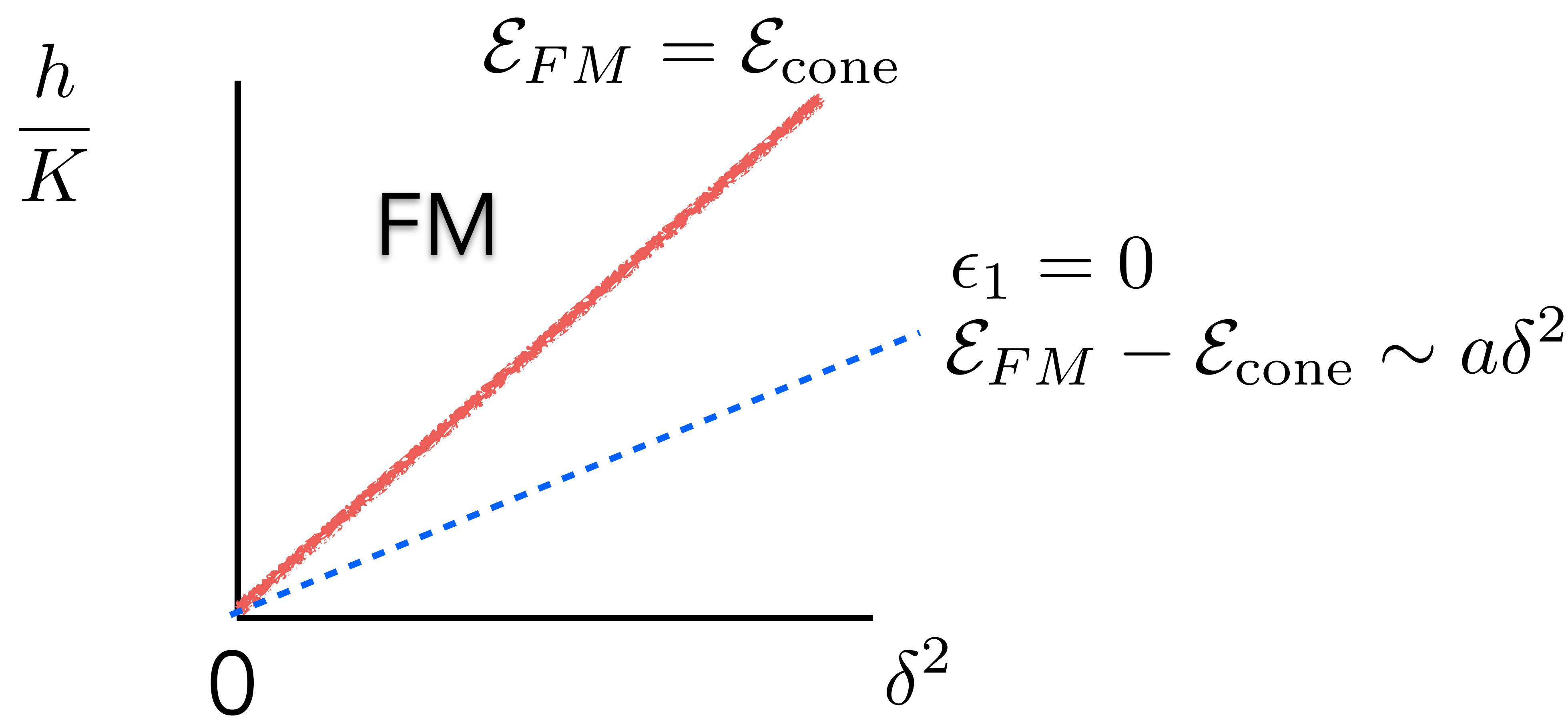


J_1-J_2

saddle point

misses metamagnetic **endpoint** and multipolar phases

Metamagnetic endpoint?



Quantum corrections penalize $E_{\rm cone}$ but not $E_{\rm FM}$

$$\sim \mathcal{E} \times \sqrt{\delta/K}$$

$$\Delta\mathcal{E}_{\rm cone} = +f(v)\delta^{5/2}$$

Quantum corrections

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

<u>transformation to rotating frame</u> $\hat{e}_1 \times \hat{e}_2 = \hat{e}_3 = \hat{m}_{\mathrm{saddle-point}}$

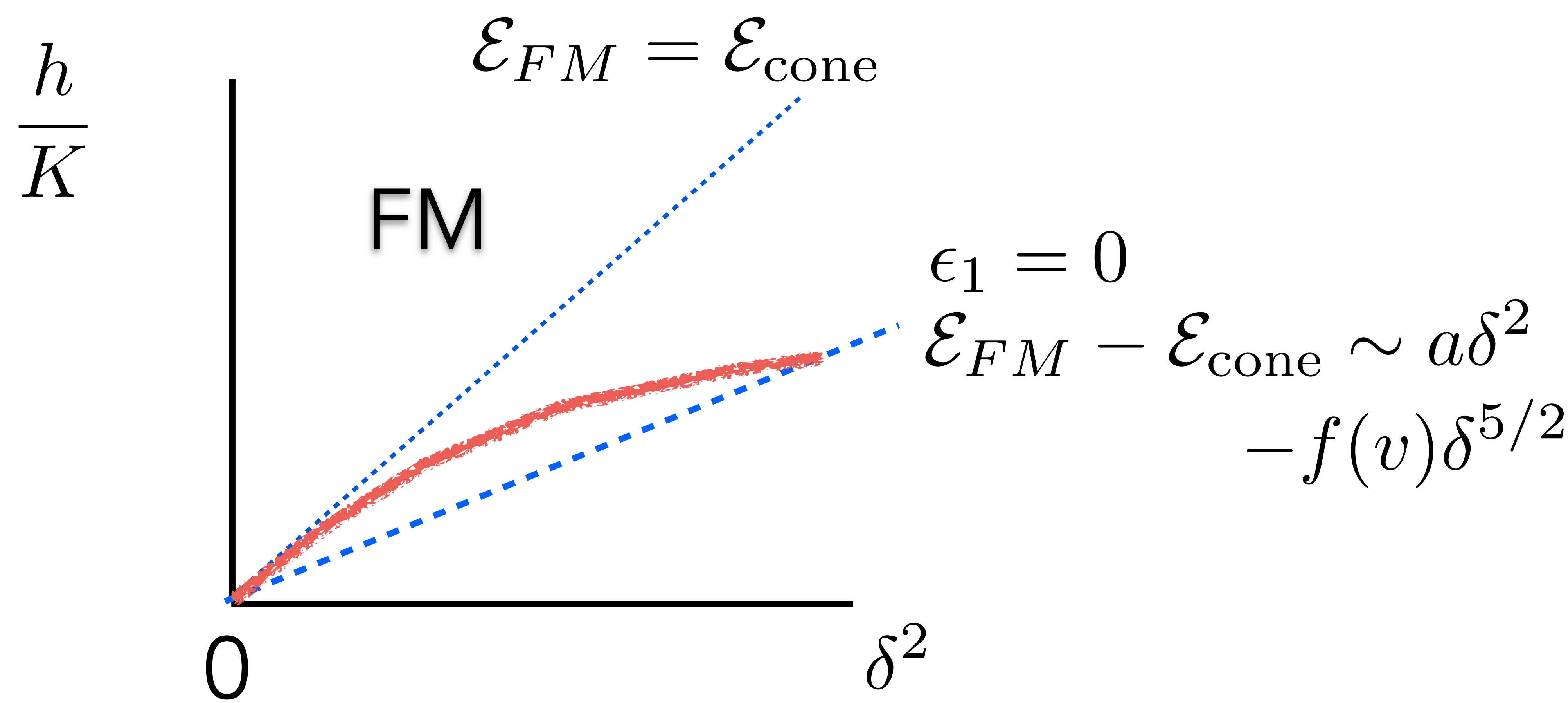
$$\hat{m} = \sqrt{2 - \frac{\bar{\eta}\eta}{s}} [\frac{\bar{\eta} + \eta}{2\sqrt{s}} \hat{e}_1 + i\frac{\bar{\eta} - \eta}{2\sqrt{s}} \hat{e}_2] + (1 - \frac{\bar{\eta}\eta}{s})\hat{e}_3,$$

<u>effective Bogoliubov Hamiltonian</u>

$$S = S_{\mathrm{sp}} + \int dx\, d\tau \ \{\overline{\eta}\partial_\tau \eta + H(\overline{\eta}, \eta)\} \quad + O(\eta^3)$$

diagonalization gives correction to GS energy

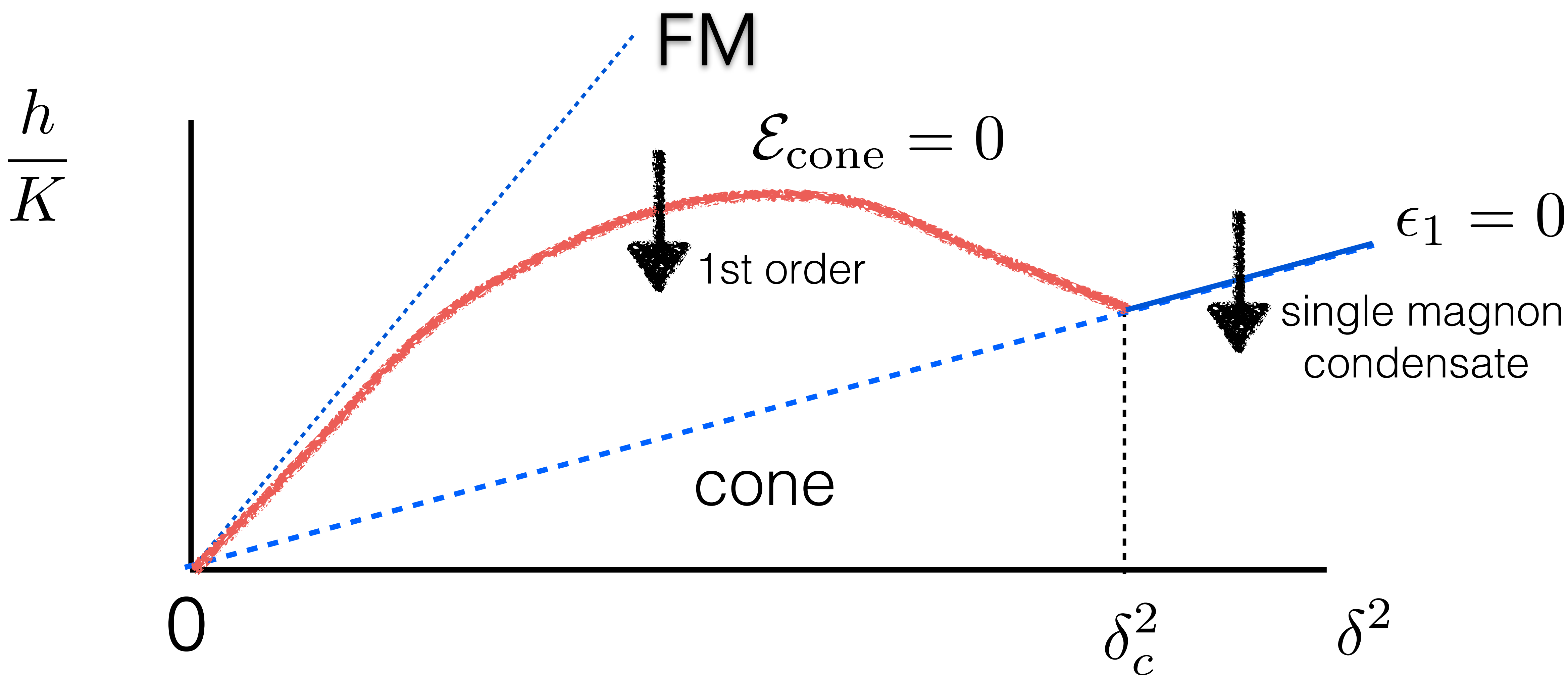
Metamagnetic endpoint?



Corrected first order curve bends slightly downward to intersect second order line

Instabilities

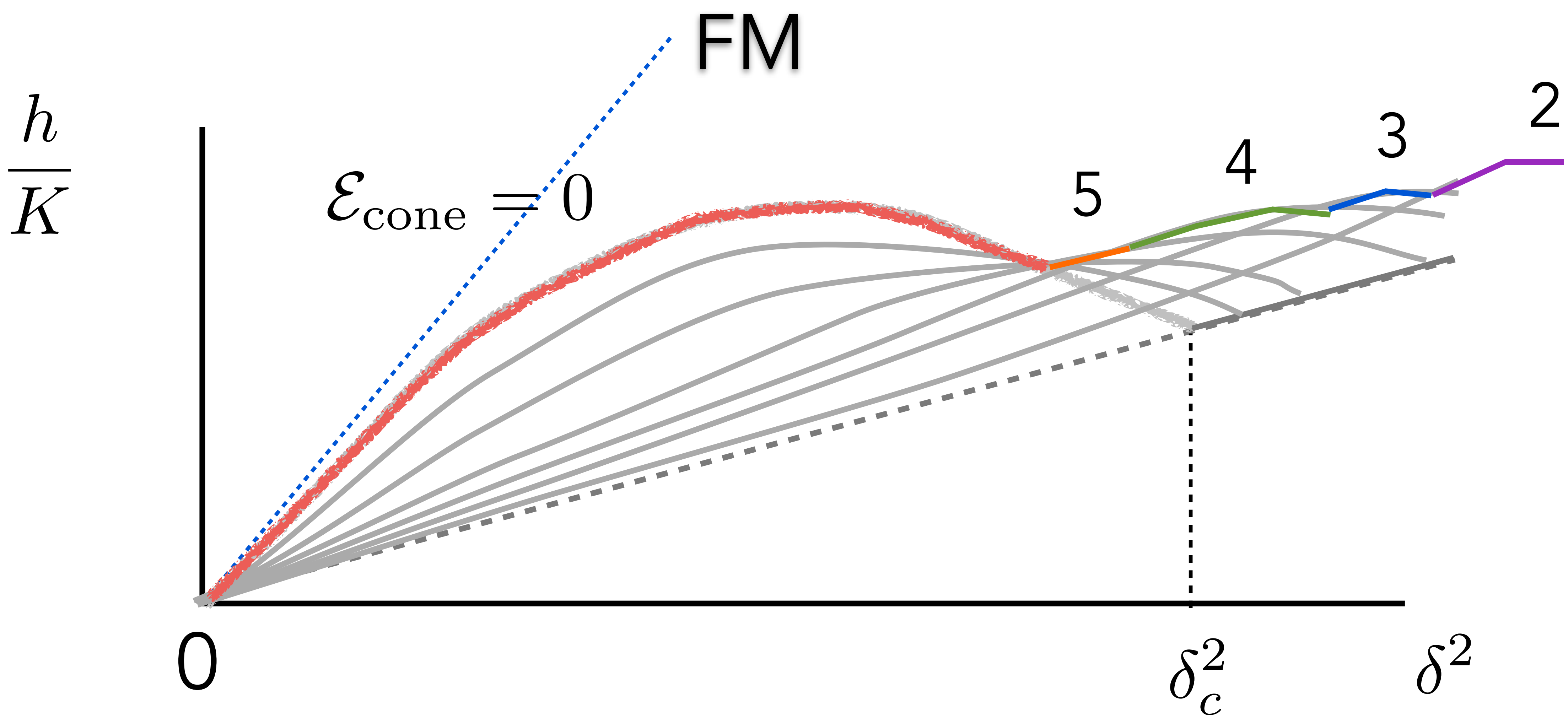
- Choose E_{FM}=0



What about multi-particle instabilities?

Instabilities

- Choose E_{FM}=0



Numerics suggests multipolar condensates beyond $\boldsymbol{\delta}_c$

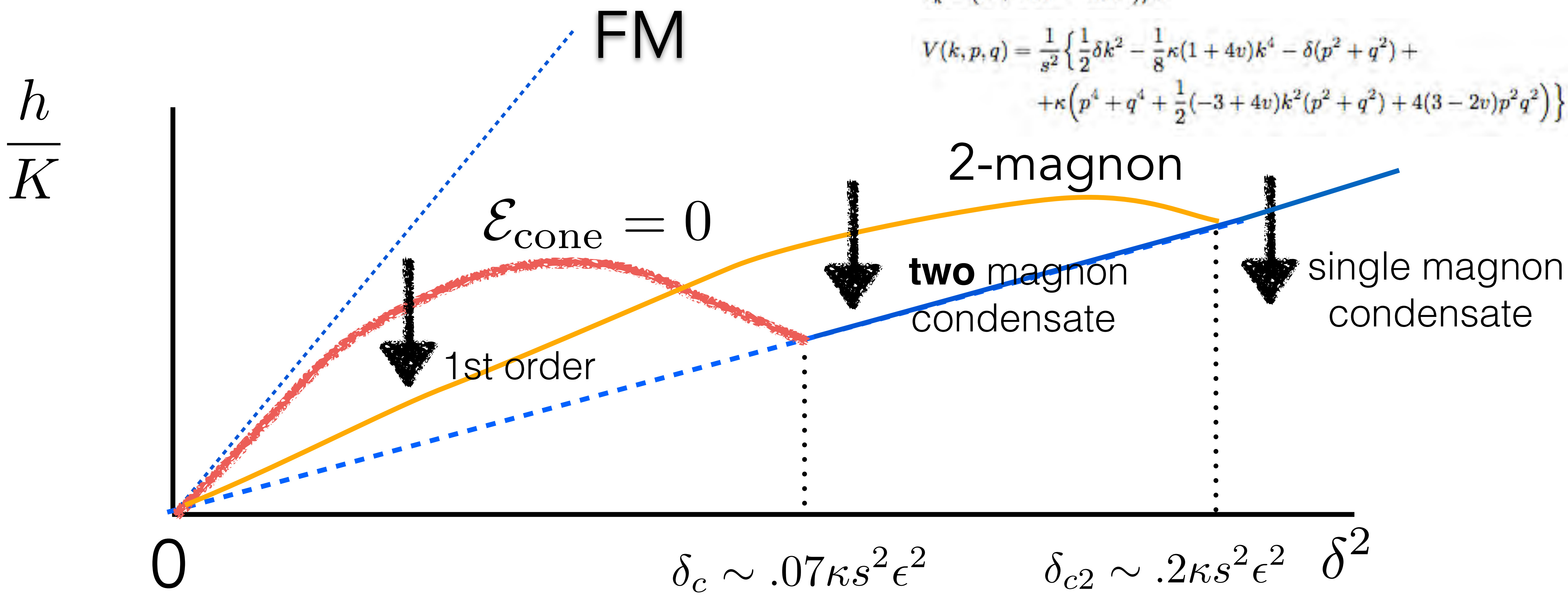
2-magnon check of the proposed scenario

$$H = \sum_k \epsilon_k \bar{\eta}_k \eta_k + \frac{1}{2L} \sum_{kpp'} V(k,p,p') \bar{\eta}_{k/2+p} \bar{\eta}_{k/2-p} \eta_{k/2-p'} \eta_{k/2+p'}$$

- Compute *exact* 2-magnon energy in QFT

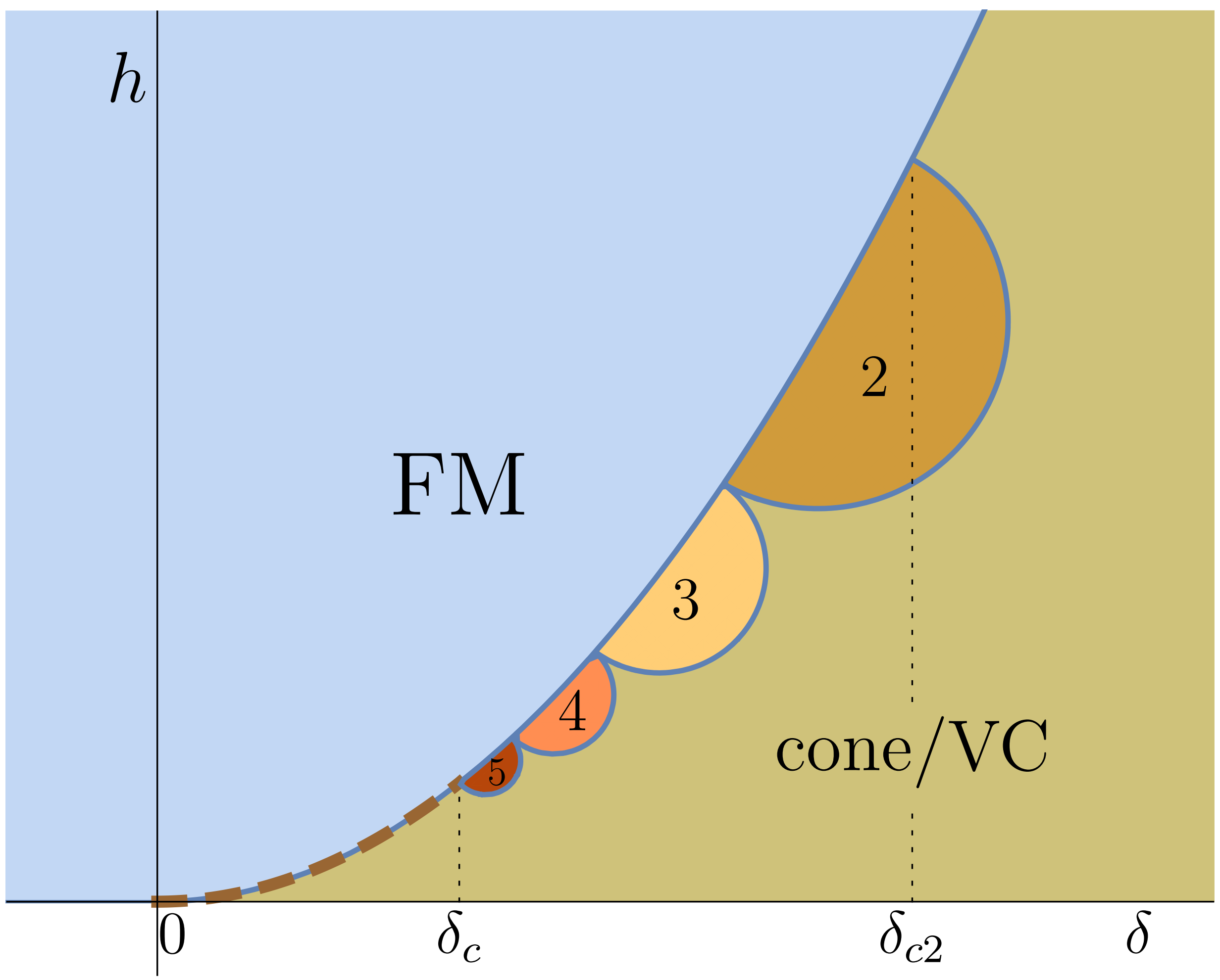
$$\epsilon_k = (h + 2\kappa k^4 - 2\delta k^2)/s$$

$$V(k,p,q) = \frac{1}{s^2}\Big\{\frac{1}{2}\delta k^2 - \frac{1}{8}\kappa(1+4v)k^4 - \delta(p^2+q^2) + {} + \kappa\Big(p^4 + q^4 + \frac{1}{2}(-3+4v)k^2(p^2+q^2) + 4(3-2v)p^2q^2\Big)\Big\}$$



Separation of metamagnetism and multipole formation

Summary



Lifshitz point is a "parent" of many phases

$$S = \int dx d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

d>1

$$S = \int dx d^{d-1} y d\tau \{ is\mathcal{A}_B[\hat{m}] + \delta|\partial_x \hat{m}|^2 + c|\partial_y \hat{m}|^2 + K|\partial_x^2 \hat{m}|^2 + u|\partial_x \hat{m}|^4 - h\hat{m}_z \}$$

- Rescaling:

$$x \rightarrow \sqrt{\frac{K}{|\delta|}} x \qquad \tau \rightarrow \frac{K}{\delta^2} \tau \qquad y \rightarrow \frac{\sqrt{cK}}{\delta} y$$

$$S = \frac{\sqrt{K^d c^{d-1}}}{\delta^{d-1/2}} \int dx d^{d-1} y d\tau \{ is\mathcal{A}_B[\hat{m}] + \mathrm{sgn}(\delta)|\partial_x \hat{m}|^2 + |\partial_x^2 \hat{m}|^2 + |\partial_y \hat{m}|^2 + v|\partial_x \hat{m}|^4 - \overline{h}\hat{m}_z \}$$

$\therefore$ Similar theory applies in d>1, and very similar conclusions apply

Outline

- Magnon BEC
- Materials
- Basic theory and some numerics
- *Field theory of the Lifshitz point*
- *Spin-current state* near the end-point of 1/3 magnetization plateau

magnon "BCS" superconductor

exotic superconductor of magnons

Question

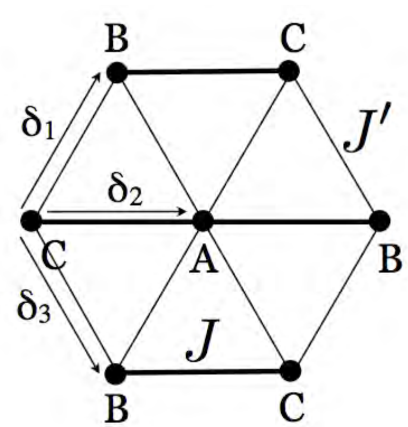
- Is magnon pairing possible in a system with purely repulsive (antiferromagnetic) interactions?

Nematic — superconductor analogy suggests positive answer: Magnon analogue of Kohn-Luttinger mechanism (e.g. pairing due to repulsive interactions)

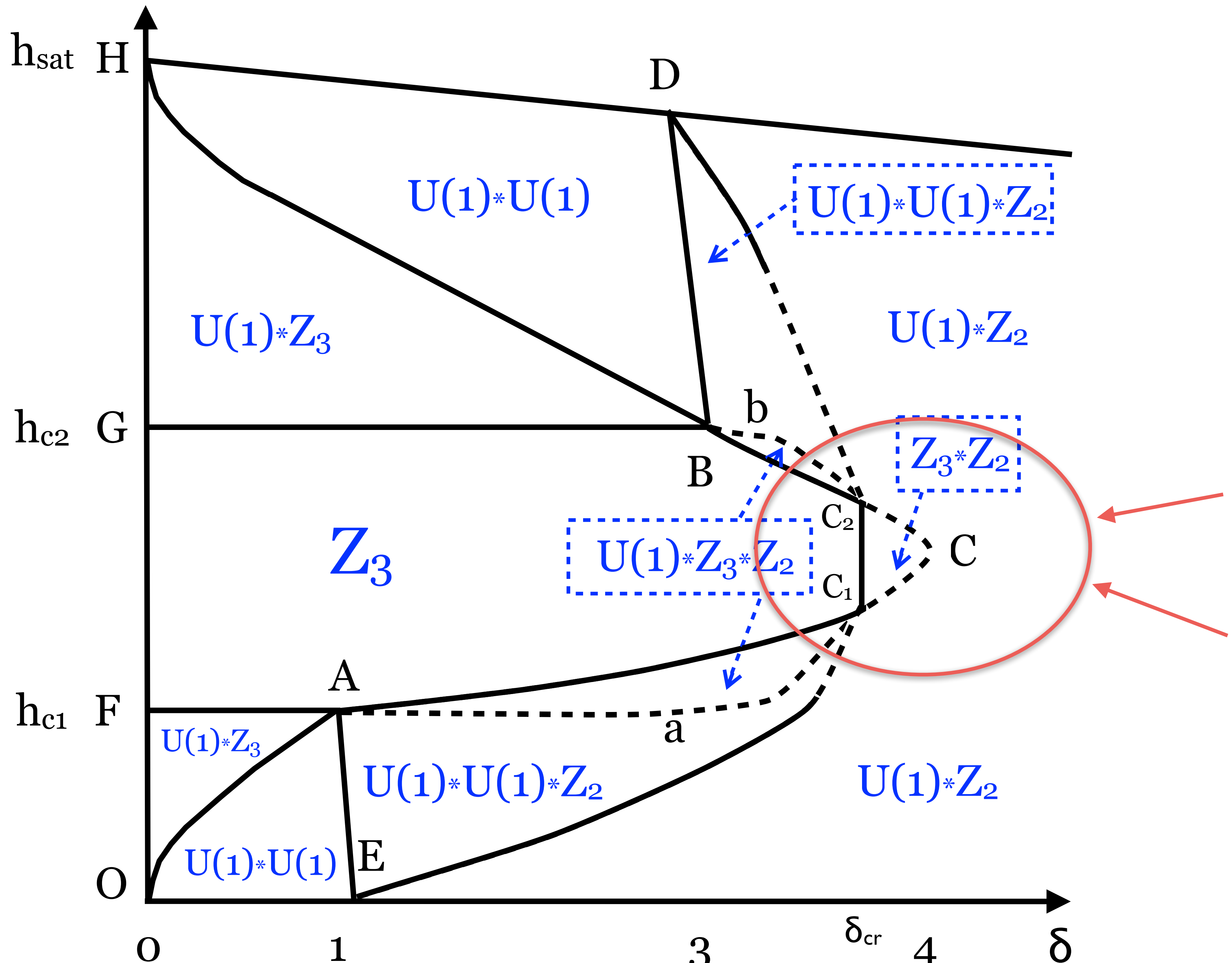
THIS TALK:

2-magnon condensate near the end-point of the 1/3 magnetization plateau

$$H = \sum_{\langle i,j\rangle} J_{ij}\vec{S}_i \cdot \vec{S}_j$$

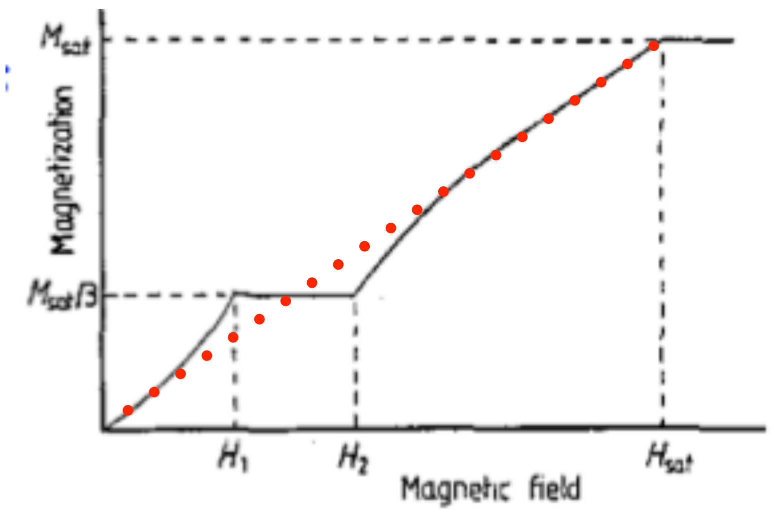


$$\delta = \frac{40}{3}S\left(\frac{J-J'}{J}\right)^2$$

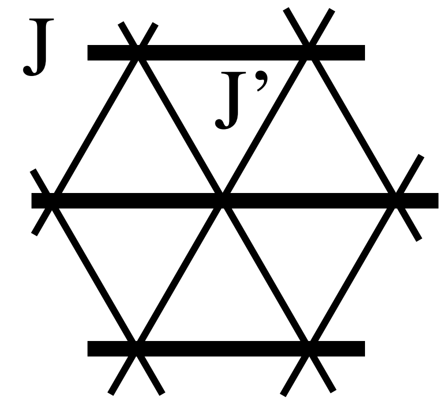


OAS, Reports on Progress in Physics 78, 052502 (2015),
OAS, Wen Jin, Chubukov, Phys. Rev. Lett. 113, 087204 (2014)

Spatially anisotropic model: classical vs *quantum*



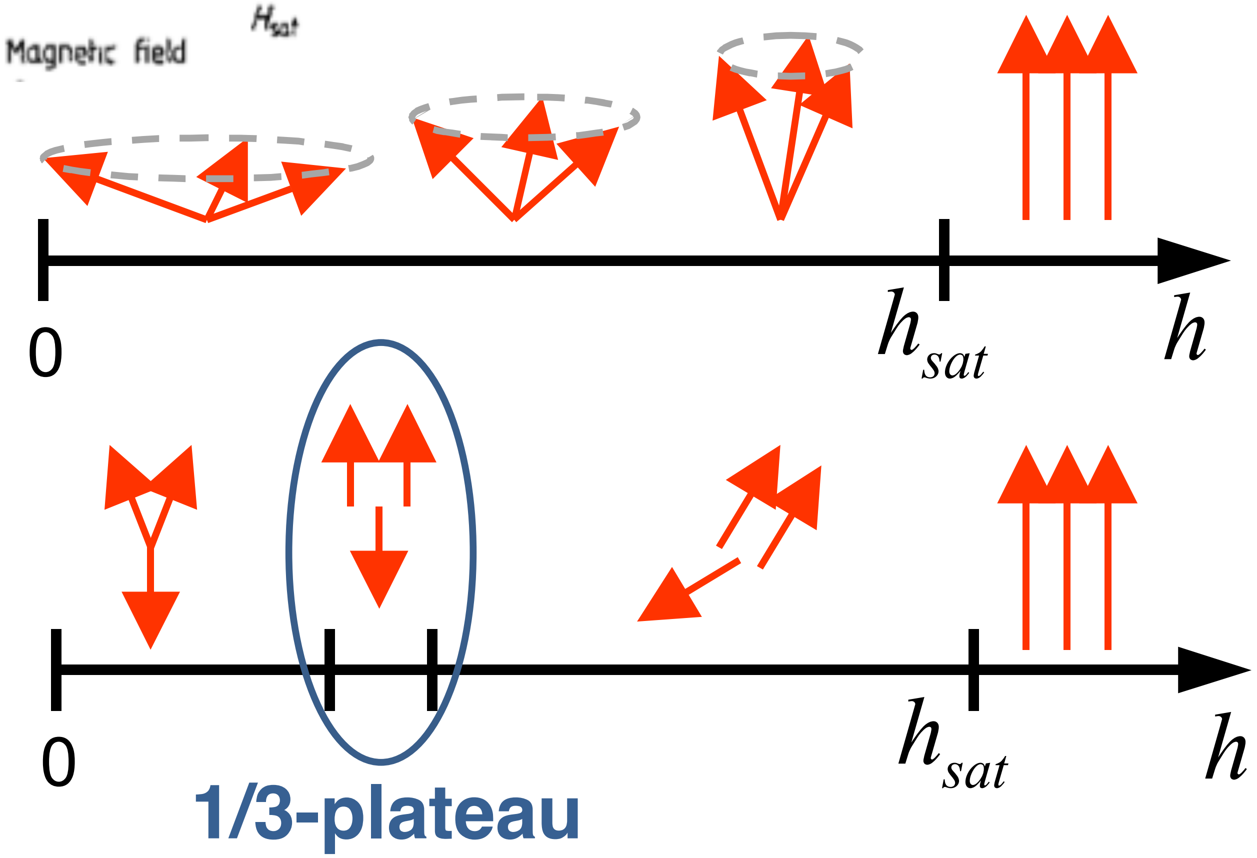
$$H = \sum_{\langle ij \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - h \sum_i \mathbf{S}_i^z$$



$S = \infty$

$\boxed{J' \neq J}$

$S = \frac{1}{2}$



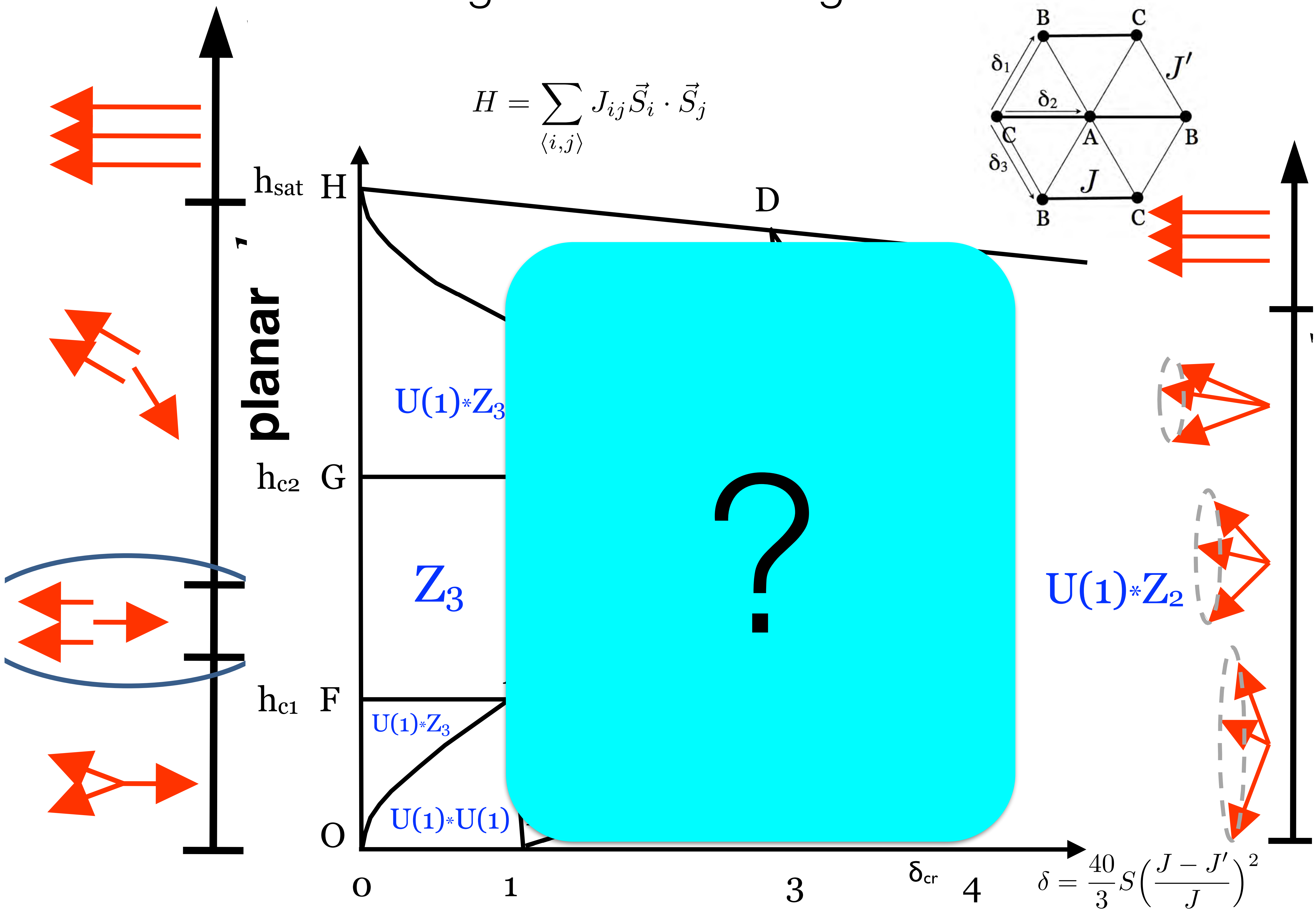
Umbrella state: favored classically; energy gain (J-J’)^2/J

Planar states: favored by quantum fluctuations; energy gain J/S

The competition is controlled by dimensionless parameter

$$\delta = S(J - J')^2 / J^2$$

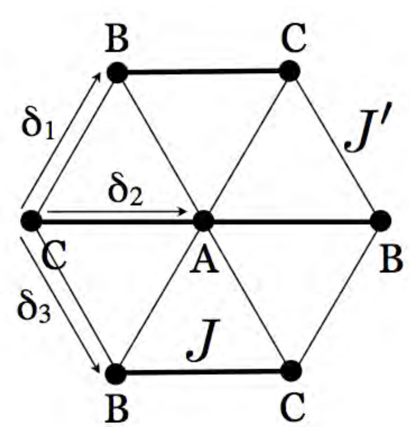
Emergent Ising orders in quantum two-dimensional triangular antiferromagnet at T=0

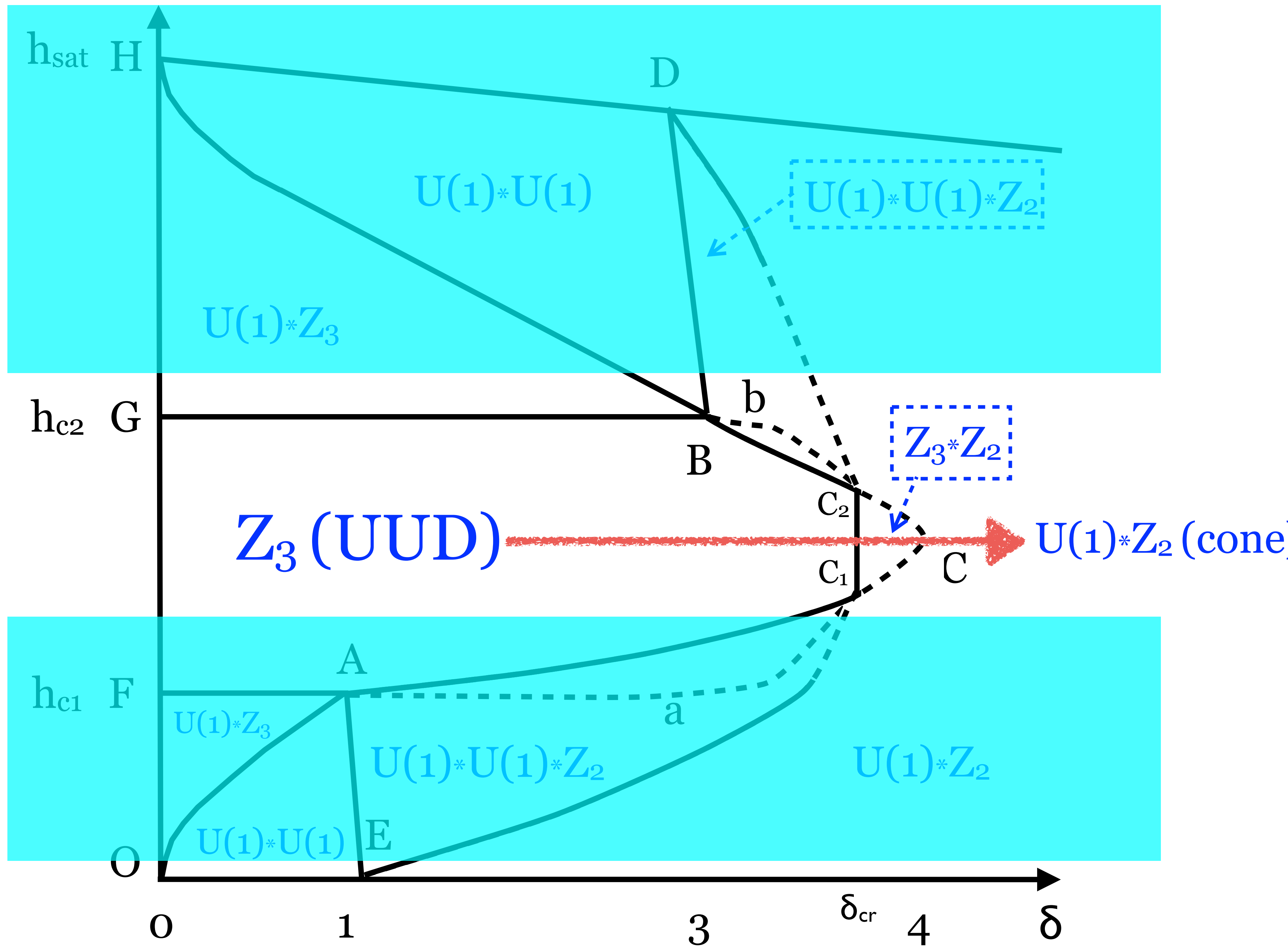


UUD-to-cone phase transition

$$Z_3 \to U(1) \times Z_2 \text{ or } Z_3 \to \text{smth else} \to U(1) \times Z_2?$$

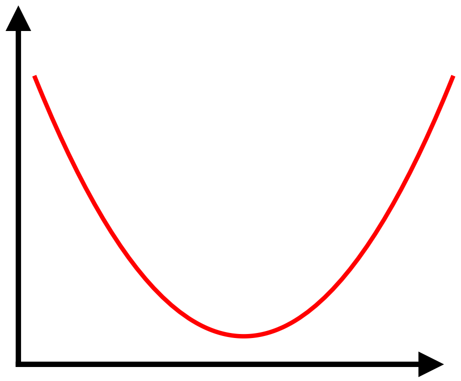
$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$





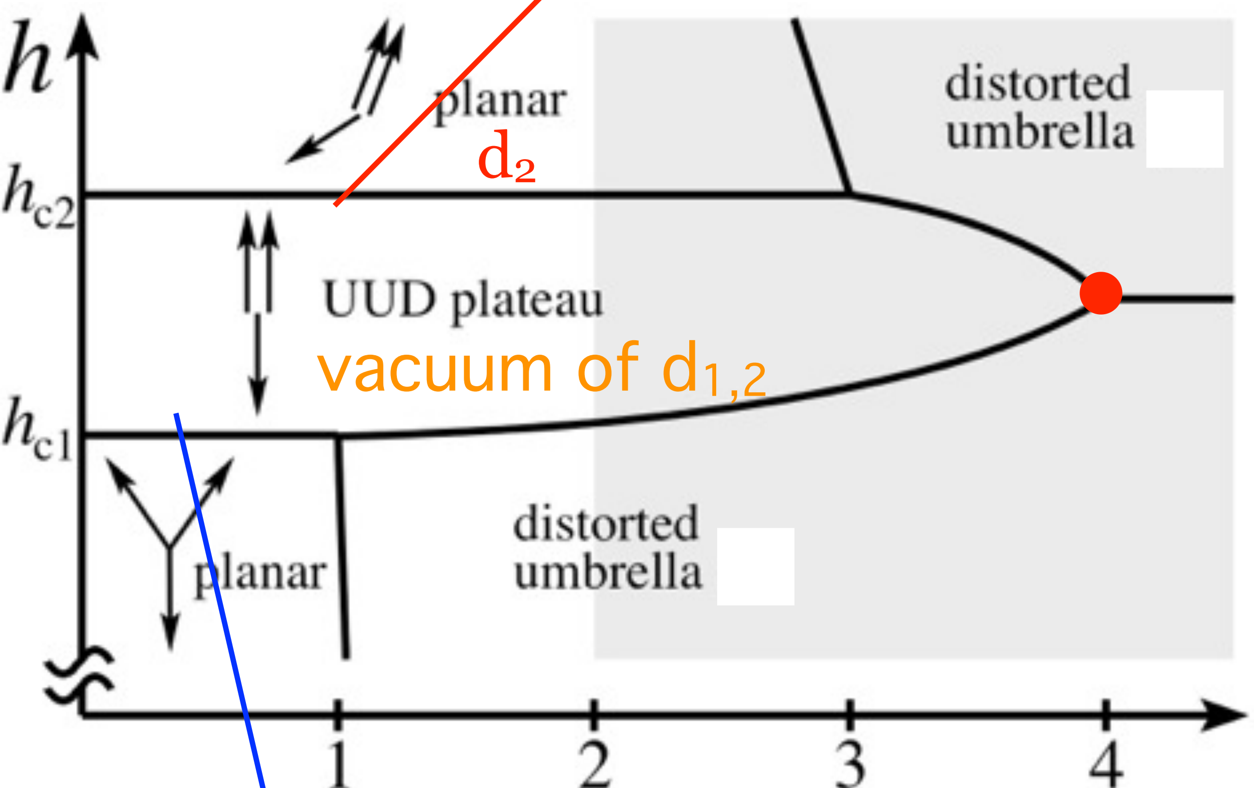
$$\delta = \frac{40}{3} S \left(\frac{J - J'}{J} \right)^2$$

Low-energy excitation spectra

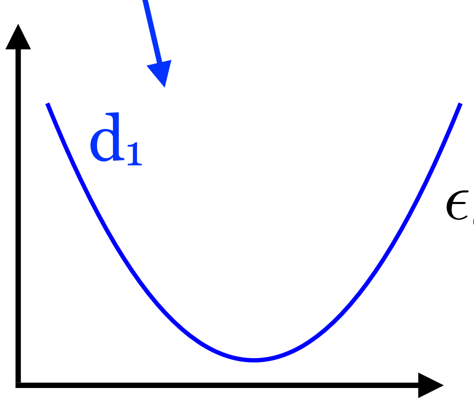


$$\epsilon_{d2} = h_{c2} - h + \frac{9Jk^2}{4}$$

for δ < 3



$$\delta = \frac{40\ S}{3}(1 - J'/J)^2$$



$$\epsilon_{d1} = h - h_{c1} + \frac{3Jk^2}{4}$$

for δ < 1

Magnetization plateau is **collinear** phase: preserves O(2) rotations about magnetic field -- no gapless spin waves. Breaks only discrete Z_3. Hence, ***very stable***.

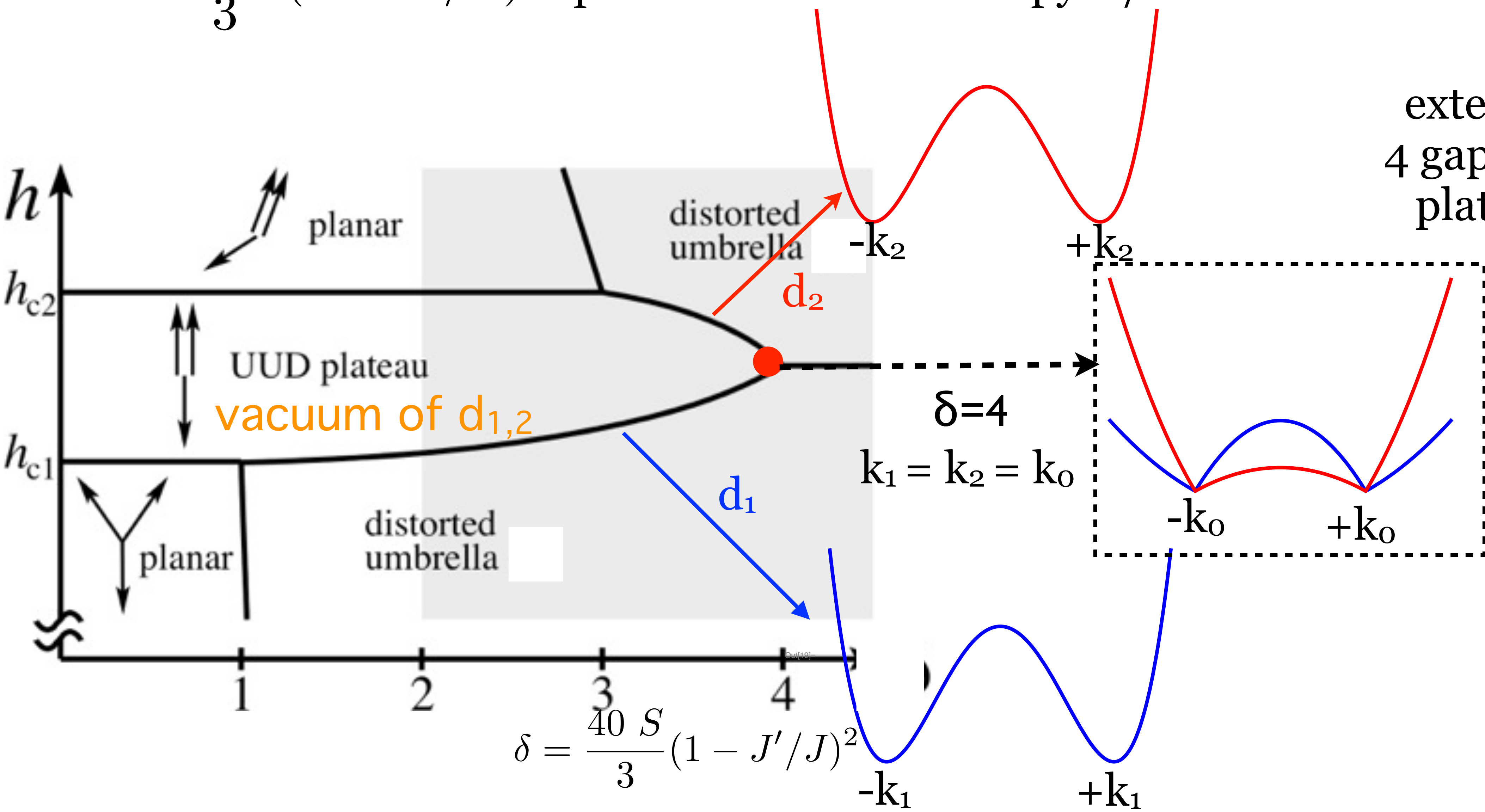
$$h_{c2} - h_{c1} = \frac{0.6}{2S} h_{\text{sat}} = \frac{0.6}{2S}(9JS)$$

Bose-Einstein condensation of d_1 (d_2) mode at k =0 leads to lower (upper) co-planar phase

Alicea, Chubukov, OS PRL 2009

Low-energy excitation spectra near the plateau's end-point

$$\delta = \frac{40\ S}{3}(1 - J'/J)^2$$ parameterizes anisotropy J'/J



extended symmetry:
4 gapless modes at the
plateau's end-point

$$k_0 = \sqrt{\frac{3}{10S}}$$

S>>1

Magnetization plateau is **collinear** phase: preserves O(2) rotations about magnetic field -- no gapless spin waves.
Breaks only discrete Z_3.

Alicea, Chubukov, OS PRL 2009

Bosonization of 2d interacting magnons

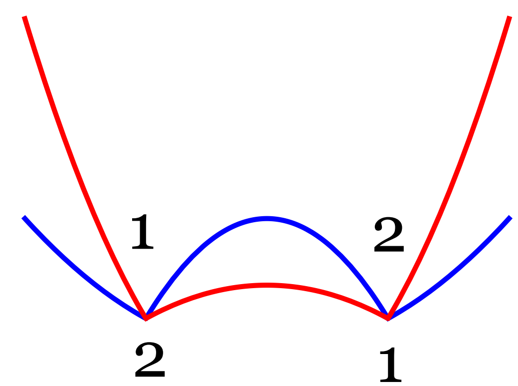
$$\mathcal{H}^{(4)}_{d_1 d_2} = \frac{3}{N}\sum_{p,q}\Phi(p,q)\Big(\underbrace{d^\dagger_{1,\mathbf{k}_0+\mathbf{p}}d^\dagger_{2,-\mathbf{k}_0-\mathbf{p}}}_{\Psi^\dagger_{1,p}\Psi_{2,q}}d_{1,-\mathbf{k}_0+\mathbf{q}}d_{2,\mathbf{k}_0-\mathbf{q}} - d^\dagger_{1,\mathbf{k}_0+\mathbf{p}}d^\dagger_{2,-\mathbf{k}_0-\mathbf{p}}\underbrace{d^\dagger_{1,-\mathbf{k}_0+\mathbf{q}}d^\dagger_{2,\mathbf{k}_0-\mathbf{q}}}_{\Psi^\dagger_{1,p}\Psi^\dagger_{2,q}}\Big) + \text{h.c.}$$

$$\Phi(p,q) \sim \frac{(-3J)k_0^2}{|\mathbf{p}||\mathbf{q}|}$$

singular magnon interaction

magnon **pair operators**

$$\begin{cases} \Psi_{1,p} = d_{1,k_0+p}d_{2,-k_0-p} \\ \Psi_{2,p} = d_{1,-k_0+p}d_{2,k_0-p} \end{cases}$$



Obey canonical Bose commutation relations in the UUD ground state

$$[\Psi_{1,\mathbf{p}}, \Psi_{2,\mathbf{q}}] = \delta_{1,2}\delta_{\mathbf{p},\mathbf{q}}\Big(1 + d^\dagger_{1,\mathbf{k}_0+\mathbf{p}}d_{1,\mathbf{k}_0+\mathbf{p}} + d^\dagger_{2,\mathbf{k}_0+\mathbf{p}}d_{2,\mathbf{k}_0+\mathbf{p}}\Big) \to \delta_{1,2}\delta_{\mathbf{p},\mathbf{q}}$$

In the UUD ground state $\langle d_1^\dagger d_1\rangle_{\text{uud}} = \langle d_2^\dagger d_2\rangle_{\text{uud}} = 0$

★ Interacting magnon Hamiltonian in terms of $\mathbf{d}_{1,2}$ bosons = non-interacting Hamiltonian in terms of $\Psi_{1,2}$ magnon pairs

Chubukov, OS PRL 2013

Two-magnon instability

Magnon pairs $\Psi_{1,2}$ condense *before* single magnons $d_{1,2}$

Equations of motion for Ψ - Hamiltonian

$$\langle \Psi^\dagger_{1,\mathbf{p}} - \Psi_{1,\mathbf{p}} \rangle = \frac{6 J f_p^2}{\Omega_p} \frac{3}{N} \sum_q f_q^2 \langle \Psi^\dagger_{2,\mathbf{q}} - \Psi_{2,\mathbf{q}} \rangle$$

$$\langle \Psi^\dagger_{2,\mathbf{p}} - \Psi_{2,\mathbf{p}} \rangle = \frac{6 J f_p^2}{\Omega_p} \frac{3}{N} \sum_q f_q^2 \langle \Psi^\dagger_{1,\mathbf{q}} - \Psi_{1,\mathbf{q}} \rangle$$

\`Superconducting' solution with *imaginary* order parameter

$$\langle \Psi_{1,p} \rangle = \langle \Psi_{2,p} \rangle \sim i \frac{\Upsilon}{\mathbf{p}^2}$$

Instability = softening of two-magnon mode @ $\delta_{cr} = 4 - O(1/S^2)$

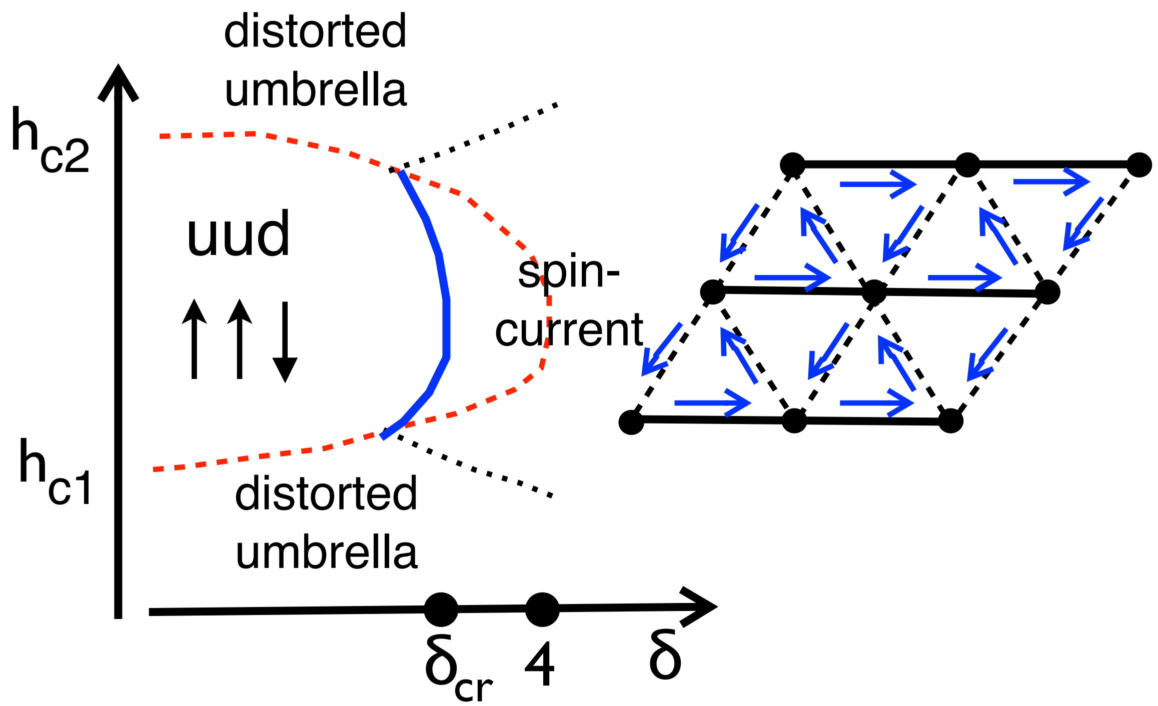
$$1 = \frac{1}{S} \frac{1}{N} \sum_p \frac{k_0}{\sqrt{|\mathbf{p}|^2 + (1 - \delta/4) k_0^2}}$$

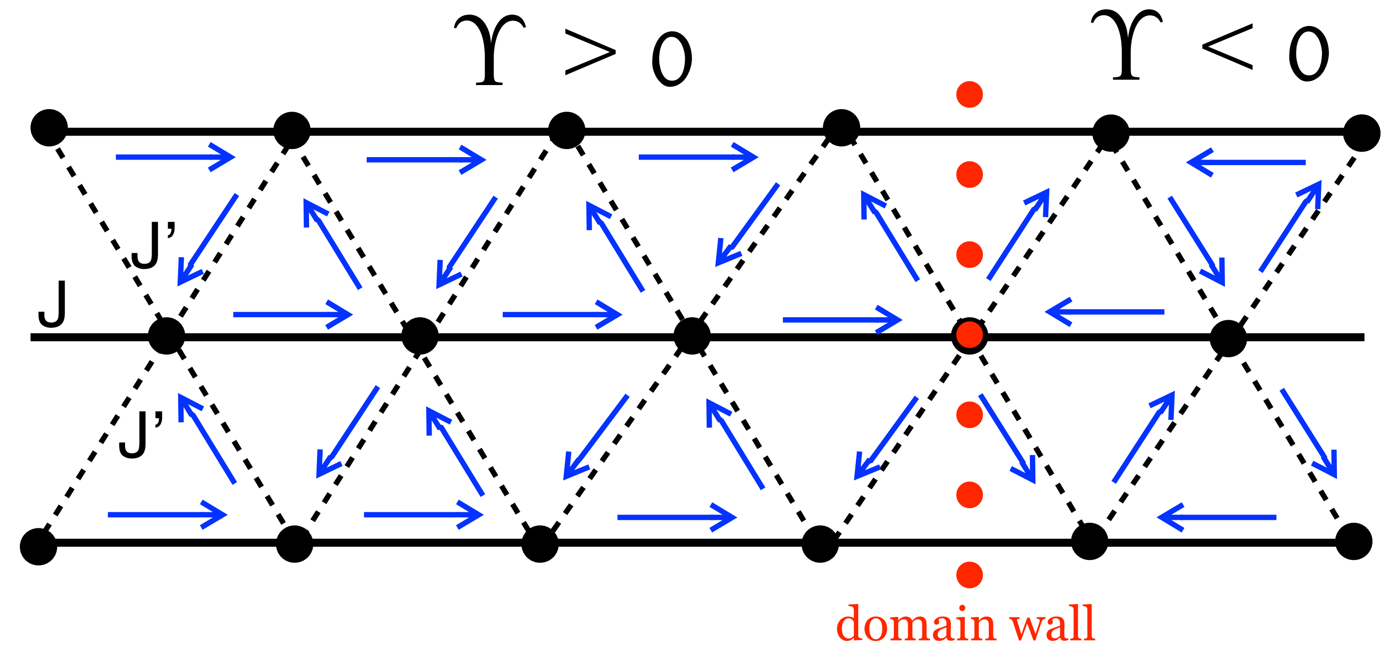
no single particle condensate

$$\langle d_1 \rangle = \langle d_2 \rangle = 0$$

Chubukov, OS PRL 2013

Two-magnon condensate = Spin-current nematic state





no transverse magnetic order $\langle \mathbf{S}_r^{x,y} \rangle = 0$ $\langle \mathbf{S}_r \cdot \mathbf{S}_{r'} \rangle$ is not affected

Finite scalar (and vector) chiralities. Sign of Υ determines sense of spin-current circulation

$$\langle \hat{z} \cdot \mathbf{S}_A \times \mathbf{S}_C \rangle = \langle \hat{z} \cdot \mathbf{S}_C \times \mathbf{S}_B \rangle = \langle \hat{z} \cdot \mathbf{S}_B \times \mathbf{S}_A \rangle \propto \Upsilon$$

Spontaneously broken Z_2 -- spatial inversion

[in addition to broken Z_3 inherited from the UUD state]

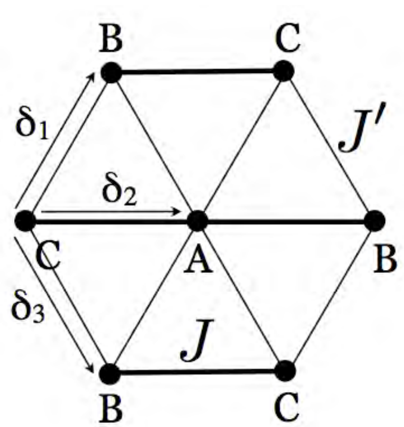
Leads to spontaneous generation of Dzyaloshisnkii-Moriya interaction

Chubukov, OS PRL 2013

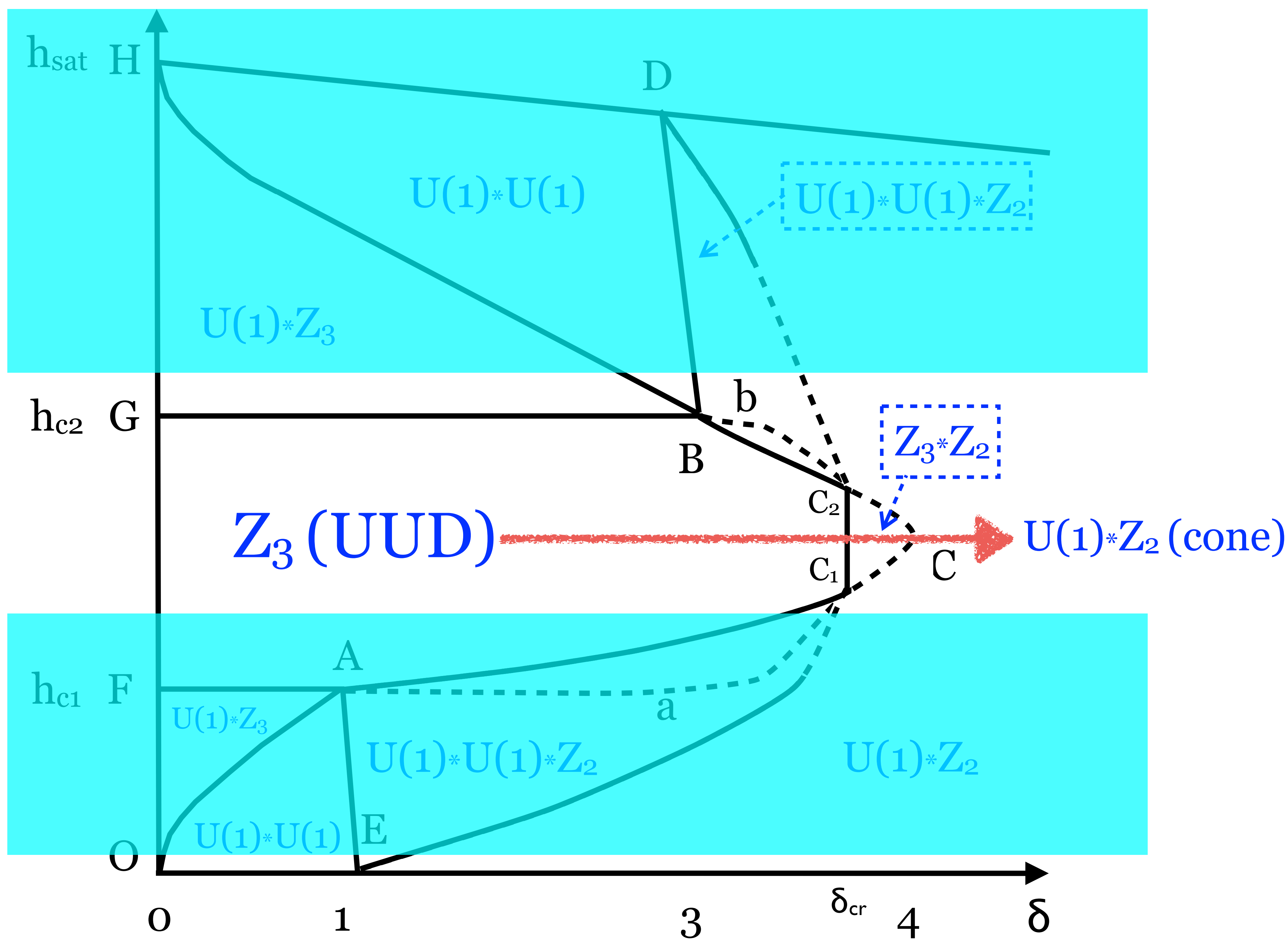
Continuous transition: plateau —> spin-current —> cone !

$$Z_3 \to Z_3 \times Z_2 \to U(1) \times Z_2$$

$$H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



$$\delta = \frac{40}{3} S \Big(\frac{J - J'}{J} \Big)^2$$



VOLUME 81, NUMBER 4 PHYSICAL REVIEW LETTERS 27 JULY 1998

Incommensurate Spin Correlations in Spin-1/2 Frustrated Two-Leg Heisenberg Ladders

Alexander A. Nersesyan,[1] Alexander O. Gogolin,[2] and Fabian H.L. Eßler[3]

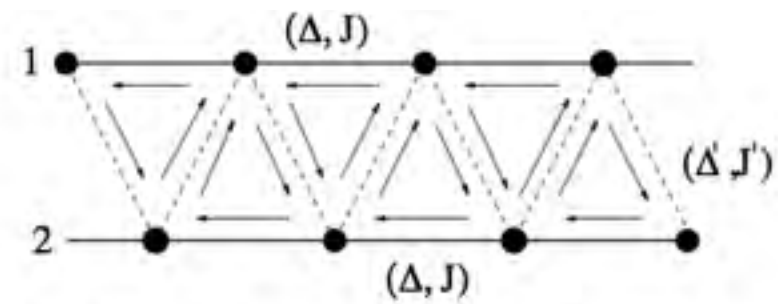


FIG. 3. Structure of the spin currents in the spin nematic phase.

PHYSICAL REVIEW B **87**, 174501 (2013)

Chiral Mott insulator with staggered loop currents in the fully frustrated Bose-Hubbard model

Arya Dhar,[1] Tapan Mishra,[2] Maheswar Maji,[3] R. V. Pai,[4] Subroto Mukerjee,[3,5] and Arun Paramekanti[2,3,6,7]

Spin-current phase = chiral Mott insulator

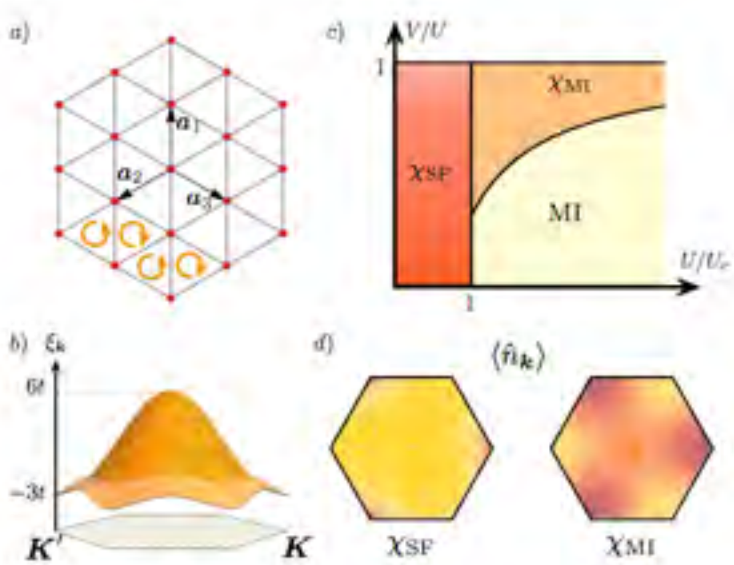


FIG. 1. **Bosons on the Frustrated Triangular Lattice.** (a) Lattice, coordinate system and sample current pattern in the χMI; (b) single-particle dispersion $\xi_{\boldsymbol{k}}$, with minima at the $\boldsymbol{K}, \boldsymbol{K}'$ points of the BZ; (c) Variational mean-field phase diagram showing χSF, χMI and MI phases tuned by the on site repulsion U and nearest neighbor repulsion V; (d) Momentum distribution $\langle \hat{n}_{\boldsymbol{k}} \rangle$ for the chiral phases.

PHYSICAL REVIEW B **89**, 155142 (2014)

Chiral bosonic Mott insulator on the frustrated triangular lattice

Michael P. Zaletel,[1] S. A. Parameswaran,[1,2] Andreas Rüegg,[1,3] and Ehud Altman[1,4]

gapped single particles;
but
spontaneously broken time-reversal
= spontaneous circulating
currents

Conclusions

Magnon pairing is a fascinating problem

Route to multipolar orders of frustrated ferromagnets
extention to d=2 problems?

Spin-current/Chiral Mott insulators

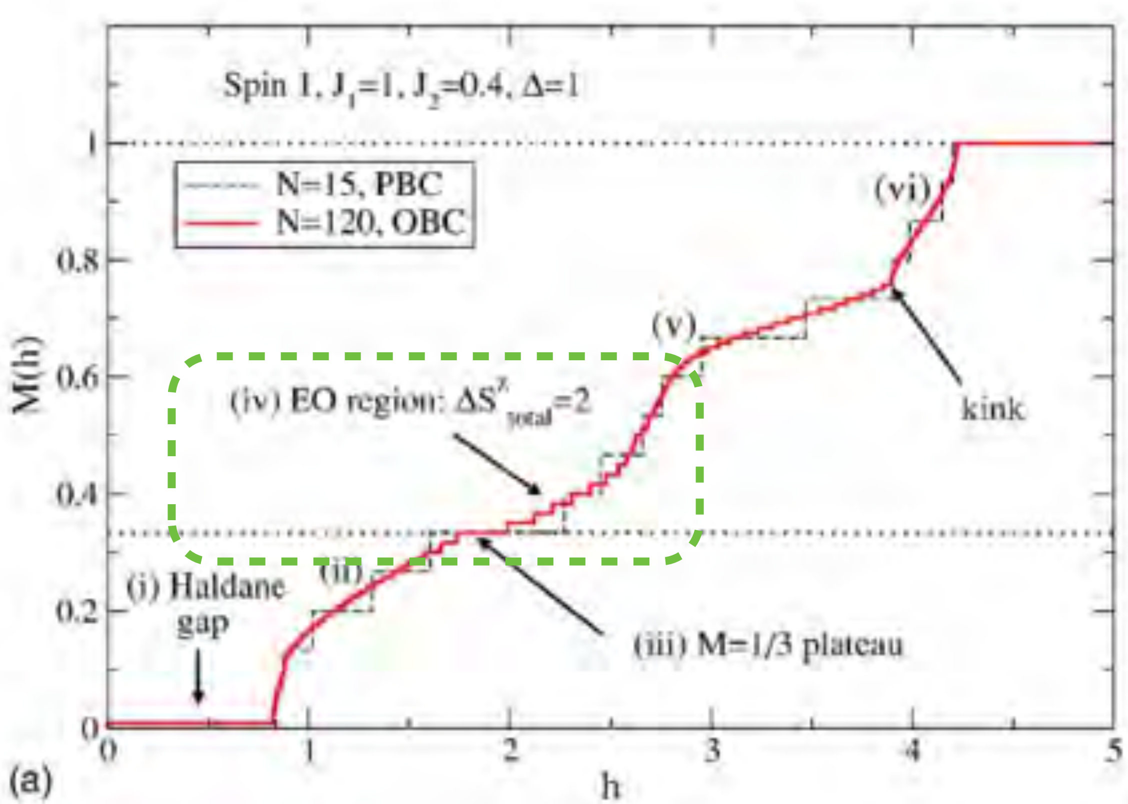
PHYSICAL REVIEW B **75**, 064413 (2007)

Universal emergence of the one-third plateau in the magnetization process of frustrated quantum spin chains

F. Heidrich-Meisner,[1] I. A. Sergienko,[1] A. E. Feiguin,[2] and E. R. Dagotto[1]

$$H = \sum_i \left[\sum_{n=1,2} J_n \left\{ \frac{1}{2}(S_i^+ S_{i+n}^- + S_i^- S_{i+n}^+) + \Delta S_i^z S_{i+n}^z \right\} - h S_i^z + D(S_i^z)^2 \right],$$

S=1,3/2,2



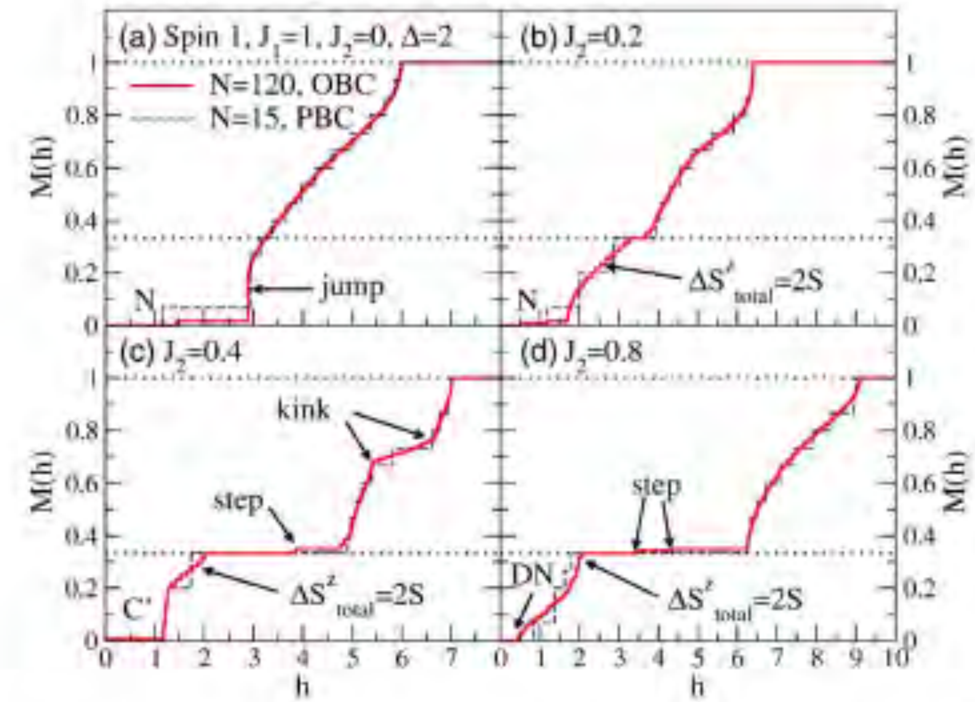


FIG. 11. (Color online) Magnetization curves of frustrated spin-1 chains with an anisotropic exchange (Δ=2) for (a) J_2=0, (b) J_2=0.2, (c) J_2=0.4, and (d) J_2=0.8. DMRG results (straight lines) are for N=60 sites, the dashed lines are ED results (PBC). The capital letters stand for: Néel phase **N**, canted phase **C**, double-Néel

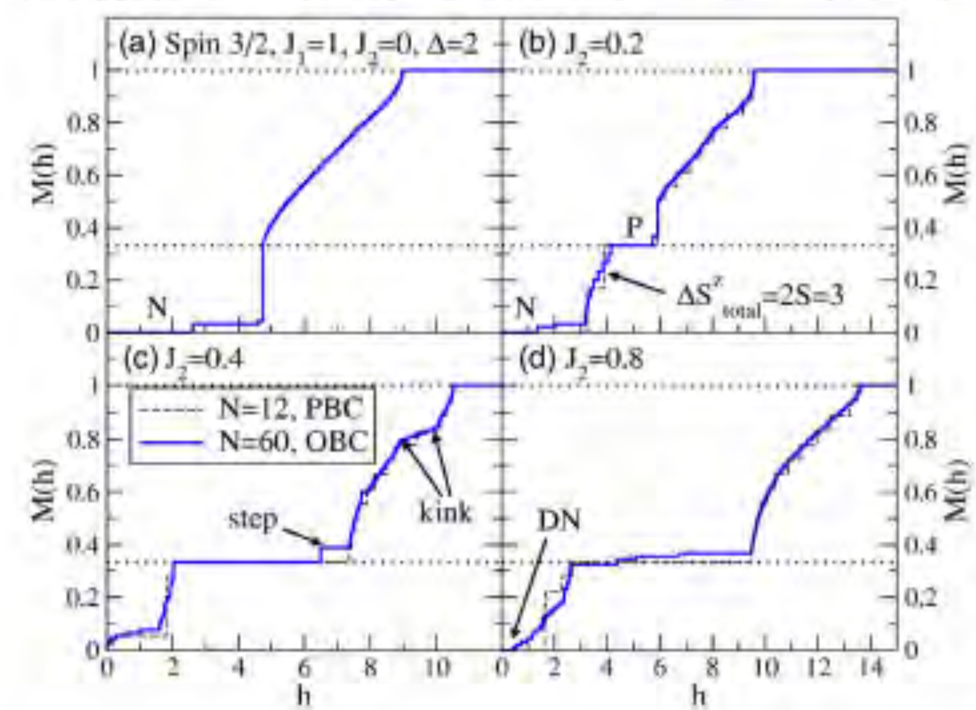


FIG. 12. (Color online) Magnetization curves for frustrated spin-3/2 chains with an anisotropic exchange (Δ=2) for (a) J_2=0, (b) J_2=0.2, (c) J_2=0.4, and (d) J_2=0.8. DMRG results (straight lines) are for N=60 sites, the dashed lines are ED results (PBC). The capital letters stand for: Néel phase **N**, canted phase **C**, double-Néel phase **DN**.