The good, the bad, and the ugly of spin superfluids: Domain walls, phase slips, and skyrmions

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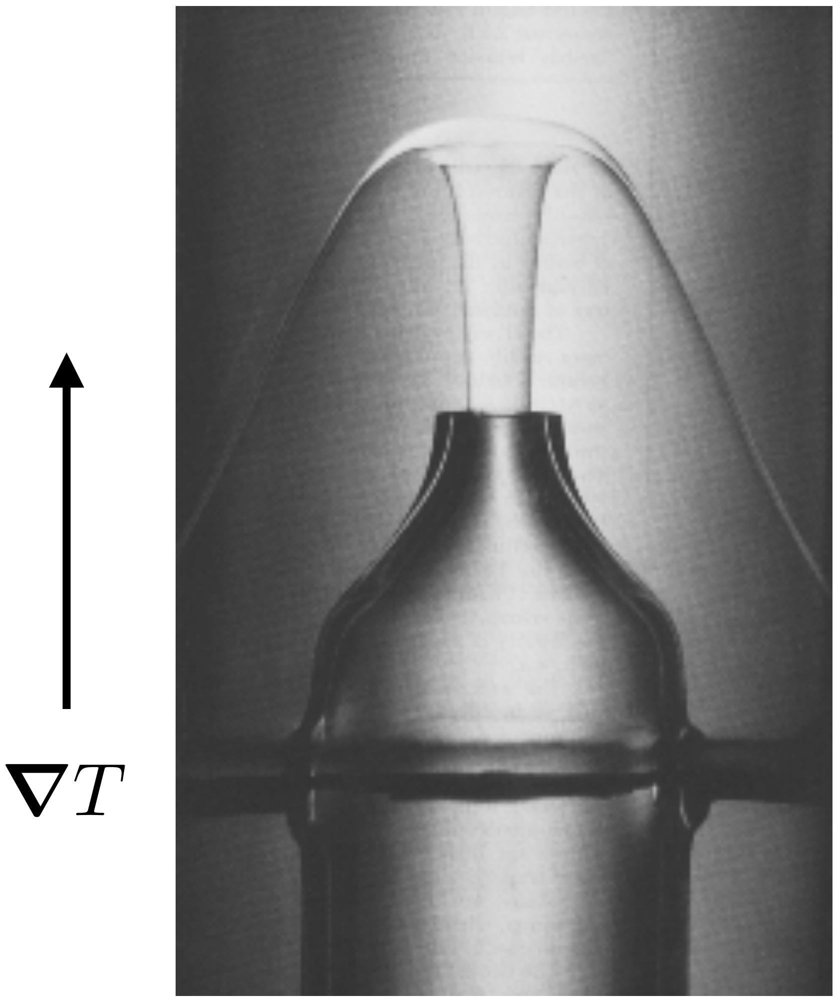
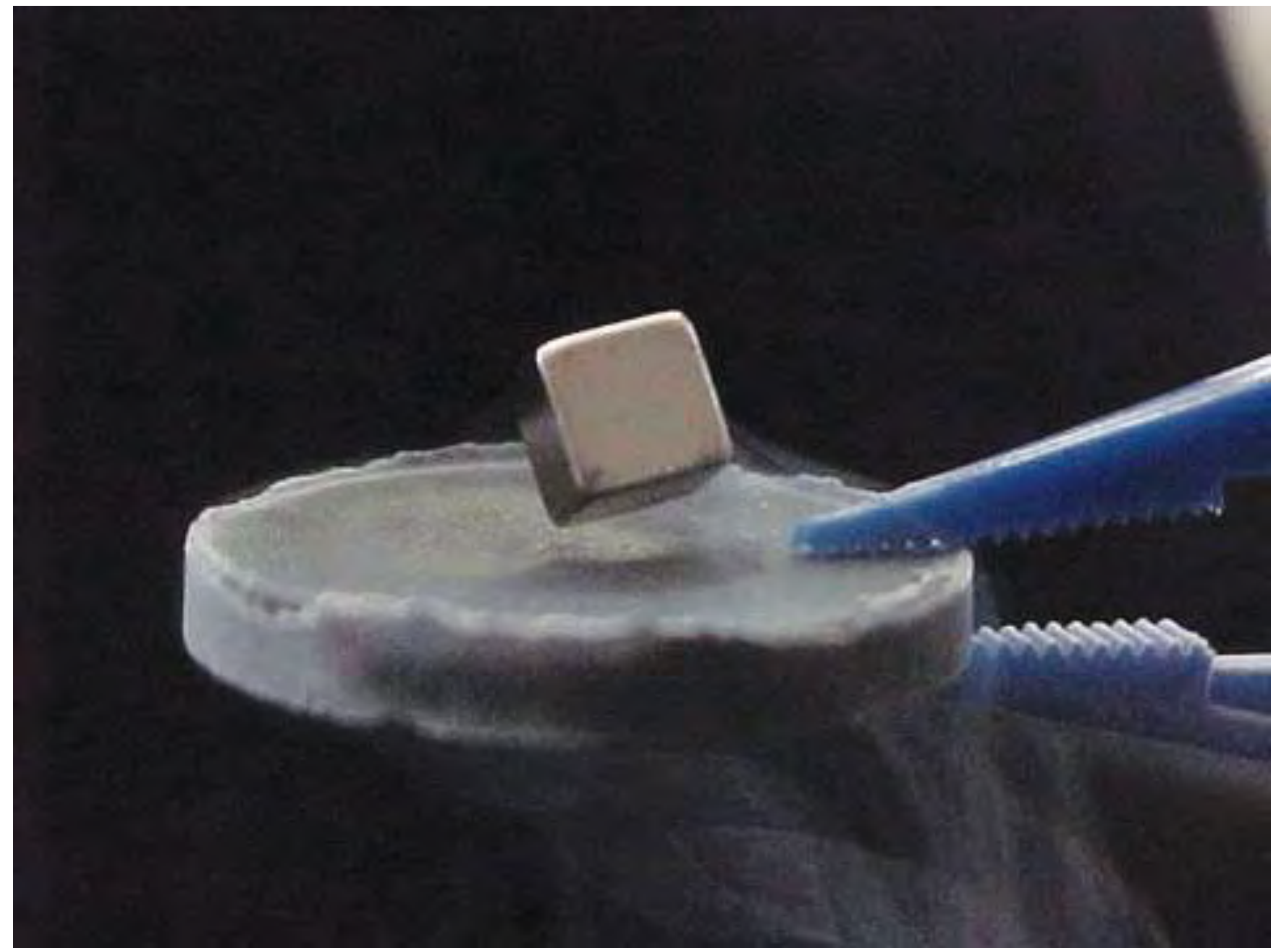
Outline

- Introduction to spin superfluidity
  - Idealized treatment
  - Dissipation and detrimental anisotropies
- From spin to topological charge hydrodynamics
  - Chiral domain walls in 1D
  - Skyrmions in 2D
- Proposals for realization and utilization of spin superfluids in ferromagnetic and antiferromagnetic materials

Superfluidity and superconductivity



(neutral) superfluid fountain



(charged) superfluid Meissner effect

spin superfluidity?

(thermal and electrical controls?)

Super-primer

- When bosonic particles condense, their collective dynamics can be described by two canonically conjugate variables: particle density ρ and condensate phase φ

  - Hamiltonian: $$H = \frac{\rho^2}{2C} + \frac{A(\boldsymbol{\nabla}\varphi)^2}{2}$$
  - Hamilton's equations: $$\dot{\varphi} \propto \frac{\rho}{C}$$ (Josephson relation)

    $$\dot{\rho} \propto A\nabla^2\varphi$$ (continuity equation)

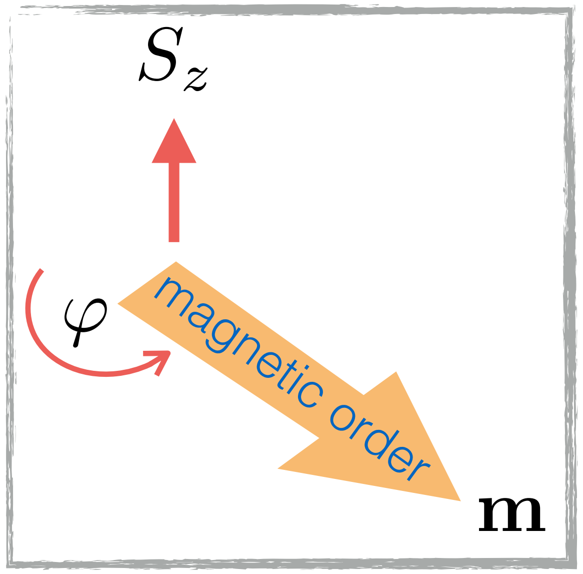
    $$\rightarrow \quad \mathbf{j} \propto -A\boldsymbol{\nabla}\varphi$$ (superflow)

(average supercurrent is determined by the overall phase winding)

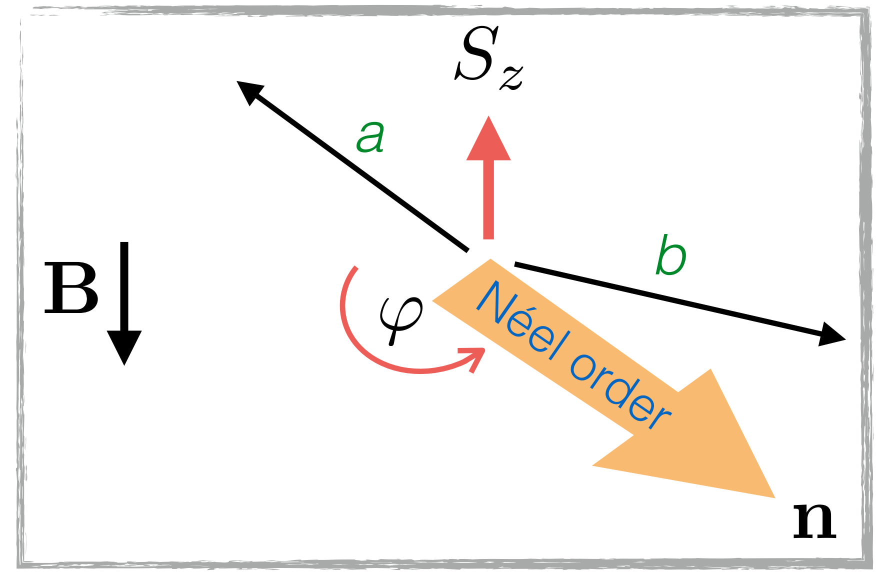
- The conservation of particle number is rooted in the gauge symmetry (here: invariance under global phase shift)

Spin superfluidity

easy-plane ferromagnet



Heisenberg antiferromagnet



$$[\varphi, S_z] = i\hbar$$

(total spin being the generator of the order-parameter rotations)

- Continuum theory:

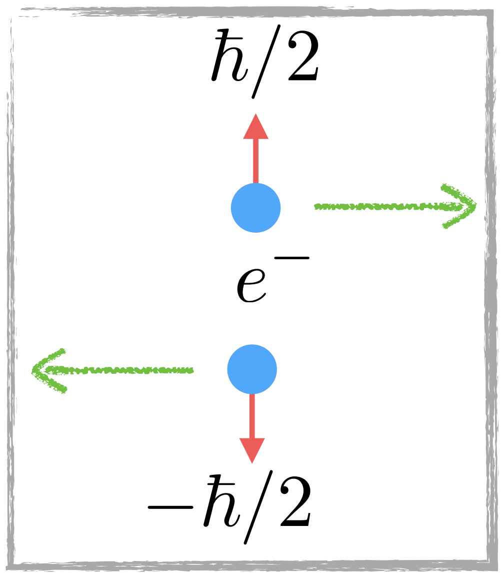
$$H = \frac{K\rho_z^2}{2} + \frac{A(\boldsymbol{\nabla}\varphi)^2}{2}$$

Halperin and Hohenberg, *PR* (1969); Sonin, *JETP* (1978) and *AP* (2010); König *et al.*, *PRL* (2001)

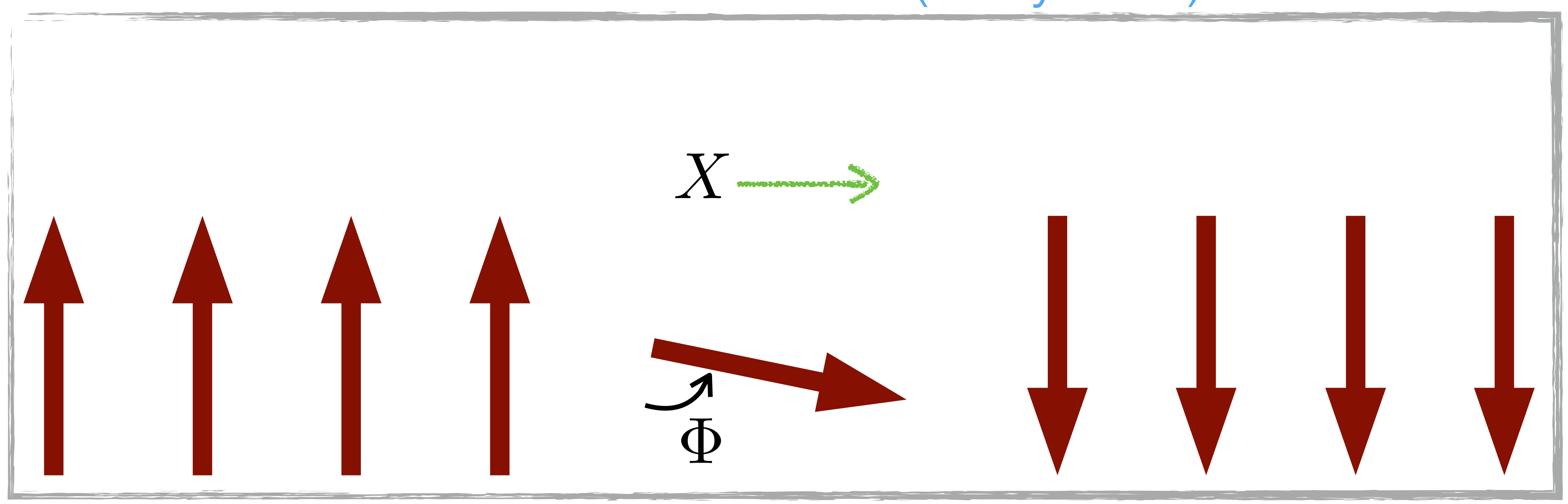
Anatomy of spin transport in solid state

(assuming spin-rotational invariance around the *z* axis)

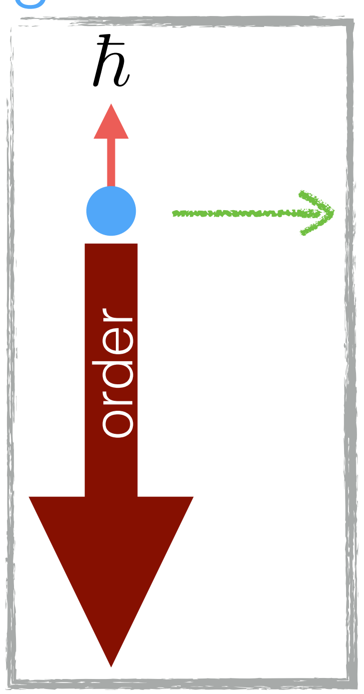
electron diffusion



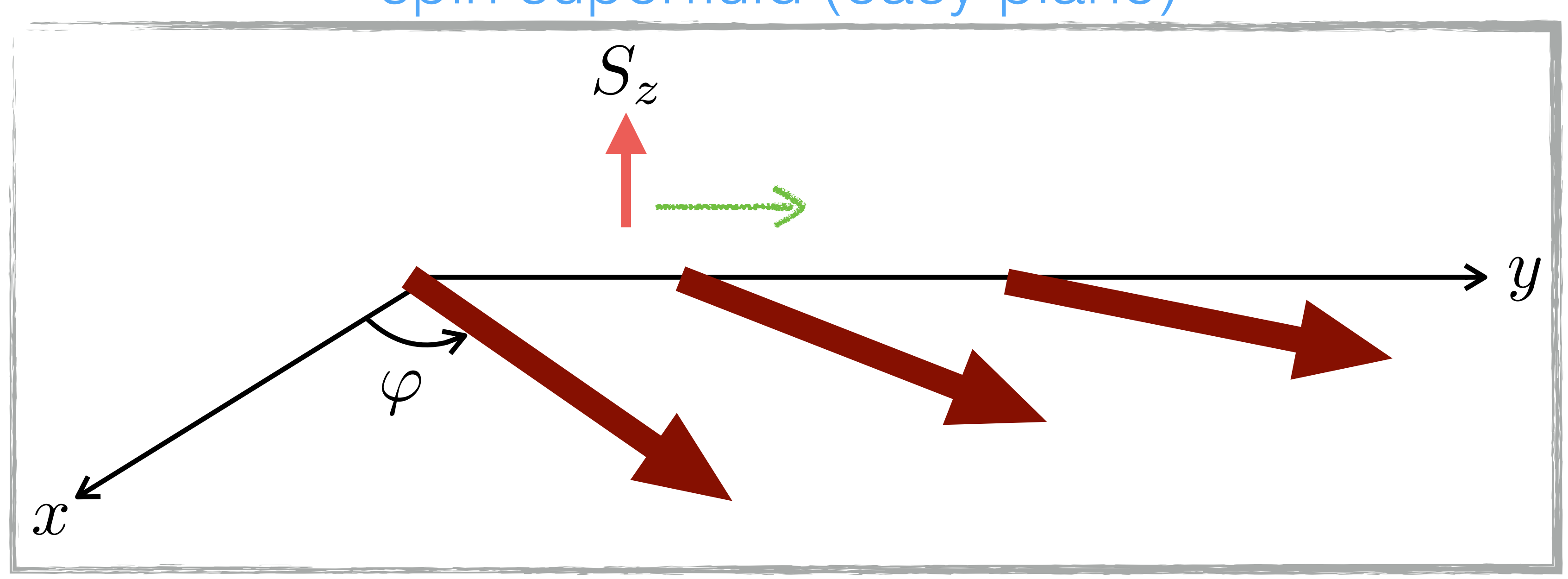
domain-wall motion (easy axis)



magnon diffusion

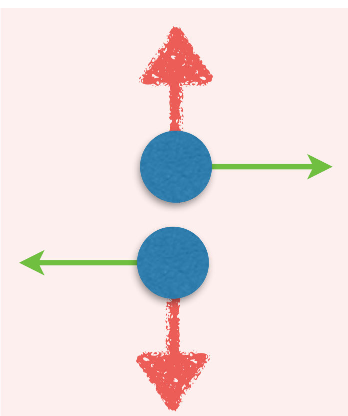


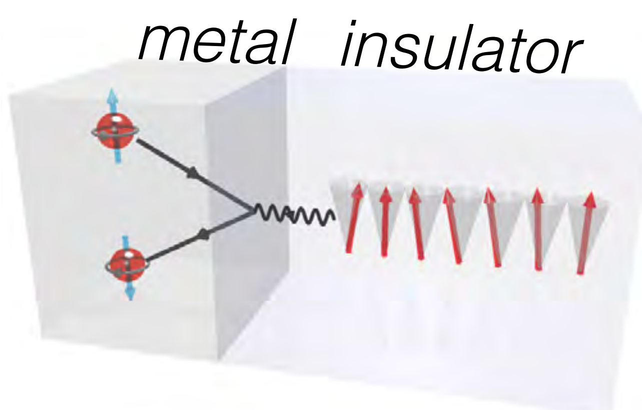
spin superfluid (easy plane)



Spin torque/pumping/Hall

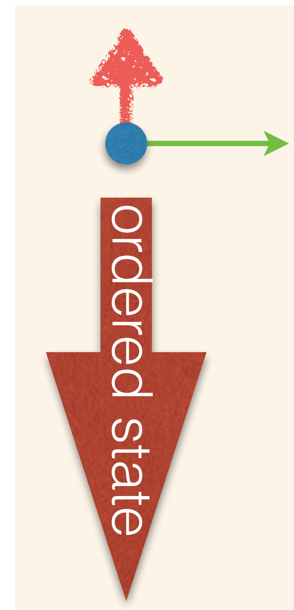
- Spin transport can be carried across (insulating) interfaces:

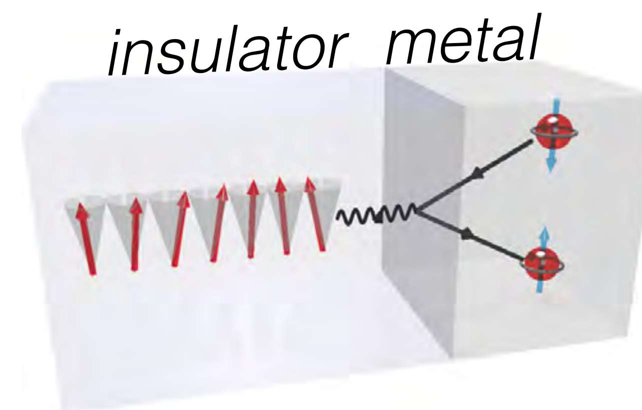




spin torque

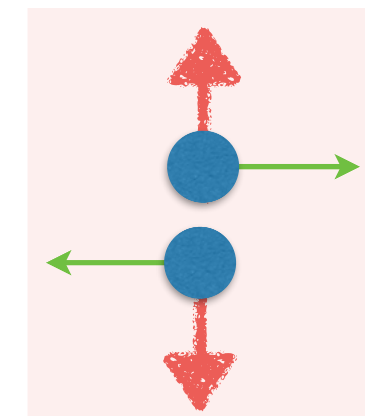
Slonczewski, *JMMM* (1996); Berger, *PRB* (1996)



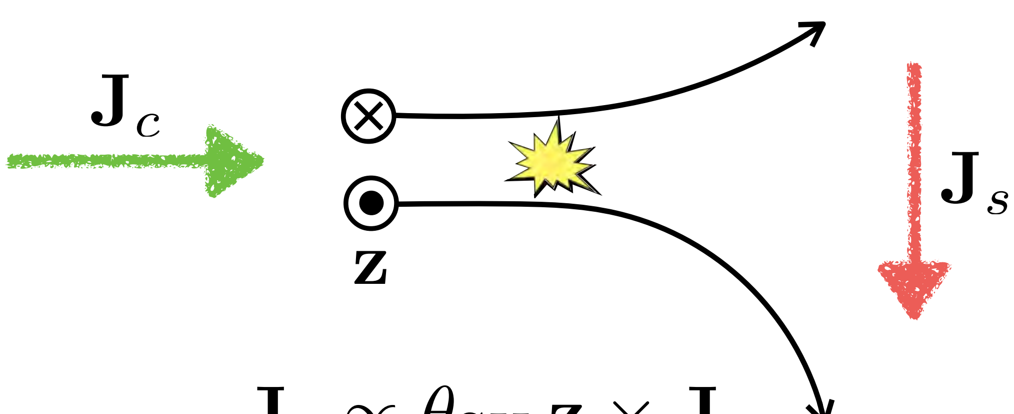


spin pumping

YT, Brataas, and Bauer, *PRL* (2002)

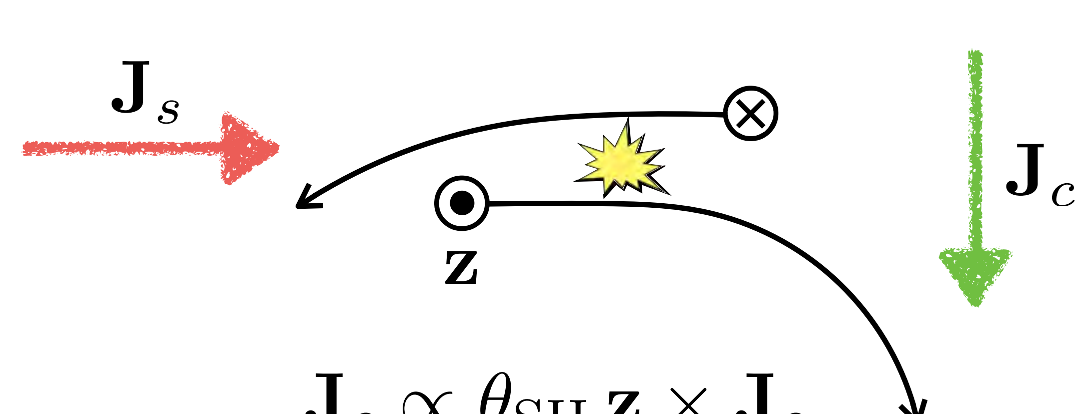


- An efficient spin-to-charge interconversion by the spin Hall effect:



$$\mathbf{J}_s \propto \theta_{\rm SH}\, \mathbf{z} \times \mathbf{J}_c$$

spin Hall



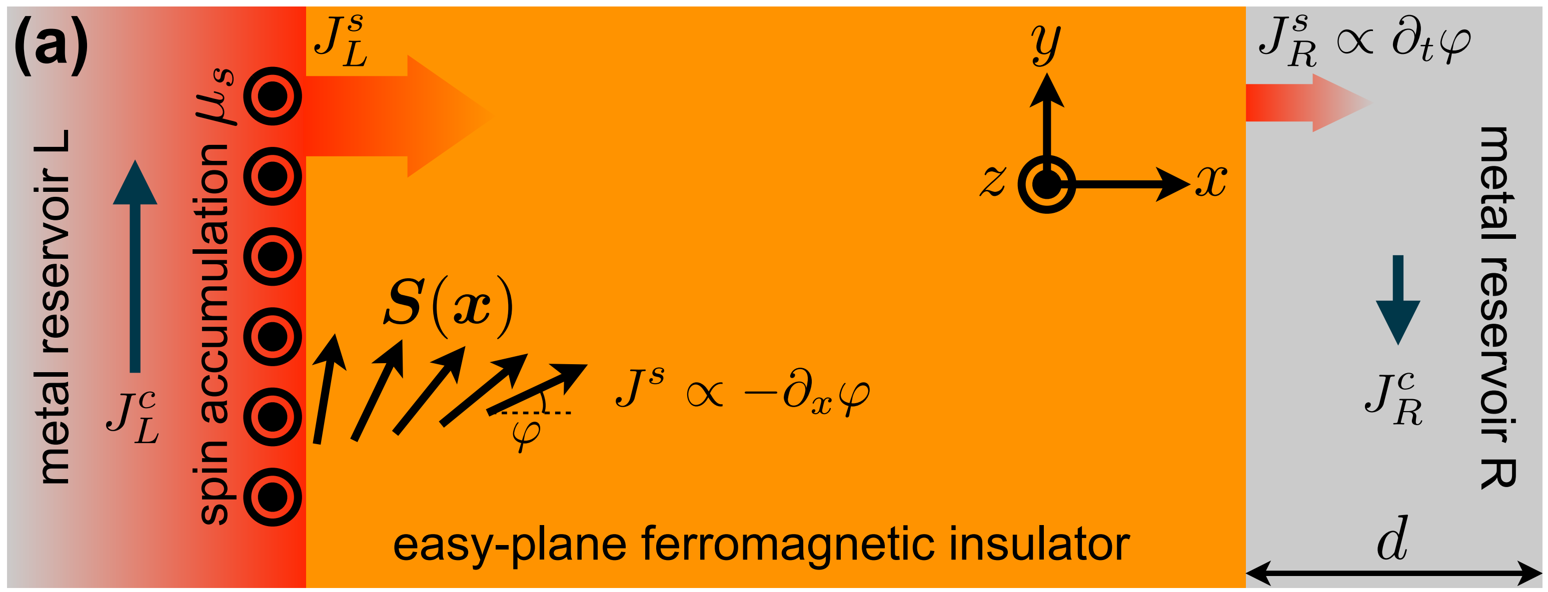
$$\mathbf{J}_c \propto \theta_{\rm SH}\, \mathbf{z} \times \mathbf{J}_s$$

inverse spin Hall

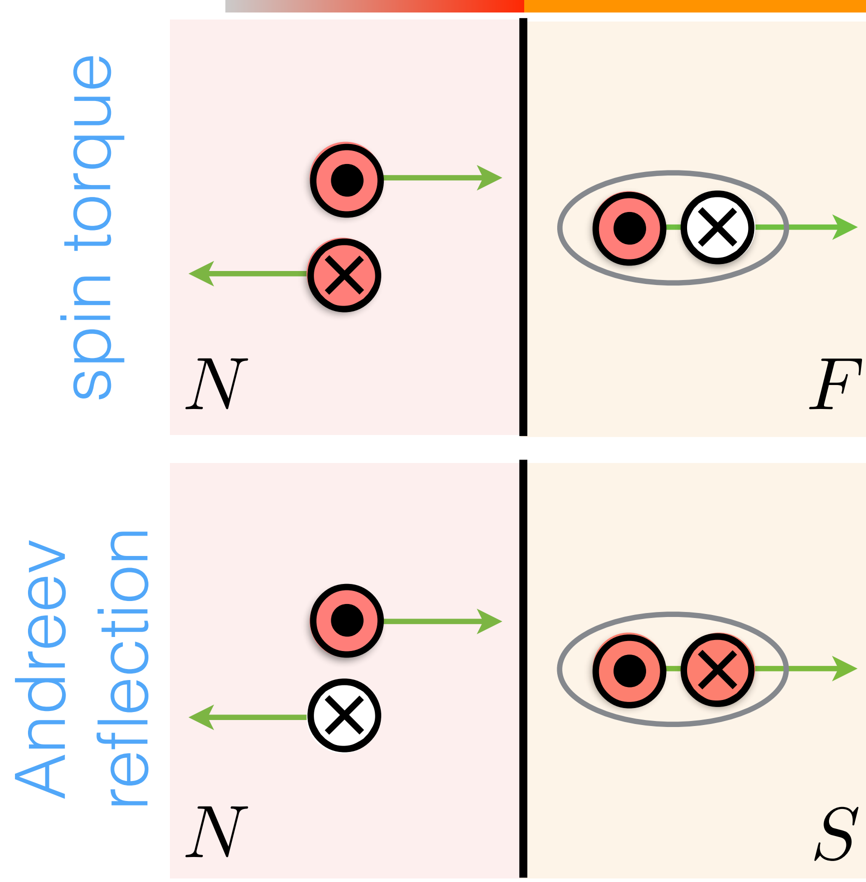
Dyakonov and Perel', *JETP* (1971); Sinova *et al.*, *RMP* (2015)

Electrical controls of spin superfluids

- The spin current can be injected and extracted directly in and out of the superflow-carrying magnetic helix:



Takei and YT, *PRL* (2014)

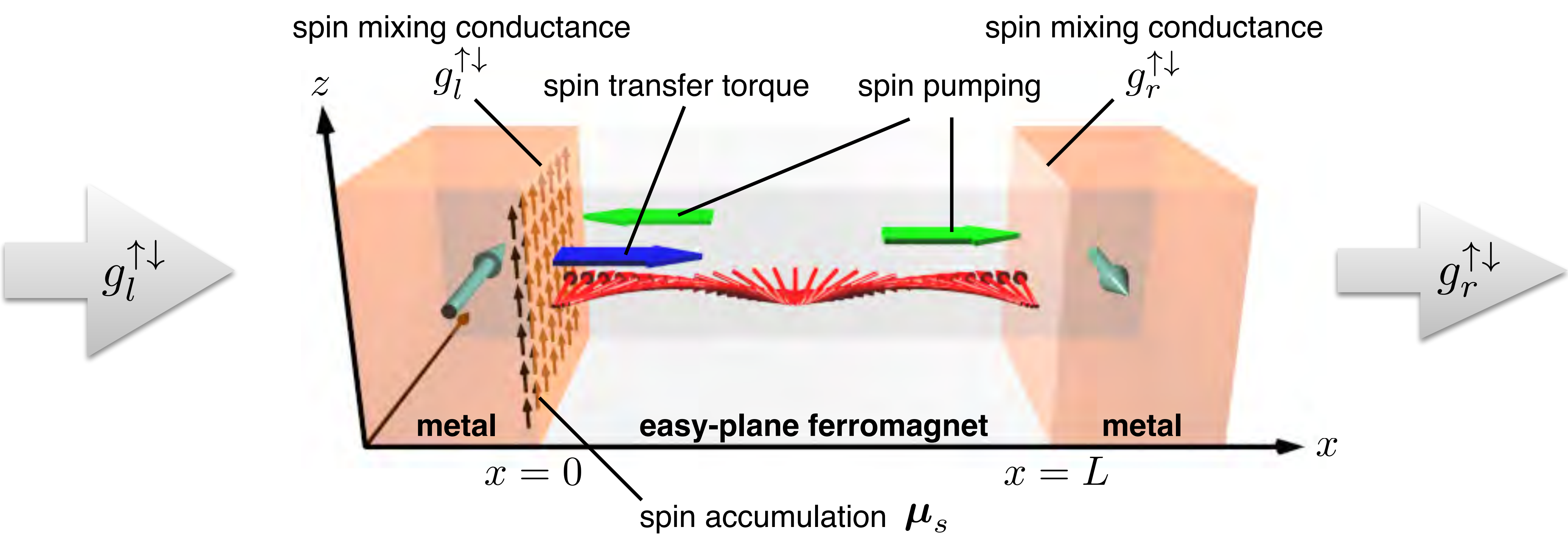


$$J_s = g^{\uparrow\downarrow}(\mu_s - \hbar \partial_t \varphi)$$

$$g^{\uparrow\downarrow} = \frac{1}{4\pi} \Re \sum_{nm} \left(\delta_{nm} - r_{nm}^{\uparrow} r_{nm}^{\downarrow *} \right)$$

YT, Brataas, Bauer, and Halperin, *RMP* (2005)

Spin-current circuit



$$g_\alpha = \frac{\alpha s L}{\hbar}$$ (mechanical torque)

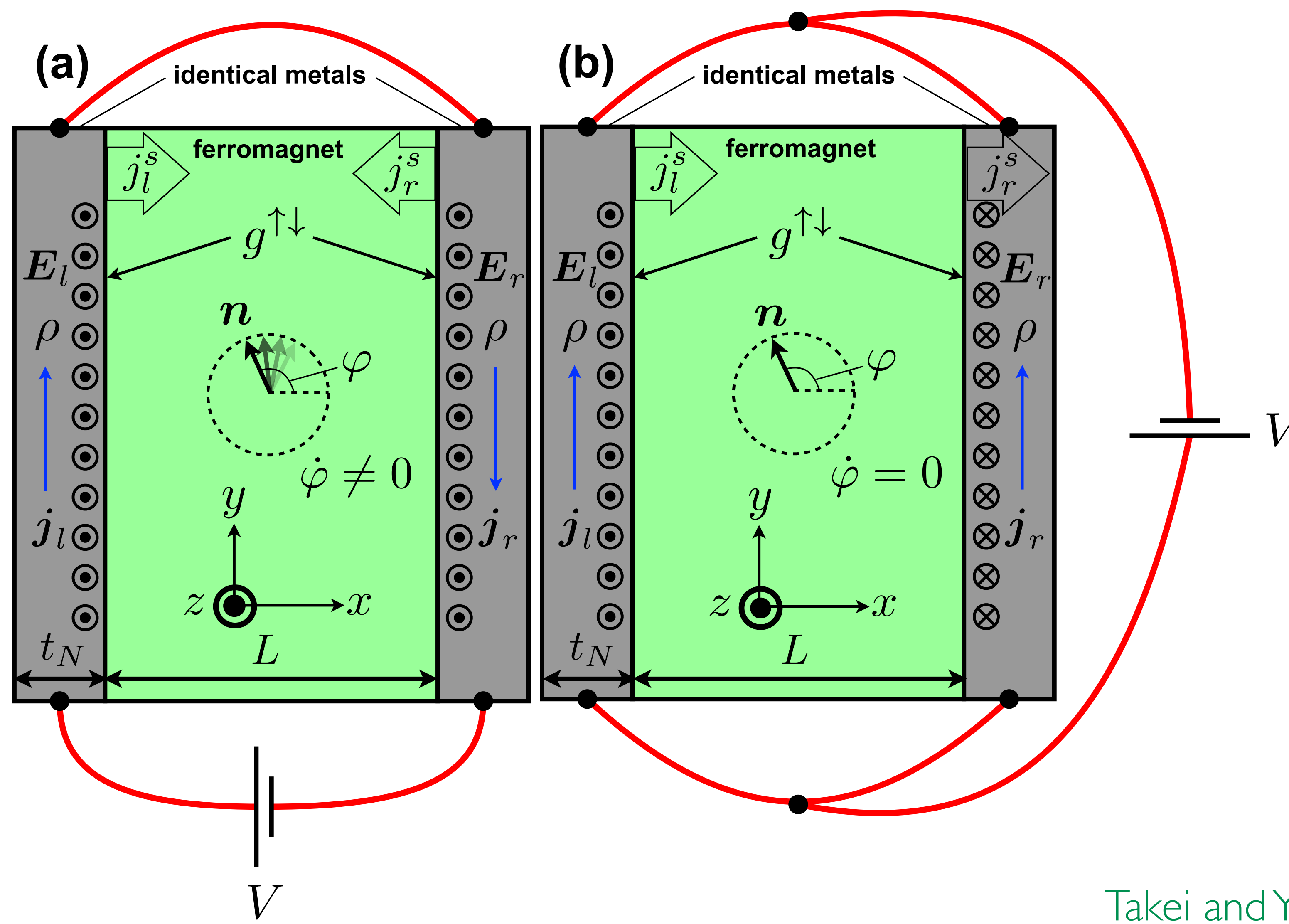
$$\hbar\Omega = \frac{g_l^{\uparrow\downarrow}}{g_l^{\uparrow\downarrow} + g_r^{\uparrow\downarrow} + g_\alpha}\mu_s\,, \quad J_r^s = \frac{g_l^{\uparrow\downarrow} g_r^{\uparrow\downarrow}}{g_l^{\uparrow\downarrow} + g_r^{\uparrow\downarrow} + g_\alpha}\mu_s$$

Takei and YT, *PRL* (2014)

Nonlocal magnetoresistance

- Circulating current through two metal films in series (a) spins the order, reducing the overall dissipation

$$\rho_m \propto -\frac{\vartheta_{\rm SH}^2}{2g^{\uparrow\downarrow}+\alpha s L}$$



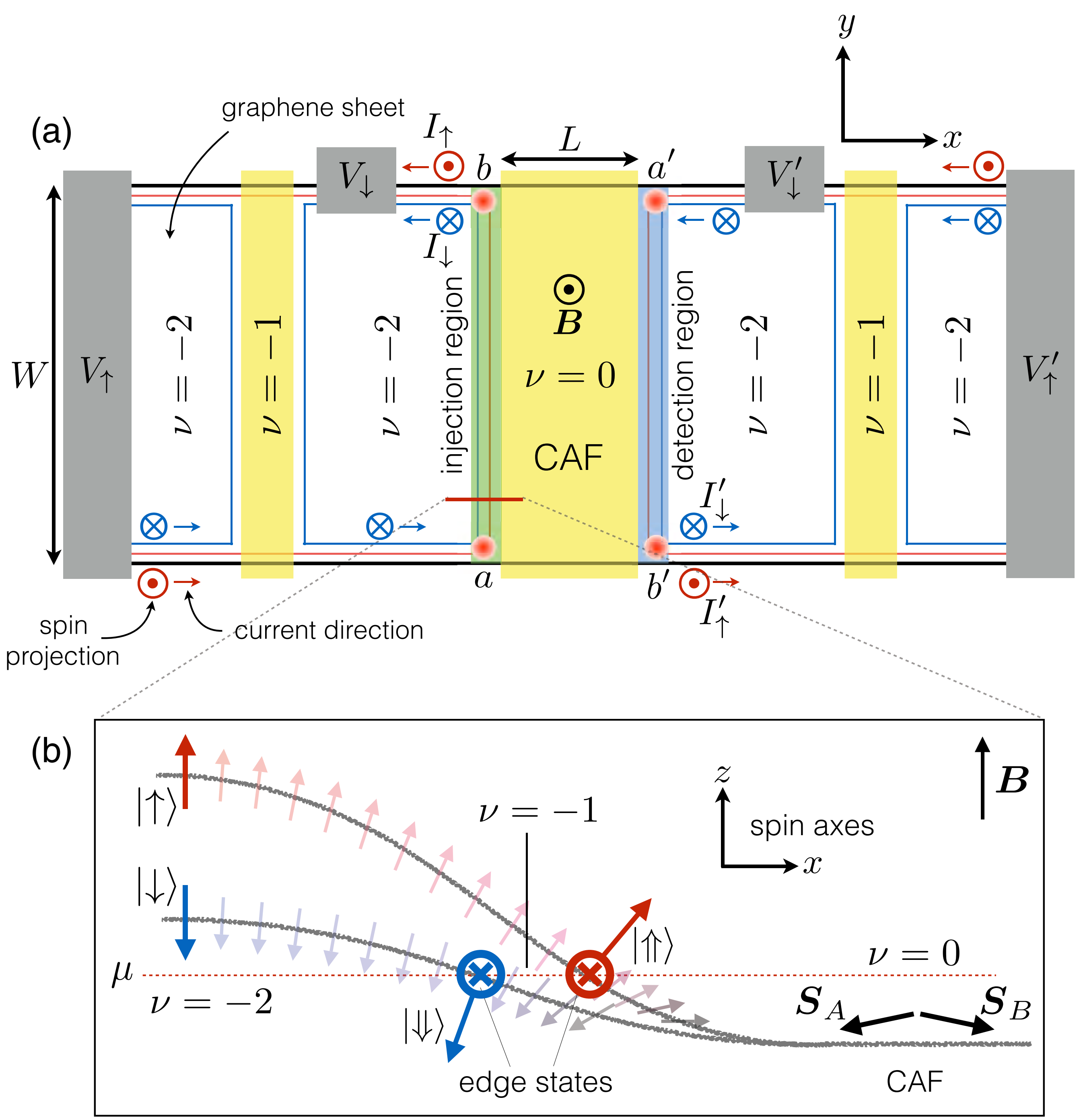
$$\rho_m = 0$$

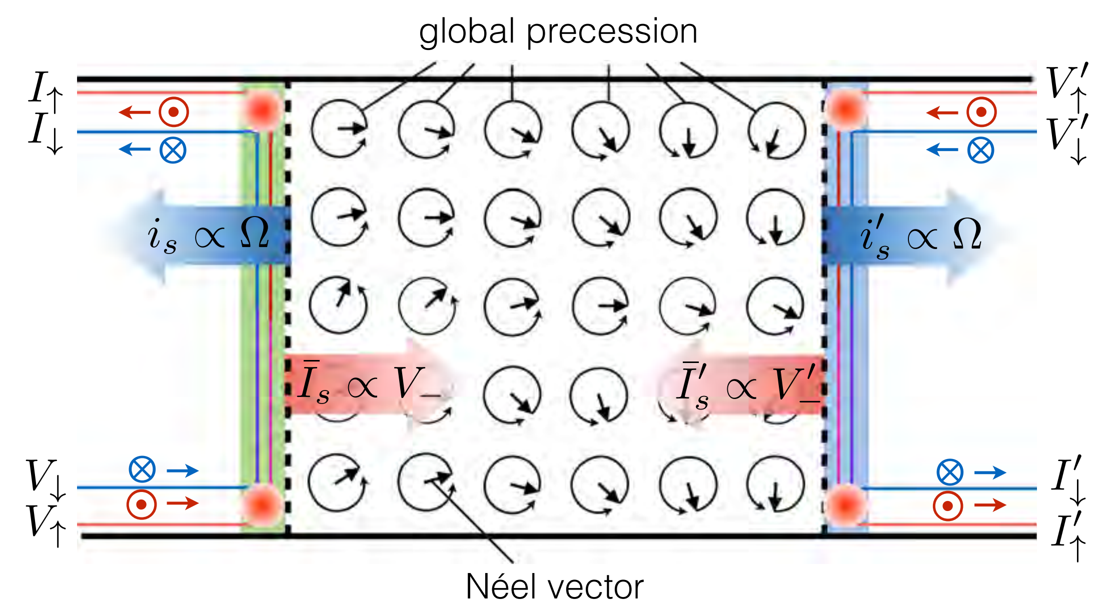
Takei and YT, *PRL* (2015)

- In the parallel configuration (b), the torques are balanced, and the magnet remains stationary, causing more friction

Cf. Eisenstein and MacDonald, *Nature* (2004) for *BEC of excitons in bilayer electron systems*

A proposed realization in the $\nu = 0$ graphene

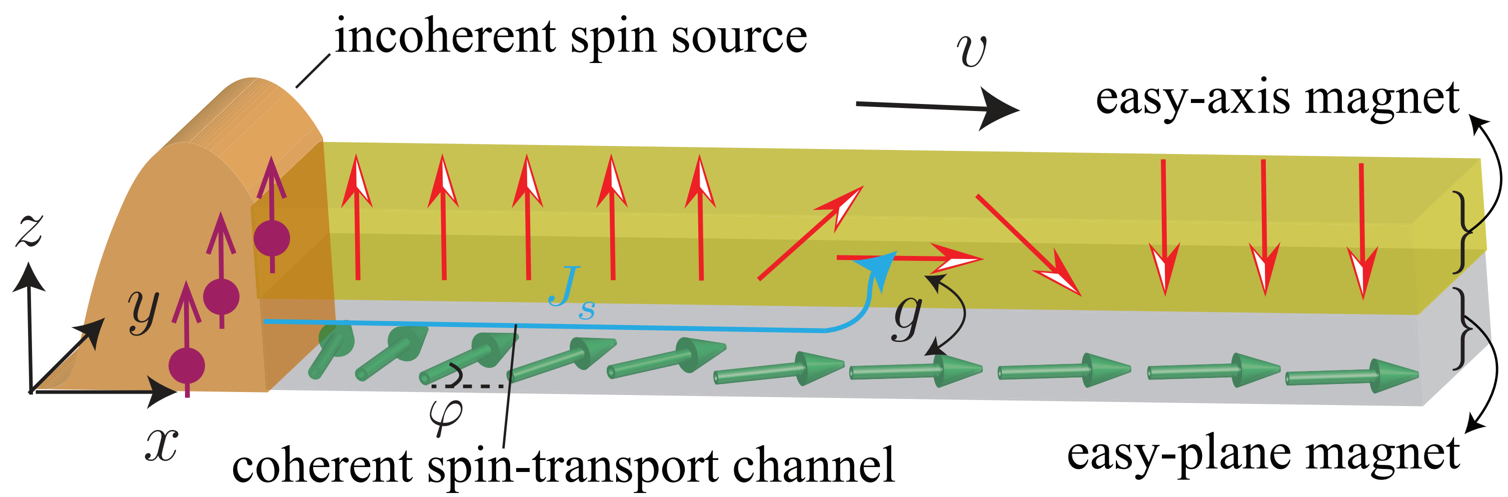


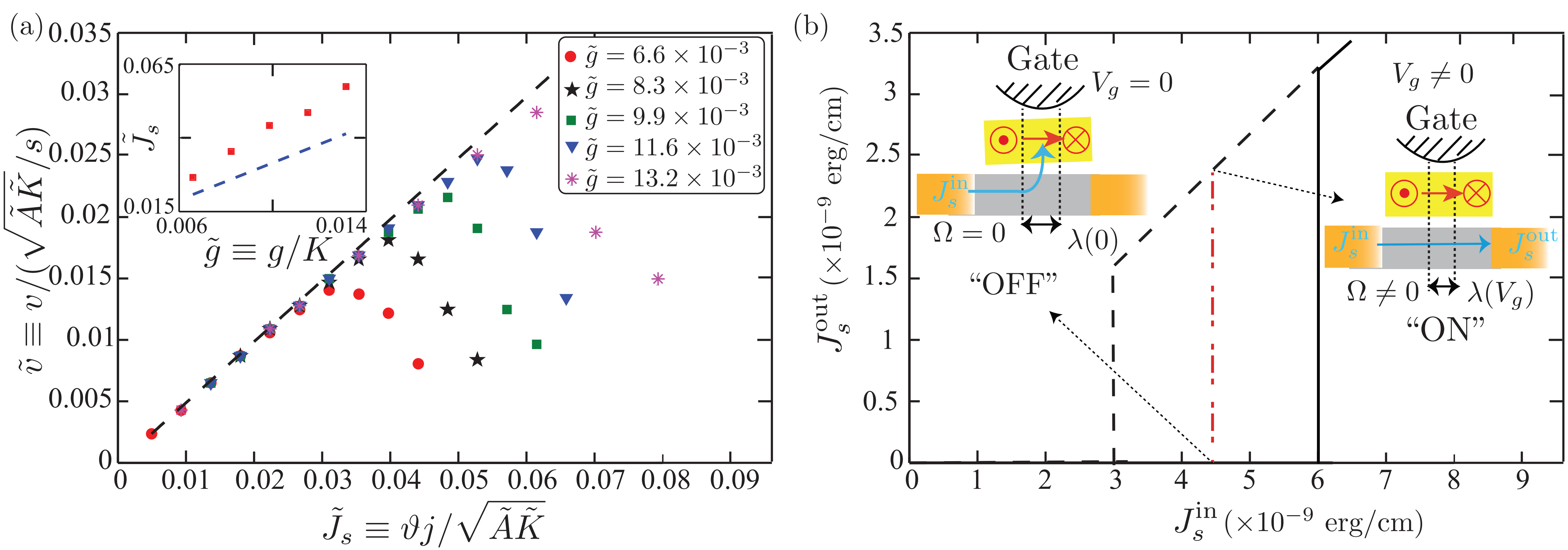


Takei, Yacoby, Halperin, and YT, *PRL* (2016)

A possible application

(domain wall floating on a spin superfluid)

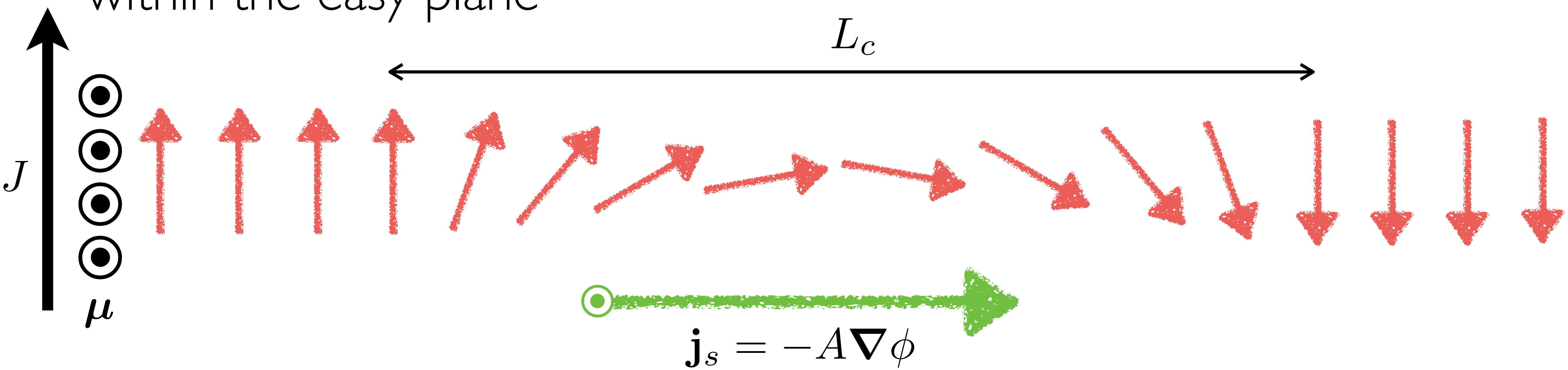




Upadhyaya, Kim, and YT, *arXiv* (2016)

Detrimental role of (inevitable) anisotropies

- A minimal bias is required to overcome pinning by an anisotropy within the easy plane



- Beyond this (lower) critical bias, the uniform magnetic state is unstable against nucleation and propagation of domain walls

- At a large enough bias, the train of domain walls coalesce into a helical superfluid

- For the minimal current to be less than the critical supercurrent, the easy-plane anisotropy needs to exceed parasitic anisotropies

Sonin, *JETP* (1978); König *et al.*, *PRL* (2001)

“Type II” boson condensates

REVIEWS OF

MODERN PHYSICS

VOLUME 42, NUMBER 1 JANUARY 1970

Two Kinds of Bosons and Bose Condensates*

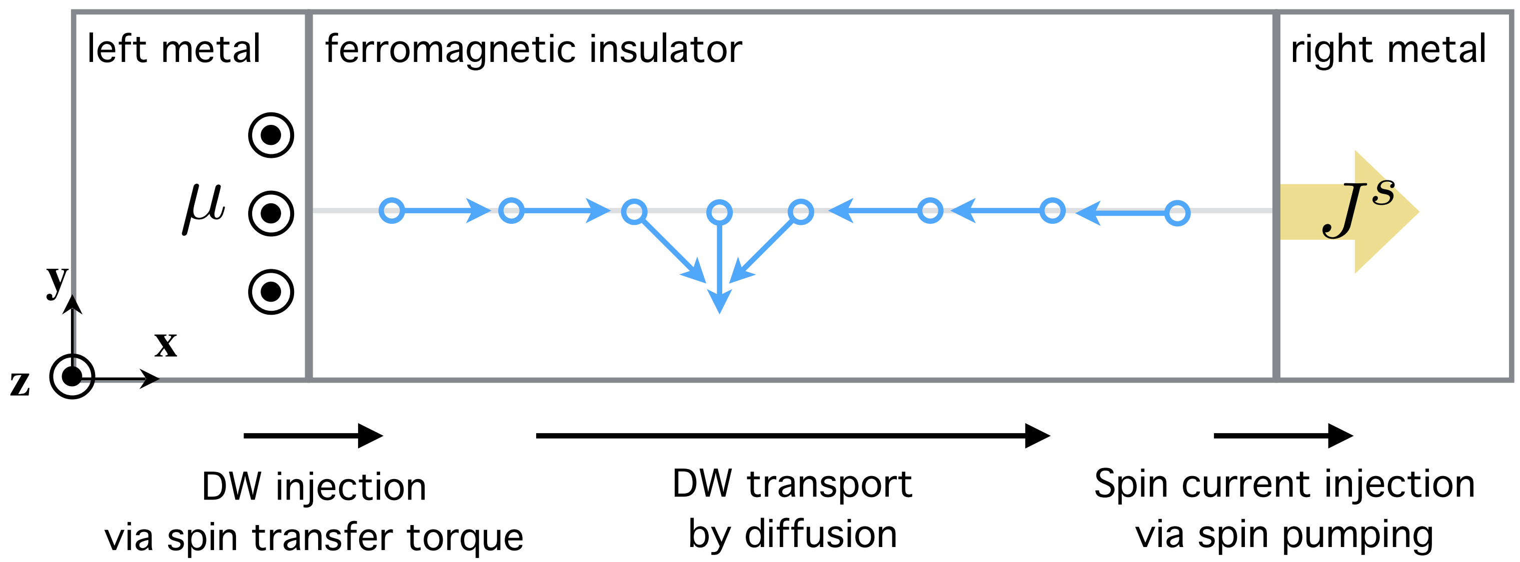
W. KOHN, D. SHERRINGTON

Department of Physics, University of California, San Diego, La Jolla, California

This paper deals with ordinary material systems whose elementary constituents are fermions. It is pointed out that in such systems there can occur two kinds of bosons with quite different physical and mathematical characteristics. Type I bosons are bound complexes of an even number of fermions (such as ^{4}He); and type II bosons are elementary excitations which are bound complexes of fermions and their holes (such as excitons). When the first type condenses, a superfluid state results with so-called off-diagonal, long-range order; while when the second type condenses, there is no superfluidity, but a change in spatial order. Thus both kinds of long-range order are related to Bose condensation.

Thermally-assisted "superfluid"

- The spin accumulation biases chirality q dependent thermal injection of domain walls:



chirality-dependent work: $$W = \int d\boldsymbol{\theta} \cdot \boldsymbol{\tau} = g^{\uparrow\downarrow} \int d\boldsymbol{\theta} \cdot (\mathbf{n} \times \boldsymbol{\mu}_s \times \mathbf{n}) = \pi g^{\uparrow\downarrow} \mu_s q$$

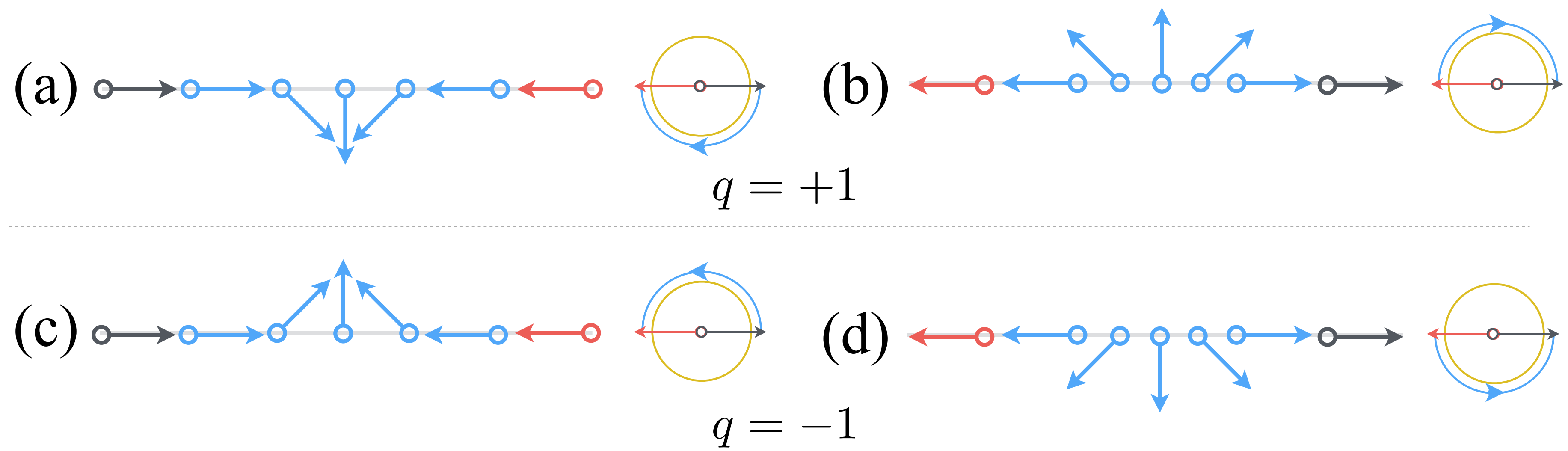
- Chiral domain walls carry this information diffusively along the length of the magnetic wire, re-emitting the spin current by spin pumping:

spin emission: $$S_z = \int dt J_s = g^{\uparrow\downarrow} \hbar \int dt \partial_t \varphi = \pi \hbar g^{\uparrow\downarrow} q$$

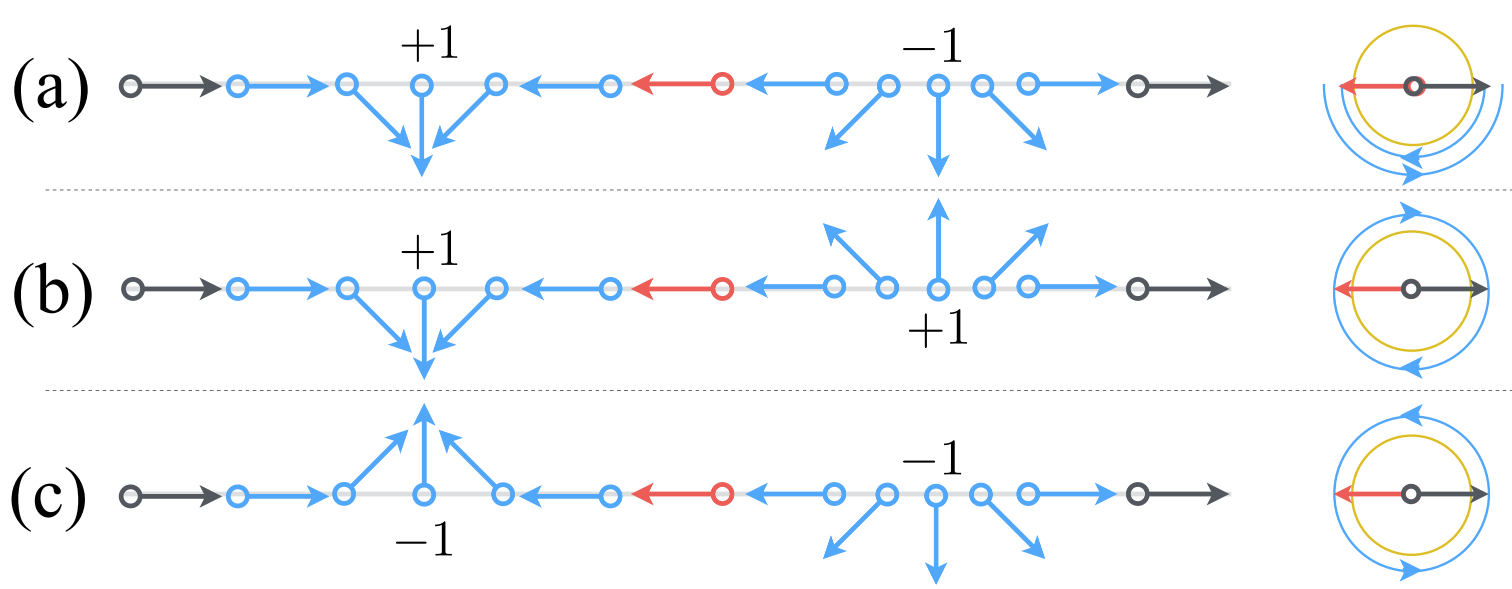
Kim, Takei, and YT, *PRB* (2015)

Spin texture book-keeping

- Four types of domain walls:



- Thermally-activated (topologically trivial) texture of zero net charge vs positively or negatively charged textures:



Diffusive transport theory

- Total topological charge:

$$q \equiv -\frac{1}{\pi} \int dx\, \partial_x \phi$$

- Continuity relation:

$$\partial_t \rho + \partial_x I = 0\,, \quad I = -D \partial_x \rho \qquad\qquad D = \lambda_{\mathrm{DW}} T / 2\alpha s \mathcal{A}$$

derived by solving stochastic Landau-Lifshitz-Gilbert equation

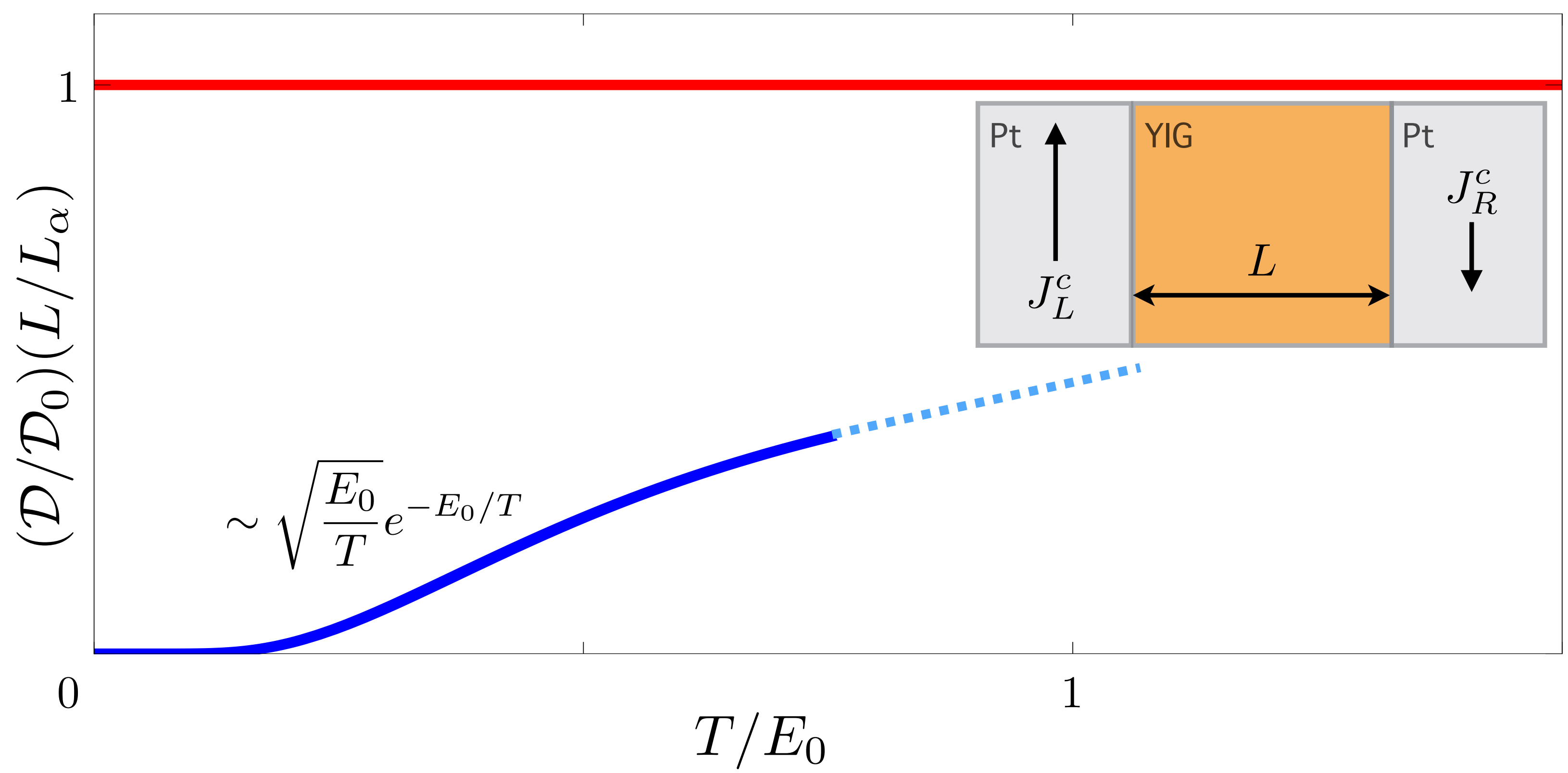
- Boundary conditions (chirality-dependent injection rate):

$$I^{\pm} = \Gamma^{\pm}(T, \mu_s) - \gamma(T) \rho^{\pm}$$

subject to the following chirality-dependent work $W = \pm \pi g^{\uparrow\downarrow} \mu_s$

Kim, Takei, and YT, *PRB* (2015)

(Thermally-activated) DC electron drag



$\mathcal{D}_0 \sim 0.1$ - drag in a perfect U(1) superfluid

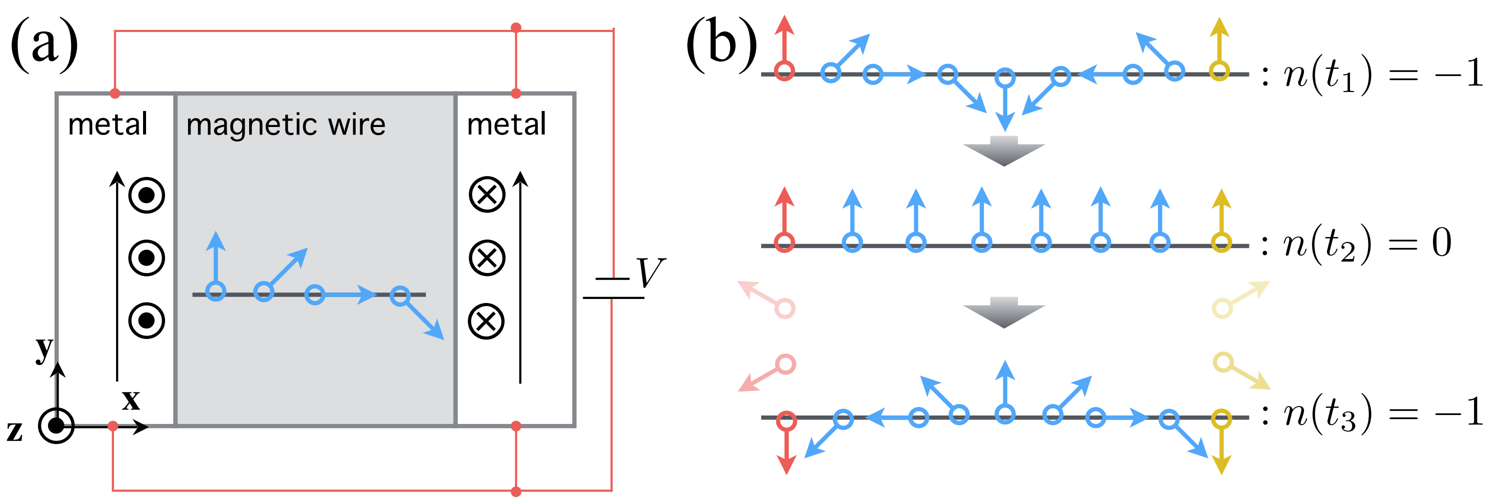
$E_0 = S\sqrt{A\kappa}$ - domain-wall energy due to parasitic anisotropy

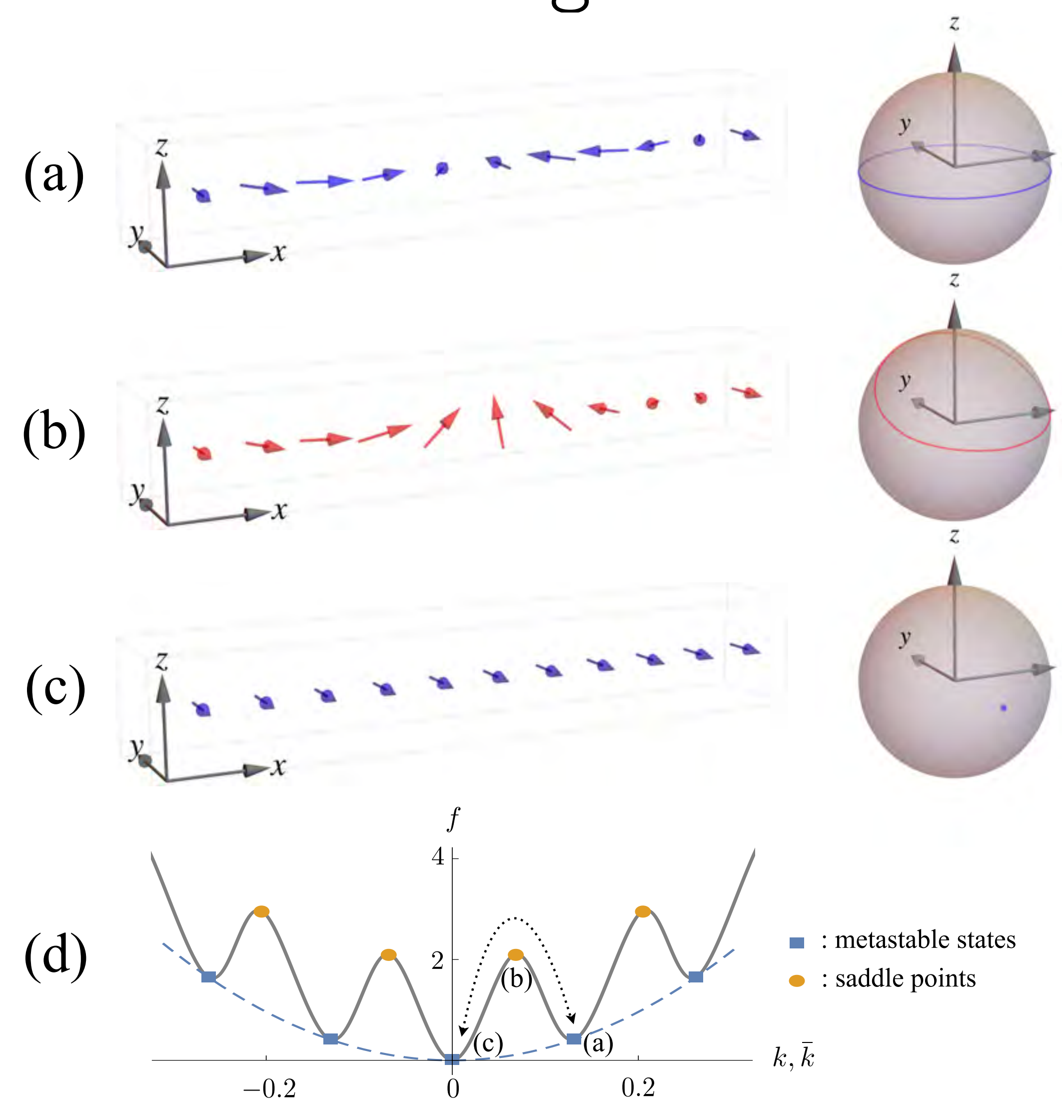
- Thermally-activated domain walls coalesce and we recover the intrinsic result when $T \sim E_0$

Kim, Takei, and YT, *PRB* (2015)

High-temperature regime: phase slips

- Out-of-plane excursions cause magnetic phase slips, unwinding the topological charge, which could be detected as a voltage in the following circuit:





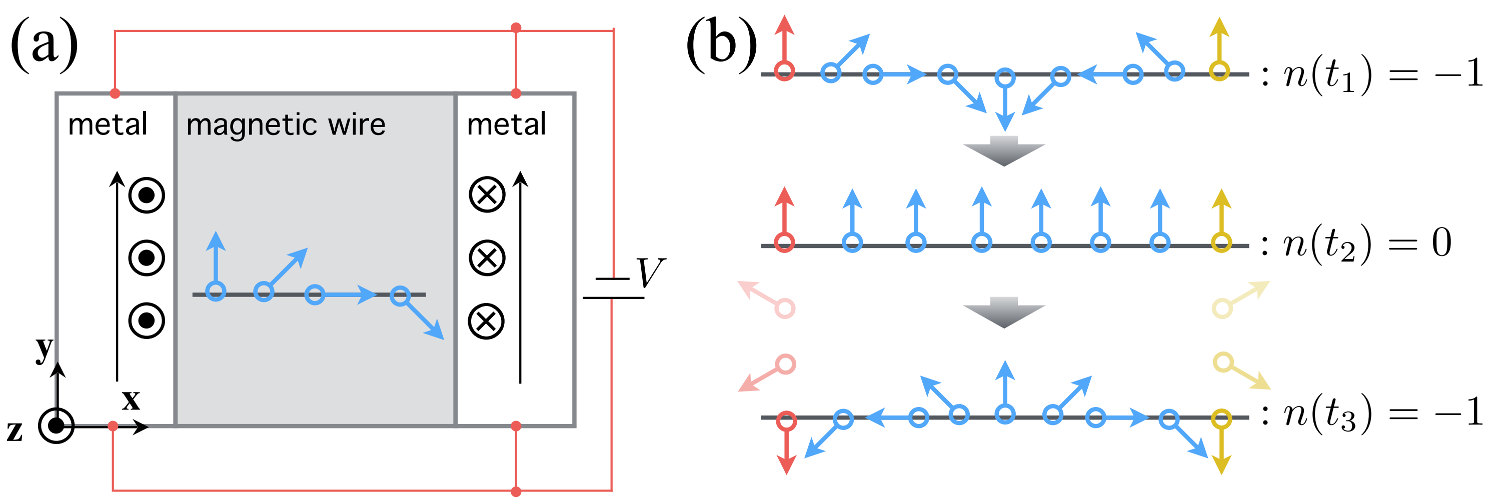
- Detrimental to the superfluid as it unwinds the helical texture

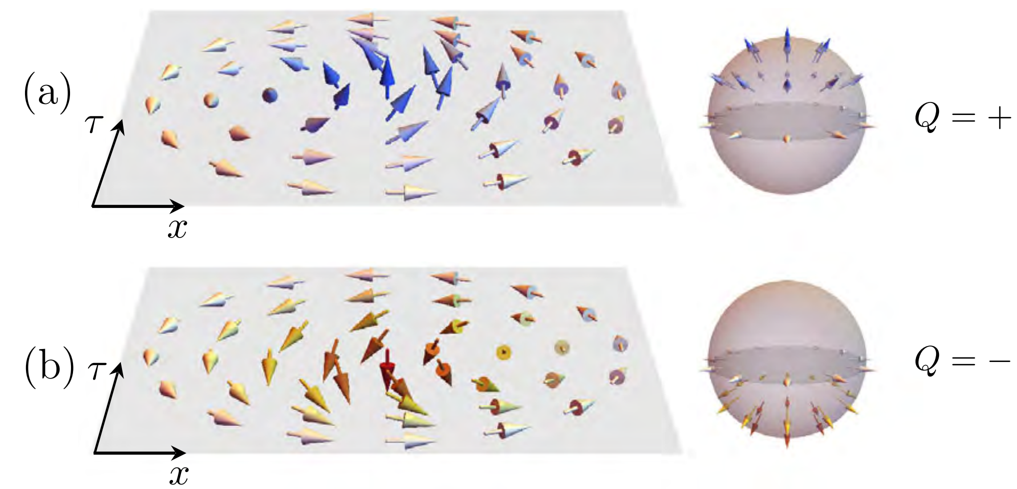
$$T \lesssim S\sqrt{A\kappa} \ll S\sqrt{AK_{xy}}$$

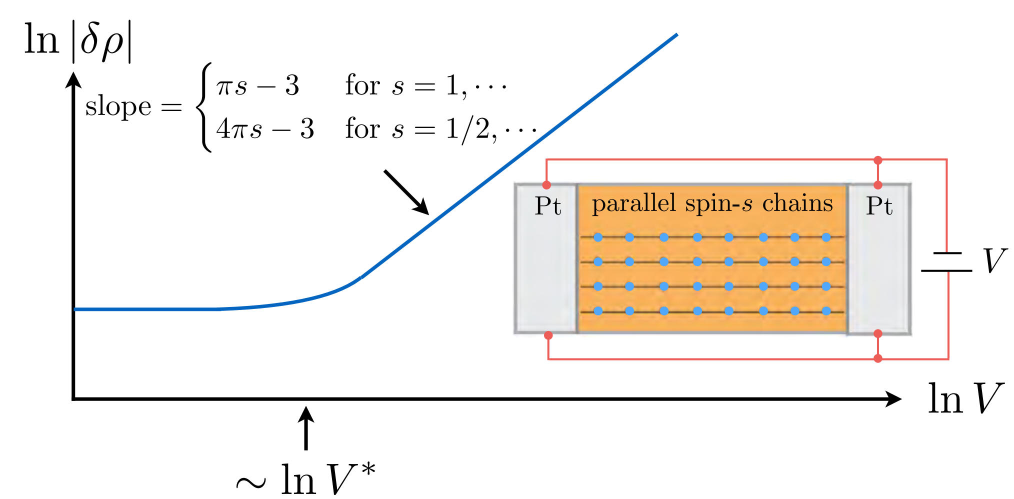
Kim, Takei, and YT, *PRB* (2016)

Quantum phase slips (antiferromagnets)

- The same (negative) magnetoresistance geometry can be used to extract the quantum phase slip rate:





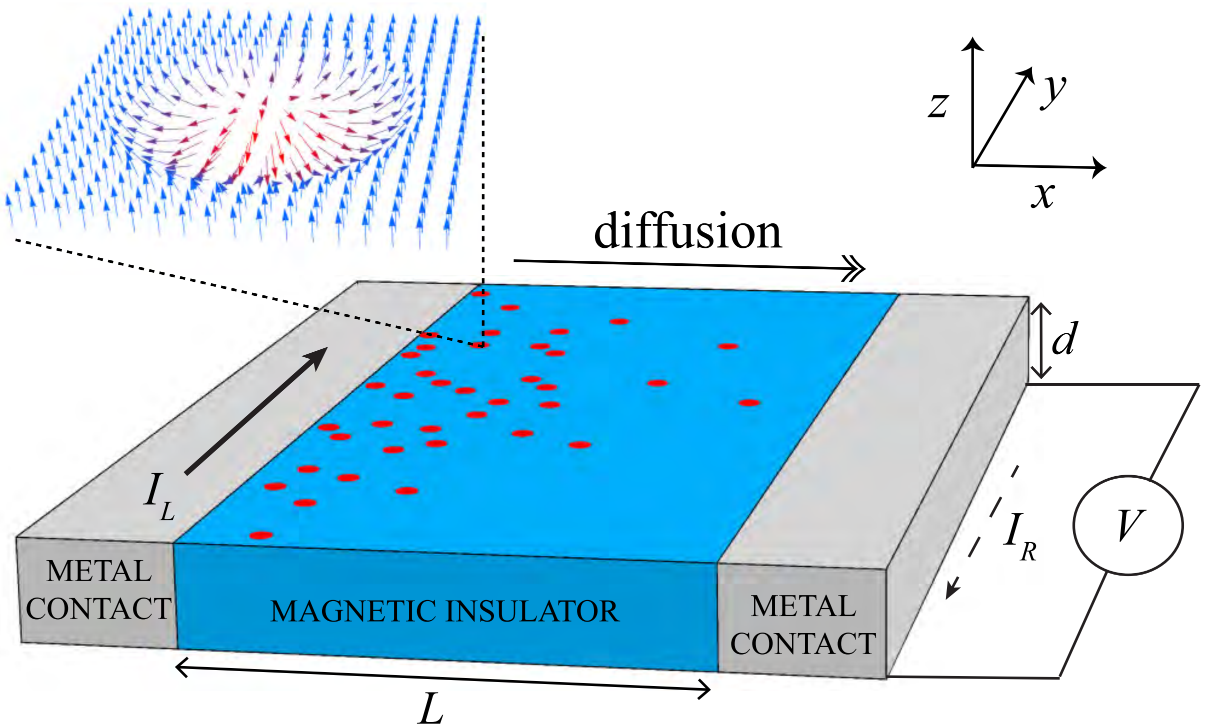


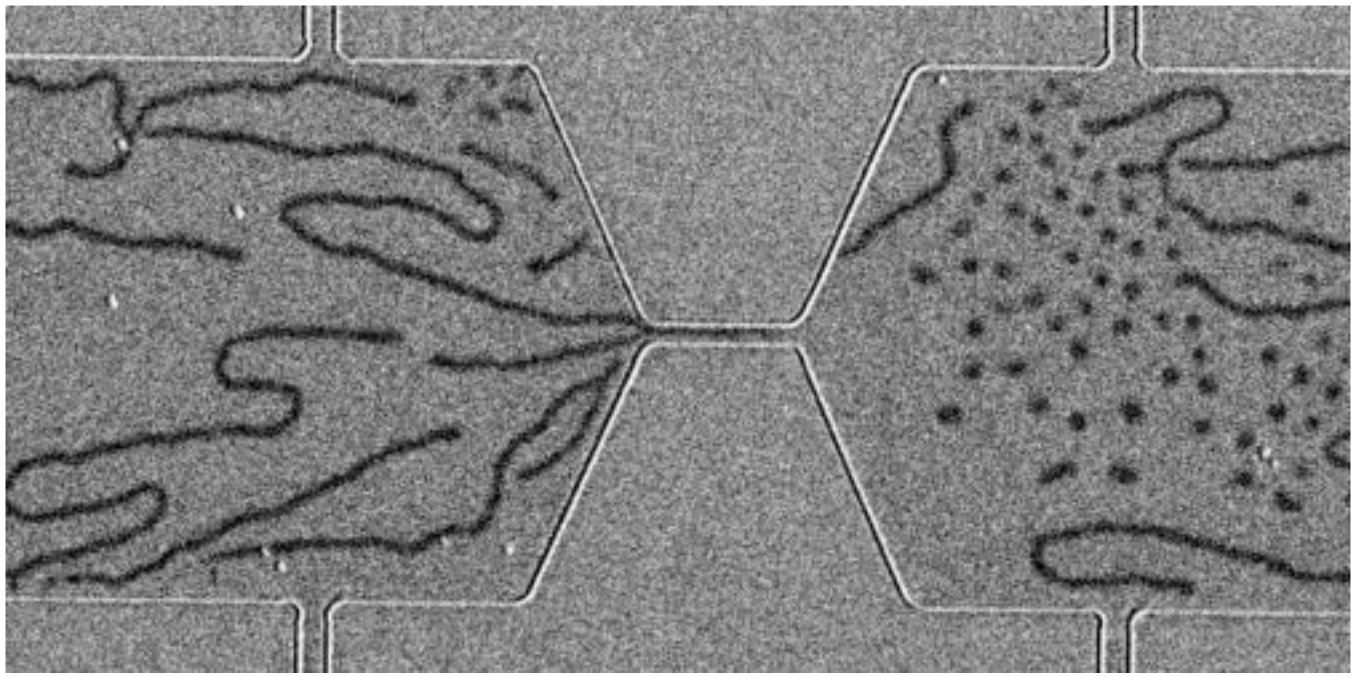
- The effective action for a gas of QPS's in the presence of a spin superflow:

$$S_{\text{eff}} = nS_{\text{core}} - \mu \sum_{i<j} q_i q_j \ln(d_{ij}/\lambda) + j_s \sum_i q_i \tau_i$$

Kim and YT, *PRL* (2016)

A 2D scenario of topo-transport: Skyrmion diffusion





Jiang *et al.*, *Science* (2015)

- Continuity equation for skyrmion density:

$$\partial_t \rho_s + \vec{\nabla} \cdot \vec{j}_s = 0$$

$$\rho_s \equiv \mathbf{n} \cdot (\partial_x \mathbf{n} \times \partial_y \mathbf{n})/4\pi \qquad \vec{j}_s = \mathbf{n} \cdot \left[\partial_t \mathbf{n} \times (\hat{z} \times \vec{\nabla}) \mathbf{n} \right] /4\pi$$

quantized Skyrmion number: $Q \equiv \int dx dy \, \rho_s$

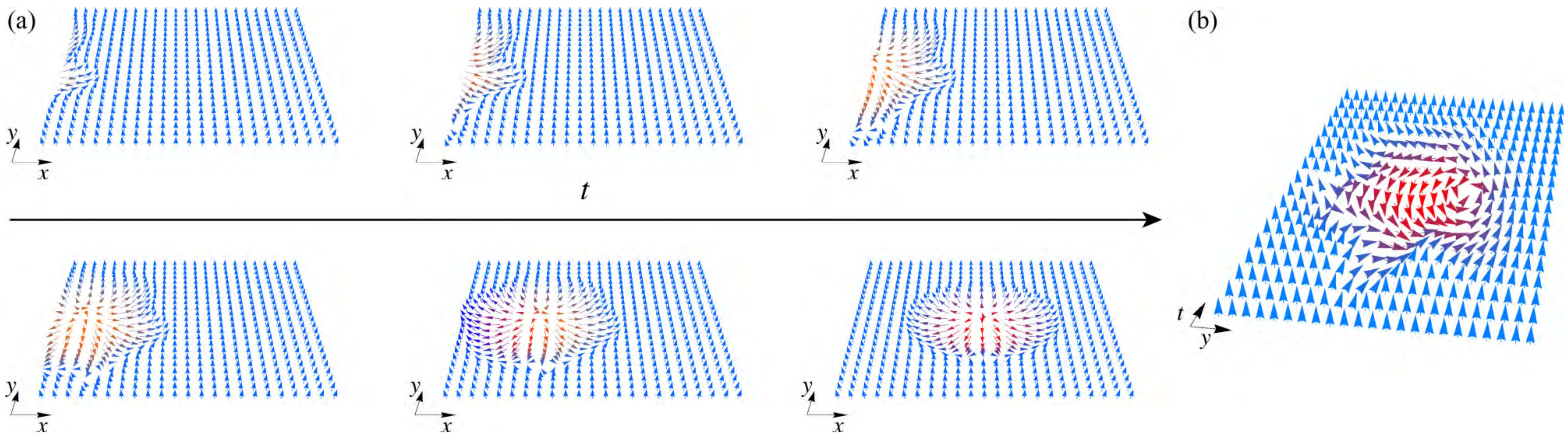
Ochoa, Kim, and YT, *PRB* (2016)

Skyrmion diffusion

- Work by the torque $\boldsymbol{\tau} = \frac{\hbar\mathcal{P}}{2e}(\vec{j}\cdot\vec{\nabla})\mathbf{n}$ on the thermally injected skyrmions:

$$w = \int \boldsymbol{\tau}\cdot\mathbf{n}\times\delta\mathbf{n} = \frac{2\pi\hbar\mathcal{P}}{e}\int dt\, \vec{j}_s\cdot\hat{z}\times\vec{j}$$

$$W = \int dy\, w = \frac{2\pi\hbar\mathcal{P}}{e} jQ$$



- Onsager-reciprocal motive force at the detection (right) side:

$$\vec{\mathcal{E}} = \frac{\mathcal{P}\hbar}{2e}\mathbf{n}\cdot\vec{\nabla}\mathbf{n}\times\partial_t\mathbf{n} = \frac{2\pi\hbar\mathcal{P}}{2}\hat{z}\times\vec{j}_s$$

Ochoa, Kim, and YT, *PRB* (2016)

Summary

- Superfluid spin flow is a collective low-dissipation phenomenon responsible for an efficient long-ranged transport of spin angular momentum

- The superfluid, which is tied to the phase winding, is topologically stable (barring phase slips); however, the spin flow is easily quenched at low temperatures by parasitic anisotropies

- Chirality diffusion of thermally-activated domain walls in 1D and skyrmion transport in 2D take over spin transport, restoring the superfluid phenomenology in linear response