The good, the bad, and the ugly of spin superfluids: Domain walls, phase slips, and skyrmions

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Outline

- Introduction to spin superfluidity
  - Idealized treatment
  - Dissipation and detrimental anisotropies

- From spin to topological charge hydrodynamics
  - Chiral domain walls in 1D
  - Skyrmions in 2D

- Proposals for realization and utilization of spin superfluids in ferromagnetic and antiferromagnetic materials
Superfluidity and superconductivity

(neutral) superfluid fountain  
(thermal and electrical controls?)

(charged) superfluid Meissner effect

spin superfluidity?
When bosonic particles condense, their collective dynamics can be described by two canonically conjugate variables: particle density $\rho$ and condensate phase $\varphi$.

- Hamiltonian: \[ H = \frac{\rho^2}{2C} + \frac{A(\nabla \varphi)^2}{2} \]

- Hamilton’s equations: \[ \dot{\varphi} \propto \frac{\rho}{C} \] (Josephson relation)

\[ \dot{\rho} \propto A \nabla^2 \varphi \] (continuity equation)

\[ \rightarrow j \propto -A \nabla \varphi \] (superflow)

(average supercurrent is determined by the overall phase winding)

The conservation of particle number is rooted in the gauge symmetry (here: invariance under global phase shift)
Spin superfluidity

**easy-plane ferromagnet**

\[ S_z \]

\[ \varphi \]

magnetic order

m

**Heisenberg antiferromagnet**

\[ S_z \]

\[ a \]

Néel order

n

\[ \left[ \varphi, S_z \right] = i\hbar \]

(totally spin being the generator of the order-parameter rotations)

**Continuum theory:**

\[
H = \frac{K \rho_z^2}{2} + \frac{A(\nabla \varphi)^2}{2}
\]

Anatomy of spin transport in solid state

(assuming spin-rotational invariance around the z axis)

electron diffusion

\[ \frac{\hbar}{2} \quad \rightarrow \quad e^- \quad \leftarrow \quad -\frac{\hbar}{2} \]

domain-wall motion (easy axis)

\[ X \quad \rightarrow \quad \Phi \]

magnon diffusion

spin superfluid (easy plane)

\[ S_z \]

\[ \varphi \]
Spin transport can be carried across (insulating) interfaces:

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- Spin Hall effect

An efficient spin-to-charge interconversion by the spin Hall effect:

- Spin Hall: $J_s \propto \theta_{SH} \mathbf{z} \times J_c$

- Inverse spin Hall: $J_c \propto \theta_{SH} \mathbf{z} \times J_s$

References:
- Slonczewski, JMMM (1996); Berger, PRB (1996)
- YT, Brataas, and Bauer, PRL (2002)
- Dyakonov and Perel', JETP (1971); Sinova et al., RMP (2015)
Electrical controls of spin superfluids

The spin current can be injected and extracted directly in and out of the superflow-carrying magnetic helix:

\[ J_s = g^{↑↓}(\mu_s - \hbar \partial_t \varphi) \]

\[ g^{↑↓} = \frac{1}{4\pi} \mathcal{R} \sum_{nm} (\delta_{nm} - r_{nm}^\uparrow r_{nm}^\downarrow*) \]

Takei and YT, PRL (2014)

YT, Brataas, Bauer, and Halperin, RMP (2005)
Spin-current circuit

Spin mixing conductance

Spin transfer torque
Spin pumping

Spin mixing conductance

Metal
Easy-plane ferromagnet
Metal

Spin accumulation $\mu_s$

$g_\alpha = \frac{\alpha s L}{\hbar}$
(mechnical torque)

$\hbar \Omega = \frac{g_l^{\uparrow \downarrow}}{g_l^{\uparrow \downarrow} + g_r^{\uparrow \downarrow} + g_\alpha} \mu_s$, $J_r^s = \frac{g_l^{\uparrow \downarrow} g_r^{\uparrow \downarrow}}{g_l^{\uparrow \downarrow} + g_r^{\uparrow \downarrow} + g_\alpha} \mu_s$

Takei and YT, PRL (2014)
Nonlocal magnetoresistance

Circulating current through two metal films in series (a) spins the order, reducing the overall dissipation.

\[ \rho_m \propto -\frac{\vartheta_{\text{SH}}^2}{2g_{\uparrow \downarrow} + \alpha s L} \]

In the parallel configuration (b), the torques are balanced, and the magnet remains stationary, causing more friction.


Takei and YT, PRL (2015)
A proposed realization in the $\nu = 0$ graphene

FIG. 1. Proposed setup for realizing and detecting superfluid QH states without antiferromagnetic ground state.

(a) Top view of a semi-infinite graphene sheet on a graphene substrate with two edge channels. The two branches of the conical valley fermion (CAF) are denoted by $\nu = \pm 2$. A voltage $V_{\uparrow}$ is applied to inject spin current into the detection region $\bar{V}_{\uparrow}$. Spin currents can also flow into the injection region $\bar{V}_{\downarrow}$. The spin currents are shown in red. The spin orientation can change dynamically during spin transport through the graphene sheet. The yellow region is the injection region, and the green region is the detection region. The current $I_{\uparrow}$ is independent of the spin state of the CAF.

(b) A cartoon of the CAF in a dynamic superfluid state. The CAF is shown in yellow, and the gray regions denote Ohmic contacts held at their respective voltages. Two independently biased spin-polarized edges exist, with a global precession frequency $\Omega$. The spin current flow is shown in red. The spin projections at the vertices labeled by $\nu = \pm 2$ are shown. Isospin $I_{\uparrow}$ is injected into the CAF while $I_{\downarrow}$ is returned back into the edge. Two analogous contributions exist due to the effective field created by the neighboring CAF.

FIG. 2. A cartoon of the CAF in a dynamic superfluid state. The CAF is shown in yellow, and the gray regions denote Ohmic contacts held at their respective voltages. Two independently biased spin-polarized edges exist, with a global precession frequency $\Omega$. The spin current flow is shown in red. The spin projections at the vertices labeled by $\nu = \pm 2$ are shown. Isospin $I_{\uparrow}$ is injected into the CAF while $I_{\downarrow}$ is returned back into the edge. Two analogous contributions exist due to the effective field created by the neighboring CAF.
A possible application

(domain wall floating on a spin superfluid)

![Diagram of spin transport](image)

Detrimental role of (inevitable) anisotropies

- A minimal bias is required to overcome pinning by an anisotropy within the easy plane.

\[ j_s = -A \nabla \phi \]

- Beyond this (lower) critical bias, the uniform magnetic state is unstable against nucleation and propagation of domain walls.

- At a large enough bias, the train of domain walls coalesce into a helical superfluid.

- For the minimal current to be less than the critical supercurrent, the easy-plane anisotropy needs to exceed parasitic anisotropies.

Sonin, JETP (1978); König et al., PRL (2001)
This paper deals with ordinary material systems whose elementary constituents are fermions. It is pointed out that in such systems there can occur two kinds of bosons with quite different physical and mathematical characteristics. Type I bosons are bound complexes of an even number of fermions (such as $^4\text{He}$); and type II bosons are elementary excitations which are bound complexes of fermions and their holes (such as excitons). When the first type condenses, a superfluid state results with so-called off-diagonal, long-range order; while when the second type condenses, there is no superfluidity, but a change in spatial order. Thus both kinds of long-range order are related to Bose condensation.
Thermally-assisted “superfluid”

- The spin accumulation biases chirality $q$ dependent thermal injection of domain walls:

\[
W = \int d\theta \cdot \tau = g^{\uparrow\downarrow} \int d\theta \cdot (n \times \mu_s \times n) = \pi g^{\uparrow\downarrow} \mu_s q
\]

- Chiral domain walls carry this information diffusively along the length of the magnetic wire, re-emitting the spin current by spin pumping:

\[
S_z = \int dt J_s = g^{\uparrow\downarrow} \hbar \int dt \partial_t \varphi = \pi \hbar g^{\uparrow\downarrow} q
\]
Spin texture book-keeping

- Four types of domain walls:

  (a) \[ q = +1 \]

  (b) \[ q = +1 \]

  (c) \[ q = -1 \]

  (d) \[ q = -1 \]

- Thermally-activated (topologically trivial) texture of zero net charge vs positively or negatively charged textures:
Diffusive transport theory

- **Total topological charge:**
  \[ q \equiv -\frac{1}{\pi} \int dx \, \partial_x \phi \]

- **Continuity relation:**
  \[ \partial_t \rho + \partial_x I = 0, \quad I = -D \partial_x \rho \]
  \[ D = \frac{\lambda_{DW} T}{2\alpha_s A} \]
  derived by solving stochastic Landau-Lifshitz-Gilbert equation

- **Boundary conditions (chirality-dependent injection rate):**
  \[ I^\pm = \Gamma^\pm (T, \mu_s) - \gamma(T) \rho^\pm \]
  subject to the following chirality-dependent work
  \[ W = \pm \pi g^{\uparrow\downarrow} \mu_s \]

Kim, Takei, and YT, PRB (2015)
(Thermally-activated) DC electron drag

\[ \mathcal{D}_0 \sim 0.1 \quad \text{- drag in a perfect U(1) superfluid} \]

\[ E_0 = S \sqrt{A \kappa} \quad \text{- domain-wall energy due to parasitic anisotropy} \]

- Thermally-activated domain walls coalesce and we recover the intrinsic result when \( T \sim E_0 \)

Kim, Takei, and YT, PRB (2015)
Out-of-plane excursions cause magnetic phase slips, unwinding the topological charge, which could be detected as a voltage in the following circuit:

Detrimental to the superfluid as it unwinds the helical texture

\[ T \lesssim S\sqrt{A\kappa} \ll S\sqrt{AK_{xy}} \]
Quantum phase slips (antiferromagnets)

- The same (negative) magnetoresistance geometry can be used to extract the quantum phase slip rate:

- The effective action for a gas of QPS's in the presence of a spin superflow:

$$S_{\text{eff}} = nS_{\text{core}} - \mu \sum_{i<j} q_i q_j \ln(d_{ij}/\lambda) + j_s \sum_i q_i \tau_i$$
A 2D scenario of topo-transport: Skyrmion diffusion

Cont. equation for skyrmion density:

$$\partial_t \rho_s + \vec{\nabla} \cdot \vec{j}_s = 0$$

$$\rho_s \equiv \mathbf{n} \cdot (\partial_x \mathbf{n} \times \partial_y \mathbf{n}) / 4\pi$$

$$\vec{j}_s = \mathbf{n} \cdot \left[ \partial_t \mathbf{n} \times (\hat{z} \times \vec{\nabla}) \mathbf{n} \right] / 4\pi$$

Quant. Skyrmion number:

$$Q \equiv \int dx dy \rho_s$$

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Ochoa, Kim, and YT, PRB (2016)
Skyrmion diffusion

- Work by the torque $\tau = \frac{\hbar \mathcal{P}}{2e} (\vec{j} \cdot \vec{\nabla}) \vec{n}$ on the thermally injected skyrmions:

$$w = \int \tau \cdot \vec{n} \times \delta \vec{n} = \frac{2\pi \hbar \mathcal{P}}{e} \int dt \vec{j}_s \cdot \hat{z} \times \vec{j}$$

$$W = \int dy w = \frac{2\pi \hbar \mathcal{P}}{e} jQ$$

- Onsager-reciprocal motive force at the detection (right) side:

$$\vec{\mathcal{E}} = \frac{\mathcal{P} \hbar}{2e} \vec{n} \cdot \vec{\nabla} \vec{n} \times \partial_t \vec{n} = \frac{2\pi \hbar \mathcal{P}}{2} \hat{z} \times \vec{j}_s$$

Ochoa, Kim, and YT, PRB (2016)
Summary

- Superfluid spin flow is a collective low-dissipation phenomenon responsible for an efficient long-ranged transport of spin angular momentum.

- The superfluid, which is tied to the phase winding, is topologically stable (barring phase slips); however, the spin flow is easily quenched at low temperatures by parasitic anisotropies.

- Chirality diffusion of thermally-activated domain walls in 1D and skyrmion transport in 2D take over spin transport, restoring the superfluid phenomenology in linear response.