Basic Theory of Antiferromagnets I

Helen Gomonay
Johannes Gutenberg Universität Mainz

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Antiferromagnetic Spintronics
Waldhausen Schloss

Take-home message

- Antiferromagnets: Neel order parameter, distinguish between **micro**- and **macro** description!
- Two types of exchange, **exchange enhancement**
- **Newtonian-like** dynamics vs **gyrotropic** dynamics in FM

Motivation

FM ⇒ AFM

Application

- High frequencies
- Zero magnetization
- Magnetomechanical coupling
- Combined with semiconductors

New physics

- Variety of structures
- Nontrivial dynamics
- Spin-orbit coupling
- Complicated, less studied

Outline

- Basics of antiferromagnetism: exchange interactions,Neel states, magnetic sublattices
- Phenomenological description, spin-flop transitions
- Magneto elastic effects
- Basics of dynamics: equation of motion

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- Basics of antiferromagnetism: exchange interactions,Neel states, magnetic sublattices
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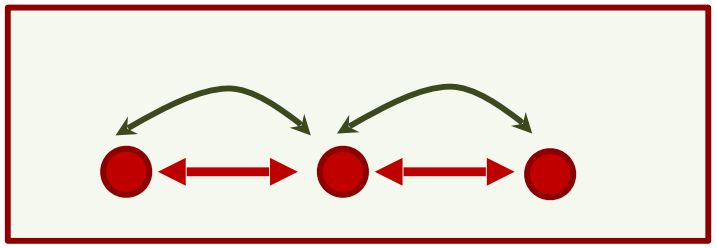
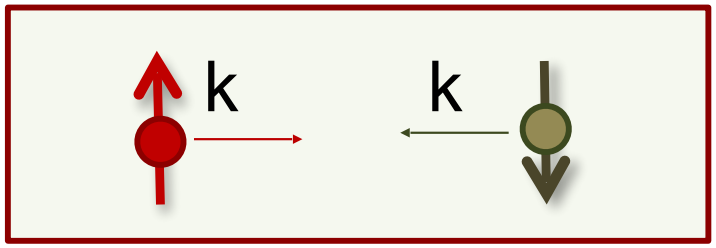
Hierarchy of atomic interactions

energy, eV



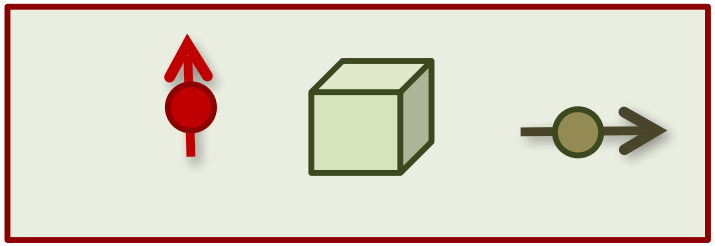
1 eV

$$\frac{e^2}{a}, \frac{\hbar^2}{ma^2}$$



1 meV

$$-\lambda_{SO}(\mathbf{SL})$$

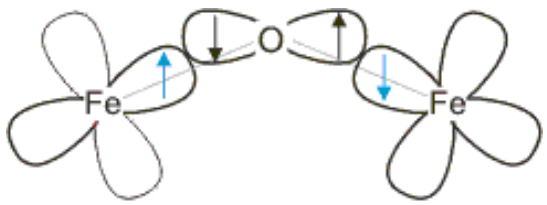


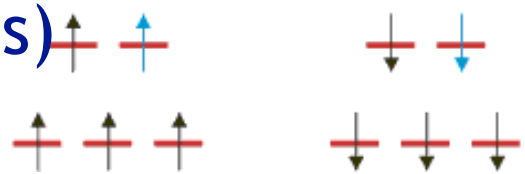
1μeV

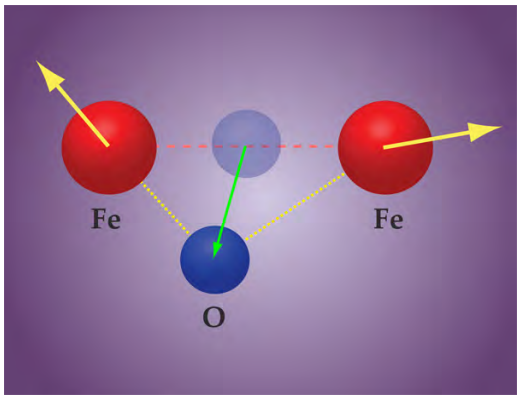
$$-K_{\mathrm{an}} S_z^2$$

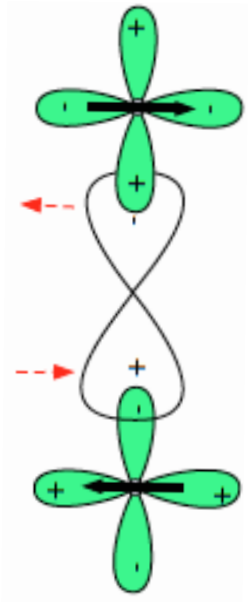
AF exchange interactions

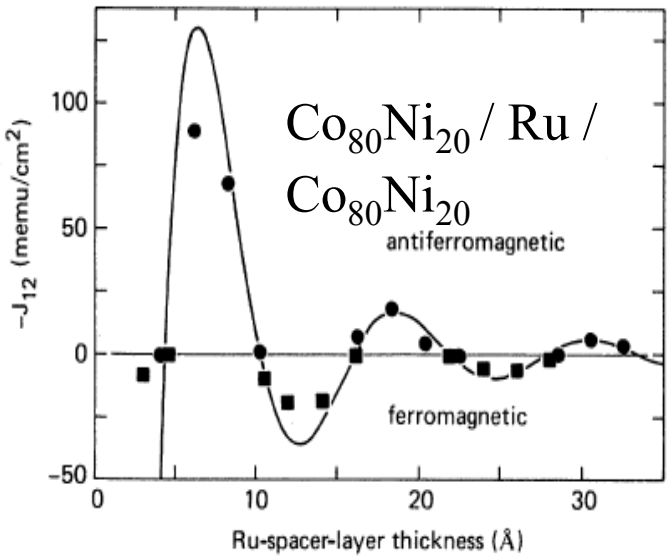
- Superexchange (insulators)
- RKKY (4-f metals)
- Exchange in 3-d metals
- Double exchange (transition metal oxides)
- **DMI** (anisotropic exchange)









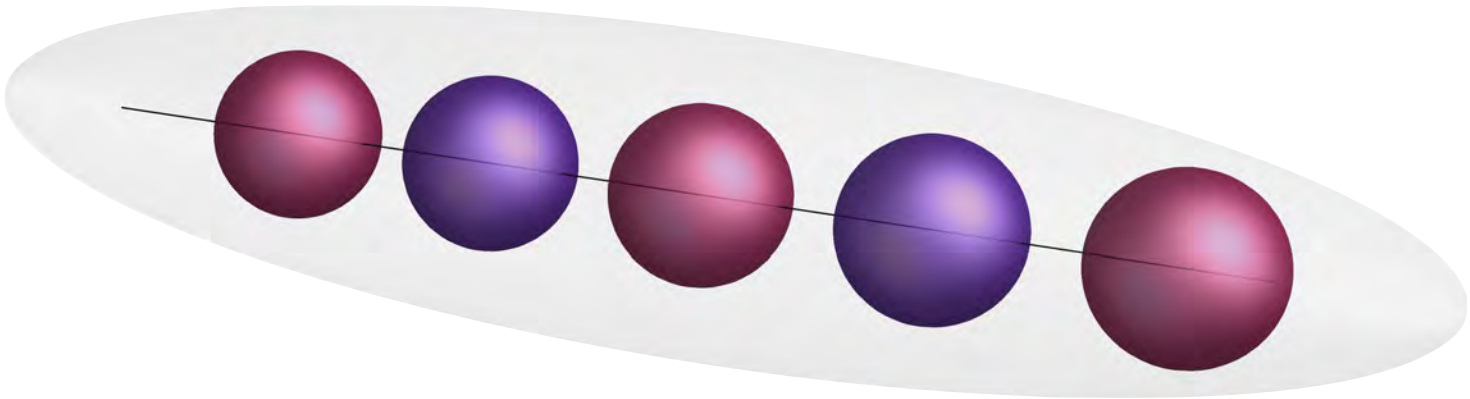


Parkin et al., Phys.Rev. B 44, 7131 (1991)

Quantum state vs Neel state

$$\hat{H} = \sum_{j,k} J_{jk} \hat{\mathbf{S}}_j \hat{\mathbf{S}}_k$$

Quantum state, T=0

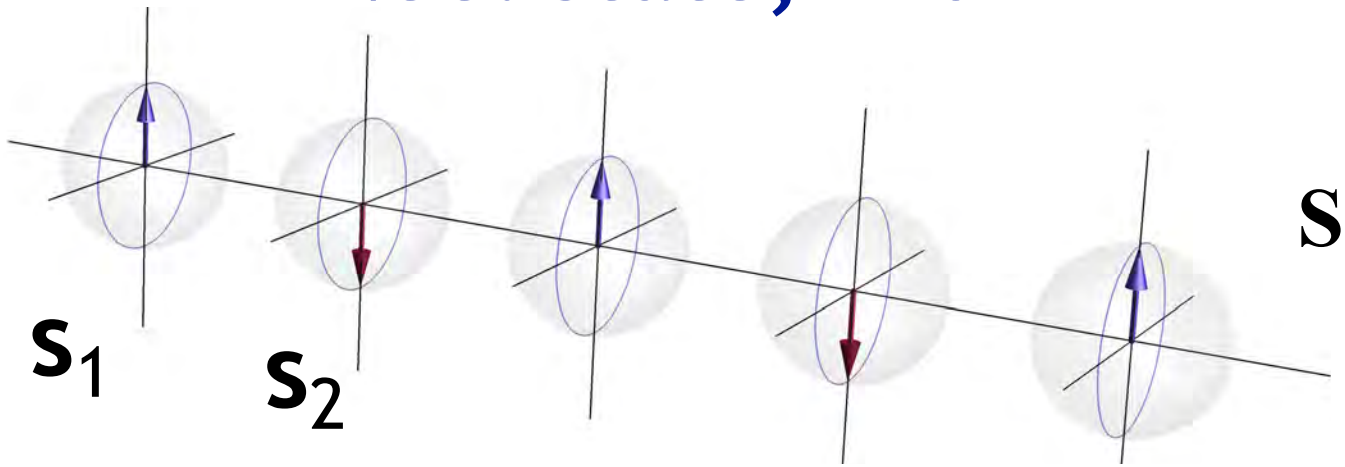


$$|\psi\rangle = \sum_{\{j\}} c_{\{j\}} |S_{z1}\rangle |S_{z2}\rangle \dots |S_{zj}\rangle |S_{zj+1}\rangle$$

Quantum state vs Neel state

$$\hat{H} = \sum_{j,k} J_{jk} \hat{\mathbf{S}}_j \hat{\mathbf{S}}_k$$

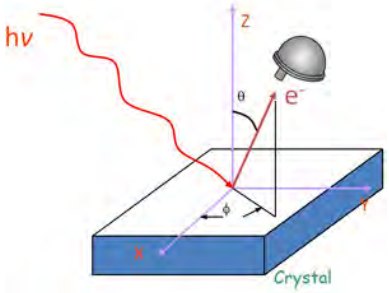
Neel state, $T \neq 0$



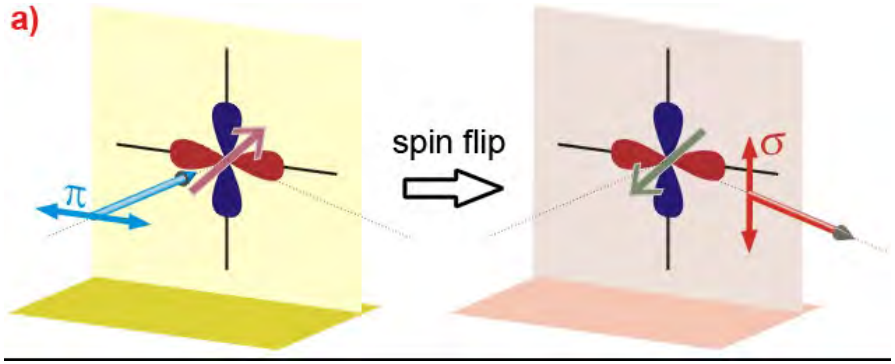
$$\mathbf{S}_1 = -\mathbf{S}_2$$

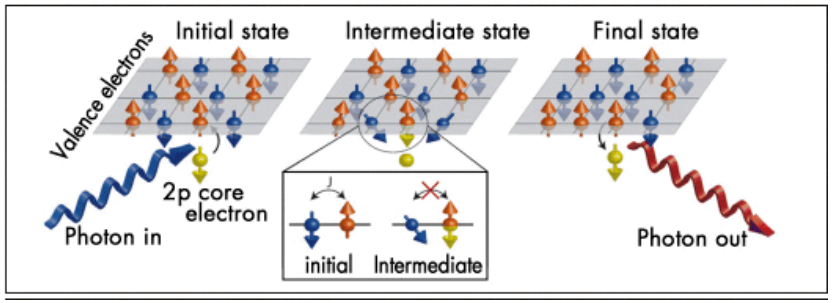
$$\{\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_j, \mathbf{s}_{j+1}\}$$

Bryan Gallagher: afternoon session

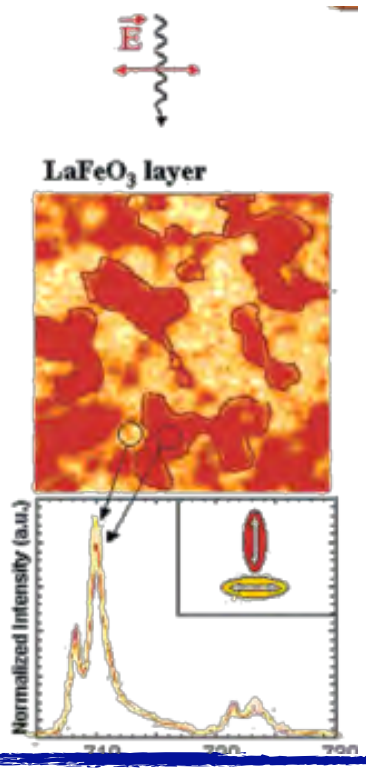


ARPES



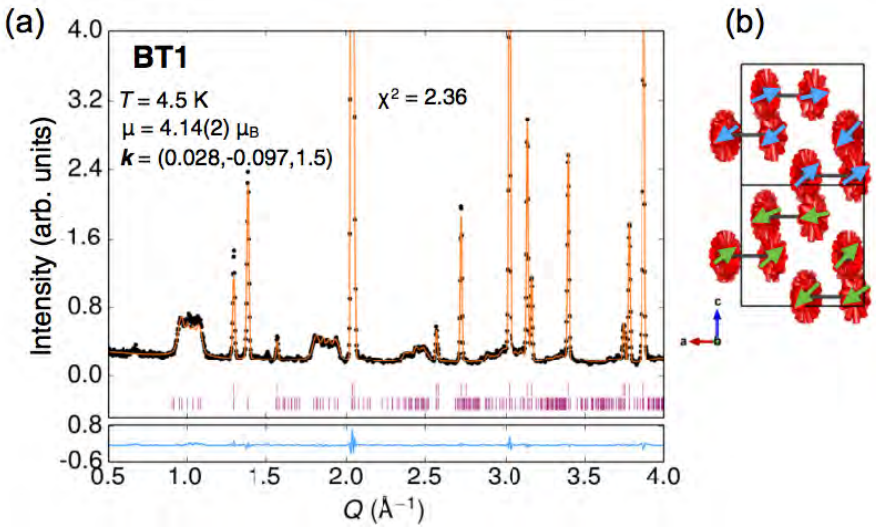


RIXS



Raman spectroscopy

XMLD

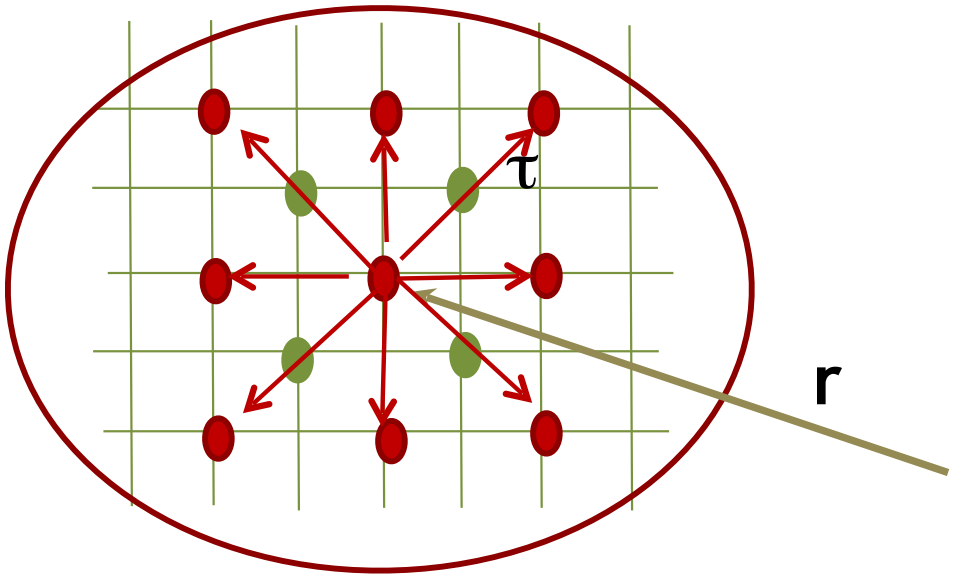


ND

Sublattices

Sublattice magnetizations (Neel, 1948)

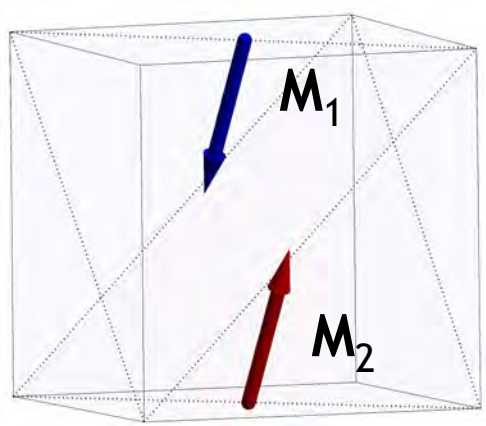
$$\mathbf{M}_k(\mathbf{r}) = \frac{g}{N}\sum_{\tau_j}\mathbf{S}_k(\mathbf{r}+\tau_j)$$



- Physically small volumes
- Macroscopic vectors
- Field variables

and order parameters

Symmetry relations: $2_{[110]}$: $\mathbf{M}_1 \leftrightarrow \mathbf{M}_2$

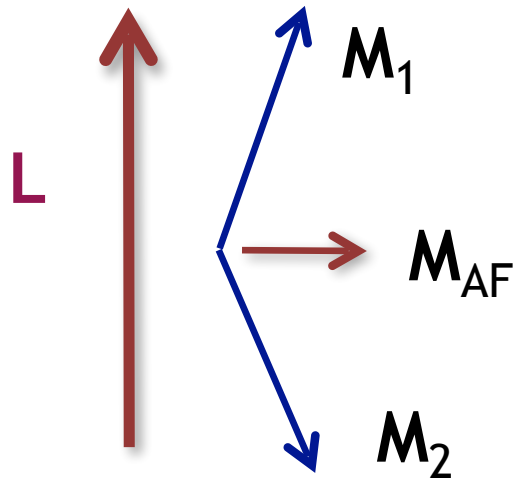


NiO, IrMn

Order parameter (Neel vector):

$$\mathbf{L} = \mathbf{M}_1 - \mathbf{M}_2$$

Magnetization: $\mathbf{M}_{AF} = \mathbf{M}_1 + \mathbf{M}_2 \approx 0$

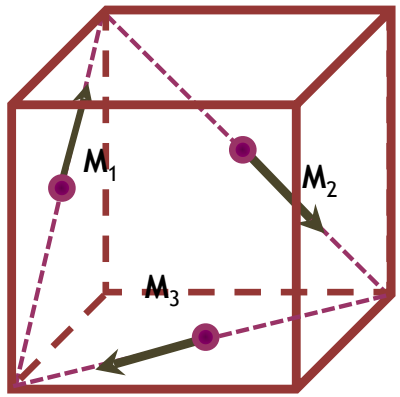


$\mathbf{L} \perp \mathbf{M}_{AF}$, $|\mathbf{L}| \approx 2M_s$

Noncollinear structures

$2_{[110]}$: $M_1 \leftrightarrow M_2$, $M_3 \leftrightarrow M_3$

$3_{[111]}$: $M_1 \rightarrow M_2 \rightarrow M_3$

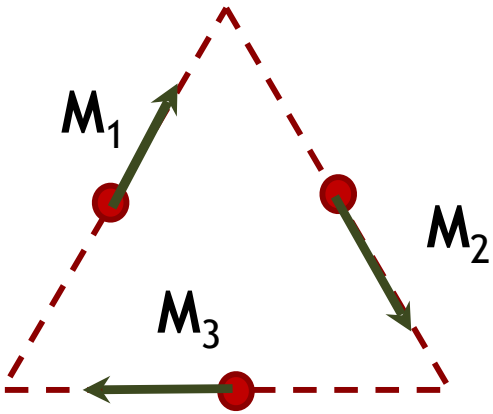


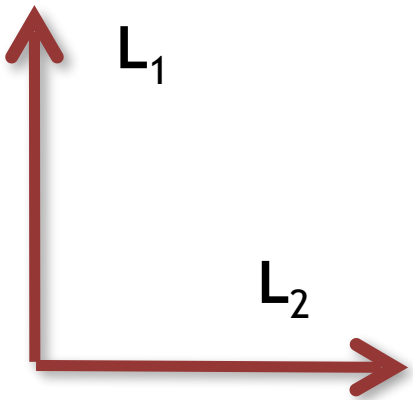
IrMn,
Mn_3NiN

Order parameters (Neel vectors):

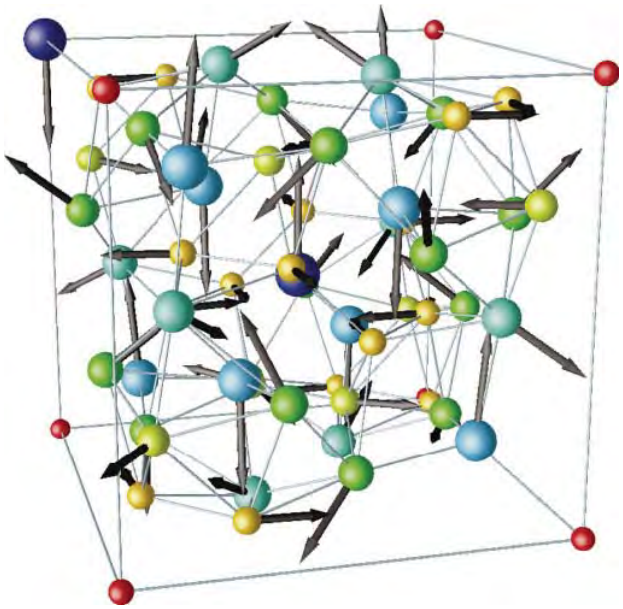
$$L_1 = M_1 + M_2 - 2M_3 \quad L_2 = M_1 - M_2$$

Magnetization: $M_{AF} = M_1 + M_2 + M_3 \approx 0$

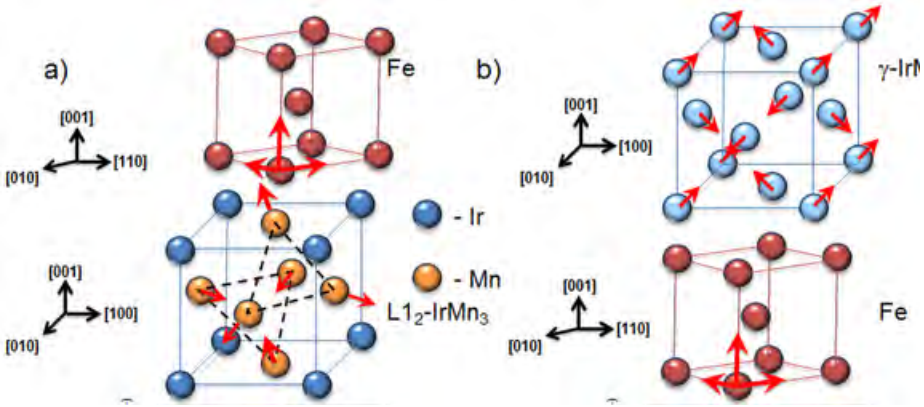


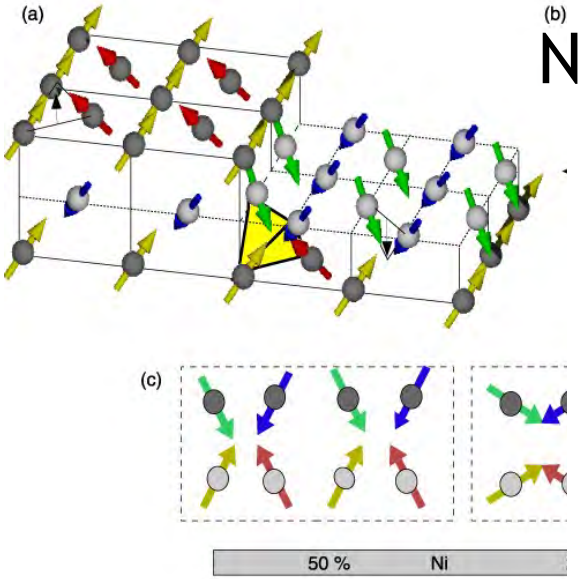


Variety of AFM structures

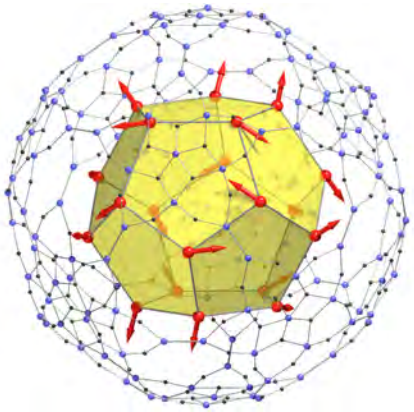


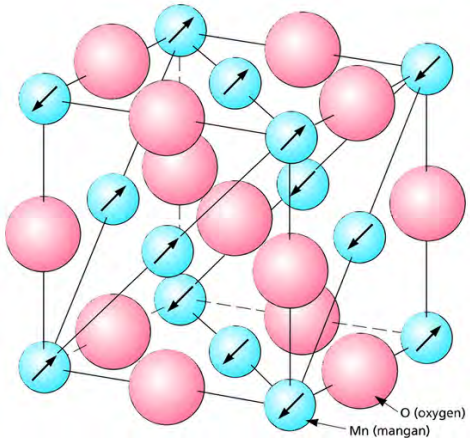
α-Mn



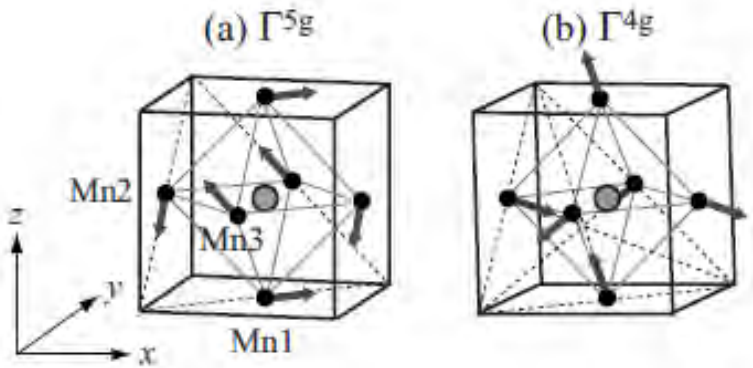


NiMn





NiO



Mn_3NiN

Take-home messages

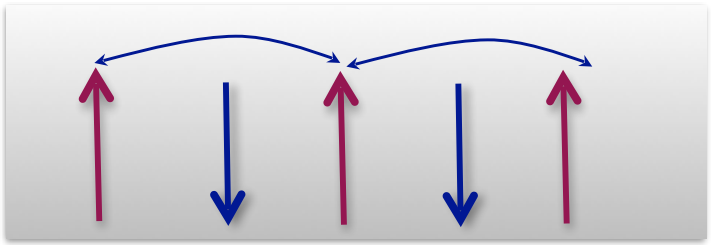
- AF = variety of exchange mechanisms
- AF = metals,insulators and in between
- AF = variety of structures
- Macroscopic description = sublattice magnetizations

Outline

- Basics of antiferromagnetism: exchange interactions,Neel states, magnetic sublattices
- **Phenomenological description, spin-flop transitions**
- Magneto elastic effects
- Basics of dynamics: equation of motion

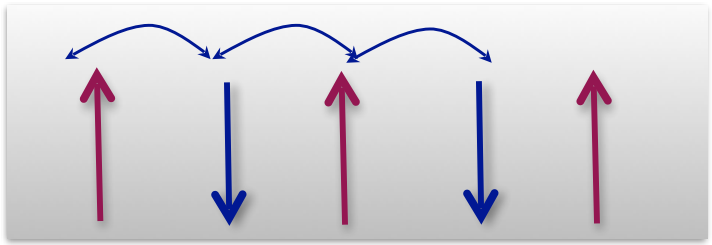
Hierarchy of interactions

H, T



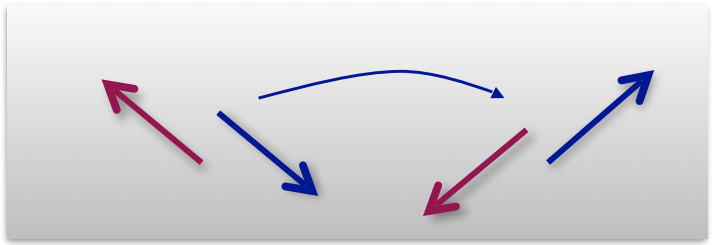
1000 T

$$J_{\text{intra}} \Leftrightarrow T_N$$



100 T

$$J_{\text{inter}} \propto 1/\chi_{\perp}$$



1 T

$$H_{\text{s-f}} \propto \sqrt{J_{\text{inter}} H_{\text{anis}}}$$

Free energy, Landau approach

Low temperature $T, H \ll J_{\text{inter}}$

$$M_{\text{AF}} \ll L, \quad L = 2M_s \approx const$$

Symmetry-based modeling

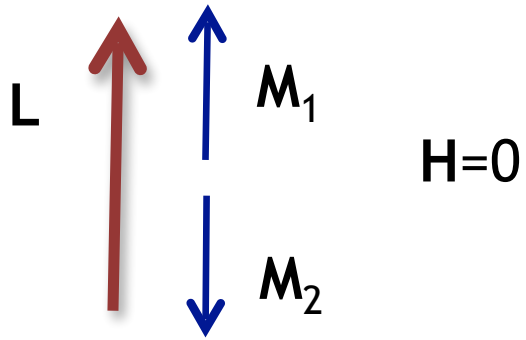
$$w = \frac{1}{2} J_{\text{inter}} \mathbf{M}_{\text{AF}}^2 - \mathbf{M}_{\text{AF}} \mathbf{H} + \frac{1}{2} A \left(\nabla \mathbf{L}\right)^2 + \frac{1}{2} K_{\text{anis}}^{(2)} L_z^2 - \frac{1}{4} K_{\text{anis}}^{(4)} \left(L_x^4 + L_y^4\right)$$

Equilibrium state

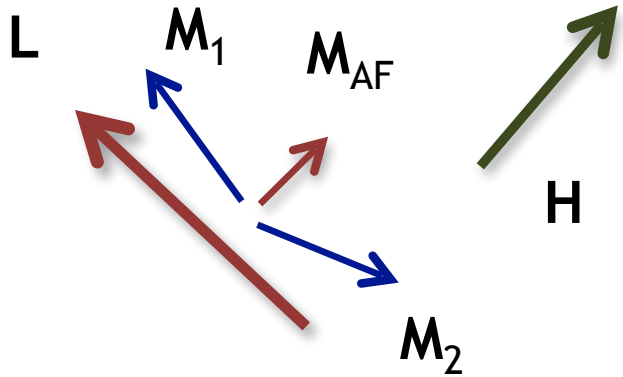
Principle of energy minimum: $\delta F = \delta \int w dV = 0$

Exchange approximation: excluding $\mathbf{M}_{AF}$

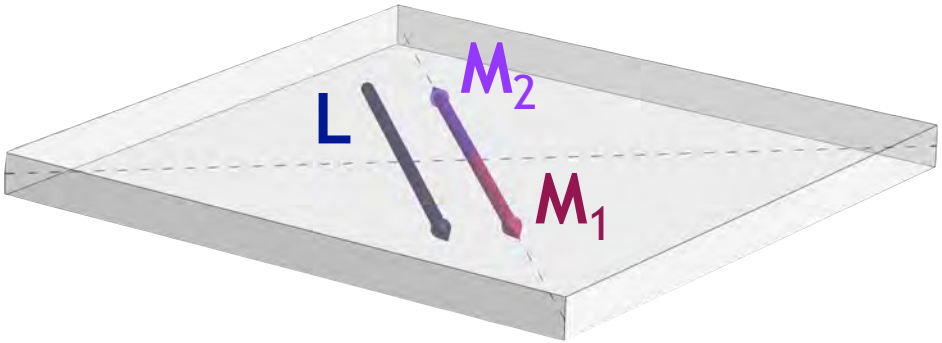
$$\mathbf{M} = \frac{1}{4J_{\text{inter}}M_s^2}\mathbf{L}\times(\mathbf{H}\times\mathbf{L})$$

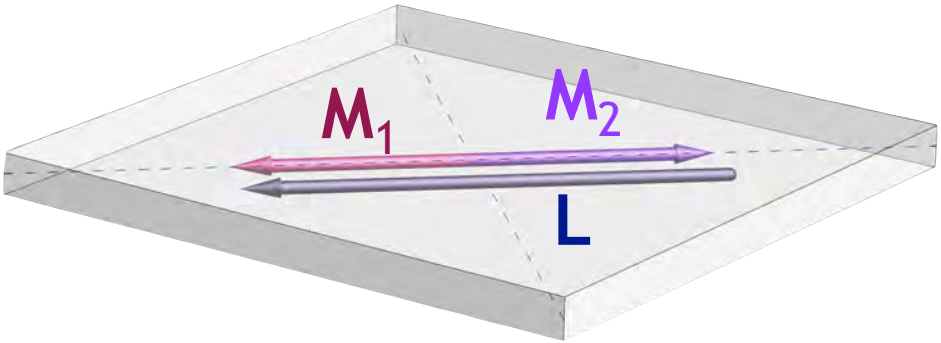


$$w_{\text{Zeeman}} = -\frac{(\mathbf{L}\times\mathbf{H})^2}{8J_{\text{inter}}M_s}$$



Spin-flop transition

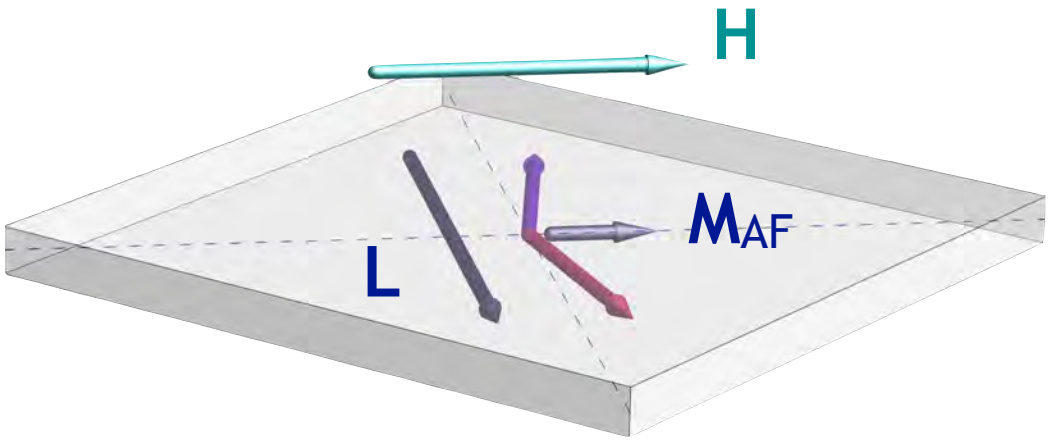


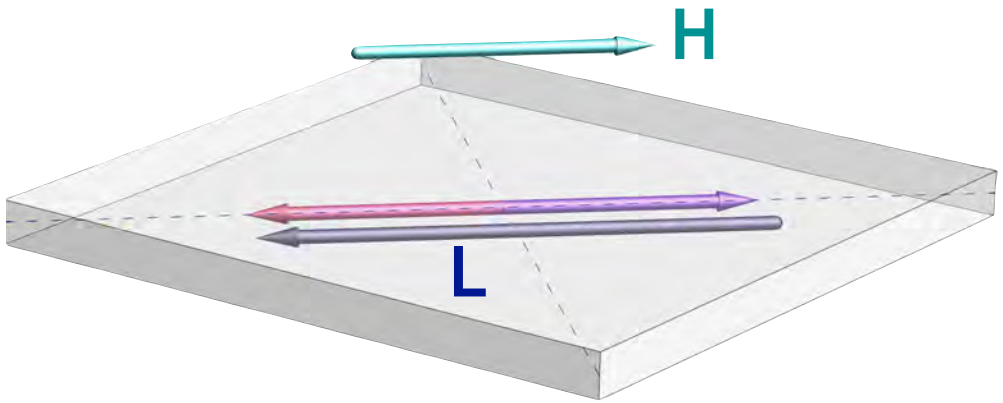


$$w = \frac{H_{\text{anis}}^{\parallel}}{8M_s} L_z^2 - \frac{H_{\text{anis}}^{\perp}}{32M_s} \left(L_x^4 + L_y^4\right)$$

Possibility for information coding!

Spin-flop transition



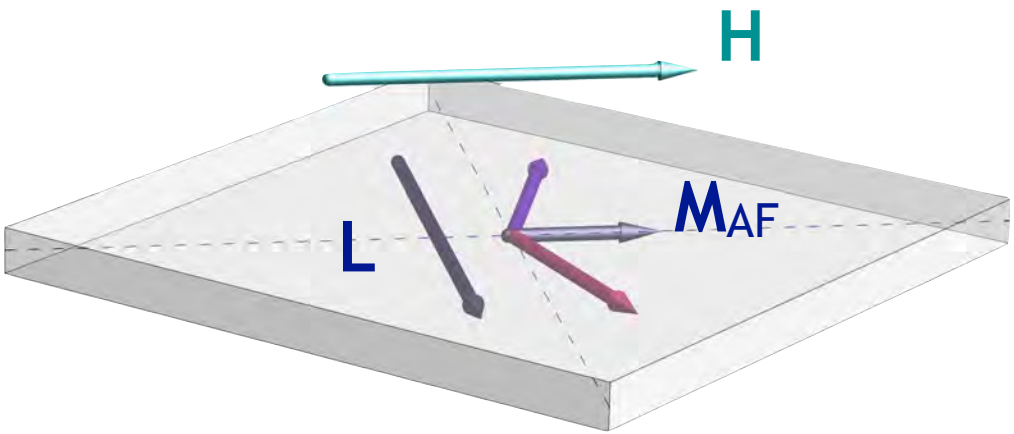


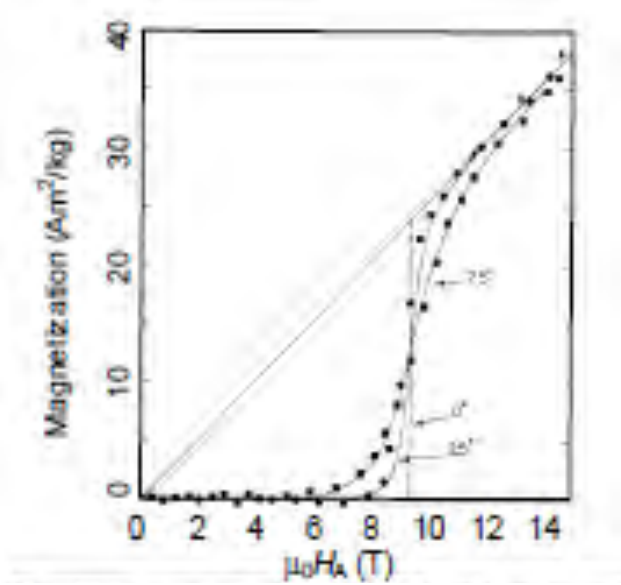
$M_{AF}=0$

$$H \leq H_{\mathrm{sf}} = \sqrt{2J_{\mathrm{inter}}H^{\perp}_{\mathrm{anis}}}$$

$$w = \frac{H^{\parallel}_{\mathrm{anis}}}{8M_s}L_z^2 - \frac{H^2L_y^2}{8J_{\mathrm{inter}}M_s} - \frac{H^{\perp}_{\mathrm{anis}}}{32M_s}\left(L_x^4 + L_y^4\right)$$

Spin-flop transition





$$H > H_{\mathrm{sf}} = \sqrt{2 J_{\mathrm{inter}} H_{\mathrm{anis}}^{\perp}}$$

Exchange enhancement

Take-home messages

- Macroscopic description: order parameters and magnetization vector
- Inter-sublattice exchange – exchange enhancement
- Static magnetic field – quadratic effects
- Equivalent states– information coding
- Spin-flop transition – information control

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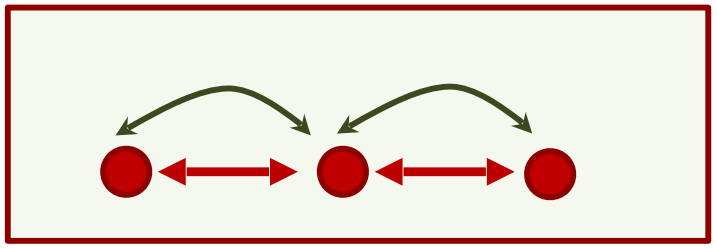
- Basics of dynamics: equation of motion

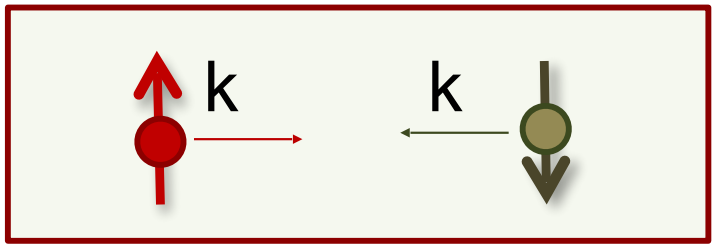
Hierarchy of atomic interactions

energy, eV



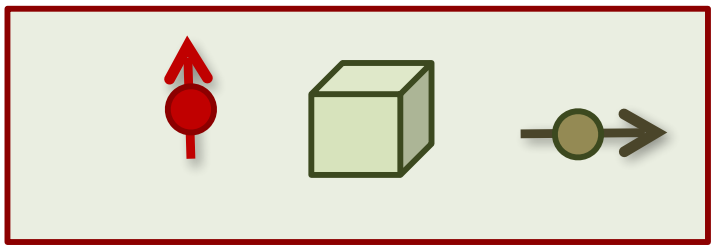
1 eV

$$\frac{e^2}{a}, \frac{\hbar^2}{ma^2}$$



1 meV

$$-\lambda_{SO}(\mathbf{SL})$$



1μeV

$$-K_{\mathrm{an}} S_z^2$$

Magnetoelastic interactions

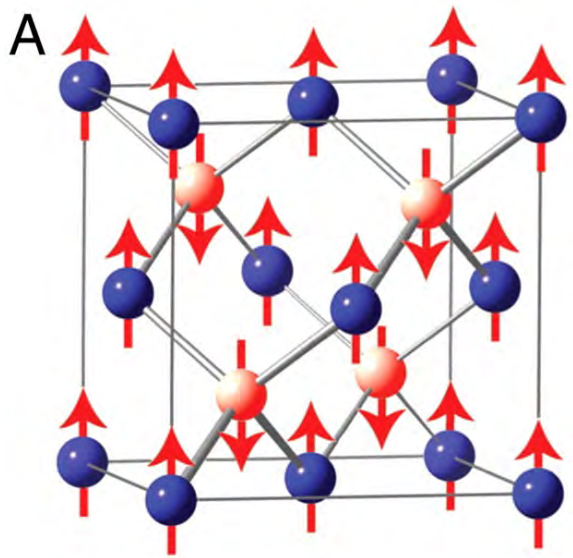
Covalent bonds ⇒ spin-orbit coupling ⇒ mag.-el.

$$w_{\mathrm{me}} = \lambda_{jklm} u_{jl} L_k L_m$$

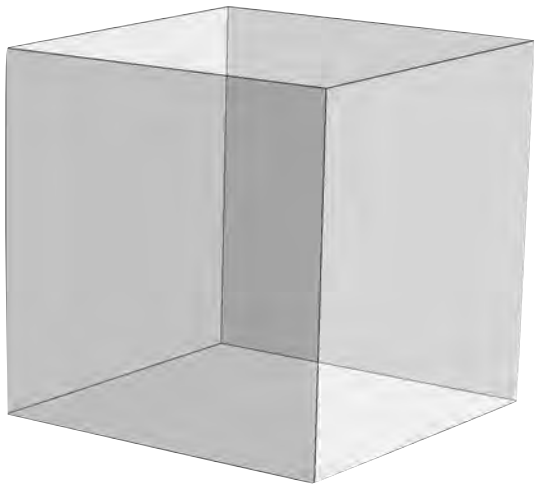
$$w_{\mathrm{el}} = \frac{1}{2} c_{jklm} u_{jl} u_{km}$$

Spontaneous striction:

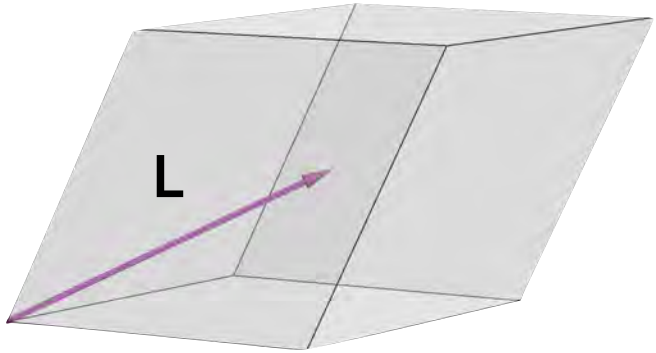
$$\hat{u}_{\mathrm{spon}} = -\frac{\hat{\lambda}_{\mathrm{me}}}{c'} \mathbf{L} \otimes \mathbf{L}$$



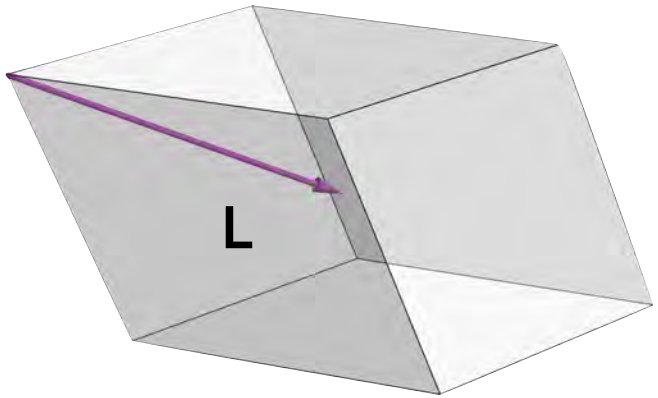
Spontaneous striction



$$u_{\text{spon}} = 0$$



$$u_{\text{spon}} = -\frac{\Lambda}{c'}\mathbf{L}^2$$



$$u_{\text{spon}} = \frac{\Lambda}{c'}\mathbf{L}^2$$

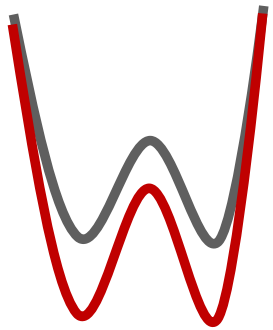
$$\hat{u}_{\text{spon}} = -\frac{\hat{\lambda}_{\text{me}}}{c'}\mathbf{L} \otimes \mathbf{L}$$

Magnetoelastic anisotropy

$$\hat{u}_{\text{spon}} = -\frac{\hat{\lambda}_{\text{me}}}{c'}\mathbf{L}\otimes\mathbf{L} \longrightarrow w_{\text{me}} = \lambda_{jklm}u_{jl}L_kL_m$$

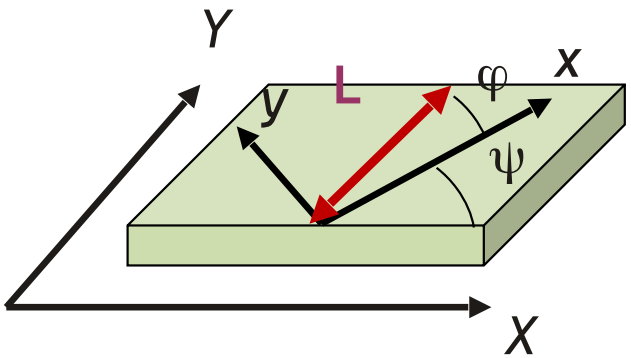
$$w = \frac{H^{\parallel}_{\text{anis}}}{8M_s}L_z^2 - \left[\frac{H^{\perp}_{\text{anis}}}{32M_s} + \frac{\lambda^2}{c'}\right](L_x^4 + L_y^4)$$

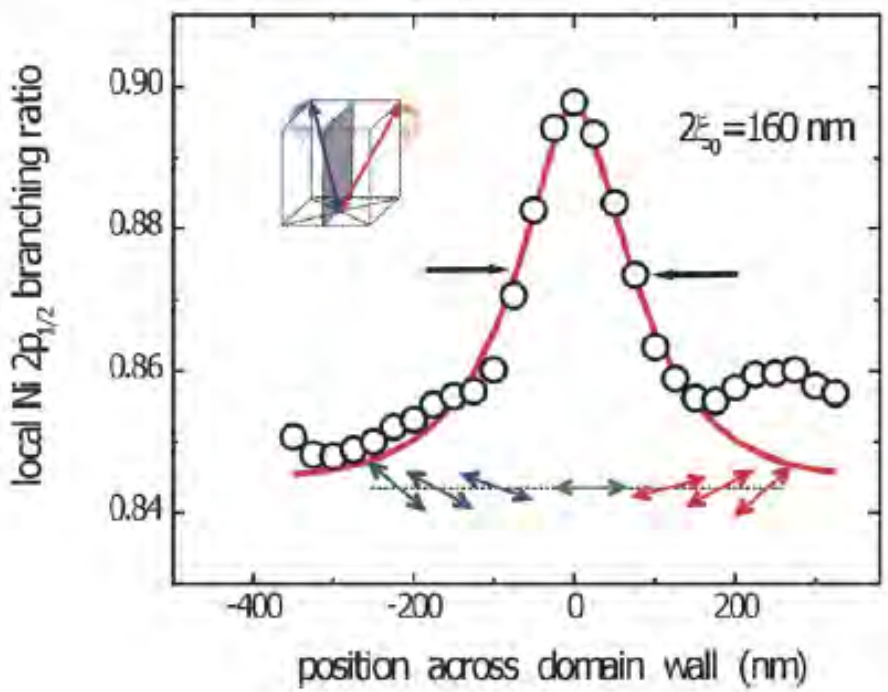
- Additional 4-th order anisotropy
- Variation of potential barrier

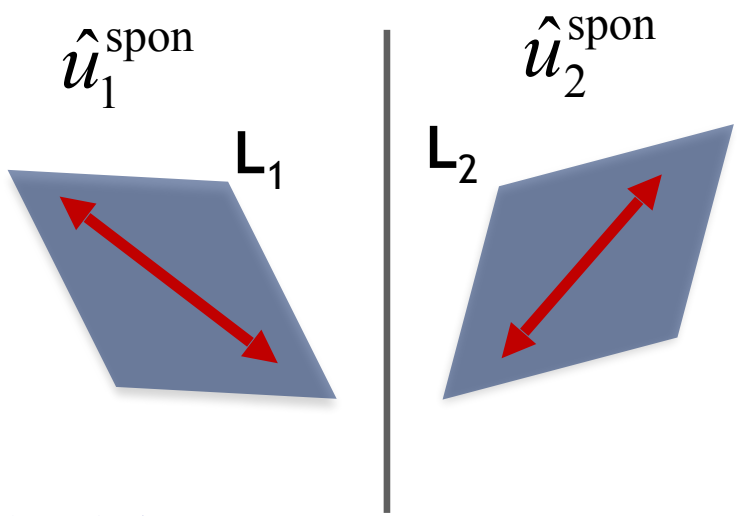


Example: domain walls

$$-A\frac{d^2\phi}{dz^2} + \left[K_{\text{anis}} + \frac{\lambda^2}{c'}f(z)\right]\sin\phi\cos\phi = 0$$

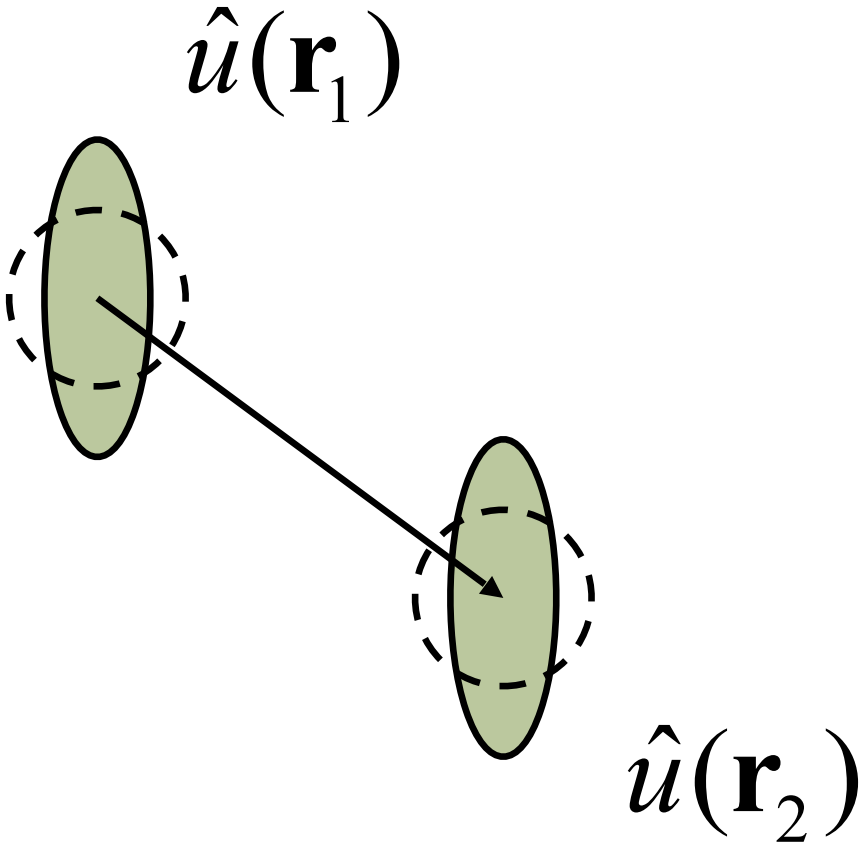


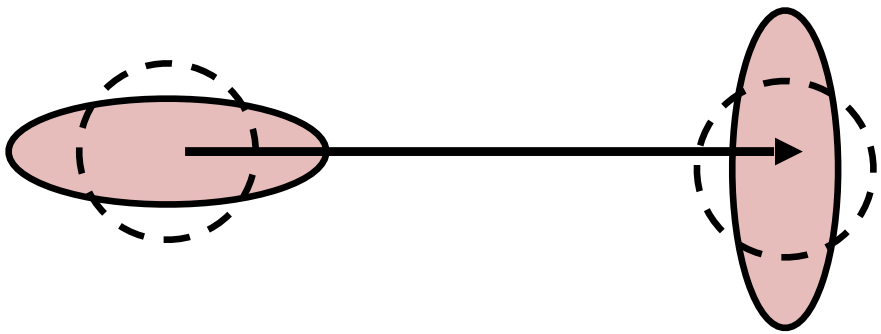




Elastic dipoles: long-range forces

$$\hat{u}_{\mathrm{spon}} = -\frac{\hat{\lambda}_{\mathrm{me}}}{c'}\mathbf{L}\otimes\mathbf{L}$$



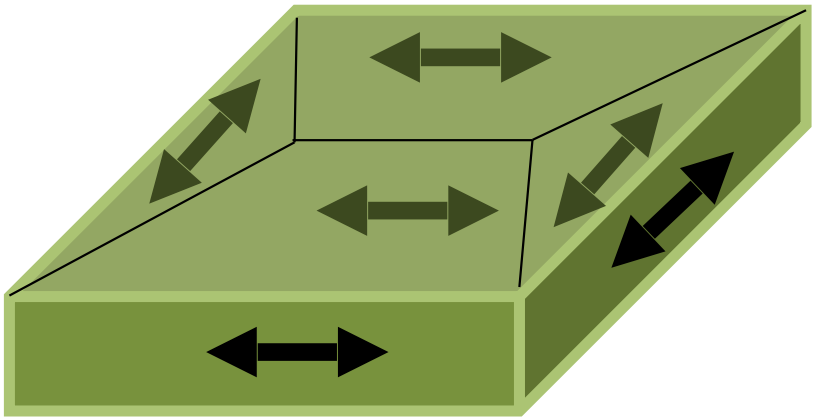


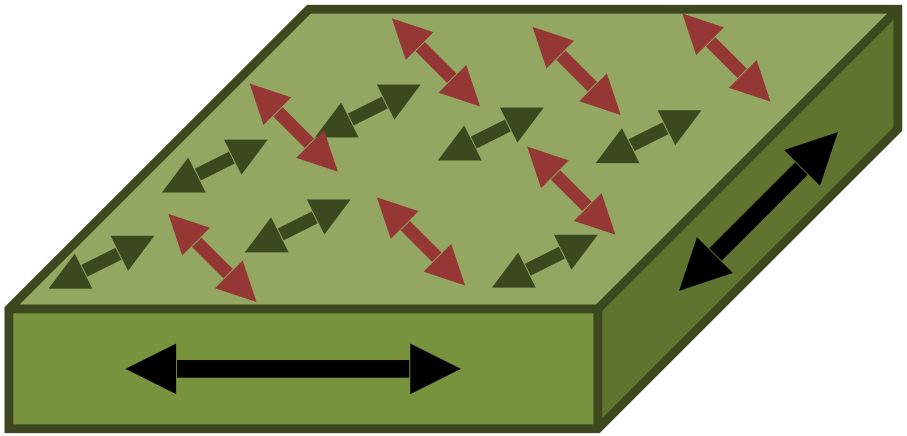
$$\Phi_{dd} \propto \frac{\hat{u}^{in}(\mathbf{r}_1)\hat{u}^{in}(\mathbf{r}_2)}{\left|\mathbf{r}_1-\mathbf{r}_2\right|^3}$$

Shape-induced effects

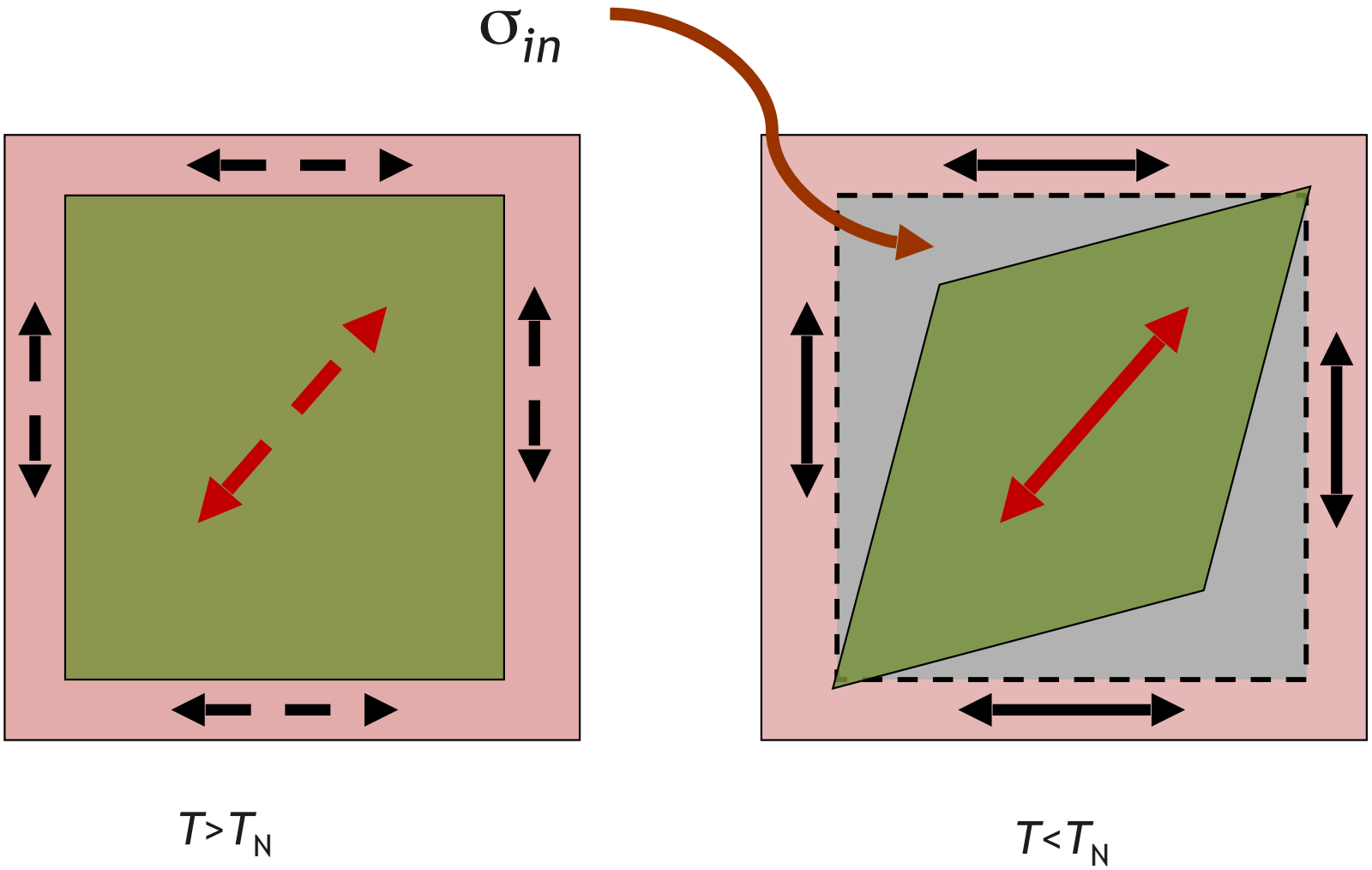
Inhomogeneous sample, destressing energypy

$$W_{\text{destr}} = K^{\text{shape}} \left(\frac{a}{b} \right) \left\langle L_x^2 - L_y^2 \right\rangle^2$$





Magnetostriction and stress



Take-home message

- Magnetostriction = source of additional anisotropy
- Orientation domains = magnetoelastic
- Shape anisotropy = magnetoelastic
- Different effects in nano and macro-samples

Outline

- Basics of antiferromagnetism: exchange interactions,Neel states, magnetic sublattices
- Phenomenological description, spin-flop transitions
- Magneto elastic effects
- **Basics of dynamics: equation of motion**

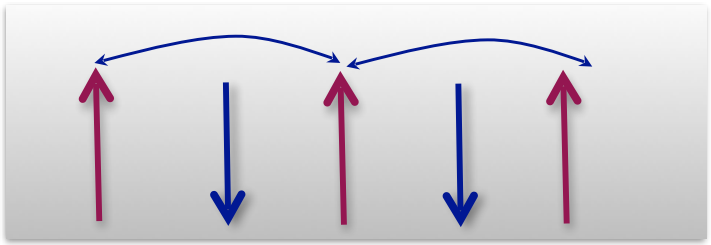
Spin Torques in antiferromagnet

$$\frac{d\mathbf{M}_1}{dt} = \boxed{\gamma \mathbf{M}_1 \times \mathbf{H}_1} + \Lambda \boxed{\mathbf{M}_1 \times \mathbf{p}_1 \times \mathbf{M}_1}$$

$$\frac{d\mathbf{M}_2}{dt} = \boxed{\gamma \mathbf{M}_2 \times \mathbf{H}_2} + \Lambda \boxed{\mathbf{M}_2 \times \mathbf{p}_2 \times \mathbf{M}_2}$$

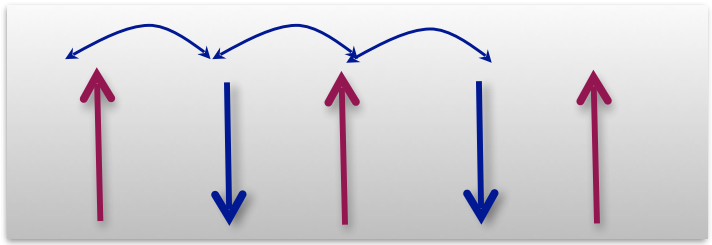
Hierarchy of interactions

H, T



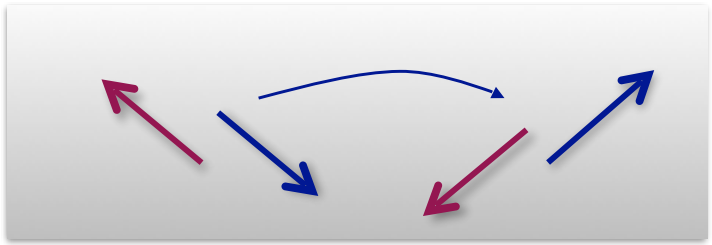
1000 T

$$J_{\text{intra}} \Leftrightarrow T_N$$



100 T

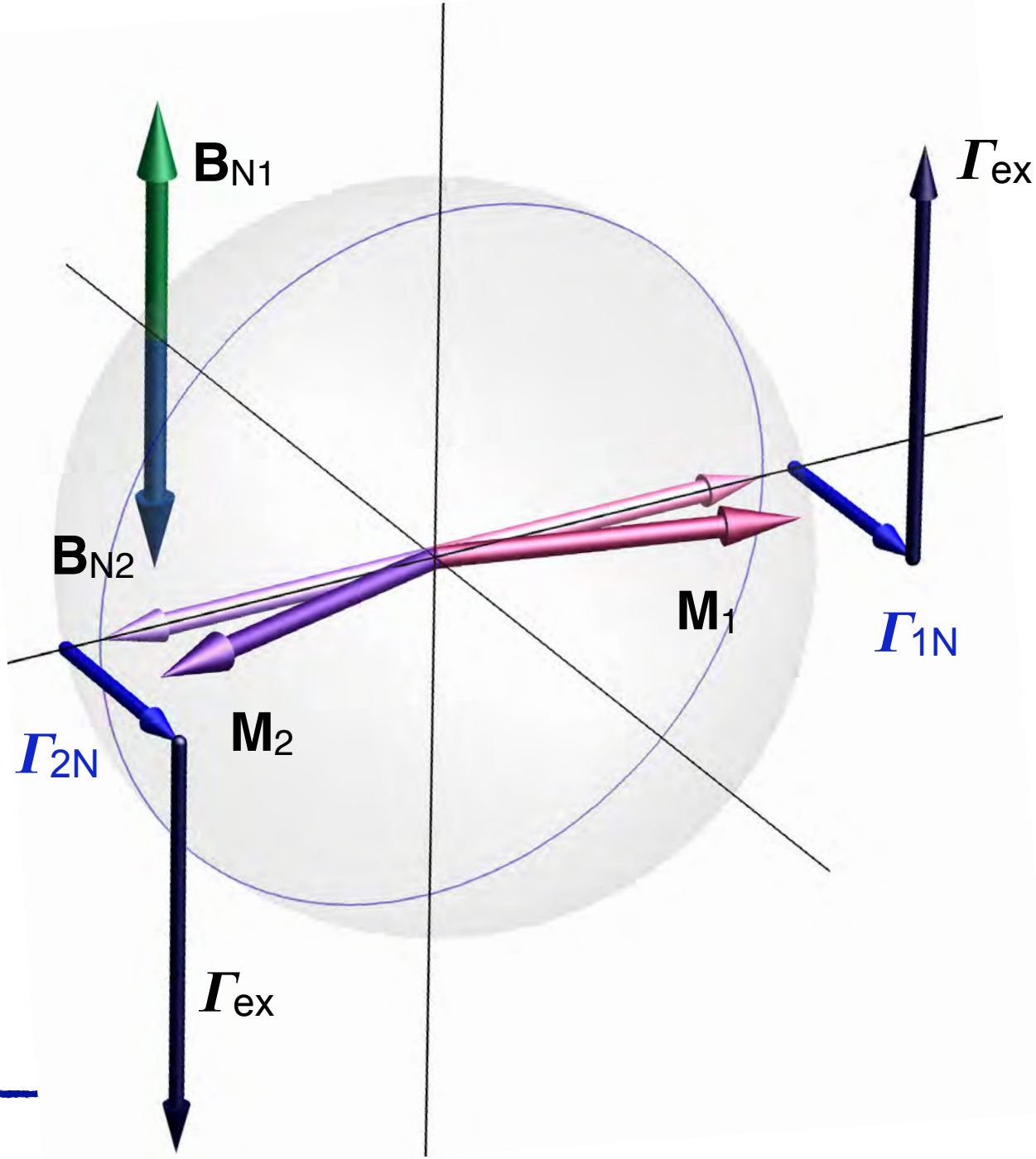
$$J_{\text{inter}} \propto 1/\chi_{\perp}$$



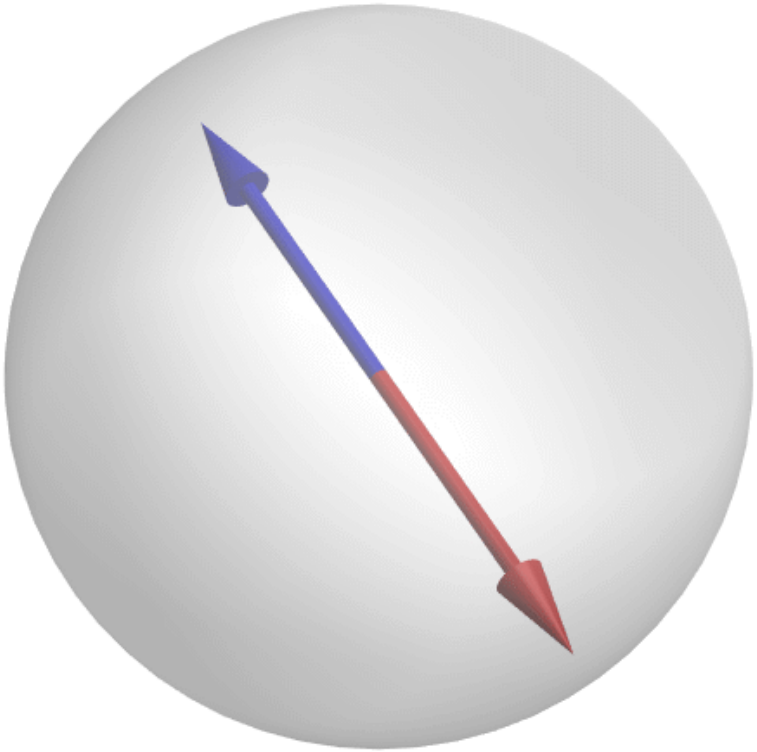
1 T

$$H_{\text{s-f}} \propto \sqrt{J_{\text{inter}} H_{\text{anis}}}$$

Exchange enhancement



Solid-like dynamics



$$T, H << J_{\text{inter}}$$

Equations of motion

Dynamic magnetisation

$$\mathbf{M} = \frac{1}{H_{\mathrm{ex}}}\mathbf{L} \times \dot{\mathbf{L}} + \frac{1}{H_{\mathrm{ex}}}\mathbf{L} \times \mathbf{H} \times \mathbf{L}$$

Magnetisation balance

$$\dot{\mathbf{M}} \equiv \frac{1}{H_{\mathrm{ex}}}\mathbf{L} \times \ddot{\mathbf{L}} = \gamma \mathbf{L} \times \mathbf{H}_{\mathrm{L}}$$

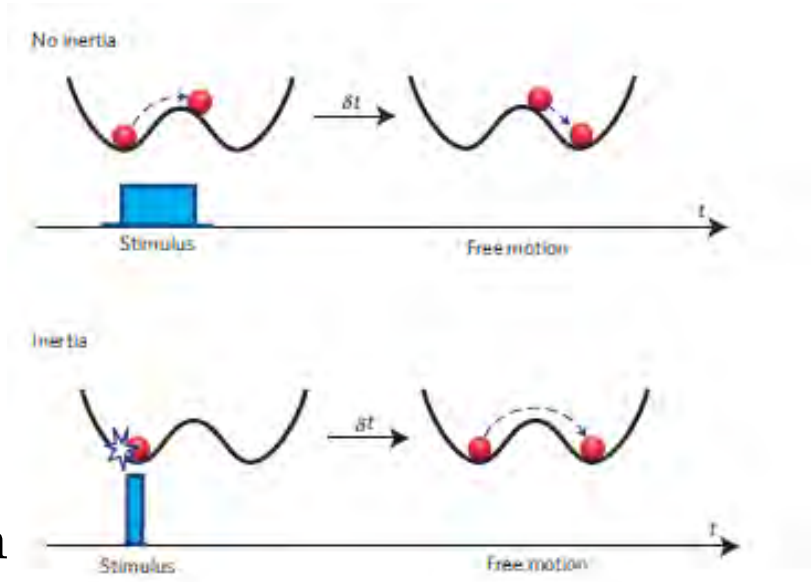
Equation of motion

AF dynamics: Newton-like

$$\underbrace{\ddot{\mathbf{L}}}_{\text{acc}} = \underbrace{-\gamma\left(\dot{\mathbf{H}} \times \mathbf{L} + 2\mathbf{H} \times \dot{\mathbf{L}}\right)}_{\text{gyroforce}} + \underbrace{\gamma^2 J_{\text{inter}} \mathbf{H}_L}_{\text{potent.force}}$$

FM dynamics: precession

$$\underbrace{\dot{\mathbf{M}}}_{\text{ang. momentum}} = \underbrace{-\gamma \mathbf{M} \times \mathbf{H}_M}_{\text{force momentum}}$$



Take-home messages

- AF: dynamics magnetisation
- AF: inertia due to exchange
- Dynamics = balance equation for magnetizations

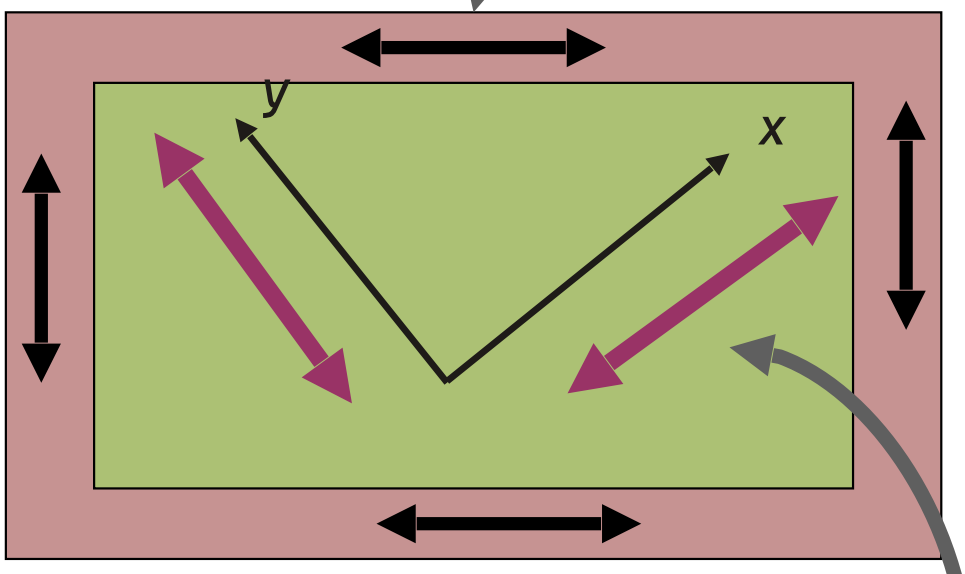
Conclusions

- AF different from FM
- Field effects => weak
- Exchange interaction => important for dynamics
- Strong magneto elastic effects

Thank you!

Surface vs bulk anisotropy

$$W_{\text{surf}} = K_{\text{surf}} \int_S (\mathbf{L} \cdot \mathbf{n})^2 dS$$



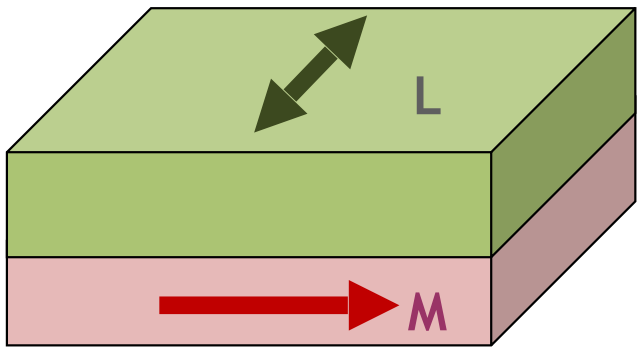
$$W_{\text{bulk}} = -\frac{1}{4} K_{\text{bulk}} \int_V \left(L_x^4 + L_y^4\right) dV$$

Shape-induced effects

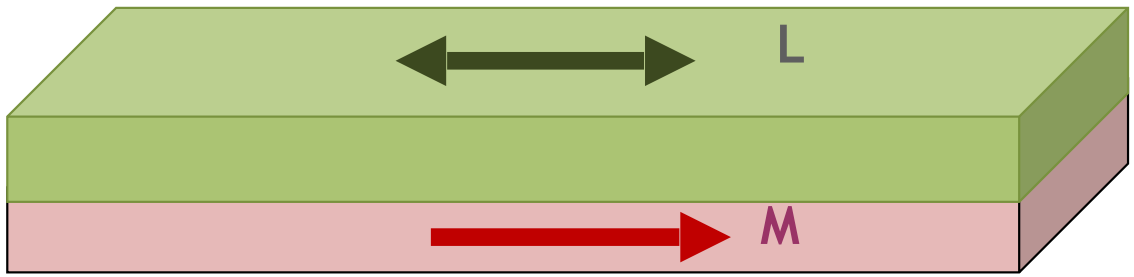
Homogeneous sample, shape-induced anisotropy

$$W = V\left[K_{\perp}^{\text{anis}}\left(L_x^4 + L_y^4\right) + K^{\text{shape}}\left(a/b\right)L_x^2\right]$$

$$K_{\text{bias}} > K_{\text{shape}}\left(\frac{a}{b}\right) = 0 \qquad K_{\text{bias}} < K_{\text{shape}}\left(\frac{a}{b}\right)$$



a=b



a>>b

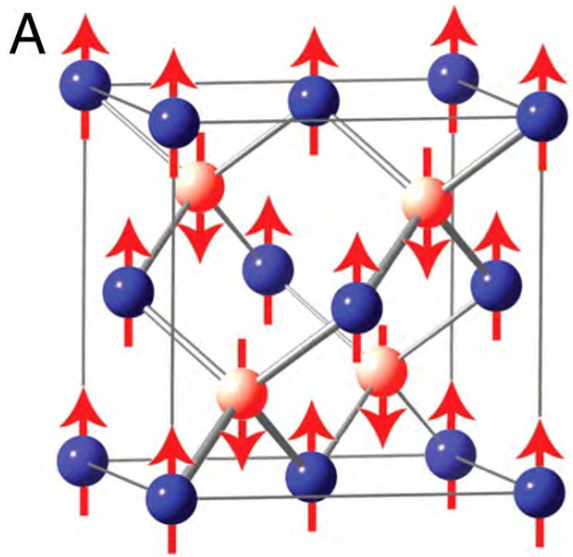
Magnetoelastic interactions

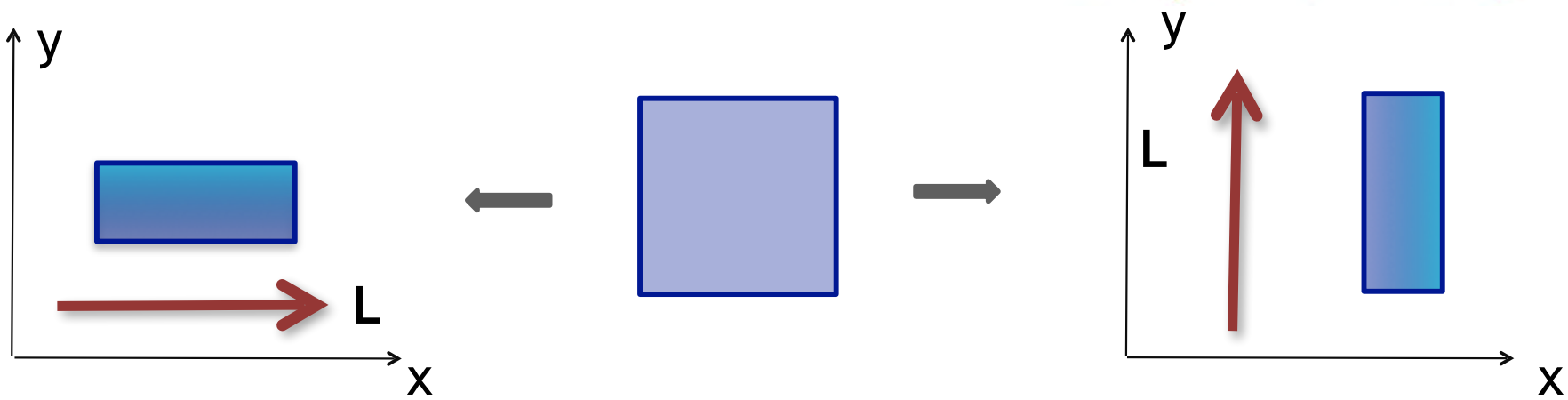
Covalent bonds ⇒ spin-orbit coupling ⇒ mag.-el.

$$w_{\text{m-e}} = \lambda_{jklm} u_{jk} L_l L_m$$

Spontaneous striction:

$$\hat{u}_{\text{spon}} = -\frac{\hat{\lambda}_{\text{me}}}{c'} \mathbf{L} \otimes \mathbf{L}$$





Basic Theory of Antiferromagnets II

Helen Gomonay
Johannes Gutenberg Universität Mainz

September 26, 2016
Antiferromagnetic Spintronics
Waldhausen Schloss

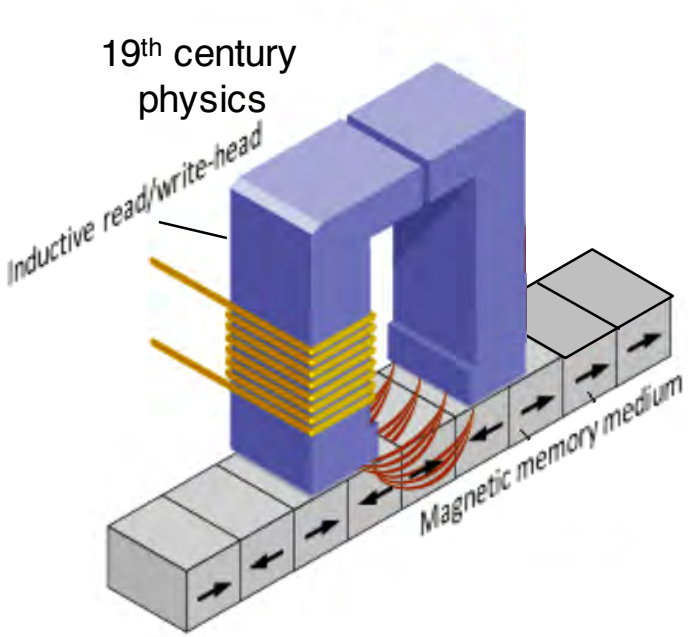
Take-home message

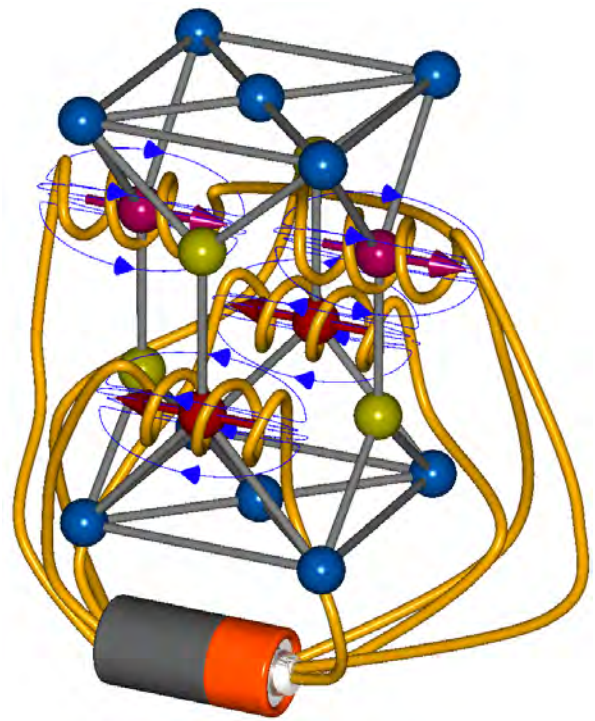
- Exchange enhancement $\Rightarrow$ fast antiferromagnetic dynamics

- Antiferromagnetic states can be **effectively** manipulated by **spin** and **charge** current

Motivation

- All-electrical control and manipulation of AF states
- Information and data storage with AF





Outline

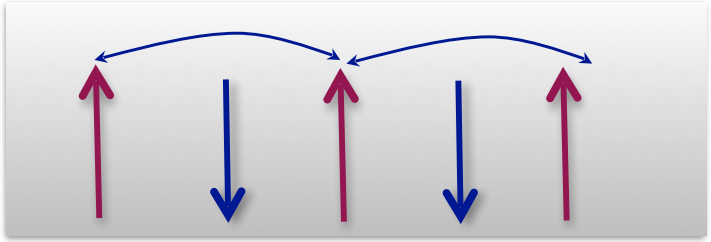
- Dynamics: spin-waves
- Dynamics: domain walls
- Current-induced dynamics
- Switching and STO

Outline

- **Dynamics: spin-waves**
- Dynamics: domain walls
- Current-induced dynamics
- What is beyond?

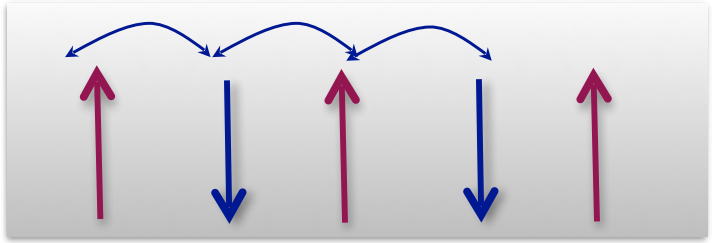
Hierarchy of interactions

H, T



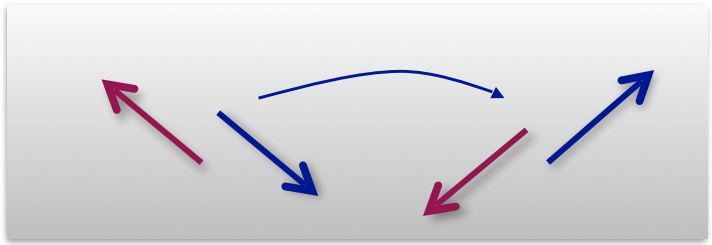
1000 T

$$J_{\text{intra}} \Leftrightarrow T_N$$



100 T

$$J_{\text{inter}} \propto 1/\chi_{\perp}$$



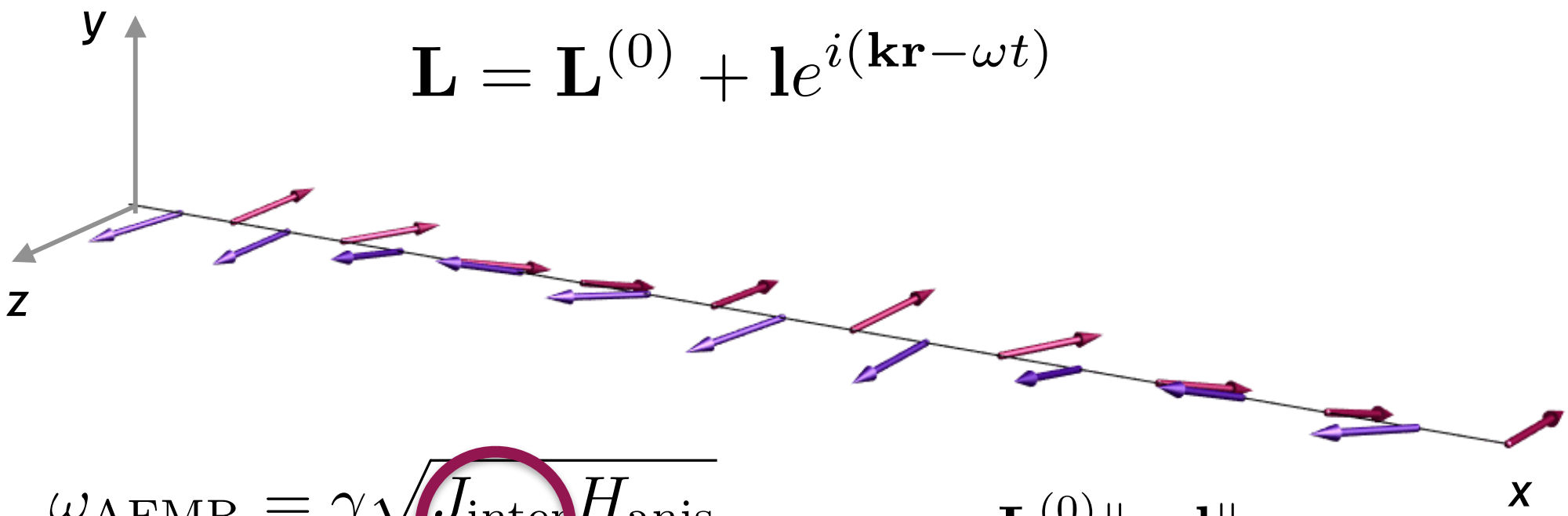
1 T

$$H_{\text{s-f}} \propto \sqrt{J_{\text{inter}} H_{\text{anis}}}$$

Spin waves as "classical" excitations

$$\mathbf{L} \times \ddot{\mathbf{L}} = \gamma^2 J_{\text{inter}} \mathbf{L} \times \mathbf{H}_L$$

$$\mathbf{L} = \mathbf{L}^{(0)} + \mathbf{l} e^{i(\mathbf{kr} - \omega t)}$$



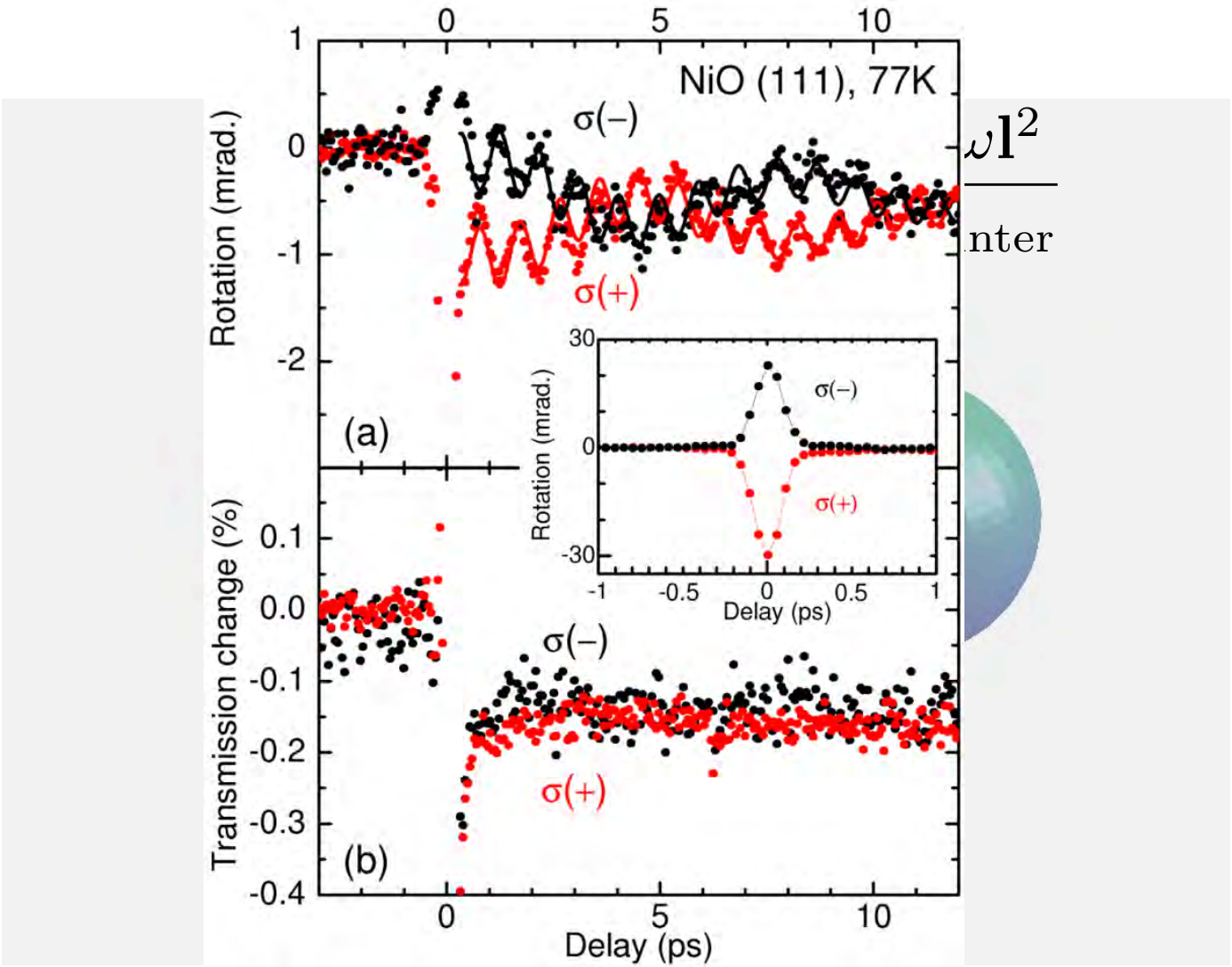
$$\omega_{\text{AFMR}} = \gamma \sqrt{J_{\text{inter}} H_{\text{anis}}}$$

$$\mathbf{L}^{(0)} \| z, \ \mathbf{l} \| y$$

$$\omega_{\text{sw}} = \sqrt{\omega_{\text{AFMR}}^2 + c^2 k^2}$$

$$\mathbf{M} \propto \frac{i\omega}{J_{\text{inter}}} \mathbf{L}^{(0)} \times \mathbf{l} e^{-i\omega t} \| x$$

Circular polarised modes



$$\frac{\omega l^2}{\text{nter}}$$

$$\omega_{\pm} = \pm\gamma H + \omega_{\mathrm{AFMR}}$$

Magnetoelastic gap

Large sample: "frozen" lattice

$$\tau_{\text{sound}} \propto \frac{d}{s}$$

d= 1 mm, $\tau_{\text{sound}} \propto 10^{-6}$ sec,
d= 30 nm, $\tau_{\text{sound}} \propto 10^{-11}$ sec

$\tau_{\text{mag}} \propto 10^{-12}$ sec

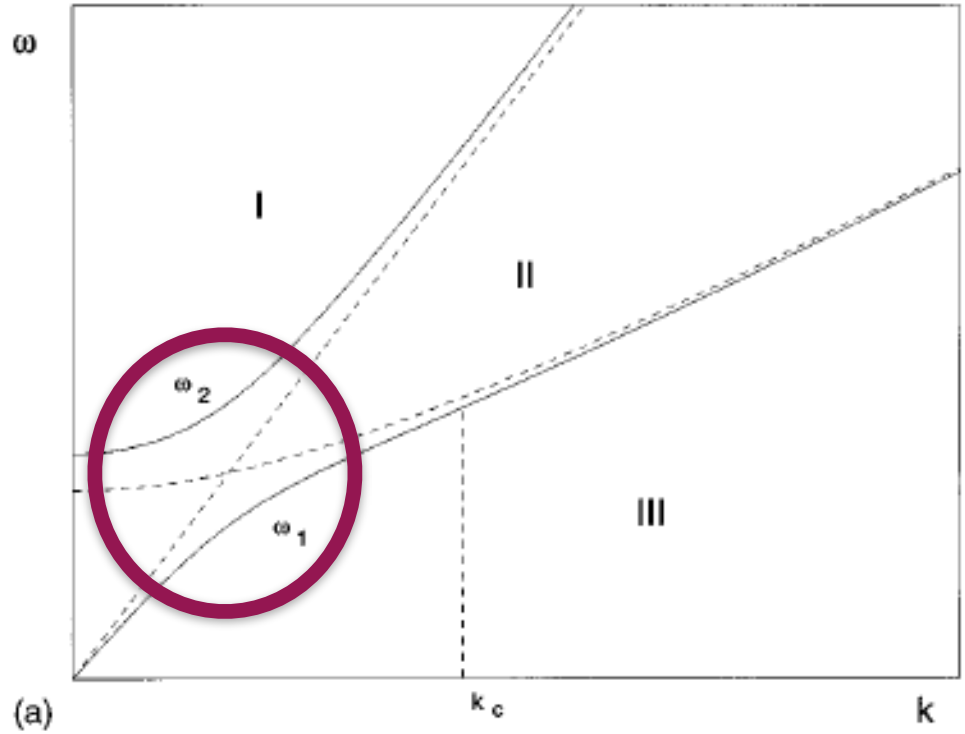
$$\hat{u}^{\text{spon}} = -\frac{\hat{\lambda}}{c'}\mathbf{L} \otimes \mathbf{L} = const$$

$$H_{\text{anis}} \rightarrow H_{\text{anis}} + 2M_s \lambda u^{\text{spon}}$$

$$\omega_{\text{AFMR}} = \gamma \sqrt{J_{\text{inter}} \left(H_{\text{anis}} + 2M_s \lambda u^{\text{spon}} \right)}$$

Magnetoelastic waves

$$\mathbf{L} = \mathbf{L}^{(0)} + \mathbf{l}e^{i(\mathbf{kr}-\omega t)}, \mathbf{u} = \mathbf{u}_j e^{i(\mathbf{kr}-\omega t)}$$



$$\omega_{1,2}^2(k) = \omega_{\mathrm{AFMR}}^2 + s^2k^2 \pm \sqrt{\left(\omega_{\mathrm{AFMR}}^2 - s^2k^2\right)^2 + 4\gamma J_{\mathrm{inter}}\lambda^2 k^2}$$

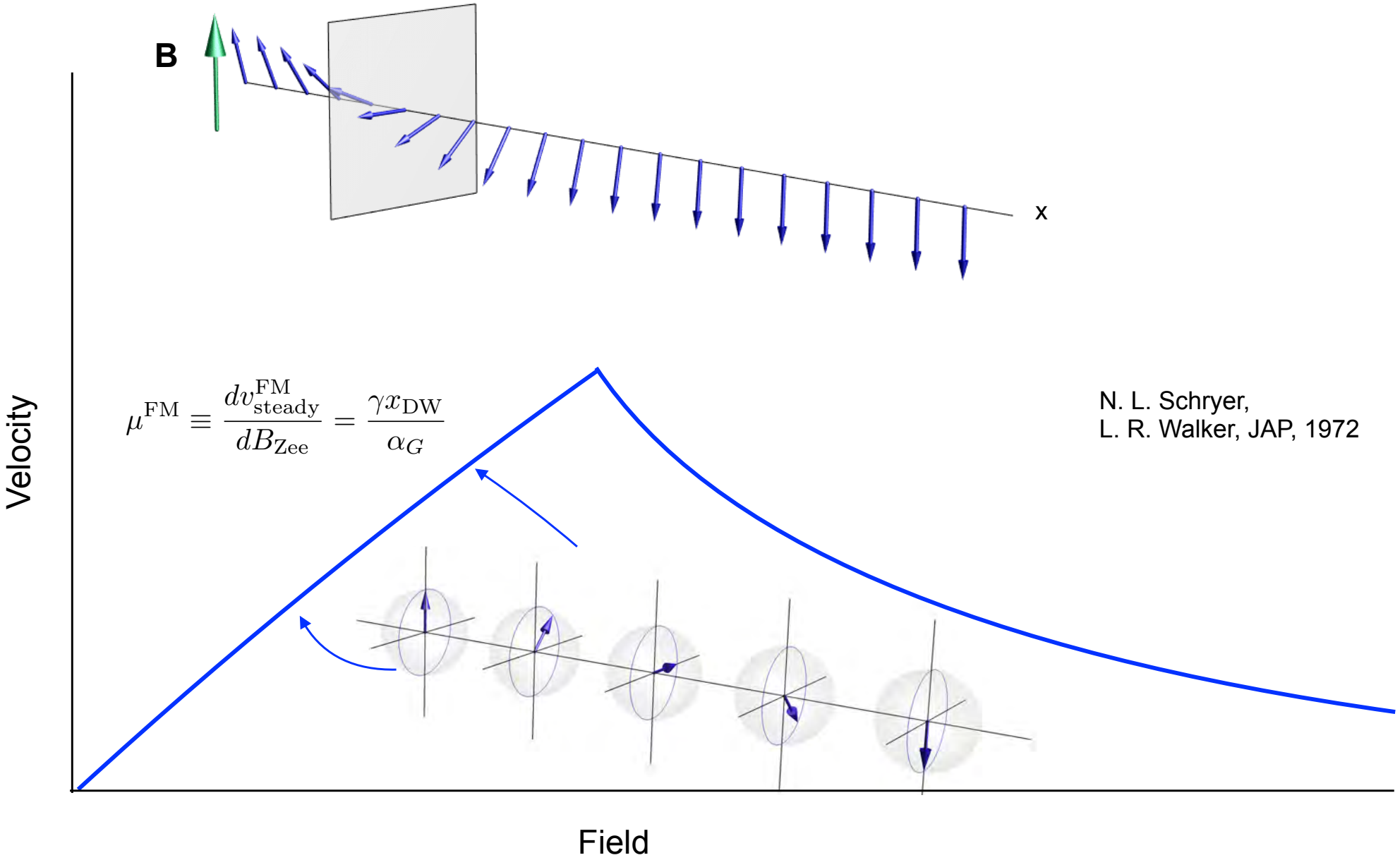
Take-home messages

- Spin-wave spectra: many modes
- Spin waves transfer magnetization
- Exchange enhancement
- Magneto elastic gap, size effects

Outline

- Dynamics: spin-waves
- **Dynamics: domain walls**
- Current-induced dynamics
- Switching and STO

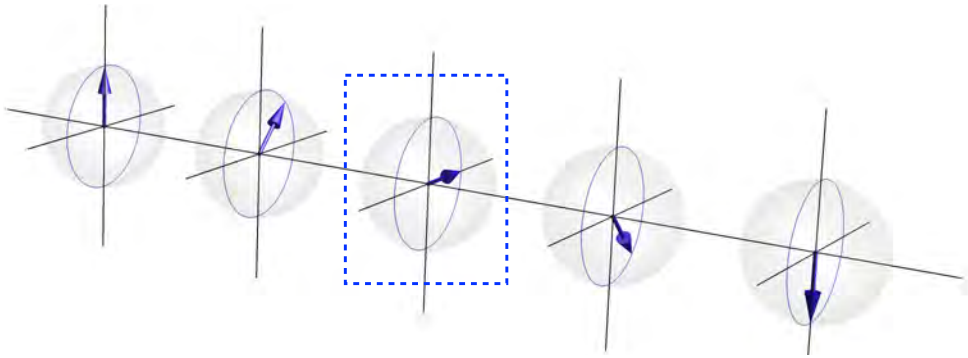
Below Walker breakdown in FM



$$\mu^{\mathrm{FM}} \equiv \frac{dv^{\mathrm{FM}}_{\mathrm{steady}}}{dB_{\mathrm{Zee}}} = \frac{\gamma x_{\mathrm{DW}}}{\alpha_G}$$

N. L. Schryer,
L. R. Walker, JAP, 1972

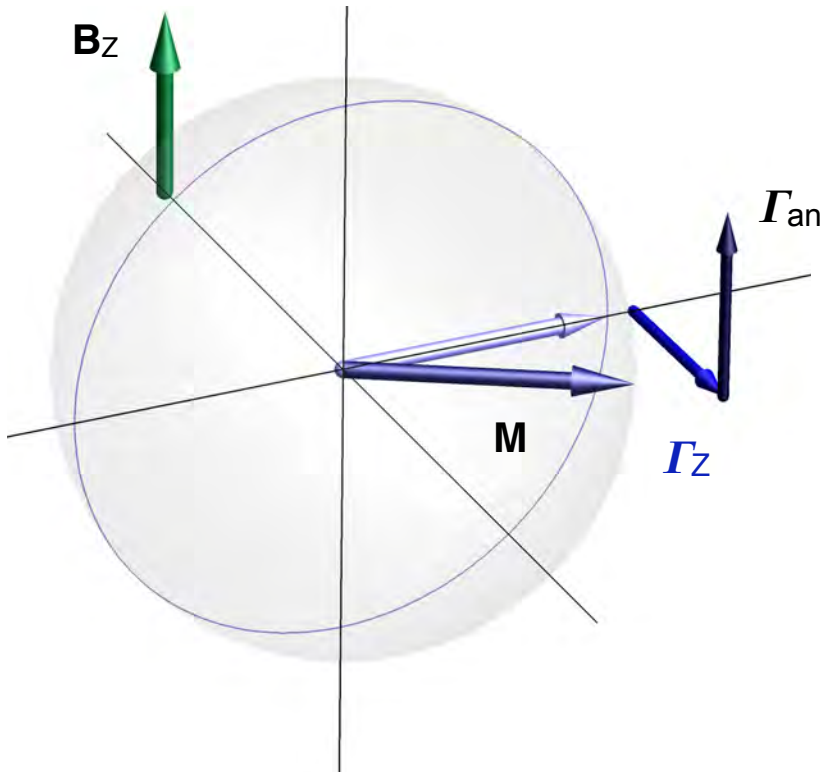
Anatomy of FM DW motion



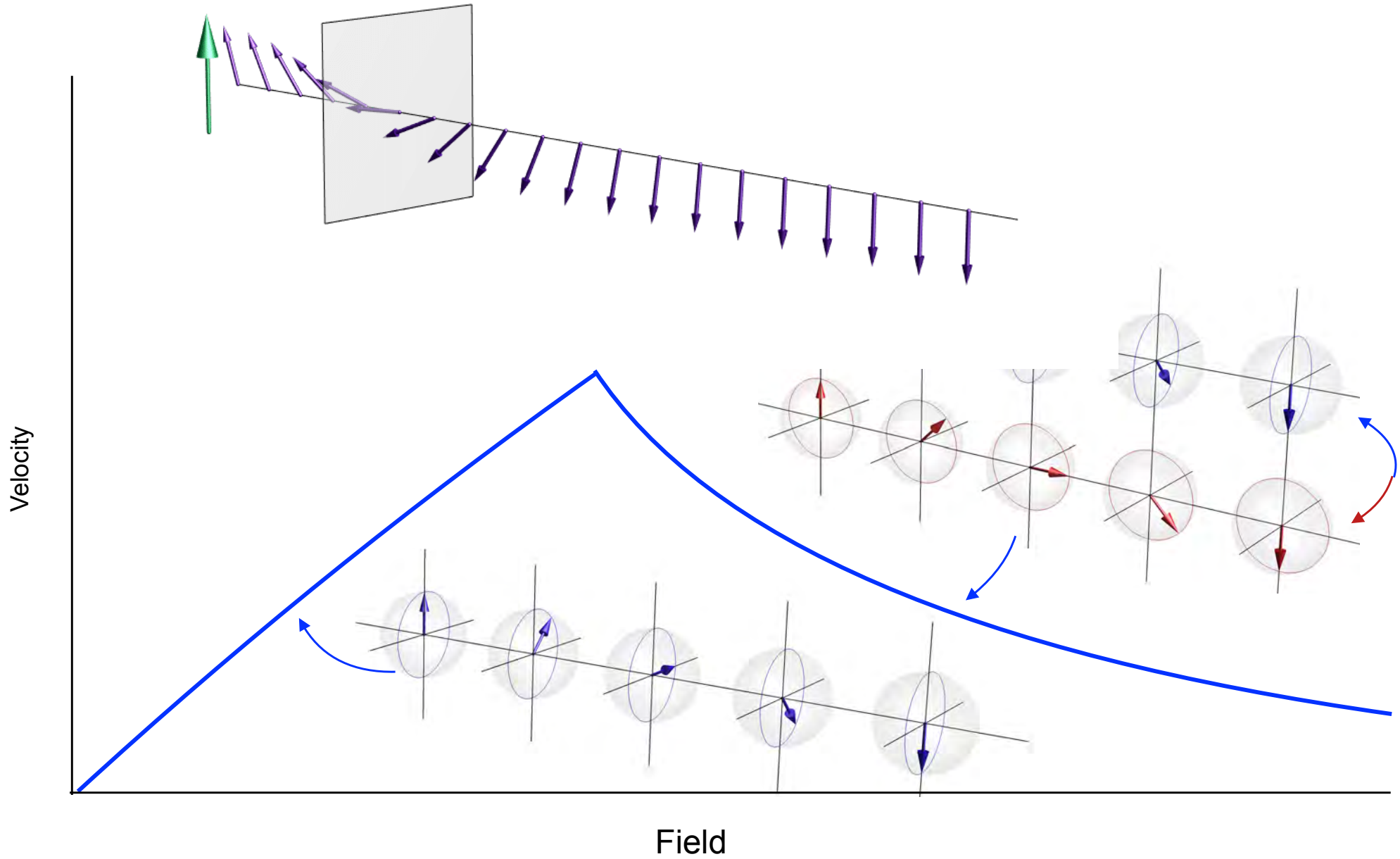
$$M_{\mathrm{FMDW}} = \frac{M_s S}{\gamma^2 H_{\mathrm{an}} x_{\mathrm{DW}}}$$

$$\tau_{\mathrm{FM}} = \frac{1}{\gamma \alpha_G H_{\mathrm{an}}}$$

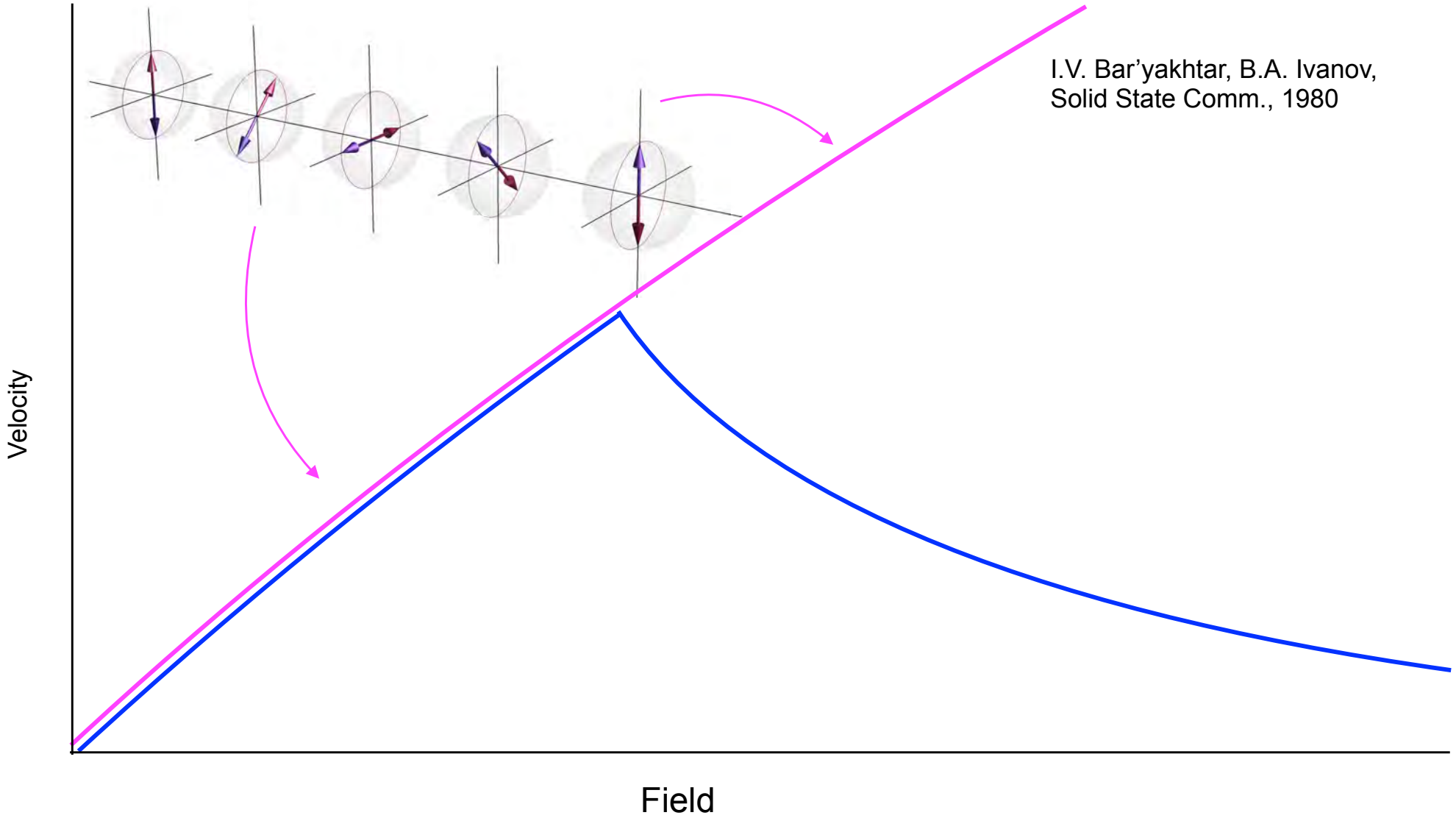
$$\boxed{\frac{M_{\mathrm{FMDW}}}{\tau_{\mathrm{FM}}} = \frac{\alpha_G M_s S}{\gamma x_{\mathrm{DW}}}}$$



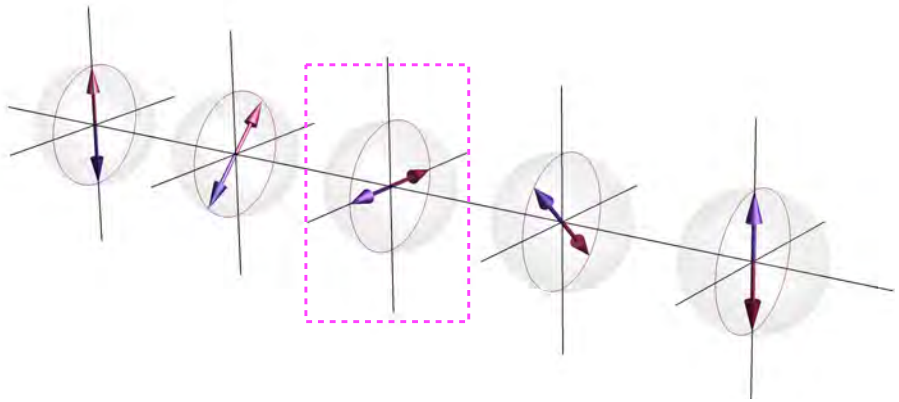
Above Walker breakdown in FM



No Walker breakdown in AFM



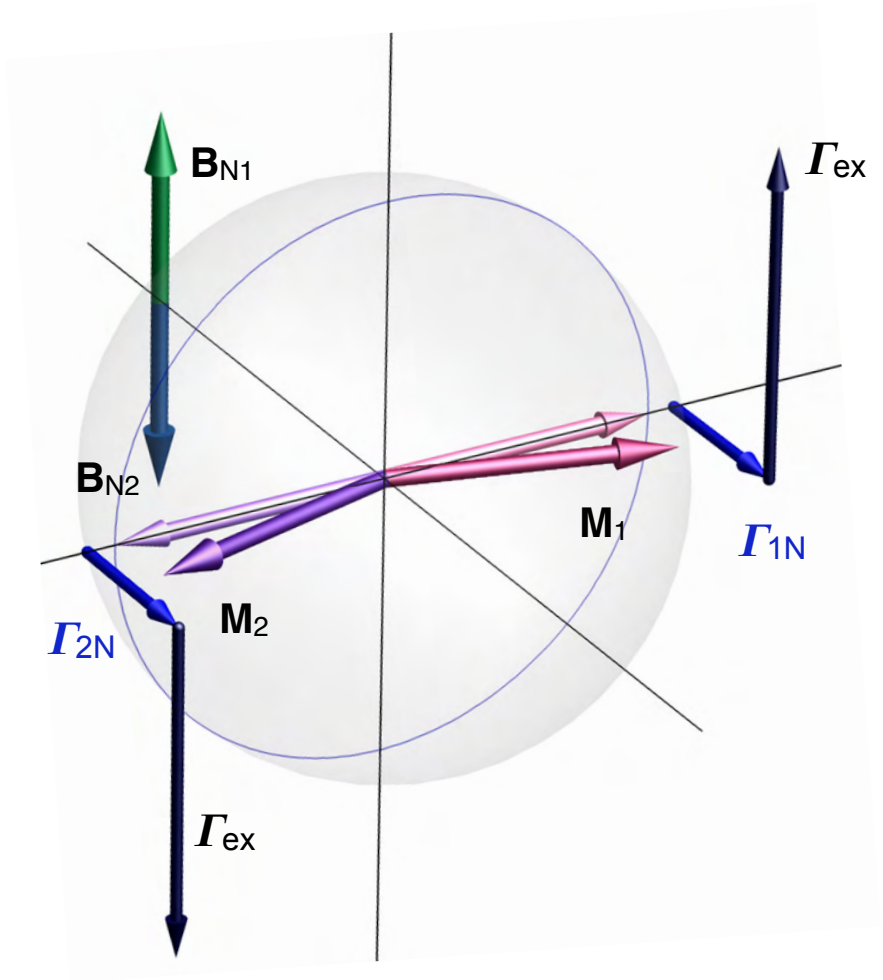
Anatomy of AF DW motion



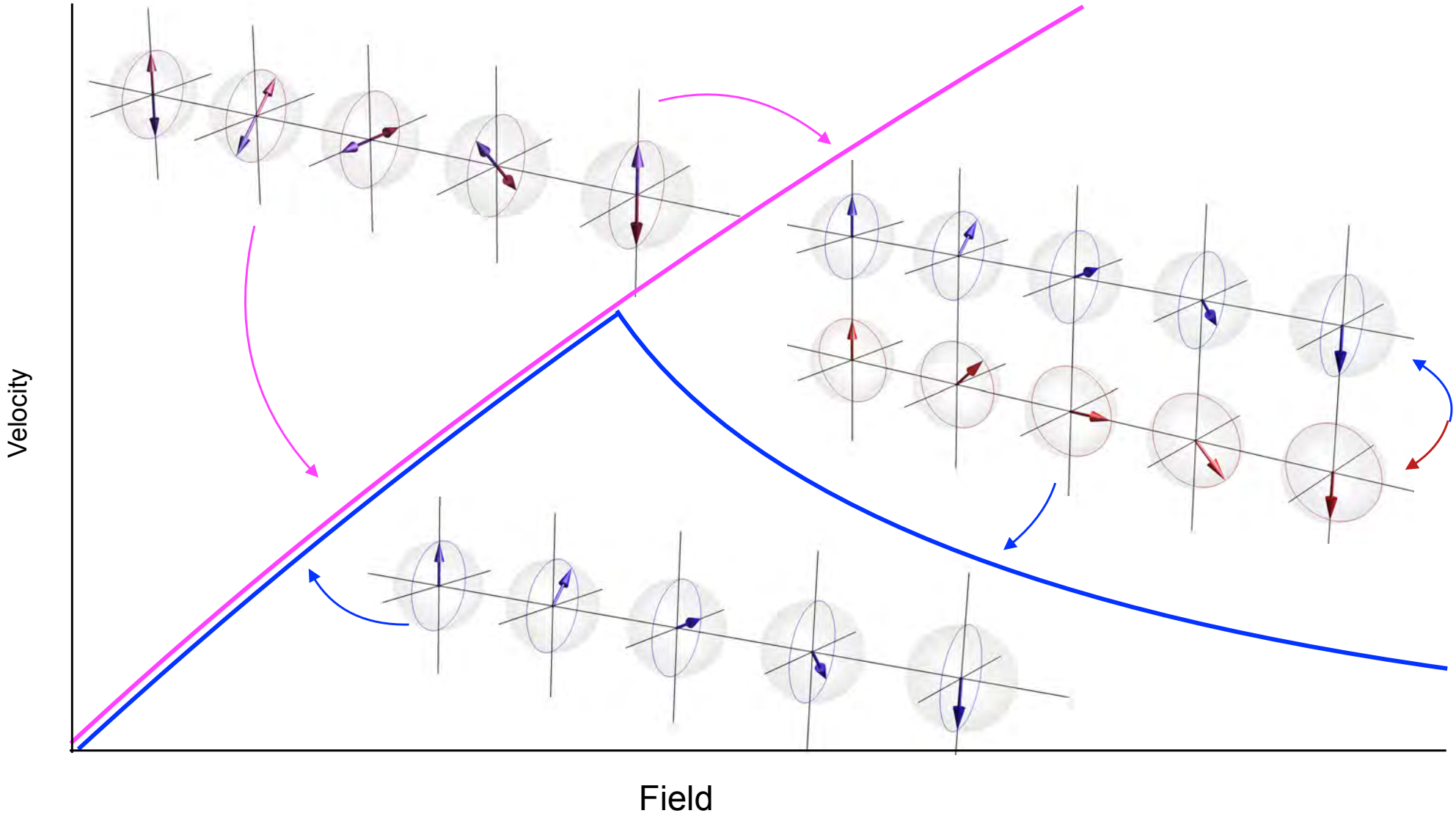
$$M_{\mathrm{AFDW}} = \frac{M_s S}{\gamma^2 H_{\mathrm{ex}} x_{\mathrm{DW}}}$$

$$\tau_{\mathrm{AF}} = \frac{1}{\gamma \alpha_G H_{\mathrm{ex}}}$$

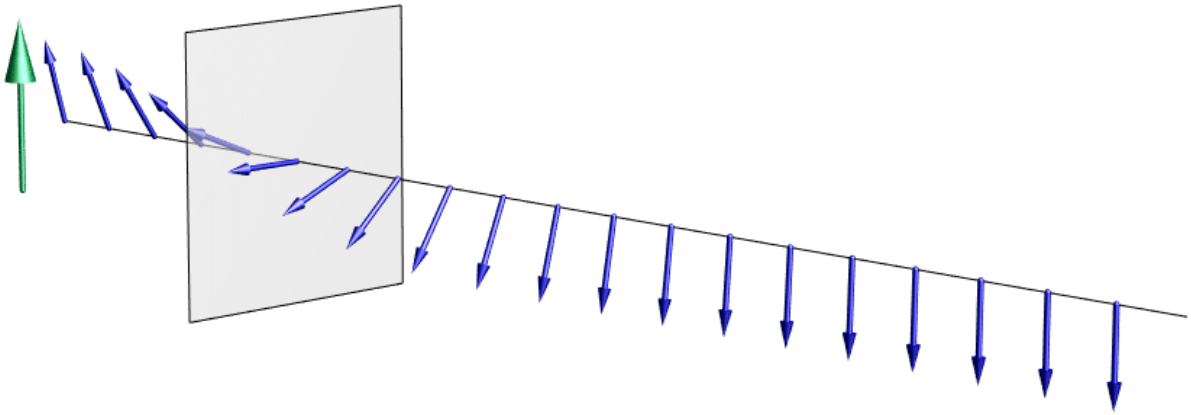
$$\frac{M_{\mathrm{AFDW}}}{\tau_{\mathrm{AF}}} = \frac{\alpha_G M_s S}{\gamma x_{\mathrm{DW}}}$$



Dynamics of DW in AF and FM



DW motion: “relativistic” dynamics



$$c^2 \frac{\partial^2 \theta}{\partial x^2} - \ddot{\theta} - \gamma^2 H_{\mathrm{ex}} H_{\mathrm{an}} \sin\theta \cos\theta = \alpha_G \gamma H_{\mathrm{ex}} \dot{\theta} + \gamma^2 H_{\mathrm{ex}} B_{\mathrm{Neel}} \sin\theta$$

$$\boxed{\frac{dP_x}{dt} = -\alpha_G \gamma H_{\mathrm{ex}} P_x + F_x}$$

$$P_x \propto -\int \frac{\partial \theta}{\partial x} \dot{\theta} dx$$

$$P_x \propto \frac{v}{\sqrt{1 - v^2/c^2}}$$

Take-home messages

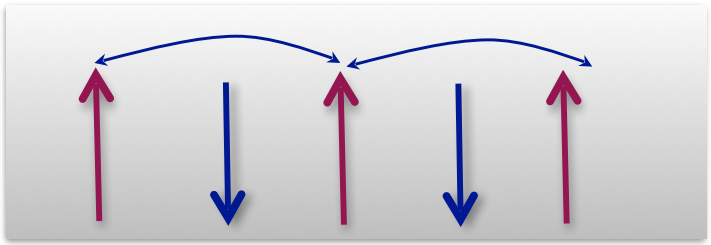
- No Walker breakdown
- Small mass and exchange enhancement
- Relativistic dynamics

Ulrich Rössler: today and tomorrow session

Outline

- Dynamics: spin-waves
- Dynamics: domain walls
- **Current-induced dynamics**
- Switching and STO

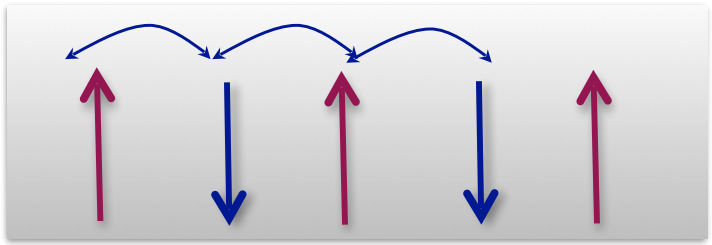
Hierarchy of interactions



H, T

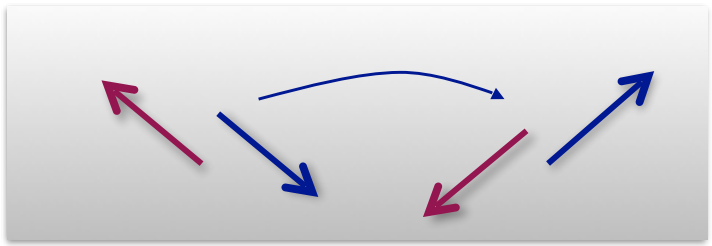
1000 T

$$J_{\text{intra}} \Leftrightarrow T_N$$



100 T

$$J_{\text{inter}} \propto 1/\chi_{\perp}$$



1 T

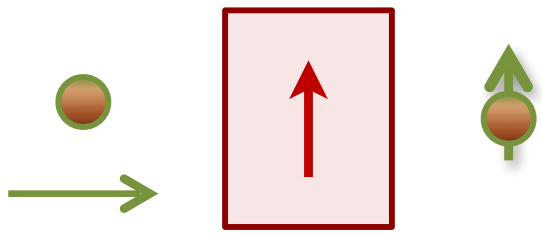
$$H_{\text{s-f}} \propto \sqrt{J_{\text{inter}} H_{\text{anis}}}$$

Spintronic: sd-exchange in FM

$$\hat{H}_{\rm sd} = -J_{\rm sd}\sum_j \hat{\mathbf{s}}_j \cdot \mathbf{S}_j \Rightarrow -J_{\rm sd}\delta\mathbf{m}\cdot\mathbf{M}$$

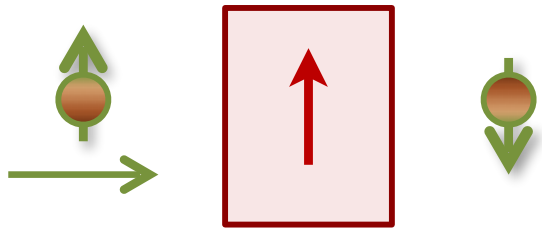
Polarization:

$$\delta\mathbf{m} \propto \langle\hat{\mathbf{s}}_j\rangle \| \mathbf{M}$$



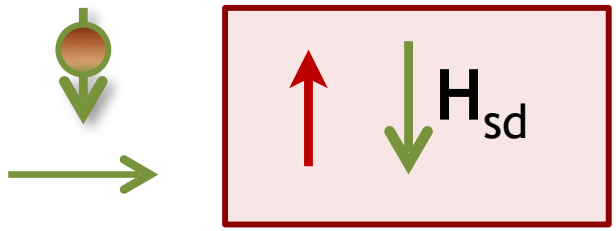
Scattering:

$$\hat{\Pi}_{\rm in} - \hat{\Pi}_{\rm out} \propto f(J_{\rm sd})\delta\mathbf{m}\otimes\mathbf{j}_e$$



Effective field:

$$\mathbf{H}_{sd} = J_{sd}\delta\mathbf{m}$$

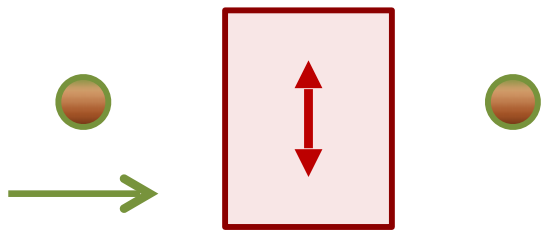


AF: sd-exchange?

$$\hat{H}_{\rm sd} = -J_{\rm sd}\sum_j \hat{\mathbf{s}}_j \cdot \mathbf{S}_j \Rightarrow -J_{\rm sd}\delta\mathbf{m}\cdot\mathbf{M}_{\rm AF}$$

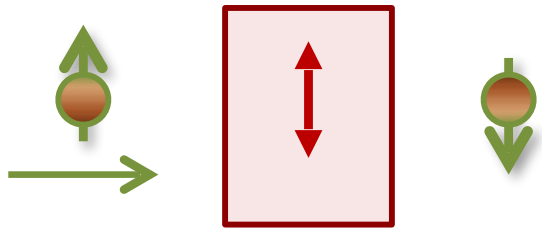
Polarization:

$$\delta\mathbf{m} \propto \langle\hat{\mathbf{s}}_j\rangle \| \mathbf{M}_{\rm AF} \to 0$$



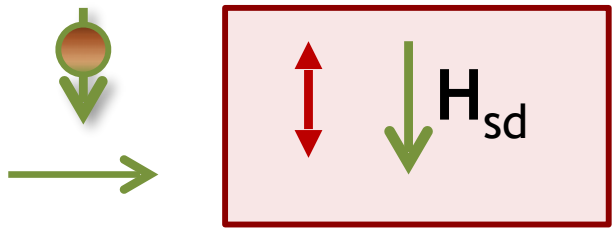
Scattering:

$$\hat{\Pi}_{\rm in} - \hat{\Pi}_{\rm out} \propto f(J_{\rm sd})\delta\mathbf{m}\otimes\mathbf{j}_e$$



Effective field:

$$\mathbf{H}_{sd} = J_{sd}\delta\mathbf{m}$$



Equations of motion

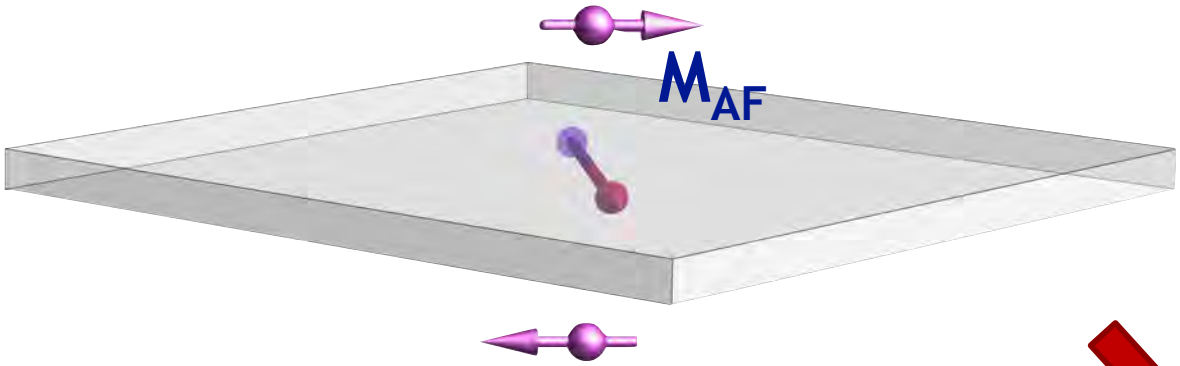
Dynamic magnetisation

$$\mathbf{M} = \frac{1}{H_{\mathrm{ex}}}\mathbf{L} \times \dot{\mathbf{L}} + \frac{1}{H_{\mathrm{ex}}}\mathbf{L} \times \mathbf{H} \times \mathbf{L}$$

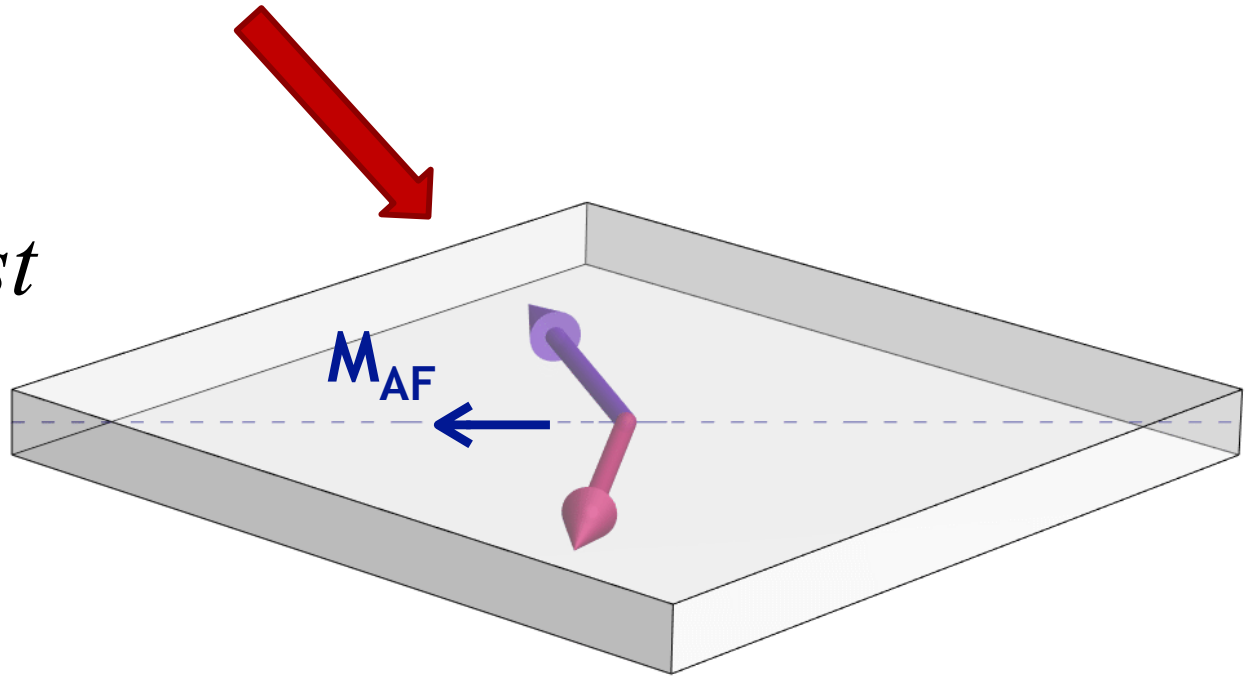
Magnetisation balance

$$\dot{\mathbf{M}} \equiv \frac{1}{H_{\mathrm{ex}}}\mathbf{L} \times \ddot{\mathbf{L}} = \gamma \mathbf{L} \times \mathbf{H}_{\mathrm{L}}$$

Magnetization⇒ rotation



$$\delta\mathbf{m} + \mathbf{M}_{\mathrm{AF}} = const$$



$$\delta\mathbf{m} \to \mathbf{M}_{\mathrm{AF}} \propto \mathbf{L} \times \dot{\mathbf{L}} \to \mathbf{L}(t)$$

Spin transfer in AF and spin balance

$$\frac{d\mathbf{M}_{\mathrm{AF}}}{dt} = \left(\hat{\Pi}_{in} - \hat{\Pi}_{out}\right)\mathbf{N} + \mathrm{sink}$$

$$\ddot{\mathbf{L}} - \gamma^2 J_{\mathrm{inter}}\mathbf{H}_L = \left(\beta\frac{dI}{dt} + J_{\mathrm{inter}} I\sigma\right)\mathbf{s}\times\mathbf{L} - \gamma\alpha_G J_{\mathrm{inter}}\mathbf{L}\times\dot{\mathbf{L}}$$

H.Gomonay, V.Loktev,2008

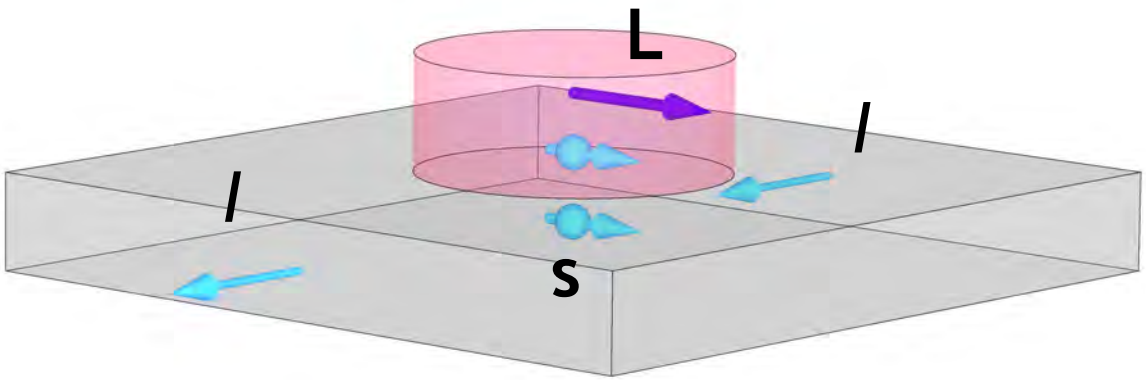
$$m\ddot{x} + 2\gamma\dot{x} + \frac{dU}{dx} = F_{diss}$$

Outline

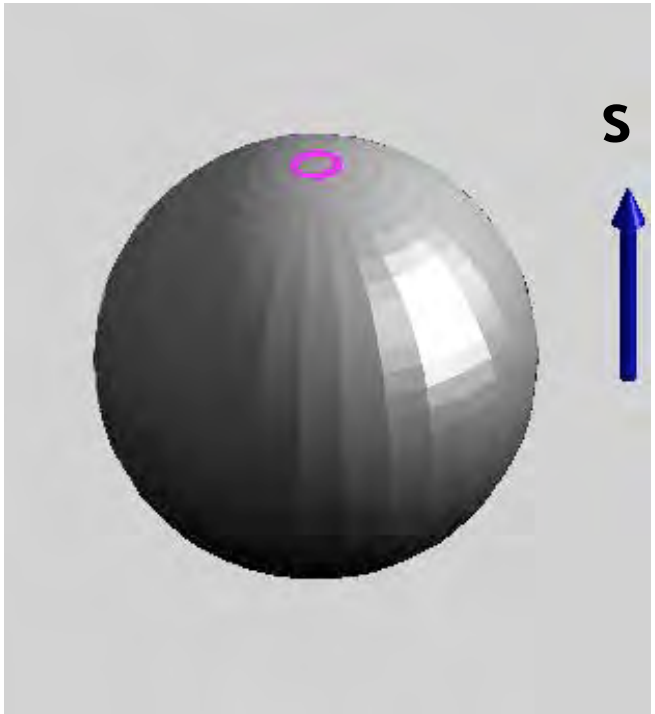
- Dynamics: spin-waves
- Dynamics: domain walls
- Current-induced dynamics
- **Switching and STO**

Dynamics in the dc spin current

$$\ddot{\mathbf{L}} - \gamma^2 J_{\text{inter}} \mathbf{H}_L = J_{\text{inter}} I \sigma \mathbf{s} \times \mathbf{L} - \gamma \alpha_G J_{\text{inter}} \mathbf{L} \times \dot{\mathbf{L}}$$

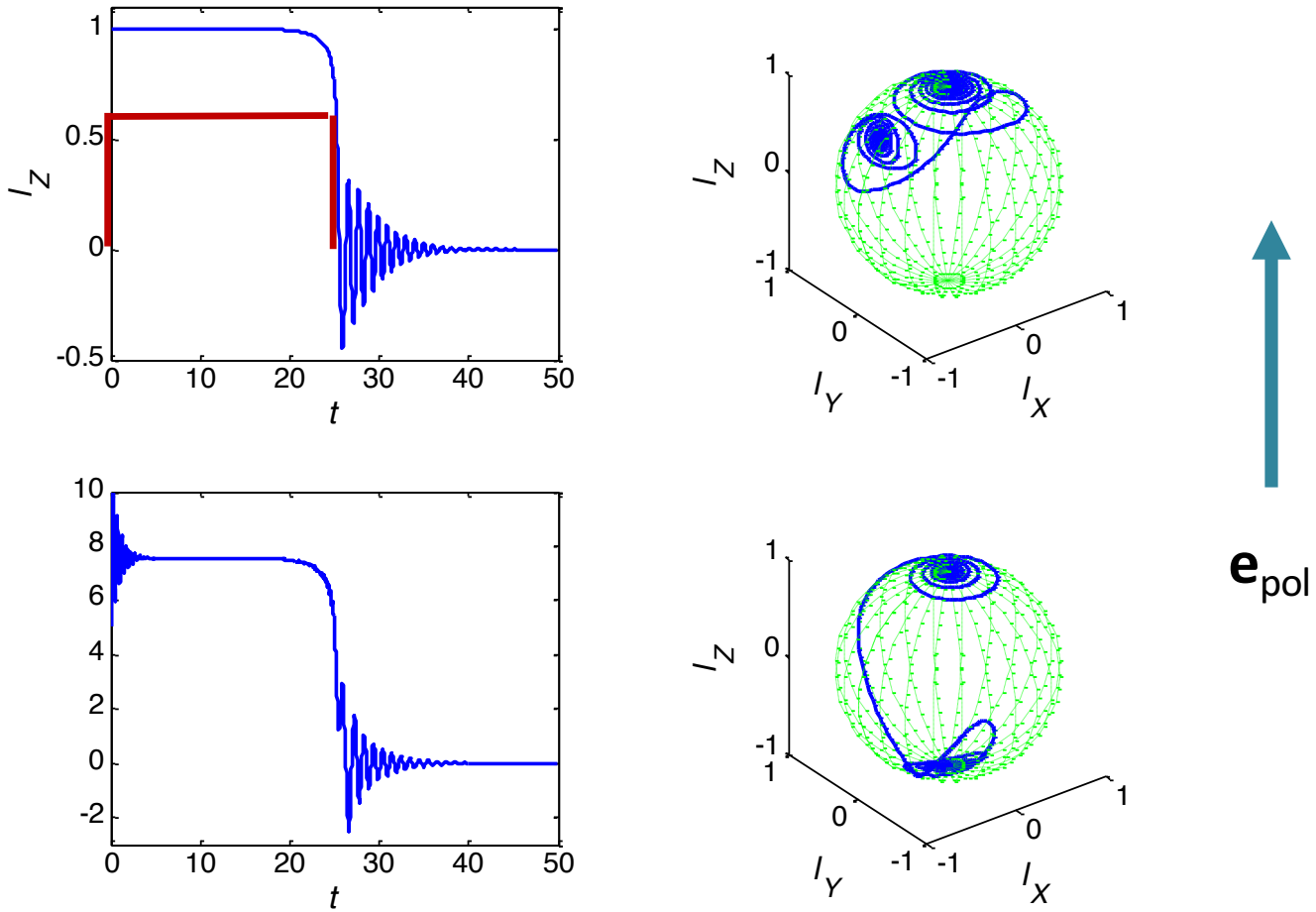


$$I_{\text{cr}} = \frac{\gamma_{\text{AF}} \omega_{\text{AFMR}}}{\gamma \sigma J_{\text{inter}}} = \frac{\alpha}{\sigma} \gamma H_{\text{an}}$$



Critical current

$$I_{cr} = \frac{\gamma_{AF}\omega_{AFMR}}{\gamma\sigma J_{inter}} = \frac{\alpha}{\sigma}\gamma H_{an}$$



Critical current

FM

$$H_{STT}^{FM} = \alpha_G H_{an}$$

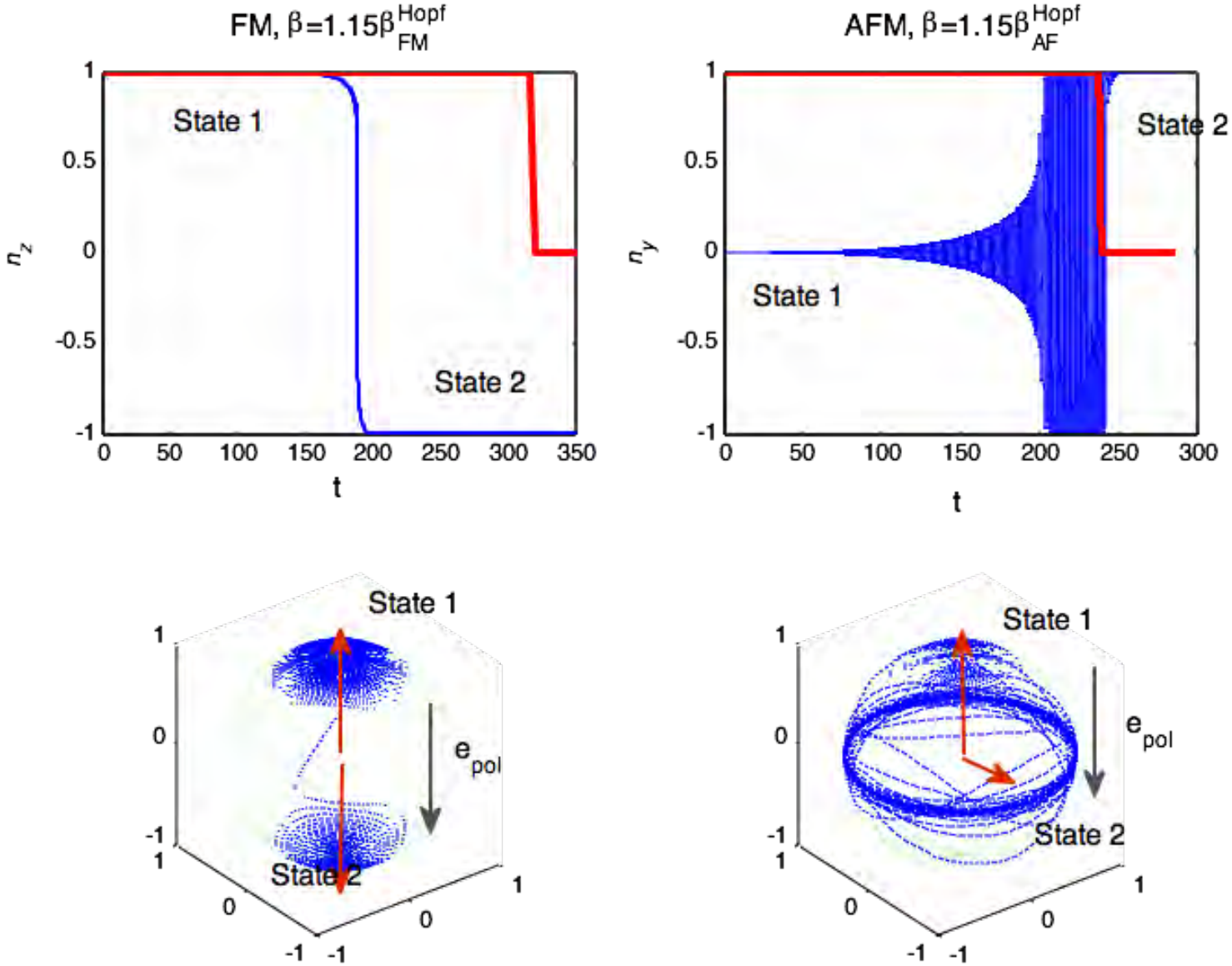
AFM, cubic

$$H_{\mathrm{STT}}^{\mathrm{AF}} = \alpha_G \sqrt{H_{\mathrm{an}} J_{\mathrm{inter}}}$$

AFM, uniaxial

$$H_{\mathrm{STT}}^{\mathrm{AF}} = \sqrt{\alpha_G^2 H_{\mathrm{an}} J_{\mathrm{inter}} + \left(H_{\mathrm{an}}^{\parallel} - H_{\mathrm{an}}^{\perp}\right)^2}$$

FM vs AFM, possible dynamics near the critical current



Take-home messages

- Spin transfer = magnetisation = dynamics
- Exchange enhancement of spin
- Different dynamics of FM and AF

What's beyond?

- Quantum fluctuations, quantum excitations
- Large fields ~ intersublattice exchange
- Ultrafast dynamics, magnetooptics
- Small AF particles (mesoscales)

Welcome to AF spintronics!