# Theory of spin transport in disordered antiferromagnets

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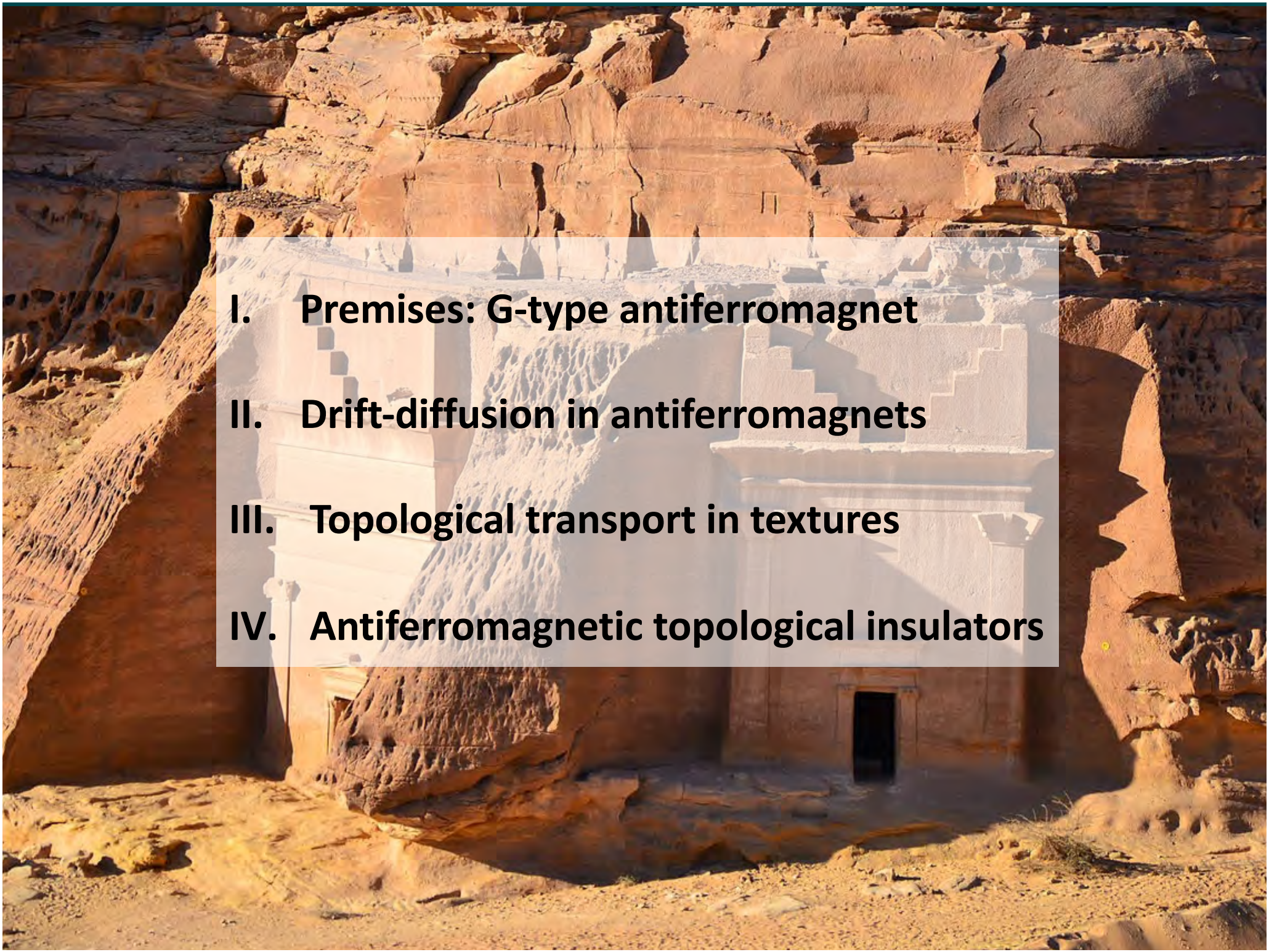
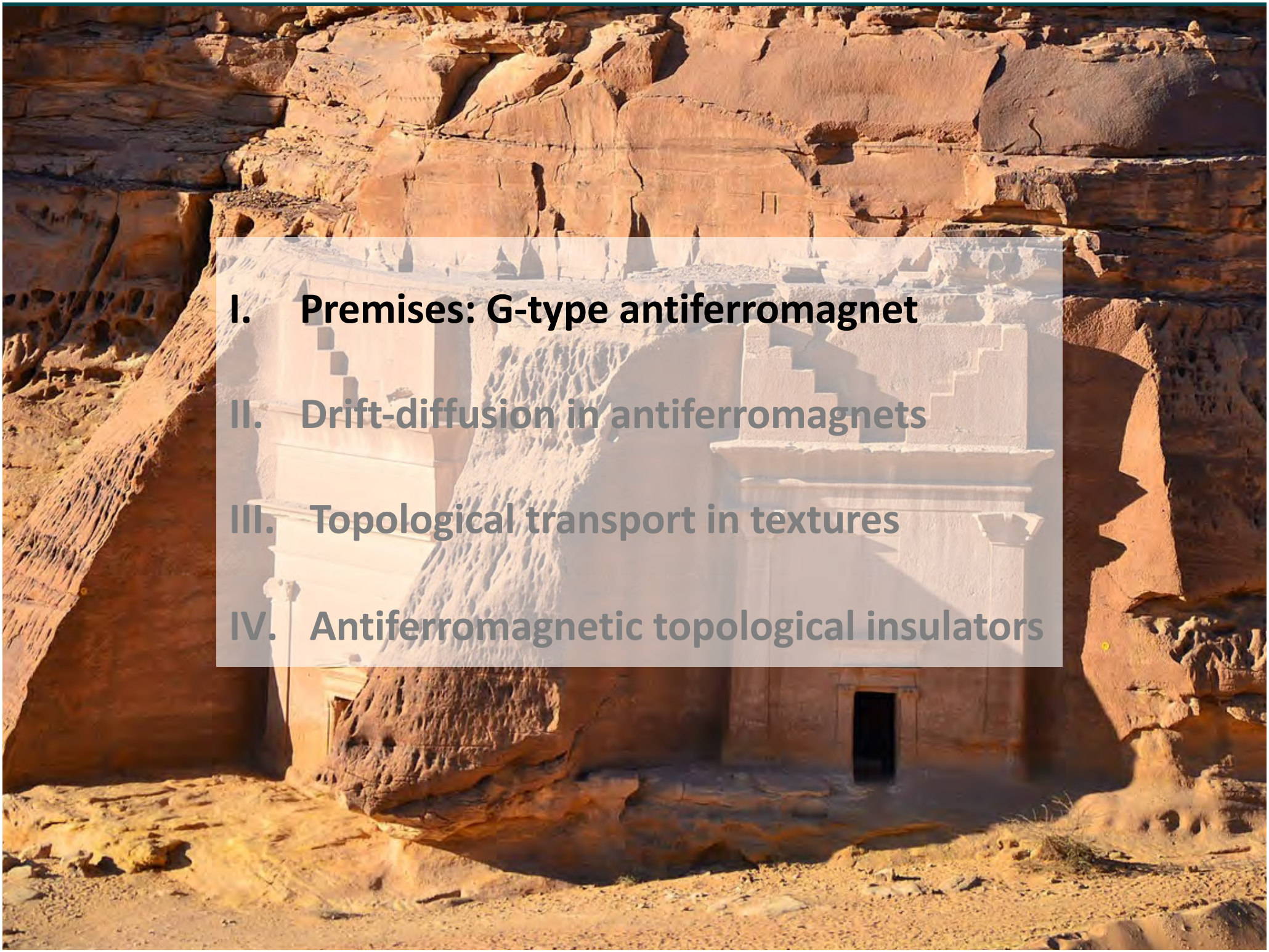


I. Premises: G-type antiferromagnet

II. Drift-diffusion in antiferromagnets

III. Topological transport in textures

IV. Antiferromagnetic topological insulators



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# I. G-type antiferromagnet

## Current-driven dynamics

Gomonay and Letkov, Low Temperature Physics 2014

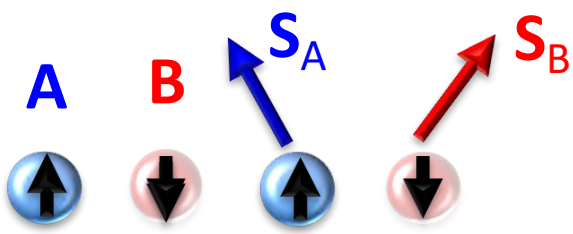
Let us consider a bipartite collinear antiferromagnet

$$\partial_t \vec{m}_A = -\gamma \vec{m}_A \times \vec{H} + (\gamma/2) H_E \vec{m}_A \times \vec{m}_B + \alpha \vec{m}_A \times \partial_t \vec{m}_A + \tau_\perp \vec{m}_A \times \vec{p} + \tau_\parallel \vec{m}_A \times (\vec{p} \times \vec{m}_A)$$

$$\partial_t \vec{m}_B = -\gamma \vec{m}_B \times \vec{H} - (\gamma/2) H_E \vec{m}_A \times \vec{m}_B + \alpha \vec{m}_B \times \partial_t \vec{m}_B - \tau_\perp \vec{m}_B \times \vec{p} + \tau_\parallel \vec{m}_B \times (\vec{p} \times \vec{m}_B)$$

Magnetic Field | Exchange | Staggered field-like | Damping-like

$$\vec{l} \times \partial_t^2 \vec{l} = \gamma \vec{l} \times (\partial_t \vec{H} \times \vec{l}) + \gamma^2 \left[\vec{l} \times (\vec{H} \times \vec{l})\right] \times \vec{H} - \alpha\gamma H_E \vec{l} \times \partial_t \vec{l} - 2\gamma (\vec{H} \cdot \vec{l}) \partial_t \vec{l}$$
$$- \gamma H_E \tau_\perp \vec{l} \times \vec{p} - \gamma H_E \tau_\parallel \vec{l} \times (\vec{p} \times \vec{l})$$

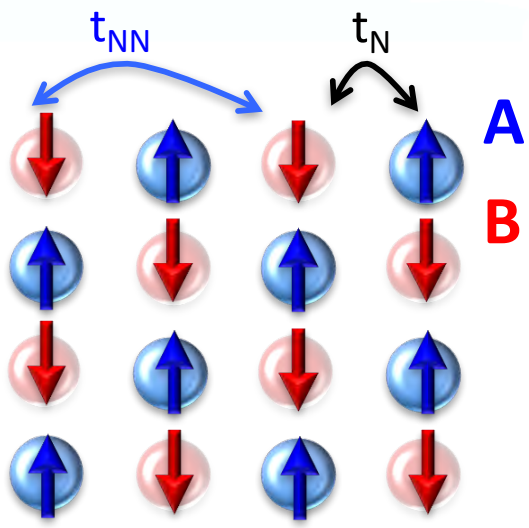



$S_A$+$S_B$: **uniform** spin accumulation->**ac Torque**

$S_A$-$S_B$: **staggered** spin accumulation->**dc Torque**

# I. G-type antiferromagnet
## Band structure and eigenstates

Let's take the simplest-minded antiferromagnet




$$\hat{H} = \begin{pmatrix} \Delta\hat{\sigma}\cdot\vec{n} + \gamma_{NN} & \gamma_N \\ \gamma_N & -\Delta\hat{\sigma}\cdot\vec{n} + \gamma_{NN} \end{pmatrix}$$

A A-B B

$$\gamma_{NN} = -2t_{NN}\sum_i \cos 2k_x a, \quad \gamma_N = -2t_N \sum_i \cos k_x a$$

Eigenenergy $\varepsilon_k^{\eta} = \gamma_{NN} + \eta\sqrt{\Delta^2 + \gamma_N^2}$

Eigenstate $\psi_k^{s,\eta} = \frac{1}{\sqrt{2}}\left(\sqrt{1 + s\eta\frac{\Delta}{\sqrt{\Delta^2+\gamma_N^2}}}\,|A\rangle + \eta\sqrt{1 - s\eta\frac{\Delta}{\sqrt{\Delta^2+\gamma_N^2}}}\,|B\rangle\right)\otimes|s\rangle$

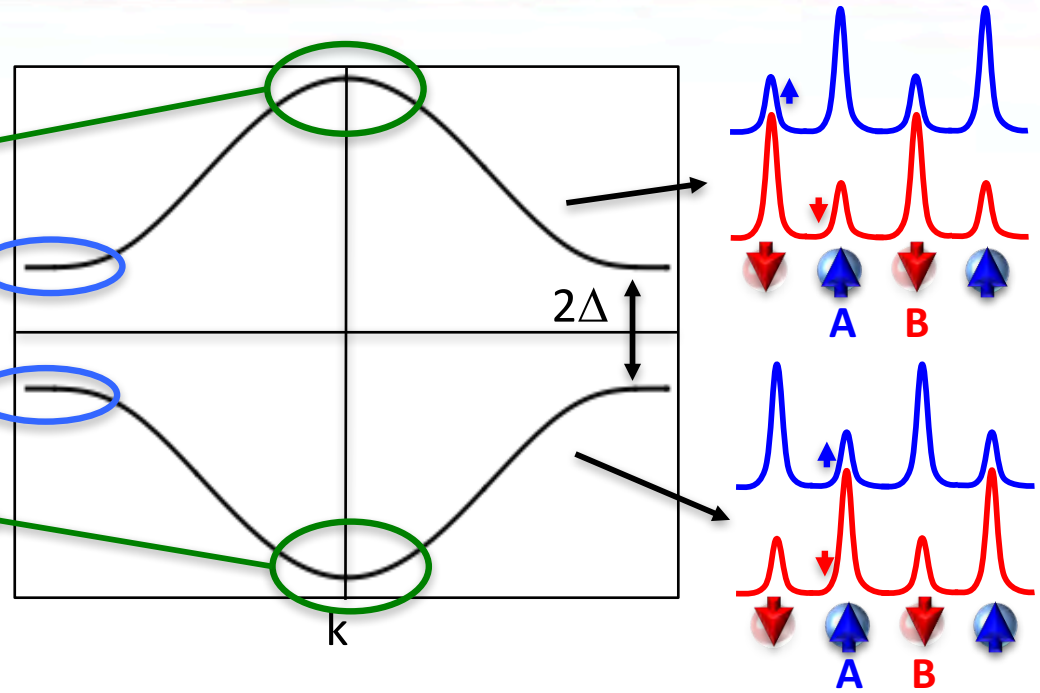
See also Cheng & Niu, PRB 86, 245118 (2012)

# I. G-type antiferromagnet
## Band structure and eigenstates

$$P = \frac{\Delta}{\sqrt{\Delta^2 + (6t_N)^2}}$$

$$P = 1$$

$$P = \frac{\Delta}{\sqrt{\Delta^2 + (6t_N)^2}}$$




$$\varepsilon_k^{\eta} = \gamma_{NN} + \eta\sqrt{\Delta^2 + \gamma_N^2}$$

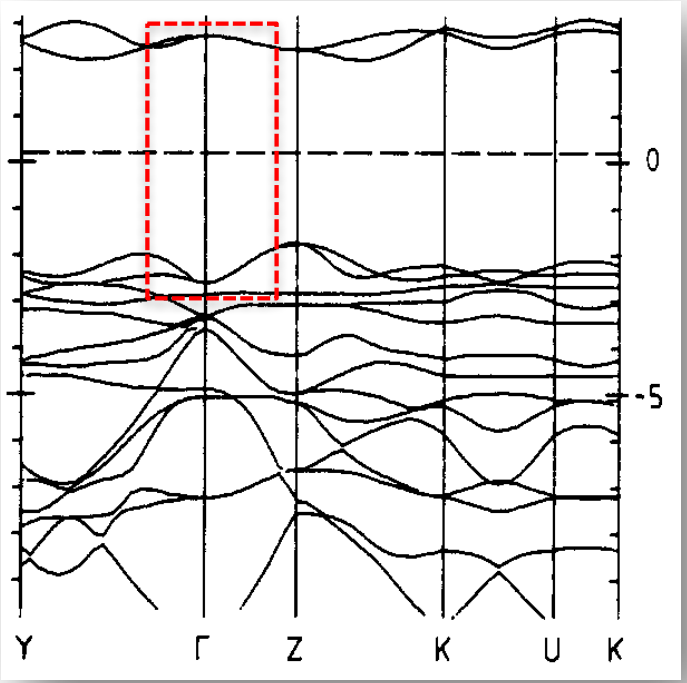
$$\psi_k^{s,\eta} = \frac{1}{\sqrt{2}}\left(\sqrt{1+s\eta P}\,|A\rangle + \eta\sqrt{1-s\eta P}\,|B\rangle\right)\otimes|s\rangle$$

$$P = \frac{\Delta}{\sqrt{\Delta^2 + \gamma_N^2}}$$

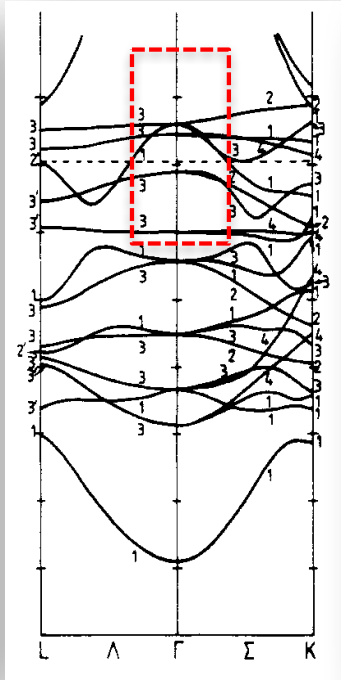
Polarization of the **local density of states**, but no spin current out of an antiferromagnet!!

# I. G-type antiferromagnet
## Band structure and eigenstates

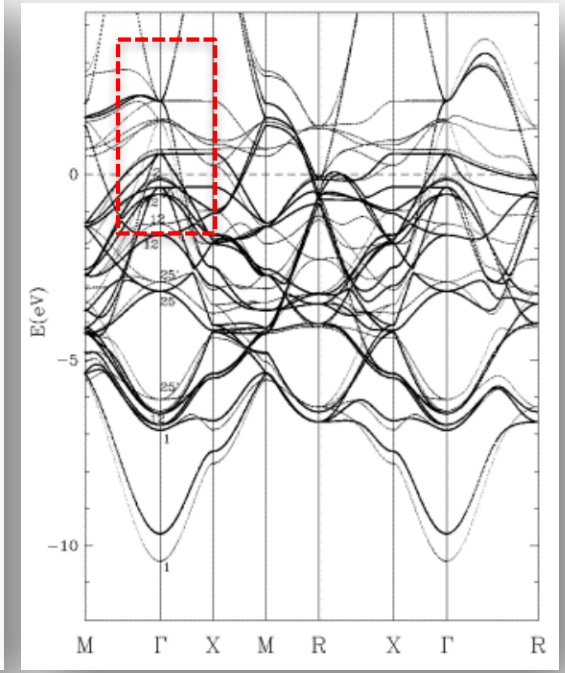
NiO (Mott insulator)




FeRh (metal)

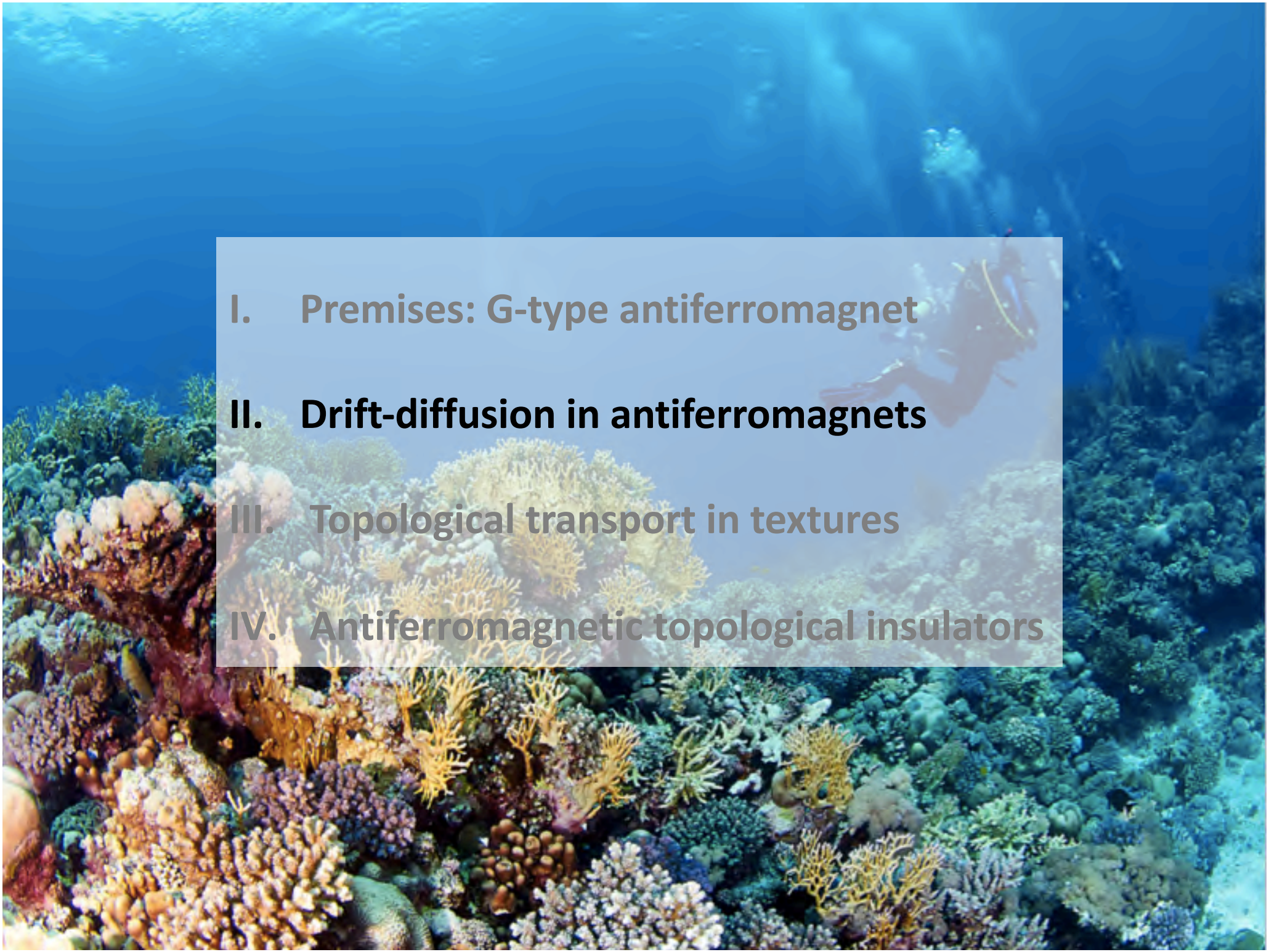



$MnPt_3$ (metal)





Anisimov, J. Phys.: Condens. Matter 2, 3973 (1990)
Koenig, J. Phys. F: Met. Phys., 12, 1123 (1982)
S Gallego, M.C Muñoz, Surface Science, 423, 324 (1999)



I. Premises: G-type antiferromagnet

II. **Drift-diffusion in antiferromagnets**

III. Topological transport in textures

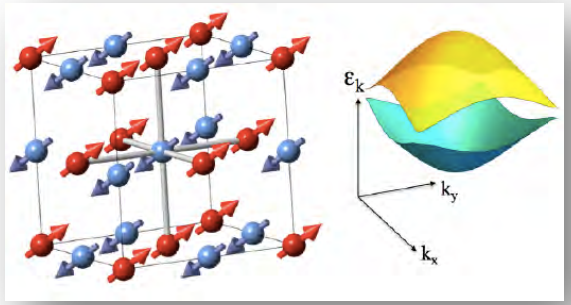
IV. Antiferromagnetic topological insulators

# II. Drift-diffusion spin transport

## From quantum kinetics to drift-diffusion

**Drift-diffusion theory in metals**

| | | |
|---|---|---|
| Ferromagnets | $\partial_t \vec{S} + \frac{1}{\tau_\Delta}\vec{S}\times\vec{m} + \frac{1}{\tau_\varphi}\vec{m}\times\left(\vec{S}\times\vec{m}\right) + \frac{1}{\tau_{sf}}\vec{S} = -\partial_i \vec{J}_i^s$ <br> $\vec{J}_i^s = -D\partial_i n\vec{m} - D\partial_i \vec{S}$ | Valet & Fert, PRB 48, 7099 (1993) <br> Zhang, Levy, Fert, PRL 88, 236601 (2002) <br> Petitjean, Luc, Waintal, PRL 109, 117204 (2012) |
| Normal metals | $\partial_t \vec{S} + \frac{1}{\tau_{sf}}\vec{S} = -\partial_i \vec{J}_i^s$ <br> $\vec{J}_i^s = -D\partial_i \vec{S} - \theta_{sh} D\left(\vec{e}_i \times \vec{\nabla}\right)n + \theta_{sw} D\vec{\nabla} S_i$ | Shchelushkin & Brataas, PRB 72, 073110 (2005) <br> Shen, Raimondi, Vignale, PRB 90, 245302 (2014) |
| Antiferromagnets | ?? | **Manchon, arXiv:1608.00140v1** |




**Objective:** derive the drift-diffusion equation in a G-type AF

$$\hat{H} = \gamma_N \hat{\tau}_x \otimes \hat{1} + \Delta \hat{\tau}_z \otimes \hat{\sigma}\cdot\vec{n} + \hat{V}$$

Quantum kinetic equation

$$i\hbar\partial_t \tilde{G}_k^< + [\tilde{G}_k^<, \tilde{H}_0] + i\{\tilde{G}_k^<, \tilde{\Sigma}\} + \frac{i}{2}\{\hat{v}_i, \partial_i \tilde{G}_k^<\} = \tilde{\Sigma}^< \tilde{G}_k^A - \tilde{G}_k^R \tilde{\Sigma}^<,$$

# II. Drift-diffusion spin transport

## From quantum kinetics to drift-diffusion

$$\tilde{G}_k^< = \underbrace{\frac{1}{2}(1+\hat{\tau}_z)\otimes\hat{g}_k^A}_{\text{Site A}} + \underbrace{\frac{1}{2}(1-\hat{\tau}_z)\otimes\hat{g}_k^B}_{\text{Site B}} + \underbrace{\hat{\tau}_x\otimes\hat{g}_k^x}_{\text{Current}} + \underbrace{\hat{\tau}_y\otimes\hat{g}_k^y}_{\text{Coupling}}.$$

$$\left(1+\frac{\xi^2}{2}-\frac{\beta^2}{2}\right)\hat{\mathcal{J}}_i + \left(\frac{\xi^2}{2}+\frac{\beta^2}{2}\right)\hat{\boldsymbol{\sigma}}\cdot\mathbf{n}\hat{\mathcal{J}}_i\hat{\boldsymbol{\sigma}}\cdot\mathbf{n} = -\mathcal{D}\partial_i(\hat{\rho}_A+\hat{\rho}_B),$$

$$\frac{\Gamma}{\hbar}(\hat{\rho}_A-\hat{\rho}_B) - i\frac{\Delta}{\hbar}[\hat{\rho}_A,\hat{\boldsymbol{\sigma}}\cdot\mathbf{n}] = -\frac{1}{2}\partial_i\hat{\mathcal{J}}_i,$$

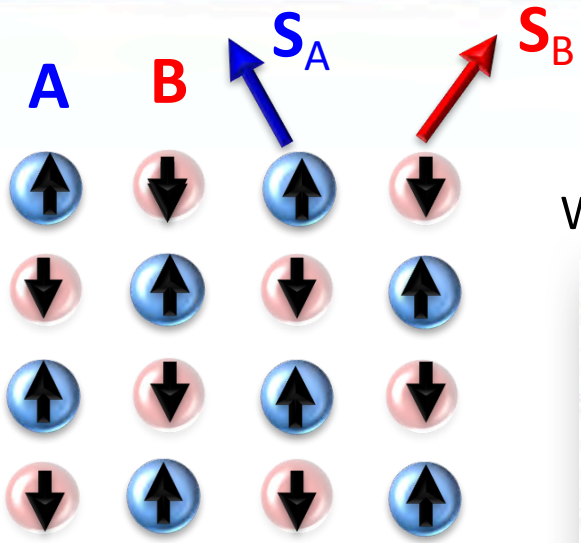
$$\frac{\Gamma}{\hbar}(\hat{\rho}_A-\hat{\rho}_B) - i\frac{\Delta}{\hbar}[\hat{\rho}_B,\hat{\boldsymbol{\sigma}}\cdot\mathbf{n}] = -\frac{1}{2}\partial_i\hat{\mathcal{J}}_i.$$

A-B Coupling

Manchon, arXiv:1608.00140v1

# II. Drift-diffusion spin transport

## "Valet-Fert" Theory for antiferromagnets




$\mathbf{S}_A+\mathbf{S}_B$: uniform spin accumulation
$\mathbf{S}_A-\mathbf{S}_B$: staggered spin accumulation

We obtain the drift-diffusion equation for the **uniform spin density**

$$\partial_t(n_A+n_B) = -\partial_i j_{c,i},\ \ j_{c,i} = -\mathcal{D}^{\parallel}\partial_i(n_A+n_B),$$

$$\mathbf{J}_i^s = -\mathcal{D}^{\parallel}\partial_i[(\mathbf{S}_A+\mathbf{S}_B)\cdot\mathbf{n}]\mathbf{n} - \mathcal{D}^{\perp}\mathbf{n}\times[\partial_i(\mathbf{S}_A+\mathbf{S}_B)\times\mathbf{n}],$$

$$\partial_t(\mathbf{S}_A+\mathbf{S}_B) + \frac{1}{\tau_\varphi}\underbrace{\mathbf{n}\times[(\mathbf{S}_A+\mathbf{S}_B)\times\mathbf{n}]}_{\text{dephasing}} + \frac{1}{\tau_{\rm sf}}\underbrace{(\mathbf{S}_A+\mathbf{S}_B)}_{\text{relaxation}} = -\partial_i\mathbf{J}_i^s,$$

$$\mathbf{S}_A-\mathbf{S}_B = \underbrace{\frac{\tau^*}{\tau_\Delta}}_{\text{Lifetime/precession time}}\mathbf{n}\times(\mathbf{S}_A+\mathbf{S}_B),$$

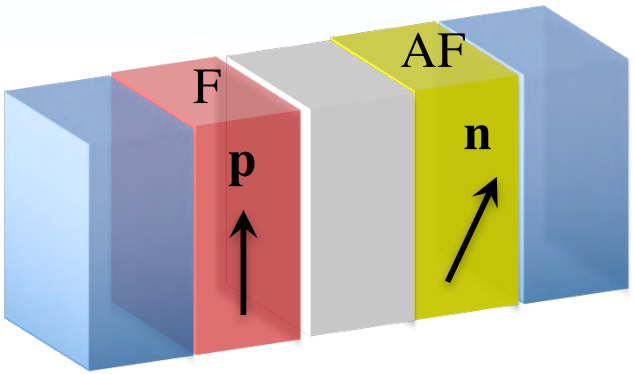
- In the diffusive regime, an antiferromagnet behaves like an **anisotropic normal metal**
- The source of **uniform** spin accumulation is the **spin current**
- The **staggered** spin accumulation is a correction to the **uniform** one

Manchon, arXiv:1608.00140v1

# II. Drift-diffusion spin transport

## Antiferromagnetic spin-valves and bilayers

### Spin Torque in F/AF spin-valve



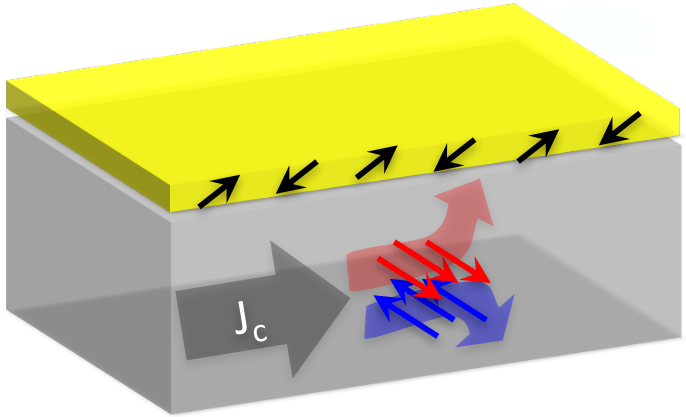

Everything works like Valet-Fert theory
The torque is robust and damping-like

$$\mathbf{T} = 2\Delta \int_{x_R}^{+\infty} dx (\mathbf{S}_A - \mathbf{S}_B) \times \mathbf{n},$$
$$= \frac{\tau^* \hbar}{\tau_\Delta^2} \int_{x_R}^{+\infty} dx \mathbf{n} \times [(\mathbf{S}_A + \mathbf{S}_B) \times \mathbf{n}],$$

$$\mathbf{T} = \frac{\tau^* \hbar}{\tau_\Delta^2} \frac{1}{e^3 \mathcal{N}} \frac{\lambda_\parallel^R \beta j_c}{\sigma_\parallel^R + \sigma_\parallel^L + r_{\rm int} \sigma_\parallel^L \sigma_\parallel^R} \mathbf{n} \times (\mathbf{p} \times \mathbf{n}).$$

Manchon, arXiv:1608.00140v1

### Antiferromagnet/normal metal bilayer




Again, the torque is damping-like and resemble the one in ferromagnetic bilayers

$$\mathbf{T} = \frac{\tau^* \hbar}{\tau_\Delta^2} \frac{1}{e^3 \mathcal{N}} \frac{\lambda_\perp^R \alpha_{\rm H} j_c}{\sigma_\perp^R + \eta_\perp \sigma_{\rm sf}^L \tanh \frac{d}{\lambda_{\rm sf}^L}} \left(1 - \cosh^{-1} \frac{d}{\lambda_{\rm sf}^L}\right) \mathbf{n} \times (\mathbf{y} \times \mathbf{n}).$$

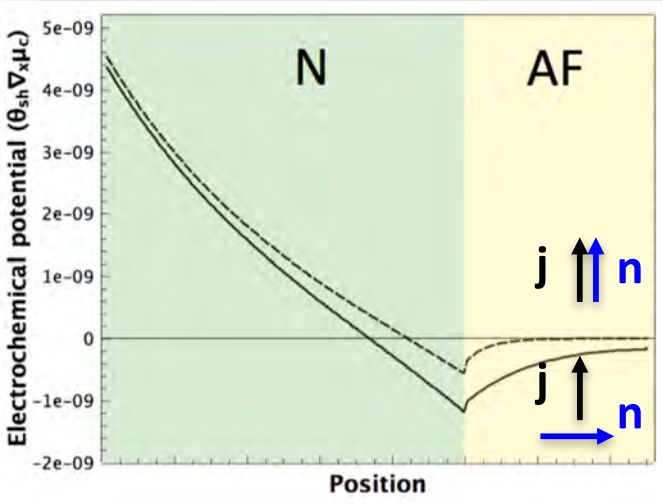
# II. Drift-diffusion spin transport

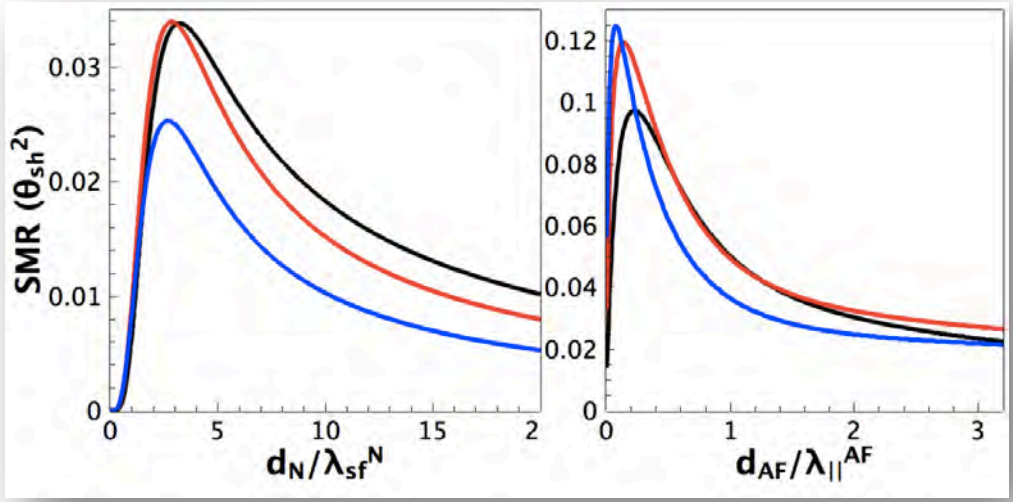
## Spin Hall Torque and Magnetoresistance

$$\Delta\sigma_{xx} = \frac{(\gamma_{\parallel}\eta_{\parallel} - \gamma_{\perp}\eta_{\perp})(1-\cosh^{-1}\frac{d_{\rm N}}{\lambda^{\rm N}_{\rm sf}})^2\lambda^{\rm N}_{\rm sf}\sigma_{\rm N}\theta^2_{\rm sh}}{(1+\gamma_{\parallel}\eta_{\parallel}\tanh^{-1}\frac{d_{\rm N}}{\lambda^{\rm N}_{\rm sf}})(1+\gamma_{\perp}\eta_{\perp}\tanh^{-1}\frac{d_{\rm N}}{\lambda^{\rm N}_{\rm sf}})}$$

$$\eta_\alpha = 1 + (r_\alpha\sigma^{\rm AF}_\alpha/\lambda^{\rm AF}_\alpha)\tanh\frac{d_{\rm AF}}{\lambda^{\rm AF}_\alpha},$$

$$\gamma_\alpha = (\lambda^{\rm AF}_\alpha\sigma_{\rm N}/\lambda^{\rm N}_{\rm sf}\sigma_\alpha)\tanh^{-1}\frac{d_{\rm AF}}{\lambda^{\rm AF}_\alpha},$$




$$\lambda^{\rm AF}_{\parallel} = \sqrt{\mathcal{D}^{\rm AF}_{\parallel}\tau^{\rm AF}_{\rm sf}},$$

$$\lambda^{\rm AF}_{\perp} = \sqrt{\mathcal{D}^{\rm AF}_{\perp}/(1/\tau^{\rm AF}_{\rm sf} + 1/\tau^{\rm AF}_{\varphi})},$$

$$2{\rm Re}G^{\uparrow\downarrow} = \sigma^{\rm AF}_{\perp}\tanh\frac{d_{\rm AF}}{\lambda^{\rm AF}_{\perp}}/\lambda^{\rm AF}_{\perp}$$

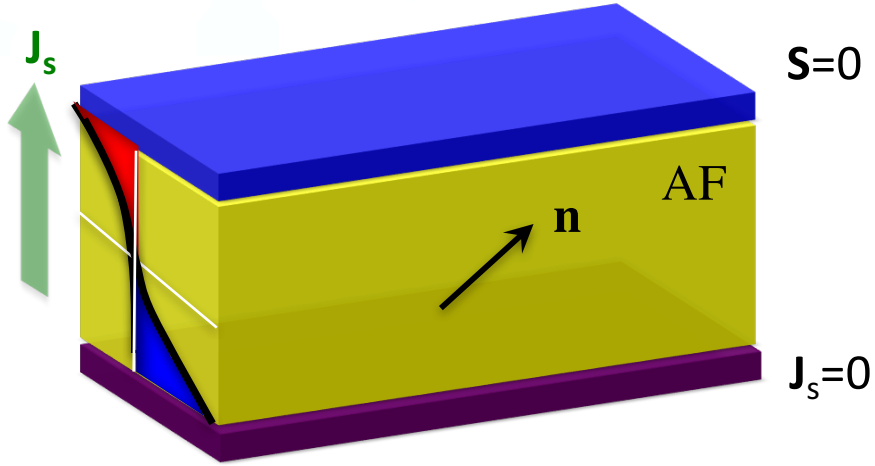
- Not limited to collinear AF!
- Also expected in NiO, CoO etc.

Manchon, arXiv:1609.06521

# II. Drift-diffusion spin transport
## Self-torque in single antiferromagnet

Let us finally consider a thin antiferromagnet embedded between dissimilar interfaces

The antiferromagnet possesses spin Hall effect




Again, we obtain the torque

Spin Hall effect

$$\mathbf{T} = \frac{\tau^* \hbar}{\tau_\Delta^2} \frac{1}{e^3 \mathcal{N}} \frac{\lambda_\perp \alpha_H j_c}{\sigma_\perp} \left(1 - \cosh^{-1} \frac{d}{\lambda_\perp}\right) \mathbf{n} \times (\mathbf{y} \times \mathbf{n}).$$

Precession (uniform-to-staggered Spin conversion)

Spin relaxation

Damping-like

I. Premises: G-type antiferromagnet

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**III. Topological transport in textures**

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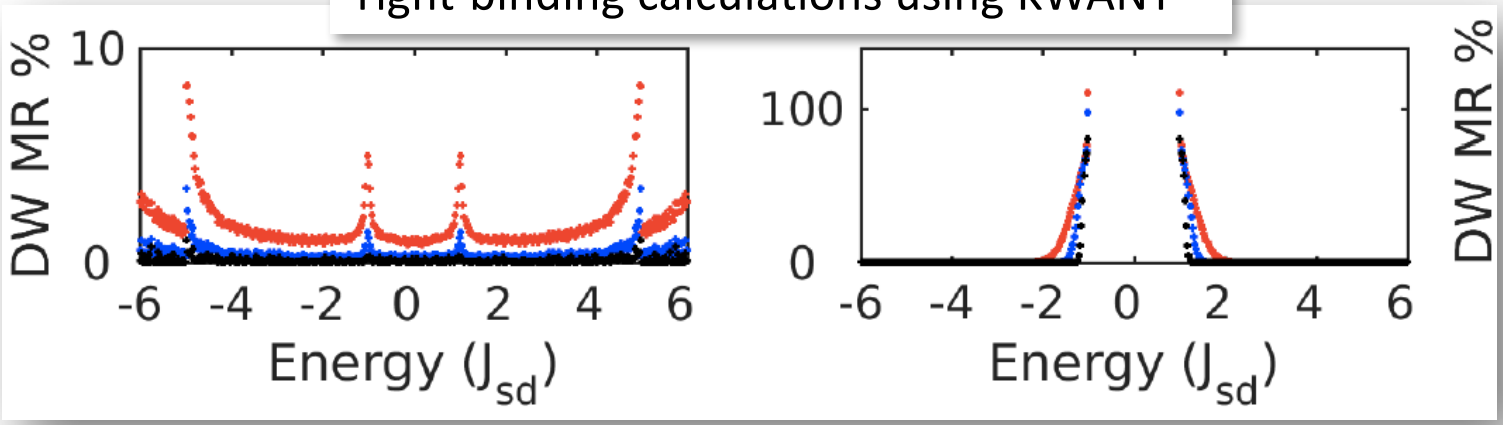
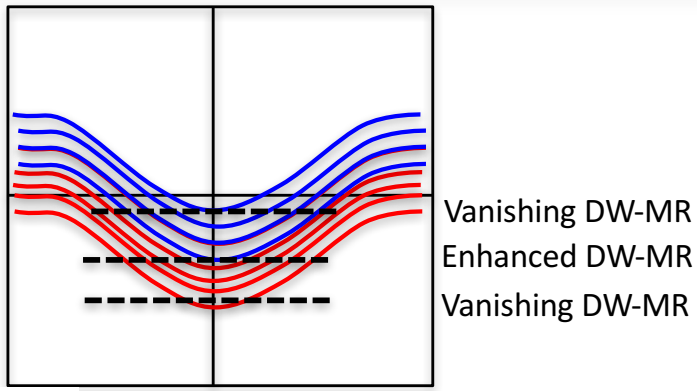
# III. Topological transport in textures

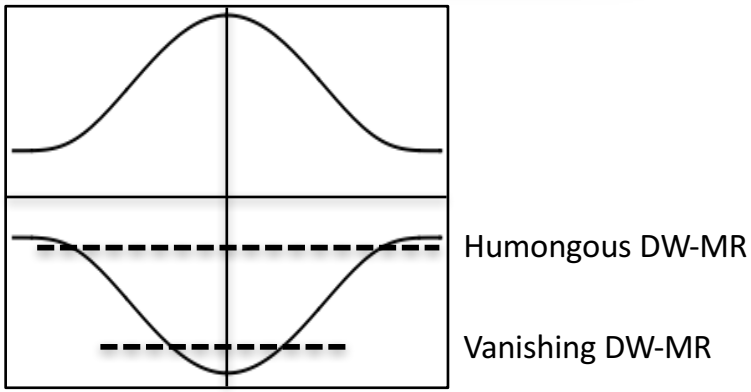
## Antiferromagnetic domain walls

C.A. Akosa

Tight-binding calculations using KWANT*

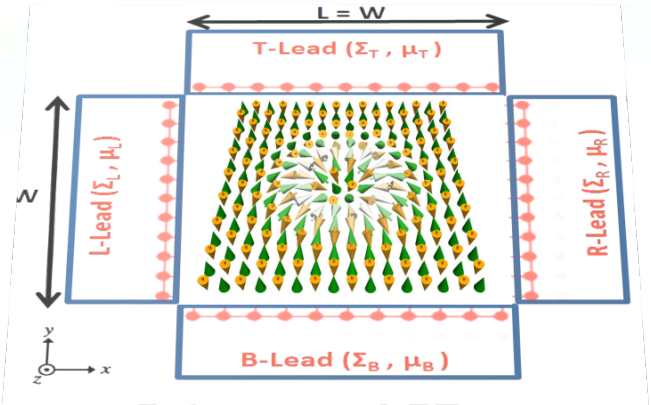


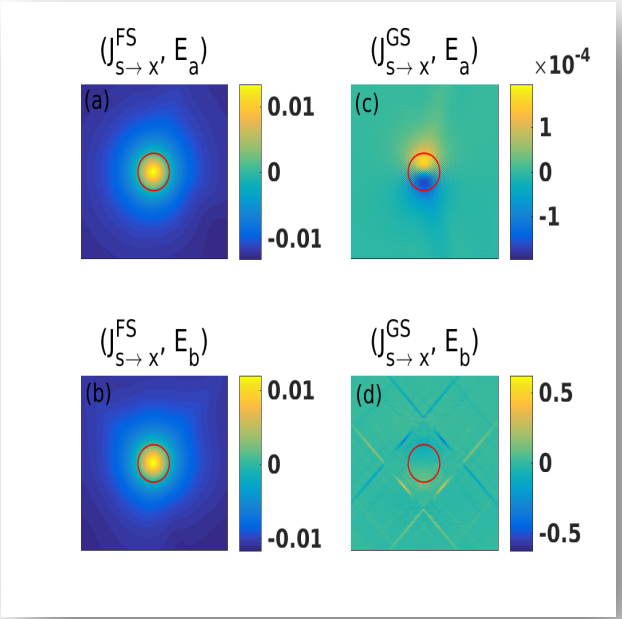




*Kwant-project.org

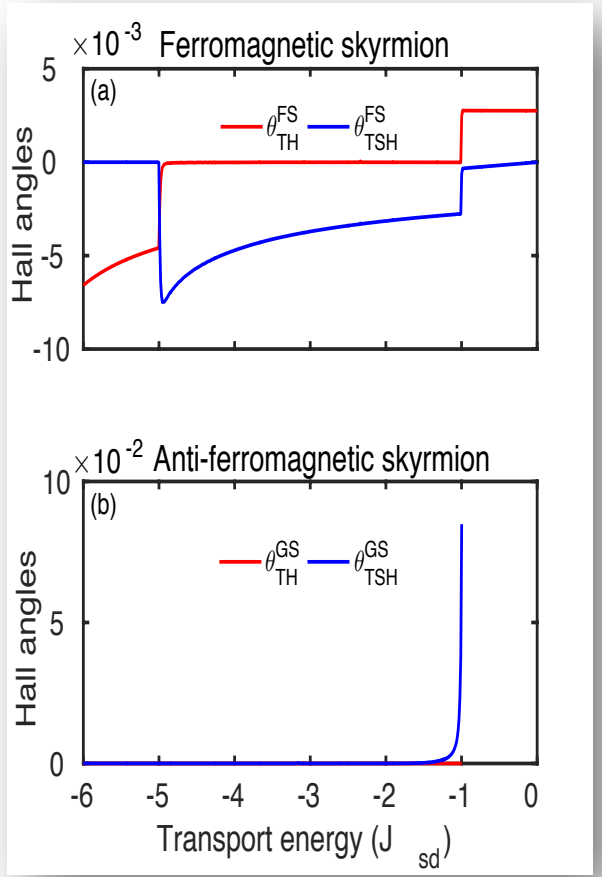
# III. Topological transport in textures
## Antiferromagnetic skyrmions



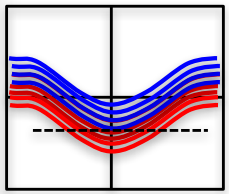

**Spin current 2D map**



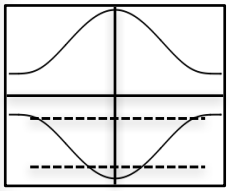

**Topological charge and spin Hall effects**




$\theta_{TH}$=0.5%



$\theta_{TSH}$=10%!!

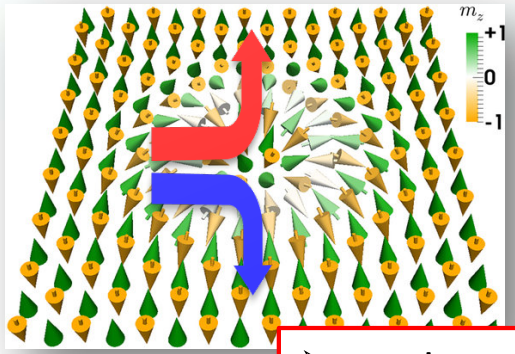


**Gigantic topological spin Hall effect!**

See also Ndiaye, Akosa, Manchon arXiv:1609.05480

# III. Topological transport in textures
## Antiferromagnetic skyrmions

Ezawa, Nat Com. 2015




**"Emergent magnetic field" model**

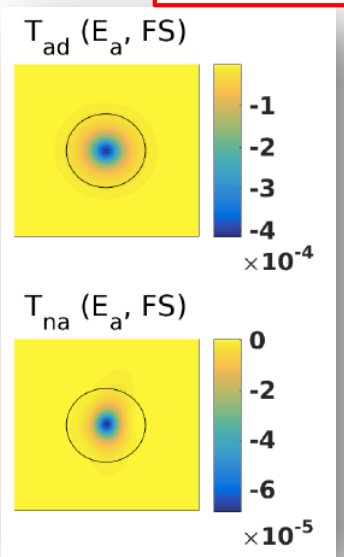
Spin current $\mathcal{J}_s^\mu = b_J^\mu \left( \mathbf{M}^\mu \otimes \mathbf{x} - qp^\mu \tilde{\lambda}_H^{\mu^2} [\mathbf{m}^\mu \cdot (\partial_x \mathbf{m}^\mu \times \partial_y \mathbf{m}^\mu)] \mathbf{M}^\mu \otimes \mathbf{y} \right)$

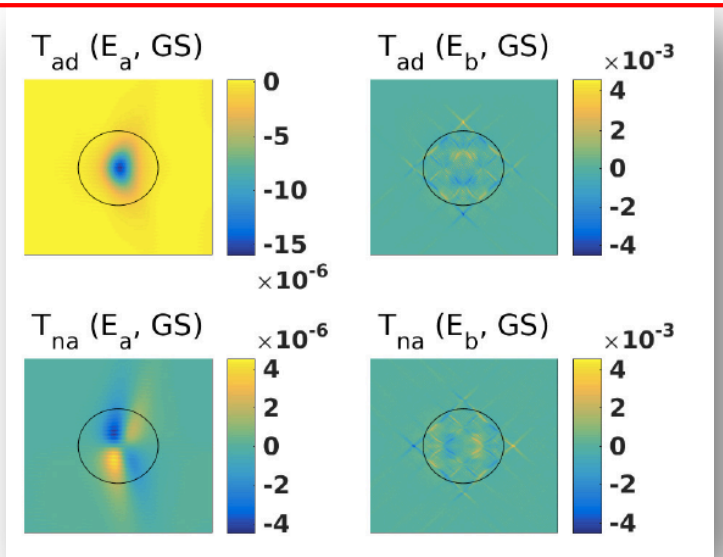
Torque $\mathbf{T}_t^\mu = -\left( \partial_x \mathbf{M}^\mu - qp^\mu \tilde{\lambda}_H^{\mu^2} [\mathbf{m}^\mu \cdot (\partial_x \mathbf{m}^\mu \times \partial_y \mathbf{m}^\mu)] \partial_y \mathbf{M}^\mu \right) b_J^\mu$

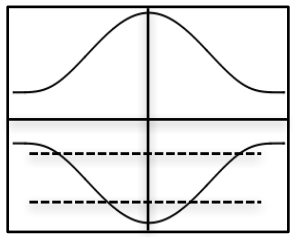
AF-skyrmion velocity $v_x = -pq \frac{\beta}{\alpha} \frac{(b_J^a + b_J^b)}{2} + pq \frac{4\mathcal{S}_2}{3\alpha} \left( \frac{\tilde{\lambda}_H^{a^2}}{2r_0^2} b_J^a + \frac{\tilde{\lambda}_H^{b^2}}{2r_0^2} b_J^b \right)$

- F-skyrmion: topological torque induces transverse motion
- AF-skyrmion: topological torque induces longitudinal motion
- The **smaller** the AF-skyrmion, the **larger** the velocity

**Torque in ferr**



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# IV. Antiferromagnetic topological insulators

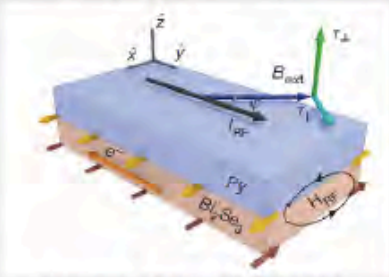
## The new frontier?

Spin-Orbit Torques with Topological Insulators

Courtesy of Hyunsoo Yang, NUS

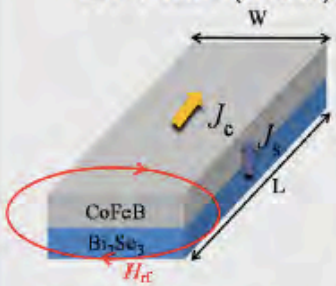
### Exotic spin Hall angles from topological insulators

$\theta_{SH}$ = 2~3.5 (room temp)
ST-FMR (Cornell)



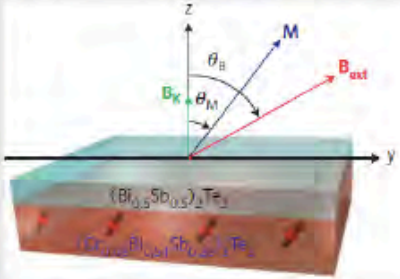

Nature **511**, 449 (2014)

$\theta_{SH}$ = 2 (low temp)
ST-FMR (NUS)



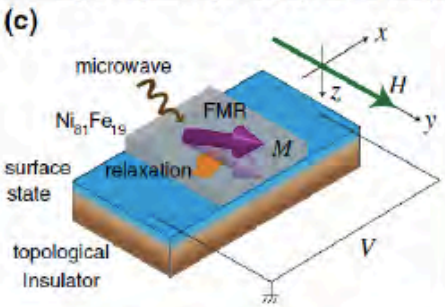

PRL **114**, 257202 (2015)

$\theta_{SH}$ = 140-425 (low temp)
spin-orbit switching (UCLA)



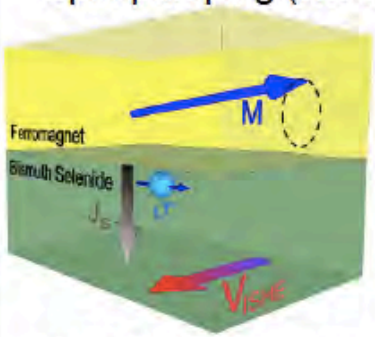

Nat. Mater. **13**, 699 (2014)

$\theta_{SH}$ = 0.01
Spin-pumping (Tohoku)



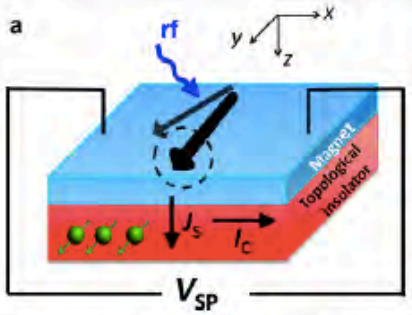

PRL **113**, 196601 (2014)

$\theta_{SH}$ = 0.01
Spin-pumping (NUS)




PRB **90**, 094403 (2014)

$\theta_{SH}$ = 0.01-0.4
Spin-pumping (Minnesota)

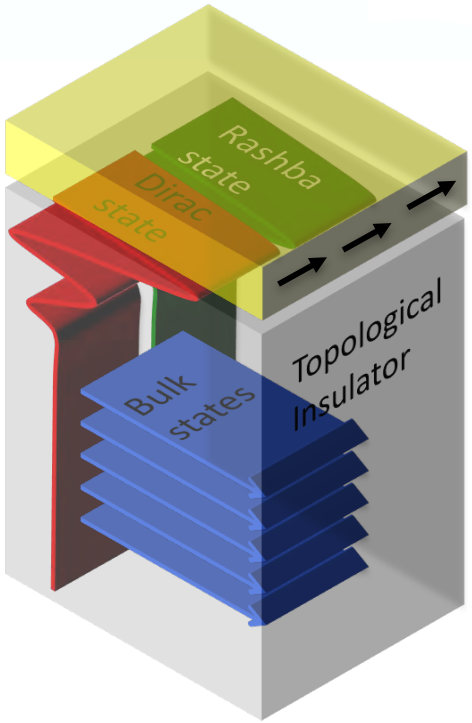



Nano Lett **15**, 7126 (2015)

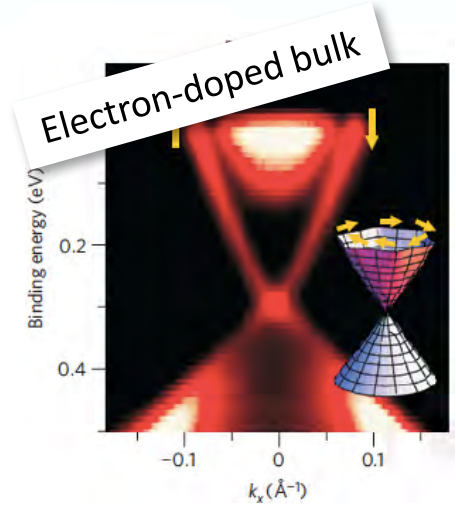
# IV. Antiferromagnetic topological insulators

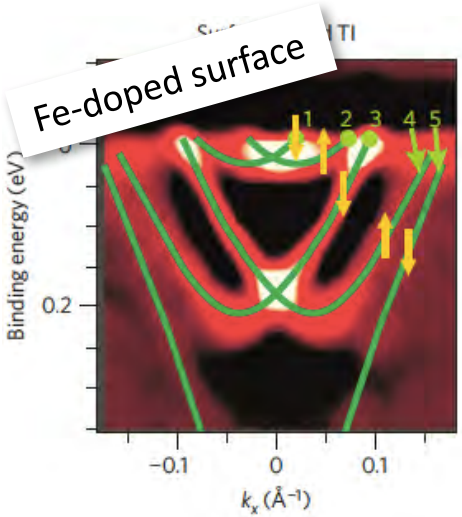
## The new frontier?

Spin-Orbit Torques with Topological Insulators




Dirac surface state
+ Rashba surface state
+SOC-coupled bulk states

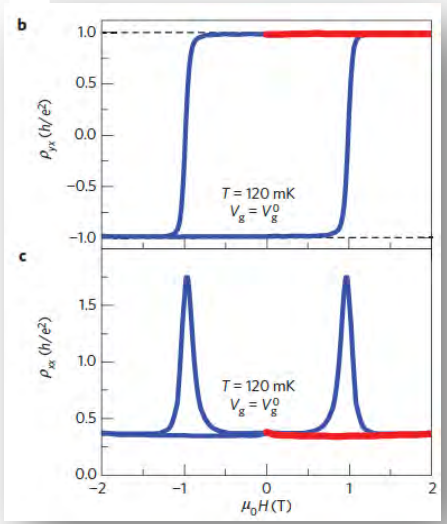



Wray, Nature Physics 7, 32 (2011)

**Quantum Anomalous Hall effect**

The "true" signature of topological states




Chang, Science 2013; Nat. Mat. 2015
Kou, PRL 2013; Nat. Comm. 2015

- What is the nature of the torque in F/TI?
- **Can we have magnetic order while keeping the topological protection**

# IV. Antiferromagnetic topological insulators

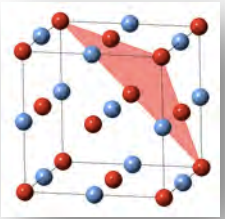
## The new frontier?

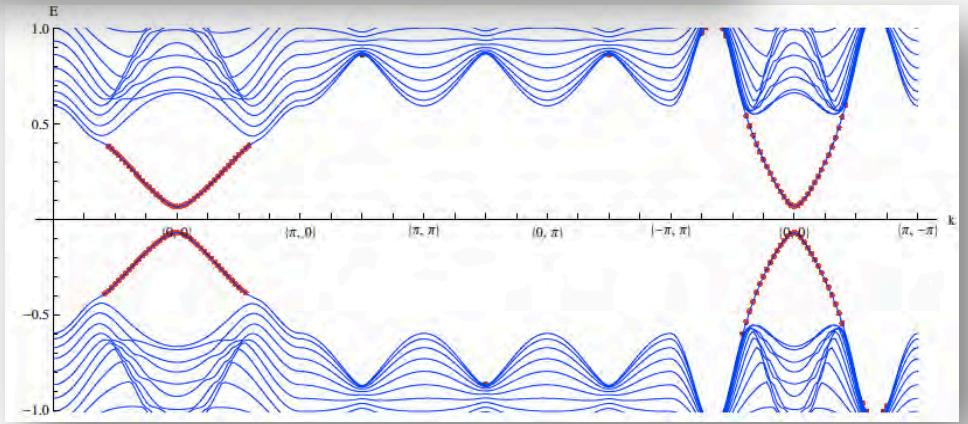
PHYSICAL REVIEW B **81**, 245209 (2010)

**Antiferromagnetic topological insulators**

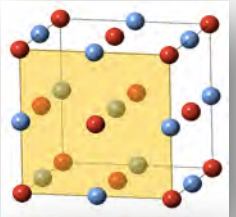
Roger S. K. Mong,[1] Andrew M. Essin,[1] and Joel E. Moore[1,2]

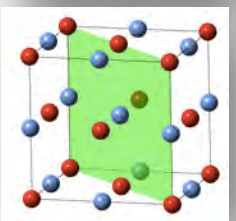
**Ferromagnetic surface**

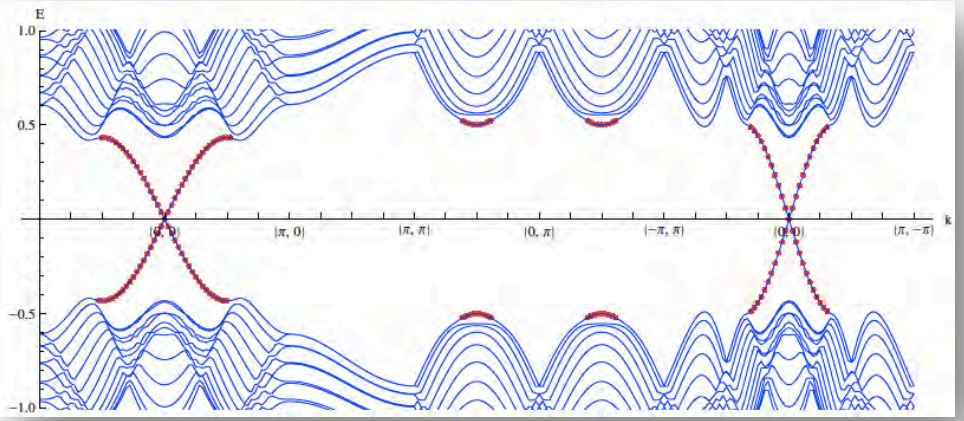





**Antiferromagnetic surface**



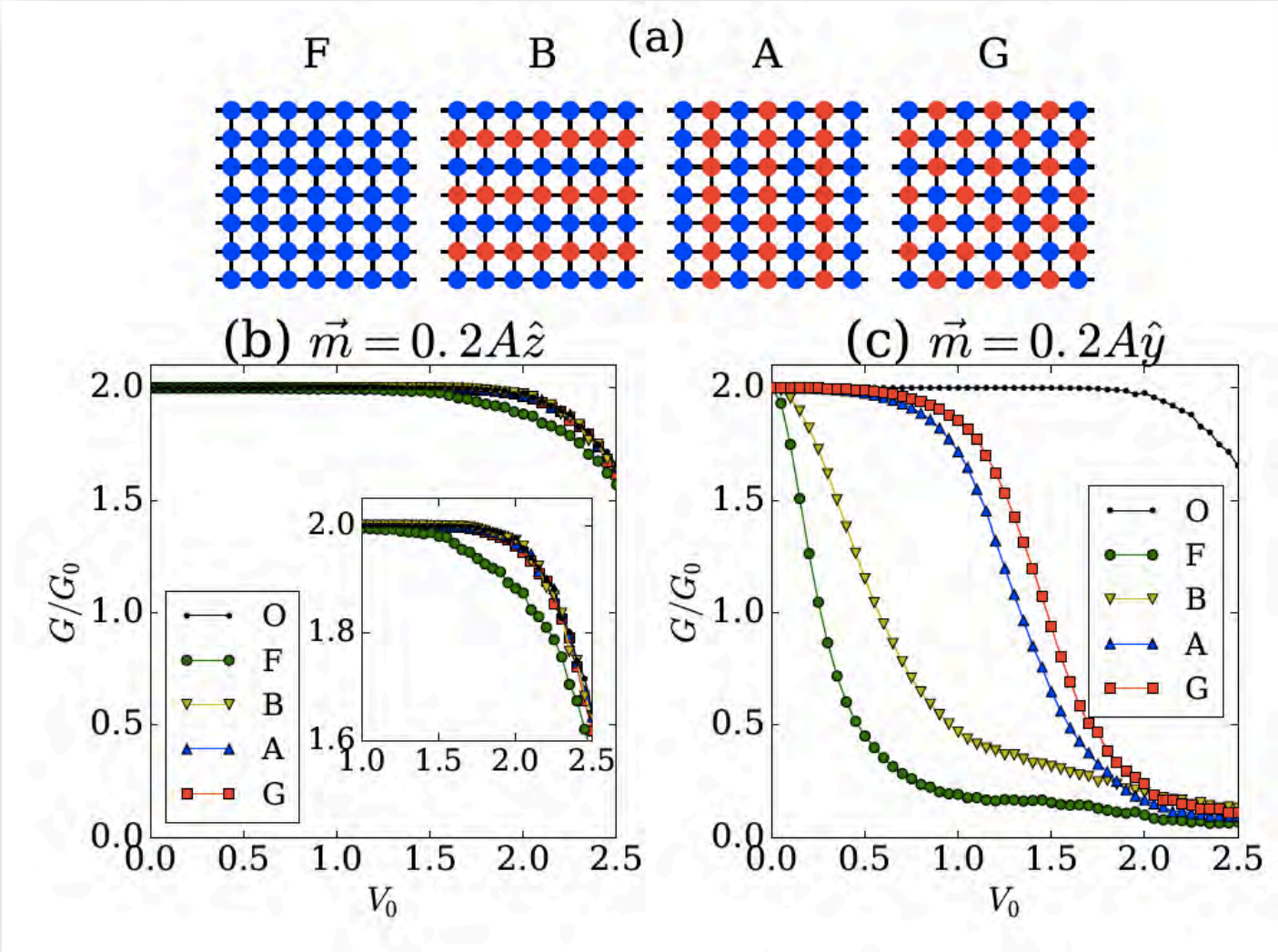




# IV. Antiferromagnetic topological insulators

## Robustness against disorder

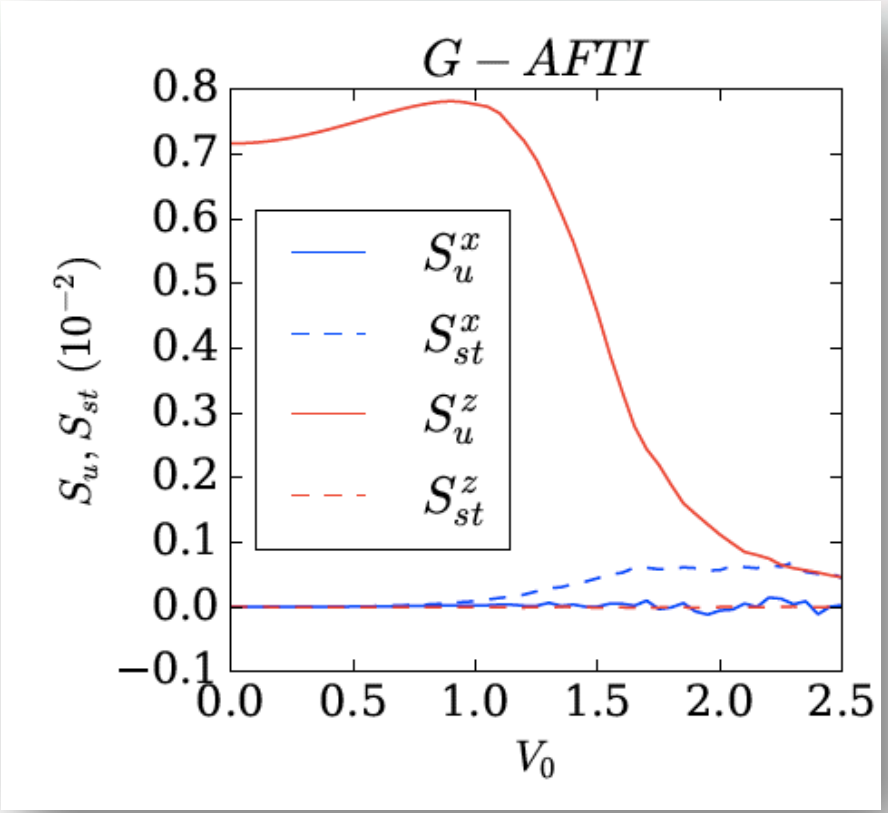
S. Ghosh

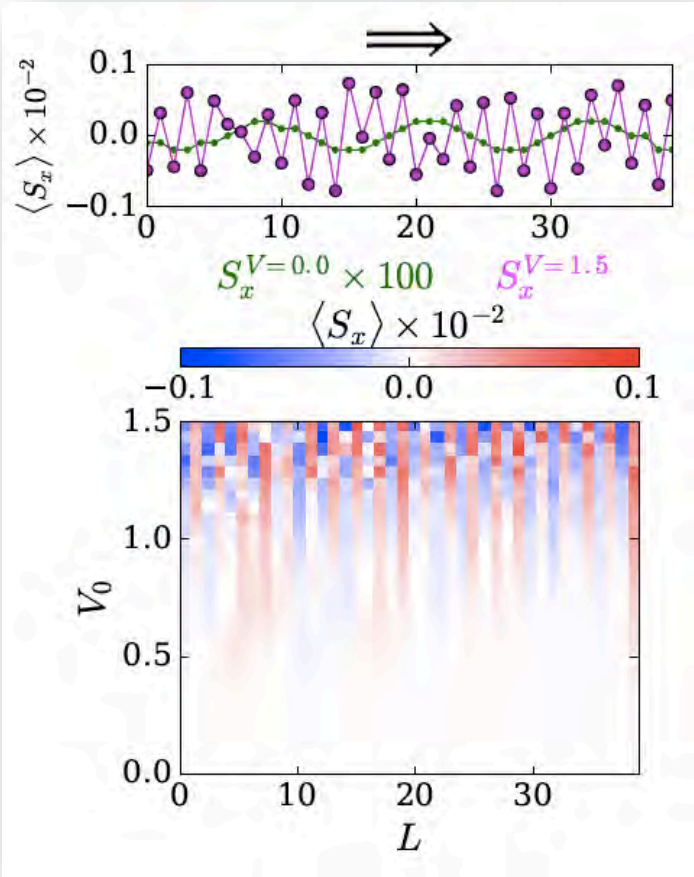



Ghosh and Manchon, arXiv:1609.01174

# IV. Antiferromagnetic topological insulators

## Spin-Orbit Torques




- Spin-polarized edge states are robust
- Onset of staggered spin density upon disorder
- **Now…3D AF-TI**

Ghosh and Manchon, arXiv:1609.01174

# Thank you for your attention!

**This presentation**

Manchon, arXiv:1608.00140v1
Manchon, arXiv:1609.06521
Zelezny et al., arXiv:1604.07590
Ghosh and Manchon, arXiv:1609.01174
Akosa…to appear soon

**Other works on aniferromagnets**

Saidaoui, Manchon, Waintal, PRB 89, 174430 (2014)
Zelezny et al., PRL 113, 157201 (2014)
Saidaoui, and Manchon, arXiv:1606.04261
Saidaoui, Manchon and Waintal, arXiv:1607.01523v2

**Review**

Baltz, Manchon, Tsoi, Moriyama, Ono and Tserkovnyak, arXiv:1606.04284