Antiferromagnetic Textures II

ULRICH K. RÖSSLE

IFW DRESDEN

Leibniz-Institut
für Festkörper- und
Werkstoffforschung
Dresden

SPICE Workshop Antiferromagnetic Spintronics 26.-30/09/2016 Schloss Waldthausen

u.roessler@ifw-dresden.de

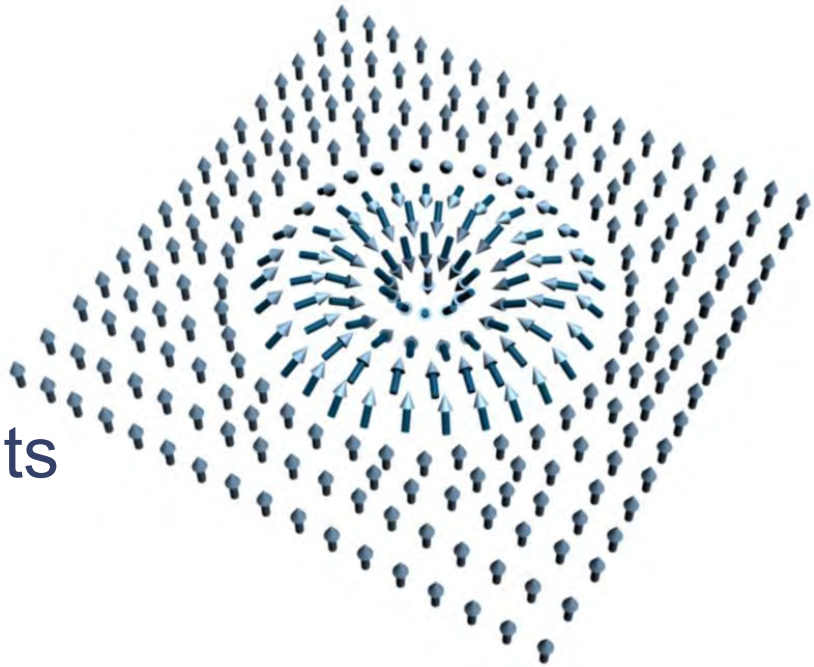
What Skyrmions ?

A Skyrmion (in generalized sense) is a

multidimensional, static, topological soliton

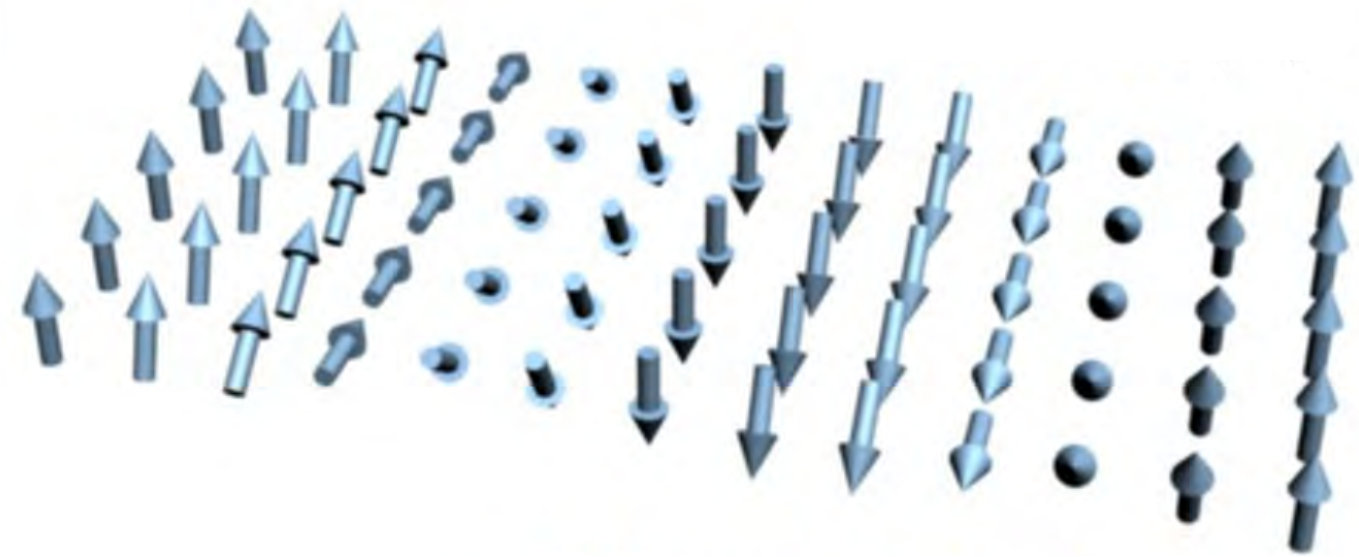
a particle-like solution of
a non-linear field equation

Here: localized spin-states
in certain 2 dimensional
non-centrosymmetric magnets



Topology of textures

Topology of textures



Topologically non-trivial texture in XY-ferromagnet or antiferromagnet

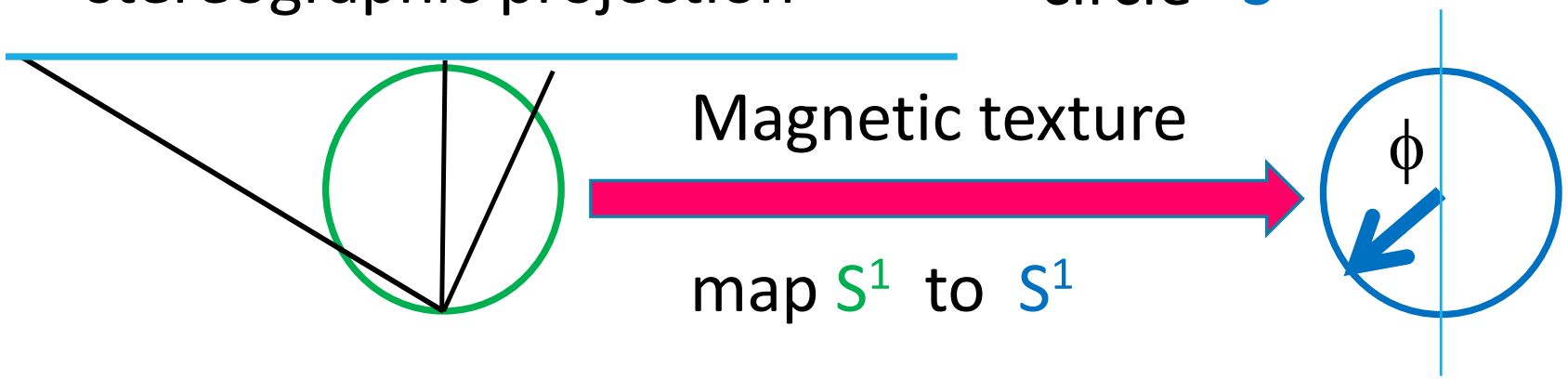
Idea of homotopy – much reduced

Go in loops after loops / how many are there ?
– see whether loops are the same or trivial

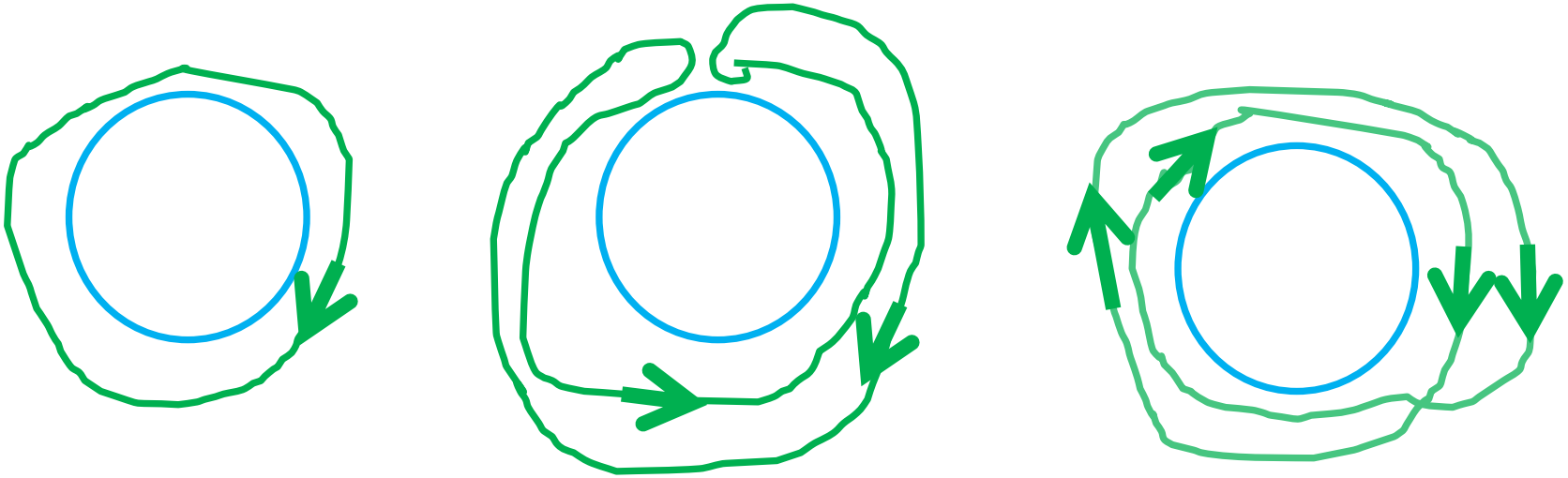
Model 1D XY magnet – 1 degree of freedom turn angle ϕ

Physical space $\mathbb{R}$ a circle by stereographic projection

Order parameter manifold circle S^1



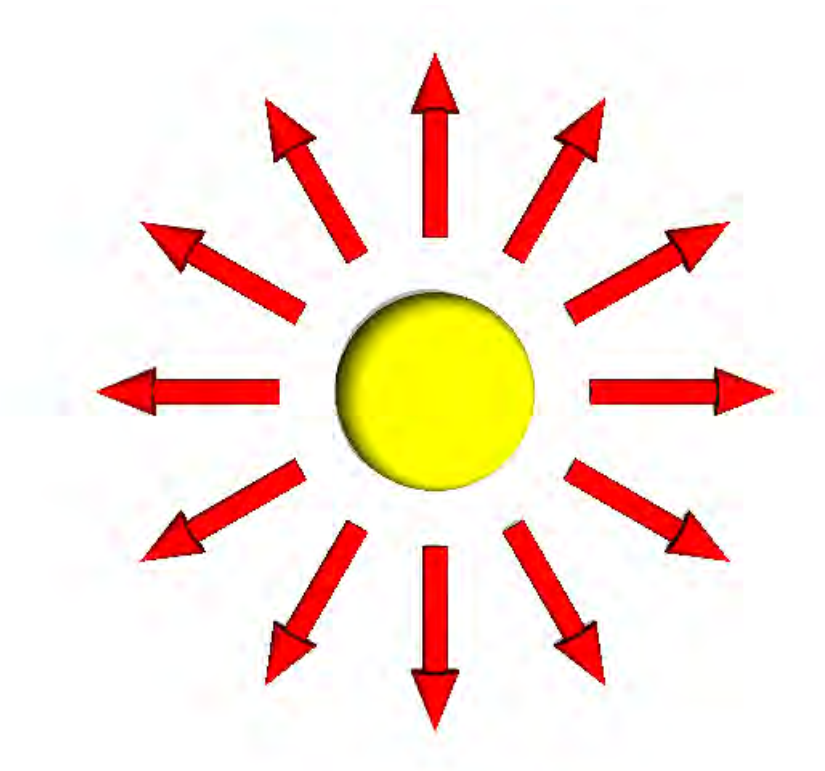
Winding number w



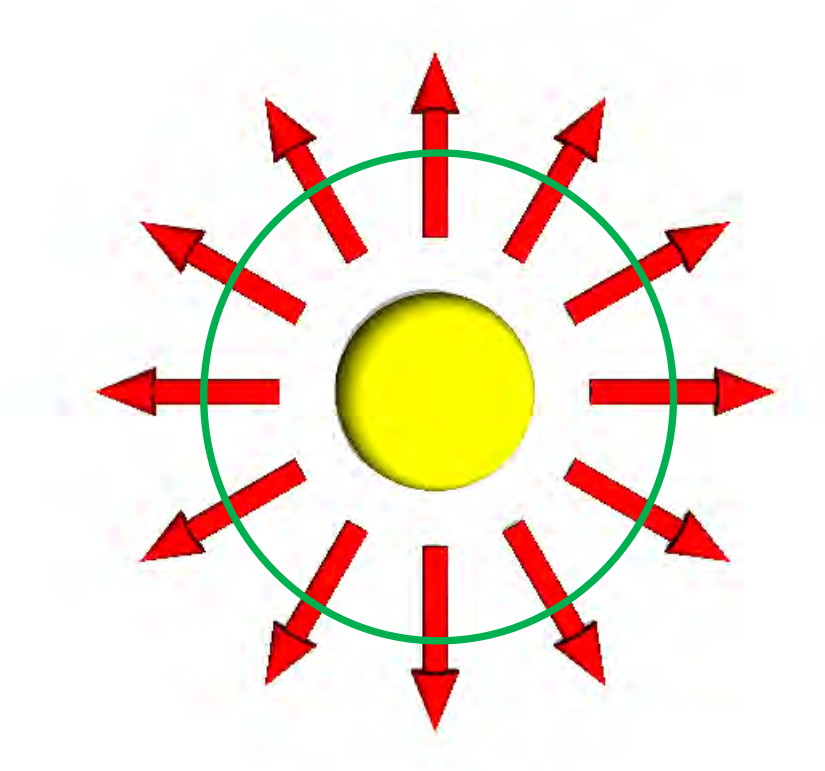
$w = -1$ 0 -2

Set of homotopically different loops form the
Fundamental group $\pi_1 (S^1)$
What are the topological invariants counted by w ? Which group is $\pi_1 (S^1)$?

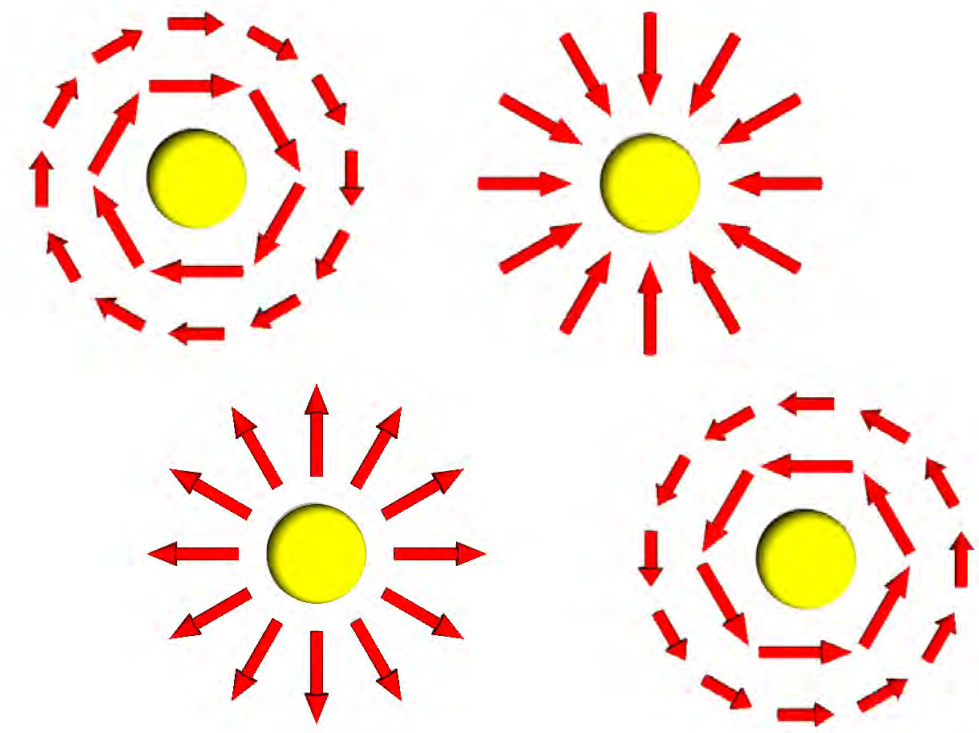
Homotopy as defect detector



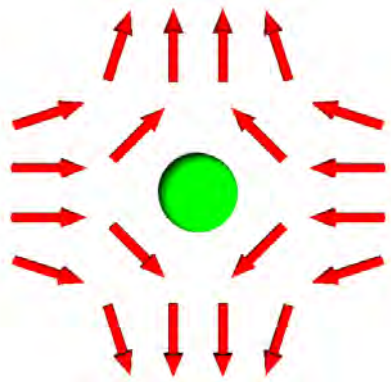
Homotopy as defect detector



Homotopy as defect detector

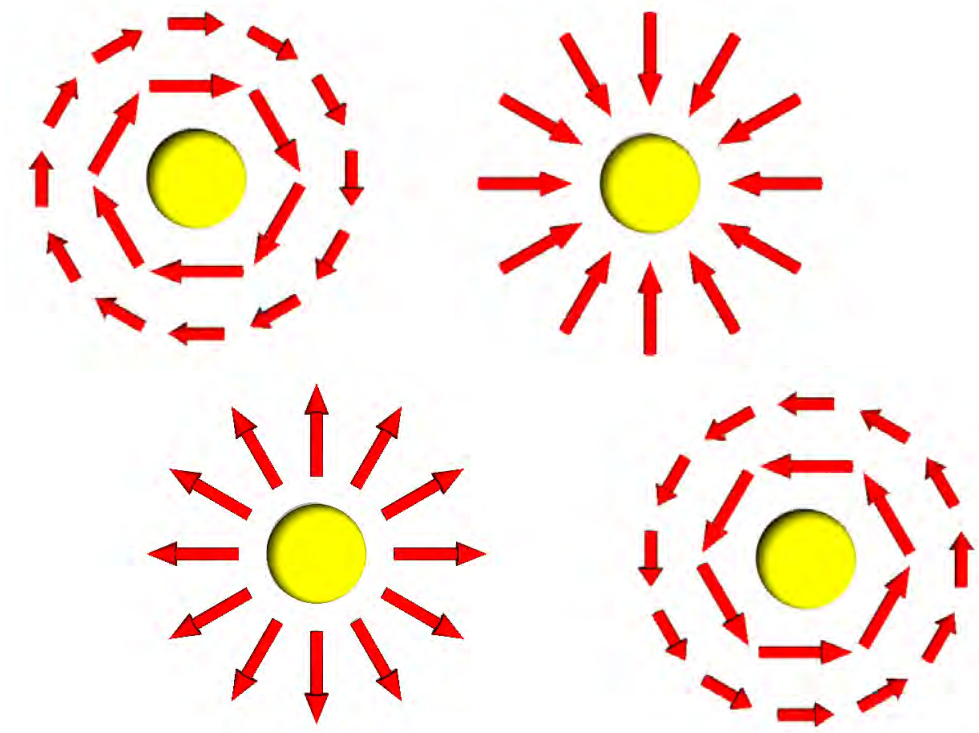


Charge 1 vortices

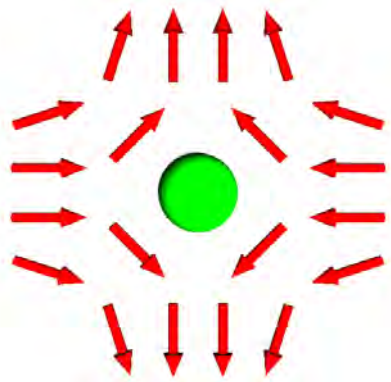


charge -1 antivortex

Homotopy as defect detector



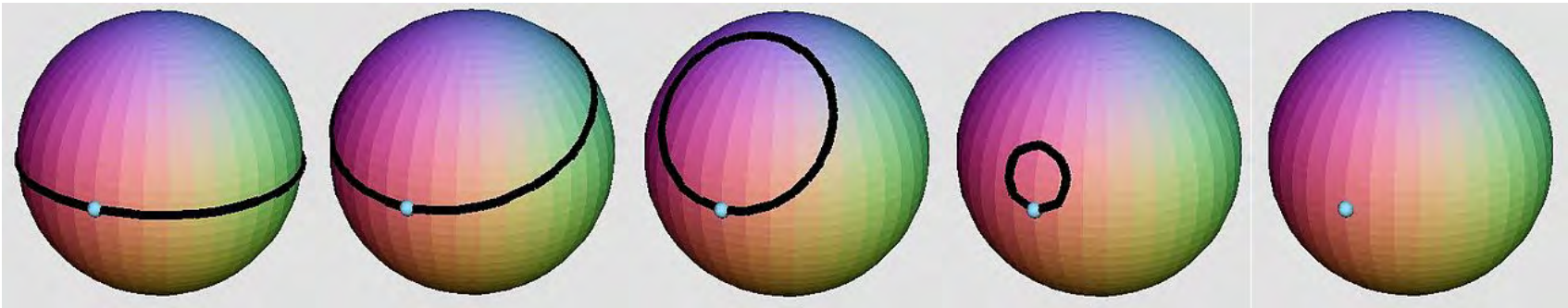
Charge 1 vortices



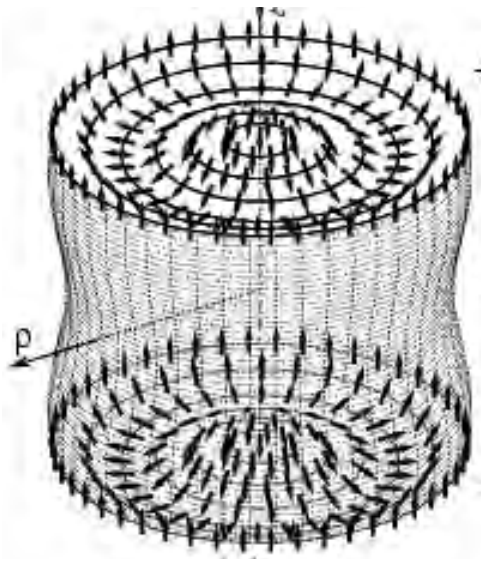
charge -1 antivortex

No line defects in Heisenberg-like magnets

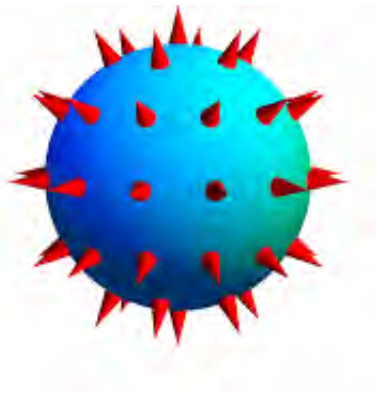
$\pi_1 (S^2) = 0$



Skyrmions



Map S^2 to S^2



„Wrapping number“

$$Q = (1/8\,\pi) \iint \varepsilon^{xy}\, \mathbf{m} \cdot (\partial_x \mathbf{m} \times \partial_y \mathbf{m})\, \mathrm{d}x\, \mathrm{d}y$$

is quantized / integer

Pontryagin index

2nd Homotopy group

$\pi_2 (S^2) = \mathbb{Z}$

Skyrmion number density

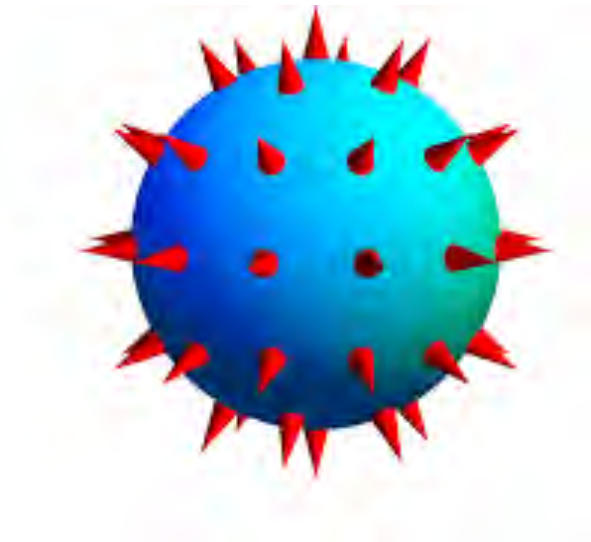
topological charge density

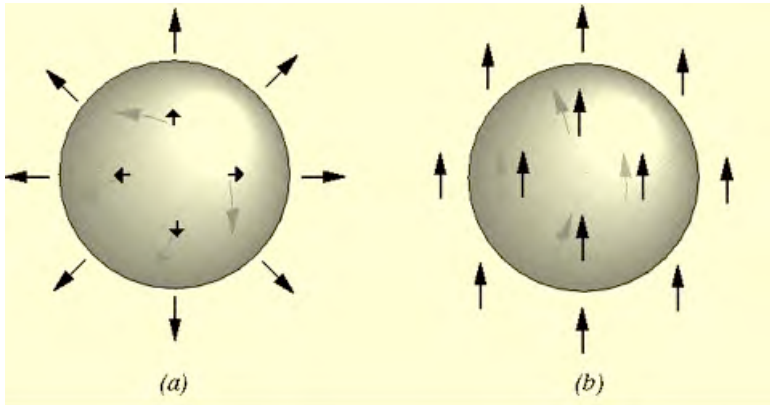
$$q = (1/8\,\pi)\,\varepsilon^{xy}\,\mathbf{m}\cdot(\partial_x\,\mathbf{m}\;\times\partial_y\,\mathbf{m})$$

a local density !

$\pi_2 (S^2) = \mathbb{Z}$ corresponding defect ?

Bloch points / monopoles



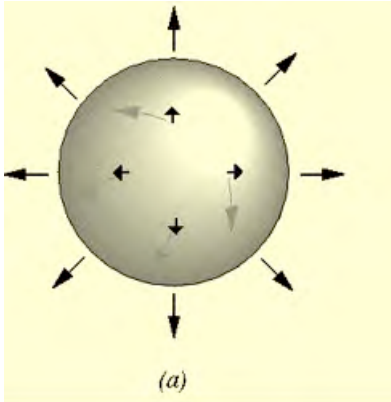


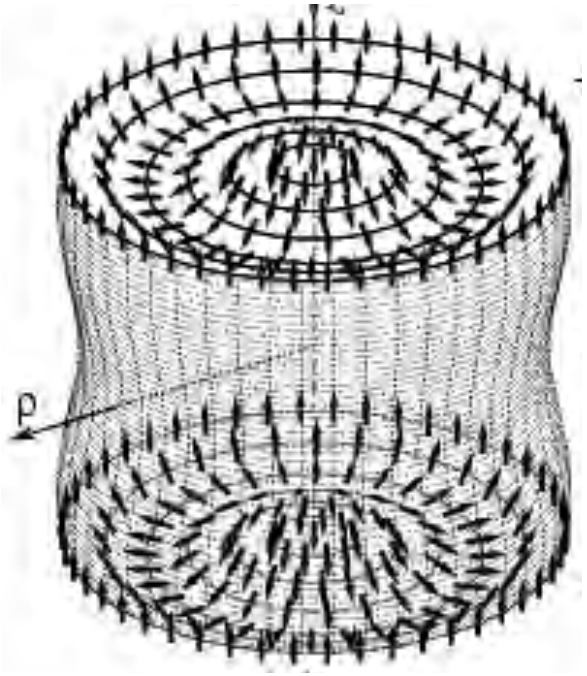
Feldtkeller 1967

Bloch points

Feldtkeller singularities

Hedgehogs





Required to create a skyrmion in a perpendicularly magnetized film

Some homotopy groups

relevant in magnetism

$\pi_1(S^1) = \mathbb{Z}$ vortices in XY magnets

$\pi_1(S^2) = 0$ no line defects in Heisenberg-like magnets

$\pi_2(S^2) = \mathbb{Z}$ skyrmions & Bloch points

$\pi_1(SO(3)) = \mathbb{Z}_2$ vortices in non-collinear magnets

$\pi_3(S^2) = \mathbb{Z}$ Hopf fibration !

still awaited

2nd set of conclusions

- Topology is a weak concept
- No statement about stability & geometry

Strength of topological classification

Defines countable units in continua

defects

textures

Skyrmions in condensed matter

Skyrmions in magnets

- Topologically non-trivial solutions in 2D Heisenberg magnets
  "Belavin-Polyakov solitons" – exact solutions

$$w = A \iint (\partial_i \mathbf{m} \cdot \partial_i \mathbf{m})\, \mathrm{d}x\, \mathrm{d}y\,, \ |\mathbf{m}| = 1$$

Indifferent stability –

scale independent finite energy solutions.

Stabilization mechanisms

- Classical Skyrme-type models

Add higher order gradient terms to the field Lagrangian, e.g.,

$$w = \iint A\,(\partial_i \mathbf{m} \cdot \partial_i \mathbf{m}) + K(\Delta\,\mathbf{m} \cdot \Delta\,\mathbf{m})\,\mathrm{d}x\,\mathrm{d}y$$

Abanov, Pokrovskii, PRB 1998

$$w = \iint A\,(\partial_i \mathbf{m} \cdot \partial_i \mathbf{m}) + B\,(F_{ij})^2\,\mathrm{d}x\,\mathrm{d}y$$

$$F_{ij} = \mathbf{m} \cdot (\partial_i \mathbf{m} \times \partial_j \mathbf{m})$$

Faddeev Niemi model 1997

Hubert model

Exchange frustrated magnetic XY-magnet

$$E = \sum_{i=1}^{N} [-J_1 \cos(\theta_i - \theta_{i+1}) - J_2 \cos(\theta_i - \theta_{i+2})],$$

$$\Phi = \int_{-\infty}^{\infty} \left[\left(\frac{d\theta}{dx}\right)^4 + a\left(\frac{d^2\theta}{dx^2}\right)^2 - 2b\left(\frac{d\theta}{dx}\right)^2 + g(\theta) \right] dx.$$

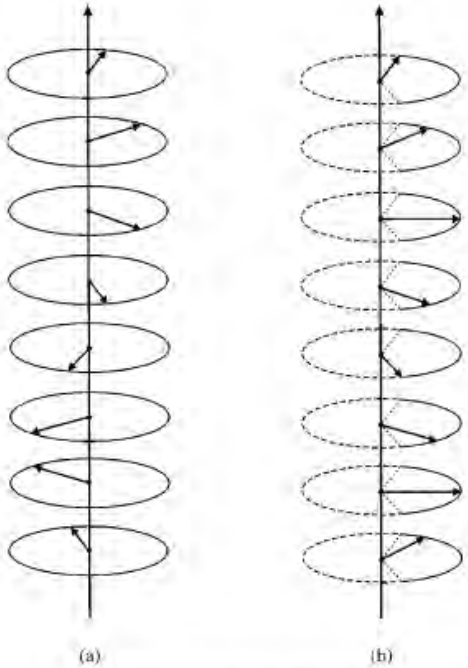
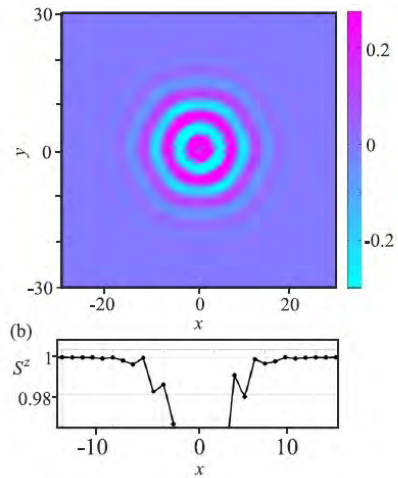


Fig. 1. Helical (a) and fan (b) structures.

Multiply modulated and skyrmionic phases

Lattice-based simulations of exchange-frustrated ferromagnets

A.O. Leonov, M. Mostovvoy, Nat. Comm. 2015



P.I. Melnichuk, UKR, A.N. Bogdanov, K.-H.Müller, JMMM 248, 142 (2002)

3rd set of conclusions

Stabilization of multidimensional solitons
by a classical Skyrme – like mechanism

- Theories introduce additional lengths scales
- no systematic expansion in gradient terms
- specific model requirements
- tremendous technical difficulties

Dzyaloshinskii models

A closer look : free energy expansion for modulated states (Dzyaloshinskii 1964)

Expansion of any modulated state

$$w_L = \phi^* \left[\alpha(T - T_0) + \delta(k - k_0) + \omega(k - k_0)^2 + \ldots\right] \phi$$

in irreps for the little group G_{k0} and surrounding irreps with the same symmetries in $\mathbf{k}$-space.

Environment of $\mathbf{k}_0$ has ν degrees of freedom.

Number μ of different Lifshitz invariants
given by coefficients δ

Weak Lifshitz criterion (Michelson 1978)

A few Lifshitz invariants are allowed,

$$\mu \leq \nu,$$

not more than degrees of freedom
for the irreps of the little group.

<u>Example</u>:
exchange J_1-J_2 helimagnets with a unique propagation vector direction
along a line of symmetry, $\mu = \nu = 1$
(hexagonal Dy …)

Violation of weak Lifshitz criterion
Chiral cubic / isotropic ferromagnets

Generalized (free) energy density

$$f = \eta A(\nabla m)^2 + Am^2 \sum_{i,j} (\partial_i n_j)^2 + f_0(m) + f_D(\mathbf{m})$$

specialized for an isotropic (cubic) ferromagnet

$$f_D = D\,\mathbf{m} \cdot \nabla \times \mathbf{m}$$

$$f_0 = a(T - T_C)m^2 + bm^4 + \text{h.o.t.}$$

$\nu = 0 < \mu = 3$

Systems with Lifshitz-type invariants

1. Landau theory for structural phase transitions, e.g. Pnma → $P2_1/m$

$$\kappa\left[\zeta_1\frac{\partial\zeta_2}{\partial x}-\zeta_2\frac{\partial\zeta_1}{\partial x}\right]$$

2. Dzyaloshinsky-Moriya interactions in magnetic materials

$$D\,\mathbf{m}\cdot\nabla\times\mathbf{m}$$

3. Lifshitz invariants in ferroelectrics with noncentrosymmetric parent

$$\frac{dP_x}{dz}P_y-\frac{dP_y}{dz}P_x$$

4. Multiferroics – magnetic order + polarization

$$\gamma\,\mathbf{P}\cdot[\mathbf{M}(\nabla\cdot\mathbf{M})-(\mathbf{M}\cdot\nabla)\mathbf{M})]+\ldots$$

5. Chiral liquid crystals

$$\tfrac{K_1}{2}(\nabla\mathbf{n})^2+\tfrac{K_2}{2}(\mathbf{n}\cdot\nabla\times\mathbf{n}-q_0)^2+\tfrac{K_3}{2}(\mathbf{n}\times\nabla\times\mathbf{n})^2$$

6. Chern-Simons terms in gauge theories …

Skyrmions in chiral magnets

Modulated states

Two classes (DeGennes 1975)

Instability of a fundamental mode Landau theory

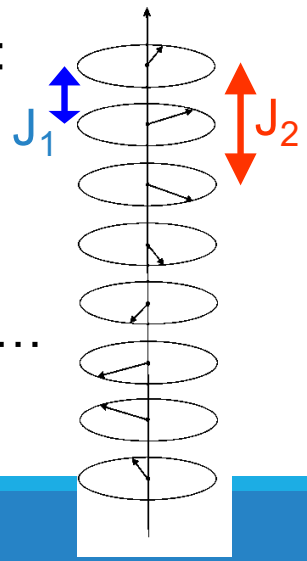
vs.

Nucleation and condensation of mesoscale structures or solitons

Realization in magnetism

Exchange frustration

J1-J2 XY-model :



Rare earths Dy

Multi-q structures Nd …

Stripes, bubbles …

Dzyaloshinskii spirals

Chiral magnetic states :

B20 Helimagnets MnSi, FeGe

$Cr_{1/3}NbSe_2$

$CsCuCl_3$

$Ba_2CuGe_2O_7$

$BiFeO_3$ …

Solitonic states in Dzyaloshinskii models

Thermodynamically stable "vortices" in magnetically ordered crystals. The mixed state of magnets

A. N. Bogdanov and D. A. Yablonskiĭ

Physicotechnical Institute, Donetsk, Academy of Sciences of the Ukrainian SSR

(Submitted 20 April 1988)
Zh. Eksp. Teor. Fiz. **95**, 178–182 (January 1989)

It is shown that in magnetically ordered crystals belonging to the crystallographic classes C_n, C_{nv}, D_n, D_{2d}, and S_4 ($n = 3, 4, 6$), in a certain range of fields, a thermodynamically stable system of magnetic vortices, analogous to the mixed state of superconductors, can be realized.

$$W = \int \left\{ \frac{1}{2} \alpha \left(\frac{\partial \mathbf{M}}{\partial x_i} \right)^2 - \frac{1}{2} \beta M_z^2 - H M_z - \frac{1}{2} \mathbf{M} \mathbf{H}_M + w' \right\} dV, \qquad (1)$$

Chiral magnets : Dzyaloshinskii 1964

Generalized (free) energy density

$$f = \eta A(\nabla m)^2 + Am^2 \sum_{i,j} (\partial_i n_j)^2 + f_0(m) + f_D(\mathbf{m})$$

for magnetization $\mathbf{m} = m\,\mathbf{n}$

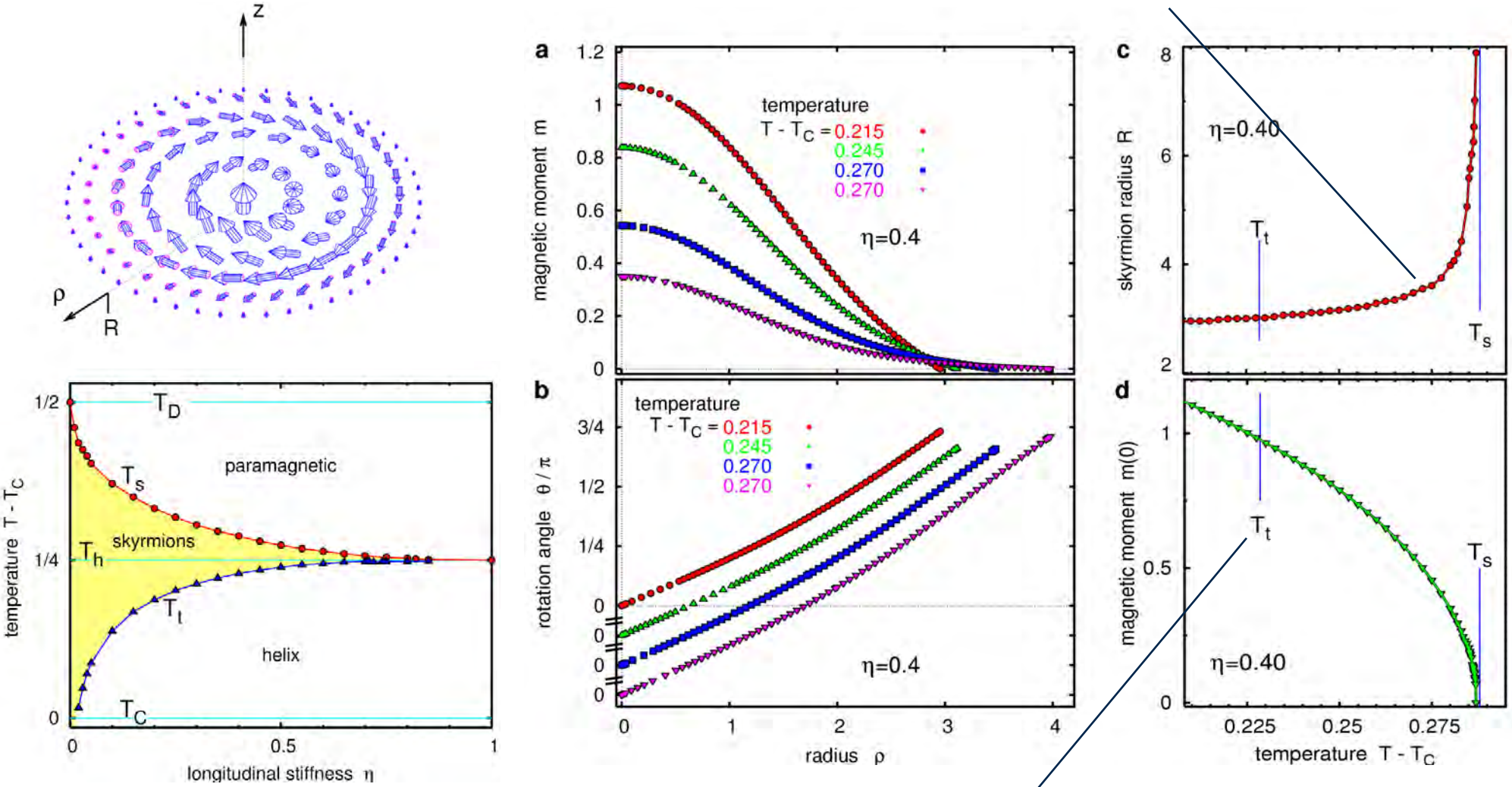
specialized for an isotropic (cubic) ferromagnet
near Curie temperature

$$f_0 = a(T - T_C)m^2 + bm^4 + \text{h.o.t.}$$

$$f_D = D\,\mathbf{m} \cdot \nabla \times \mathbf{m}$$

Skyrmions near transition temperature – soft modulus

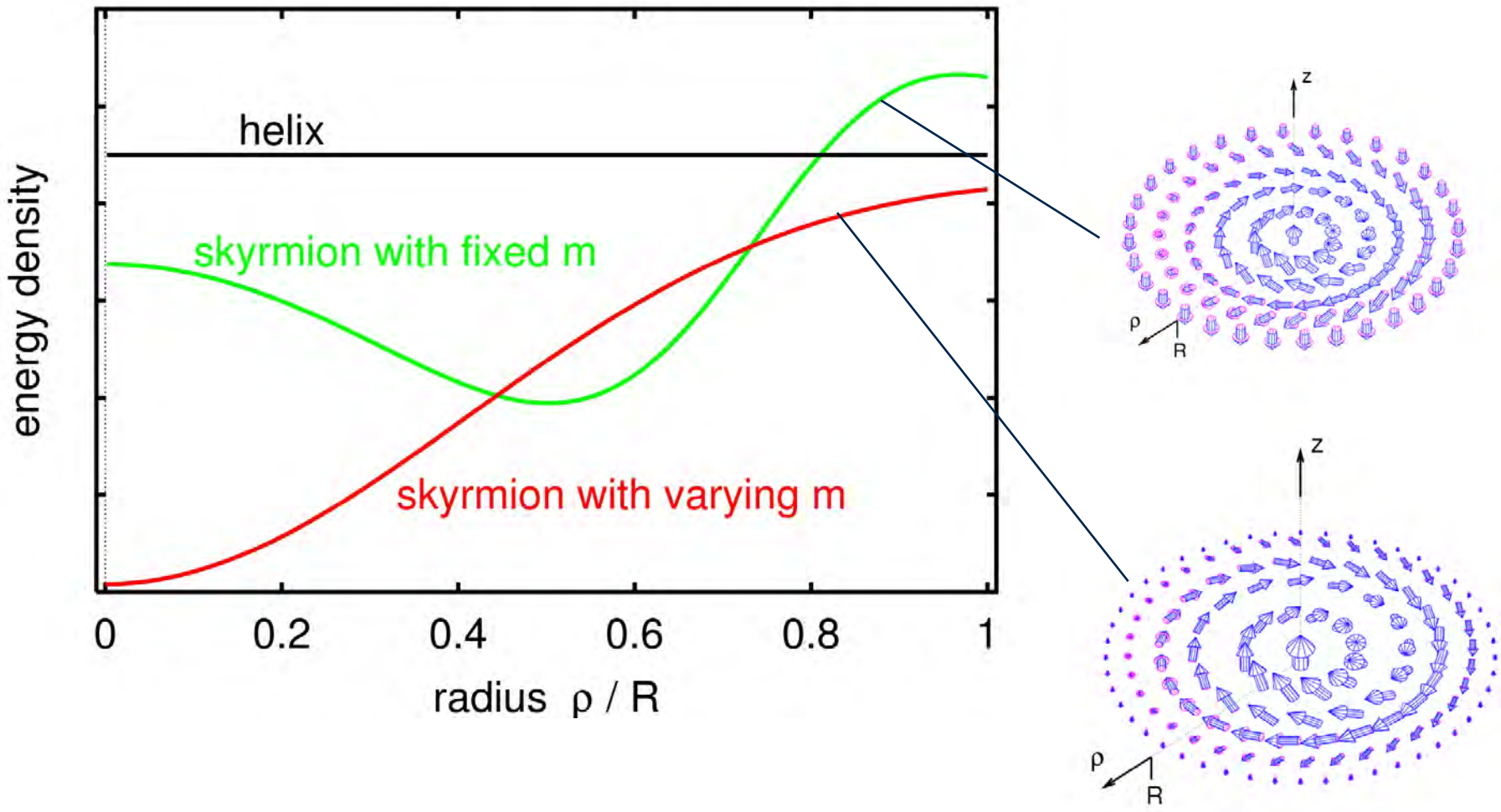
Condensed Skyrmion-phase



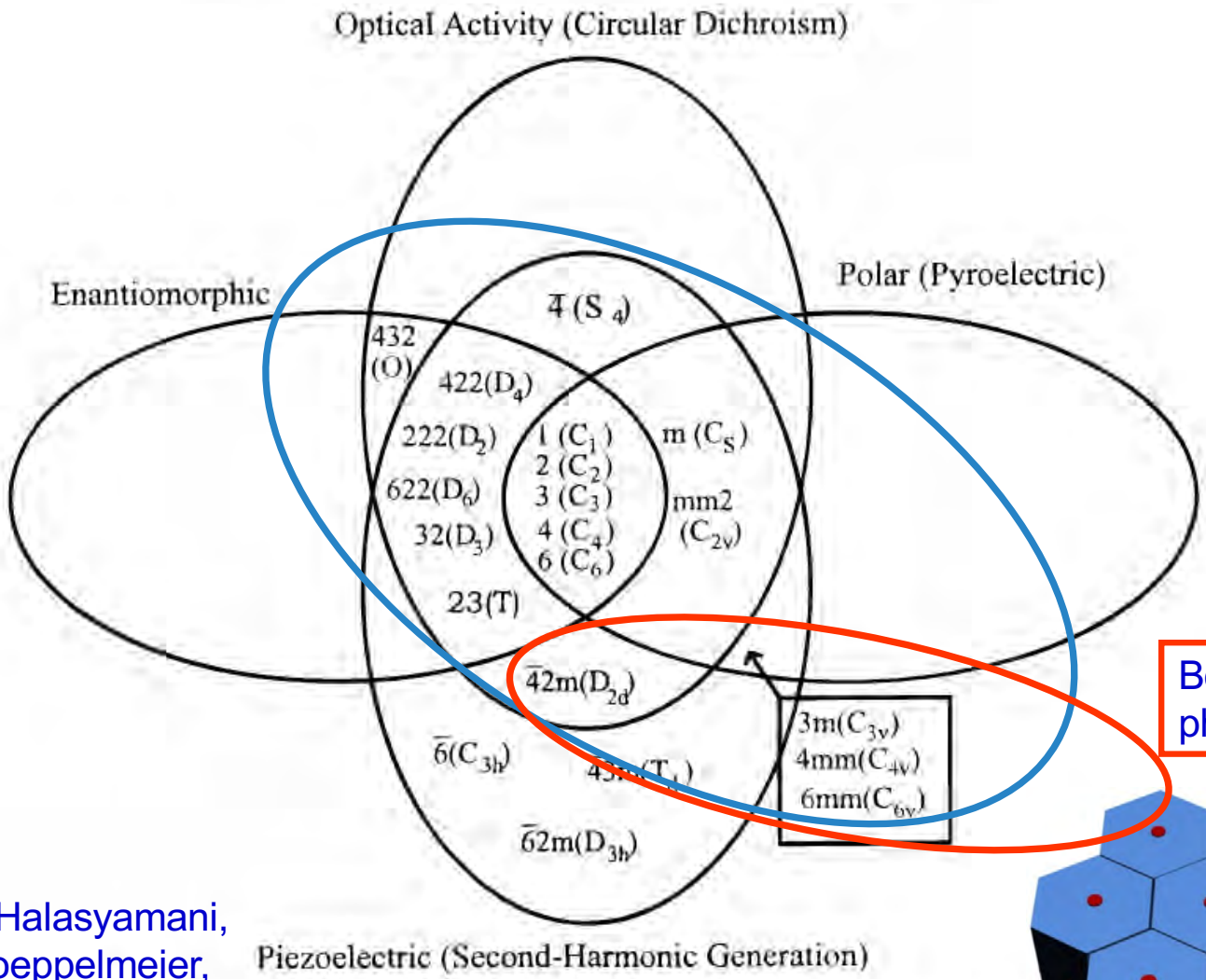
Helix phase reached at a first-order transition

UKR, A.N. Bogdanov, C. Pfleiderer, Nature 442 (2006) 797

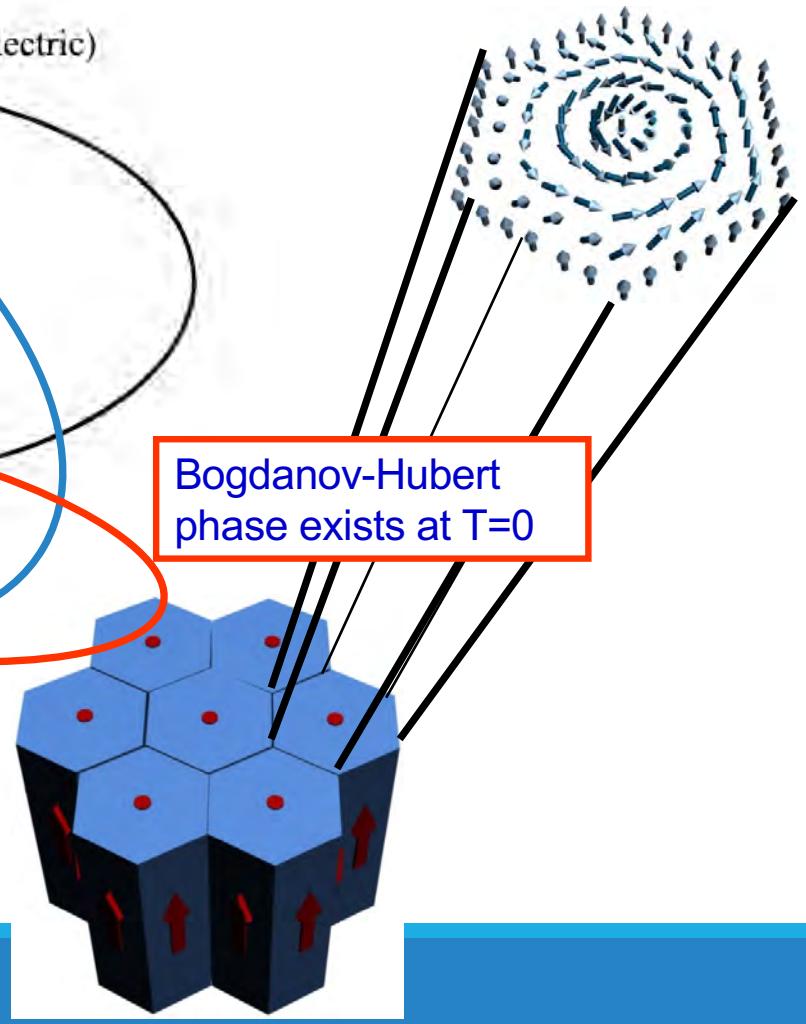
Circularly averaged energy densities



Noncentrosymmetric crystal classes

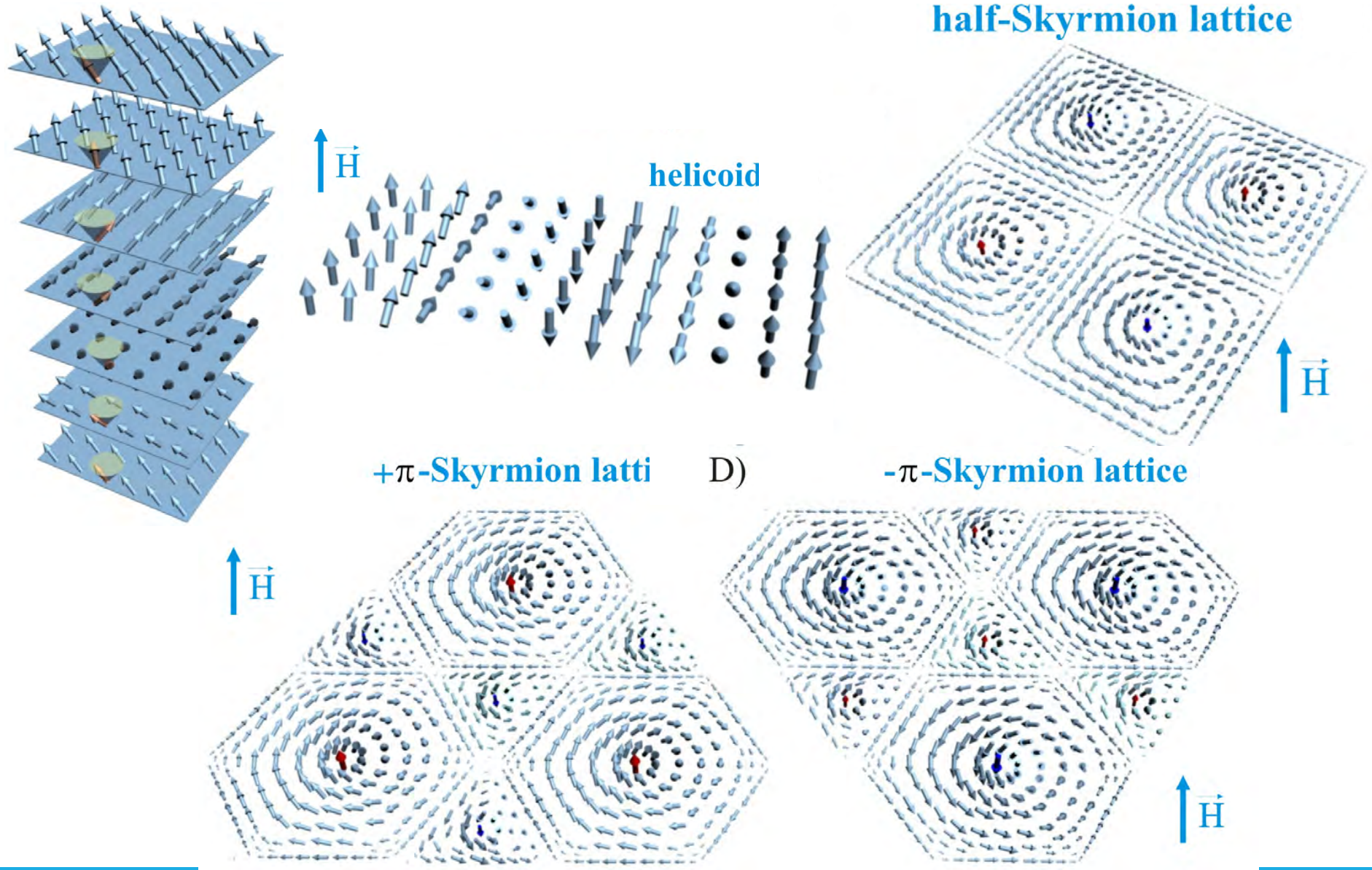


Bogdanov-Hubert phase exists at T=0



P.S.Halasyamani, K.Poeppelmeier, Chem. Mater. 10 (1998) 2753

Extended textures

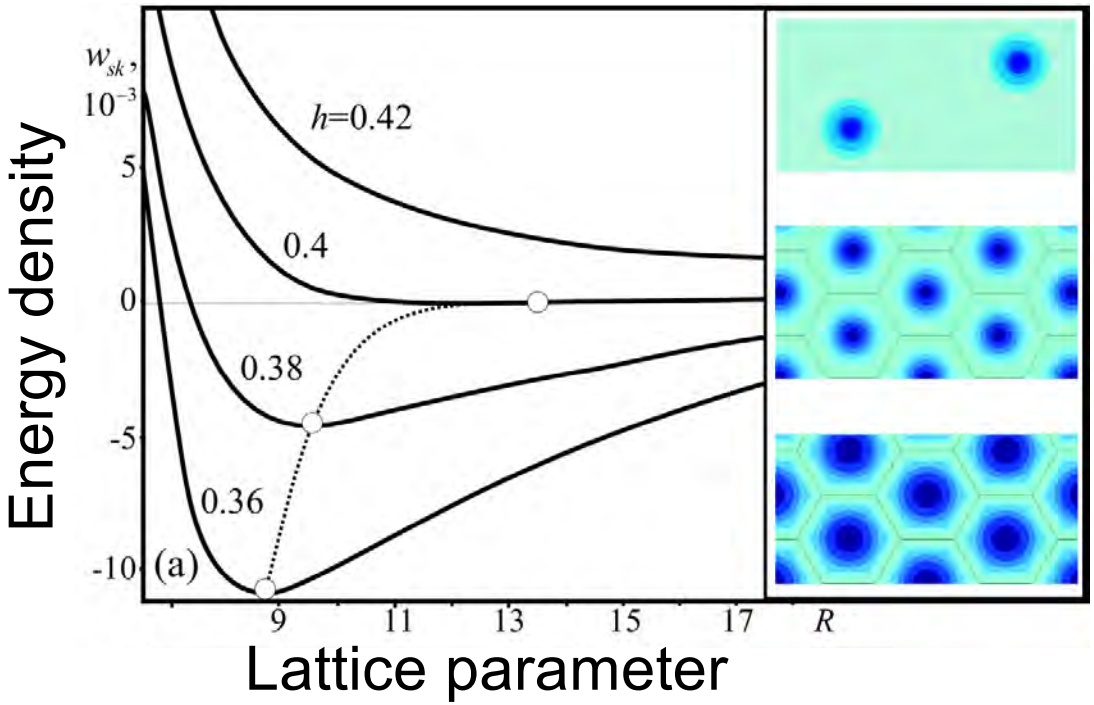


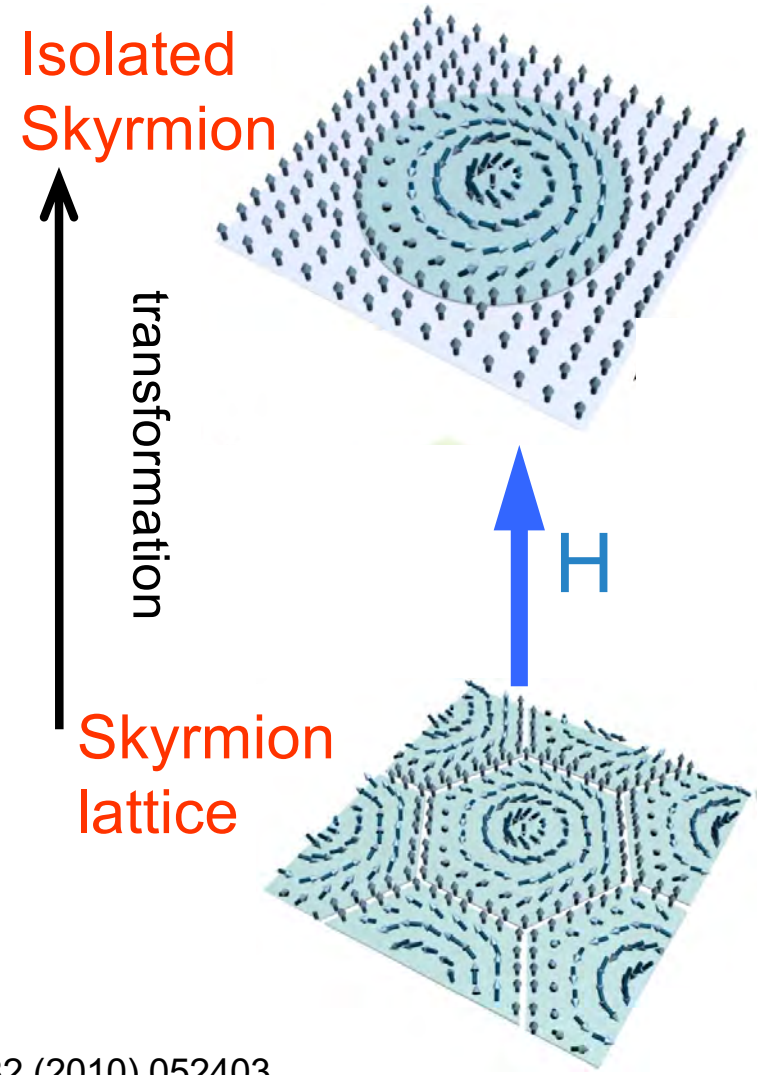
Modulated states

DeGennes (1974) Classification of (continuous) transitions into modulated states

instability type	vs.	nucleation type
Fundamental harmonic mode wavevector **q**		Solitonic units with infinite radius condense into "lattices"
	Fourier amplitudes	
$\phi(nq)/\phi(q) \rightarrow 0$		$\phi(nq)/\phi(q) \rightarrow \text{const}$

Skyrmion lattices transformed into free “particles”





A.B. Butenko, A.A. Leonov, UKR, A.N. Bogdanov, UKR, Phys. Rev. B 82 (2010) 052403

4th set of conclusions

Chiral magnets

- Condensed phases of 1D and 2D modulated & localized states
- narrow competition between “mesophases”
- close analogy with (chiral) liquid crystal systems

Magnetic ordering transition

Bulk cubic chiral helimagnets

A-phase

paramagnetic

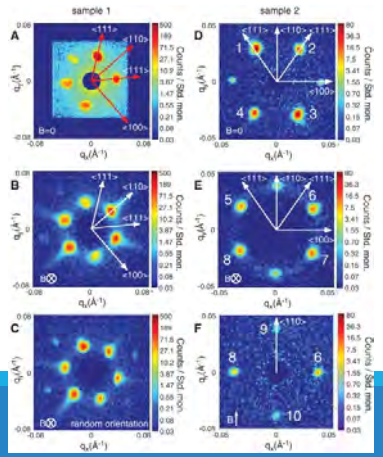
Arai, Ishikawa et al. 1984

phase with transverse modulation

Lebech et al., 1994

-π skyrmion triple-q structure somewhere here

S.Mühlbauer etal., Science 2009



MnSi (B20)

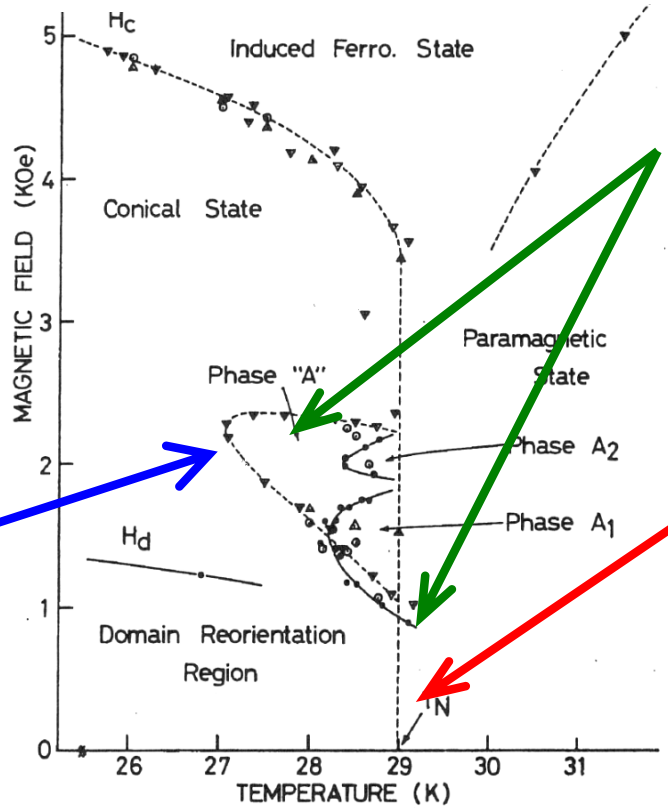


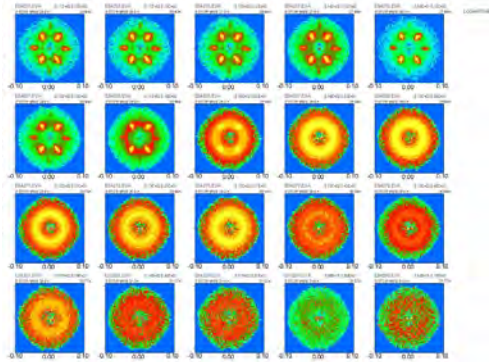
Fig. 11. Phase diagram of MnSi near T_N.

K.Kadowaki, K. Okuda, M. Date, 1982

Traditional approach only 1-q spirals no skyrmions

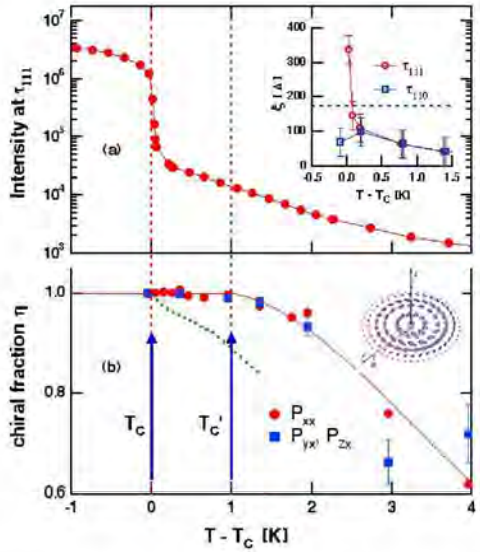
S.V. Maleyev, S.V. Grigoriev et al.

Possibly multiply modulated structures



S.A. Sorensen, B. Lebech, Riso 1999

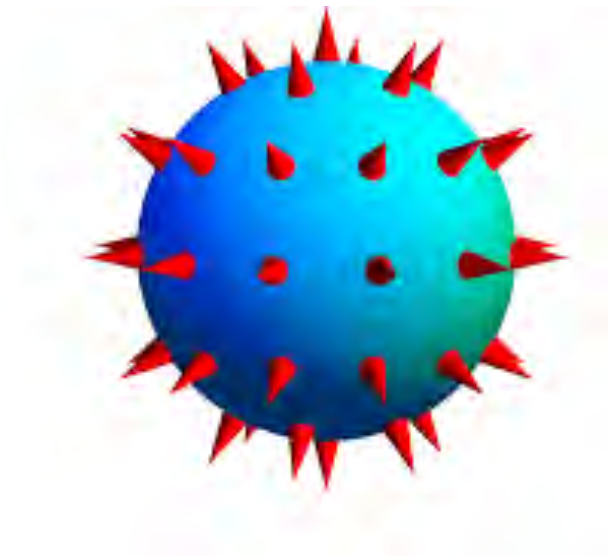
Fully chiral precursor at H=0

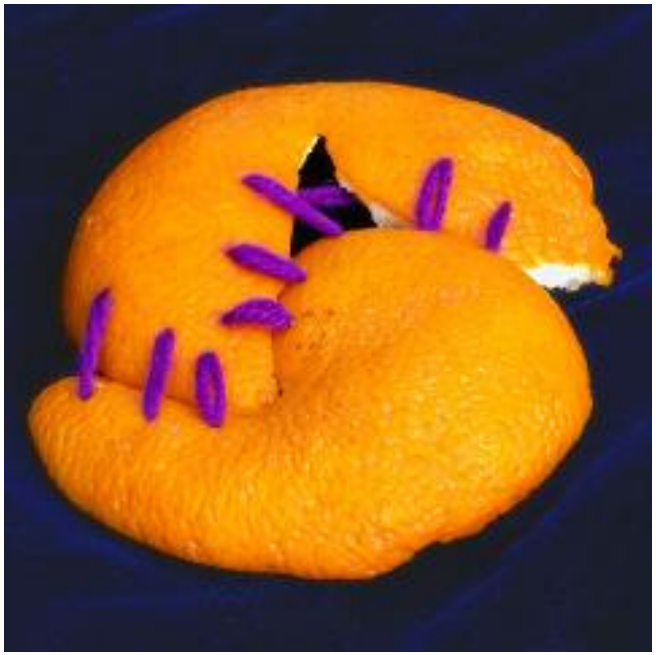


C.Pappas et al., PRL 2009; PRB 2011

Prehistory

Geometric frustration – orange peel carpet





Sadoc, Kleman P.W. Anderson, Sethna, Kivelson ....

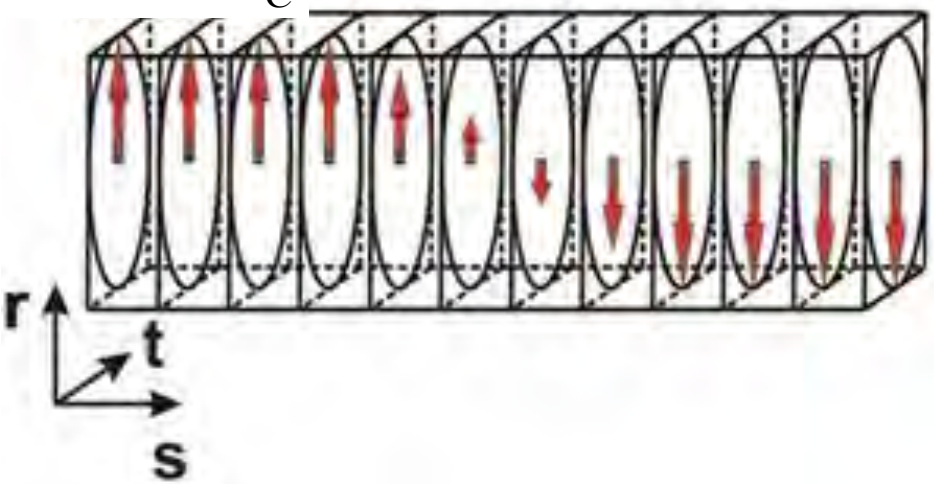
Magnetic walls near T_C

Transformation of Bloch walls into Ising walls
in strong easy axis ferromagnets

low $T << T_C$

$+ a_0 (T\text{-}T_C) m^2 + b\, m^4$

T near T_C



$$f = A\,(\nabla \boldsymbol{\mu})^2 + \mathrm{A}\,\eta\,(\nabla \mathrm{m})^2$$
$$- k_1\,(\mathbf{m}\cdot\mathbf{r})^2$$
$$+ a_0\,(T\text{-}T_C)\,m^2 + b\,m^4$$
$$- \mathbf{m}\cdot\mathbf{h}^{(e)} - (1/2)\,\mathbf{m}\cdot\mathbf{h}^{(d)}$$

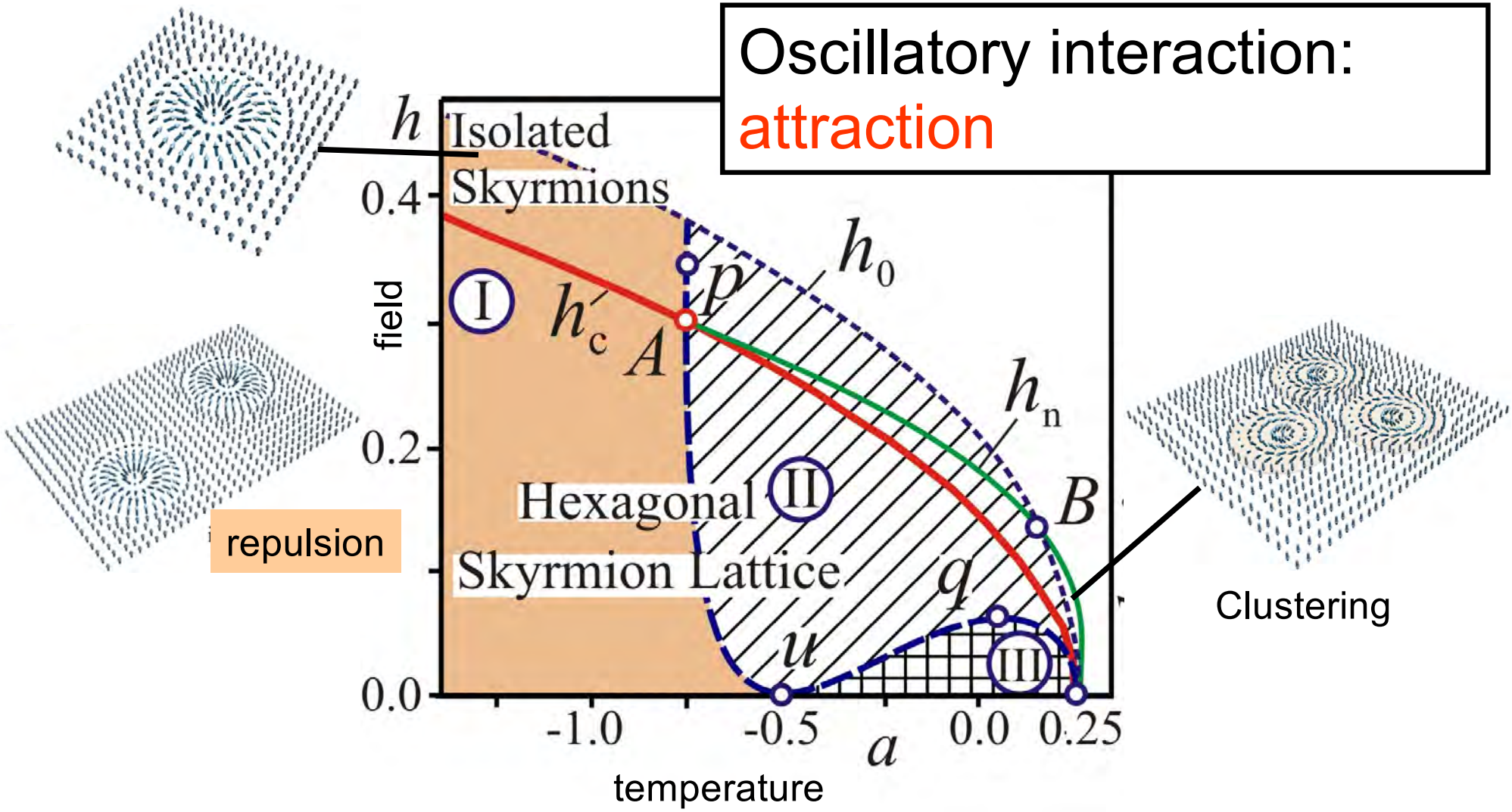
Bulaevskii Ginzburg

$$a_0\,(T\text{-}T_C) \sim k_1$$

easy axis

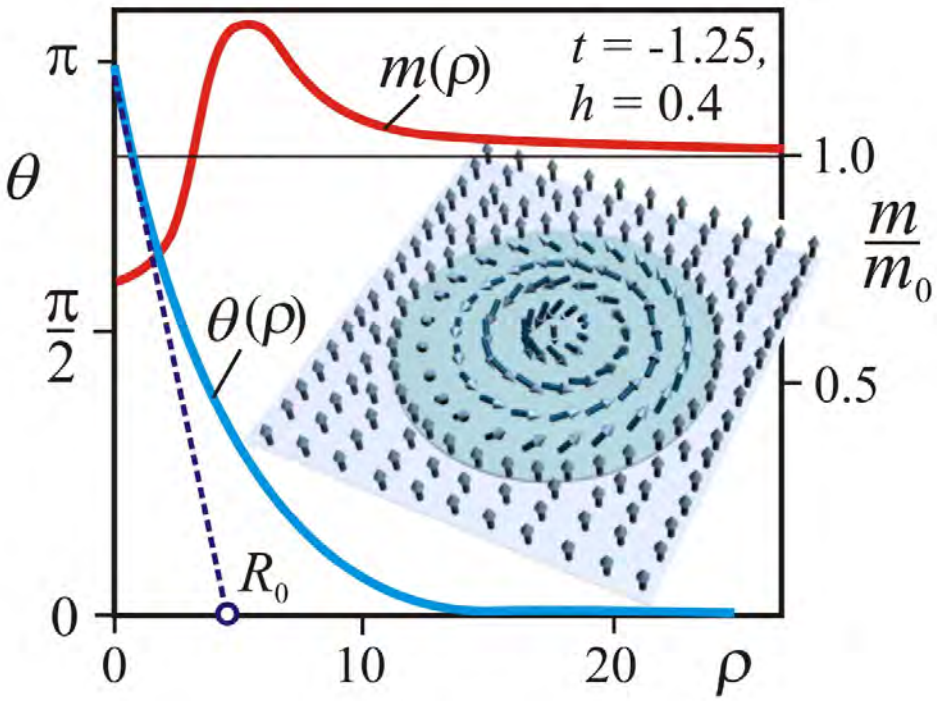
Confinement of Skyrmions

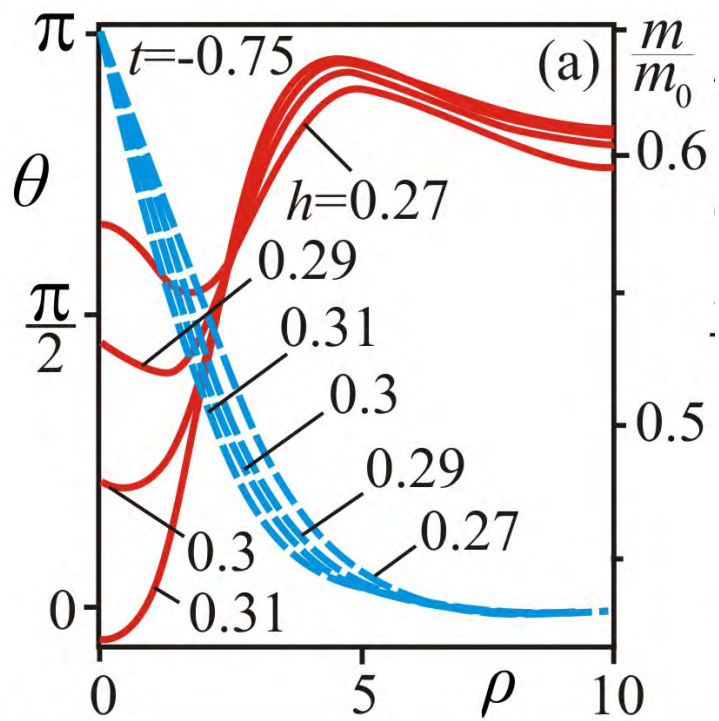
Oscillatory interaction:
attraction



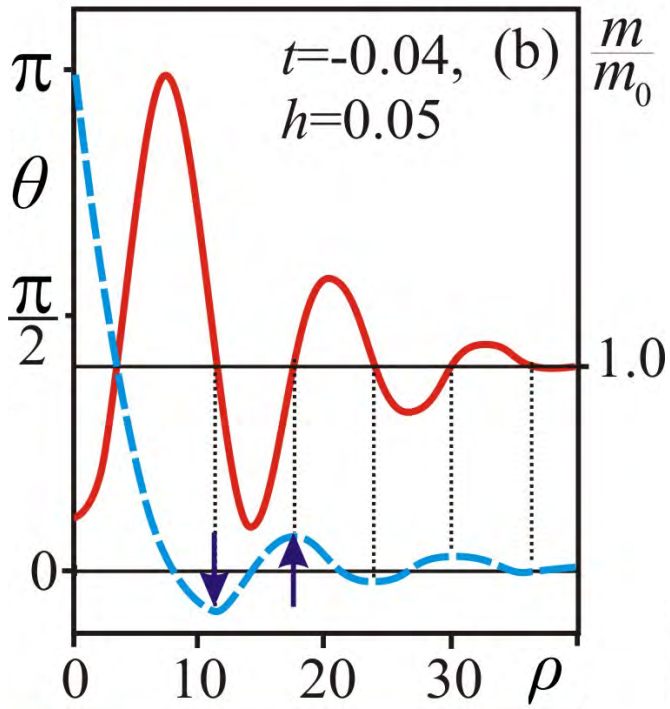
Isolated chiral skyrmion

Longitudinal and angular part of magnetization always coupled



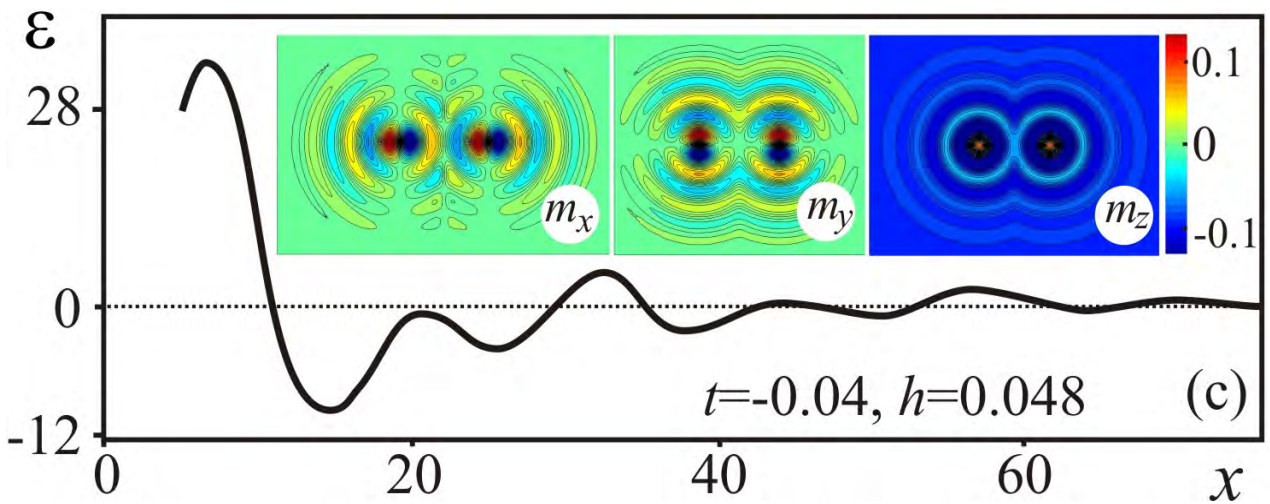


Isolated Skyrmion in the precursor regime



Oscillatory
Skyrmion-Skyrmion couplings

- frustration
- defects



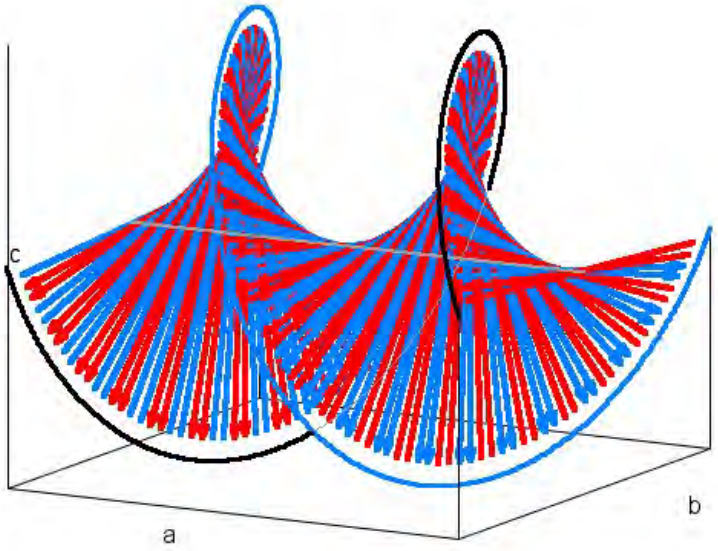
5th set of conclusions

Precursor range of chiral magnets

- Harmonic contents / Fourier modes of condensed phases are never trivial
- localized non-linear excitations of paramagnetic background
- Longitudinal & directional parts of order-parameter are intertwined / cannot be separated
- Defect-ordered states exist

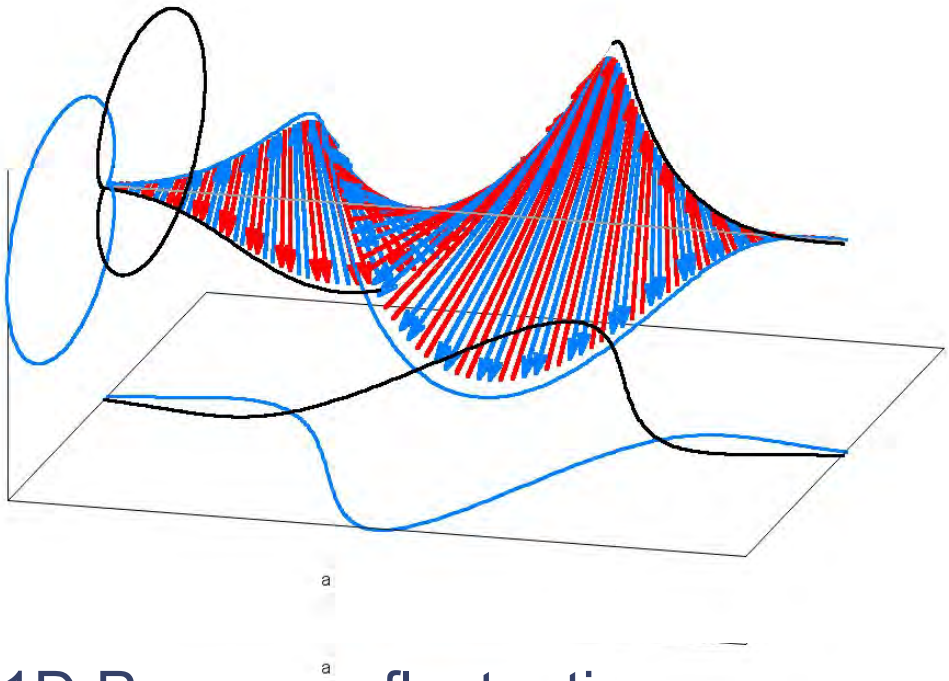
Beyond simple magnetic order

A more messy case : Incommensurate (magnetic) phases



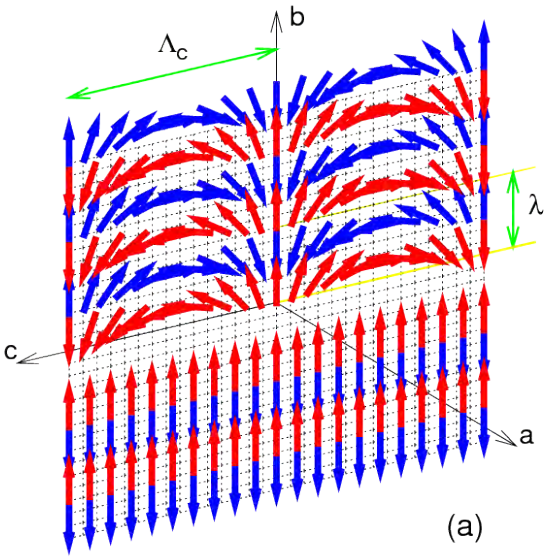
OP-textures

Basic 1D incommensurate state twisted by relativistic DM coupling



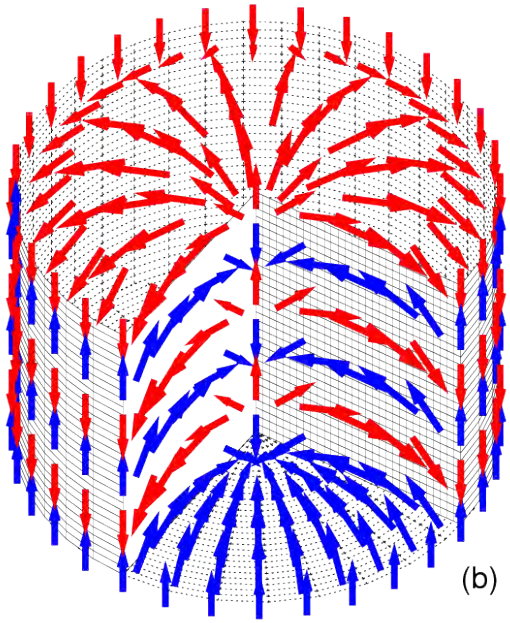
1D Precursor fluctuation with inhomogeneous magnitude of the OP

Possible textures

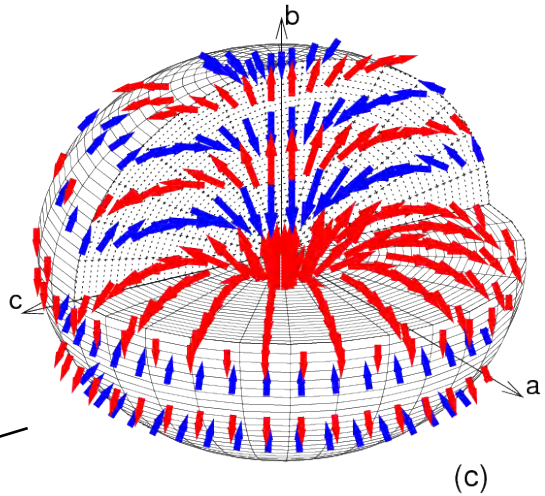


helical
incommensurate
1D-phase

Double twist



Ball soliton



Solitonic constituents / molecules of precursor or meso-phases

Ph. Materne, C. Koz, UKR, M. Doerr, T. Goltz, H. H. Klauss, U. Schwarz, S. Wirth , S. Rößler
PRL 115, 177203 (2015)

Proper & improper Dzyaloshinskii textures

Proper textures: simple continuum model with Lifshitz invariants affecting a “primary” order parameter.

Best-known examples :

Cholesterics and blue phases of chiral nematics

Chiral spiral states and skyrmionic textures in acentric / gyrotropic cubic crystals

Improper Dzyaloshinskii textures

- Coupling of different order parameters via Lifshitz-type coupling

Lowest-order Lifshitz-type terms only operative near multicritical points

Example :

multisublattice centrosymmetric magnet

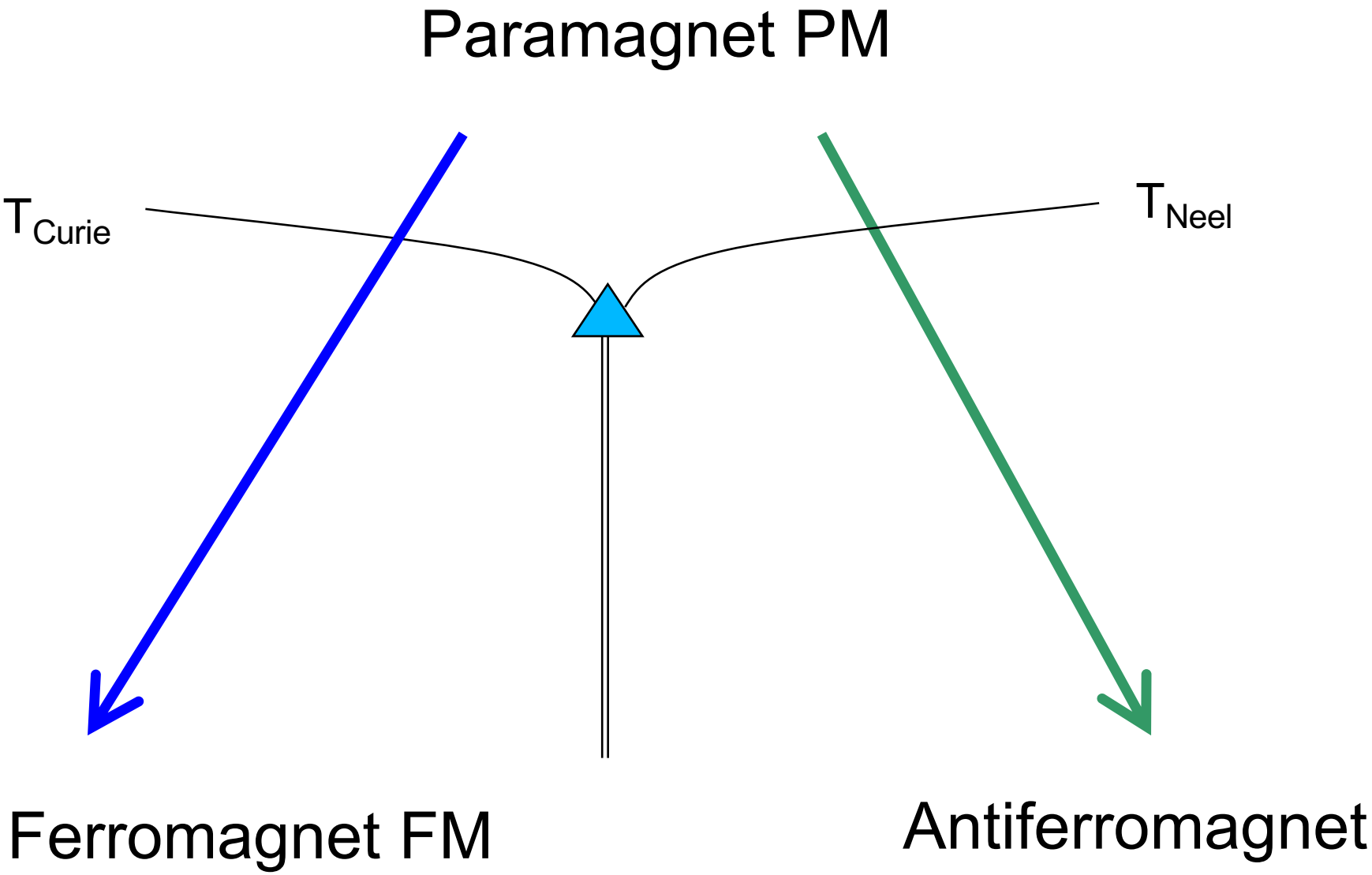
L Antiferromagnetic mode odd under inversion

F Ferromagnetic mode axial vector

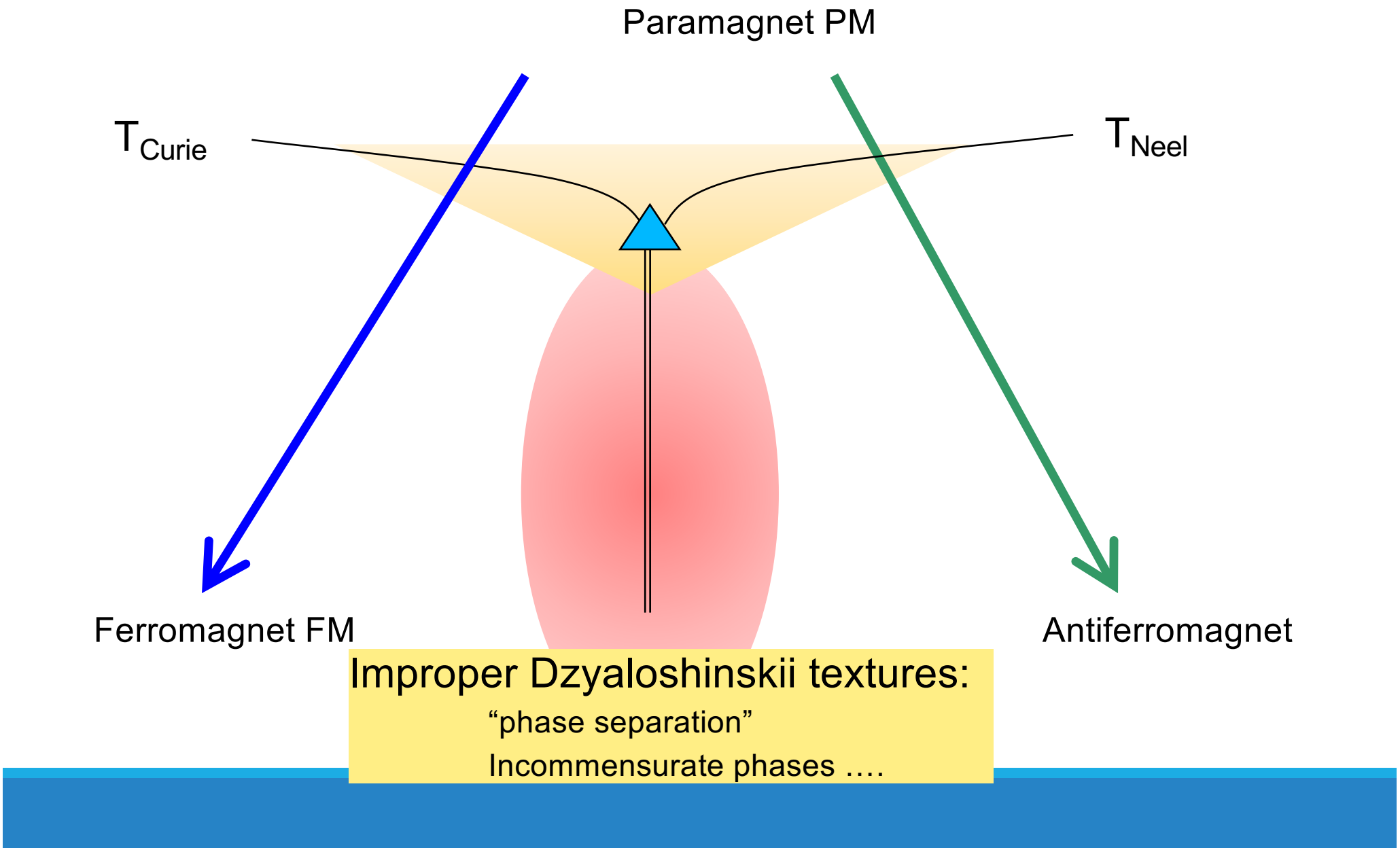
Lifshitz-type invariant $\mathrm{F}_\alpha \partial_\beta \mathrm{L}_\gamma - \mathrm{L}_\alpha \partial_\beta \mathrm{F}_\gamma$

Mechanism for phase coexistence

Bicritical phase diagram



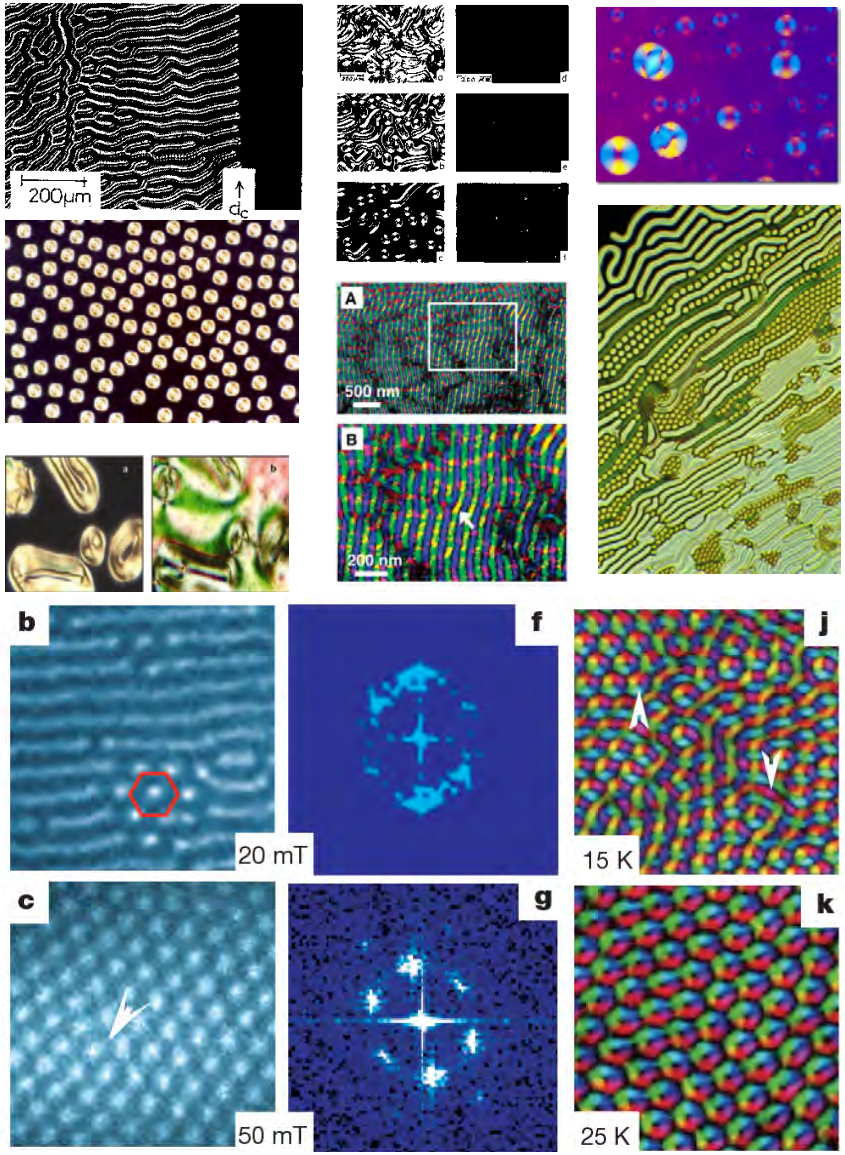
Impact of Lifshitz-type invariants



“…the fairest universe is but a heap of rubbish piled up at random ...”

MD-simulated metallic glass $Ni_{37.5}Zr_{37.5}V_{25}$

Many thanks



A.N. Bogdanov, A.A. Leonov,
A.B. Butenko, N.S. Kiselev

T.L. Monchesky and team U Dalhousie

H. Wilhelm, M. Baenitz et al.

Diamond & MPI-CPFS Dresden

D. Makarov and his team IFW Dresden

O. Janson, I. Rouchosatzkis, H. Rosner

A.A. Tsirlin et al.

A.J. Nayak, S. Chadov et al. C.Felser

MPI-CPFS Dresden

S. Rößler, Ph. Materne. H.H.Klauss et al.

S.Blügel, M.Bode, H.Eschrig,
I.Keszmarki, A.Möbius, K.-H.Müller,
C.Pfleiderer, M.Richter, A.A.Shestakov,
R.Schäfer, R.L.Stamps, J. van den Brink,
M.Wolf

and support by DFG, DAAD, EU …