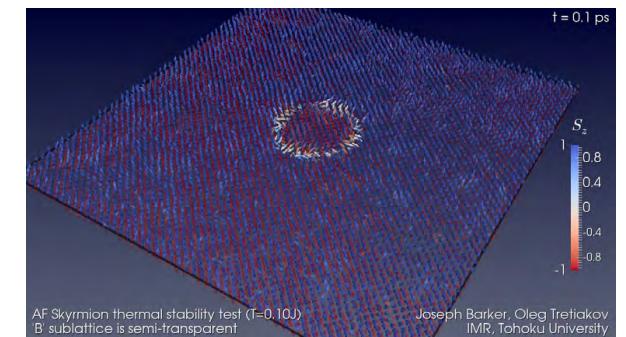
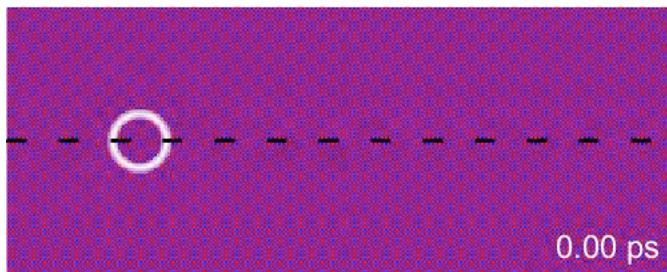


# Antiferromagnetic Skyrmions



***Oleg A. Tretiakov***

*Institute for Materials Research, Tohoku University, Japan*

# Collaborators:

*Koji Sato, Nailul Hasan (Tohoku University, Japan)*

*Artem Abanov (Texas A&M University, USA)*

*Geoff Beach (MIT, USA)*

*Ivan Ado, Misha Titov (Radboud University, Netherlands)*

*Kai Litzius, Benjamin Kruger, Mathias Klaeui (University of Mainz, Germany)*



Joe Barker  
(Tohoku Univ.)



Ksenia Chichai  
(IKBFU, Russia)

## Grant Support:

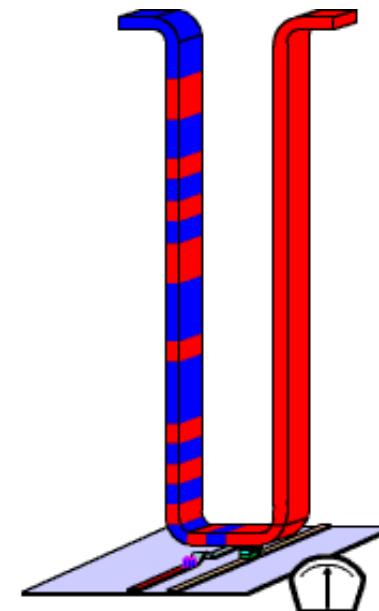


# Future Memory Trends

In the 1980's

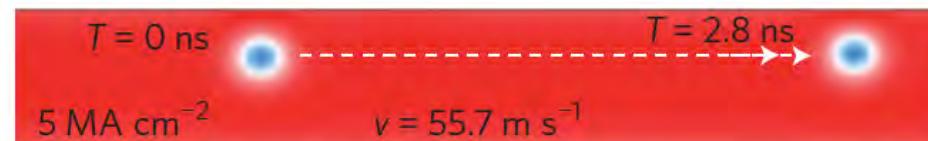


In the 2010's ?



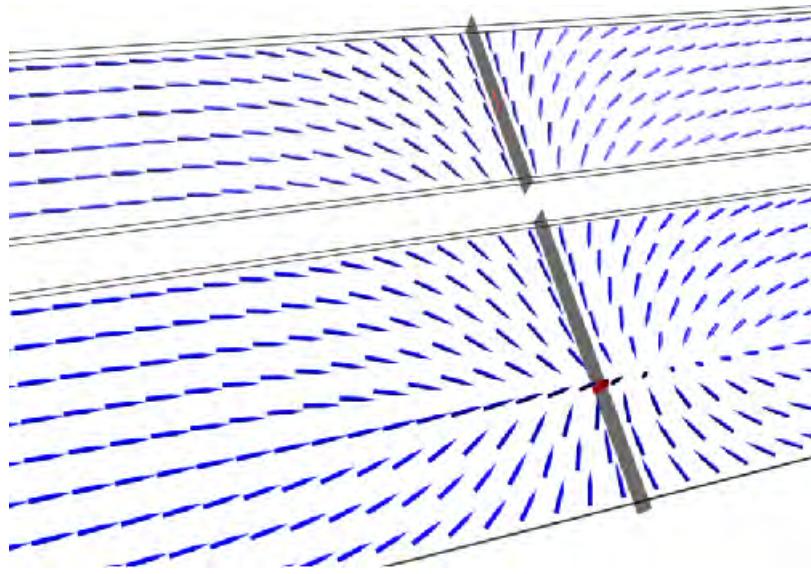
S. Parkin, US patent

In the 2020's ?

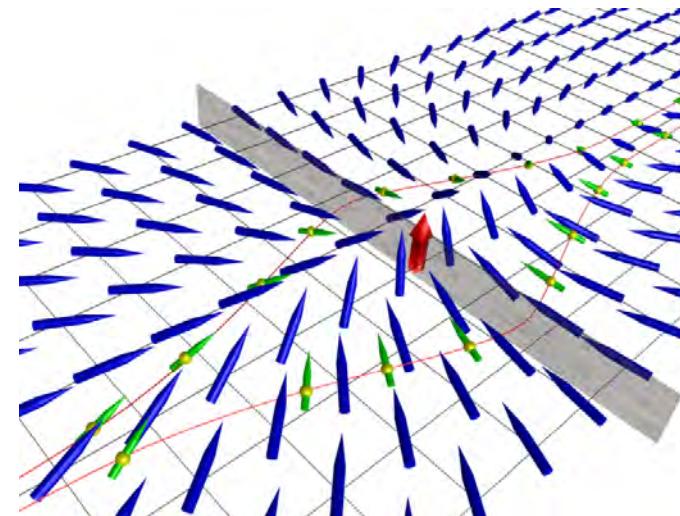


A. Fert et al., Nature Nano (2013)

# Topological Domain Walls and Skyrmions



meronic spin texture:

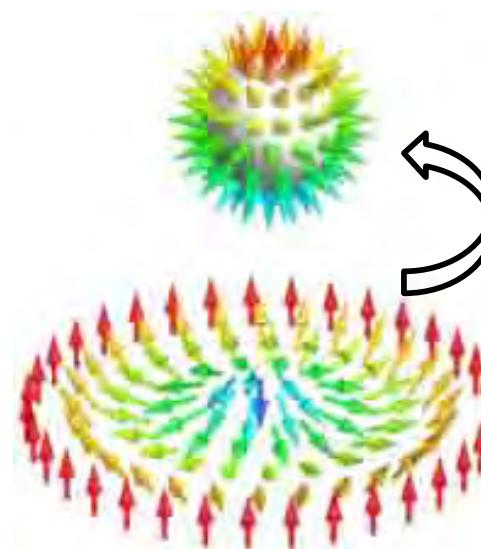


skyrmionic spin texture:

Winding (skyrmion) number:

$$Q = \frac{1}{4\pi} \int d^2r \mathbf{m} \cdot \frac{\partial \mathbf{m}}{\partial x} \times \frac{\partial \mathbf{m}}{\partial y}$$

- Rigid textures
- Different soft modes



Bogdanov & Yablonskii,  
JETP 68, 101 (1989)

# Types of Skyrmions



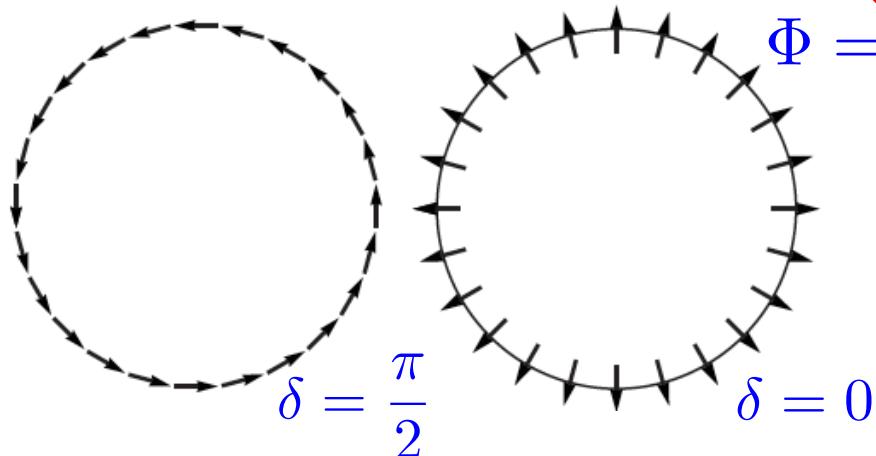
Winding number:

$$Q = \frac{1}{4\pi} \int d^2r \mathbf{m} \cdot \frac{\partial \mathbf{m}}{\partial x} \times \frac{\partial \mathbf{m}}{\partial y}$$

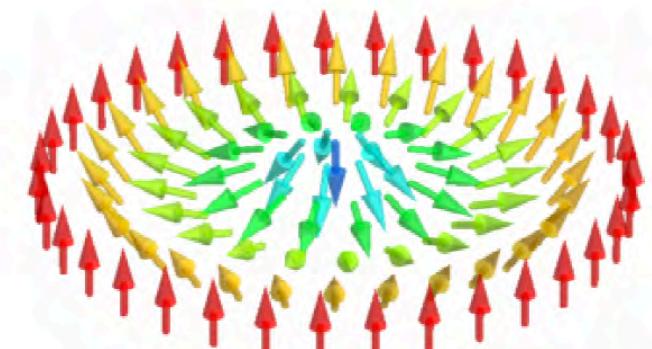
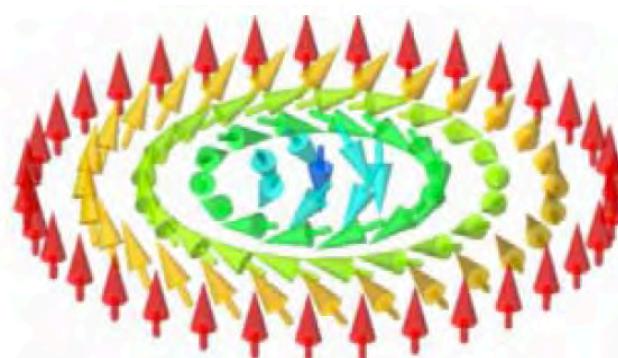
$$\mathbf{m} = (\cos \Phi \sin \theta, \sin \Phi \sin \theta, \cos \theta)$$

$$Q = pq$$

Two types of winding:



Neel and Bloch Skyrmions:

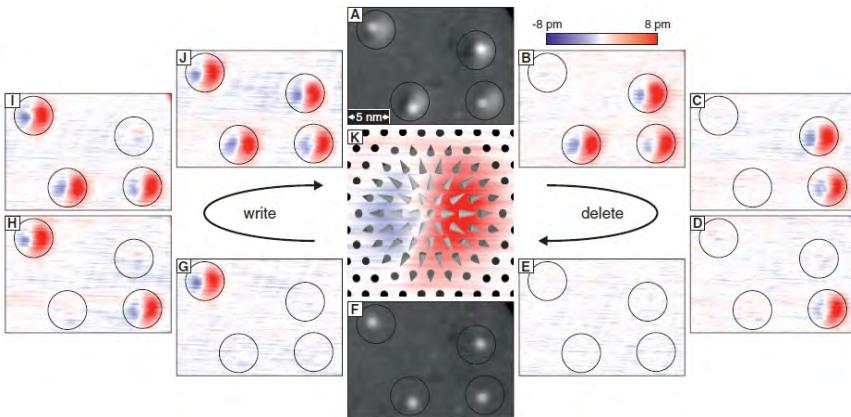


Stabilized by dipolar interaction

Stabilized by DMI

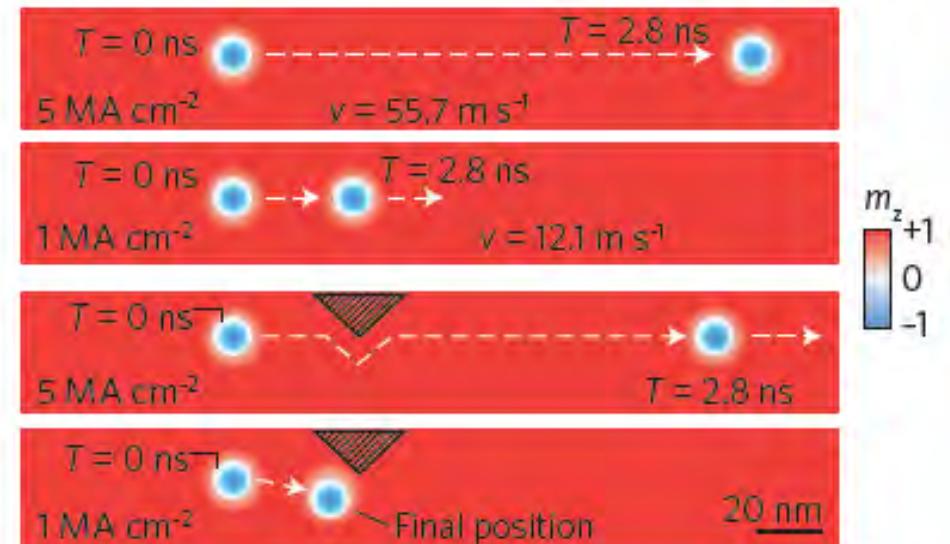
# Recent Progress: Single Skyrmions

## Skyrmion Creation (Experiment)



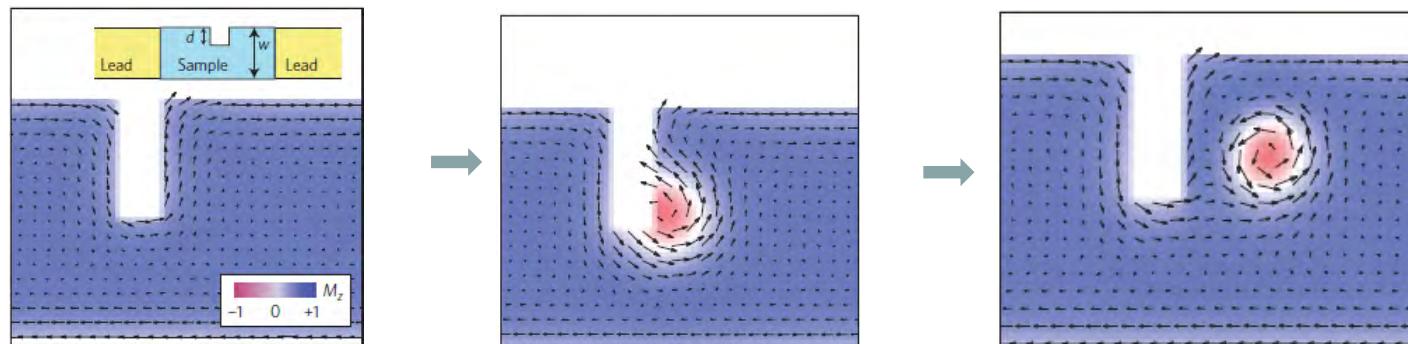
N. Romming et.al, Science (2013)

## Skyrmion motion induced by current (simulations)



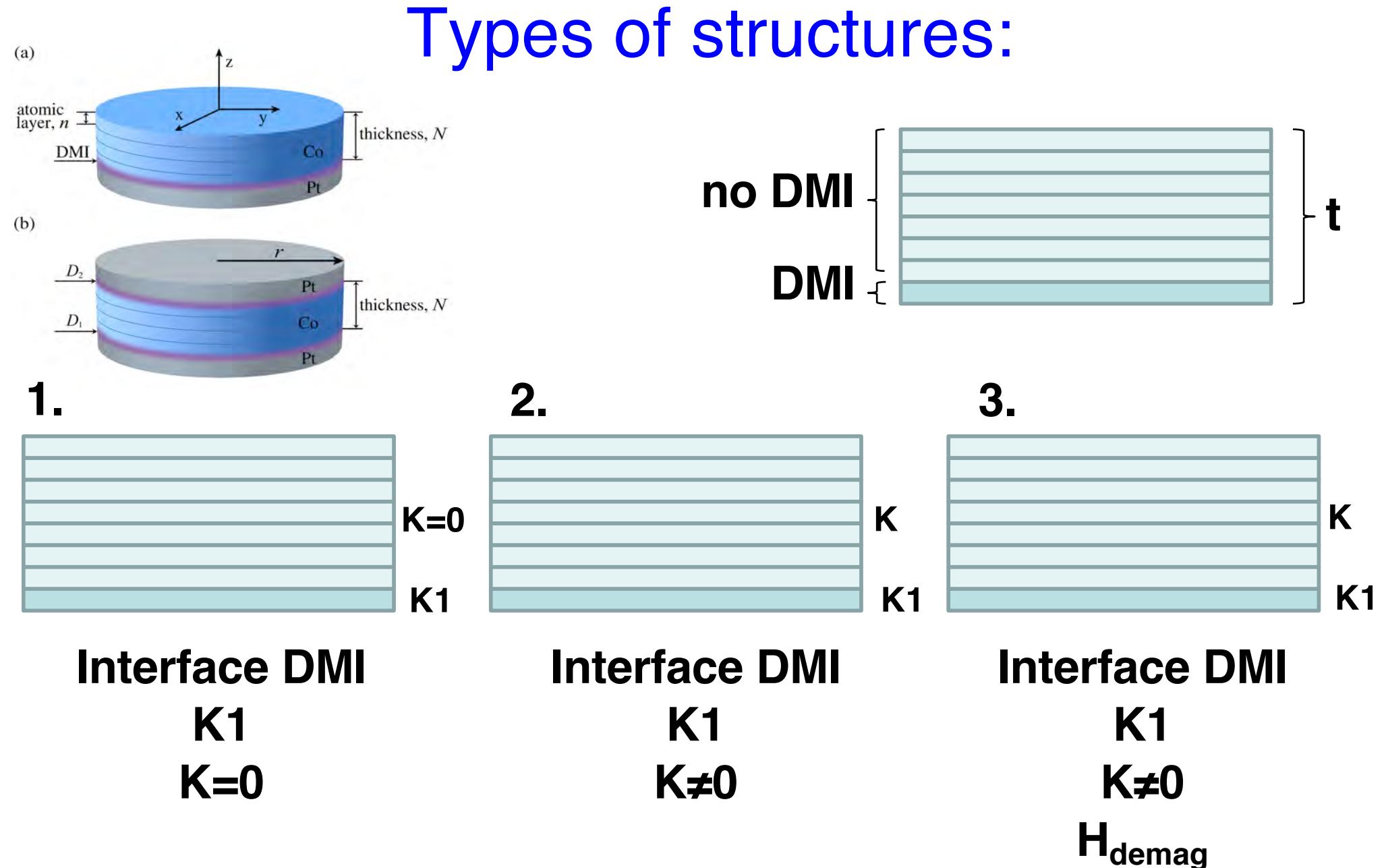
J. Sampaio et.al, Nat. Nanotech. (2013)

## Skyrmion Creation (simulations)



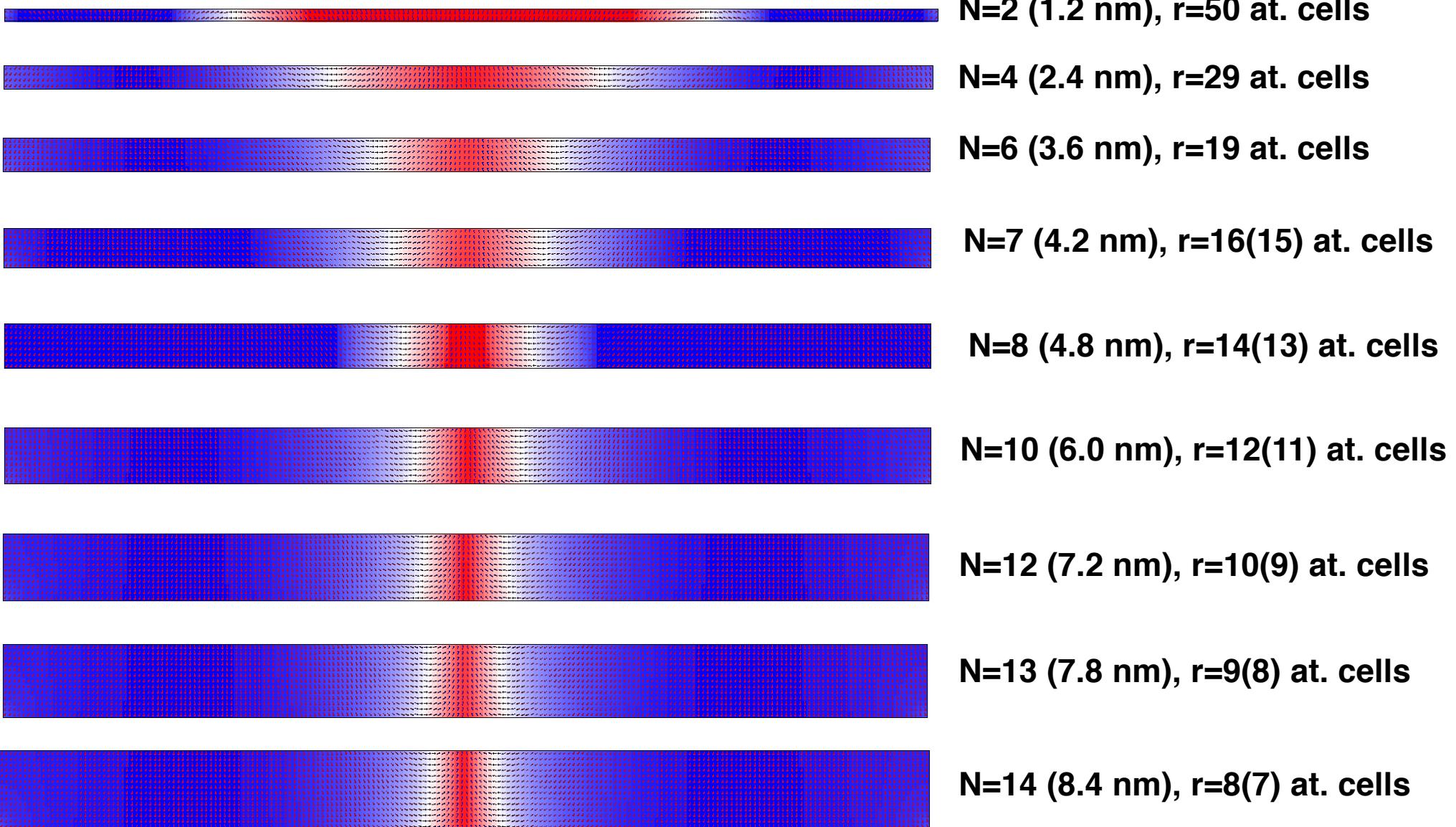
J. Iwasaki et.al, Nat. Nanotech. (2013)

# Skyrmions in Multilayers



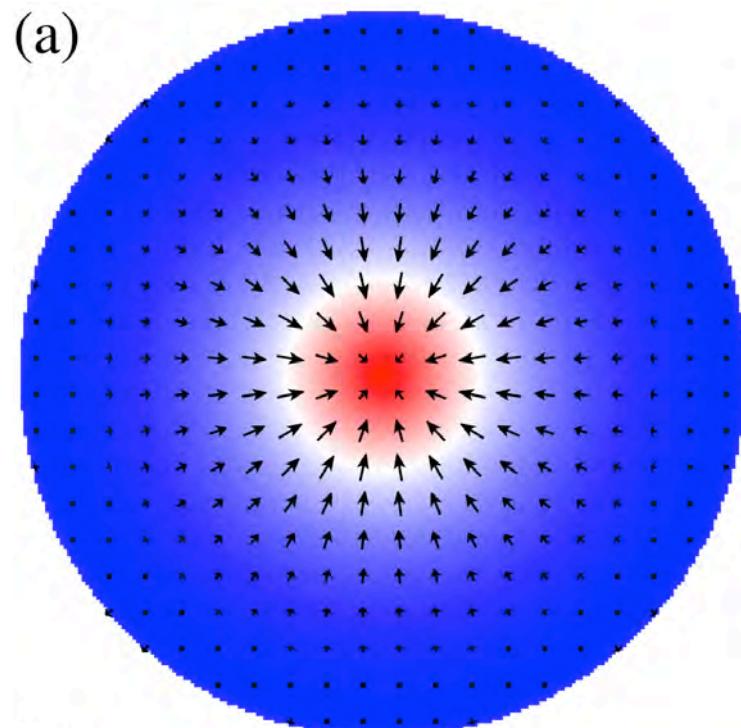
# Multilayer skyrmions: thickness dependence

$D/D_C=1.95$  ( $D=7.0$ )

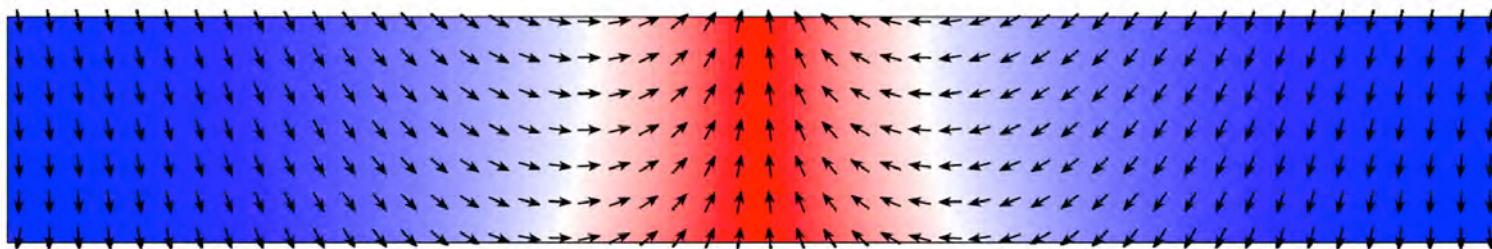


# Skyrmions in Disks with interfacial DMI

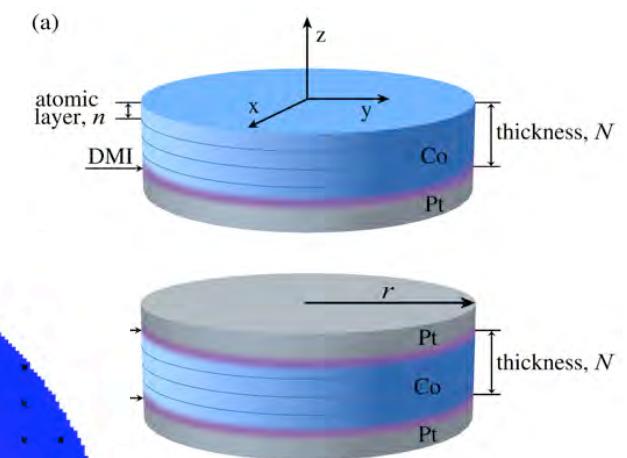
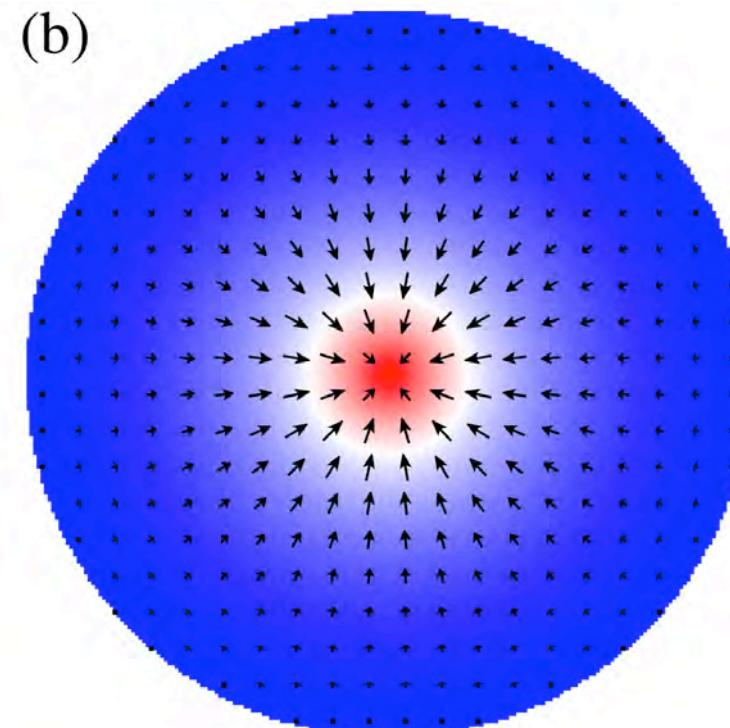
**Bottom (DMI)**



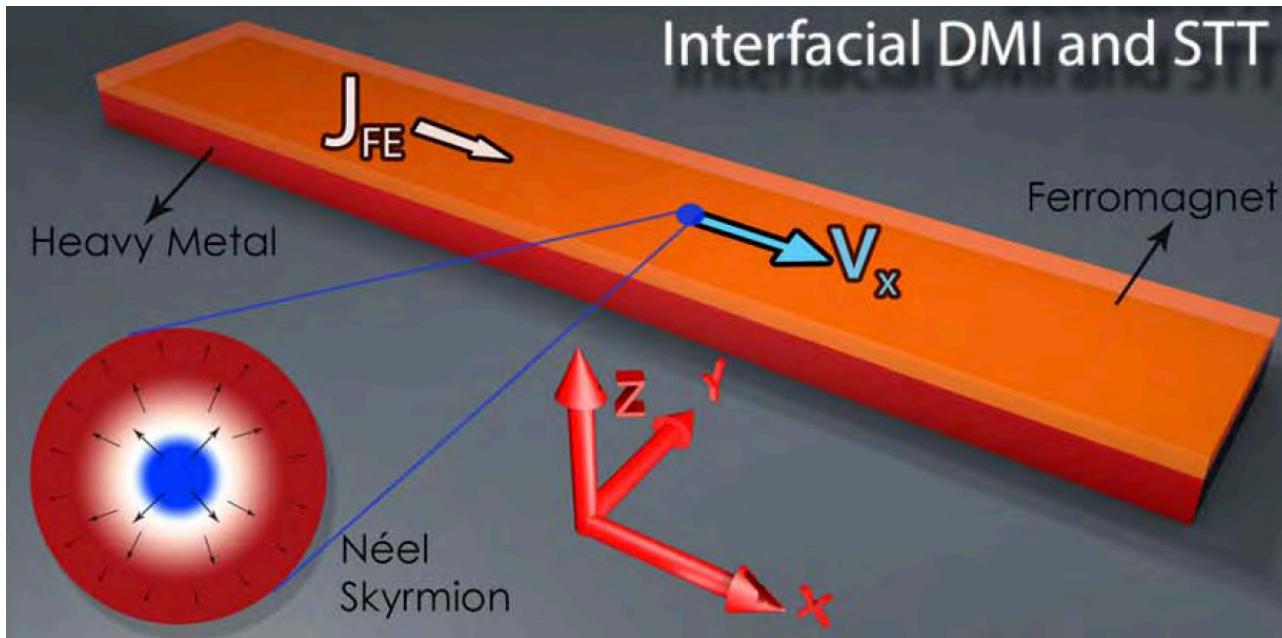
(c)



**Top**



# Spin-orbit Torques



from Tomasello *et al.* (2015)

Two main types of spin-orbit torques: Damping and field-like

Heavy-metal/FM Hamiltonian:

$$H_p = \xi_p + \alpha_R (\sigma \times p)_z + M_x \sigma_x + M_z \sigma_z + V(\mathbf{r})$$

disorder potential

# Magnetic Hamiltonian

Atomistic Formalism:

$$\mathcal{H} = - \sum_{ij} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - \sum_{ij} (\mathbf{d}_{ij} \times \hat{z}) \cdot (\mathbf{S}_i \times \mathbf{S}_j) - \sum_i k_z S_{i,z}^2$$

Exchange      Dzyaloshinskii-Moriya      Uniaxial anisotropy  
Interaction



Competition gives skyrmion

Micromagnetic Formalism:

$$E = \int A(\nabla \mathbf{m})^2 d^3 \mathbf{r} - \int K m_z^2 d^3 \mathbf{r} + t \int D \left[ \left( m_x \frac{\partial m_z}{\partial x} - m_z \frac{\partial m_x}{\partial x} \right) + \left( m_y \frac{\partial m_z}{\partial y} - m_z \frac{\partial m_y}{\partial y} \right) \right] d^2 \mathbf{r}$$

# Spin-Orbit Torques & Skyrmion Dynamics

LLG equation with spin-orbit torques:

$$\dot{\mathbf{m}} = \mathbf{f} \times \mathbf{m} + \alpha_G \mathbf{m} \times \dot{\mathbf{m}}$$

with

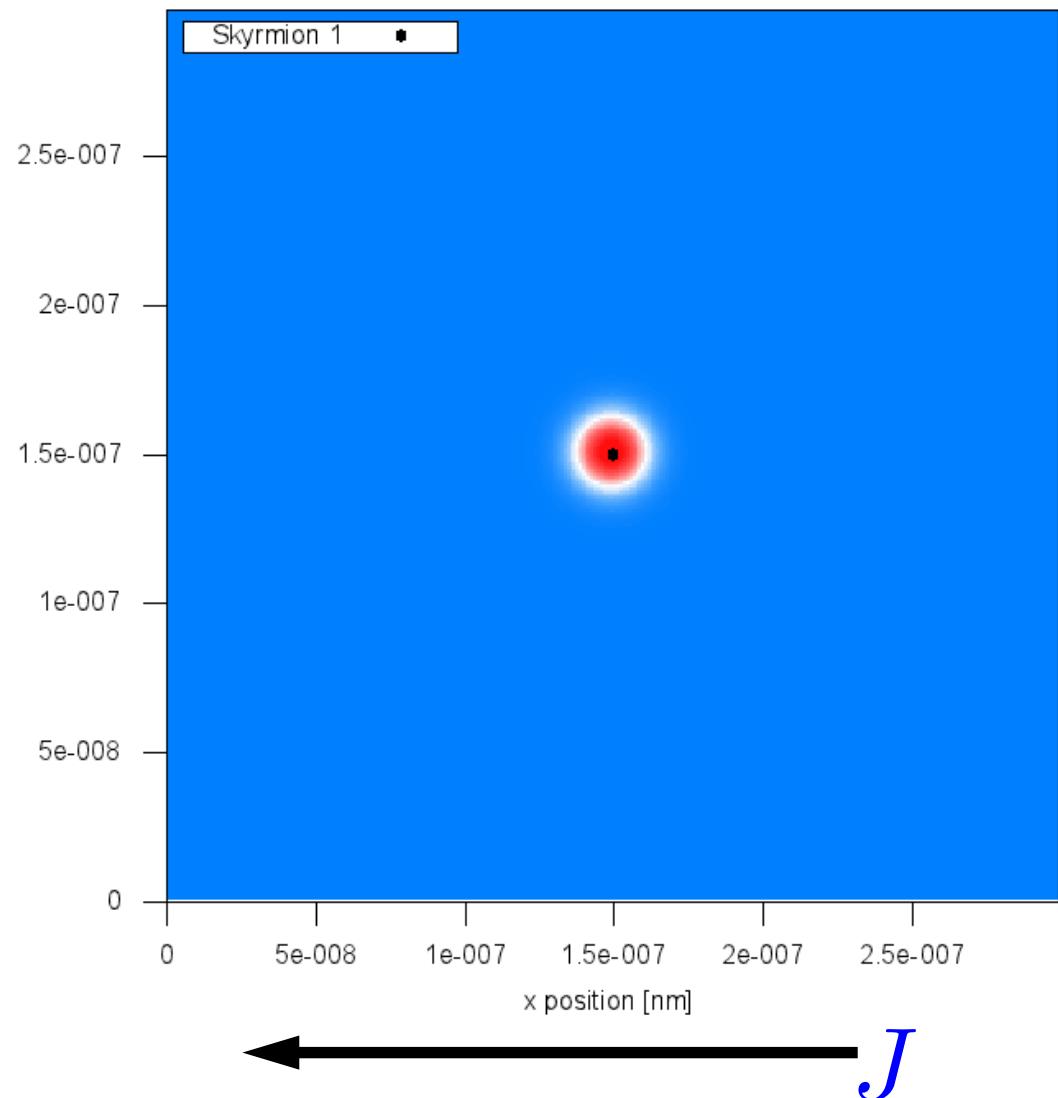
$$\mathbf{f} \times \mathbf{m} = \gamma [\mathcal{H} \times \mathbf{m}] + \mathbf{T}$$

Generalized Thiele equations of motion:

$$(Q\hat{\epsilon} - \hat{D})\mathbf{v} = \mathbf{F}$$

$\mathbf{F}$  is force from SOT

## SOT Skyrmion Dynamics



# Skymion equations of motion

Thiele's equation (collective coordinates approach):

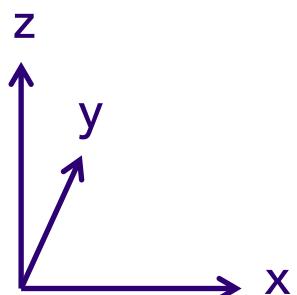
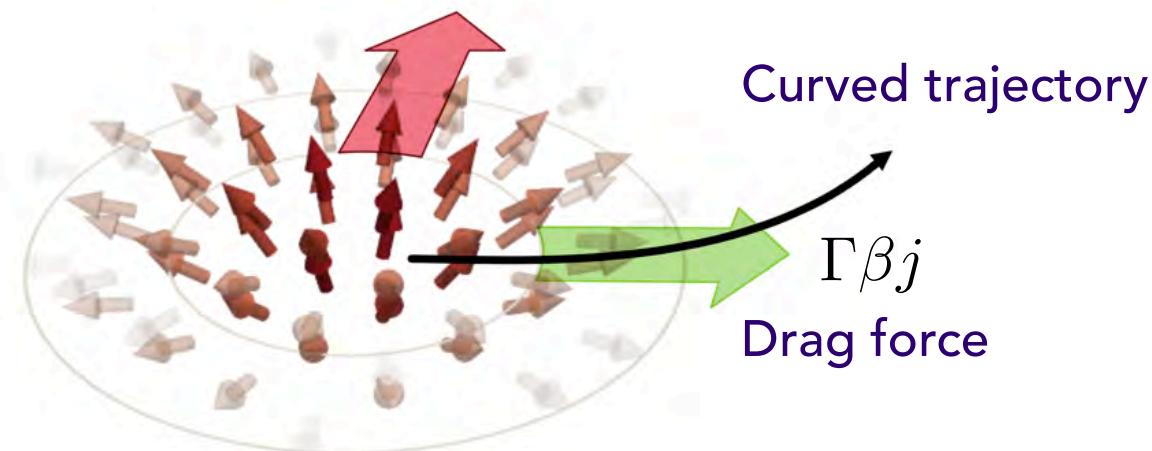
$$G \times (\mathbf{j} - \mathbf{v}) + \Gamma(\beta\mathbf{j} - \alpha\mathbf{v}) = 0$$

$\Gamma$  Dissipative tensor

Magnus force

$G$  Gyrocoupling vector

$G\hat{\mathbf{z}} \times \mathbf{j}$



# Spin-Orbit torque driven Skyrmions

From symmetries the spin-orbit torques are

$$\begin{aligned}\mathbf{T}_J^{\parallel} &= a_J \mathbf{m} \times (\hat{\mathbf{z}} \times \mathbf{J}) + \bar{a}_J (\mathbf{m} \cdot \mathbf{J}) \mathbf{m} \times (\hat{\mathbf{z}} \times \mathbf{m}), \\ \mathbf{T}_J^{\perp} &= b_J \mathbf{m} \times (\mathbf{m} \times (\hat{\mathbf{z}} \times \mathbf{J})) + \bar{b}_J (\mathbf{m} \cdot \mathbf{J}) \hat{\mathbf{z}} \times \mathbf{m}\end{aligned}$$

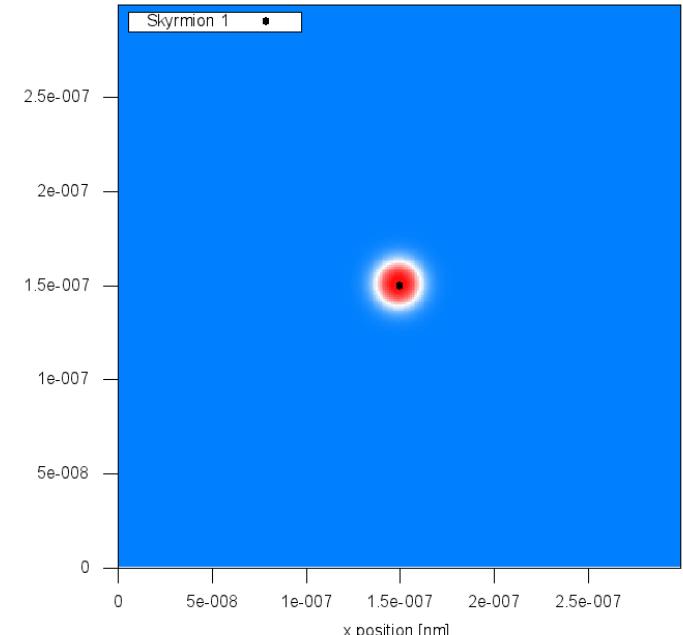
## SOT Skyrmion dynamics

Thiele equation:  $(Q\hat{\epsilon} - \hat{D})\mathbf{v} = \mathbf{F}$

$$Q = \frac{1}{4\pi} \int d^2\mathbf{r} \mathbf{m} \cdot [(\nabla_x \mathbf{m}) \times (\nabla_y \mathbf{m})],$$

$$D_{\alpha\beta} = \frac{\alpha_G}{4\pi} \int d^2\mathbf{r} (\nabla_\alpha \mathbf{m}) \cdot (\nabla_\beta \mathbf{m}),$$

$$F_\alpha = \frac{1}{4\pi} \int d^2\mathbf{r} (\nabla_\alpha \mathbf{m}) \cdot \mathbf{f}_s$$



# Spin-Orbit torque driven Skyrmions

Topological Hall angle (ratio of velocities):

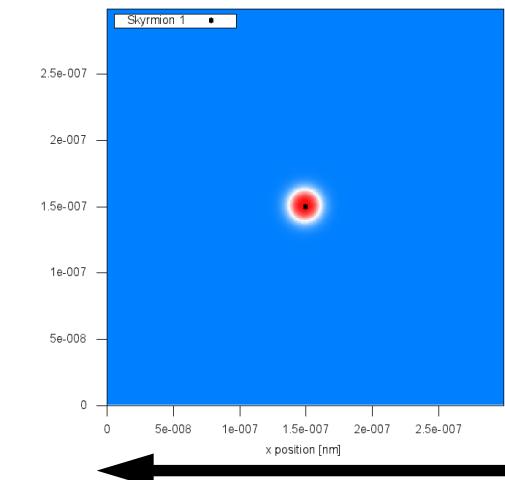
$$\frac{\nu_y}{\nu_x} = \frac{DF_y - QF_x}{DF_x + QF_y}, \quad D = \alpha_G \int_0^\infty d\rho \frac{\sin^2 \theta + (\rho \theta')^2}{4\rho}$$

Thiele equation:  $(Q\hat{\epsilon} - \hat{D})\nu = \mathbf{F}^\perp + \mathbf{F}^\parallel$

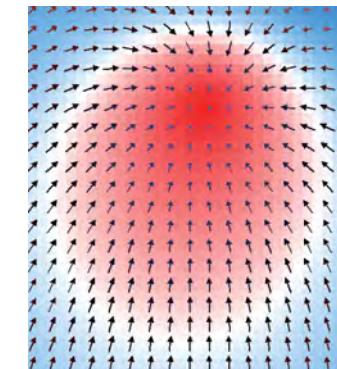
For  $Q=1$ :

$$\mathbf{F}^\parallel = \frac{\hat{\mathbf{z}} \times \mathbf{J}_\delta}{4} \int_0^\infty d\rho \left[ \frac{\partial a_J}{\partial \rho} - \bar{a}_J \sin^2 \theta \right] \sin \theta,$$

$$\mathbf{F}^\perp = \frac{\mathbf{J}_\delta}{4} \int_0^\infty d\rho \left[ \frac{b_J}{2} \sin 2\theta + (b_J - \bar{b}_J \sin^2 \theta) \rho \theta' \right]$$



deformed skyrmion:



# Antiferromagnetic Skyrmions

*In collaboration with Joe Barker*

Phys. Rev. Lett. 116, 147203 (2016)



JSPS

Japan Society for the Promotion of Science

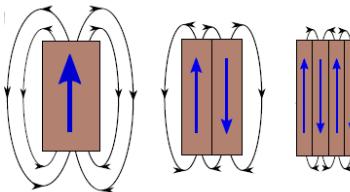


# Why Antiferromagnets?



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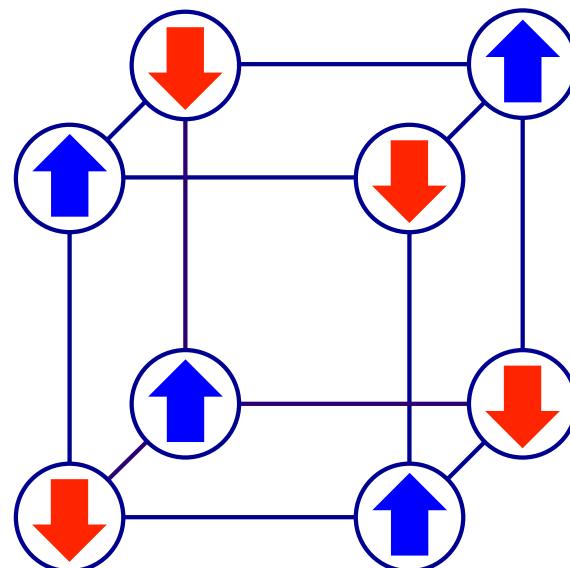
- ❖ No stray fields
- ❖ More abundant in nature
- ❖ Large range of different ground states



The system we consider

- ❖ Individual skyrmions
- ❖ Thin films
- ❖ Stabilized by uniaxial anisotropy competing with DMI (no applied field)

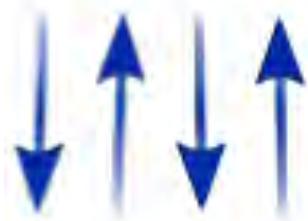
**KMnF<sub>3</sub>**  
 $M_s = 3.76 \times 10^5 \text{ A/m}$   
 $A = -6.59 \times 10^{-12} \text{ J/m}$   
 $K = 1.16 \times 10^5 \text{ J/m}^3$   
 $D = 7 \times 10^{-4} \text{ J/m}^2$



G-type ground state

# Texture Dynamics in AFMs

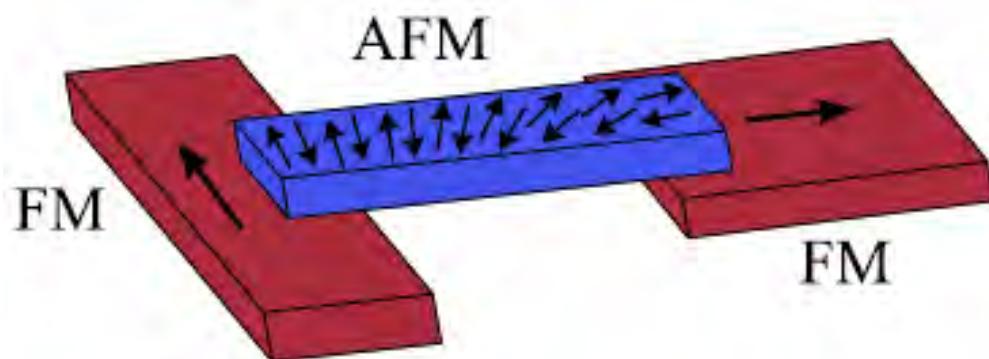
Antiferromagnet:



AFM equations:

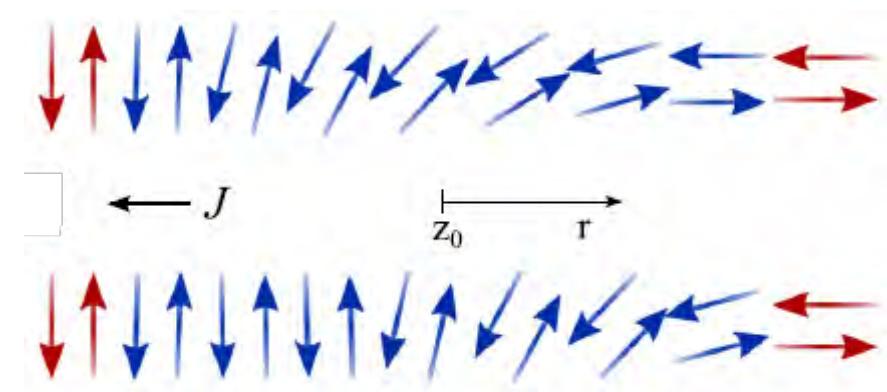
$$\dot{\mathbf{n}} = (\gamma \mathbf{f}_m - G_1 \dot{\mathbf{m}}) \times \mathbf{n} + \eta \gamma (\mathbf{J} \cdot \nabla) \mathbf{n},$$

$$\dot{\mathbf{m}} = [\gamma \mathbf{f}_n - G_2 \dot{\mathbf{n}} + \beta \gamma (\mathbf{J} \cdot \nabla) \mathbf{n}] \times \mathbf{n} + T_{nl},$$



Damped harmonic oscillator:

$$M\ddot{r} + \Gamma\dot{r} + M\omega_0^2 r = F_J + F_H,$$



Tveten, Qaiumzadeh, Tretiakov, Brataas, PRL (2013)  
Kim, Tserkovnyak, Tchernyshyov, PRB (2014)



# Hamiltonian

Atomistic Formalism:

$$\mathcal{H} = - \sum_{ij} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - \sum_{ij} (\mathbf{d}_{ij} \times \hat{z}) \cdot (\mathbf{S}_i \times \mathbf{S}_j) - \sum_i k_z S_{i,z}^2$$

Exchange

Dzyaloshinskii-Moriya  
Interaction

Uniaxial anisotropy

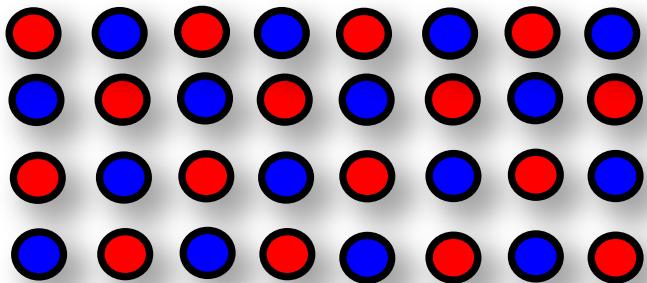
Competition gives skyrmion

Micromagnetic Formalism:

$$\begin{aligned} E = & \int A(\nabla \mathbf{m})^2 d^3 \mathbf{r} - \int K m_z^2 d^3 \mathbf{r} \\ & + t \int D \left[ \left( m_x \frac{\partial m_z}{\partial x} - m_z \frac{\partial m_x}{\partial x} \right) + \left( m_y \frac{\partial m_z}{\partial y} - m_z \frac{\partial m_y}{\partial y} \right) \right] d^2 \mathbf{r} \end{aligned}$$

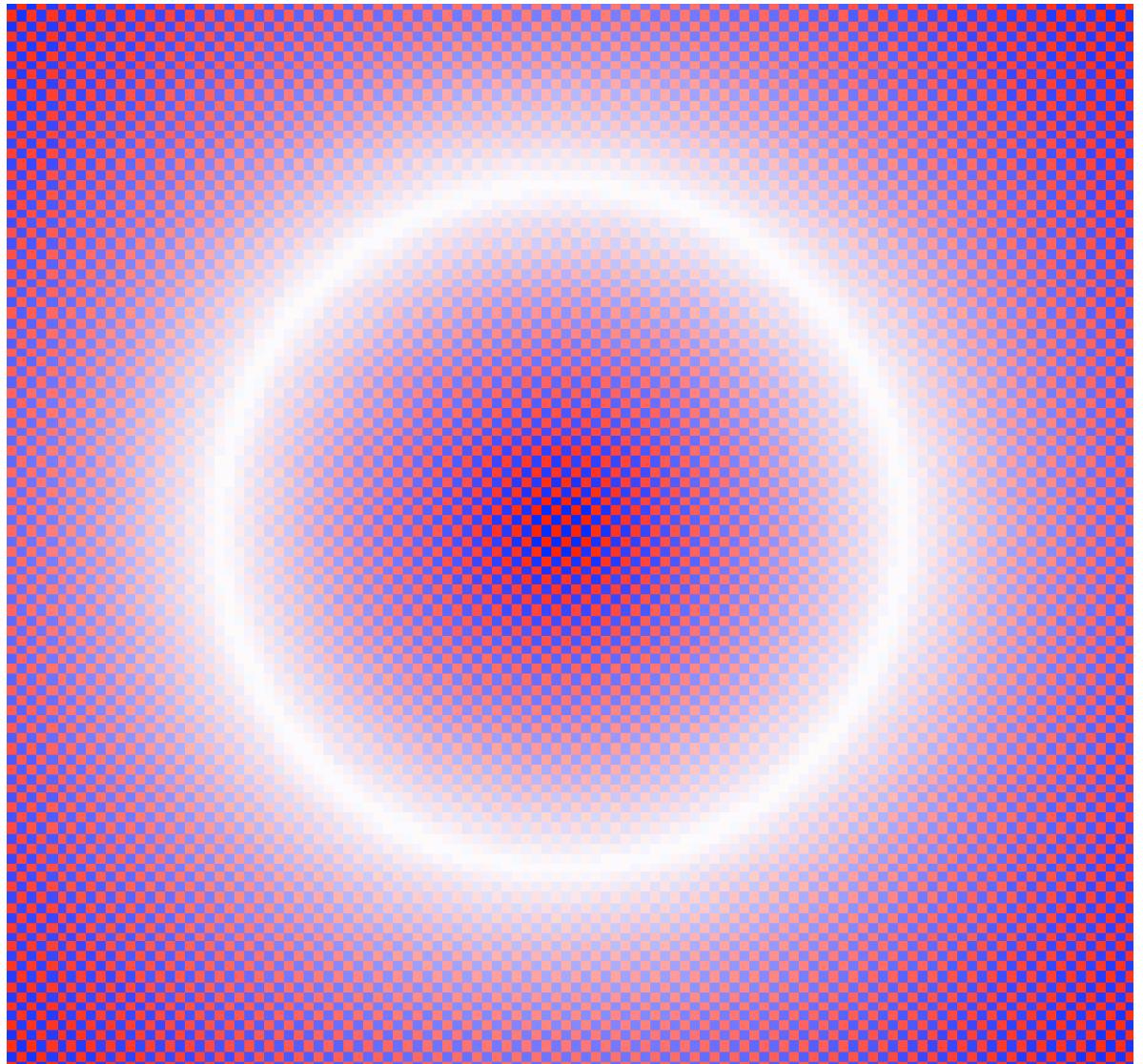
# AFM Skyrmi<sup>on</sup>

G-type  
antiferromagnet:



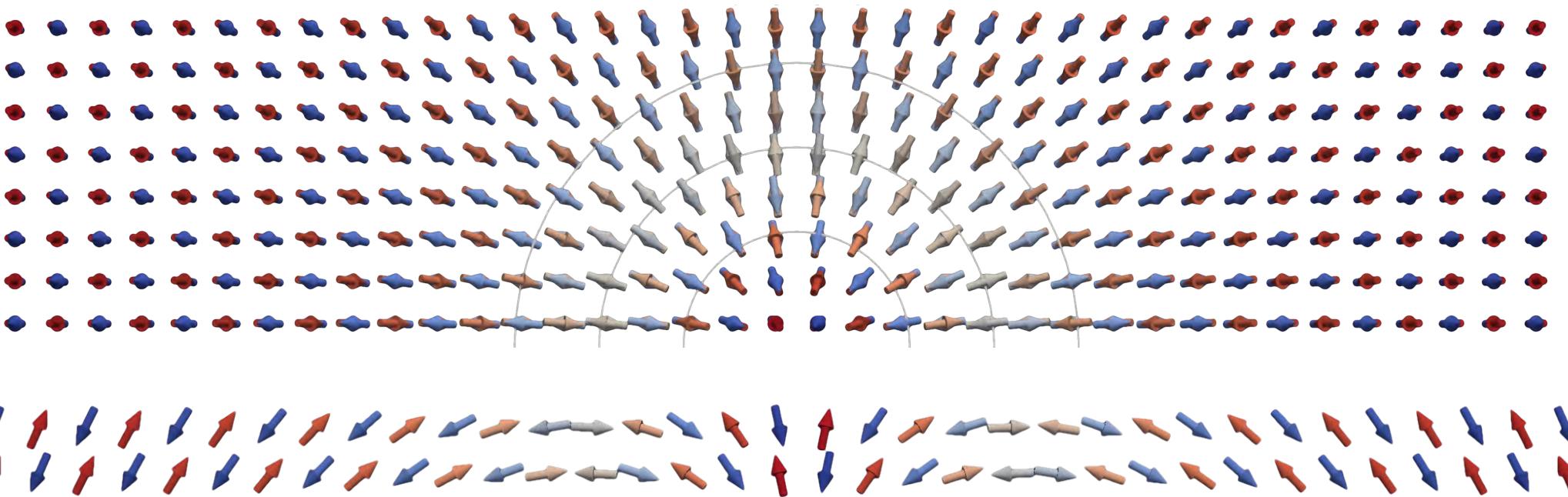
Winding number:

$$W = \frac{1}{4\pi} \int d^2r \Omega \cdot \frac{\partial \Omega}{\partial x} \times \frac{\partial \Omega}{\partial y}$$

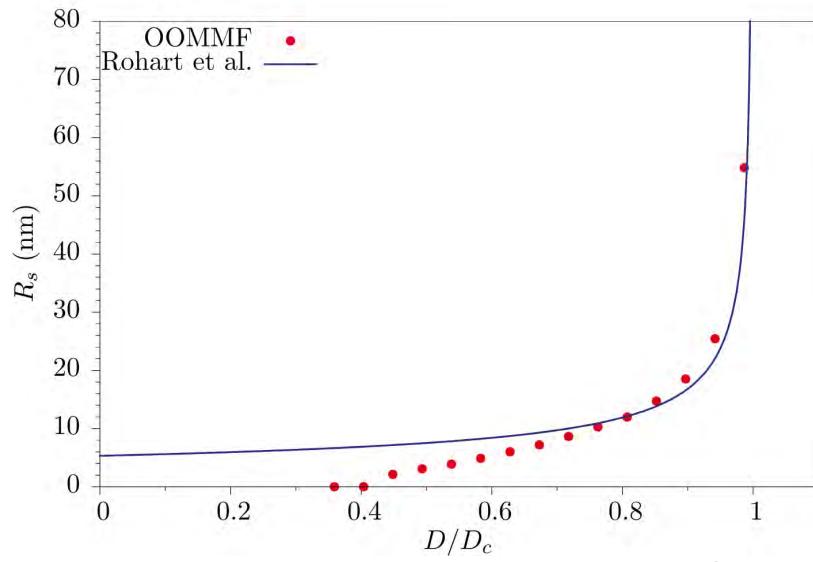


Bogdanov et al., 1998

# AFM Skyrmion (hedgehog type)

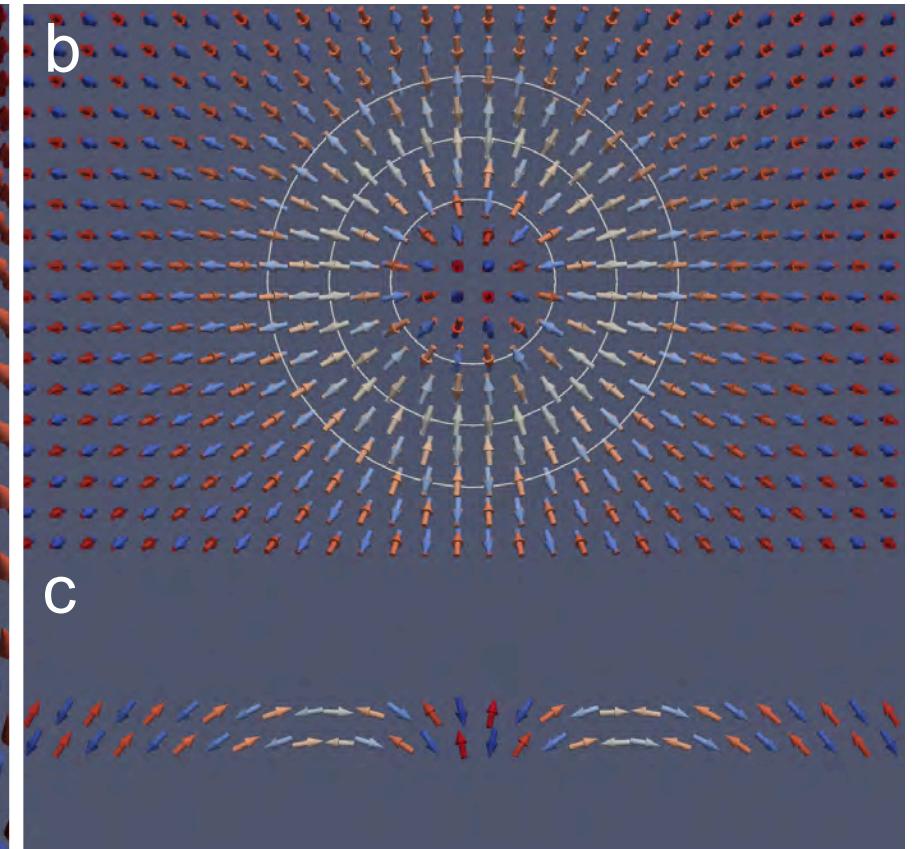
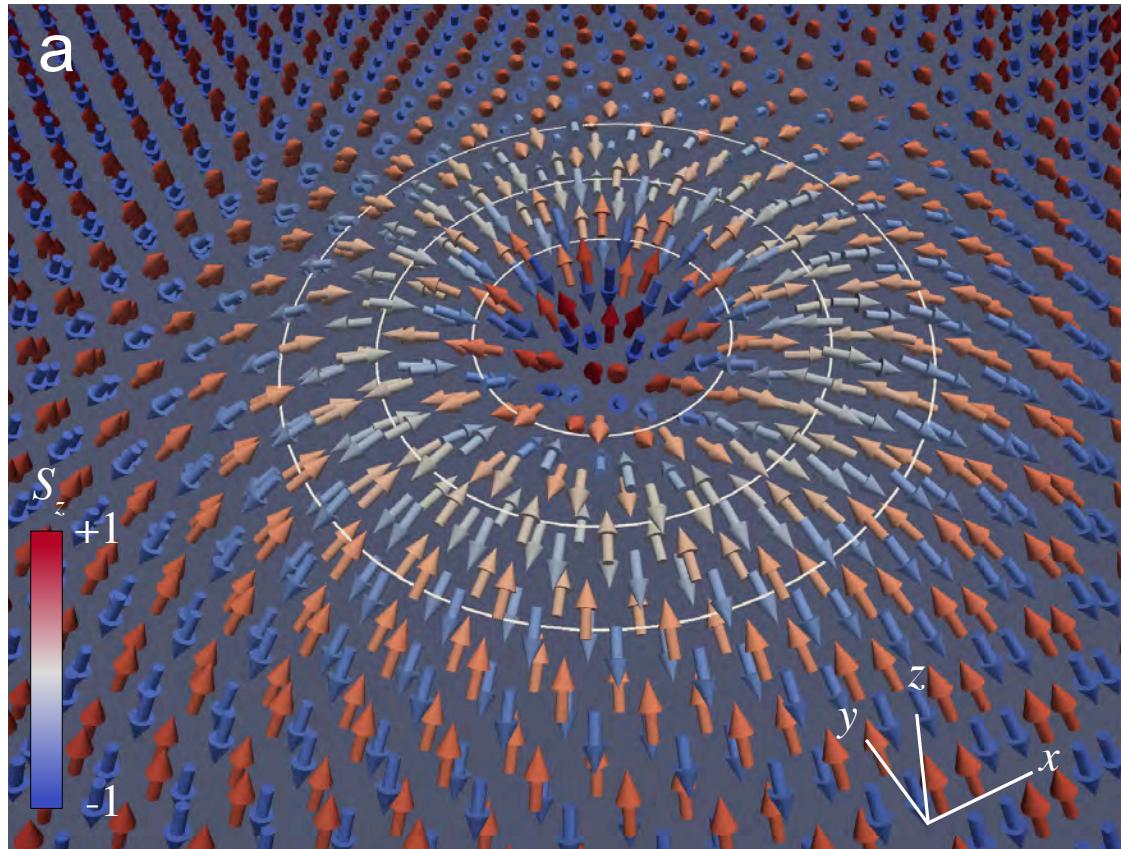


Neel field profile of AFM skyrmion  
is the same as magnetization profile  
of FM skyrmion



Rohart & Thiaville  
Phys. Rev B, 88, 184422

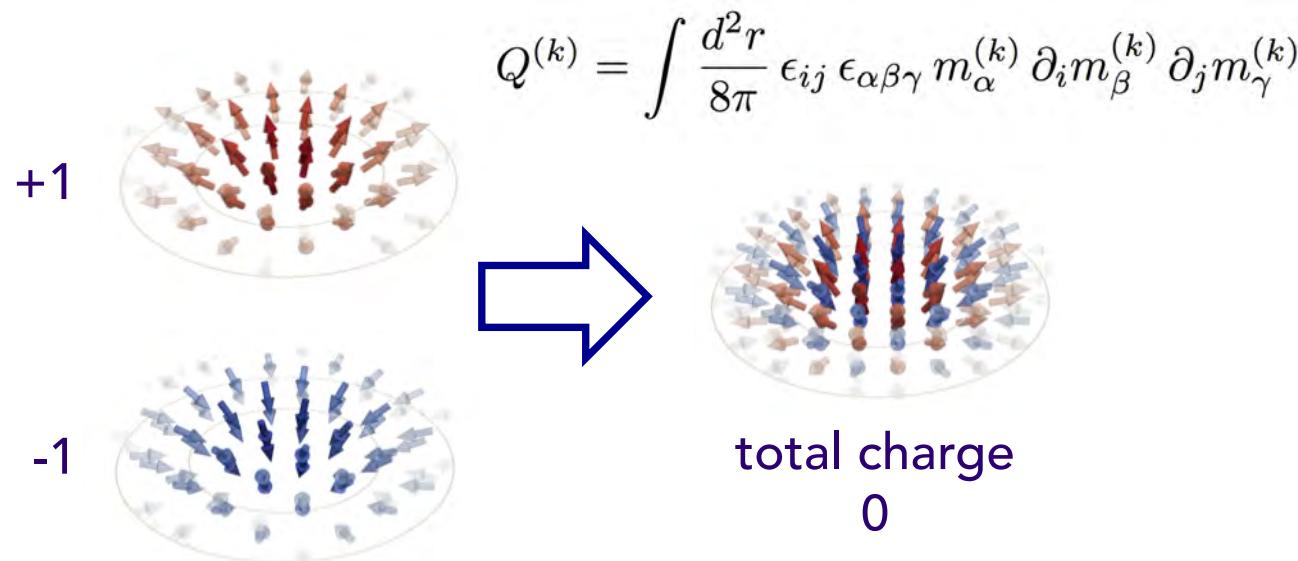
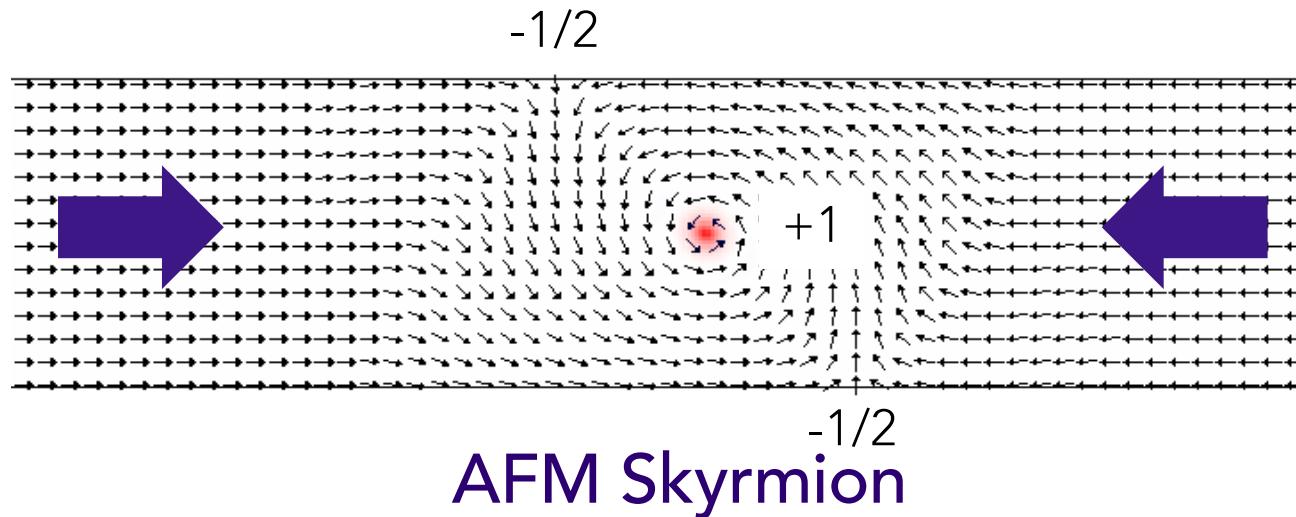
# AFM Skyrmion Structure



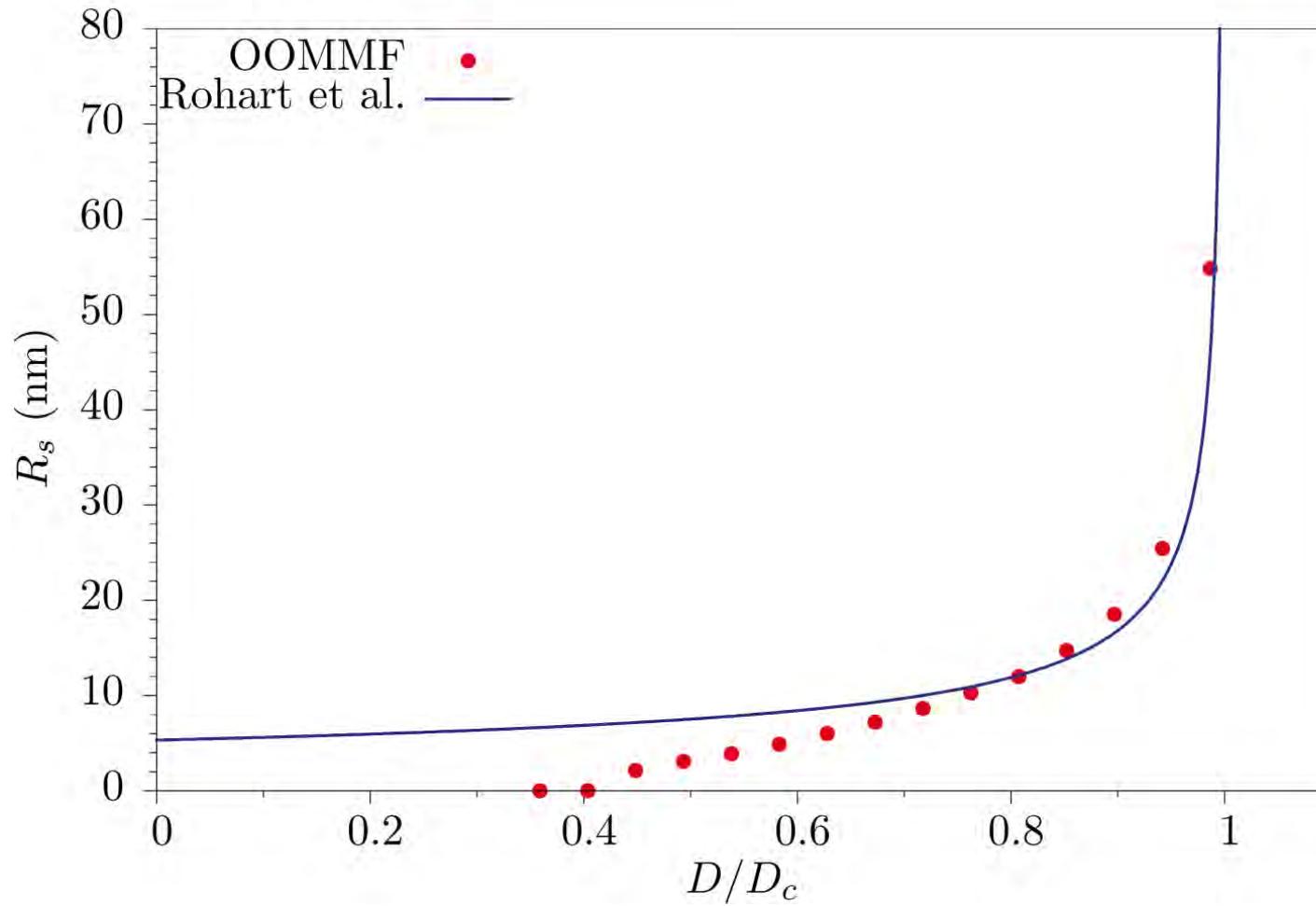
Winding number:  $W = \frac{1}{4\pi} \int d^2r \Omega \cdot \frac{\partial \Omega}{\partial x} \times \frac{\partial \Omega}{\partial y}$

# AFM Skyrmion: Compound topological object

## Composite vortex domain wall



# Skyrmion Radius vs. DMI



Rohart et.al, PRB (2013 )

$$R_s \approx \frac{\Delta}{\sqrt{2(1 - D/D_c)}}$$

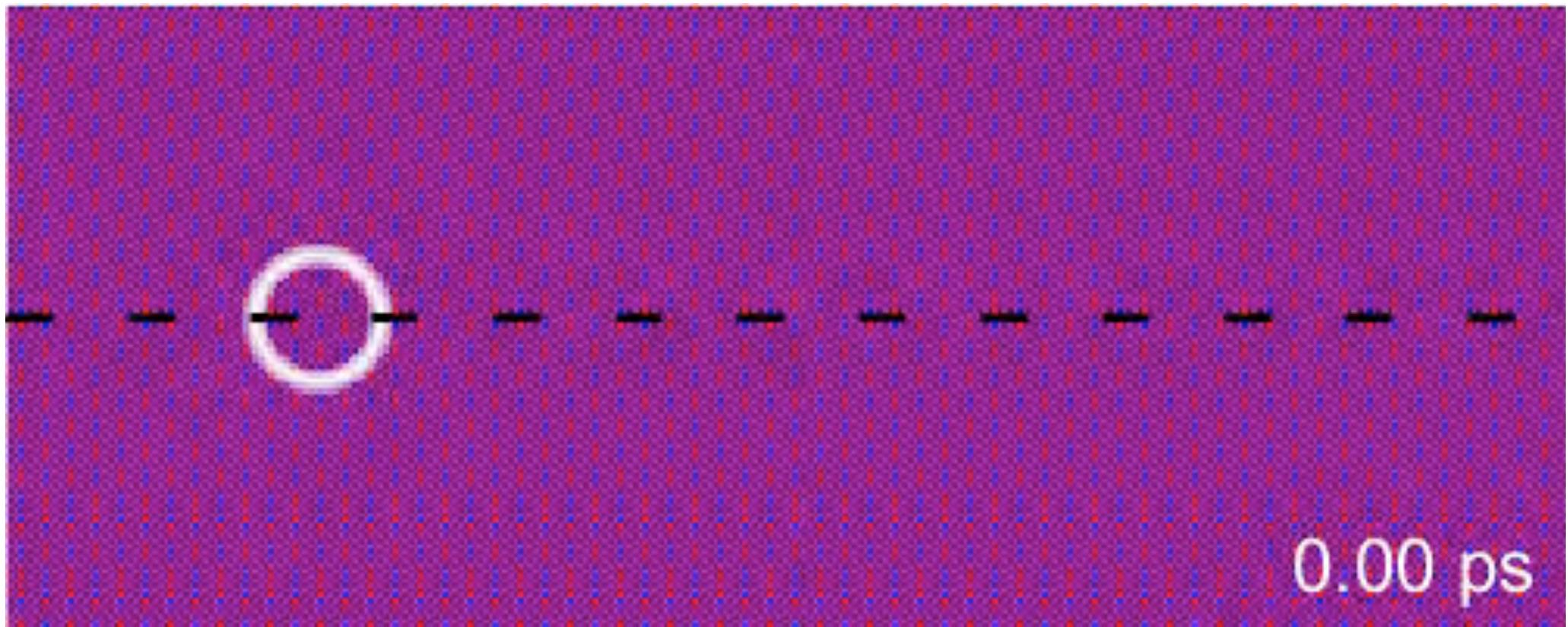
Bogdanov et al.,  
Phys. Rev B 66, 214410 (2002)

Large AFM/FM skyrmions described well by continuous model

# Current induced dynamics



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# FM Skyrmion equation of motion



Thiele's equation (collective coordinates approach)

$$G \times (\mathbf{j} - \mathbf{v}) + \Gamma(\beta\mathbf{j} - \alpha\mathbf{v}) = 0$$

$\Gamma$  Dissipative tensor

Magnus force

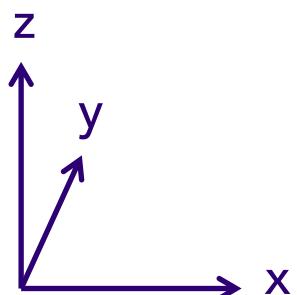
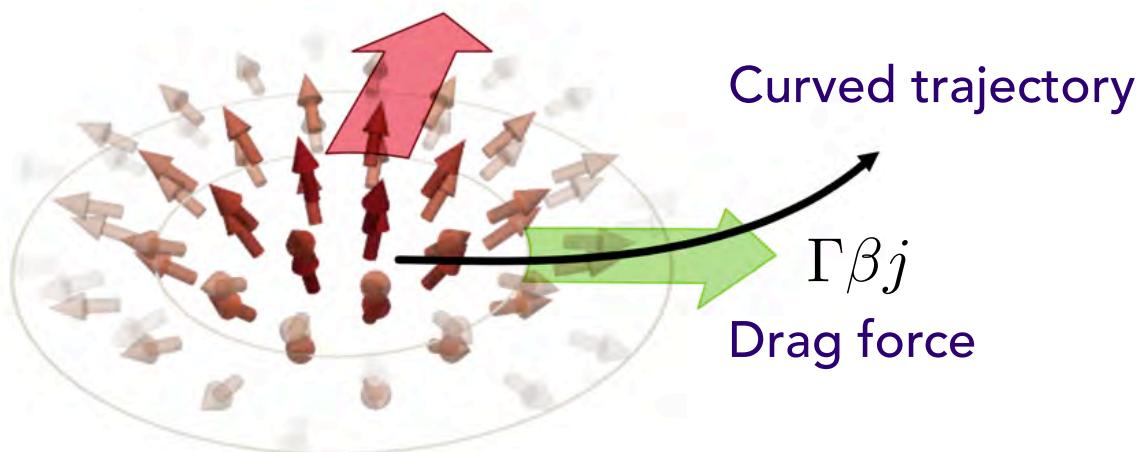
$G$  Gyrocoupling vector

$$G\hat{\mathbf{z}} \times \mathbf{j}$$

Curved trajectory

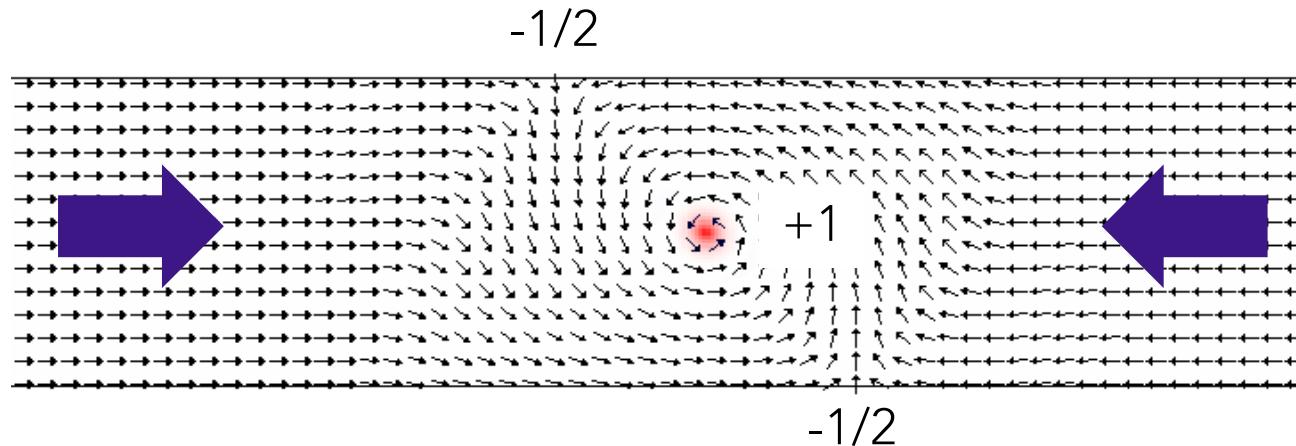
$$\Gamma\beta j$$

Drag force

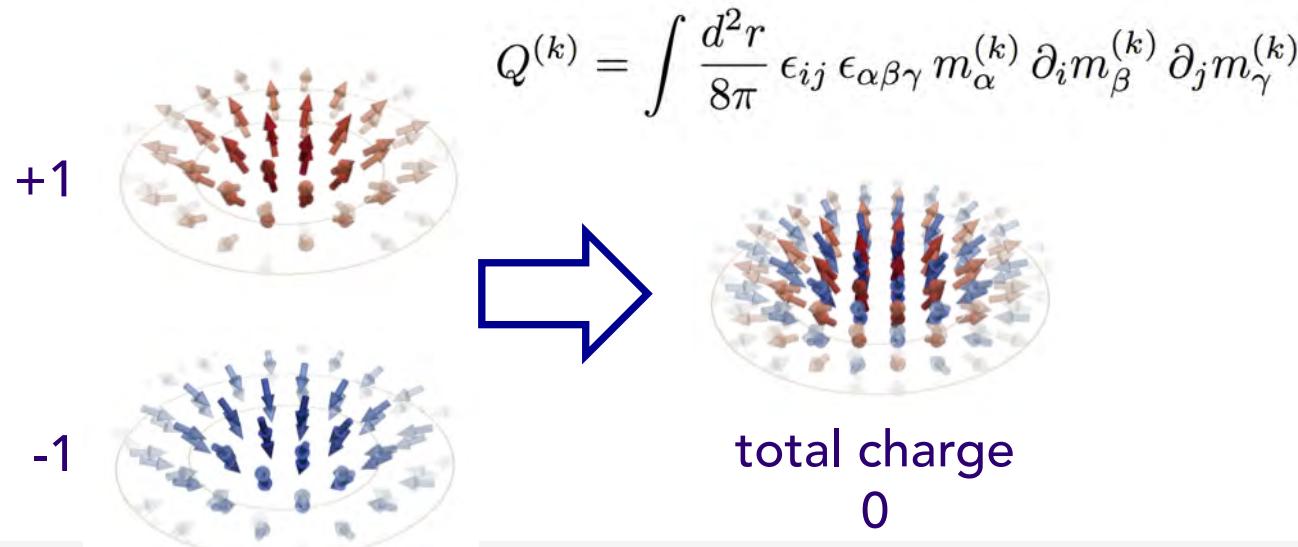


# AFM Skyrmion: Compound topological object

## Composite vortex domain wall



## AFM Skyrmion

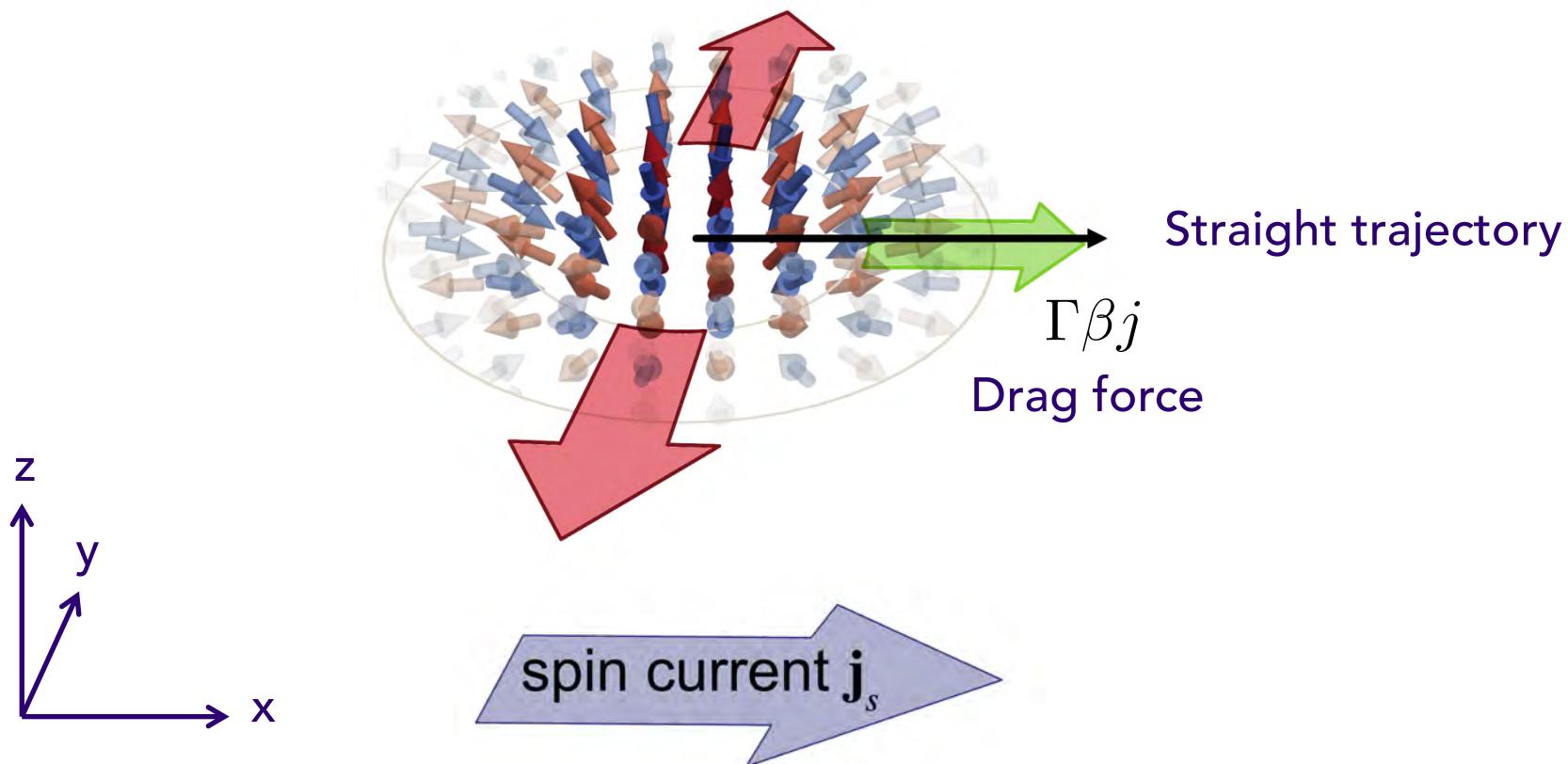


# AFM Skyrmion Equation of Motion

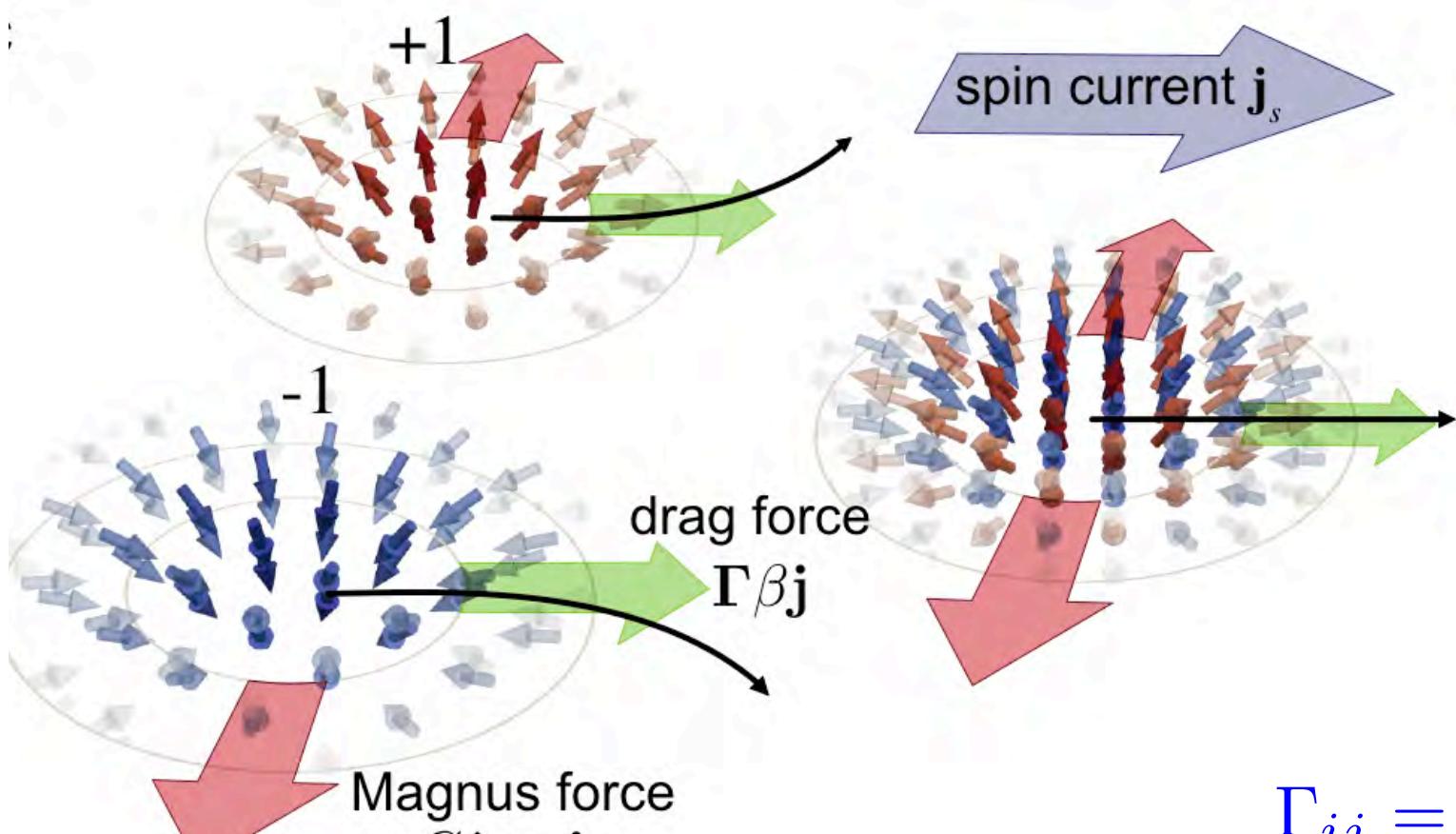
Thiele's equation (collective coordinates approach):

$$\cancel{G} \times (\mathbf{j} - \mathbf{v}) + \Gamma(\beta\mathbf{j} - \alpha\mathbf{v}) = 0$$

Exact cancellation of  
Magnus force



# AFM Skyrmion Equation of Motion



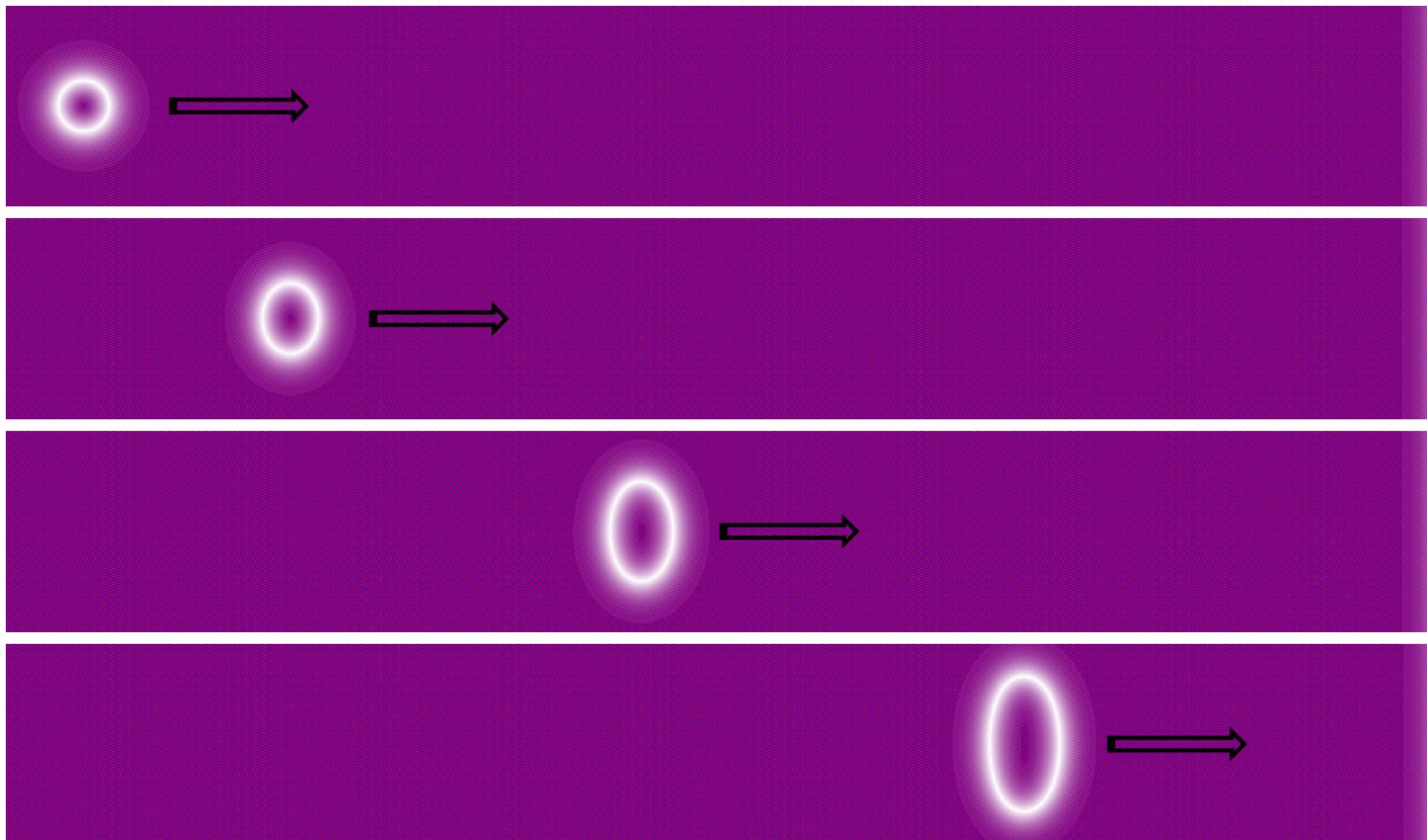
$$\alpha \Gamma \mathbf{v}(t) = \mathbf{F}$$

$$\Gamma_{ij} = \int d^2r \frac{\partial \vec{\Omega}}{\partial x_i} \frac{\partial \vec{\Omega}}{\partial x_j}$$

Equation of motion

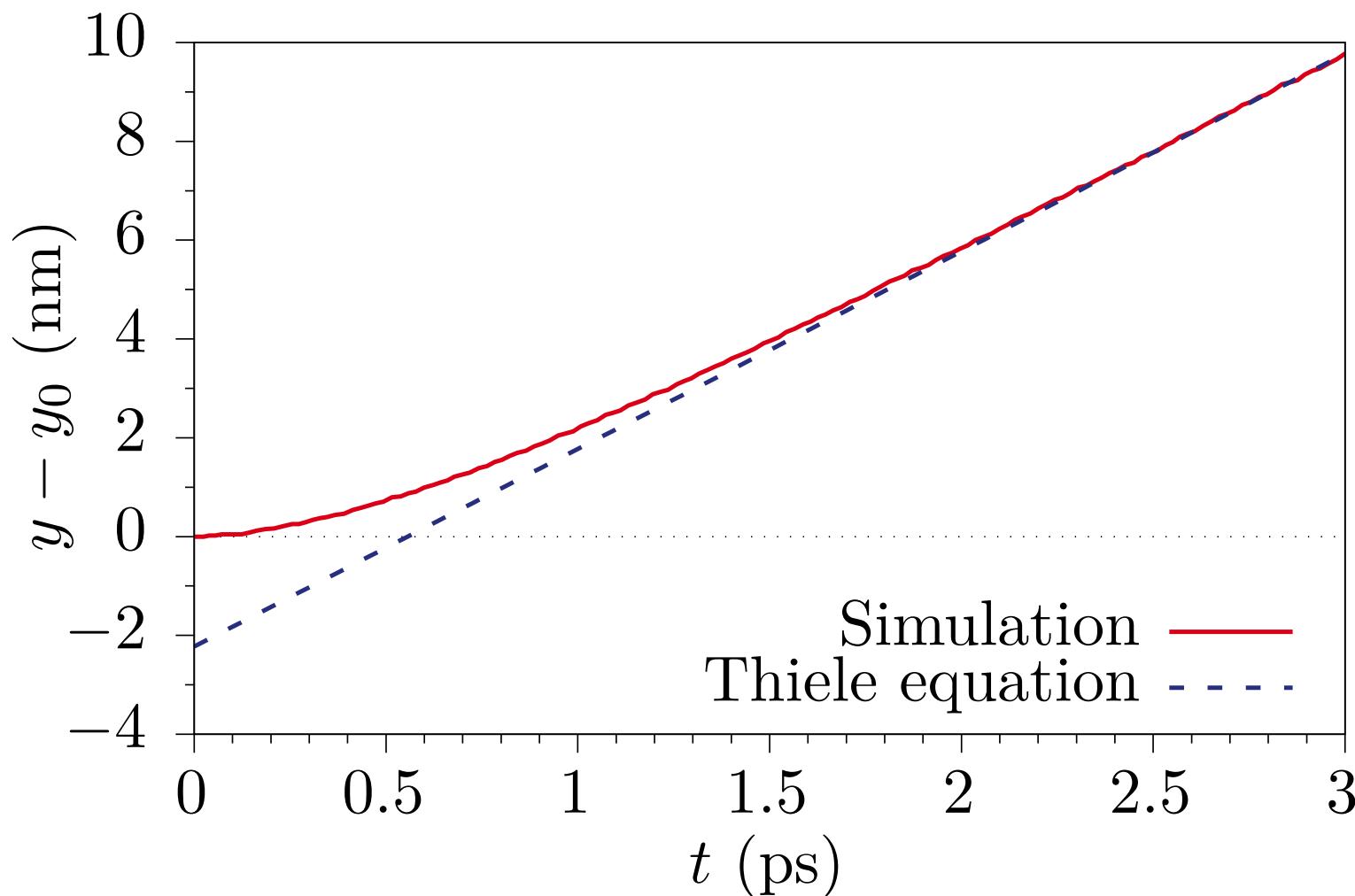
$$v_{||} = \frac{\beta}{\alpha} j$$

# Motion of Deformed AFM Skyrmion



Big advantage: for high currents, even though skyrmion deforms  
– it still moves strictly parallel to the current!

# AFM Skyrmion Dynamics



AFM skyrmion quickly (~2ps)  
reaches terminal velocity

# Inclusion of Temperature



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## Landau-Lifshitz-Gilbert

Equation of motion for a classical spin in a local field.

$$\frac{\partial \mathbf{S}_i}{\partial t} = -\frac{\gamma_i}{(1 + \alpha_i^2)} (\mathbf{S}_i \times \mathbf{H}_i + \alpha_i \mathbf{S}_i \times \mathbf{S}_i \times \mathbf{H}_i)$$

## Langevin thermostat

Equation of motion for a classical spin in a local field.

$$\mathbf{H}_i(t) = -\frac{\partial \mathcal{H}}{\partial \mathbf{S}_i} - \eta \int_{-\infty}^t \varphi(t-t') \frac{d\mathbf{S}_i}{dt'} dt' + \boldsymbol{\xi}_i(t)$$

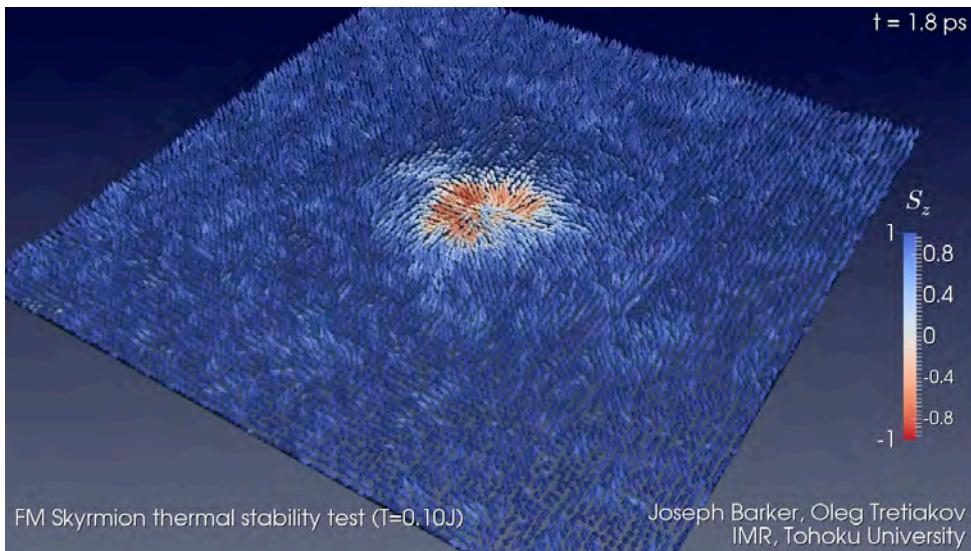
$$\langle \xi_{i,a}(t) \rangle = 0$$

$$\langle \xi_{i,a}(t), \xi_{j,b}(t') \rangle = 2k_B T \eta \varphi(|t-t'|) \delta_{ij} \delta_{ab}$$

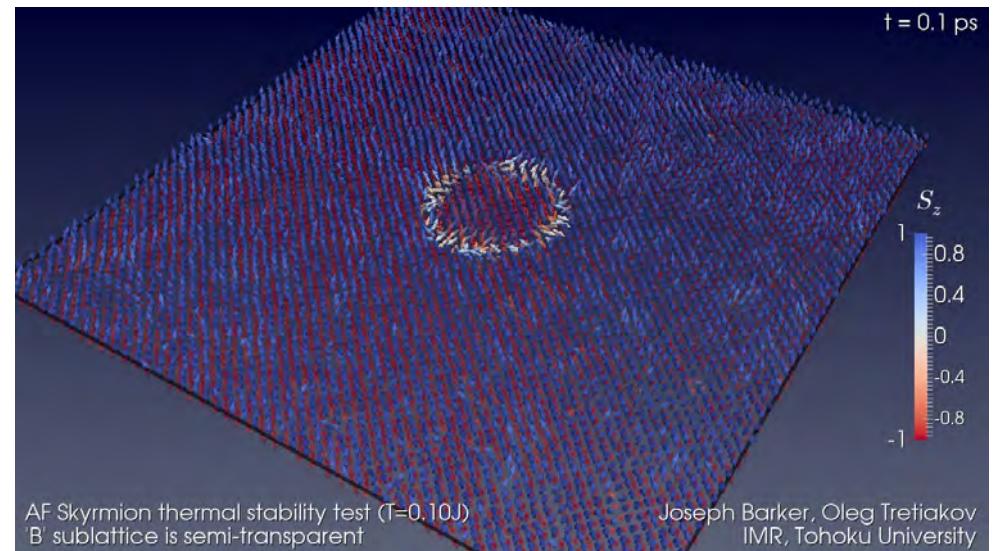
$$\varphi(|t-t'|) = \delta(|t-t'|)$$

# Diffusion: AFM vs FM Skyrmi

FM



AFM

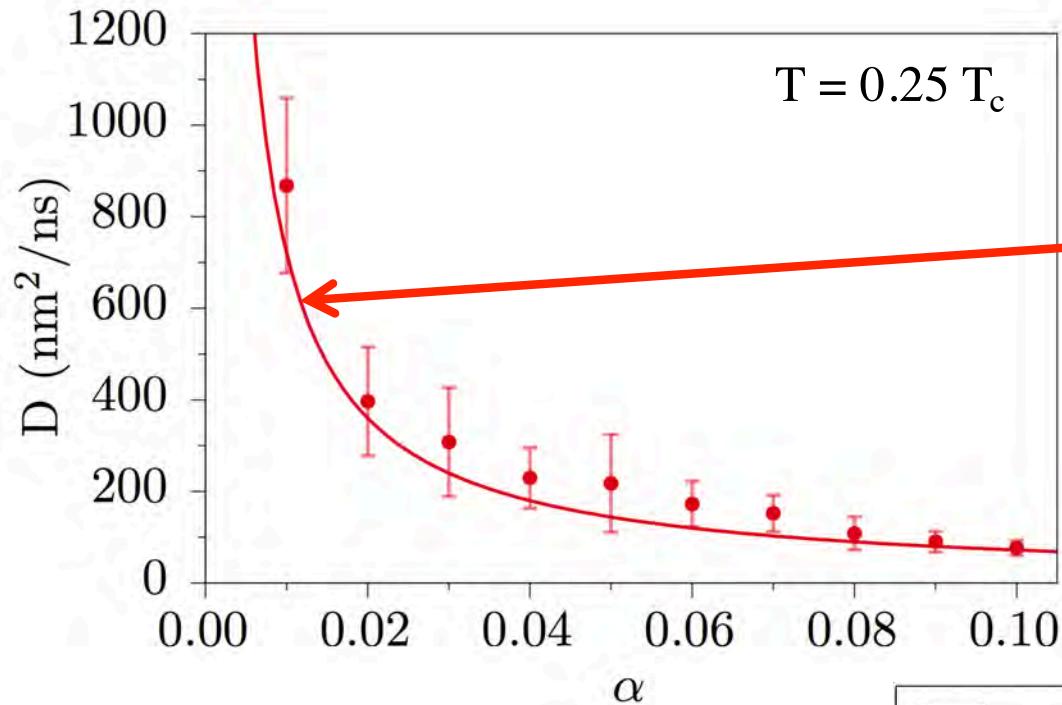


Diffusion coefficient for FM skyrmion:

$$\mathcal{D} = \lambda k_B T \frac{\alpha \Gamma}{G^2 + (\alpha \Gamma)^2}$$

AFM Skyrmions have  
higher diffusion const!

# Diffusion of AFM Skyrmions



AFM skyrmion

$$\mathcal{D} \propto 1/\alpha$$

$$\mathcal{D} = k_B T \frac{\lambda}{2\alpha s\sigma}$$

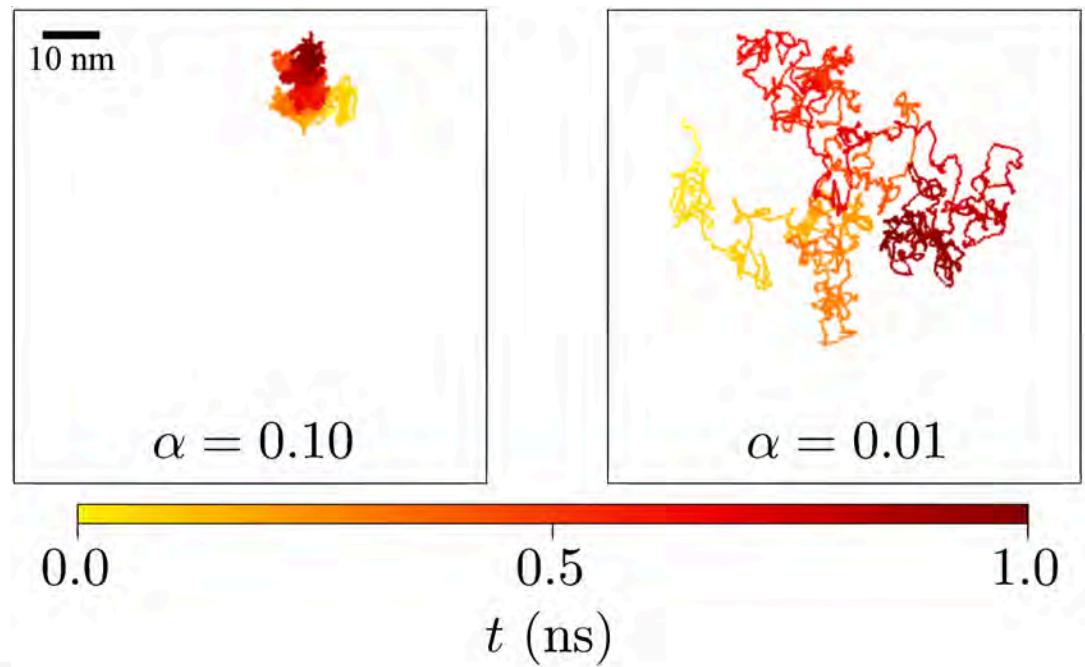
Kim, Tchernyshyov & Tserkovnyak, PRB (2015)

Good agreement with no fitting

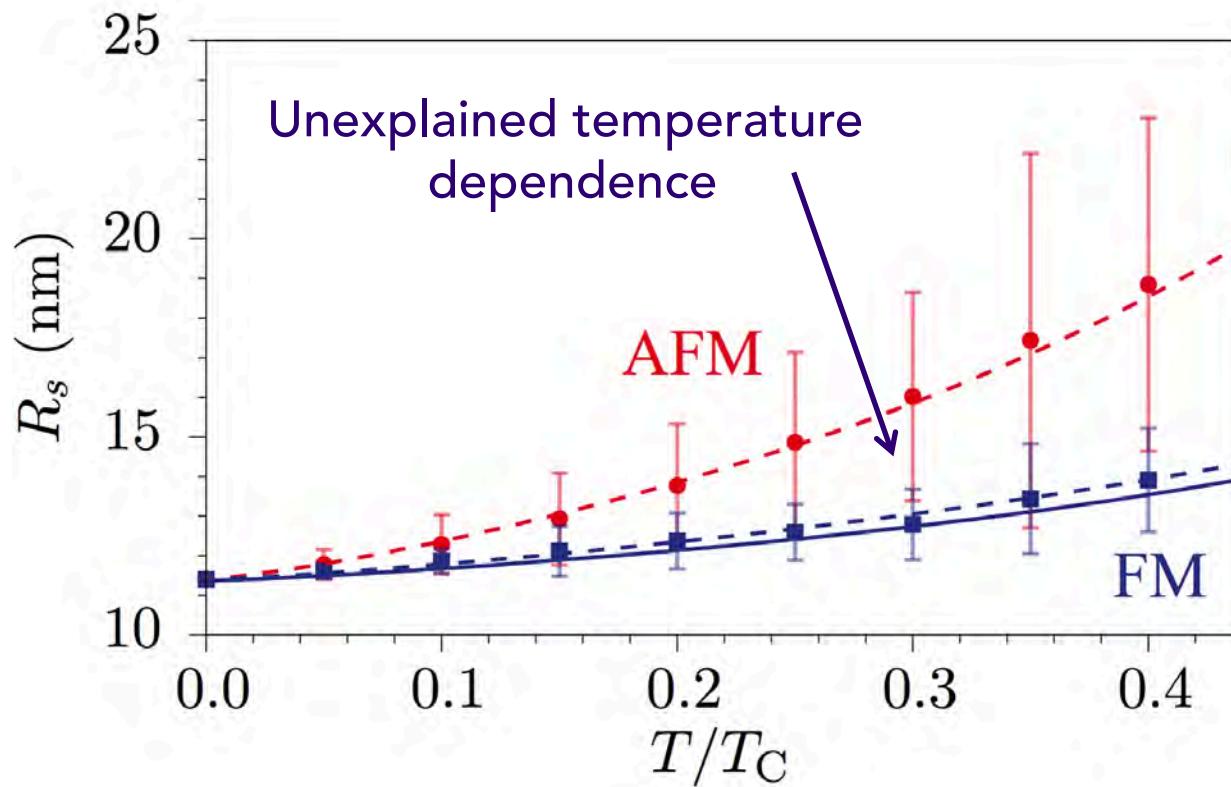
FM skyrmion:  $\mathcal{D} \propto \alpha$

$$\mathcal{D} = k_B T \frac{\alpha \Gamma}{\cancel{\Gamma^2} + (\alpha \Gamma)^2}$$

Schütte, Iwasaki, Rosch & Nagaosa  
Phys. Rev. B 90, 174434 (2014)



# Temperature dependence of Skyrmion radius



Large thermal fluctuations

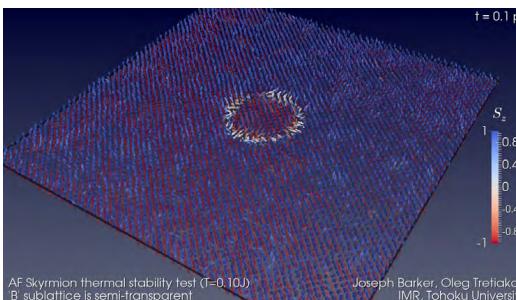
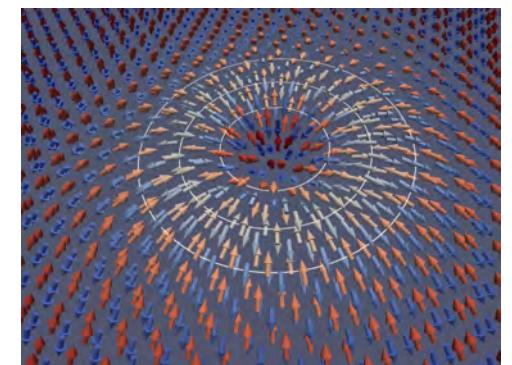
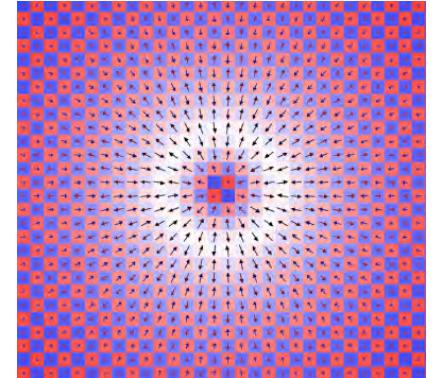
Higher temperature dependence

$$R_s(T) = \sqrt{\frac{2A\lambda}{4\sqrt{AK}m(T) - \pi Dm^{-3/2}(T)}}$$

Callen-Callen theory  
describes FM at low T

# Summary

- AFM skyrmions are stable objects. The effect of *Dzyaloshinskii-Moriya interaction* on the AFM skyrmion stability/radius were studied.
- Skyrmiон dynamics in AFMs obeys *generalized Thiele's equations*.
- Thermal effects on AFM skyrmions were studied. Diffusion constant for AFM skyrmions is larger than for FM skyrmions.
- *AFM skyrmions move only along the current.*



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