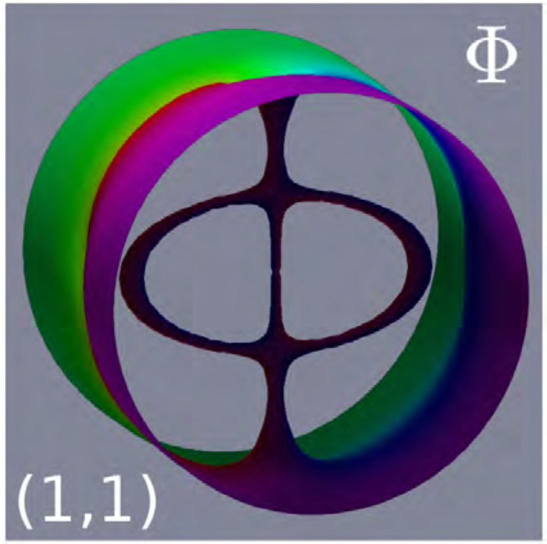
New Zealand INSTITUTE for Advanced Study

CTCP Centre for Theoretical Chemistry and Physics

Dynamics of solitons and vortices in strongly-correlated and topological superfluids

Joachim Brand



The Dodd-Walls Centre for Photonics and Quantum Technology

Massey University COLLEGE OF SCIENCES

SPICE workshop Nonequilibrium Quantum Matter, Mainz, May 30 – June 2, 2017

ATOMIC DARK SOLITONS

Quantum canaries learn to fly

Dark solitons in Bose–Einstein condensates have been made to live long enough for their dynamical properties to be observed. They might serve as a sensitive probe of the rich physics at the mesoscale.

James Anglin
is at the Technische Universität Kaiserslautern, OPTIMAS Research Centre and Fachbereich Physik, Erwin-Schrödinger-Straße, D-67663 Kaiserslautern, Germany.
e-mail: janglin@physik.uni-kl.de

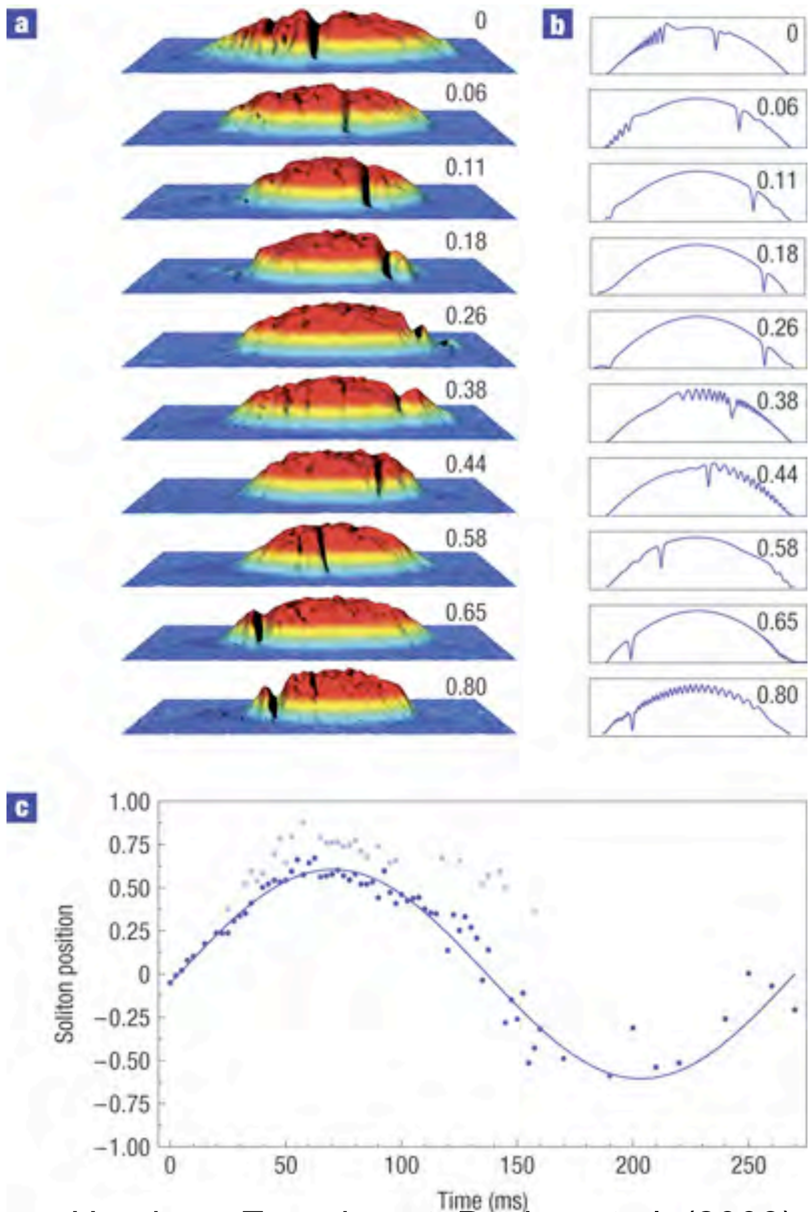
Miners once carried canaries down the mineshaft to warn of poisonous fumes; the birds are especially susceptible to toxic gases, and they quickly died if any were present. Writing on page 496 of this issue, Christoph Becker and colleagues[1] report experiments with 'creatures' that are sensitive not to subtle changes in the atmosphere, but to the elusive physics of the frontiers between classical and quantum mechanics, and between dynamics and thermodynamics. Their 'quantum canaries' are dark solitons in a Bose–Einstein condensate (BEC): narrow pulses of sharply reduced density, travelling slowly back and forth through the background condensate cloud.

Solitons are isolated waves that manage, through a dynamic balance between dispersion and nonlinearity, to keep their form intact as they move. John Scott Russell made the first reported observation of a soliton in 1834, when he noticed a remarkably durable 'heap of water' rolling along a canal. Solitons have since

Canary in a coalmine. The little birds were once used for detecting small amounts of harmful gases in mineshafts. By way of analogy, dark solitons in a BEC can be considered 'quantum canaries' — they sensitively probe the elusive physics at the mesoscale, between classical and quantum mechanics.

What can be learned about quantum gases from observing solitary wave and vortex dynamics?

Dark solitons in a trapped BEC



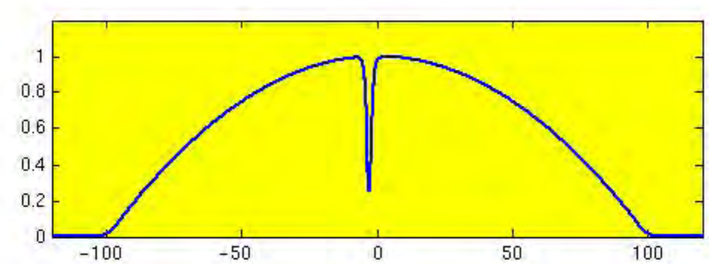
Hamburg Experiment: Becker et al. (2008)

Solitons in trapped BEC oscillate more slowly than COM

$$\left(\frac{T_s}{T_{\rm trap}}\right)^2 = \frac{m^*}{m_{ph}} = 2$$

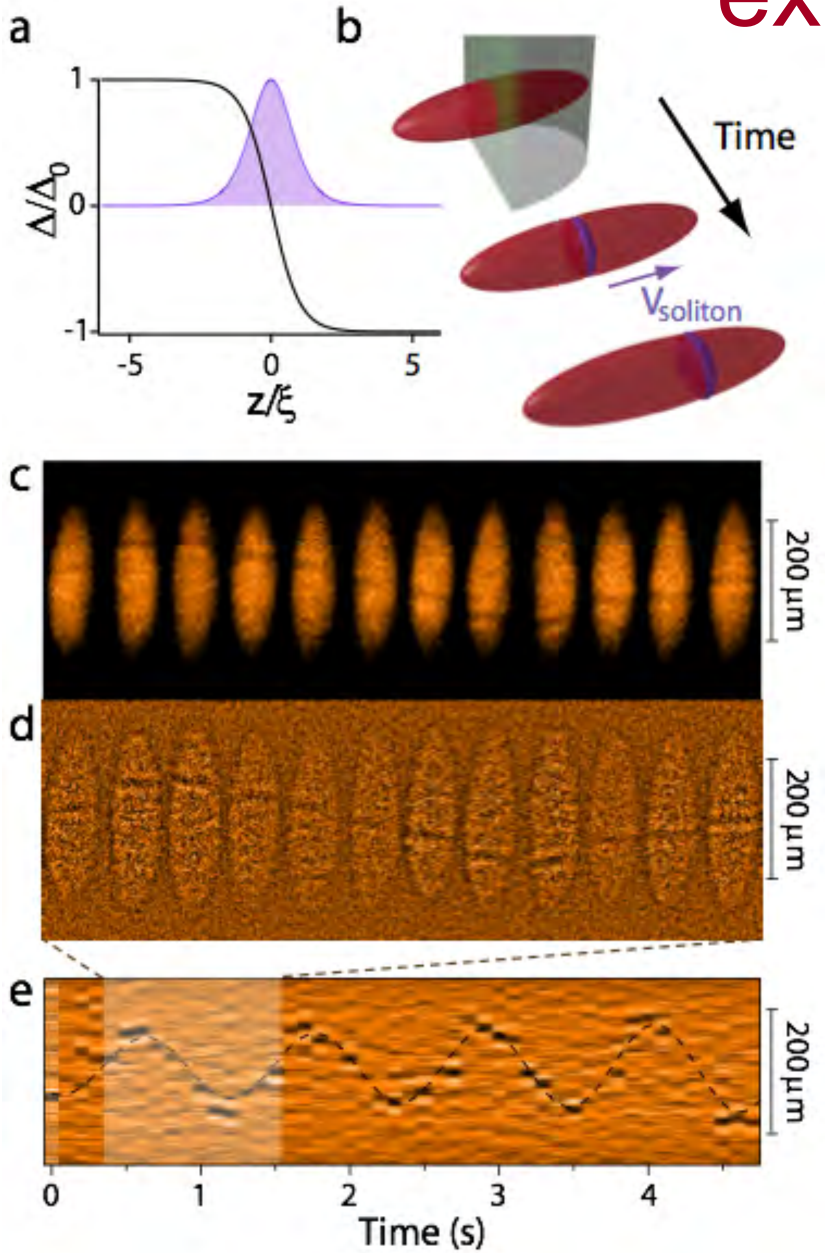
Theory:
- Busch, Anglin PRL (2000)
- Konotop, Pitaevskii, PRL (2004)

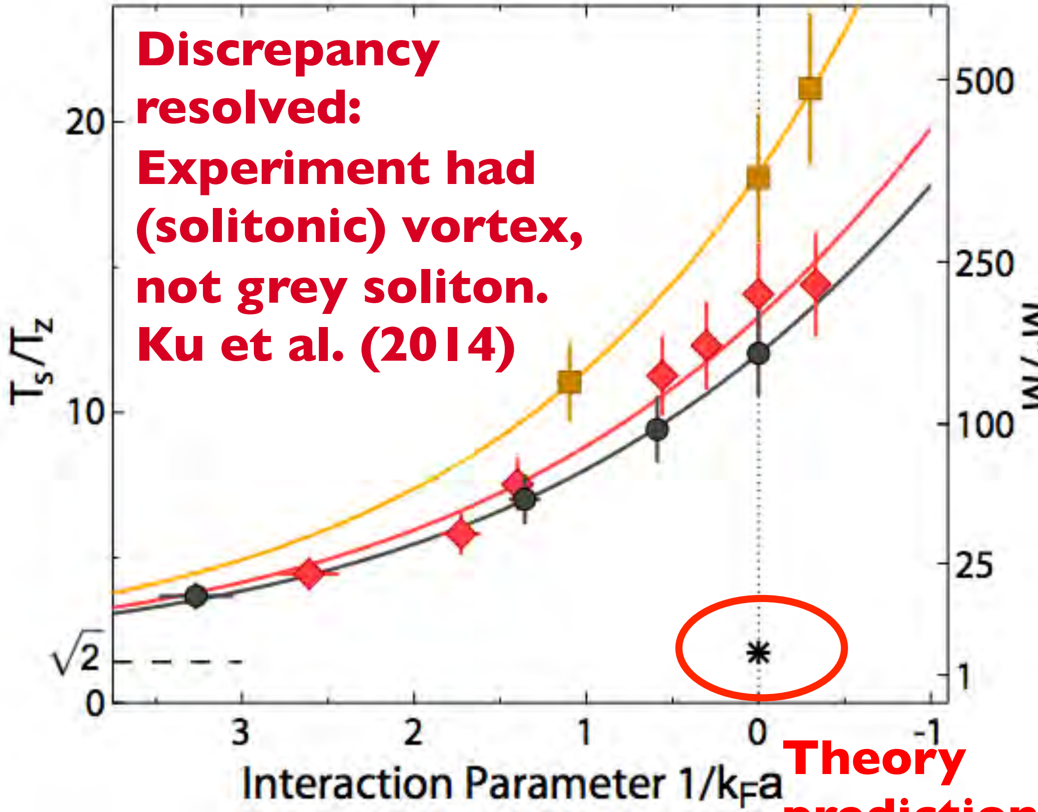
Experiment:
- Becker et al. Nat. Phys. (2008)
- Weller et al. PRL (2008)



Movie credits: Nick Parker, Univ. Leeds

Dark soliton in superfluid Fermi gas experiment





Discrepancy resolved: Experiment had (solitonic) vortex, not grey soliton. Ku et al. (2014)

Theory prediction

Experiment: Yefsah et al., Nature (2013)
Theory: Liao, Brand, PRA (2011),
Scott, Dalfovo, Pitaevskii, Stringari, PRL (2011)

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  - Slab model – asymptotically solvable for compressible fluids
  - Snaking instability and Chladni solitons

Soliton dynamics from dispersion relation

If a soliton moves in a slowly changing environment (density, chemical potential, etc.), then its dynamics can be predicted from its dispersion relation, i.e. the properties of constant solutions on a homogeneous background.

This is the **Landau** picture of **quasi-particle dynamics**.

The **effective mass** is a particularly helpful concept.

Konotop, Pitaevskii, PRL 93, 240403 (2004)

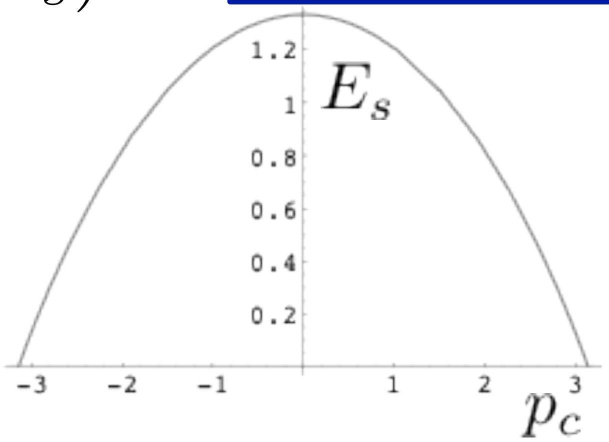
Soliton dispersion

Soliton energy: $E_s(\mu, v_s, g) = \langle \hat{H} - \mu \hat{N} \rangle - E_h$

Canonical momentum: $v_s = \dfrac{dE_s}{dp_c}$

Effective (inertial) mass: $m^* = 2\dfrac{\partial E_s}{\partial (v_s)^2}$

$$E_s \approx E_0 + \frac{p_c^2}{2m^*}$$



Physical (heavy) mass:

$$m_{ph} = mN_s$$

$$N_s = \int (n_s - n_0) d^3 r = -\frac{\partial E_s}{\partial \mu}$$ (for *v* = 0)

Landau quasiparticle dynamics

Konotop, Pitaevskii, PRL (2004)
Scott, Dalfovo, Pitaevskii, Stringari, PRL(2011)

- soliton moves on a slowly varying background, locally conserving energy

$$\frac{dE_s(v_s, \mu(z))}{dt} = 0 \longrightarrow \text{equation of motion}$$

- For *harmonic trapping potential* obtain small amplitude oscillations with

$$\left(\frac{T_s}{T_{\text{trap}}}\right)^2 = \frac{m^*}{m_{ph}}$$

  – BEC solitons: also locally conserve particle number

$$\frac{m^*}{m_{ph}} = 2 \qquad N_s = f(E_s(v_s, \mu))$$

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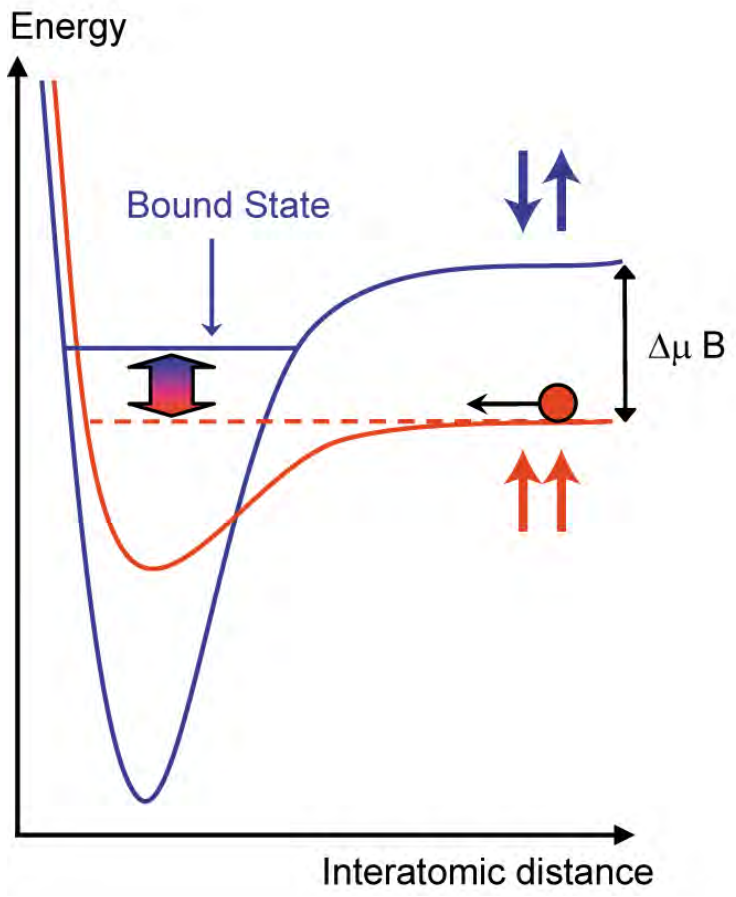
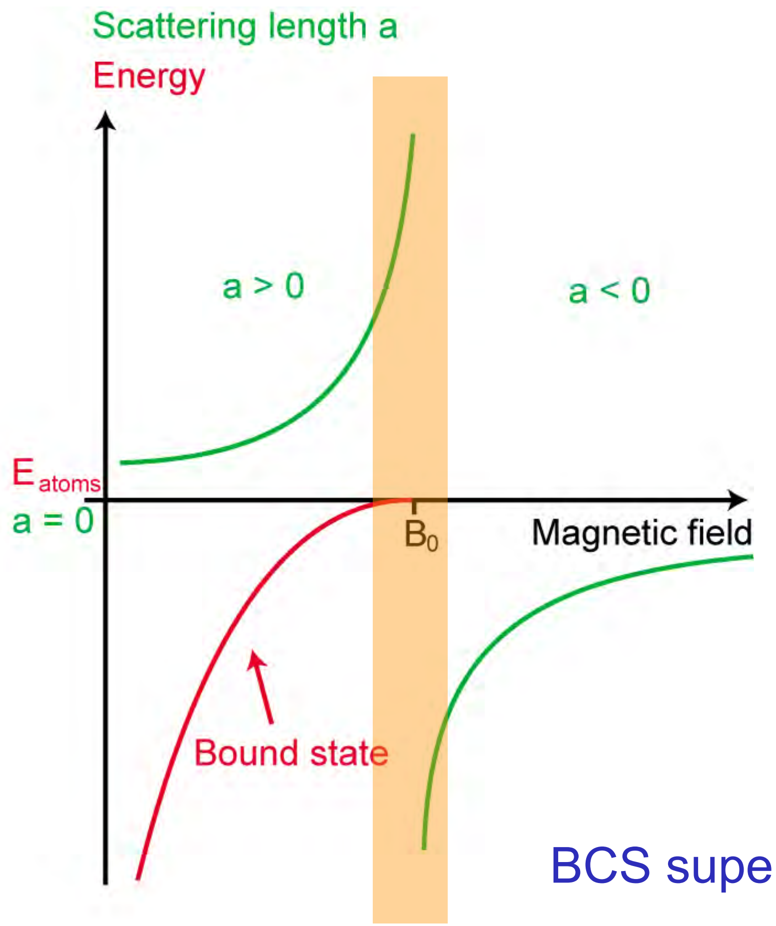
What happens to solitons in the resonant Fermi superfluid?



credit: MIT group



BCS superfluid

BEC of
preformed pairs

unitarity

Can solitons probe strongly-interacting physics beyond hydrodynamics?

Hydrodynamics: equation of state

Bose-Einstein condensate $\mu_{BEC} = gn \propto n^{\frac{2}{2}}$

Unitary Fermi gas $\mu_{UFG} = (1+\beta)\frac{\hbar^2 k_F^2}{2m} \propto n^{\frac{2}{3}}$

Phonons (sound waves):

- Sound speed determined by equation of state
- Zero effective mass (linear dispersion)
- In a harmonically trapped gas, they traverse the gas with sinusoidal oscillations

$$\left(\frac{T_{\mathrm{BEC\ phon's}}}{T_{\mathrm{trap}}}\right)^2 = 2 \qquad \left(\frac{T_{\mathrm{UFG\ phon's}}}{T_{\mathrm{trap}}}\right)^2 = 3$$

…determined by exponent in equation of state

Solitons in unitary Fermi gas

- Analytical arguments based on scaling arguments and few (unproven) assumptions suggest [1]

$$\left(\frac{T_{\mathrm{s,UFG}}}{T_{\mathrm{trap}}}\right)^2 = \frac{m^*}{m_{ph}} = 3$$

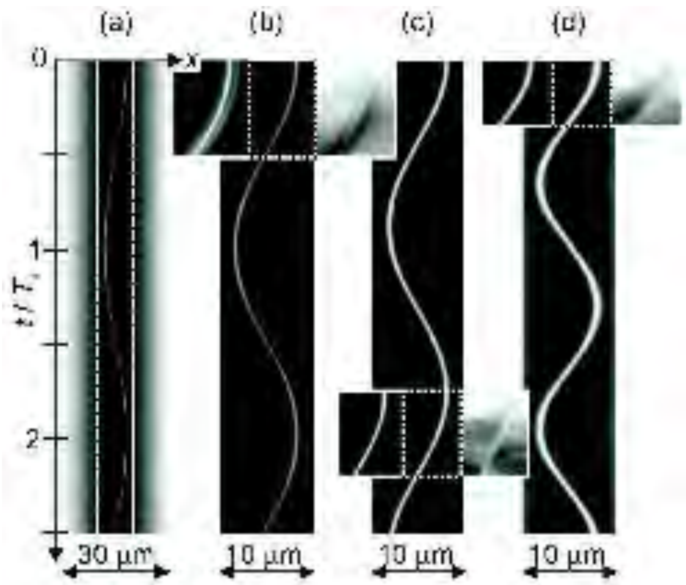
- Numerical solutions of the time-independent [1] and time-dependent [2] Bogoliubov-de Gennes equations (interpolating mean-field theory) give a value consistent with 3.0.

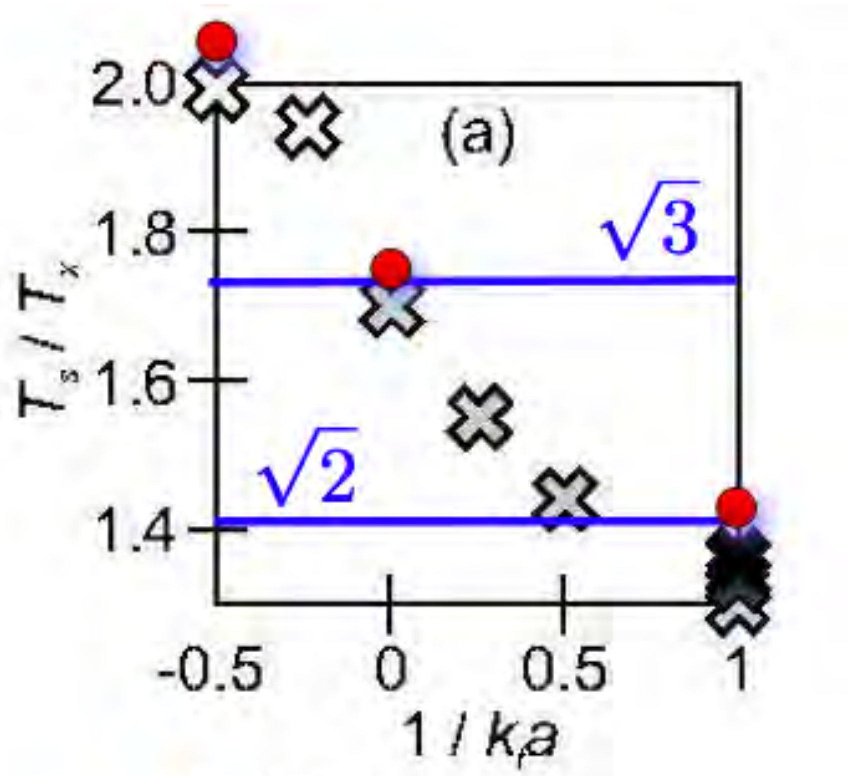
Does the equation of state completely determine the mass ratio of dark solitons? If so, why?

[1] Liao, Brand PRA 83, 041604(R) (2011)
[2] Scott, Dalfovo, Pitaevskii, Stringari PRL (2011)

Oscillation period: BdG numerics

Time-dependent simulations were performed by the Trento group





- Analytics
- Our numerics: (Liao PRA 2011)
  - 2.083 at eta = -0.5 (BCS)
  - 1.748 at eta = 0 (unitarity)
  - 1.456 at eta = +1 (BEC)
- Trento data (time dependent) Scott et al. PRL (2011)

Scott, Dalfovo, Pitaevskii, Stringari PRL (2011)

Liao, Brand PRA 83, 041604(R) (2011)

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One-dimensional Fermi superfluid

The previous theory applied to the 3D Fermi superfluid (unitary FG and crossover regime).

What happens in a 1D (tightly confined) situation?

Efimkin and Galitski found an analytic solution of the Bogoliubov-de Gennes equations. They predict:

$$m^* = -\frac{4m}{\pi}\frac{E_F}{\Delta_0} \to -\infty \quad \text{when} \quad \Delta_0 \to 0$$

$$m_{\text{ph}} \to +0 \quad \text{in pure 1D or} \quad m_{\text{ph}} \to -0 \quad \text{in quasi-1D}$$

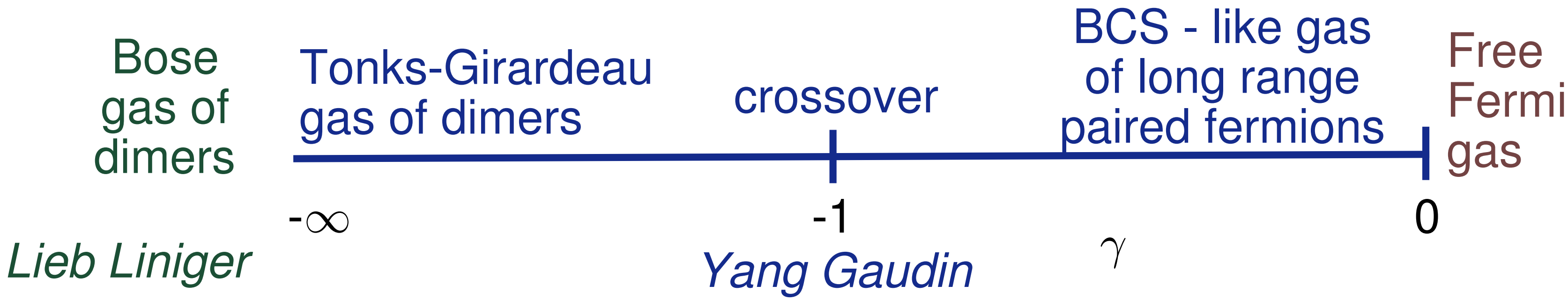
But is BdG theory applicable to a one-dimensional gas?

Efimkin, Galitski, PRA 91, 023616 (2015)

Purely one-dimensional model

$$H = -\frac{\hbar^2}{2m}\sum_{j=1}^{N}\frac{d^2}{dx_j^2} + g\sum_{\langle i,j\rangle}\delta\left(x_i - x_j\right)$$

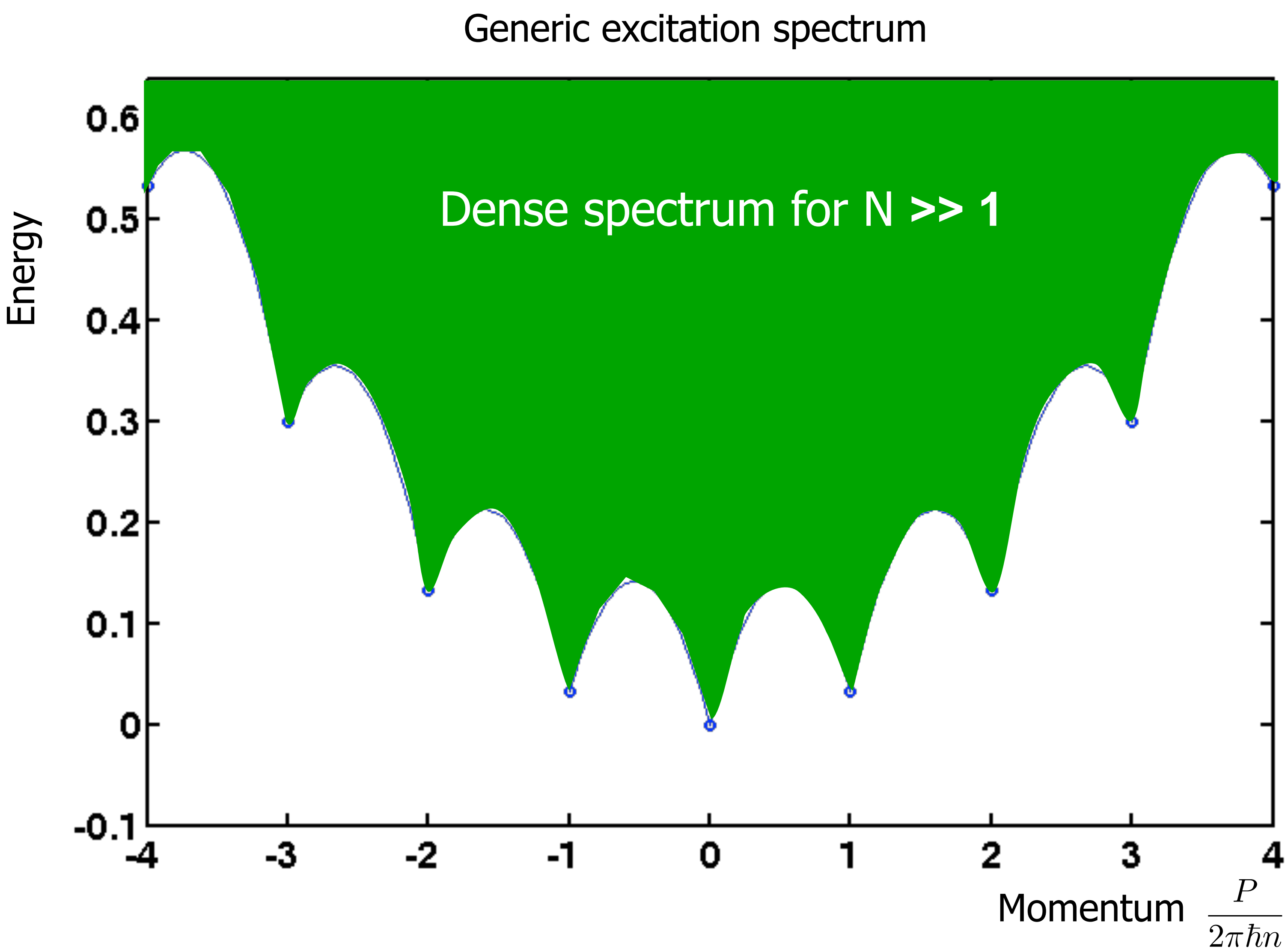
- Consider N particles (bosons or fermions) with identical mass and interactions
- Bethe ansatz provides exact description of ground and excited states
- Lieb-Liniger model: Bosons with repulsive interactions (g>0)
- **Yang-Gaudin model: spin-1/2 fermions (here: attractive g<0)**



$$\gamma = \frac{gm}{n_0\hbar^2} = \frac{c}{n_0}$$

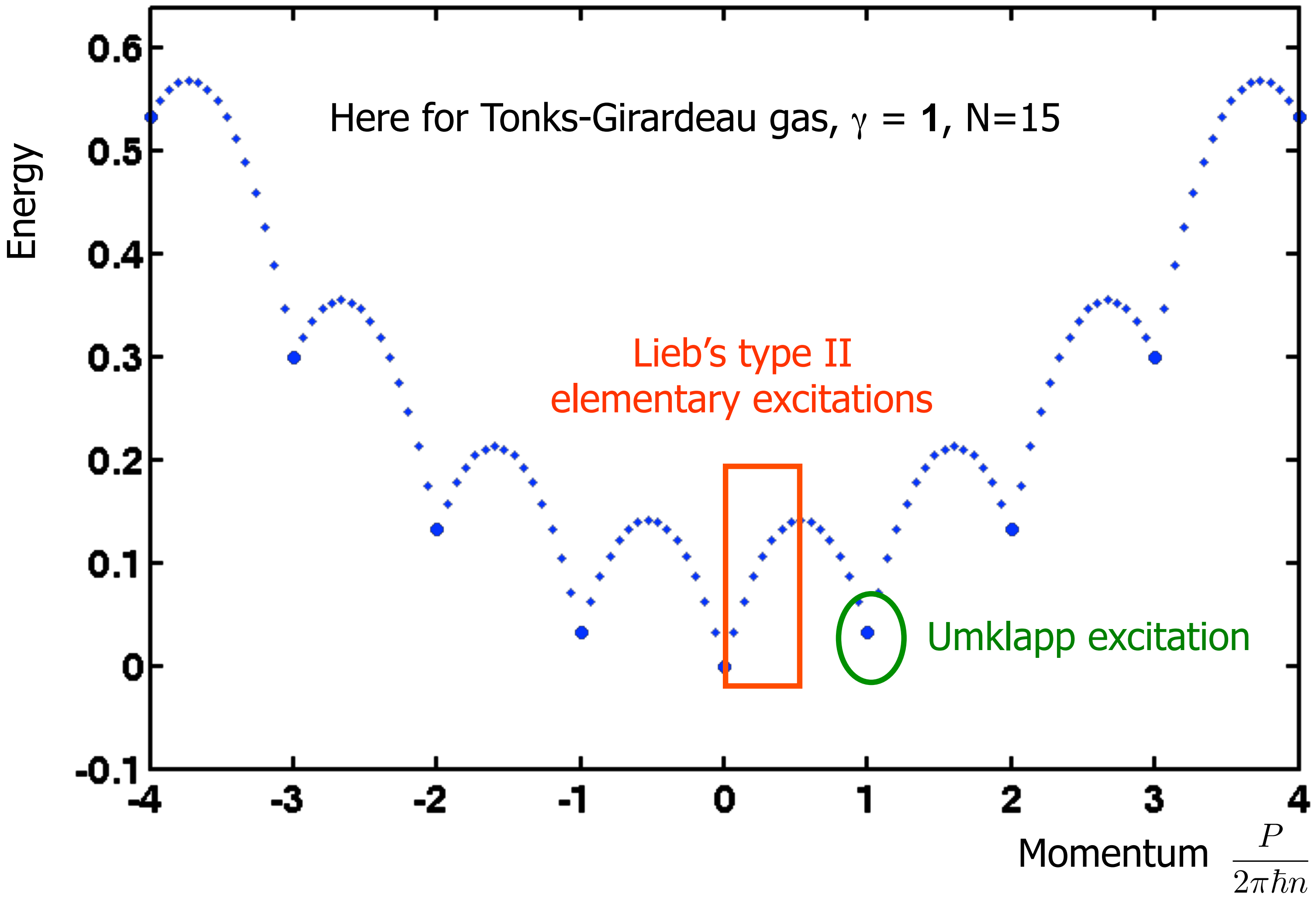
See also poster by A. Ayet on quantum bright solitons in attractive Bose gas

Where are the solitons?



Identify solitons with the *yrast* states

Low-lying excitation spectrum (yrast states)



Bethe ansatz equations

$$\exp(ik_jL) = \prod_{n=1}^{M} \frac{k_j - \alpha_n + ic/2}{k_j - \alpha_n - ic/2},$$

$$\prod_{j=1}^{N} \frac{\alpha_m - k_j + ic/2}{\alpha_m - k_j - ic/2} = -\prod_{n=1}^{M} \frac{\alpha_m - \alpha_n + ic}{\alpha_m - \alpha_n - ic}.$$

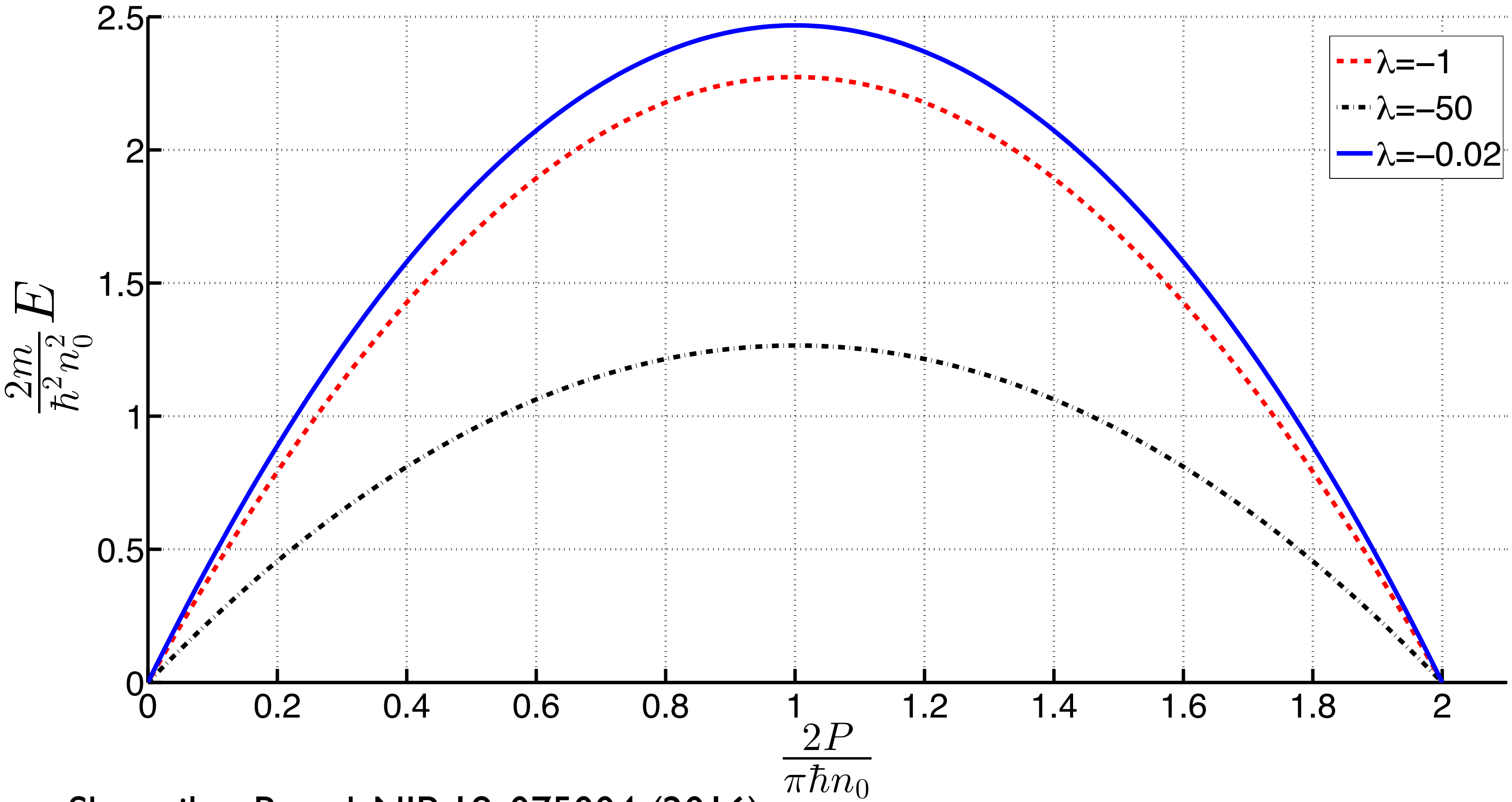
k_j - charge rapidities
α_j - spin rapidities
N - number of fermions
M - number of spin-up fermions

$$P_{\text{tot}} = \hbar \sum_{j=1}^{N} k_j,$$

$$E_{\text{tot}} = \frac{\hbar^2}{2m} \sum_{j=1}^{N} k_j^2.$$

Yrast dispersion thermodynamic limit

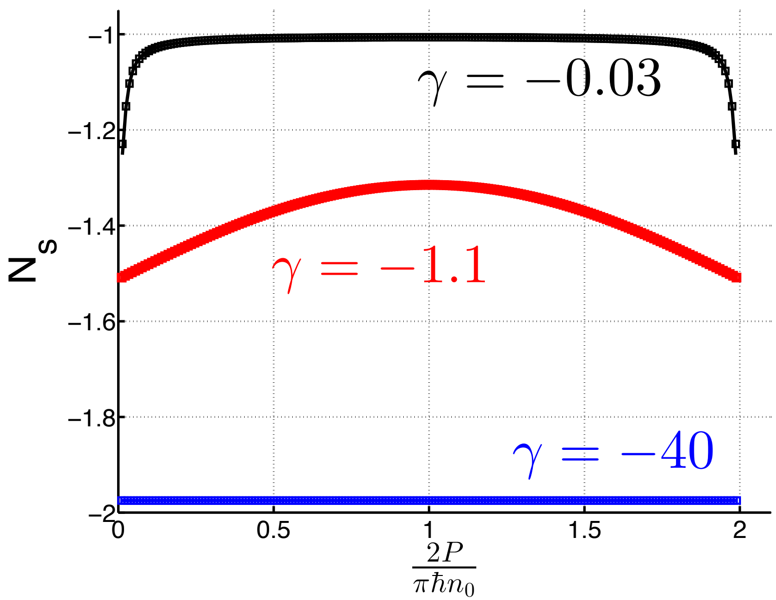
Bethe ansatz equations yield Fredholm integro-differential equations in the thermodynamic limit



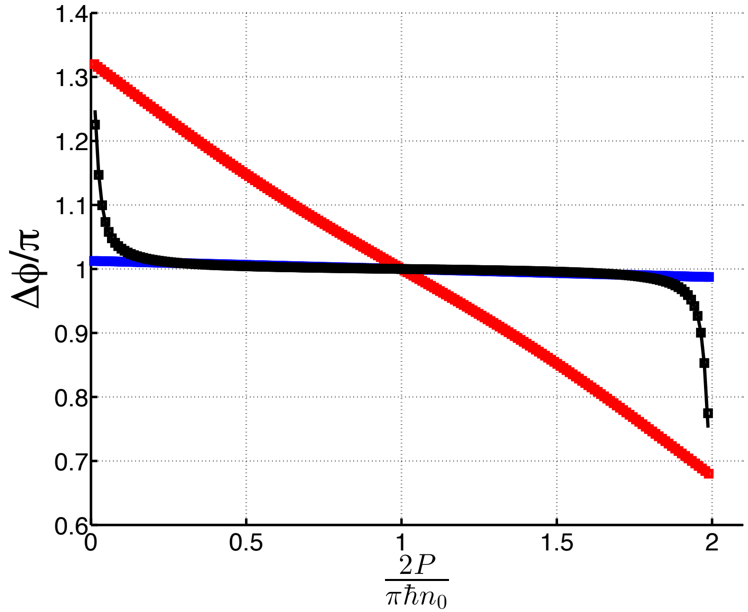
Shamailov, Brand, NJP 18, 075004 (2016)

Yrast dispersion properties

Missing particle number



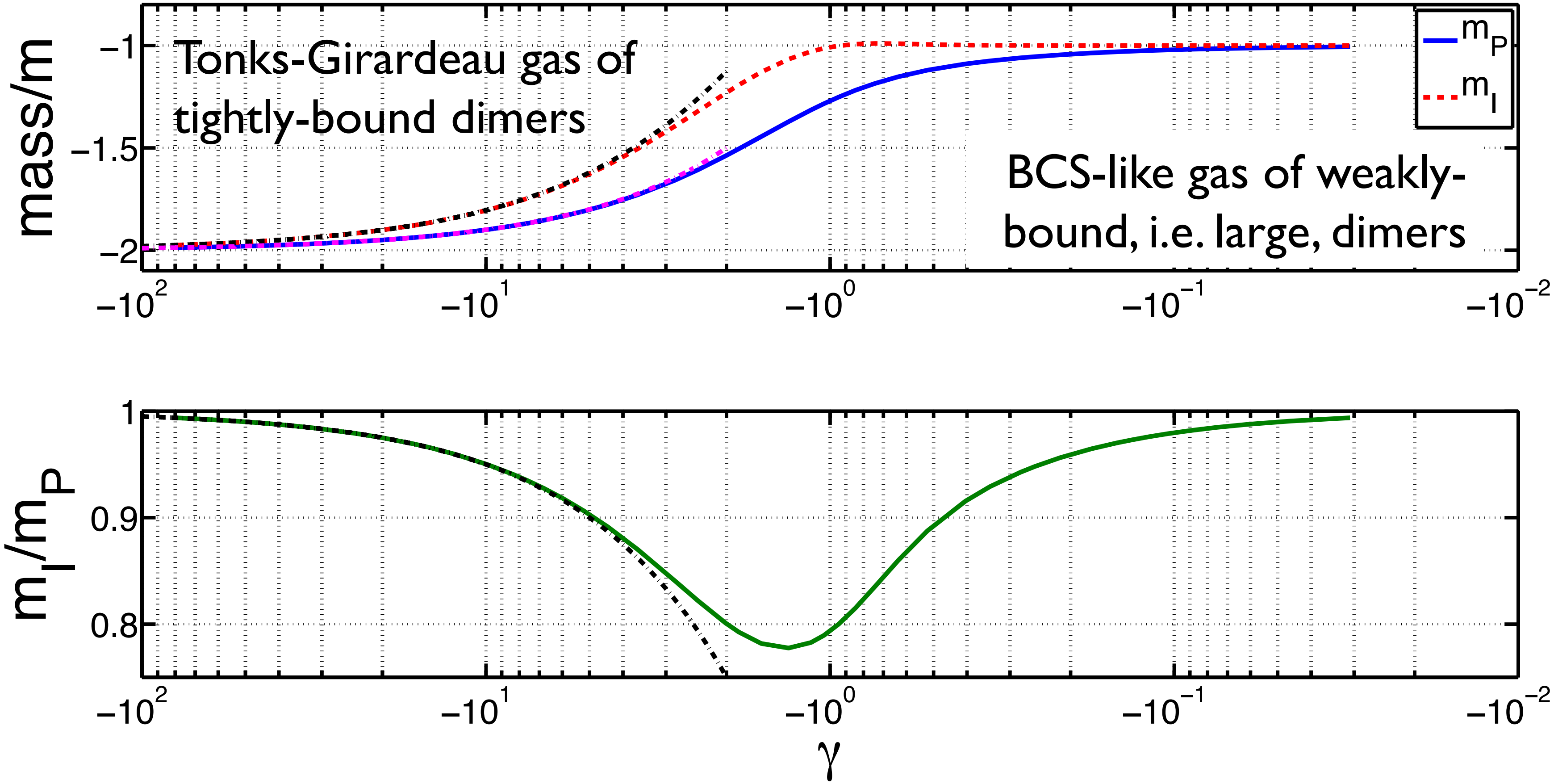
Phase step



$$P = m v_s N_s - \frac{1}{2}\hbar n_0 \Delta\phi$$

Shamailov, Brand, NJP 18, 075004 (2016)

Inertial and physical mass



Contrast: mean-field theory predicts divergent mass ratio

Shamailov, Brand, NJP 18, 075004 (2016)

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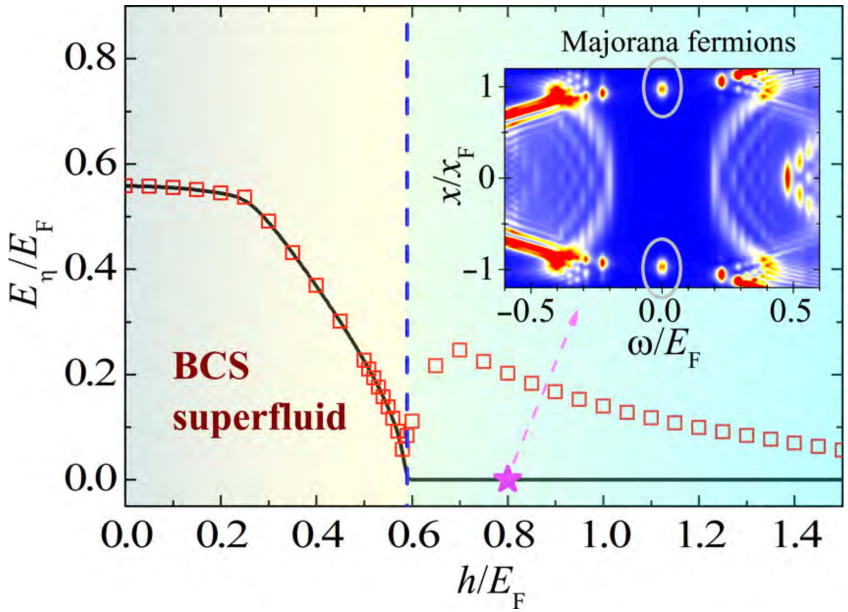
Fermi superfluid with spin-orbit coupling

Fermi superfluid with spin-orbit coupling and spin-imbalance (Zeeman) has a topological phase transition.

This means that Majorana quasiparticles appear as edge states – and as Andreev bound states in dark solitons.

Majorana quasiparticles are their own anti-particles.



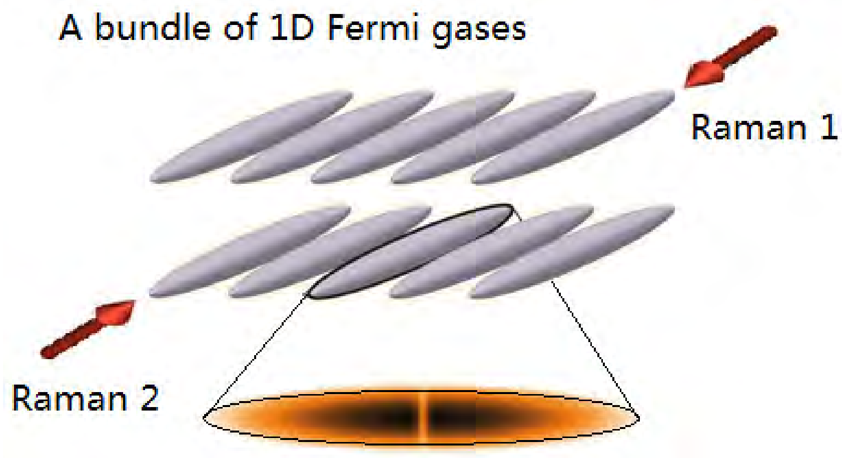
Topological phase transition as the Zeeman splitting h is changed.

Mizushima, Machida PRA 81, 053605 (2010)
Xu et al. PRL 113, 130404 (2014)
Xia-Ji Liu PRA 91, 023610 (2015)

Moving solitons in spin-orbit coupled Fermi superfluid

Choose 1D system and Bogoliubov-de Gennes approach

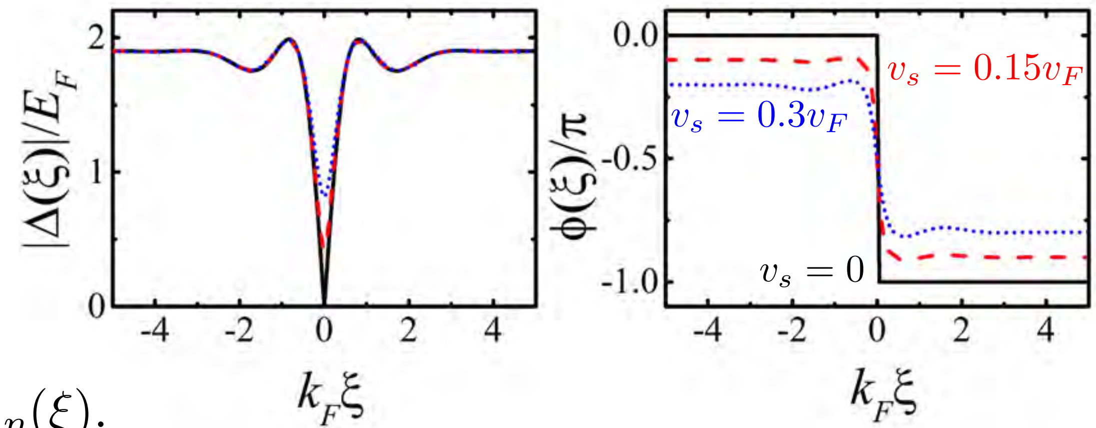
$$\mathcal{H}_{\rm BdG} \equiv \begin{bmatrix} \mathcal{H}_s - h & -\lambda\partial/\partial x & 0 & -\Delta \\ \lambda\partial/\partial x & \mathcal{H}_s + h & \Delta & 0 \\ 0 & \Delta^* & -\mathcal{H}_s + h & \lambda\partial/\partial x \\ -\Delta^* & 0 & -\lambda\partial/\partial x & -\mathcal{H}_s - h \end{bmatrix}$$



Solve BdG equation for moving solitons.

$$\Delta(x,t) = \Delta(x - v_s t) = \Delta(\xi)$$

$$\mathcal{H}_{\rm BdG}(\xi)\boldsymbol{\Phi}_\eta(\xi) = \left[E_\eta - i\hbar v_s \frac{\partial}{\partial \xi}\right]\boldsymbol{\Phi}_\eta(\xi).$$



The phase step changes with velocity in the **non-topological** regime. h=0.5E_F

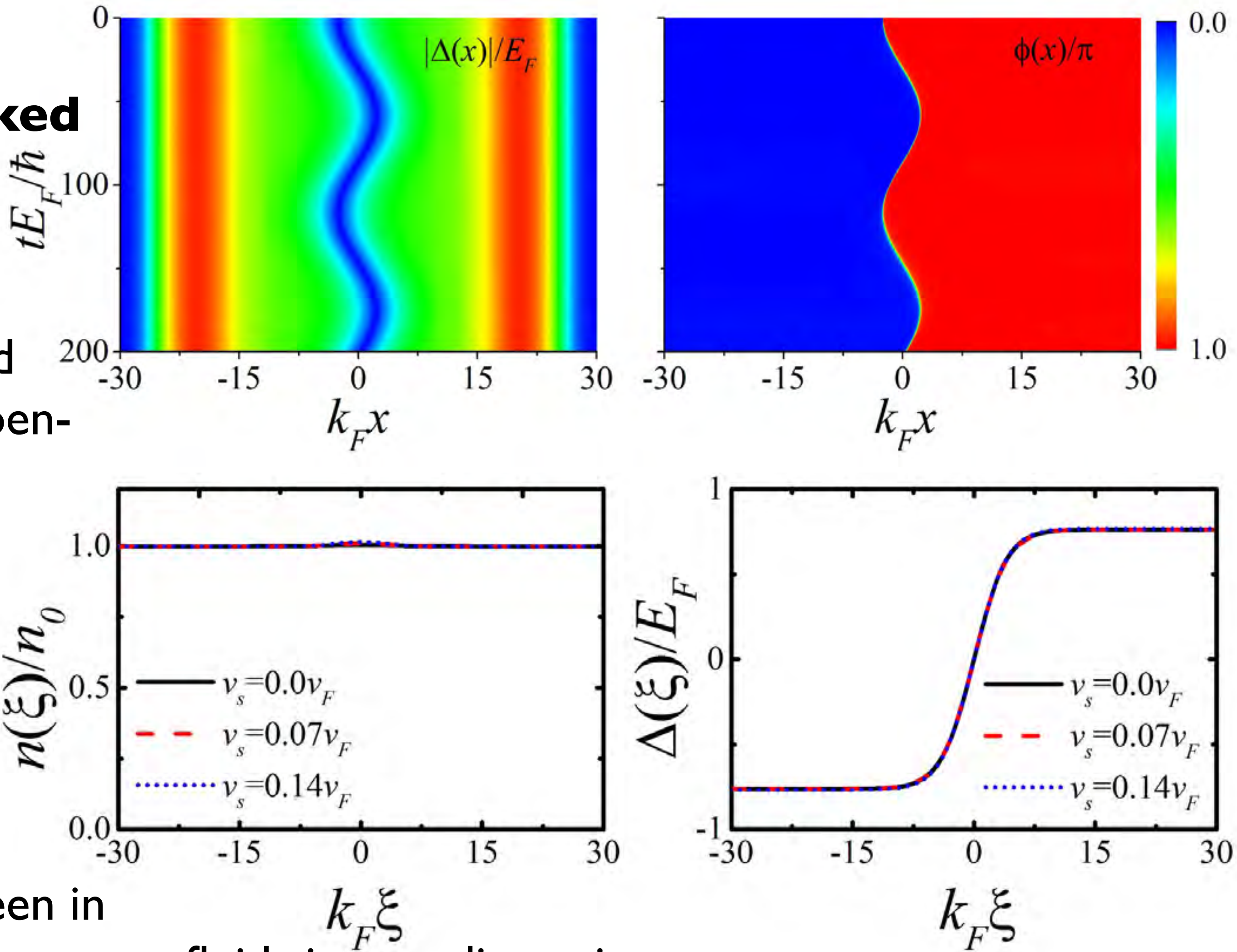
Zhou, Brand, Liu, Hu, PRL (2016) PRL 117, 225302 (2016)

Majorana soliton in topological superfluid

In the topological regime the **phase step is locked to π**, independent of velocity.

The order parameter and density profiles are indepen-dent as well.

In a harmonic trap, the Majorana soliton oscillates, but the phase step remains π.



The same behaviour is seen in Rashba-coupled and p-wave superfluids in two dimensions.

Zhou, Brand, Liu, Hu, PRL (2016) PRL 117, 225302 (2016)

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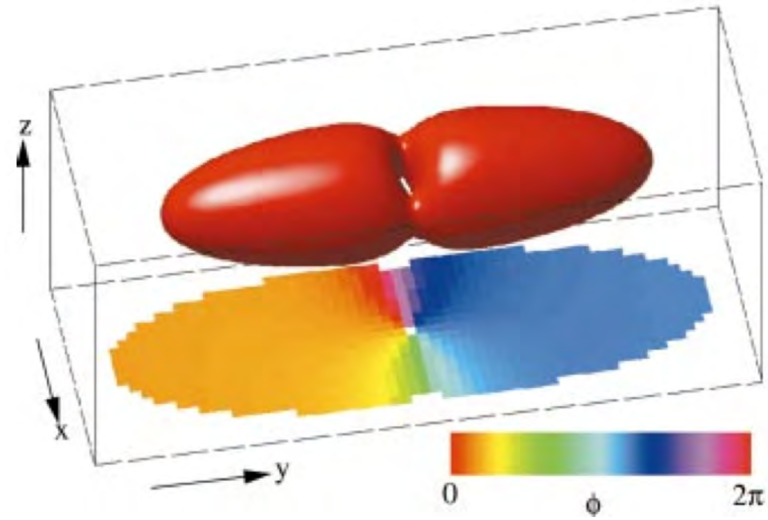
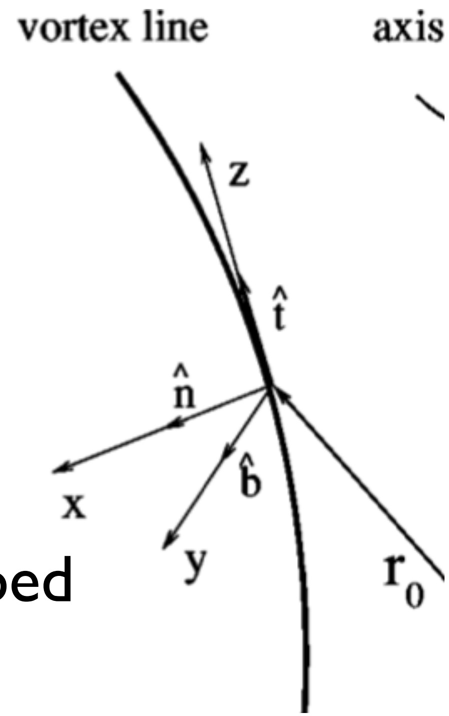
What is a solitonic vortex?

1. … a solitary wave that is localised (exponentially) in the long dimension of a fluid that is confined in the other two dimensions.
2. … a single vortex filament.



Brand, Reinhardt, JPB 37, L113 (2001)
Brand, Reinhardt, PRA 65, 043612 (2002)

Vortex dynamics from hydrodynamics



Local evolution of vortex filament

$$\vec{v} = \left(\kappa \hat{b} + \frac{\hat{t} \times \vec{\nabla} V}{\mu_{\rm loc}} \right) \frac{\Lambda}{2}$$

Svdizinski, Fetter, PRA (2000)
Horng, Gou, Lin PRA (2006)

Dynamics of the solitonic vortex in harmonically trapped (cylindrical) superfluid was solved by Ku et al. (2014).

However, the hydrodynamic solutions suffer from **'logarithmic (in-) accuracy'**.

Ku et al. PRL 113, 065301 (2014)

Dynamics of solitonic vortex

$$\left(\frac{T_s}{T_{\text{trap}}}\right)^2 = \frac{m^*}{m_{ph}}$$

Oscillation frequency can be measured accurately in experiments

Effective (inertial) mass:

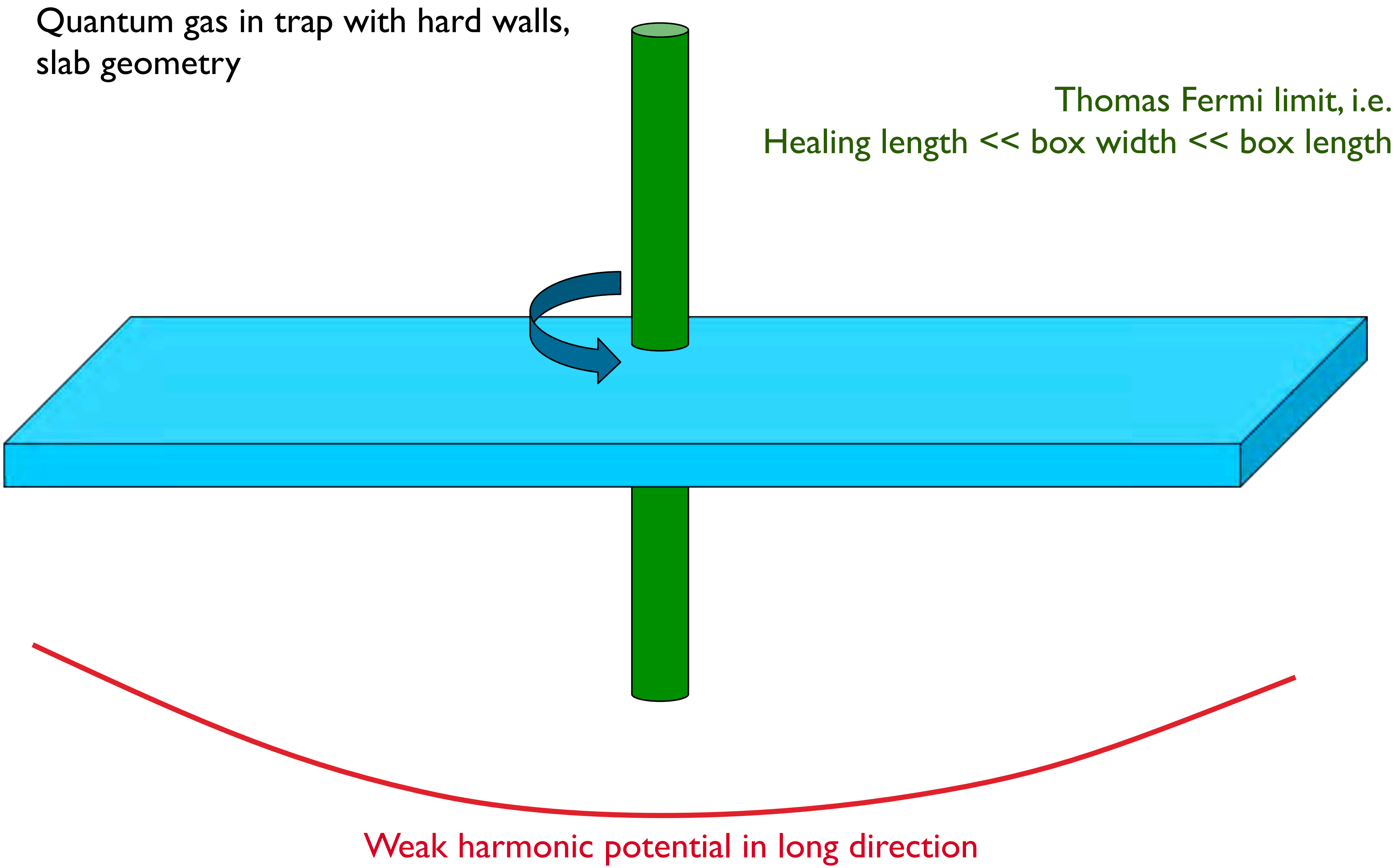
- Property of the velocity field
  - length scale > healing length
  - determined by hydrodynamics

Physical (heavy) mass:

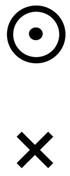
- Integral over expelled mass density
  - This happens at healing length scale
  - not captured accurately by hydrodynamics

Can we solve the hydrodynamic problem exactly?
Measuring the oscillation frequency then gives access to the physical mass, i.e. the vortex core 'size'.

Solitonic vortex in a slab geometry



Method of images – incompressible fluid

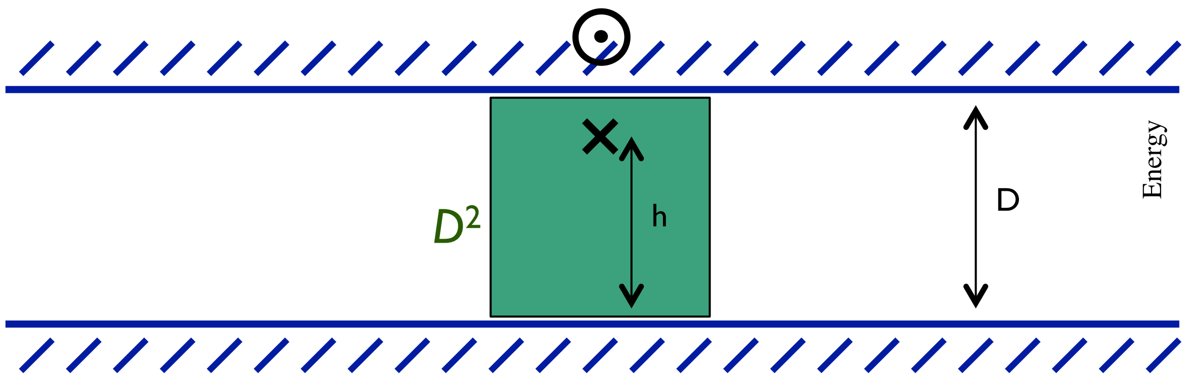


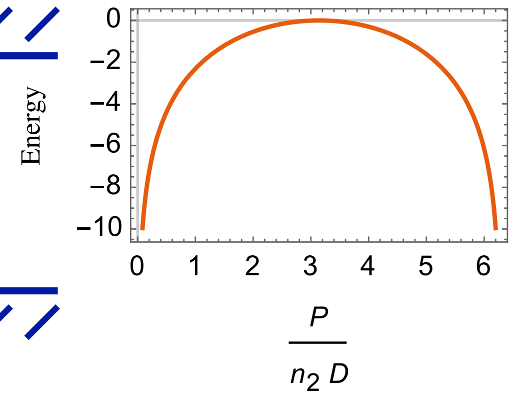
Velocity potential

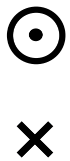
$$w = i \ln \frac{\sinh \frac{\pi}{2D}(z - ih)}{\sinh \frac{\pi}{2D}[z - i(2D - h)]}$$

Energy-momentum dispersion relation

$$E_s = \pi n_2 \ln \sin\left(\frac{P}{2n_2 D}\right)$$







The solitonic vortex is indeed **exponentially** localised.

Effective mass (v=0)

$$m^* = -m\frac{4}{\pi}D^2 n_2$$

All particles in volume D^2 contribute to the effective/inertial mass

Toikka, Brand NJP 19, 023029 (2017)

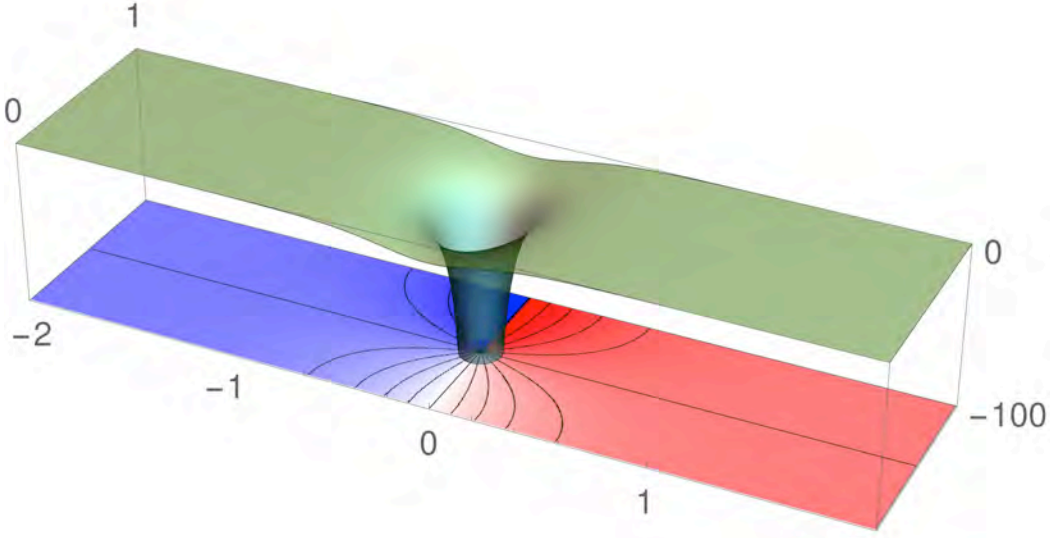
Compressible fluid – perturbation theory

- Idea: View kinetic energy density in Euler equation as 'perturbation'.
- Series expansion for density and phase lead to Poisson equations.
- These can be solved by Greens function methods but need to be renormalised.
- Leads to series expansion for dispersion relation in powers of ξ^2/D^2

$$\left(\frac{T_0}{T_{\text{trap}}}\right)^{-2} = \frac{M_{\text{ph}}}{M^*}\bigg|_{h=\frac{D}{2}} = a_2 \frac{\pi^2 q^2}{2}\frac{\xi^2}{D^2} + a_4 \frac{\pi^2 q^4}{2}\frac{\xi^4}{D^4} + \mathcal{O}\left(\frac{\xi^6}{D^6}\right),$$

$$a_2 = \ln\left(\frac{D}{\xi}\right) - F,$$

$$a_4 = -\left[F + 9\ln\left(\frac{D}{\xi}\right)\right] + \left[\ln\left(\frac{D}{\xi}\right) - F\right]\left[\pi^2 \ln\left(\frac{\pi\xi}{2D}\right) + \frac{\pi^2}{2}\right]$$



The series can be solved, order-by-order, beyond logarithmic accuracy – exactly. Precision experiments that reveal deviations could help us **understand vortex-core physics**.

Toikka, Brand NJP 19, 023029 (2017)

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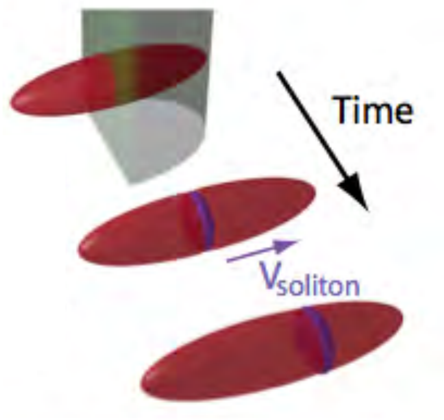
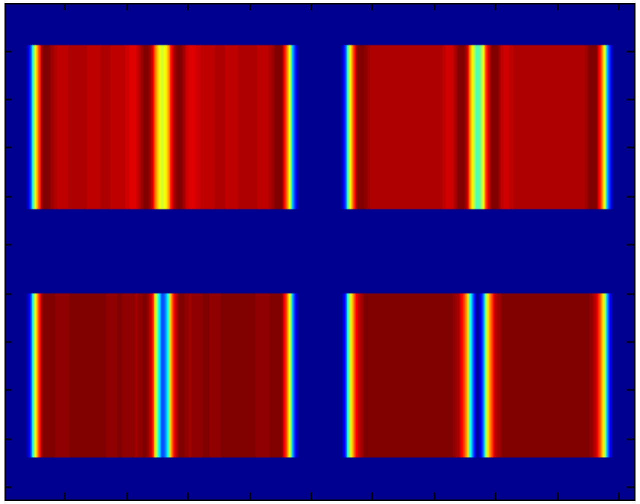
How do solitonic vortices form?

1. Phase imprinting generates dark soliton



2. Dark soliton decays via snaking instability

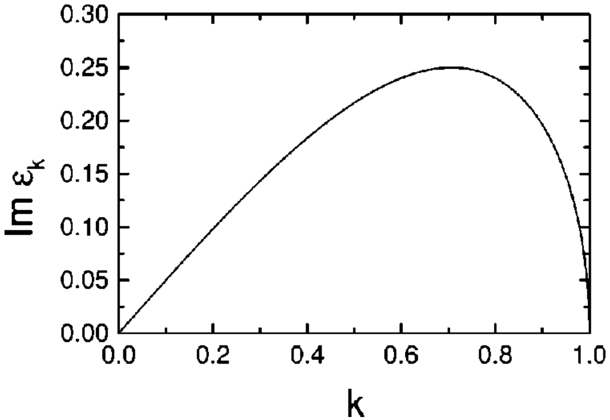
Snaking instability for homogeneous gas

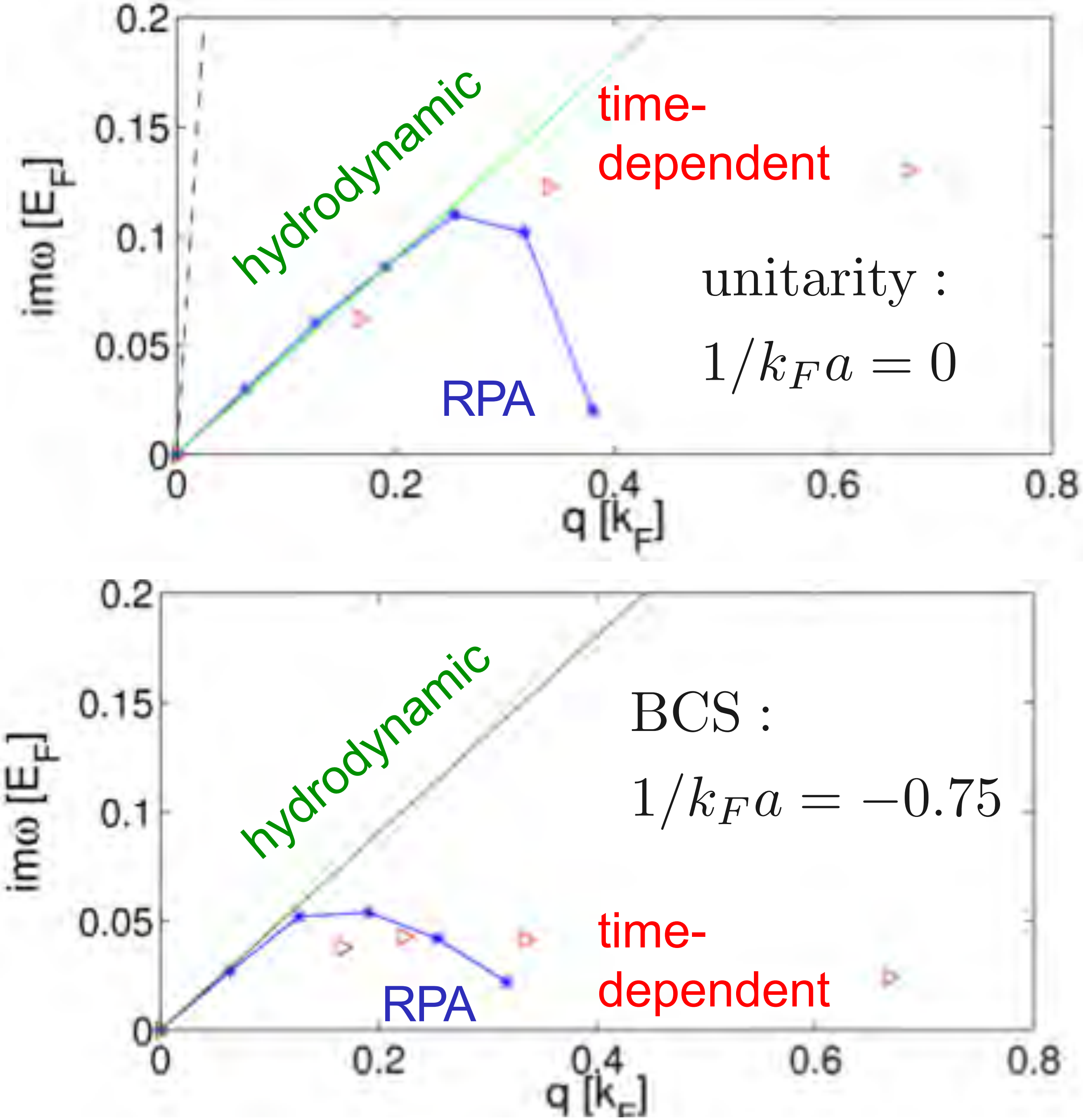


BEC (theory):
Kuznetsov and Turitsyn JETP (1988)
Muryshev et al. PRA (1999)

- BEC experiment

Anderson et al. PRL (2001)



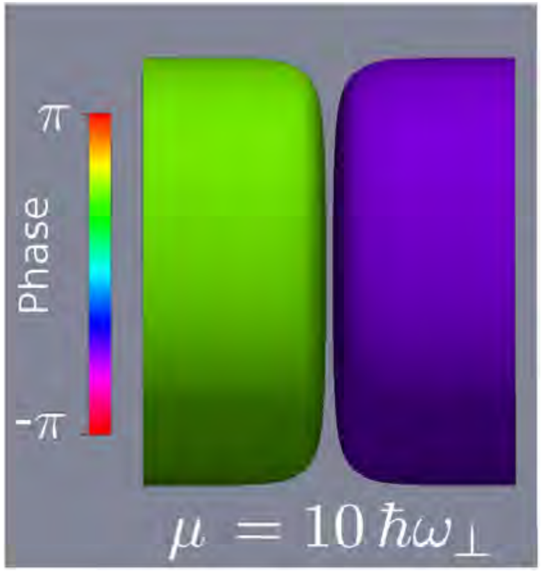


Cetoli, Brand, Scott, Dalfovo, Pitaevskii, PRA (2013)

Solution: Approach from hydrodynamics

Hydrodynamic picture of the snaking instability:
Dark soliton is a membrane that "vibrates" under the influence of *surface tension* (and negative mass density).

Kamchatnov, Pitaevskii PRL (2008)



Thus, we should expect the vibration modes of a circular membrane …

Entdeckungen

über die

Theorie des Klanges

"Discoveries about the Theory of Chimes"

von

Ernst Florens Friedrich Chladni,

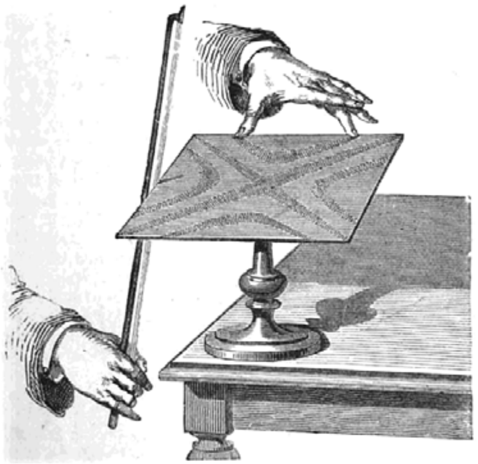
der Philosophie und Rechte Doctor zu Wittenberg.

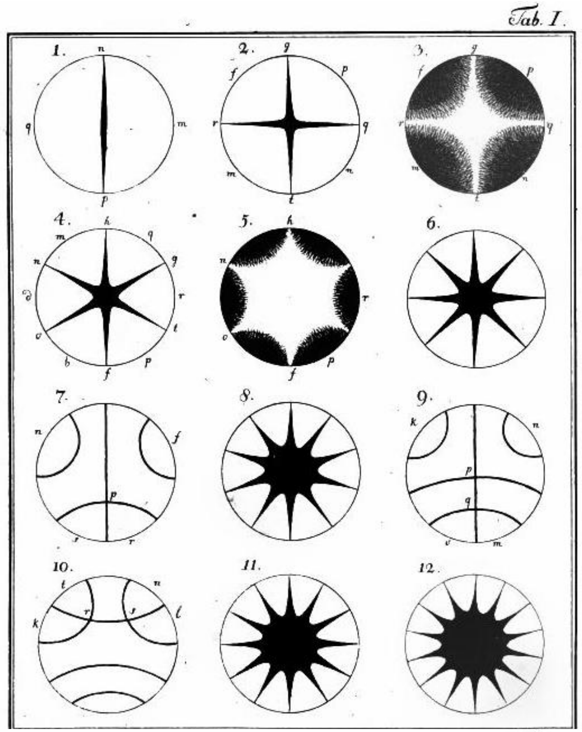
Mit eilf Kupfertafeln.

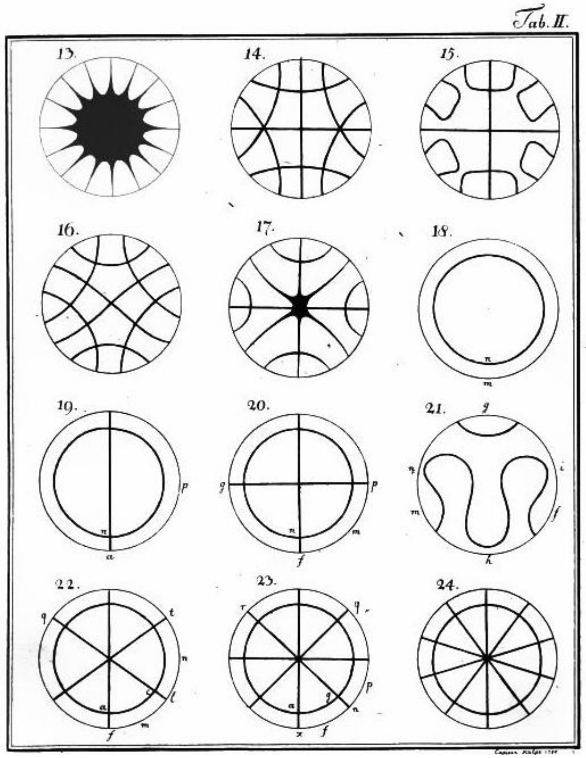
Leipzig,

bey Weidmanns Erben und Reich.

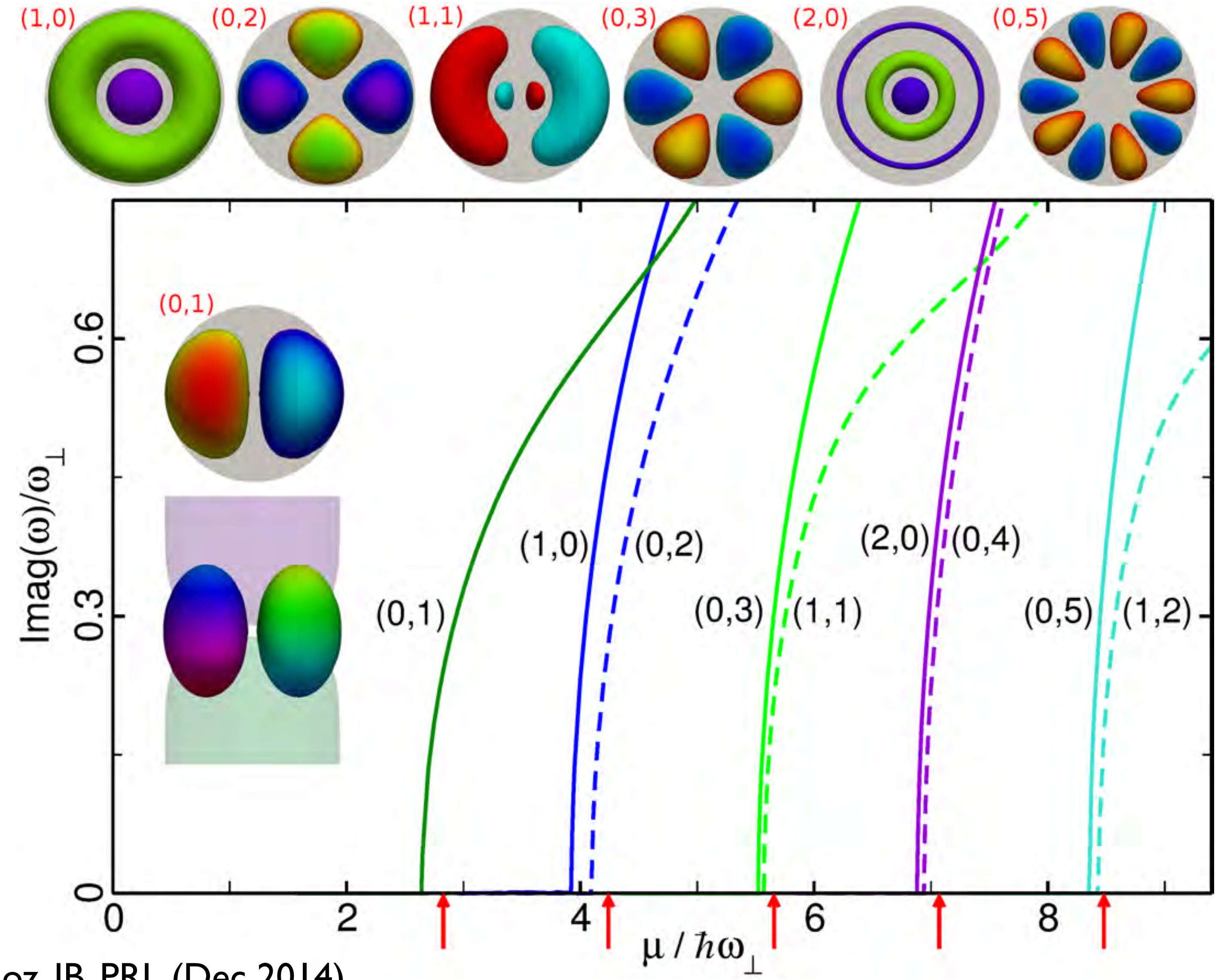
1787.







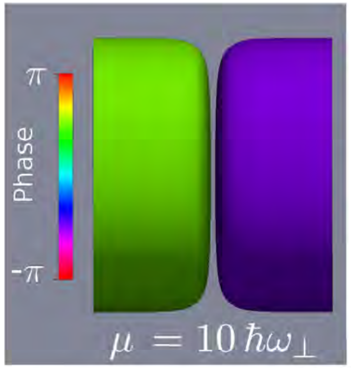
Unstable modes of the dark soliton (numerics)

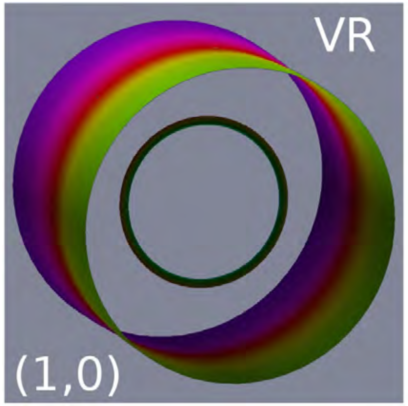


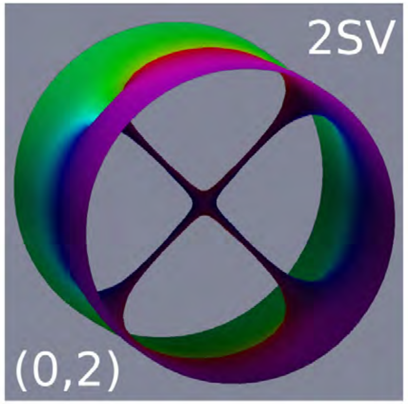
A. Mateo Munoz, JB, PRL (Dec 2014)

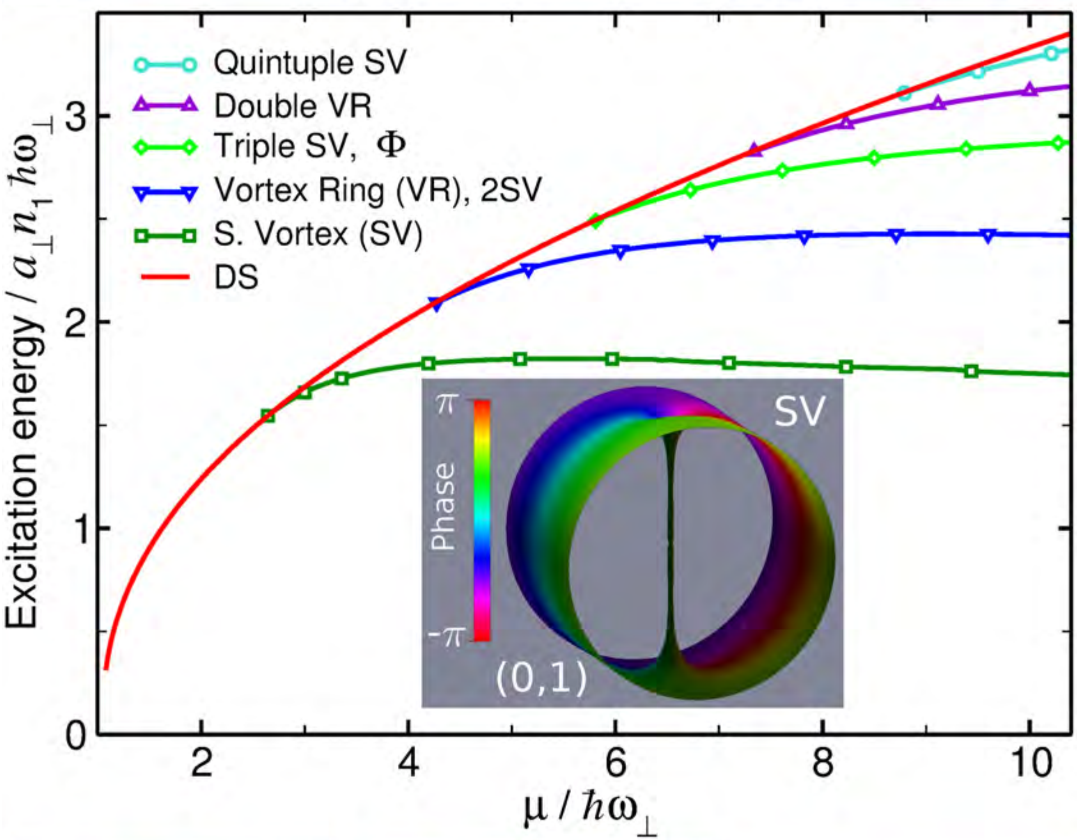
Chladni Solitons: Numerics (GPE)

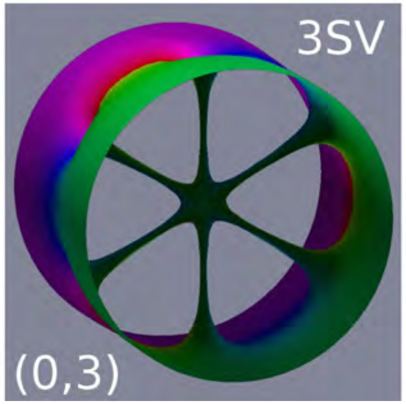
Dark soliton (DS)

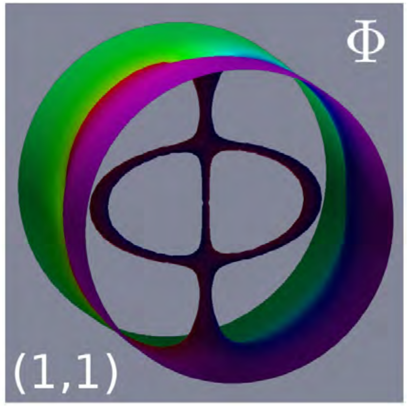


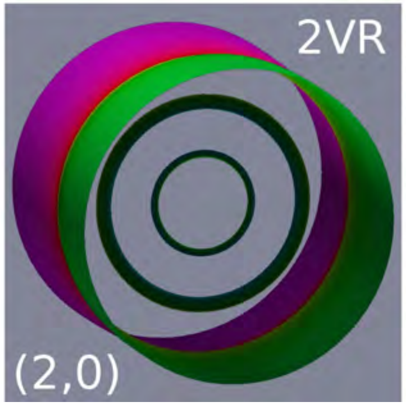


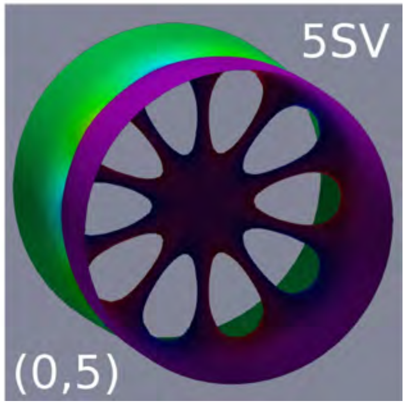










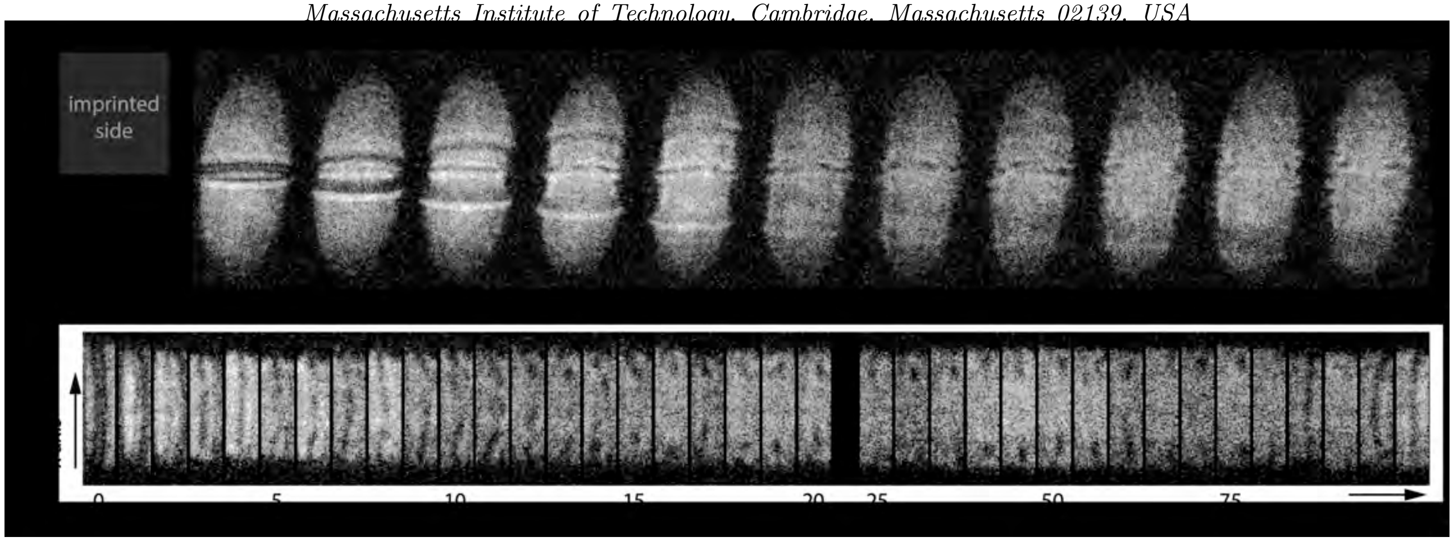


A. Mateo Munoz, JB, PRL (Dec 2014)

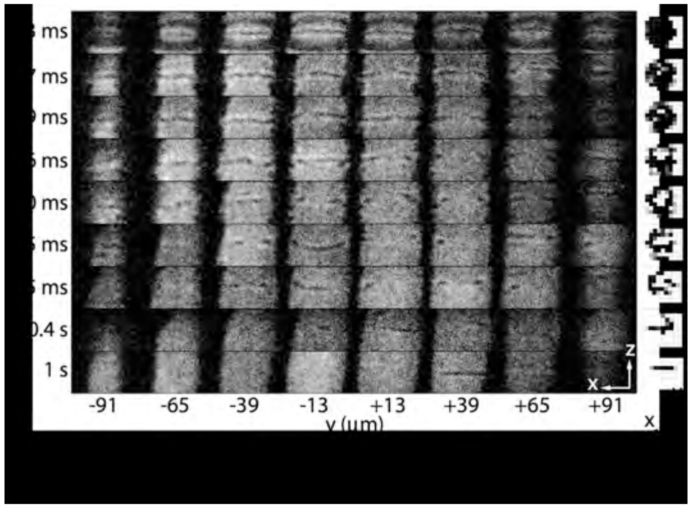
Decay of planar dark solitons observed in the unitary Fermi gas

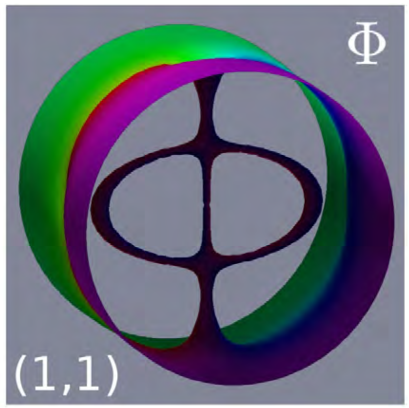
**From Planar Solitons to Vortex Rings and Lines:
Cascade of Solitonic Excitations in a Superfluid Fermi Gas**

Mark J.H. Ku, Biswaroop Mukherjee, Tarik Yefsah, Martin W. Zwierlein
MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics, and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA



Evidence for observation of the Phi soliton.





Ku et al. PRL 116, 045304 (2016)

Outline

- Solitons
  - Dynamics from dispersion relations
  - Dark solitons in the crossover Fermi superfluid
  - One-dimensional Fermi superfluid
  - Spin-orbit coupled topological Fermi superfluid

- Vortices as solitary waves – solitonic vortices
  - What is a solitonic vortex?
  - Slab model – asymptotically solvable for compressible fluids
  - Snaking instability and Chladni solitons

What can we learn from solitary waves about strongly-correlated superfluids?

Dynamics is interesting.

Consider solitonic vortex: Hydrodynamics (almost) completely determines dynamics, determines inertial mass.

But the physical mass (buoyant, or 'heavy' mass) is determined by micro/mesoscopic properties of the core structure.

The ratio inertial mass / physical mass can be measured by oscillation experiments (like at MIT).

The end!

Colloborators:

At Massey University:
Sophie Shamailov
Antonio Muñoz Mateo
Renyuan Liao
Oleksandr Fialko
Alberto Cetoli

Elsewhere:
Peng Zou
Xia-Ji Liu
Hui Hu
Robin Scott
Franco Dalfovo
Lev Pitaevskii
Sandro Stringari