Dynamical Cooper pairing in non-equilibrium electron-phonon systems

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Outline

Physical picture: enhanced electron-phonon interaction in systems with driven phonons

Floquet-Migdal-Eliashberg analysis

Optical response of photo-excited superconductor: Nonequilibrium dynamics with variational states

Conclusions

Motivation: photoinduced superconducitivty in K3C60



M. Mitrano et al., Nature 530, 461 (2016)

Response of photoexcited system







How to increase electron-phonon interaction and enhance superconductivity, CDW, ... by creating a non-equilibrium state of phonons

This talk: focus on superconductivity

Other experiments: optical control of Mott insulators, charge and spin density wave states, superconductivity in high Tc cuprates

Other proposals for photoinduced superconductivity: Jaksch, Galitski, Komnik, Thorwart, Georges, Kollath, Mathey, Millis, Chamon, Sentef, ... Can one gain from non-equilibrium state of phonons?



interaction via phonon emission

 $-\frac{g^2(1+n)}{\epsilon_p + \omega_{k-p} - \epsilon_k} \approx -\frac{g^2(1+n)}{\omega_{\rm ph}}$



interaction via phonon absorption

$$-\frac{g^2 n}{\epsilon_p - \omega_{p-k} - \epsilon_k} \approx +\frac{g^2 n}{\omega_{\rm ph}}$$

Effective interaction

$$V_{
m eff}\,=\,-\,rac{g^2}{\omega_{
m ph}}$$

This is the usual argument that real photons do not help to increase effective pairing strength

Electron-phonon interactions in systems with driven phonons

$$\mathcal{H} = \sum_{q} \left(\frac{P_q^2}{2M} + \frac{M\omega_q^2}{2} Q_q^2 \right) + \mathcal{H}_{\text{drive}}(t)$$

Example: parametric drive of phonons

$$\mathcal{H} = \sum_{q} \left(\frac{P_q^2}{2M} + \frac{M\Omega_q^2(t)}{2} Q_q^2 \right) \qquad \qquad \Omega_q^2(t) = \omega_q^2 \left(1 + 2\alpha \cos(2\Omega_{\rm drv}t) \right)$$

Effective electron-electron interaction assuming slow electron dynamics

$$\begin{array}{ll} \text{nteraction} & \mathcal{H}_{\text{eff}} \,=\, U(t)\,\hat{\rho}_{\text{el}}(t)\,\hat{\rho}_{\text{el}}(t)\\ \text{amics} & \\ U(t) = \frac{|\tilde{g}_{\mathbf{q}}|^2}{\hbar}\int_{-\infty}^t \mathrm{d}t' \;\; \mathcal{D}^R_{QQ}(t,t')\\ \mathcal{D}^R_{QQ}(t,t') = -i\theta(t-t')\langle \hat{Q}(t)\,\hat{Q}(t') - \hat{Q}(t')\,\hat{Q}(t)\rangle \end{array}$$

Electron-phonon interactions in systems with driven phonons

Compare to

$$\begin{aligned} \mathcal{H}' &= \mathcal{H} - \phi \, \hat{Q} \\ \langle \hat{Q} \rangle &= \chi \, \phi \quad \text{Fluctuation-dissipation theorem} \quad \chi = \langle \hat{Q} \, \hat{Q} \rangle \\ \Delta E &= -\frac{\chi \phi^2}{2} \end{aligned}$$

From electron-phonon coupling to effective electron-electron interaction

$$\mathcal{H}_{\text{el-phon}} = \sum_{k q \sigma} g_q Q_q c_{k-q\sigma}^{\dagger} c_{k\sigma}$$
$$\mathcal{H}_{\text{eff}} = U(t) \hat{\rho}_{\text{el}}(t) \hat{\rho}_{\text{el}}(t)$$
$$U(t) = \frac{|\tilde{g}_{\mathbf{q}}|^2}{\hbar} \int_{-\infty}^t dt' \ \mathcal{D}_{QQ}^R(t,t')$$

$$\mathcal{D}_{QQ}^{R}(t,t') = -i\theta(t-t')\langle \hat{Q}(t)\,\hat{Q}(t') - \hat{Q}(t')\,\hat{Q}(t)\rangle$$

Phonon response function

 \mathcal{H}

Parametric drive

Harmonic oscillator equations of motion

From linearity of equations

Response function

$$\begin{aligned} \mathcal{H} &= \sum_{q} \left(\frac{P_{q}^{2}}{2M} + \frac{M\Omega_{q}^{2}(t)}{2} Q_{q}^{2} \right) \\ &\quad \frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}t} = \frac{\hat{P}(t)}{M} \\ &\quad \frac{\mathrm{d}\hat{P}(t)}{\mathrm{d}t} = -M\omega_{\mathbf{q}}^{2} \big[1 + 2\alpha \cos(2\Omega_{\mathrm{drv}}t) \big] \hat{Q}(t) \\ &\text{ns} \qquad \hat{Q}(t) = \mathfrak{M}_{QQ}(t,t') \, \hat{Q}(t') - \mathfrak{M}_{QP}(t,t') \, \frac{\hat{P}(t')}{M\Omega_{\mathrm{drv}}} \\ &\quad \mathcal{D}_{QQ}^{R}(t,t') = -i \, \theta(t-t') \langle [\hat{Q}(t'), \hat{P}(t')] \rangle \, \times \frac{-\mathfrak{M}_{QP}(t,t')}{M\Omega_{\mathrm{drv}}} \\ &\quad = -\frac{\hbar}{M\Omega_{\mathrm{drv}}} \, \theta(t-t') \, \mathfrak{M}_{QP}(t,t') \end{aligned}$$

Response function does not depend on the initial state of phonons. It is determined by the Hamiltonian only. It is enhanced near parametric resonance.

Electron-phonon interactions in systems with driven phonons

Parametrically driven phonons $\Omega_q^2(t) = \omega_q^2 \left(1 + 2\alpha \cos(2\Omega_{\rm drv}t)\right)$

$$\frac{U(t)}{U_{\rm eq}} = 1 - \frac{2\alpha\,\omega_{\mathbf{q}}^2\,\cos(2\Omega_{\rm drv}t)}{\omega_{\mathbf{q}}^2 - 4\Omega_{\rm drv}^2} + \frac{2\alpha^2\,\omega_{\mathbf{q}}^2\left[\omega_{\mathbf{q}}^2 - 16\Omega_{\rm drv}^2 + \omega_{\mathbf{q}}^2\cos(4\Omega_{\rm drv}t)\right]}{(\omega_{\mathbf{q}}^2 - 16\Omega_{\rm drv}^2)(\omega_{\mathbf{q}}^2 - 4\Omega_{\rm drv}^2)} + \mathcal{O}(\alpha^4)$$

Effective interaction: time average and variance. α = 0.2



Strong enhancement near resonance. Large response function near "instability"

Exponential dependence of Tc: gain on increase in U is larger than suppression due to decrease

Photo-induced superconductivity Floquet-Keldysh-Migdal-Eliashberg approach

M. Babadi, M. Knap, G. Refael, I. Martin, E. Demler



Driven electron-phonon system



This talk: focus on dynamical effects

Microscopic model

$$\mathcal{L}[\varphi, \Psi](t) = \sum_{\mathbf{k}} \Psi_{\mathbf{k}}^{\dagger} \left(i\partial_{t} \mathbb{I} - \xi_{\mathbf{k}} \hat{\sigma}_{3} \right) \Psi_{\mathbf{k}} - \frac{1}{2} \sum_{\mathbf{q}} \frac{1}{2\omega_{\mathbf{q}}} \varphi_{\mathbf{q}} \left(\partial_{t}^{2} + \omega_{\mathbf{q}}^{2} \right) \varphi_{-\mathbf{q}}$$
$$- \sum_{j \in \text{lattice}} \mathcal{V}^{\text{ph}}(\varphi_{j}) - \frac{1}{\sqrt{N}} \sum_{\mathbf{k}, \mathbf{k}'} g_{\mathbf{k}, \mathbf{k}'} \varphi_{\mathbf{k} - \mathbf{k}'} \Psi_{\mathbf{k}'}^{\dagger} \hat{\sigma}_{3} \Psi_{\mathbf{k}} + \frac{\Lambda}{2} |F(t)|^{2} \sum_{j \in \text{lattice}} \varphi_{j}$$

Phonons are driven only at q=0: e.g. $\Lambda \varphi_{\text{IR},q=0}^2(t) \varphi_{q=0}$

This analysis can be applied to the IR modes themselves, if they couple to electrons We assume phonon-nonlinearities given by κ_3 and κ_4 .

$$\mathcal{V}^{\mathrm{ph}}(\varphi) = -\frac{\kappa_3}{3!}\,\varphi^3 - \frac{\kappa_4}{4!}\,\varphi^4$$

We assume finite small dissipation γ_0 for phonons at q=0 due to other modes. Additional dissipation is generated by electrons

Phonon dynamics

Equations of motion for phonons: coherent part and fluctuations

$$\begin{split} \frac{1}{2\omega_0} \left(\partial_t^2 + \omega_0^2 + \gamma_0 \partial_t\right) \varphi(t) - \frac{\kappa_4}{6} \varphi^3(t) - \frac{\kappa_3}{2} \varphi^2(t) - \frac{\kappa_4}{2} \chi(t) \varphi(t) = \frac{\Lambda}{2} |F(t)|^2 + \frac{\kappa_3}{2} \chi(t) \\ \chi(t) &\equiv \frac{1}{N} \sum_{\mathbf{q}} i \mathcal{D}_{\mathbf{q}}(t, t) \quad \text{Force from} \\ \text{finite q phonons} \\ - \frac{1}{2\omega_{\mathbf{q}}} \left[\partial_{t_1}^2 + \omega_{\mathbf{q}}^2\right] \mathcal{D}_{\mathbf{q}}(t_1, t_2) = \delta_{\mathcal{C}}(t_1, t_2) + V(t_1) \mathcal{D}_{\mathbf{q}}(t_1, t_2) + \int_{\mathcal{C}} \mathrm{d}\tau \, \Pi_{\mathbf{q}}(t_1, \tau) \mathcal{D}_{\mathbf{q}}(\tau, t_2) \\ V(t) &\equiv -\frac{\kappa_4}{2} \chi(t) - \frac{\kappa_4}{2} \varphi^2(t) - \kappa_3 \varphi(t) \qquad \Pi_{\mathbf{q}}(t_1, t_2) = \frac{1}{N} \sum_{\mathbf{k}} |g_{\mathbf{k}, \mathbf{k} + \mathbf{q}}|^2 \operatorname{tr} \left[\hat{\mathcal{G}}_{\mathbf{k} + \mathbf{q}}(t_1, t_2) \hat{\sigma}_3 \, \hat{\mathcal{G}}_{\mathbf{k}}(t_2, t_1) \hat{\sigma}_3\right] \\ & \text{Drive from} \\ & \text{go phonon} \qquad \qquad \text{Force from electrons} \\ \end{split}$$

and frequency renormalization

Floquet-Wigner representation



Evolution of phonons

Coherent amplitude at q=0

Ramped-up external drive from with Amax=0.75



Propagators for finite q

Note the red shift in the spectrum of phonons: contributes to SC enhancement

The real time Migdal-Eliashberg theory

We want to compare enhancement of electron-phonon interaction with shortening of electron lifetime



The real time Migdal-Eliashberg theory

No superconductivity yet

$$\hat{\Sigma}_{\mathbf{k}}(t,t') = \frac{i}{N} \sum_{\mathbf{k}'} \hat{\sigma}_3 \,\hat{\mathcal{G}}_{\mathbf{k}'}(t,t') \,\hat{\sigma}_3 \,|g_{\mathbf{k}\mathbf{k}'}|^2 \,D_{\mathbf{k}-\mathbf{k}'}(t,t')$$



Renormalization of electron energy, quasiparticle weight, lifetime

spectral/Keldysh decomposition $i\hat{\mathcal{G}}_{\mathbf{k}}^{>}(\omega,T) = \frac{1}{2} \left[i\hat{\mathcal{G}}^{K}(\omega,T) + \hat{\mathsf{A}}(\omega,T) \right]$ $i\hat{\mathcal{G}}_{\mathbf{k}}^{<}(\omega,T) = \frac{1}{2} \left[i\hat{\mathcal{G}}^{K}(\omega,T) - \hat{\mathsf{A}}(\omega,T) \right]$

 $\hat{\Sigma}^{R}(\omega,T) = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \int_{-\infty}^{+\infty} \frac{d\nu}{\omega - \omega' - \nu + i0^{+}} \left\{ iF^{K}(\nu,T) \check{A}(\omega',T) + F^{\rho}(\nu,T) i\check{\mathcal{G}}^{K}(\omega',T) \right\}, \quad \text{Renormalization of effective mass}$ $i\hat{\Sigma}^{K}(\omega,T) = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \int_{-\infty}^{+\infty} d\nu (2\pi)\delta(\omega - \omega' - \nu) \left\{ iF^{K}(\nu,T) i\check{\mathcal{G}}^{K}(\omega',T) + F^{\rho}(\nu,T) \check{A}(\omega',T) \right\} \quad \text{Coherence of quasiparticles}$

spectral Eliashberg $F_{\xi,\xi'}^{\rho}(\nu,T) \equiv \frac{\nu(0)}{\nu(\xi)\,\nu(\xi')} \frac{1}{N^2} \sum_{\mathbf{k},\mathbf{k'}} |g_{\mathbf{k}\mathbf{k'}}|^2 \frac{1}{2\pi} \rho_{\mathbf{k}-\mathbf{k'}}(\nu,T) \,\delta(\xi_{\mathbf{k}}-\xi) \,\delta(\xi_{\mathbf{k'}}-\xi'),$ Keldysh Eliashberg $iF_{\xi,\xi'}^{K}(\nu,T) \equiv \frac{\nu(0)}{\nu(\xi)\,\nu(\xi')} \frac{1}{N^2} \sum_{\mathbf{k},\mathbf{k'}} |g_{\mathbf{k}\mathbf{k'}}|^2 \frac{1}{2\pi} \,iD_{\mathbf{k}-\mathbf{k'}}^{K}(\nu,T) \,\delta(\xi_{\mathbf{k}}-\xi) \,\delta(\xi_{\mathbf{k'}}-\xi')$

Out of equilibrium phonons

The real time Migdal-Eliashberg theory

Predictions for ARPES



Mass enhancement indicates increase in effective electron-phonon interaction: good for pairing

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Decrease of electron coherence: detrimental for pairing

Onset of pairing in non-equilibrium Floquet system

, retarded interaction

Introduce off-diagonal component of self-energy. Require self-consistency.

1. Calculate the Q-matrix—

$$(2\omega - m\Omega) \,\delta \mathcal{F}_{n,m}^{R} = -2\pi i \,\phi_{n,m} + \sum_{n'=-N_D}^{N_D} \left(\sum_{n',m-n+n'}^{R} \delta \mathcal{F}_{n-n',m+n'}^{R} + \sum_{n',m+n-n'}^{R} \delta \mathcal{F}_{n-n',m-n'}^{R} \right)$$

$$\delta \mathcal{F}_{n,m}^{R}(\omega) = \sum_{n',m'} \mathbf{Q}_{n',m'}^{n,m}(\omega) \,\phi_{n',m'}(\omega)$$

2. Solve the functional eigenvalue equation-

$$\Delta_{n}(\omega) = \frac{i\omega}{2\pi} \sum_{n'=-N_{\phi}}^{N_{\phi}} \sum_{n''=-N_{D}}^{N_{D}} \sum_{m'} \left\{ \mathsf{Q}_{n',m'}^{n,0}(\omega) \int_{0}^{+\infty} \frac{\mathrm{d}\omega'}{\omega'} K_{n''}(\omega - m'\Omega/2, \omega') \Delta_{n'-n''}(\omega') \right\}$$

dynamical self-energy effects (scattering, qp renormalization)

Compare to simple BCS
$$\Delta = -V
u(0)\int d\xi rac{\Delta}{|\xi|}$$
 $V = -rac{g^2}{\omega_{
m ph}}$



Pairing instability in a driven electron-phonon system



Optical response of photo-excited superconductor

Mean-field as Gaussian states

Bogoliubov theory of the superfluid state of bosons $|\Psi_{\text{Bogoliubov}}\rangle = e^{i\alpha_0 b_{k=0}^{\dagger} + \sum_k f_k b_k^{\dagger} b_{-k}^{\dagger}} |0\rangle$

BCS theory of superconductivity

$$|\Psi_{\rm BCS}\rangle = \prod_{k} (u_k + v_k c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow})|0\rangle = e^{\sum_k \frac{v_k}{u_k} c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow}}|0\rangle$$

Mean-field as Gaussian states



General Gaussian state of fermions $A = \begin{pmatrix} c_1^{\dagger} + c_1 \\ i(c_1^{\dagger} - c_1) \\ \dots \\ c_N^{\dagger} + c_N \\ i(c_N^{\dagger} - c_N) \end{pmatrix}$ $| \Psi_{\text{Fermion Gauss}} \rangle = e^{\frac{i}{4}RA^T}\xi_m A$ $\langle [A_i, A_j]_- \rangle = e^{i\xi_m}$

General Gaussian state

 $|\Psi_{\mathrm{Gauss}}\rangle = |\Psi_{\mathrm{Boson\,Gauss}}\rangle \times |\Psi_{\mathrm{Fermion\,Gauss}}\rangle$

Beyond Gaussian states

$$\mathcal{H} = -t \sum \langle ij \rangle \sigma c_{i\sigma}^{\dagger} c_{j\sigma} - \mu \sum_{i\sigma} c_{i\sigma}^{\dagger} c_{i\sigma} + \sum_{nm} \omega_{nm} b_n^{\dagger} b_m + g \sum_{nm} c_{n\sigma}^{\dagger} c_{n\sigma} (b_n + b_n^{\dagger})$$

Flow in imaginary time to find the ground state Tao Shi, Ignacio Cirac, ED $|\Psi_{\text{Variational}}(\tau)\rangle = e^{iS(\tau)} \times |\Psi_{\text{Gauss}}(\tau)\rangle$

$$S(\tau) = i \sum_{nm} \lambda_{nm}(\tau) (b_n - b_n^{\dagger}) c_{m\sigma}^{\dagger} c_{m\sigma} \qquad \text{Lang-Firsov transformation}$$

Geometrical approach to variational imaginary time flow

$$\begin{split} |\Psi_{j}\rangle &= \partial_{\xi_{j}} |\Psi(\xi)\rangle \\ |\mathbf{R}_{\Psi}\rangle &= -(H-E) |\Psi(\xi)\rangle \\ \mathbf{G}_{ij} &= \langle \Psi_{i} | \Psi_{j}\rangle \\ d_{\tau}\xi_{i} &= \sum_{j=1}^{2} \mathbf{G}_{ij}^{-1} \langle \Psi_{j} | \mathbf{R}_{\Psi}\rangle \end{split}$$

Beyond Gaussian states

Variational approach to real time dynamics

 $|\Psi_{\text{Variational}}(t)\rangle = e^{iS(t)} \times |\Psi_{\text{Gauss}}(t)\rangle$

$$\mathcal{H} = -t \sum \langle ij \rangle \sigma c_{i\sigma}^{\dagger} c_{j\sigma} - \mu \sum_{i\sigma} c_{i\sigma}^{\dagger} c_{i\sigma} + \sum_{nm} \omega_{nm} b_n^{\dagger} b_m + g \sum_{nm} c_{n\sigma}^{\dagger} c_{n\sigma} (b_n + b_n^{\dagger}) + \mathcal{H}_J(t)$$
$$H_J(t) = 2J \sum_{k,\sigma,\alpha} A_{\alpha}(t) \sin k_{\alpha} c_{k\sigma}^{\dagger} c_{k\sigma},$$

Calculation of optical conductivity in parametrically driven superconductors Work in progress ...

Beyond Gaussian states

Variational approach to real time dynamics in the Kondo model Tao Shi, Ignacio Cirac, ED

$$|\Psi_{\text{Variational}}(t)\rangle = e^{iS(t)} \times |\Psi_{\text{Gauss}}(t)\rangle$$

$$H_{\mathrm{K}} = \sum_{k\sigma} k c_{k\sigma}^{\dagger} c_{k\sigma} + \frac{J_{\perp}}{2} [\sigma_{+} \psi_{\downarrow}^{\dagger}(0) \psi_{\uparrow}(0) + \mathrm{H.c.}] + \frac{J_{\parallel}}{4} \sigma_{z} [\psi_{\uparrow}^{\dagger}(0) \psi_{\uparrow}(0) - \psi_{\downarrow}^{\dagger}(0) \psi_{\downarrow}(0)]$$

Equilibrium phase diagram

Non-equilibrium dynamics is Challenging problem due to many crossovers



Fermion spin structure corresponding to spin up impurity state (d) (e) (f) (f) (f) (f)



Conclusions

External drive leads to enhancement of electronphonon interaction. It can be understood as parametric amplification. This leads to an increase in the effective BCS coupling constant

Nonequilibrium state of phonons also results in additional scattering of electrons that leads to pairbreaking

We find that increase in BCS coupling can dominate and find possible increase of instability temperature by 150%. Floquet aspects are crucial.