Quantum soliton friction and soliton diffusion

Johannes Hofmann

work with Dmitry Efimkin and Victor Galitski

Theory of Condensed Matter Group, Cavendish Laboratory

Mainz, May 2017

Outline

1. Introduction: Excitations in Bose-Einstein condensates

2. Absence of classical soliton friction

3. Quantum friction of bright solitons

4. Conclusion

Outline

1. Introduction: Excitations in Bose-Einstein condensates

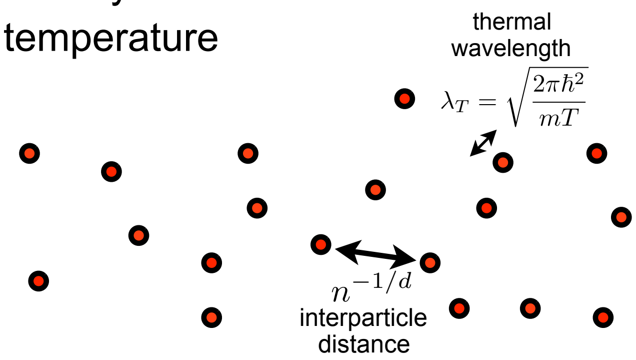
2. Absence of classical soliton friction

3. Quantum friction of bright solitons

4. Conclusion

Interacting Bose gases

n : density
T : temperature



Universal many-body system:
independent of microscopic details

Lagrangian:

$$L = \int dx \left[\phi^* i\hbar\partial_t \phi - \frac{\hbar^2}{2m} |\nabla\phi|^2 + \mu|\phi|^2 - \frac{g}{2}|\phi|^4 \right]$$

g : strength of contact interaction

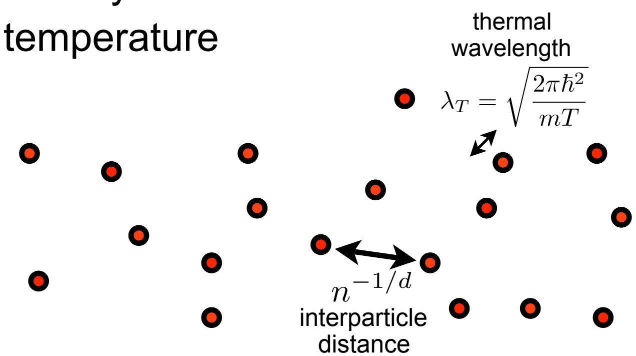
$\phi^\dagger$: Bose creation operator

m : atom mass

<u>Feshbach resonance</u>: tune interaction strength with external magnetic field

Interacting Bose gases

n : density
T : temperature



Universal many-body system:
independent of microscopic details

T

quantum degeneracy: $n\lambda_T^3 \gtrsim 1$

Bose-Einstein condensate

Lagrangian:

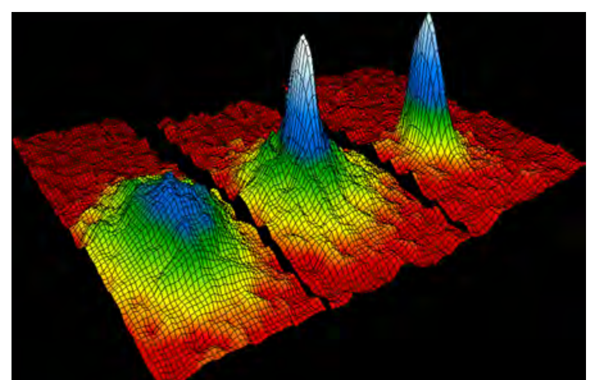
$$L = \int dx \left[\phi^* i\hbar\partial_t \phi - \frac{\hbar^2}{2m}|\nabla\phi|^2 + \mu|\phi|^2 - \frac{g}{2}|\phi|^4 \right]$$

g : strength of contact interaction

$\phi^\dagger$: Bose creation operator

m : atom mass

Feshbach resonance: tune interaction strength with external magnetic field



Cornell, J. Res. Natl. Inst. Stand. Technol. (1996)

Bogoliubov excitations

Bose-Einstein condensate: $\phi = \phi_0 + \delta\phi$

$$\left[-\frac{\hbar^2\partial_x^2}{2m} - \mu + g|\phi_0(x)|^2\right]\phi_0(x) = 0$$

Gross-Pitaevskii equation

excitations:

$$\delta L = \frac{1}{2}\int dx\, \Psi^\dagger \left(i\hbar\sigma_3\partial_t - K_{\mathrm{BdG}}\right)\Psi \qquad \Psi = \begin{pmatrix} \delta\phi \\ \delta\phi^* \end{pmatrix}$$

$$K_{\mathrm{BdG}} = \begin{pmatrix} -\frac{\hbar^2\nabla^2}{2m} - \mu + 2g_1|\phi_0|^2 & g_1\phi_0^2 \\ g_1\phi_0^{*2} & -\frac{\hbar^2\nabla^2}{2m} - \mu + 2g_1|\phi_0|^2 \end{pmatrix}$$

repulsive interaction: $g > 0$

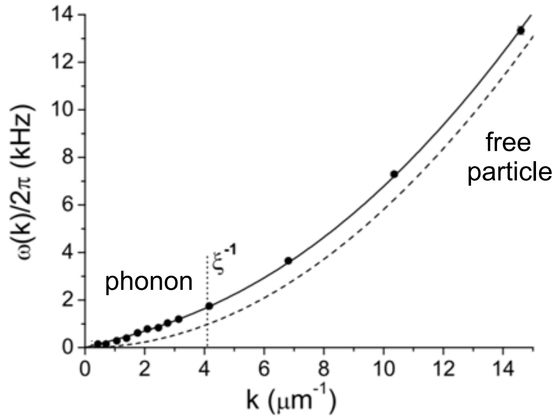
Bogoliubov dispersion: $\varepsilon_k = \sqrt{\frac{\hbar^2k^2}{2m}\left(\frac{\hbar^2k^2}{2m} + 2gn_0\right)}$

$g = \frac{4\pi\hbar^2 a}{m}$ interaction strength

n_0 condensate density

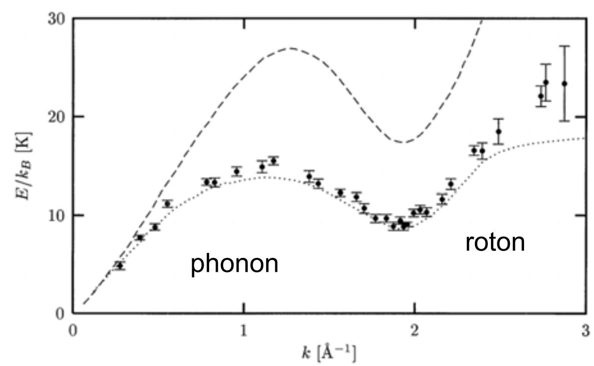
Bogoliubov excitations

dilute quantum gas



Steinhauer et al., Phys. Rev. Lett (2002)

quantum fluid ^{4}He



Cowley et al., J. Low Temp. Phys. (1986)

repulsive interaction: $g > 0$

Bogoliubov dispersion: $$\varepsilon_k = \sqrt{\frac{\hbar^2 k^2}{2m}\left(\frac{\hbar^2 k^2}{2m} + 2gn_0\right)}$$

$g = \frac{4\pi\hbar^2 a}{m}$ interaction strength

n_0 condensate density

Outline

1. Introduction: Excitations in Bose-Einstein condensates

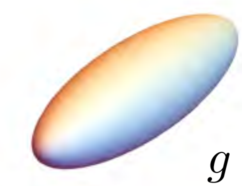
2. **Absence of classical soliton friction**

3. Quantum friction of bright solitons

4. Conclusion

Bright solitons

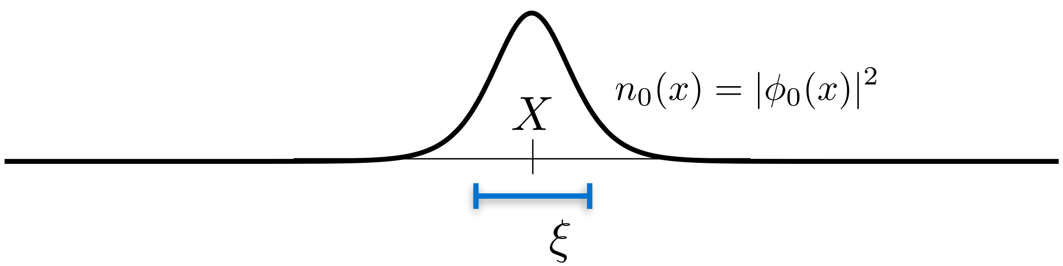
attractive interaction: $g < 0$



$$g \sim \frac{a}{l_\perp^2}$$

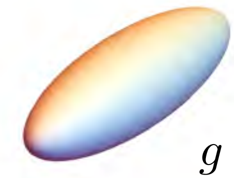
1D: bright soliton $\qquad \phi(x, \mathbf{r}_\perp) = \Phi_{\mathrm{ho}}(\mathbf{r}_\perp)\phi_0(x)$

$$\phi_0(x) = \sqrt{\frac{N_0}{2\xi}}\, e^{i\theta} \mathrm{sech}\left(\frac{x - X}{\xi}\right)$$



Bright solitons

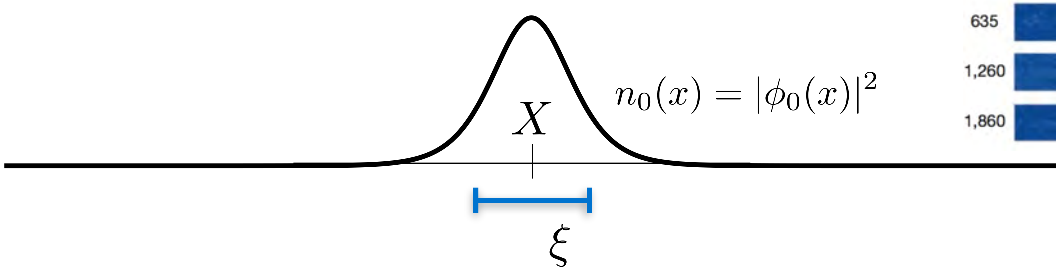
attractive interaction: $g < 0$

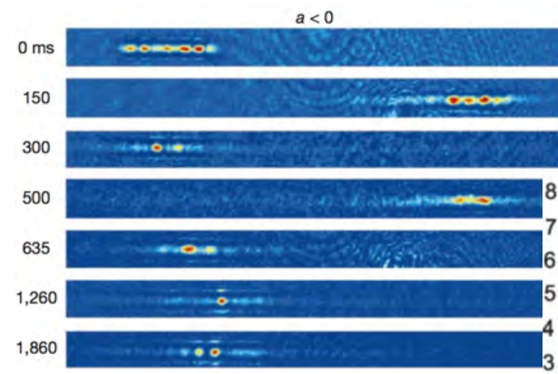


$$g \sim \frac{a}{l_\perp^2}$$

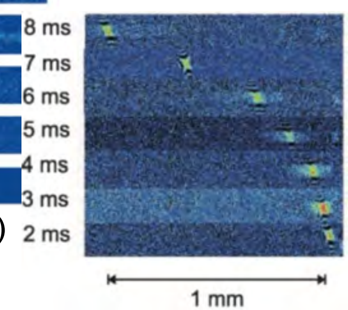
1D: bright soliton $\qquad \phi(x, \mathbf{r}_\perp) = \Phi_{\text{ho}}(\mathbf{r}_\perp)\phi_0(x)$

$$\phi_0(x) = \sqrt{\frac{N_0}{2\xi}}\, e^{i\theta} \mathrm{sech}\left(\frac{x - X}{\xi}\right)$$





Strecker et al., Nature (2002)



Khaykovich et al., Science (2002)

soliton mass: $M_{\text{eff}} = N_0 m$

soliton motion in a trap: $\quad M_{\text{eff}}\ddot{X} = -\partial_X U + F[X(t)]$

trapping potential:

$$U(X) = \frac{M_{\text{eff}}\omega_t^2}{2} X^2$$

soliton friction

Ohmic friction: $\quad F[X(t)] = -\gamma \dot{X}$

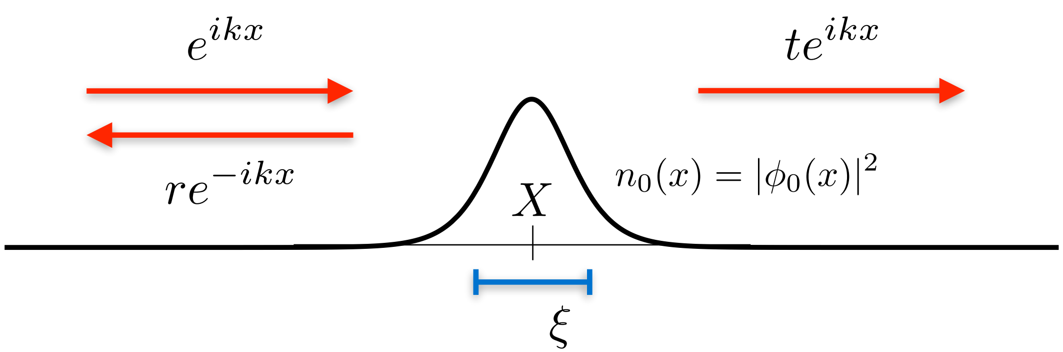
Absence of Ohmic friction

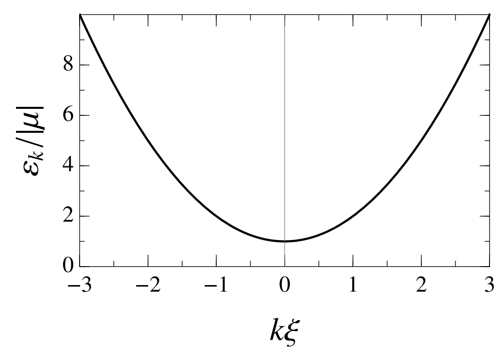
Bogoliubov excitations:

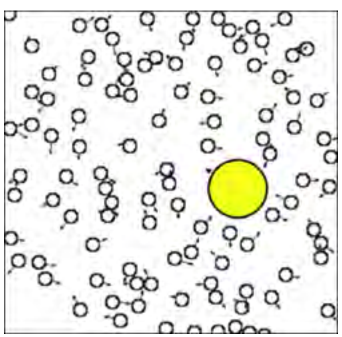
1) scattering solutions

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m} + |\mu|$$

$$\phi_0(x) = \sqrt{\frac{N_0}{2\xi}}\, e^{i\theta} \mathrm{sech}\left(\frac{x-X}{\xi}\right)$$



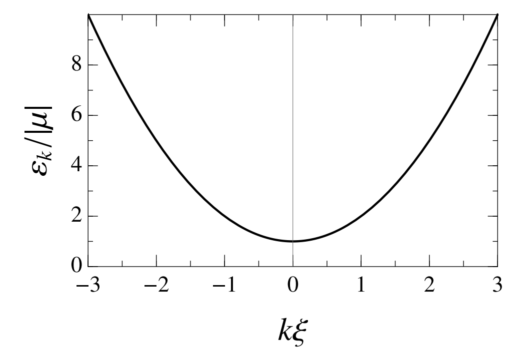




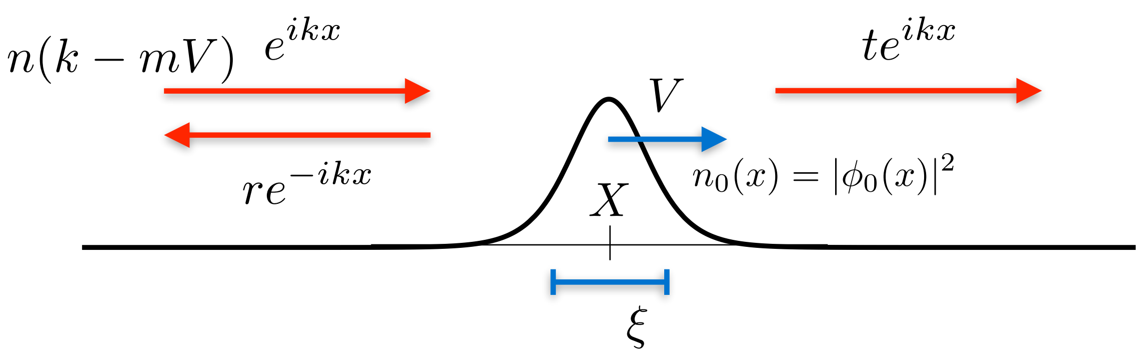
Absence of Ohmic friction

Bogoliubov excitations:

1) scattering solutions $\varepsilon_k = \frac{\hbar^2 k^2}{2m} + |\mu|$



$$\phi_0(x) = \sqrt{\frac{N_0}{2\xi}}\, e^{i\theta} \mathrm{sech}\left(\frac{x-X}{\xi}\right)$$



$$t = \frac{(1-ik)^2}{(1+ik)^2} \qquad |t|^2 = 1$$

$$|r|^2 = 1 - |t|^2 = 0$$

<u>'Classical friction'</u>: Brownian motion of heavy particle

effective Fokker-Planck description $\gamma \sim \int dk\, |r|^2 (\ldots)$

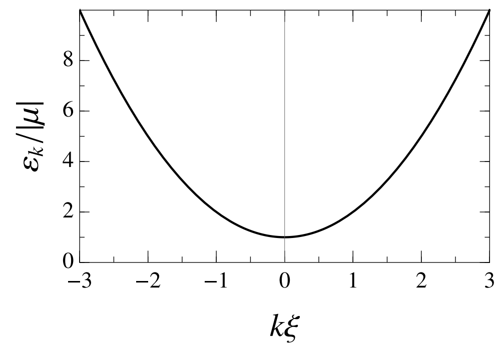
BUT: integrable problem, soliton reflectionless potential

Absence of Ohmic friction in the integrable setup.

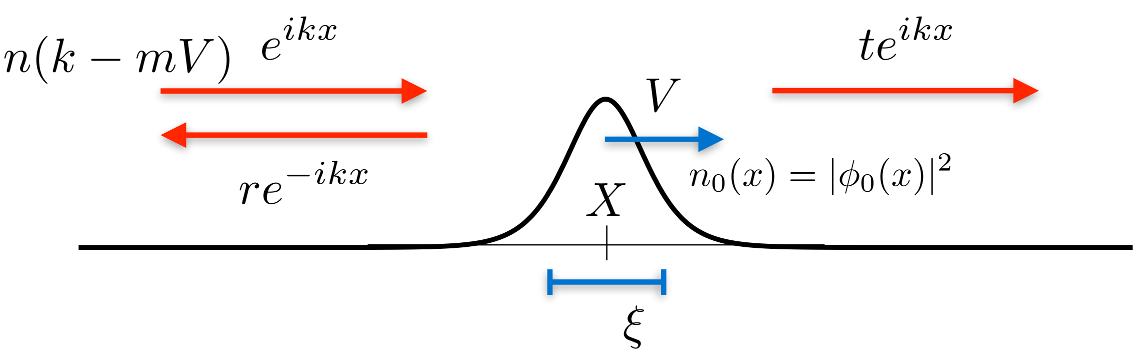
Absence of Ohmic friction

Bogoliubov excitations:

1) scattering solutions $\varepsilon_k = \dfrac{\hbar^2 k^2}{2m} + |\mu|$



$$\phi_0(x) = \sqrt{\frac{N_0}{2\xi}}\, e^{i\theta} \mathrm{sech}\left(\frac{x-X}{\xi}\right)$$



$$t = \frac{(1-ik)^2}{(1+ik)^2} \qquad |t|^2 = 1$$

$$|r|^2 = 1 - |t|^2 = 0$$

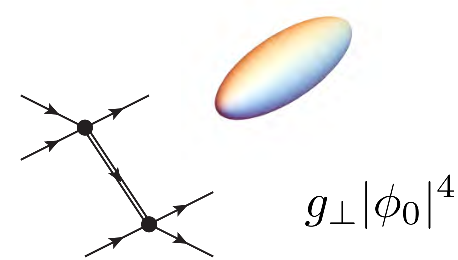
<u>‘Classical friction’</u>: Brownian motion of heavy particle

effective Fokker-Planck description $\gamma \sim \int dk\, |r|^2 (\ldots)$

BUT: integrable problem, soliton reflectionless potential

Absence of Ohmic friction in the integrable setup.

Classical friction for explicit breaking of integrability



Brand, Shlyapnikov,...

Outline

1. Introduction: Excitations in Bose-Einstein condensates

2. Absence of classical soliton friction

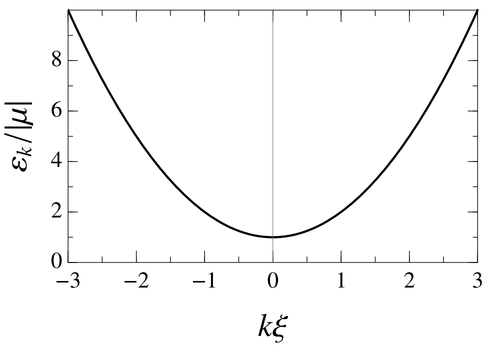
3. **Quantum friction of bright solitons**

D. Efimkin, J. Hofmann, and V. Galitski, Phys. Rev. Lett. **116**, 225301 (2016)

4. Conclusion

Zero modes and collective coordinates

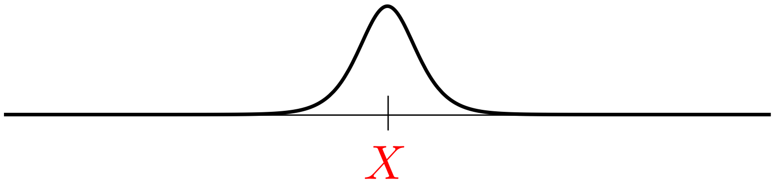
Bogoliubov excitations:



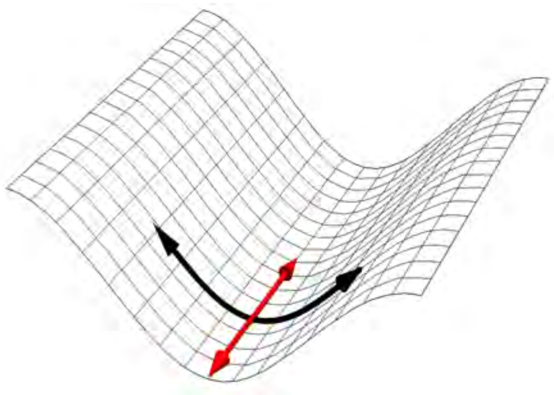
1) scattering solutions $$\varepsilon_k = \frac{\hbar^2 k^2}{2m} + |\mu|$$

$$\phi_0(x) = \sqrt{\frac{N}{2\xi}}\, e^{i\theta} \mathrm{sech}\left(\frac{x - X}{\xi}\right)$$

2) zero modes



large degeneracy of ground state configurations



collective coordinates:

$$X \to X(t) \qquad\qquad \theta \to \theta(t)$$

promote to dynamical variables

Collective coordinate quantization

promote to dynamical variables $\phi(x,t) = \phi_0(x - X(t)) + \delta\phi(x,t)$

$$\delta L = \frac{1}{2}\int dx\, \Psi^\dagger \big(i\hbar\sigma_3\partial_t - K_{\rm BdG}\big)\Psi \qquad \Psi = \begin{pmatrix}\delta\phi \\ \delta\phi^*\end{pmatrix}$$

effective Lagrangian:

$$L = \underline{\frac{M_{\rm eff}\dot{X}^2}{2}} + \pi_{\rm qp}\dot{X} + \sum_k c_k^*[i\hbar\partial_t - \epsilon_k]c_k$$

<u>phase</u>:
Lewenstein and You, PRL (1997)
<u>dark soliton coordinate</u>:
Dziarmaga, PRA (2004)

Collective coordinate quantization

promote to dynamical variables $\phi(x,t) = \phi_0(x - X(t)) + \delta\phi(x,t)$

$$\delta L = \frac{1}{2}\int dx\, \Psi^\dagger \big(i\hbar\sigma_3\partial_t - K_{\rm BdG}\big)\Psi \qquad \Psi = \begin{pmatrix}\delta\phi \\ \delta\phi^*\end{pmatrix}$$

effective Lagrangian:

interaction between moving soliton and quasiparticles

$$L = \frac{M_{\rm eff}\dot{X}^2}{2} + \pi_{\rm qp}\dot{X} + \sum_k c_k^*[i\hbar\partial_t - \epsilon_k]c_k$$

$$\pi_{\rm qp} = \frac{1}{2}\sum_{k,k'}(c_k^*, c_k)\begin{pmatrix}\langle k|\sigma_z\hat{p}|k'\rangle & -\langle k|\sigma_z\hat{p}|\overline{k'}\rangle \\ -\overline{\langle k|}\sigma_z\hat{p}|k'\rangle & \overline{\langle k|}\sigma_z\hat{p}|\overline{k'}\rangle\end{pmatrix}\begin{pmatrix}c_{k'} \\ c_{k'}^*\end{pmatrix}$$

Collective coordinate quantization

promote to dynamical variables $\quad \phi(x,t) = \phi_0(x - X(t)) + \delta\phi(x,t)$

$$\delta L = \frac{1}{2}\int dx\, \Psi^\dagger \big(i\hbar\sigma_3\partial_t - K_{\mathrm{BdG}}\big)\Psi \qquad \Psi = \begin{pmatrix} \delta\phi \\ \delta\phi^* \end{pmatrix}$$

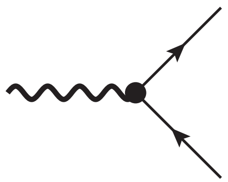
effective Lagrangian:

interaction between moving soliton and quasiparticles

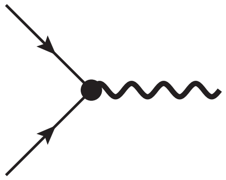
$$L = \frac{M_{\mathrm{eff}}\dot{X}^2}{2} + \pi_{\mathrm{qp}}\dot{X} + \sum_k c_k^*[i\hbar\partial_t - \epsilon_k]c_k$$

$$\pi_{\mathrm{qp}} = \frac{1}{2}\sum_{k,k'}(c_k^*, c_k)\begin{pmatrix} \langle k|\sigma_z\hat{p}|k'\rangle & -\langle k|\sigma_z\hat{p}\overline{|k'\rangle} \\ -\overline{\langle k|}\sigma_z\hat{p}|k'\rangle & \overline{\langle k|}\sigma_z\hat{p}\overline{|k'\rangle} \end{pmatrix}\begin{pmatrix} c_{k'} \\ c_{k'}^* \end{pmatrix}$$

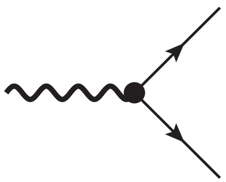
soliton-quasiparticle coupling: $\qquad (\pi^{\mathrm{sc}}_{k,-k} = 0)$



scattering



absorption

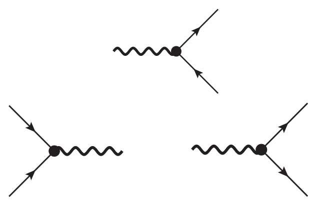


emission

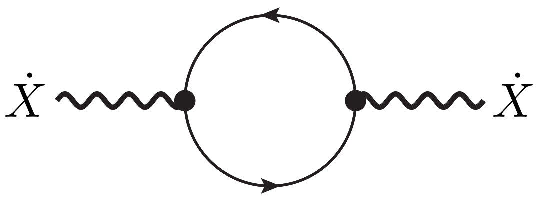
Effective soliton action

System — **coupling** $\pi_{\rm qp}\dot{X}$ — **Bath**

Soliton | **Thermal gas of quasiparticles**



integrate out quasiparticles:



The Theory of a General Quantum System Interacting with a Linear Dissipative System

R. P. FEYNMAN

itute of Technology, Pasadena, California

AND

F. L. VERNON, JR.*

Quantum Tunnelling in a Dissipative System

A. O. CALDEIRA

leb Wataghin" Universidade Estadual de Campinas, Barao Geraldo, 13-100 Campinas, Sao Paulo, Brazil

AND

A. J. LEGGETT*

f Mathematical and Physical Sciences, Sussex, Falmer, Brighton BN1 9QH, England

uly 22, 1982; revised December 20, 1982

PATH INTEGRAL APPROACH TO QUANTUM BROWNIAN MOTION*

A.O. CALDEIRA

Instituto de Fisica "Gleb Wataghin", Universidade Estadual de Campinas, 13.100 Campinas – SP – Brasil

and

A.J. LEGGETT

School of Mathematical and Physical Sciences, University of Sussex, Brighton BN1 9QH, Sussex, UK

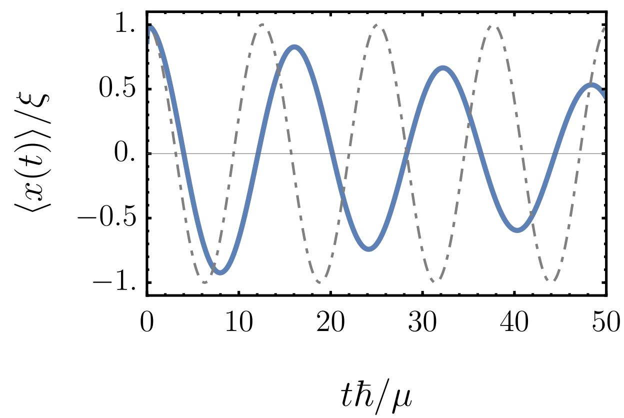
Received 25 August 1982
Revised 4 January 1983

Effective soliton action

System — coupling — **Bath**

Soliton | **Thermal gas of quasiparticles**

$$M_{\mathrm{eff}}\ddot{X}(t) + \underbrace{\int_0^t dt'\eta(t-t')\dot{X}(t')}_{\text{Non-Markovian friction}} = -\partial_{\mathrm{X}} U + \underbrace{f_{\mathrm{s}}(t)}_{\text{stoch. force}}$$



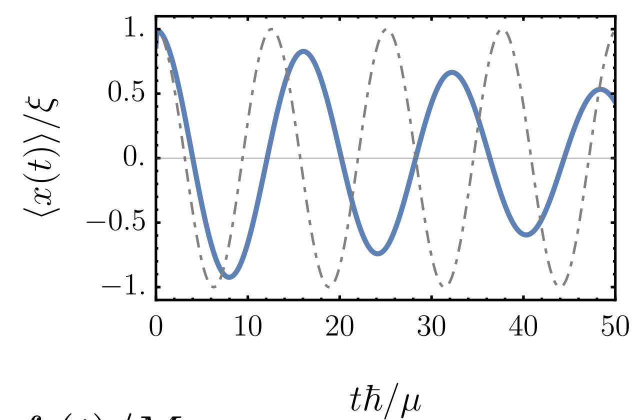
Effective soliton action

System — **coupling** — **Bath**

Soliton | **Thermal gas of quasiparticles**

$$M_{\text{eff}}\ddot{X}(t) + \int_0^t dt'\eta(t-t')\dot{X}(t') = -\partial_X U + f_s(t)$$

Non-Markovian friction **stoch. force**



Meaning? Markovian approximation:

$$\eta(t) = \cancel{\gamma\delta(t)} + \tau_{\text{AL}}\delta''(t) + \ldots \qquad \ddot{X} - \underline{\tau_{\text{AL}}\dddot{X}} + \omega_t^2 X = f_s(t)/M_{\text{eff}}$$

no Ohmic term!

new type of friction

Abraham-Lorentz force

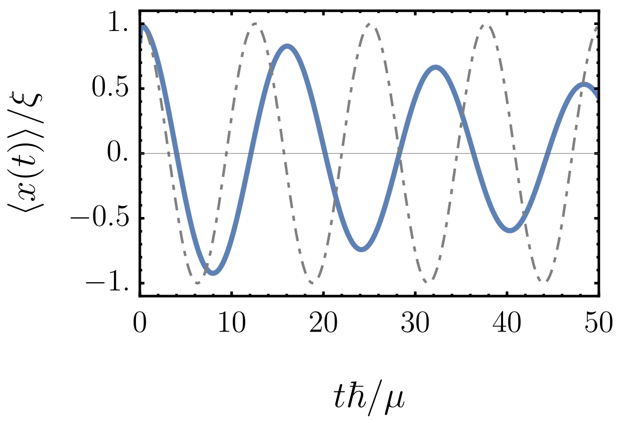
electrodynamics: radiation field backreaction of accelerated charge

here: scattering of quasiparticles

Soliton motion in a trap

$$\ddot{X} - \underline{\tau_{\mathrm{AL}}\dddot{X}} + \omega_{\mathrm{t}}^2 X = f_{\mathrm{s}}(t)/M_{\mathrm{eff}}$$

Abraham-Lorentz force



Landau-Lifshitz: $\ddot{X} + \omega_t^2 \tau_{\mathrm{AL}} \dot{X} + \omega_{\mathrm{t}}^2 X = f_{\mathrm{s}}(t)/M$

effective damping term: $\gamma = \omega_t^2 \tau_{\mathrm{AL}}$

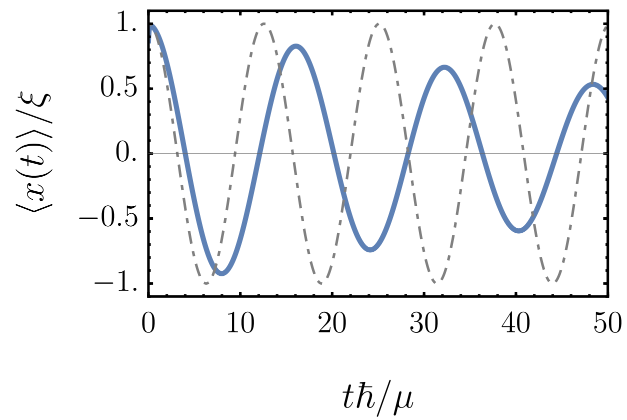
scales with trap frequency

damping time $1/\gamma$ should be observable

Soliton motion in a trap

$$\ddot{X} - \tau_{\mathrm{AL}}\dddot{X} + \omega_{\mathrm{t}}^2 X = f_{\mathrm{s}}(t)/M_{\mathrm{eff}}$$

Abraham-Lorentz force

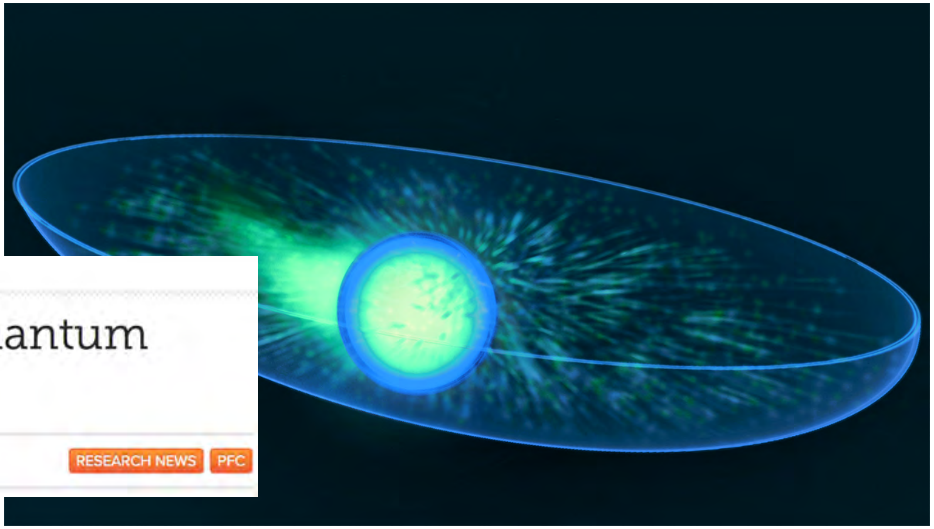


Landau-Lifshitz: $\ddot{X} + \omega_t^2\tau_{\mathrm{AL}}\dot{X} + \omega_{\mathrm{t}}^2 X = f_{\mathrm{s}}(t)/M$

effective damping term: $\gamma = \omega_t^2\tau_{\mathrm{AL}}$

scales with trap frequency

damping time $1/\gamma$ should be observable




HOME > NEWS

Ultra-cold atoms may wade through quantum friction

JUNE 24, 2016

RESEARCH NEWS PFC

JQI popular article

Outline

1. Introduction: Excitations in Bose-Einstein condensates

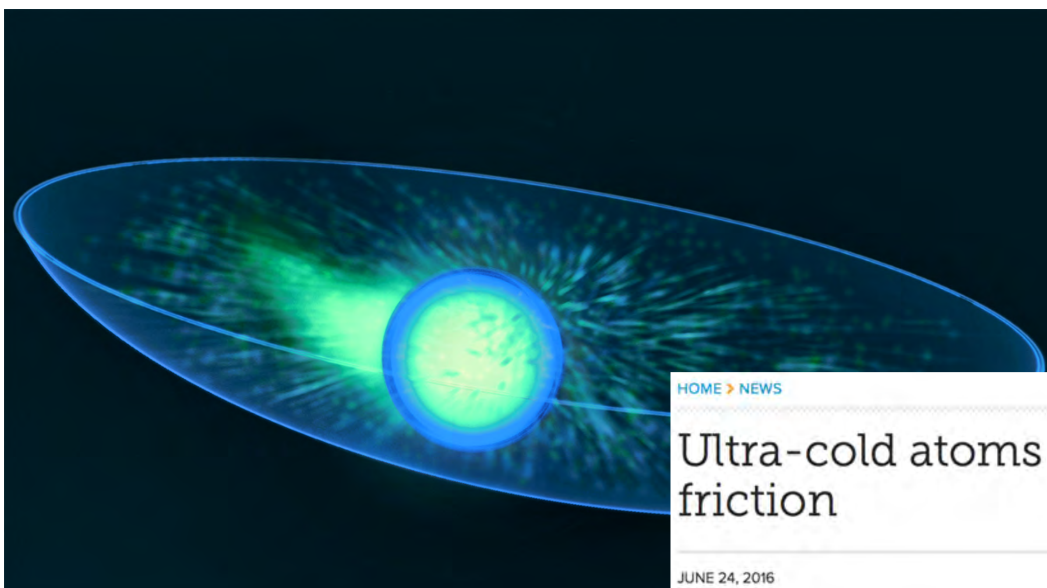
2. Absence of classical soliton friction

3. Quantum friction of bright solitons

4. Conclusion

Summary

- The motion of solitons is damped due to a retarded interaction with thermal quasiparticles: Non-Markovian quantum friction.
- The damping resembles the Abraham-Lorentz force of a charged particle.
- The damping for motion in a trap has a distinct signature that distinguishes it from damping due to an explicit breaking of integrability.



HOME > NEWS

Ultra-cold atoms may wade through quantum friction

JUNE 24, 2016

RESEARCH NEWS PFC

JQI popular article