Superconductivity following a quantum quench

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Initially system of free electrons. Quench involves turning on attractive pairing interactions.

PART I: Quench to the superconducting critical point. Results for how electron spectral density evolves in time. (With Dr. Yonah Lemonik) **arXiv:1705.09200**

Part II: Many competing superconducting order-parameters in realistic systems. By tuning the quench amplitude, superconducting order of different symmetries can be realized. (With Hossein Dehghani) **arXiv:1703.01621**

PART I: Critical Quenches

$$H_i = \sum_{k,\sigma=\uparrow,\downarrow,\alpha=1\ldots N} \epsilon_k c^\dagger_{k\sigma\alpha} c_{k\sigma\alpha}.$$

Above k is the momentum, $\sigma = \uparrow, \downarrow$ denotes the spin, and α is an orbital quantum number that takes N values.

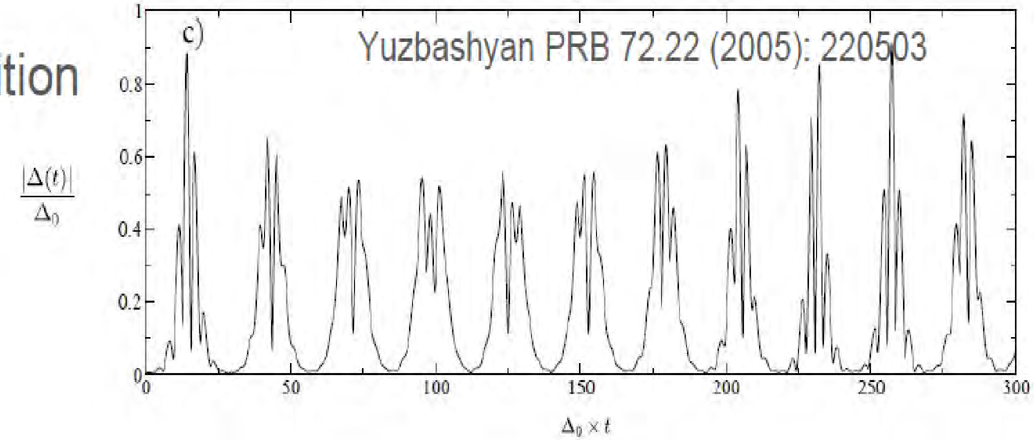
$$H_f = H_i + \frac{u}{N} \sum_q \bar{\Delta}_q \Delta_q,$$

$$\Delta_q = \sum_{k\alpha} c_{k,\uparrow,\alpha} c_{-k+q,\downarrow,\alpha}; \bar{\Delta}_q = \sum_{k,\alpha} c^\dagger_{-k+q,\downarrow,\alpha} c^\dagger_{k\uparrow\alpha}.$$

Prior work

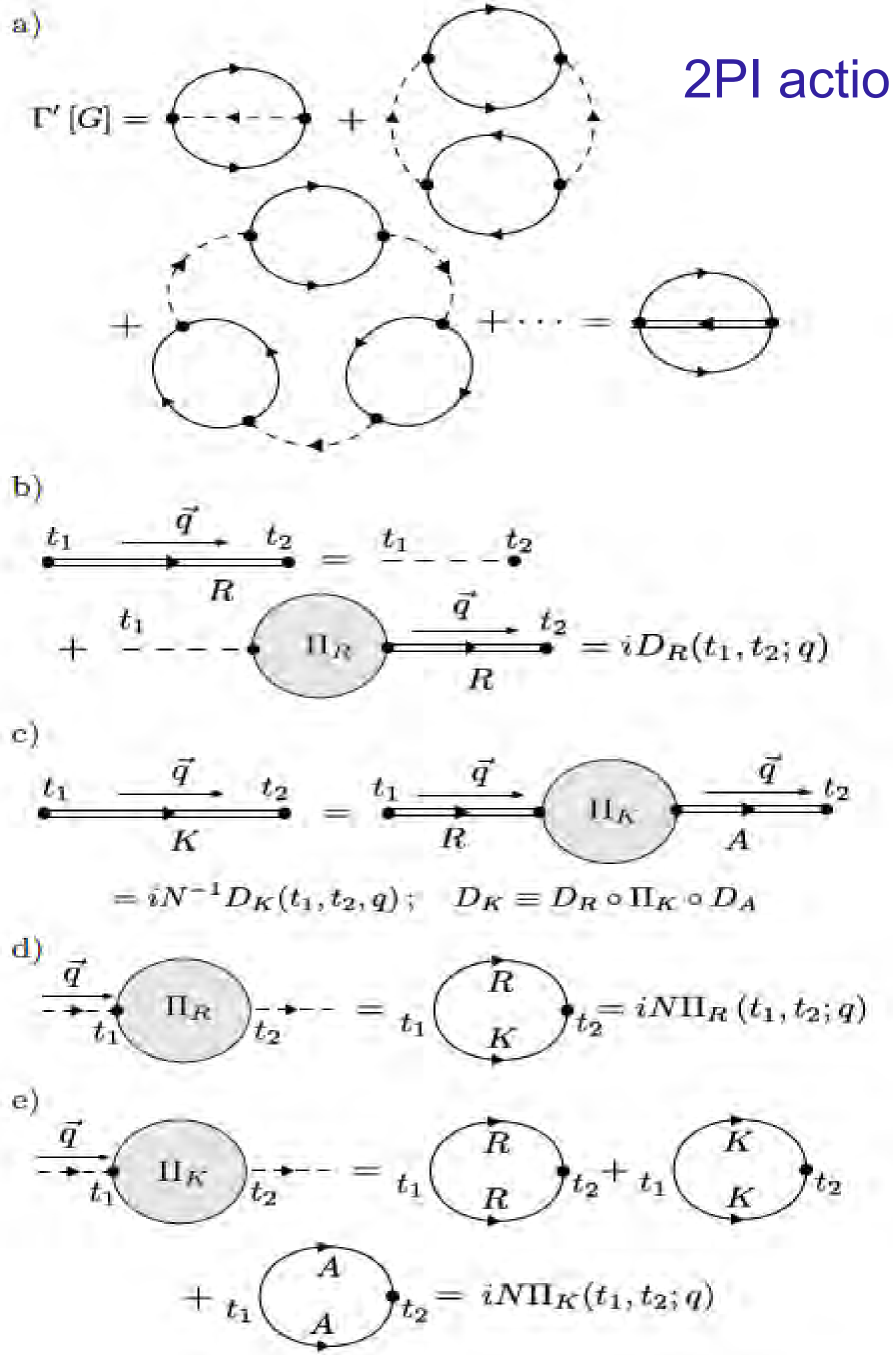
Within ordered phase, $N \to \infty$ (mean field), evolution of $\langle\Delta(t)\rangle$

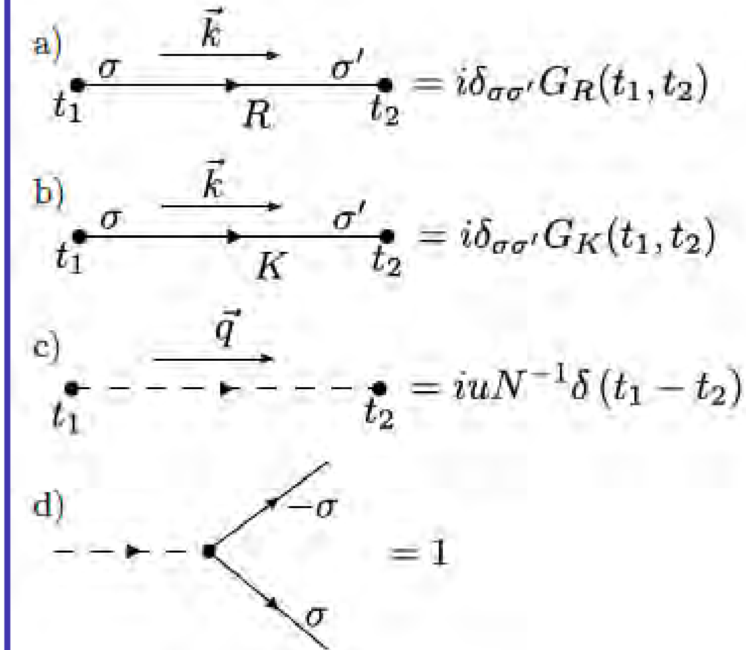
No fluctuations, no phase transition



In contrast for us, we are always in the normal phase, $\langle\Delta_q\rangle = 0$
so mean-field is trivial.

2PI action

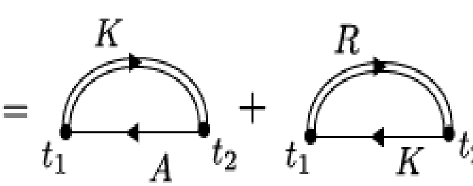




$$\Sigma_R[G] \equiv \delta\Gamma'/\delta G_A,$$

$$G_R{}^{-1} = g_R{}^{-1} - \Sigma_R[G],$$

$i\Sigma_R(t_1, t_2; q) =$



$$\Sigma_R(1,2) = \frac{i}{N}\left[D_K(1,2)G_A(2,1) + D_R(1,2)G_K(2,1)\right]$$

$$D_R^{-1} \equiv u^{-1} - \Pi_R.$$
$$D_K \equiv D_R \circ \Pi_K \circ D_A.$$

$$i\Pi^K(q,t,t') = \langle\{\Delta_q(t), \bar{\Delta}_q(t')\}\rangle,$$
$$i\Pi^R(q,t,t') = \theta(t-t')\langle[\Delta_q(t), \bar{\Delta}_q(t')]\rangle,$$

Non-trivial set of equations as G has to be determined self-consistently.

Step1: Short time perturbative calculation.
Step 2: Self-consistent calculation by mapping Cooperon (D propagator) dynamics to Model A.

Short time approximation: Replace full Green's function by non-interacting Green's function

$$\Pi_R(v_F|q| \ll T, \omega \ll T) = \nu\left(a - ib\frac{\omega}{T} + c\frac{v_F^2}{T^2}\right),$$

$$i\Pi_K(v_F|q| \ll T, \omega \ll T) = 4b\nu,$$

$$\Rightarrow i\Pi_K(q=0,t,t') \sim \delta(t-t'),$$

$$D_R^{-1} \equiv u^{-1} - \Pi_R.$$

$$D_K \equiv D_R \circ \Pi_K \circ D_A.$$

$$\left[\partial_t + \gamma_q\right] D_R(q,t,t') = -Z\delta(t-t'),$$

$$\gamma_q = T\left(l^2 q^2 + r\right). \quad l \sim v_F/T$$

→ Model-A

$$D_R(q,t,t') = -Z\theta(t-t')e^{-(t-t')\gamma_q}$$

$$iD_K(q,t,t') = Z\frac{T}{\gamma_q}\left[e^{-\gamma_q|t-t'|} - e^{-\gamma_q(t+t')}\right]$$

→ Broken time-translational Invariance due to quench

At the critical point $r = 0$, $iD_K(q,t,t)$ for $v_F q \ll T$ can be written in the scaling form

$$iD_K(q,t,t) = ZTtF_K^0\left(Tl^2q^2t\right); \quad v_F q \ll T.$$

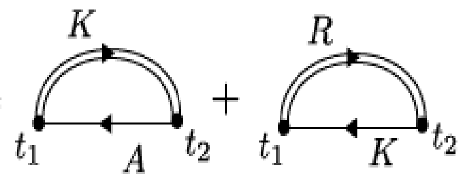
$$F_K^0(x) = \frac{1 - e^{-2x}}{x},$$

At the critical point, only two scales in the problem. $t,\ l\sqrt{Tt}.$

The first is time after quench, and the second is the associate length scale

Electron self-energy in Wigner coordinates and using the fact that the Cooperon varies on a very slow time scale relative to the temperature

$i\Sigma_R(t_1, t_2; q) =$



$$\Sigma_R^{WT}(k, \omega; t) = -\int \frac{d^d q}{(2\pi)^d} \frac{iD_K(q, t, t)}{\omega + \varepsilon_{q-k} + i\delta}.$$

$$\Sigma^R(k, \omega = \varepsilon_k, t) = (Tt)^{\frac{3-d}{2}} \frac{Z}{Tl^d} S_d^0(4\varepsilon_k^2 t/T)$$

Spectral density or electron lifetime

$$\tau^{-1}(k, t) = (Tt)^{\frac{3-d}{2}} \frac{Z}{Tl^d} \mathrm{Im} S_d^0(\varepsilon_k^2 t/T)$$

$$\mathrm{Im} S_d^0(x) = \frac{1}{2} \int \frac{d^{d-1}y}{(2\pi)^{d-1}} F_K^0([y^2 + x])$$

Perturbative results for the electron spectral density:

$d = 2$

$$\begin{aligned} \tau^{-1}(k,t) &\sim \sqrt{t}; & t\varepsilon_k^2/T \ll 1 \\ &\sim \frac{1}{\varepsilon_k}; & t\varepsilon_k^2/T \gg 1. \end{aligned}$$

$d = 3$

$$\begin{aligned} \tau^{-1}(k,t) &\sim \log(Tt)\,; & t\varepsilon_k^2/T \ll 1 \\ &\sim -2\log\left(\frac{\varepsilon_k}{T}\right); & t\varepsilon_k^2/T \gg 1. \end{aligned}$$

Perturbation theory breaks down at long times as it gives a diverging electron life-time

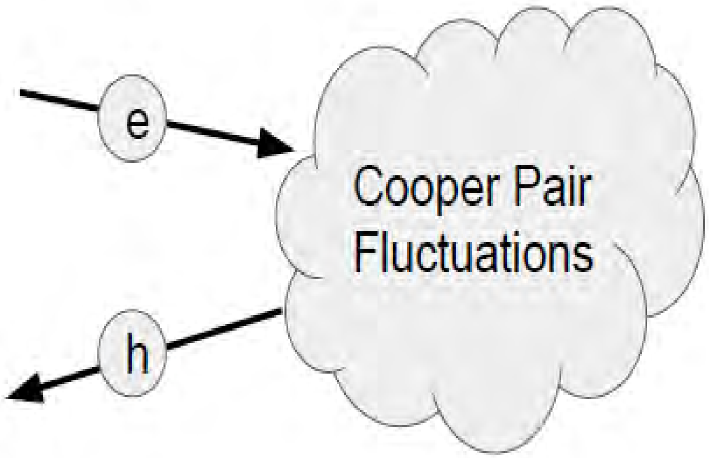
Electron Andreev reflects into a hole. This process is resonant for electrons at the Fermi energy.

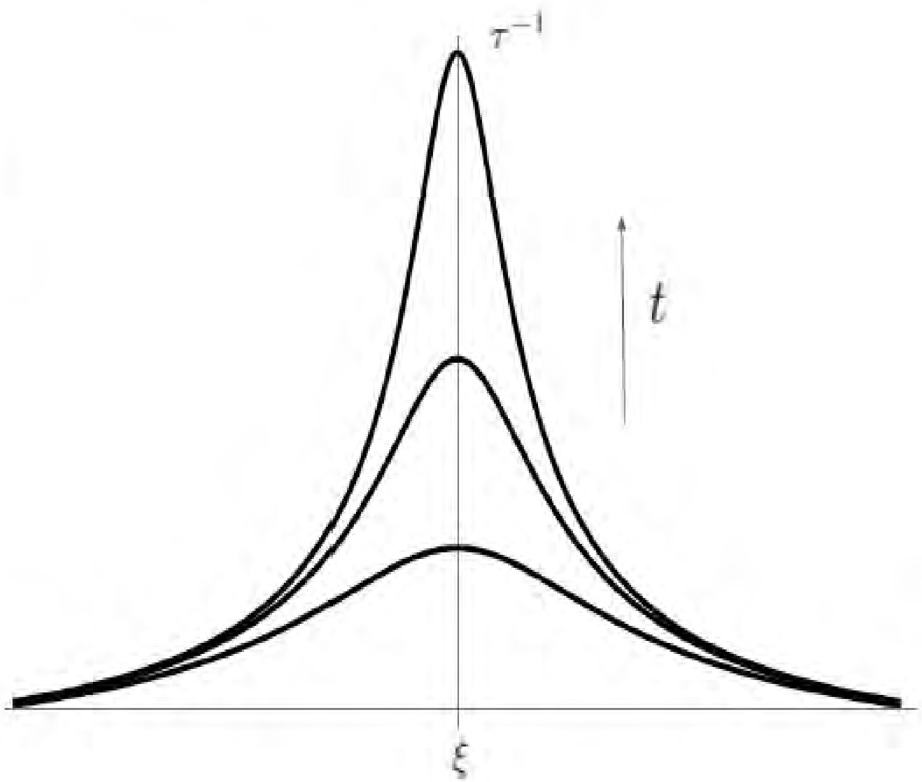
Lifetime

Universal

Can we see this in observables?

$$Im\Sigma \equiv \tau^{-1} = t^{\frac{3-d}{2}} g_d(\xi^2 T t)$$





Going beyond perturbation theory: Interacting bosons obeying Model A.

The equations of motion are,

$$\partial_t D_R(k,t,t') + \left[k^2 + r_{\rm eff}(t)\right] D_R(k,t,t') = -\delta(t-t'),$$

$$\Rightarrow D_R(k,t,t') = -\theta(t-t')e^{-k^2(t-t')}e^{-\int_{t'}^{t} dt_1 r_{\rm eff}(t_1)}, \quad \text{(C1)}$$

where the mass obeys the equation of motion

$$r_{\rm eff}(t) = r + u\int \frac{d^dq}{(2\pi)^d} iD_K(q,t,t), \quad \text{(C2)}$$

$$D_K = D_R \circ \Pi_K \circ D_A. \quad \text{(C3)}$$

Scaling results can be obtained from RG or Hartree-Fock

Going beyond perturbation theory: Interacting bosons obeying Model A.

$$\prod_q e^{-\frac{u}{N}\bar{\Delta}_q\Delta_q} = \int\left[\phi_q, \phi_q^*\right] \times e^{-\frac{N}{u}|\phi_q|^2+\phi_q\bar{\Delta}_q+\phi_q^*\Delta_q}.$$

$$\partial_t\phi_i(t) = -\sum_i \mathbf{D}_{ij}\frac{\delta\mathcal{H}[\phi]}{\delta\phi_j(t)} + \zeta_i(t)\,.$$

Model-A dynamics

$$\langle\zeta(\mathbf{x},t)\zeta(\mathbf{x}',t')\rangle = 2\mathcal{N}\delta(t-t')\delta(\mathbf{x}-\mathbf{x}')\,,$$

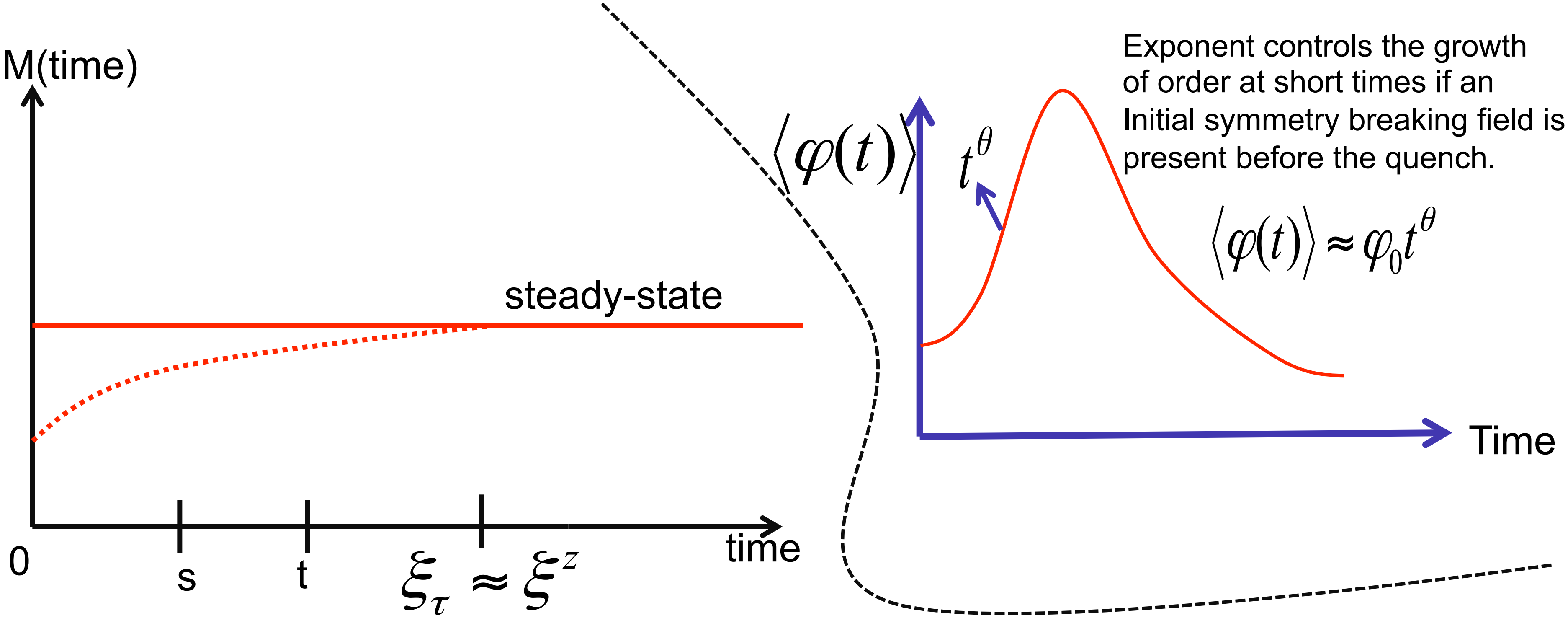
White-noise

$$D = \frac{N}{T}$$

Fluctuation-dissipation theorem

Dynamics arising from a temperature quench from Ti>> Tc to Tf=Tc

H. Janssen, B. Schaub, and B. Schmittmann, Z. Phys. B **73**, 539 (1989).
D. A. Huse, Phys. Rev. B **40**, 304 (1989).
P. Calabrese and A. Gambassi, J. Phys. A: Math. Gen. **38**, R133 (2005).



t>> s and s >> microscopic times but s,t << than correlation time $\xi_t \approx \xi^z$

Singularities whenever the field is at time s=0. Scaling dimension of the field on the temporal boundary different from that in the temporal bulk.

$$D_K(1,2) = C_{\mathbf{q}}(t,s) = (t-s)^{(2-\eta)/z}(t/s)^{\theta-1}\tilde{F}_C(q(t-s)^{1/z}, s/t) \, ,$$
$$D_R(1,2) = R_{\mathbf{q}}(t,s) = (t-s)^{(2-\eta-z)/z}(t/s)^{\theta}\tilde{F}_R(q(t-s)^{1/z}, s/t) \, .$$

See also: Quantum quench in an open system (Gagel, Orth, Schmalian, PRL 2014)

Quench dynamics of the Cooperon from Model-A:

$$iD_K(q,t,t') = Tt'e^{-q^2(t-t')}\,(t/t')^{\theta}\,F_K(2q^2t'); \quad ql \ll 1,$$

$$iD_K(q,t,t) \propto tF_K(2q^2t).$$

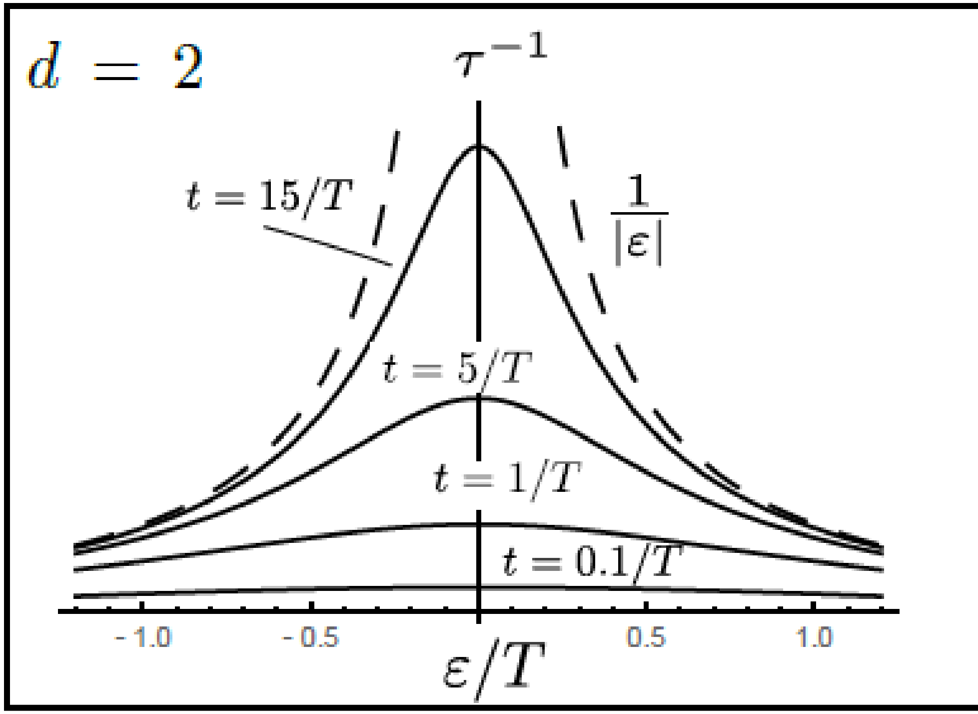
$$F_K(x) \equiv \int_0^1 dy e^{-xy}(1-y)^{-2\theta}$$

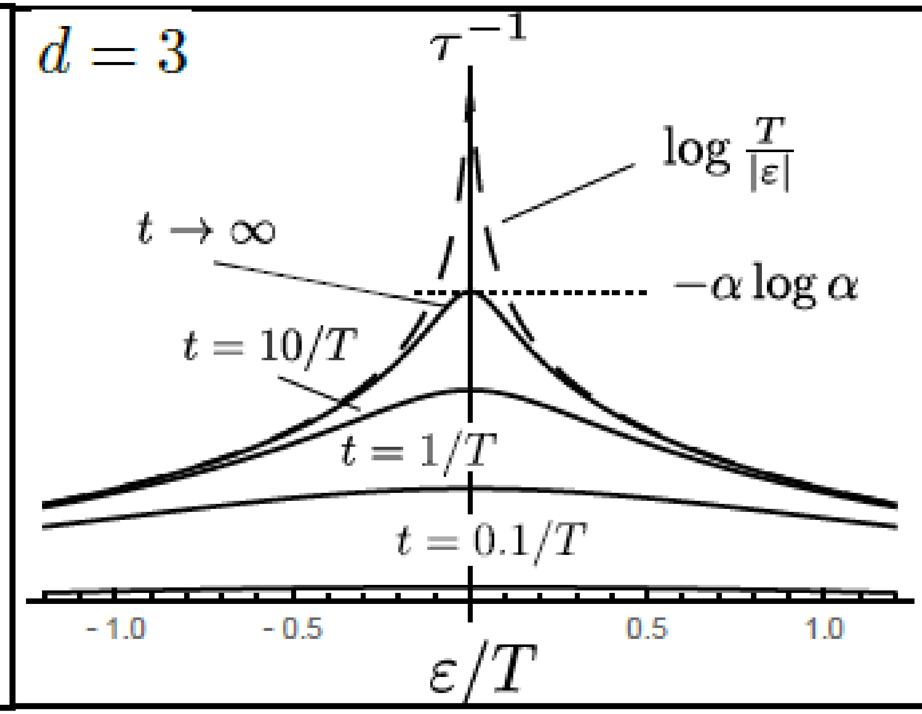
And solving the self-consistent equation for the electron spectral density:

$$\Sigma^R(k,\omega,t) = i\int \frac{d^dq}{(2\pi)^d}\frac{D_K(q,t,t)}{\omega+\epsilon_{k+q}+\Sigma^R(k+q,\omega,t)}$$

$$S_d(x) \equiv \int_0^{\sqrt{Tt}} \frac{dy y^{d-1}}{(2\pi)^d}\int d\hat{n}\frac{F_K(y^2)}{y\cos\theta' + \sqrt{x} + i\delta}$$

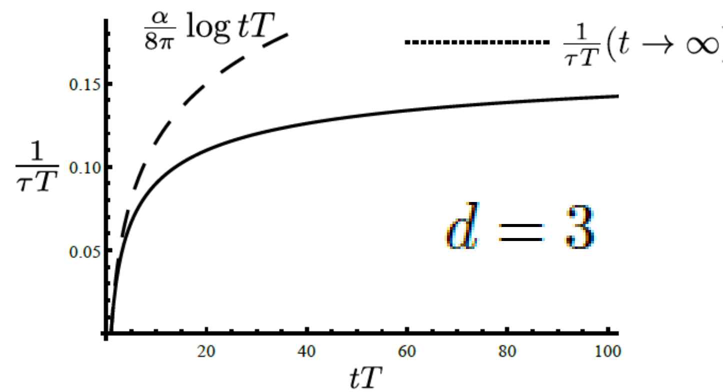
$$\tau^{-1} = t^{(3-d)/2} S_d(\varepsilon^2 t)$$





In d=2, results are still divergent, but weakly as a logarithm in time. This is because d=2 is the lower critical dimension.

$$\delta\Sigma_R \sim T\sqrt{\alpha \log(\alpha T t)}.$$



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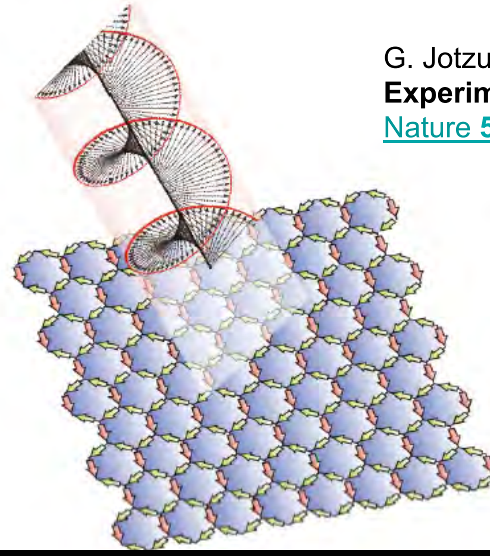
Graphene irradiated with circularly polarized laser



G. Jotzu, T. Esslinger et al.:
Experimental realization of the topological Haldane model
Nature **515**, 237-240 (2014).

Oka and Aoki PRB 2009 in graphene
Other models:
Kitagawa et al PRB 2011,
Lindner et al Nature 2011
Yao, MacDonald, Niu et al PRL 2007

$$U(T+t,t) = e^{-iH_{eff}T}$$

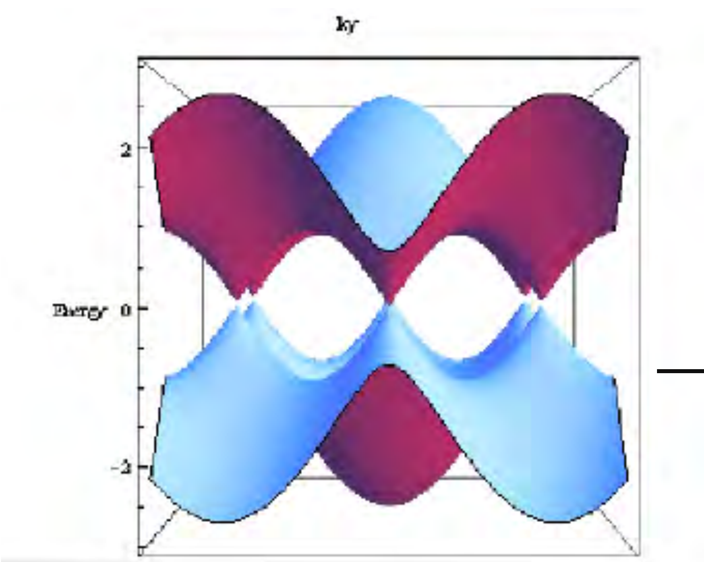
$$H_{eff} \approx k_x \sigma_x \tau_z + k_y \sigma_y + \frac{A_0^2}{\Omega} \sigma_z \tau_z$$

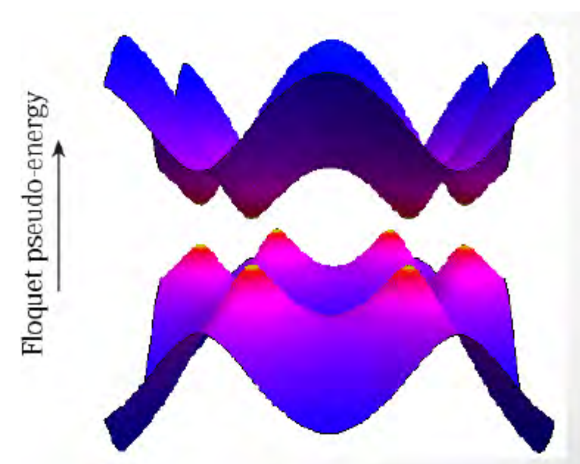
σ: Sublattice index

τ: K,K' points

Breaks time-reversal

H_{eff} **Maps onto the Haldane model**





Initial Hamiltonian Heff: Graphene and/or Haldane model
Final Hamiltonian H: Attractive BCS interactions

$$V = J \sum_{<ij>} \left[\vec{S}_i \cdot \vec{S}_j - \frac{1}{4} n_i n_j \right]$$

$$\Delta_\alpha = J \left\langle b_{i+\alpha\downarrow} a_{i\uparrow} - b_{i+\alpha\uparrow} a_{i\downarrow} \right\rangle$$

$\alpha = 1, 2, 3$ denote the three nearest neighbor bonds

$$H = H_{\text{eff}} - \sum_{k,\alpha} \left[\Delta_\alpha e^{i\vec{k}\cdot\vec{a}_\alpha} \left(a^\dagger_{k\uparrow} b^\dagger_{-k\downarrow} - a^\dagger_{k\downarrow} b^\dagger_{-k\uparrow} \right) + h.c. \right].$$

We will monitor how an initial superconducting fluctuation evolves in time.

$$\Delta_\alpha(t) = J \sum_\beta \int_0^t dt' \Pi^R_{\alpha\beta}(q = 0, t, t') \Delta_\beta(t').$$

SC in doped graphene in equilibrium

R. Nandkishore, L. Levitov, and A. Chubukov, Nature Physics **8**, 158 (2012).
A. Black-Schaffer and C. Honerkamp, Journal of Physics Condensed Matter **26** (2014).

$$\Delta_s = \frac{1}{\sqrt{3}}[1,1,1]$$

$$\Delta_{d+id} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ e^{2\pi i/3} \\ e^{4\pi i/3} \end{pmatrix}$$

$$\Delta_{d-id} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ e^{4\pi i/3} \\ e^{2\pi i/3} \end{pmatrix}$$

Graphene: C3 symmetry and time-reversal symmetry

$$\Pi^R(t) = \begin{pmatrix} A(t) & B(t) & B(t) \\ B(t) & A(t) & B(t) \\ B(t) & B(t) & A(t) \end{pmatrix}.$$

Eigenvalues (EV): s-wave. And two degenerate EV: d+id and d-id

Haldane model: C3 symmetry but broken time-reversal symmetry

$$\Pi^R(t) = \begin{pmatrix} A(t) & B(t) & C(t) \\ C(t) & A(t) & B(t) \\ B(t) & C(t) & A(t) \end{pmatrix}.$$

3 Non-degenerate eigenvalues: s-wave, d+id and d-id.

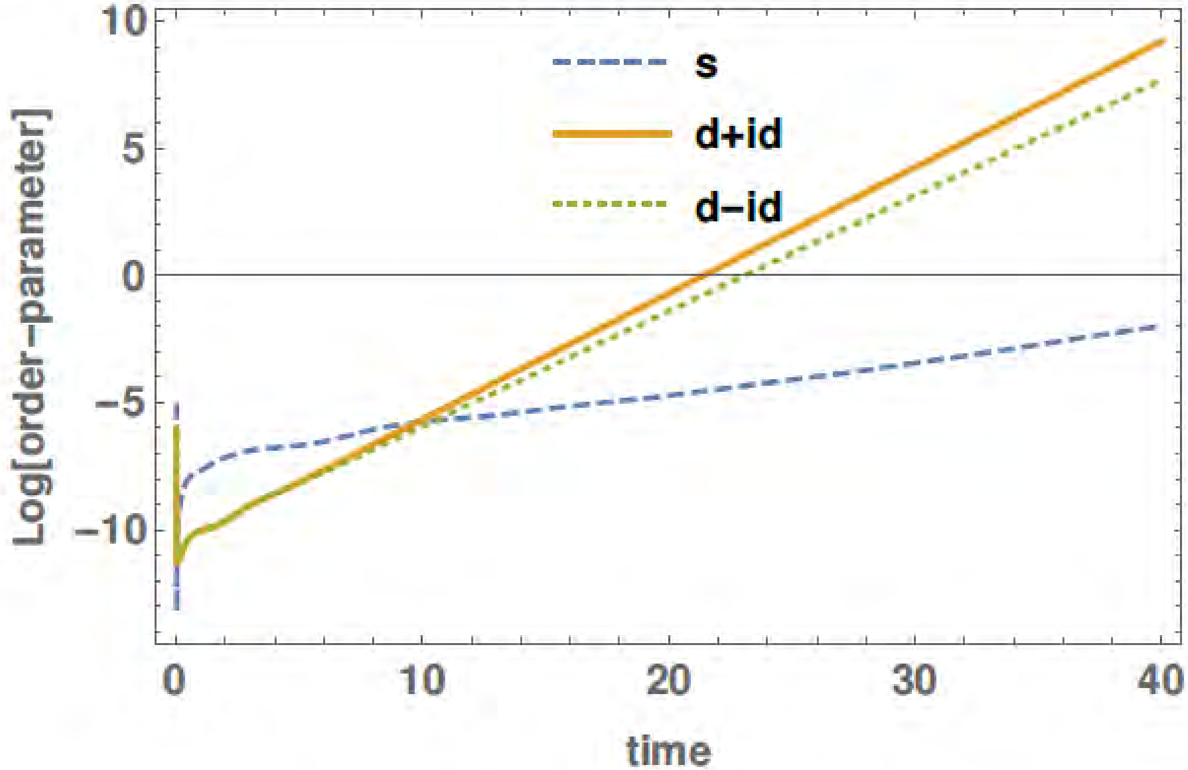
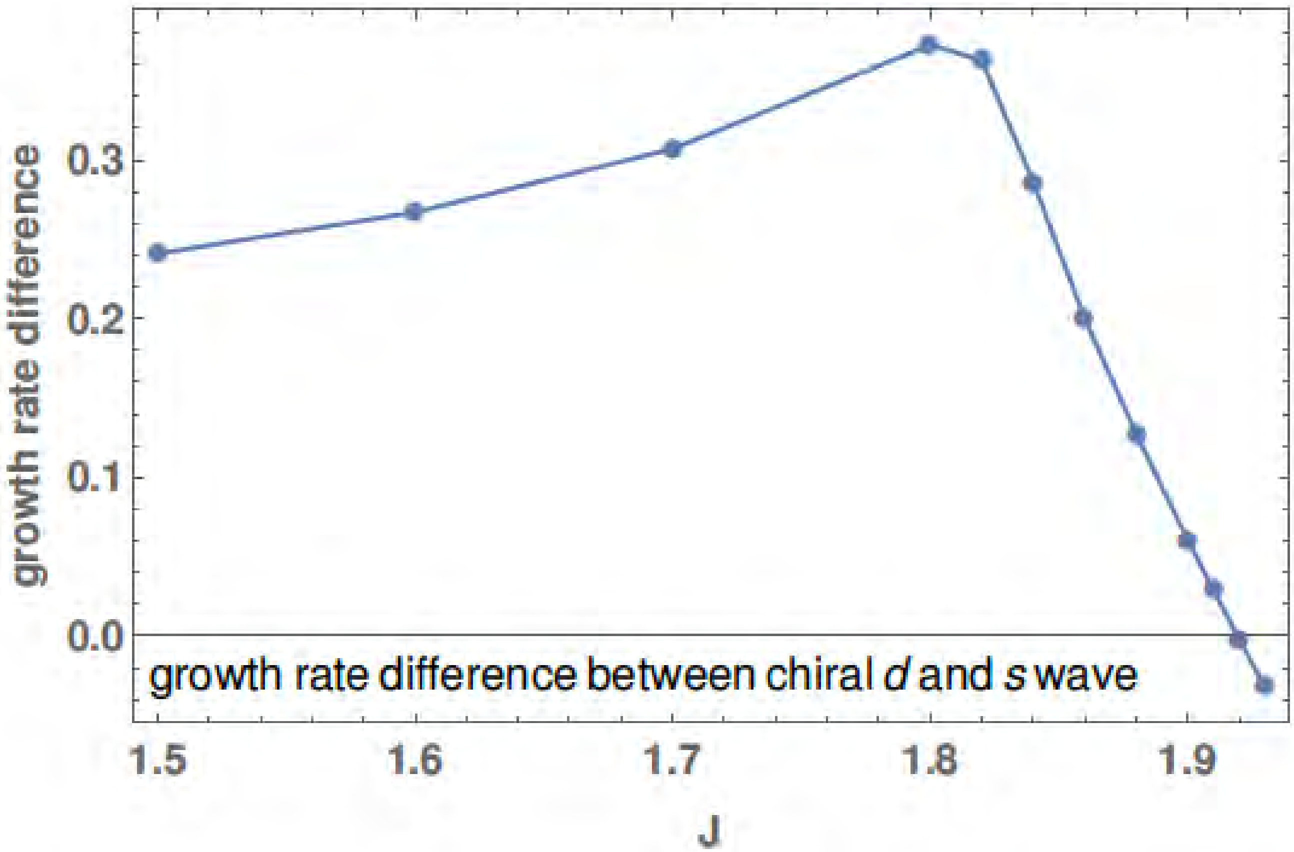
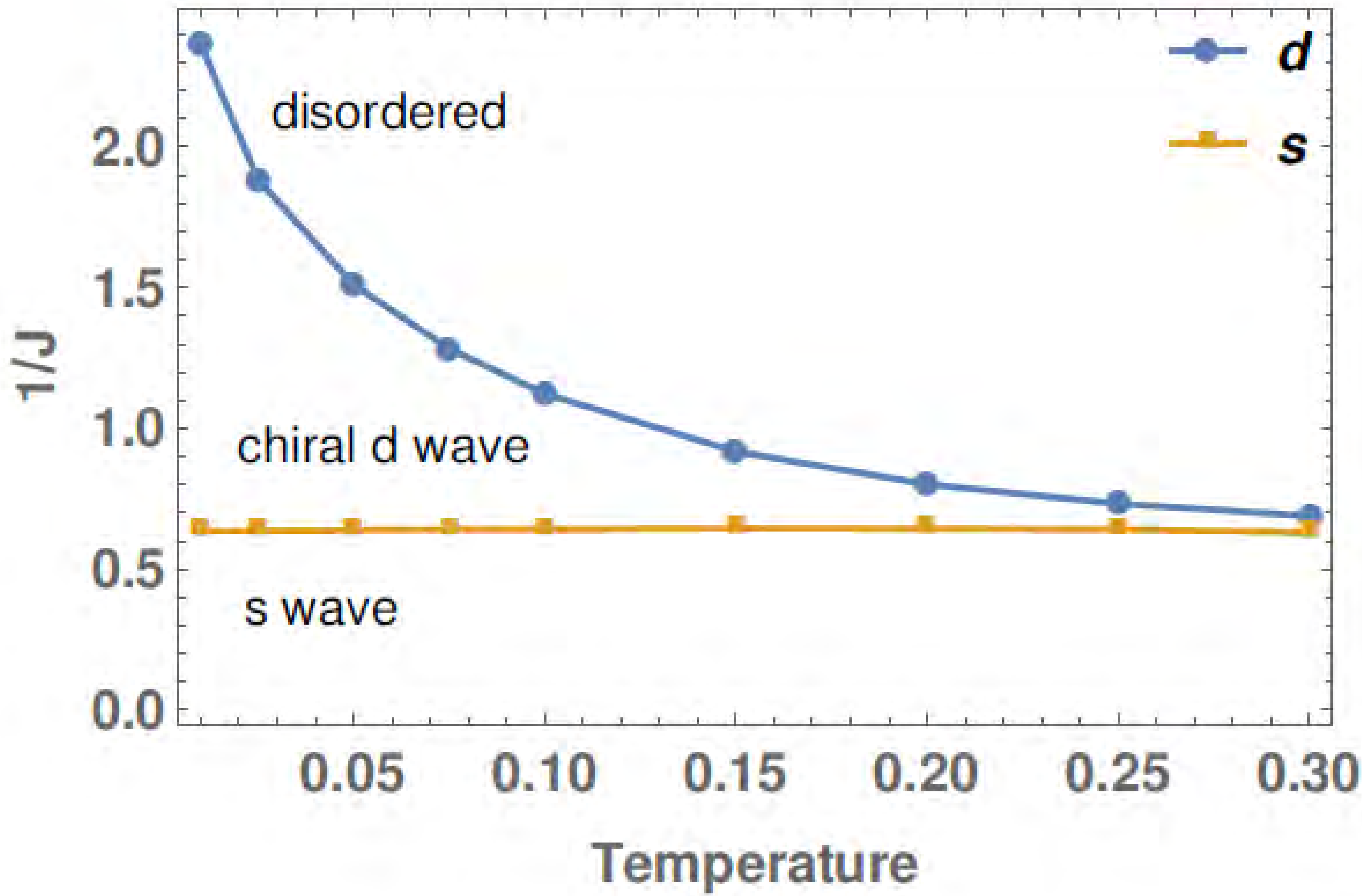
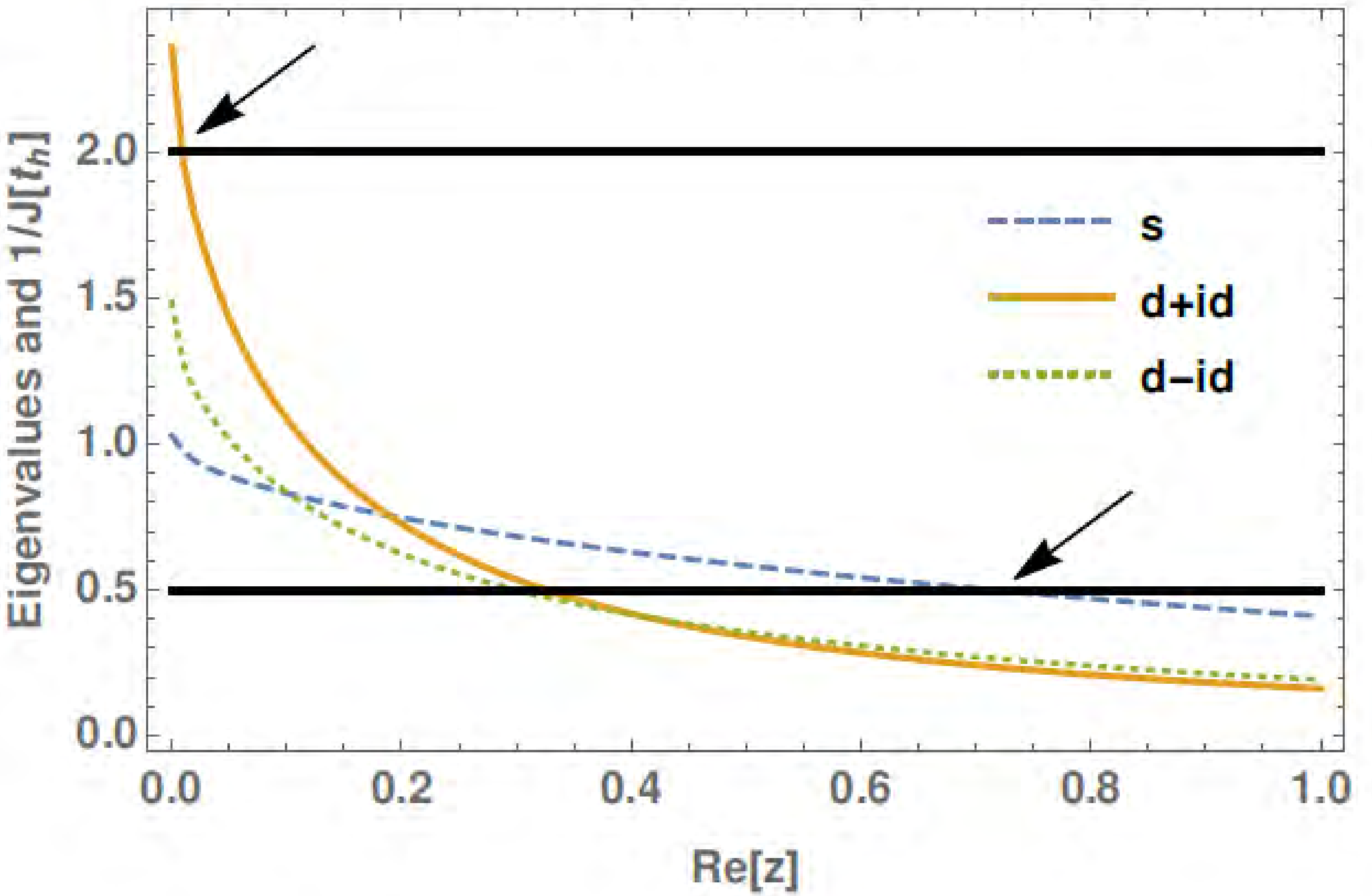


FIG. 1: Haldane model
$A_0 a = 0.5, \Omega = 10t_h, \delta = 0.1, J = 1.82t_h, T = 0.01t_h$.
Time-evolution of the logarithm of an initial random vector. The time-evolution is projected along the three orthogonal directions with s, $d + id$, $d - id$ symmetry. The slopes indicate that for the chosen parameters $d + id$ is the fastest growing instability, followed by $d - id$ and then s. Time is in units of t_h^{-1}.


growth rate difference
0.3
0.2
0.1
0.0
growth rate difference between chiral d and s wave
1.5
1.6
1.7
1.8
1.9
J



Dephasing effects due to excited quasi-particles detrimental for d-wave pairing


Eigenvalues and 1/J[t_h]
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Re[z]
s
d+id
d−id

Part I: Quenches from the normal phase to the superconducting critical point. A theory that takes into account fluctuations self-consistently to O(1/N) was presented. Results for the time-evolution of the spectral density was given. Can it be observed with TR-ARPES??

Ongoing work: Transport following a quench, ramifications for optical conductivity.

Part-II: A mean-field theory for determining the dominant superconducting order after a quench was presented. By tuning the quench amplitude, the symmetry of the dominant order-parameter may be tuned.