DYNAMICS OF IMPURITIES IN QUANTUM GASES

MEERA PARISH

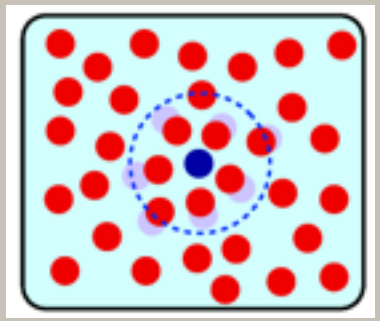
Monash University

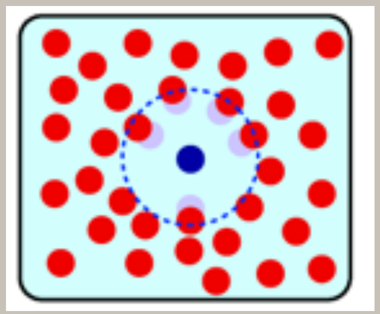
FLEET

ARC CENTRE OF EXCELLENCE IN FUTURE LOW-ENERGY ELECTRONICS TECHNOLOGIES

SPICE workshop, 2 June 2017

WHY IMPURITIES?

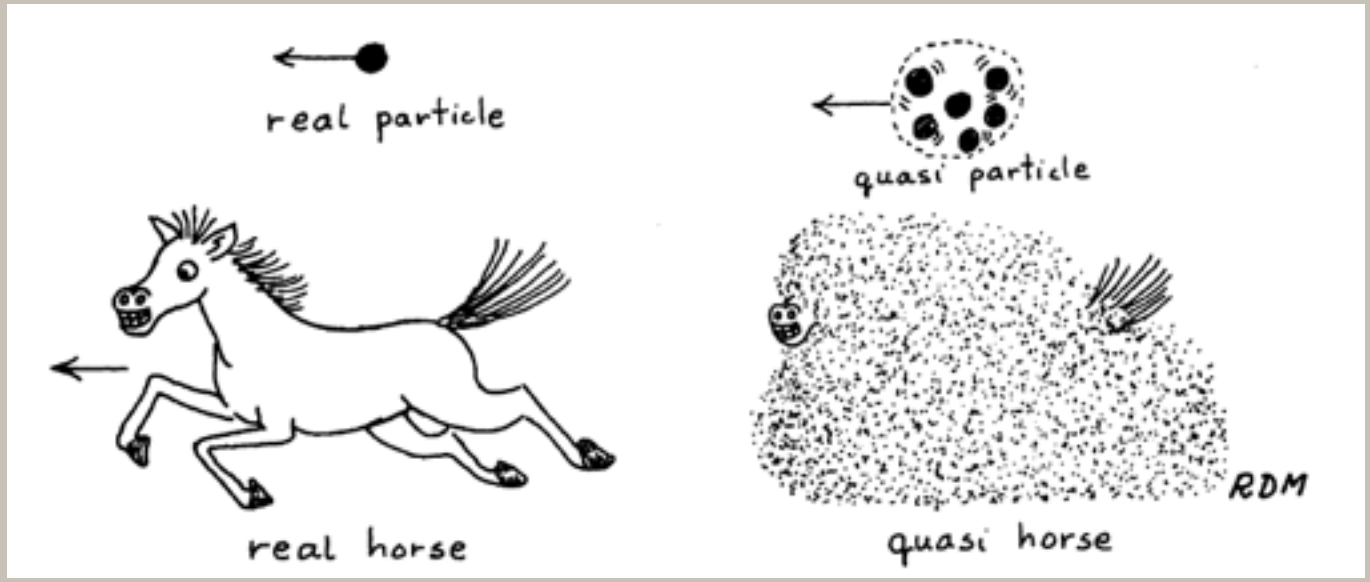




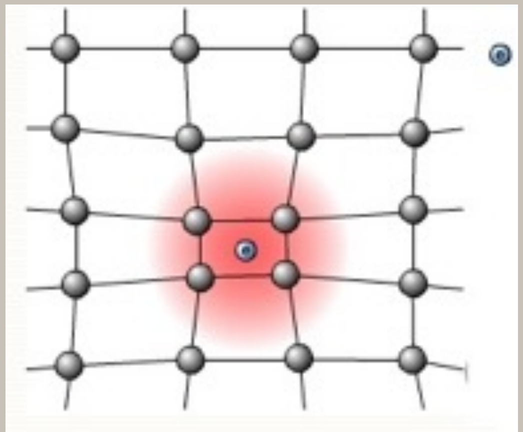
- Prototypical many-body system
  - Gain insight into more complicated quantum systems
- Mobile impurity:
  - "Dressed" impurity, quasiparticle, polaron ...
- Fixed impurity:
  - Orthogonality catastrophe $\boxed{\langle \Psi_0 | \Psi_{\rm int} \rangle = 0}$
- Possibility of doing controlled experiments with cold atoms
  - Quasiparticle states and dynamics
  - Bosonic or fermionic mediums

The “dressed” impurity (polaron)

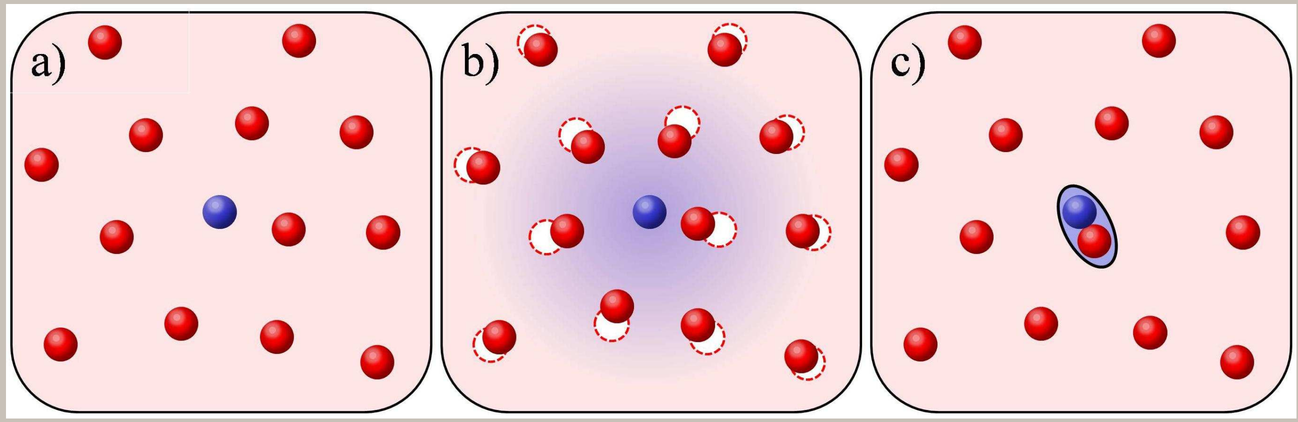
- A mobile impurity will move in a cloud of excitations of its environment, yielding a quasiparticle or “dressed” impurity



- How an impurity changes its character (mass, charge …) is a fundamental problem, relevant to many areas of physics



electrons in a solid



interaction-driven transitions

OUTLINE

- Quench dynamics of Fermi polaron
- Li-K experiments
- Theoretical description
  - Quasiparticle states & spectrum

Innsbruck experimental group

Marko Cetina

Rudi Grimm

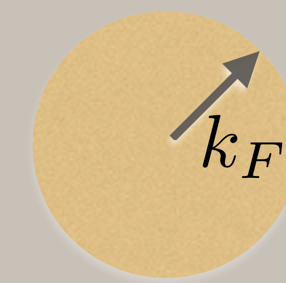
Jesper Levinsen
(Monash)

Eugene Demler
Richard Schmidt
Michael Knap
(Harvard)

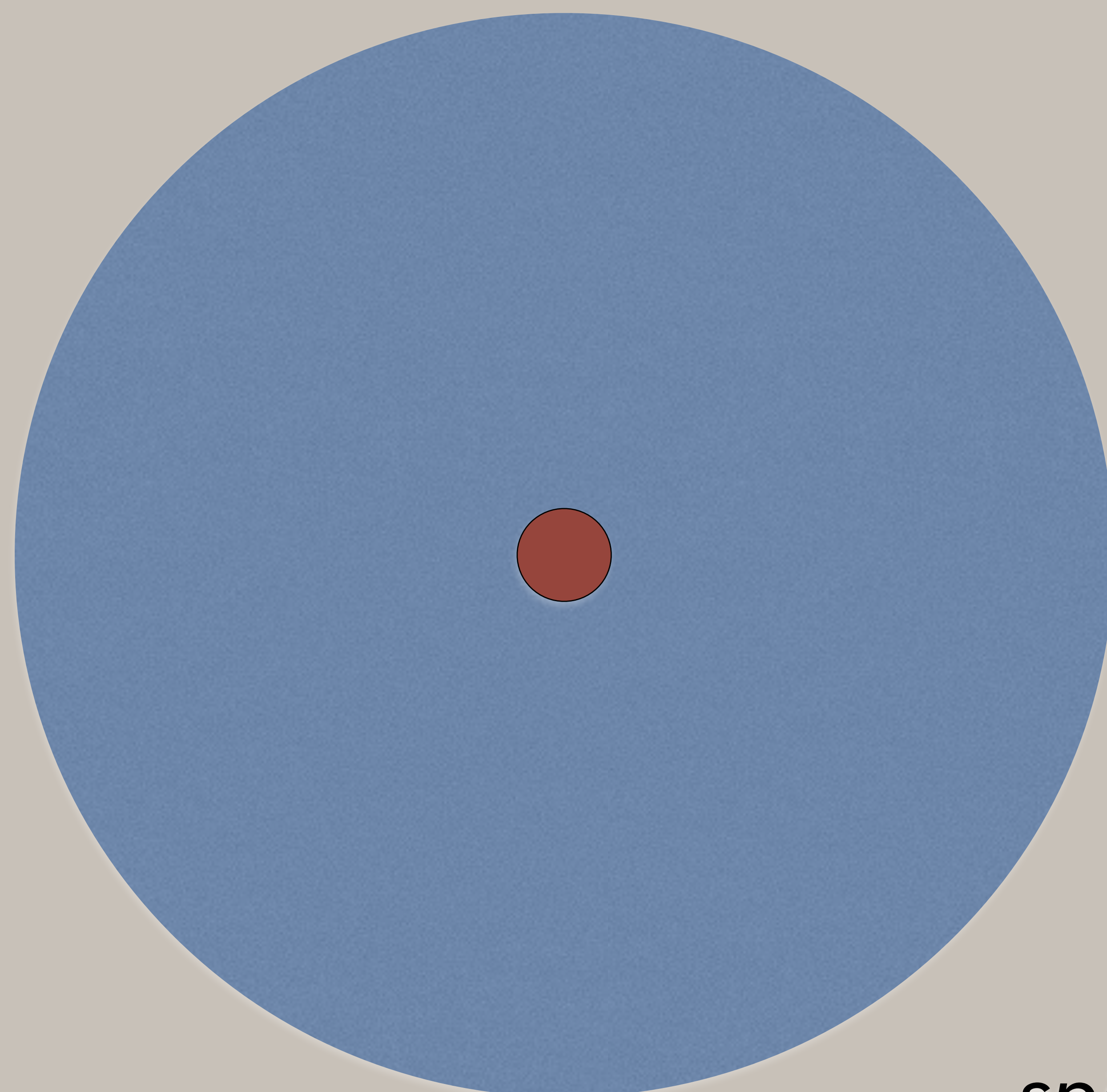
M. Cetina, …, J. Levinsen, MMP et al., Science 354, 96 (2016)

MMP & J. Levinsen, PRB 94, 184303 (2016)

Impurity in a Fermi sea



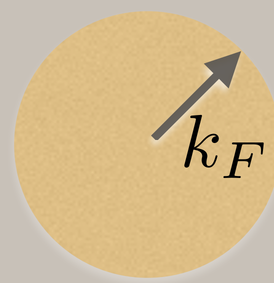
$$\boxed{\varepsilon_F = \frac{\hbar^2 k_F^2}{2m}}$$



$t = 0$

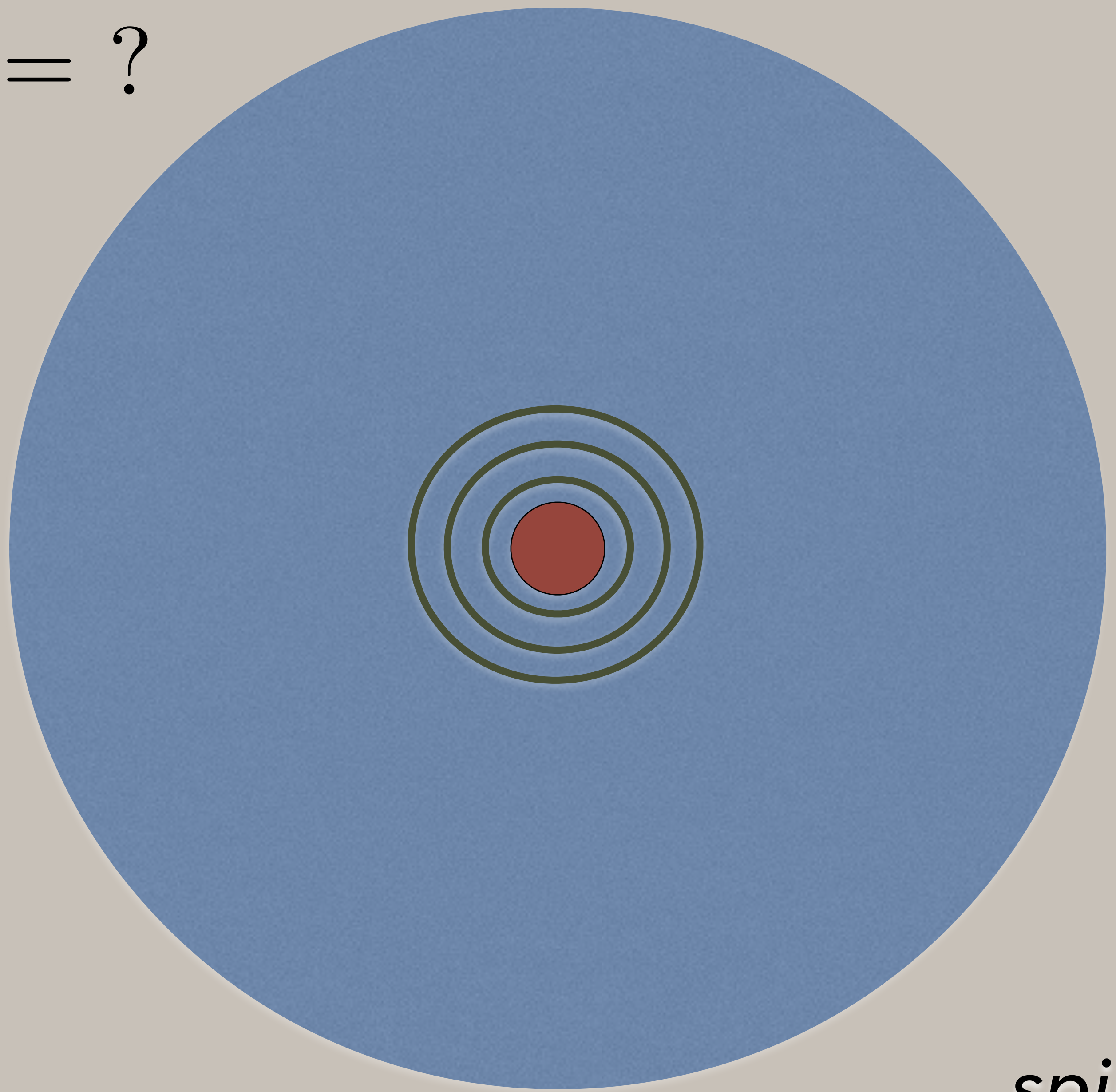
spinless impurity
non-interacting

Impurity in a Fermi sea



$$\boxed{\varepsilon_F = \frac{\hbar^2 k_F^2}{2m}}$$

$$\langle \Psi(0) | \Psi(t) \rangle = \, ?$$



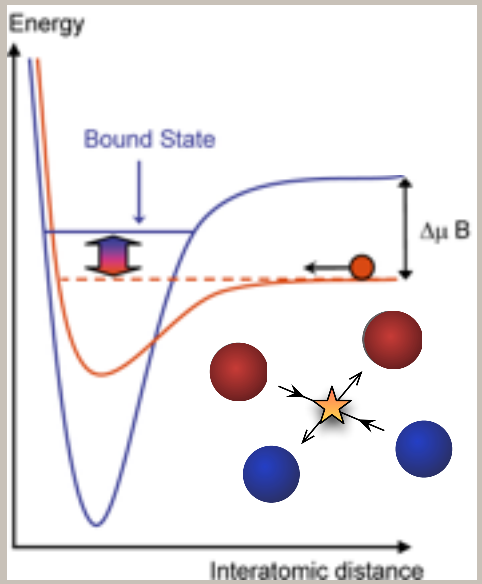
$$t > 0$$

spinless impurity

$\boxed{\textit{strongly interacting}}$

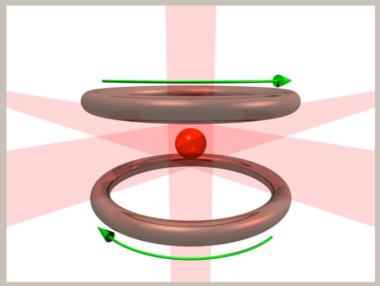
The cold-atom system

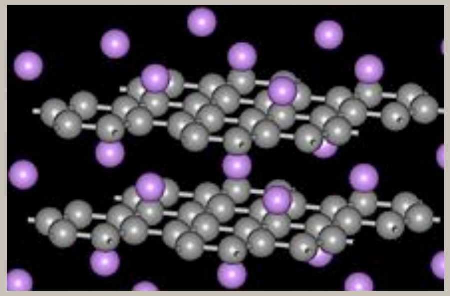
- Low energies compared to electron systems
  - Low particle density: 10^{13}cm^{-3} (NB. air has 10^{19}cm^{-3})
  - fast time scales $t \lesssim \hbar/\varepsilon_F$



- Tunable short-range interactions

Probe “ultrafast” dynamics of fermionic systems

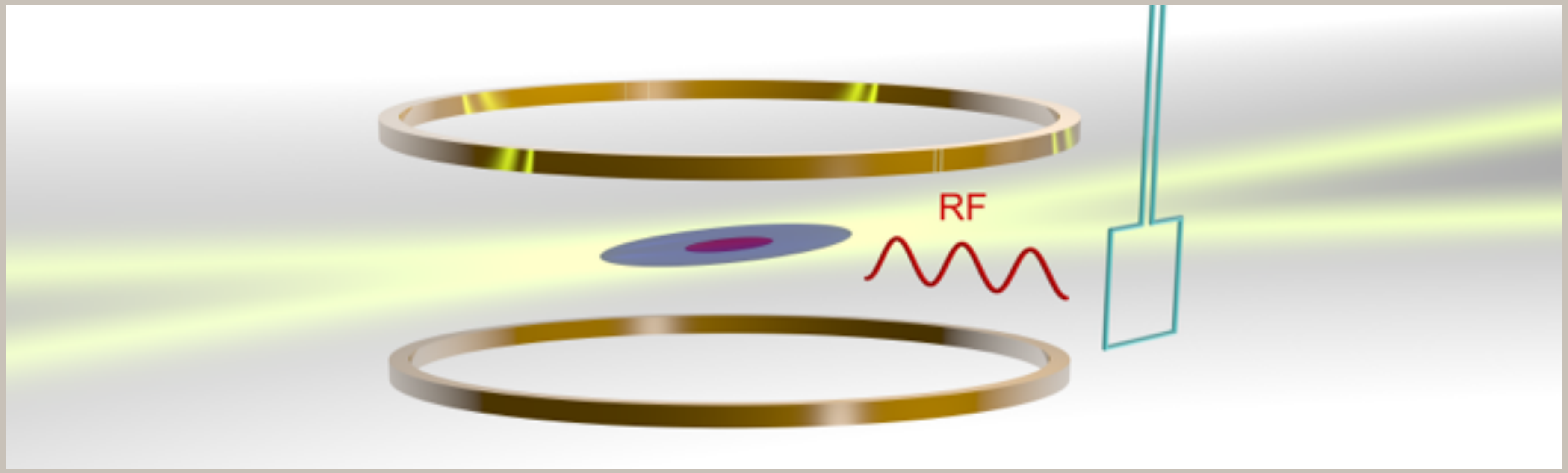




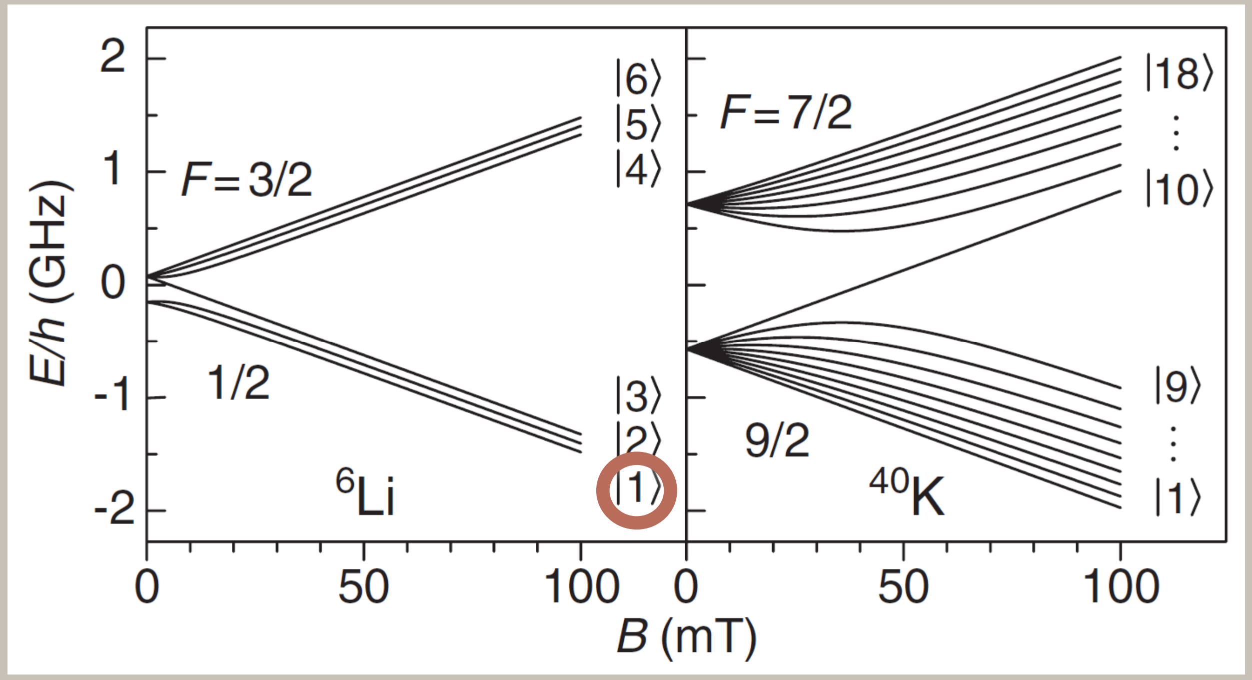
microseconds ⟷ *~100 attoseconds*

Innsbruck experiment

- Small cloud of ^{40}K atoms in a ^{6}Li Fermi gas

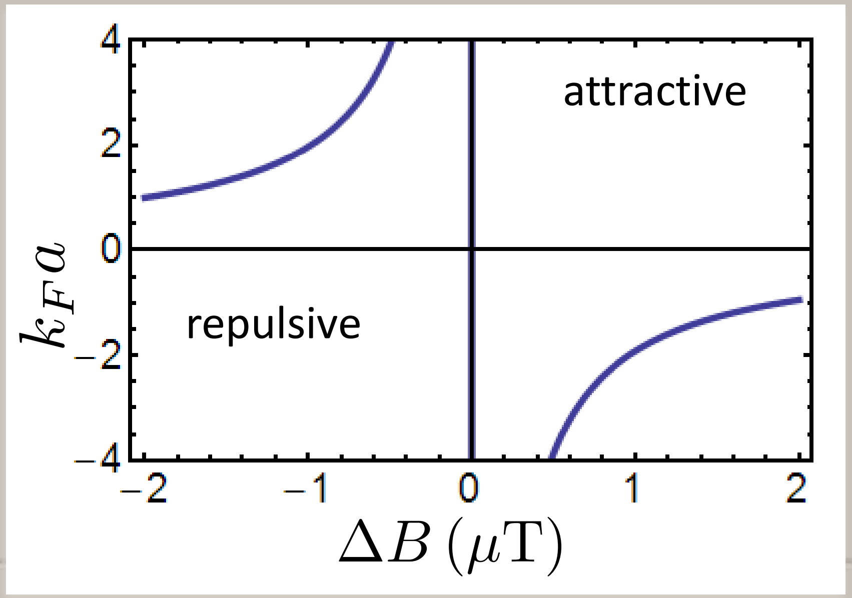


- Hyperfine states:



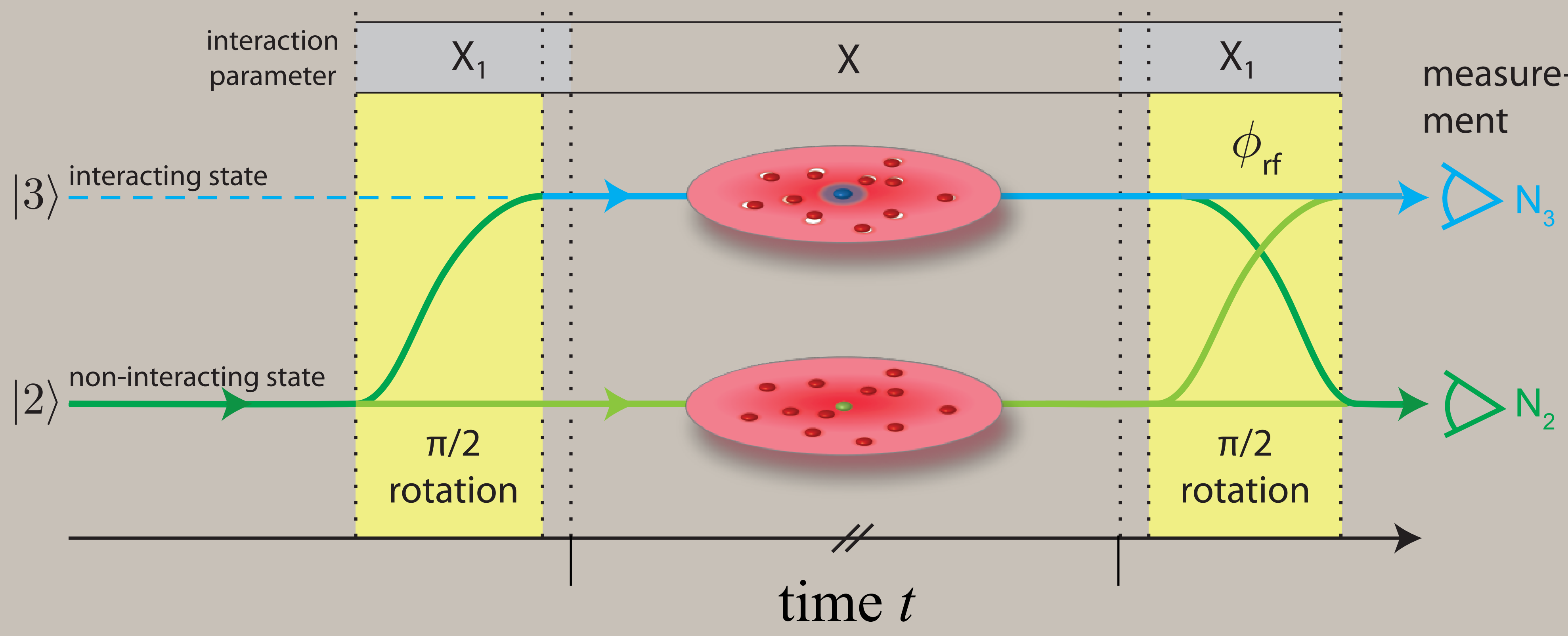
^{6}Li $|1\rangle$-^{40}K $|2\rangle$: non-interacting

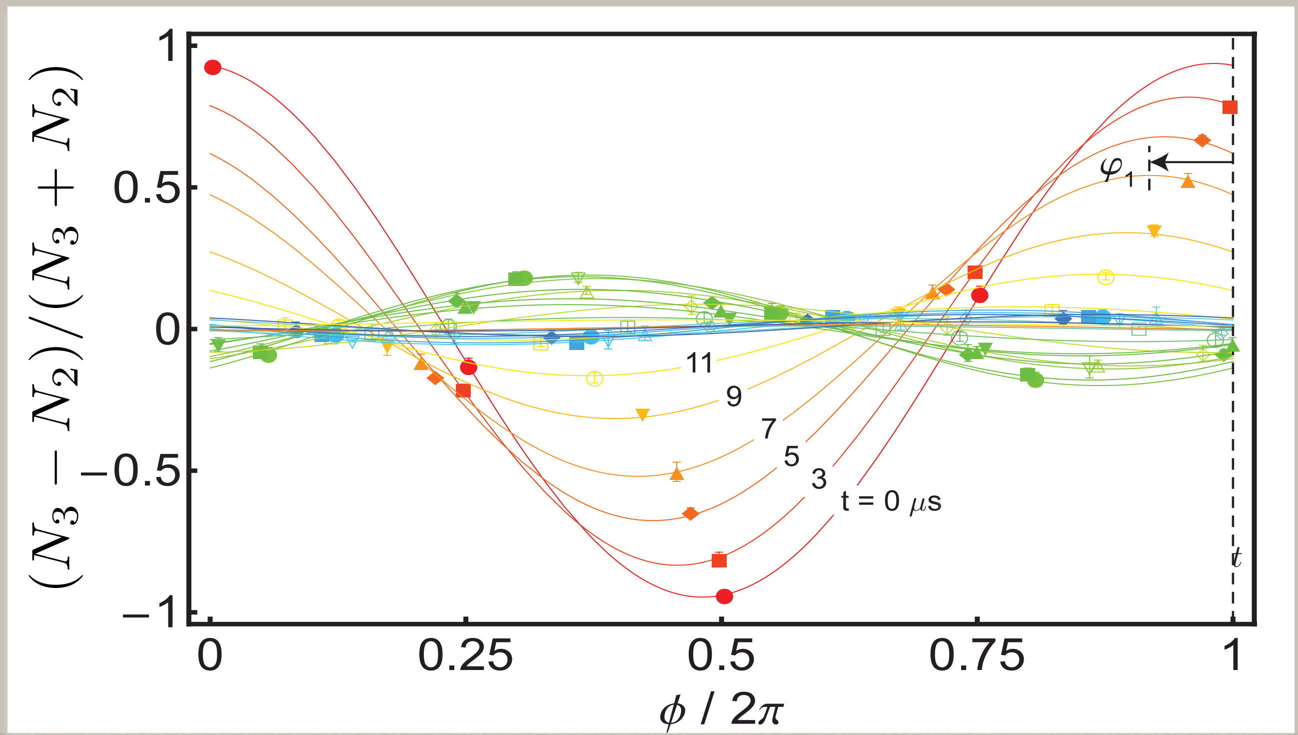
^{6}Li $|1\rangle$-^{40}K $|3\rangle$: tunable interactions



C. Kohstall, et al., Nature 485, 615 (2012)

Innsbruck experiment





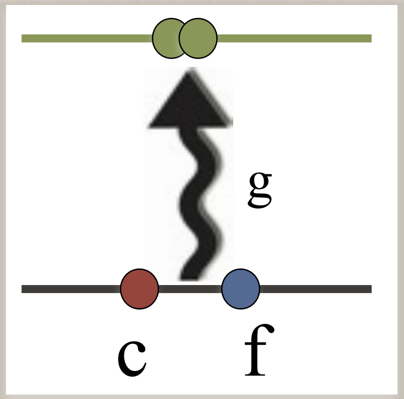
- Interference of states gives:

$$S(t) = \langle \psi_0 | \, e^{-i\hat{H}t/\hbar} \, | \psi_0 \rangle$$

$$= |S(t)| e^{-i\phi(t)}$$

M. Cetina et al., Science 354, 96 (2016)

Theoretical description



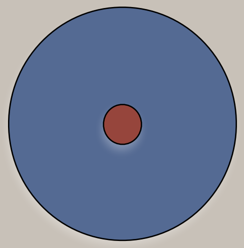
- Hamiltonian for ^{40}K atom in a ^{6}Li Fermi gas:

$$\hat{H} = \sum_{\mathbf{k}} \left[\epsilon_{\mathbf{k}} f^{\dagger}_{\mathbf{k}} f_{\mathbf{k}} + \epsilon^{\mathrm{im}}_{\mathbf{k}} c^{\dagger}_{\mathbf{k}} c_{\mathbf{k}} + (\epsilon^{\mathrm{d}}_{\mathbf{k}} + \nu_0) d^{\dagger}_{\mathbf{k}} d_{\mathbf{k}} \right] + g \sum_{\mathbf{k},\mathbf{q}} \left(d^{\dagger}_{\mathbf{q}} c_{\mathbf{q}-\mathbf{k}} f_{\mathbf{k}} + h.c. \right)$$

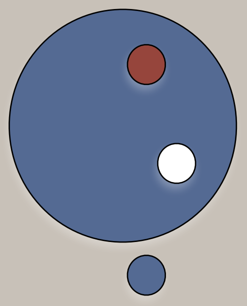
  – Closed channel (*d)* fixes *a* and effective range R^*

- Restrict Hilbert space to wavefunctions of the form:

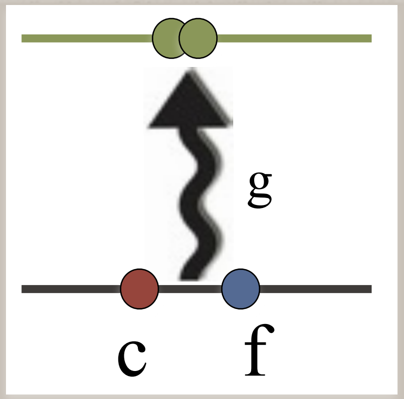
$$|\phi\rangle =$$



$$+ \; \dots$$



Theoretical description



- Hamiltonian for ^{40}K atom in a ^{6}Li Fermi gas:

$$\hat{H} = \sum_{\mathbf{k}} \left[\epsilon_{\mathbf{k}} f^\dagger_{\mathbf{k}} f_{\mathbf{k}} + \epsilon^{\mathrm{im}}_{\mathbf{k}} c^\dagger_{\mathbf{k}} c_{\mathbf{k}} + (\epsilon^{\mathrm{d}}_{\mathbf{k}} + \nu_0) d^\dagger_{\mathbf{k}} d_{\mathbf{k}} \right] + g \sum_{\mathbf{k},\mathbf{q}} \left(d^\dagger_{\mathbf{q}} c_{\mathbf{q}-\mathbf{k}} f_{\mathbf{k}} + h.c. \right)$$

  – Closed channel (*d*) fixes *a* and effective range R^*

- Restrict Hilbert space to wavefunctions of the form:

$$|\phi_{\boldsymbol{\alpha}}\rangle = \left(\alpha_0 c^\dagger_0 + \sum_{\mathbf{q}} \alpha_{\mathbf{q}} d^\dagger_{\mathbf{q}} f_{\mathbf{q}} + \sum_{\mathbf{kq}} \alpha_{\mathbf{kq}} c^\dagger_{\mathbf{q}-\mathbf{k}} f^\dagger_{\mathbf{k}} f_{\mathbf{q}} \right) |FS\rangle$$

F. Chevy, PRA 74, 063628 (2006)

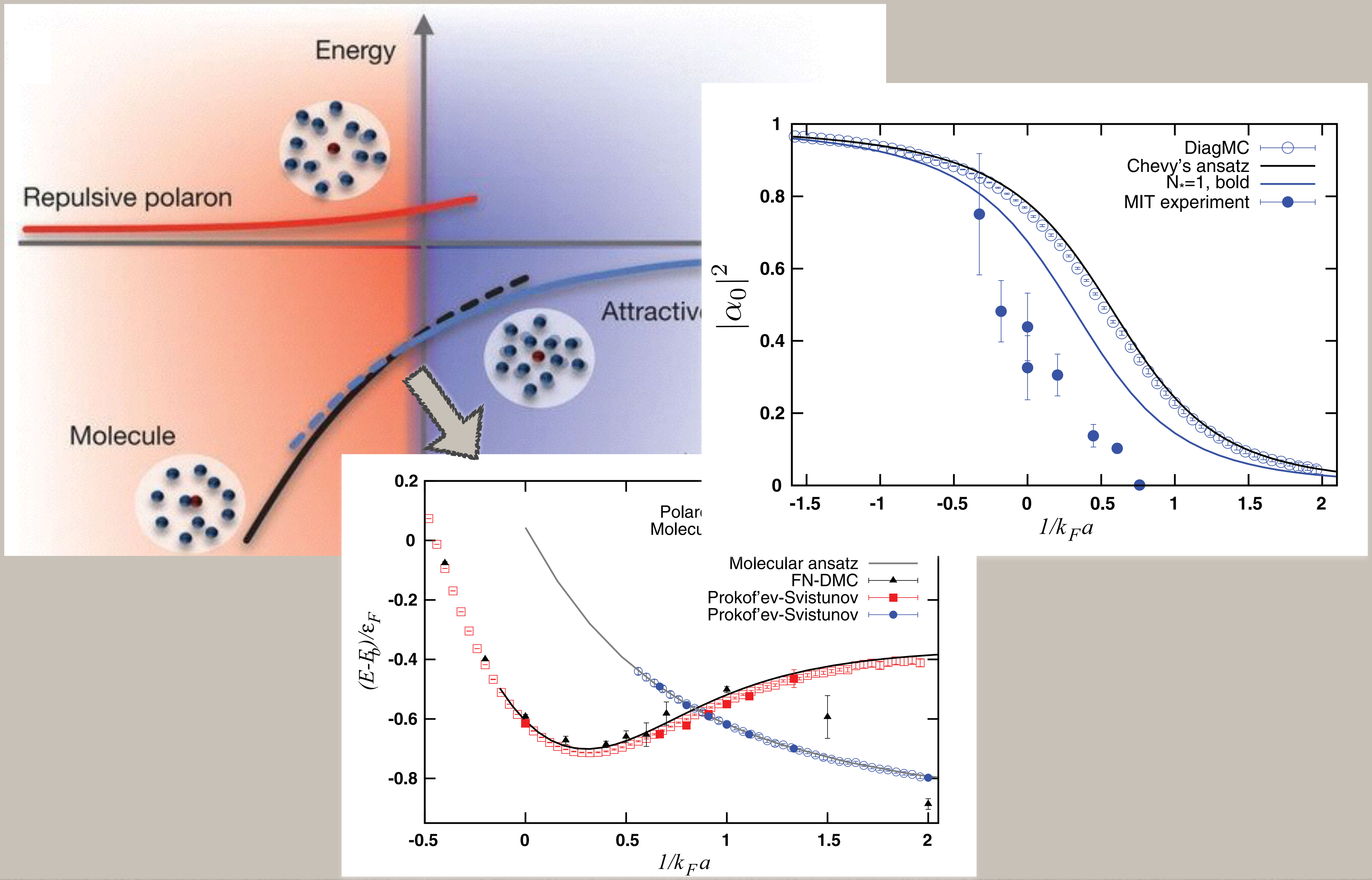
- Diagonalize Hamiltonian within this truncated basis and determine response (at *zero T*):

$$S(t) = \langle \psi_0 | e^{-i\hat{H}t/\hbar} | \psi_0 \rangle = \sum_j |\langle \psi_0 | \phi_j \rangle|^2 \, e^{-iE_j t/\hbar} \;\; = |S(t)| e^{-i\phi(t)}$$

where non-interacting state $|\psi_0\rangle \equiv c^\dagger_0 |FS\rangle$

Static case

$$|\phi_{\boldsymbol{\alpha}}\rangle = \left(\alpha_0 c_0^\dagger + \sum_{\mathbf{q}} \alpha_{\mathbf{q}} d_{\mathbf{q}}^\dagger f_{\mathbf{q}} + \sum_{\mathbf{kq}} \alpha_{\mathbf{kq}} c_{\mathbf{q-k}}^\dagger f_{\mathbf{k}}^\dagger f_{\mathbf{q}} \right) |FS\rangle$$



Vlietinck et al., PRB 87, 115133 (2013)

Short-time dynamics

- Truncated basis method accurately describes behaviour at short times
  - few-body correlations expected to dominate

$$S(t) = \langle\psi_0| e^{-i\hat{H}t/\hbar} |\psi_0\rangle \simeq 1 - \frac{it}{\hbar} \langle\psi_0| \hat{H} |\psi_0\rangle - \frac{t^2}{\hbar^2} \langle\psi_0| \hat{H}^2 |\psi_0\rangle + \dots$$

- For small t, the coherent dynamics is determined by *2-body properties*:

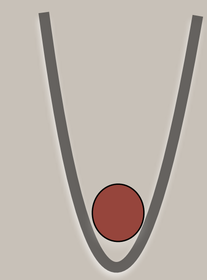
$$\boxed{S(t) \simeq 1 - \frac{(m/m_r)^2}{3\pi k_F R^*} \left(\frac{t}{\tau_F}\right)^2 + \frac{16 e^{i\pi/4} (m/m_r)^{5/2}}{45\pi^{3/2} (k_F R^*)^2} \left(\frac{t}{\tau_F}\right)^{\frac{5}{2}}} \qquad k_F R^* \gtrsim 1$$

$$\boxed{S(t) \simeq 1 - \frac{8 e^{-i\pi/4} (m/m_r)^{3/2}}{9\pi^{3/2}} \left(\frac{t}{\tau_F}\right)^{\frac{3}{2}}} \qquad k_F R^* = 0$$

- *Non-analytic* dependence on t that depends on range of interaction

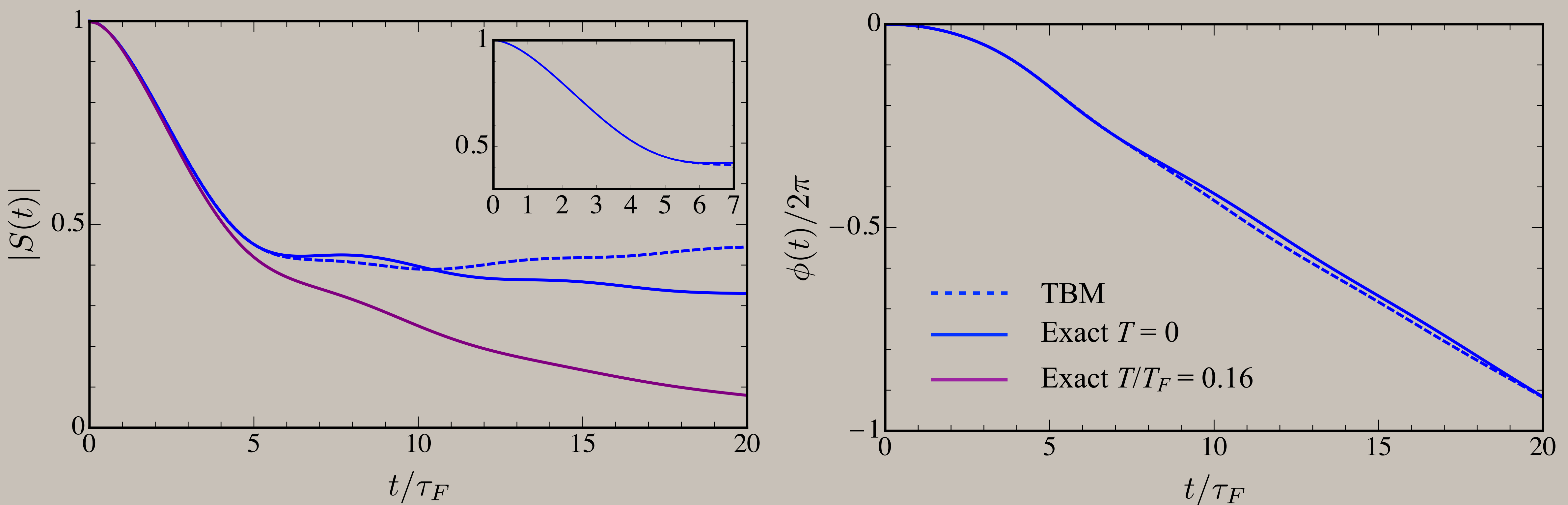
MMP & J. Levinsen, PRB 94, 184303 (2016)

Accuracy of approach



- Comparison with exact result for an infinitely massive impurity:

M. Knap et al., PRX 2, 041020 (2012)



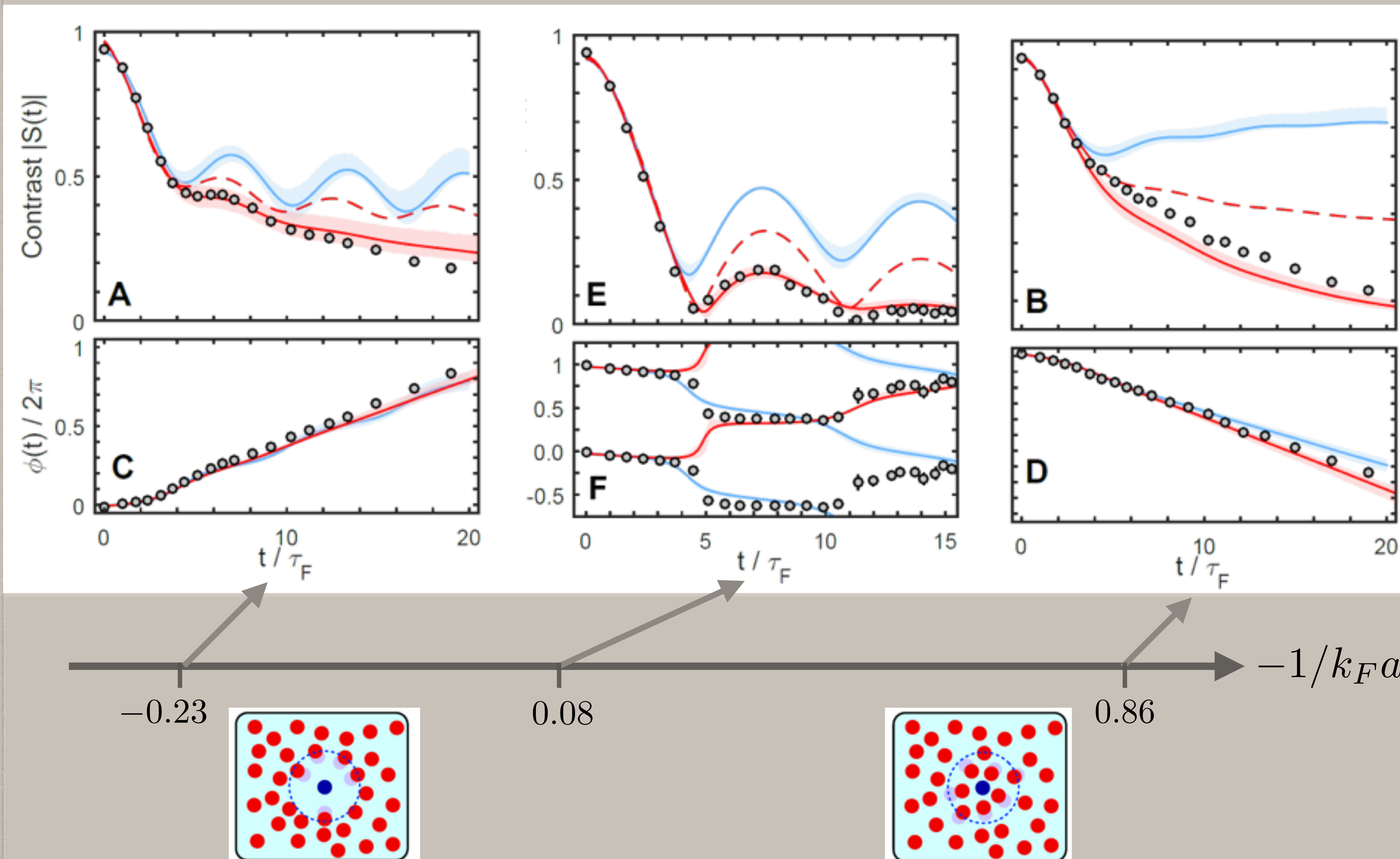
  - Excellent agreement at short times
  - Multiple particle-hole pair excitations become relevant at longer times
  - Truncated basis method cannot capture orthogonality catastrophe

$\boxed{k_F R^* \sim 1}$

M. Cetina et al., Science 354, 96 (2016)

Dynamics of an impurity

$\boxed{\tau_F = \hbar/\varepsilon_F}$

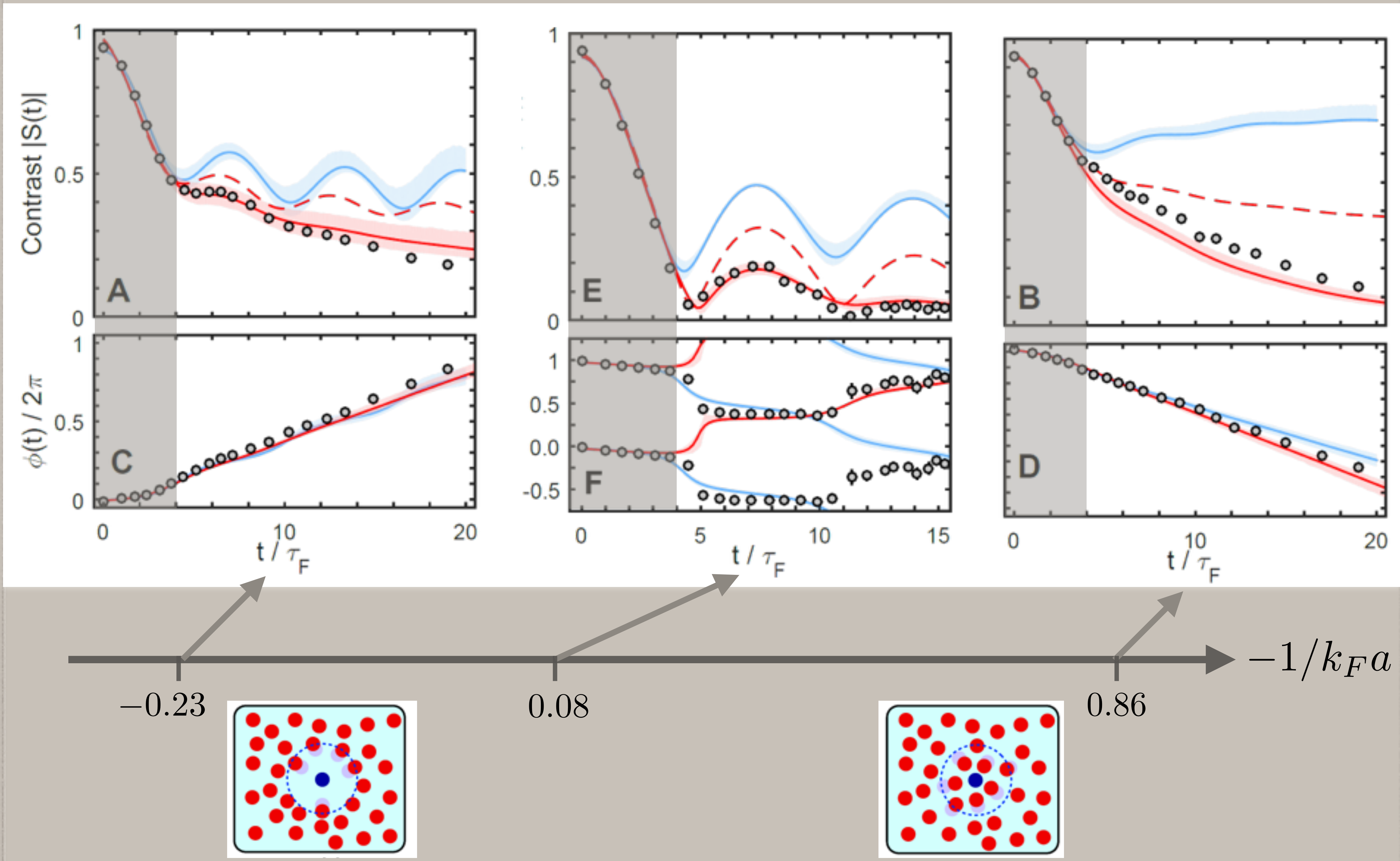


$\boxed{k_F R^* \sim 1}$

M. Cetina et al., Science 354, 96 (2016)

Dynamics of an impurity

$\boxed{\tau_F = \hbar/\varepsilon_F}$



$\boxed{k_F R^* \sim 1}$

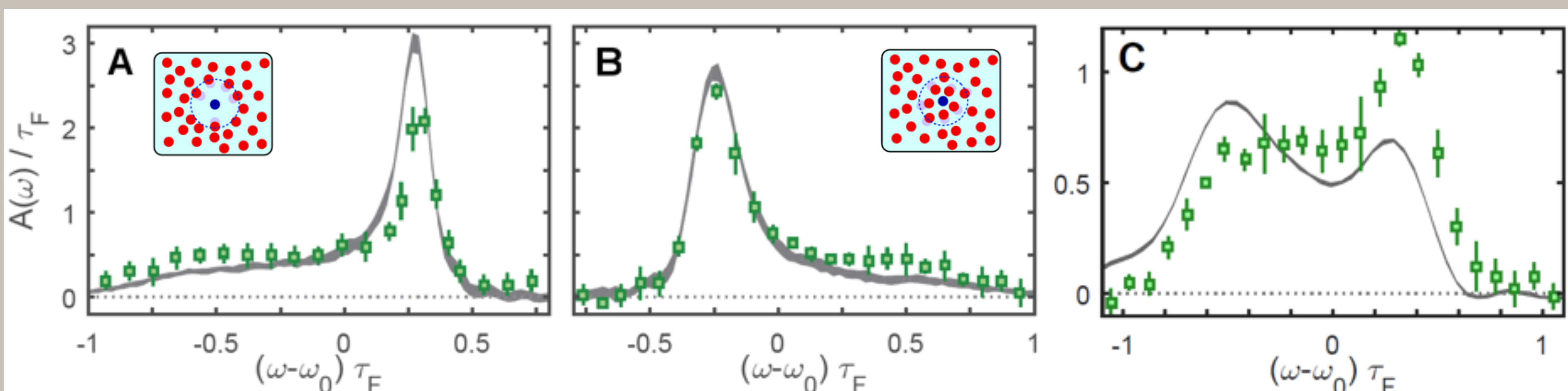
$\boxed{T/T_F \sim 0.2}$

Relationship to spectrum

- In ideal case, interference signal is Fourier transform of spectral function:

$$S(t) = \int d\omega \; e^{-i\omega t} A(\omega)$$

- Comparison with measurement of spectrum:



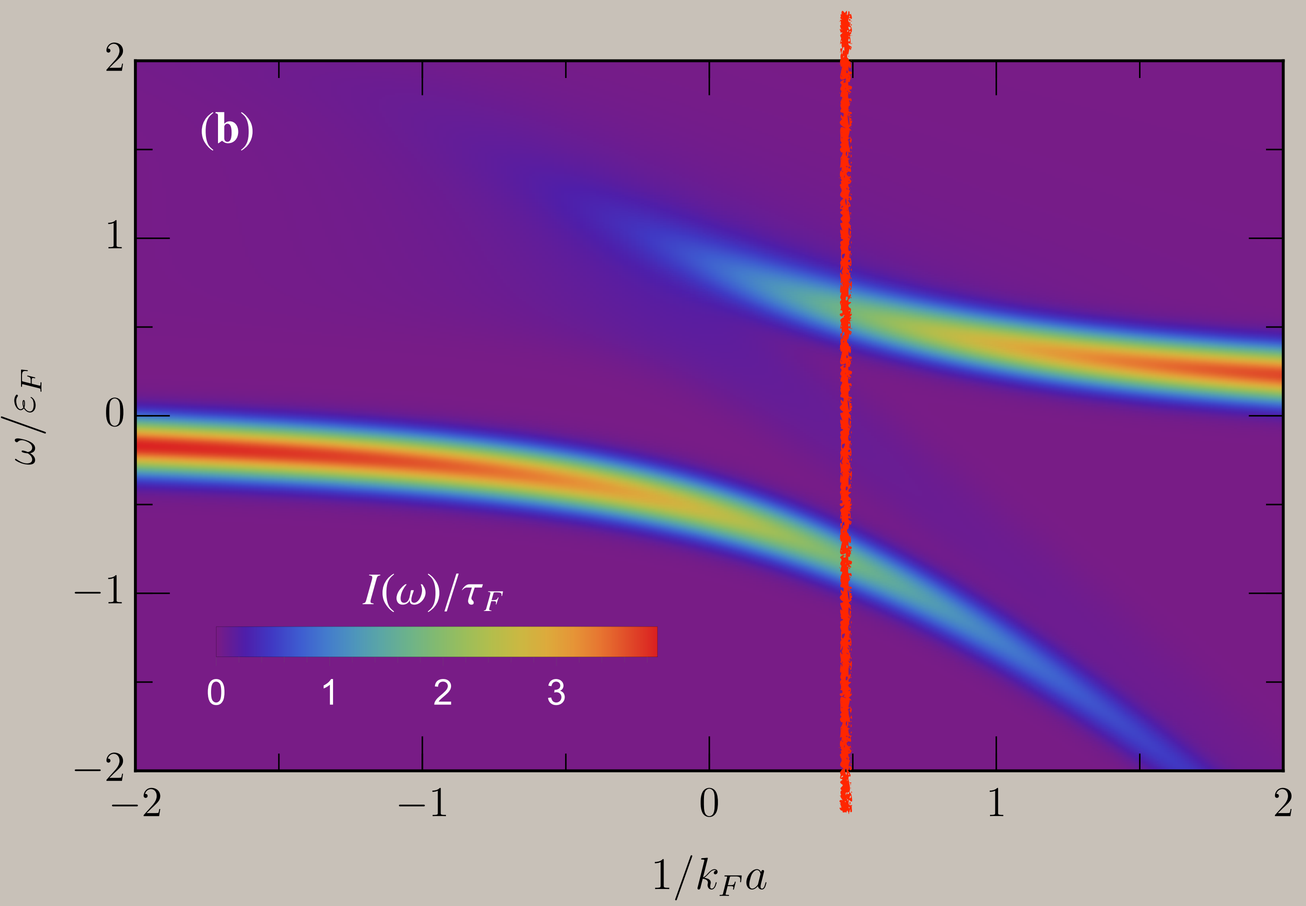
  - Discrepancy due to (weak) interactions during $\pi/2$ pulse
  - Initial state is dressed impurity: $\langle\psi_0|\, e^{-iHt/\hbar}\, |\psi_0\rangle \;\longrightarrow\; \langle\psi_P|\, e^{-iHt/\hbar}\, |\psi_P\rangle$

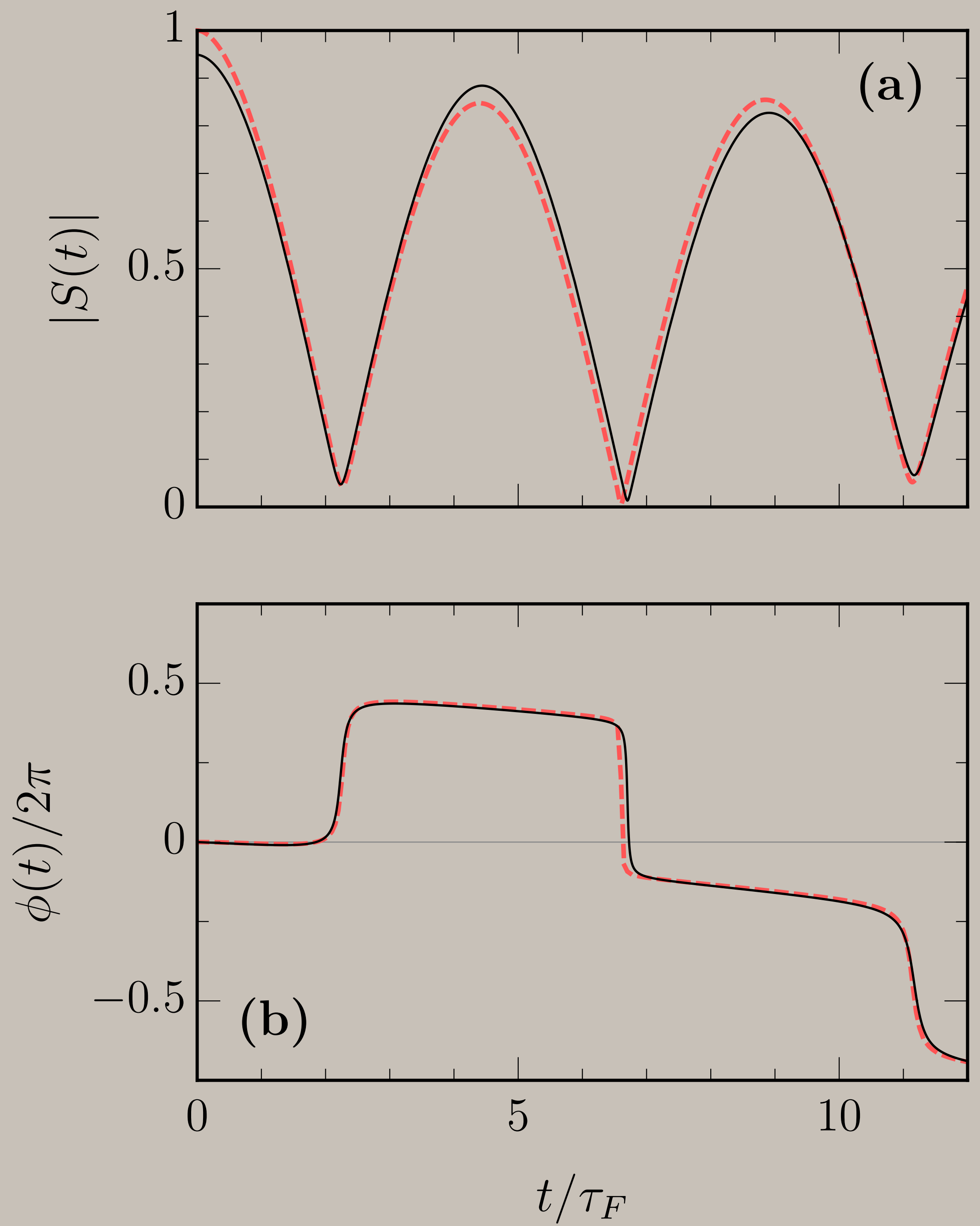
M. Cetina et al., Science 354, 96 (2016)

Polaron interference

$k_F R^* \sim 1$

- Case of equal masses:





- Well approximated by two quasiparticle branches

$$S(t) \simeq Z_{\text{att}}\, e^{-iE_{\text{att}}t} + Z_{\text{rep}}\, e^{-iE_{\text{rep}}t}$$

MMP & J. Levinsen, PRB 94, 184303 (2016)

IMPURITY IN A BEC

Jan Arlt
(Aarhus expt)

Jesper Levinsen
(Monash)

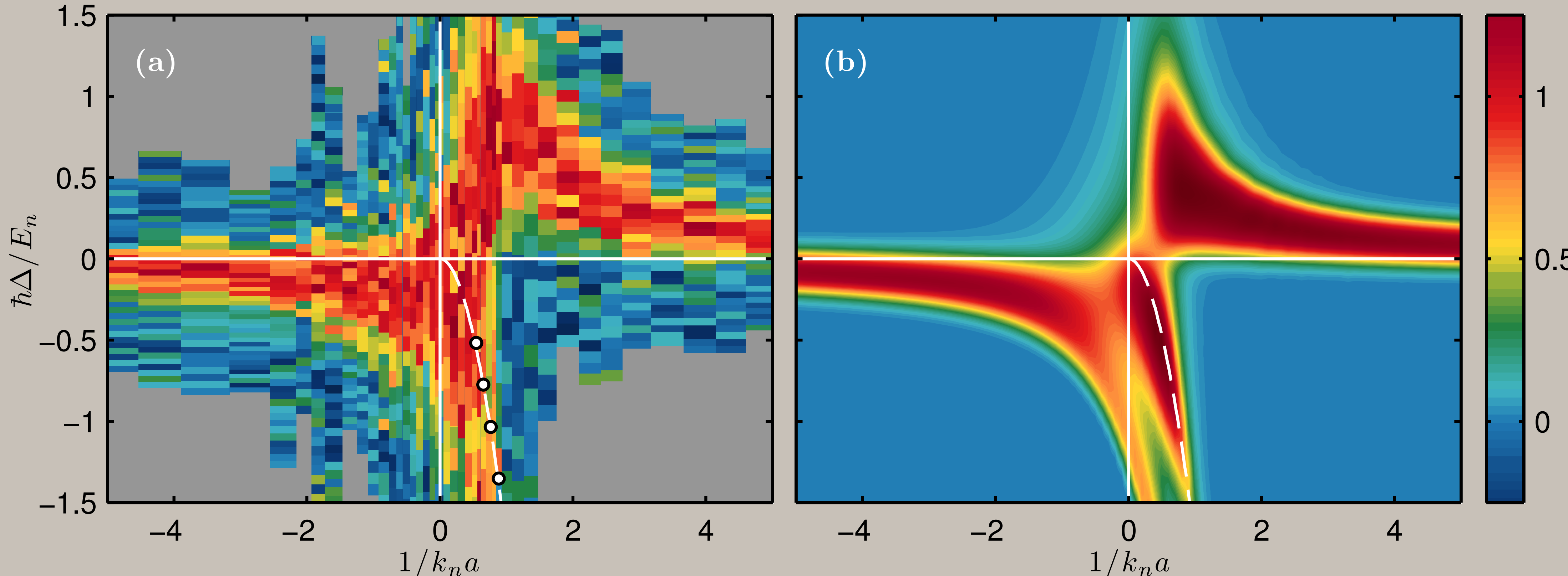
Georg Bruun
(Aarhus)

N. Jørgensen, …, MMP, J. Levinsen et al., Phys. Rev. Lett. 117, 055302 (2016)

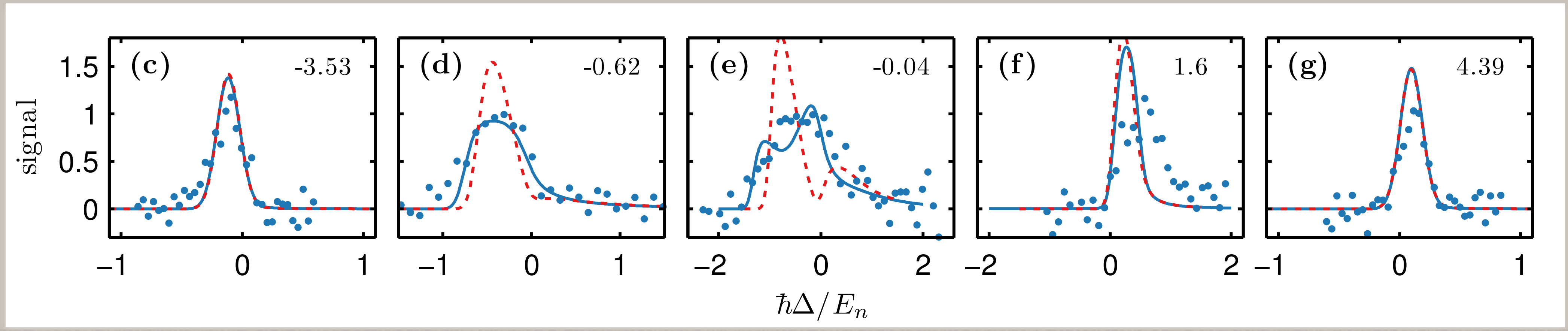
See also: M.-G. Hu et al., Phys. Rev. Lett. 117, 055301 (2016)

Observation of a Bose polaron

- Spectral response:



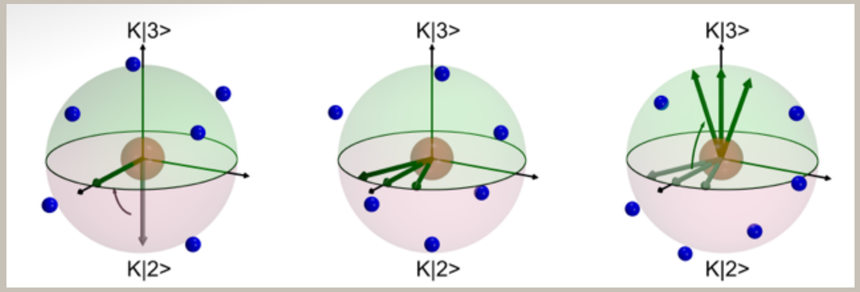
- Excellent agreement between theory and experiment



N. Jørgensen, …, MMP, J. Levinsen et al., Phys. Rev. Lett. 117, 055302 (2016)

Concluding remarks

- Cold-atom experiments can probe ultrafast non-equilibrium dynamics of fermionic systems
  - e.g. impurity problem



- Observe formation of quasiparticles and quantum interference of different many-body branches

- Accurate theoretical description involving only very few particle-hole excitations of Fermi sea

- Universal short-time dynamics governed by few-body physics

- Outlook: prepare and probe highly excited many-body states

MMP & J. Levinsen, PRB 94, 184303 (2016)

M. Cetina, …, J. Levinsen, MMP et al., Science 354, 96 (2016)