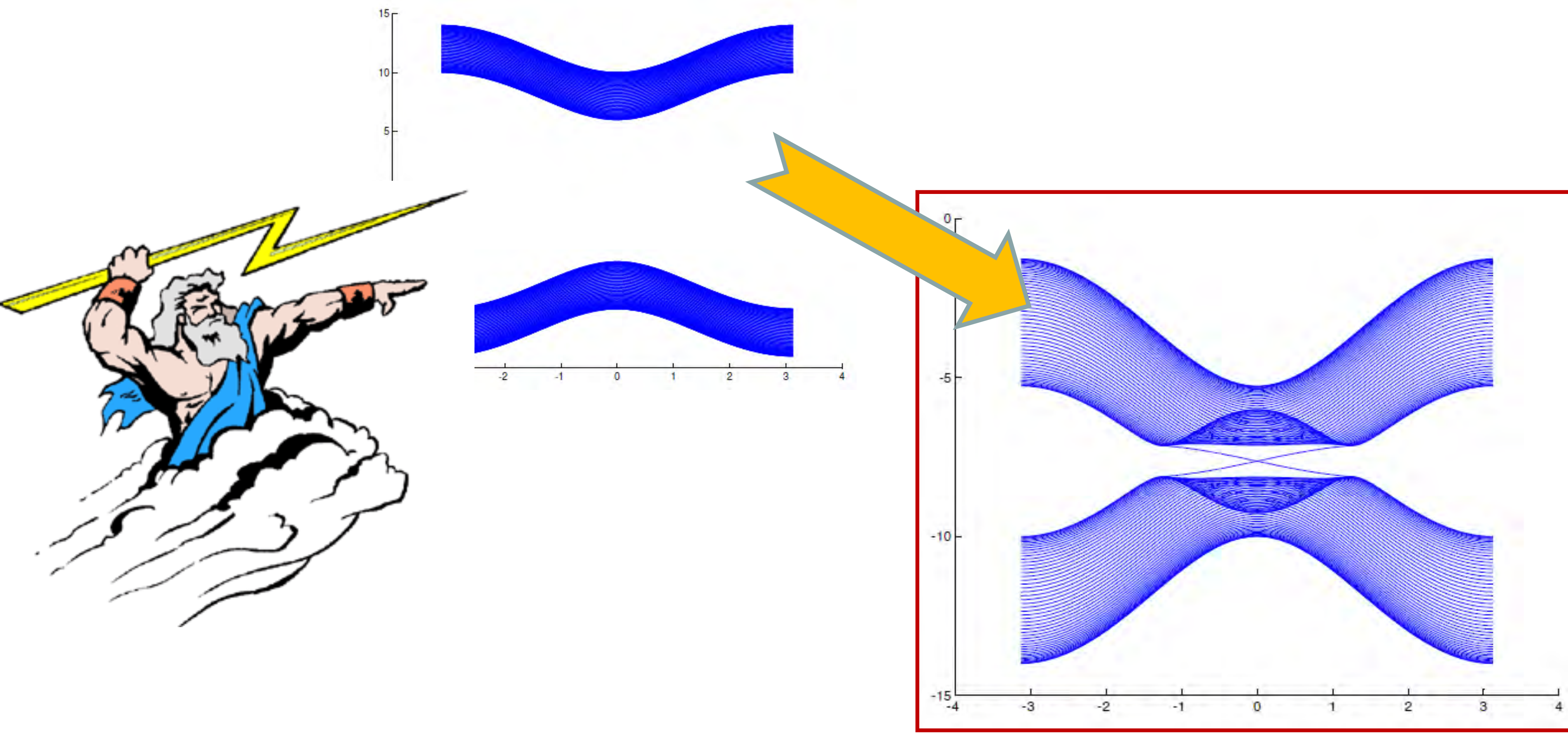
CALIFORNIA INSTITUTE OF TECHNOLOGY 1891

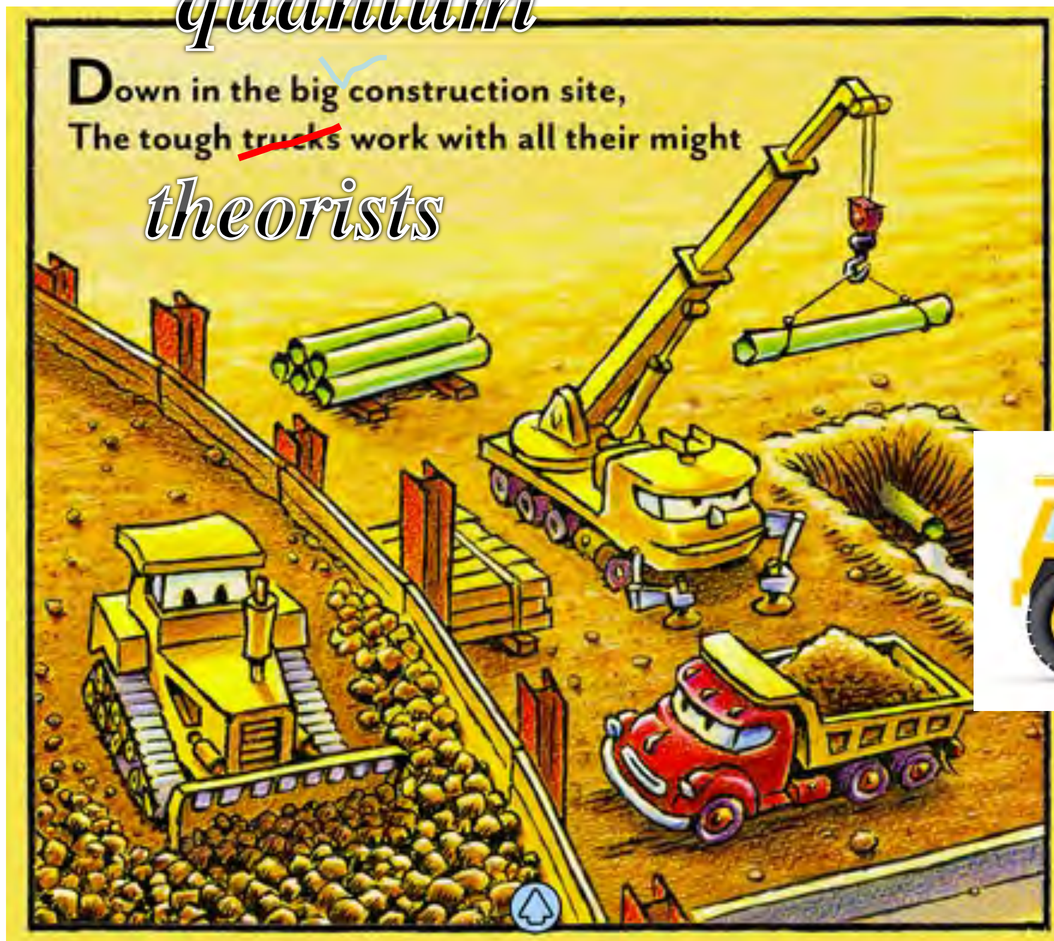
INSTITUTE FOR QUANTUM INFORMATION AND MATTER



Quantum Floquet Systems:
Topology, steady states, and more

Funding:
Darpa (FENA), NSF,
Packard foundation

Hamiltonian construction



$$\hat{H} = (hopping) + (interactions)$$

(taken from: Good night (quantum) construction site)

Hamiltonian construction

$$\hat{H} = (hopping) + (interactions) + (something\ missing...)$$

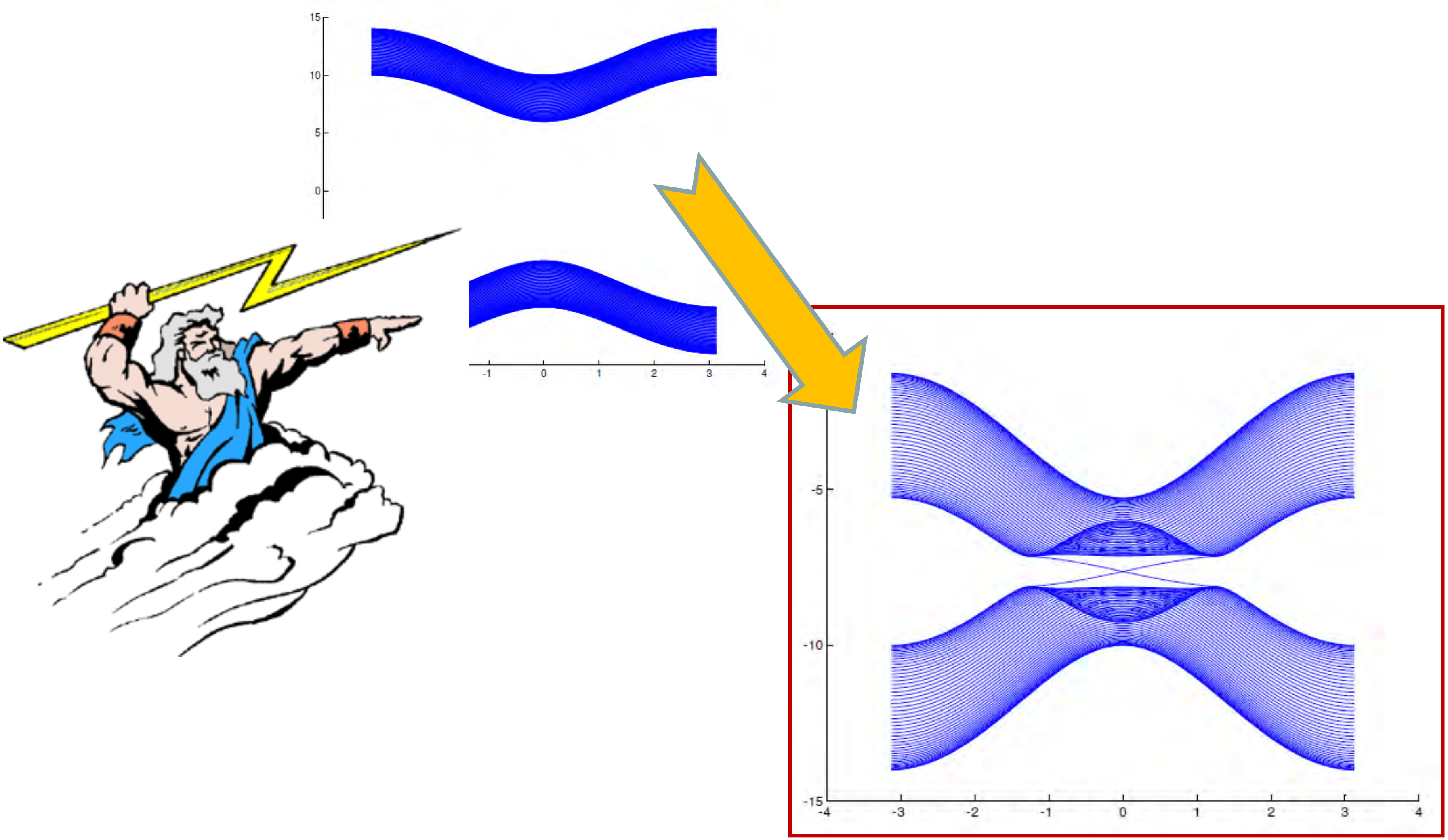
$$\hat{H} = (hopping) + (interactions)$$

Hamiltonian construction

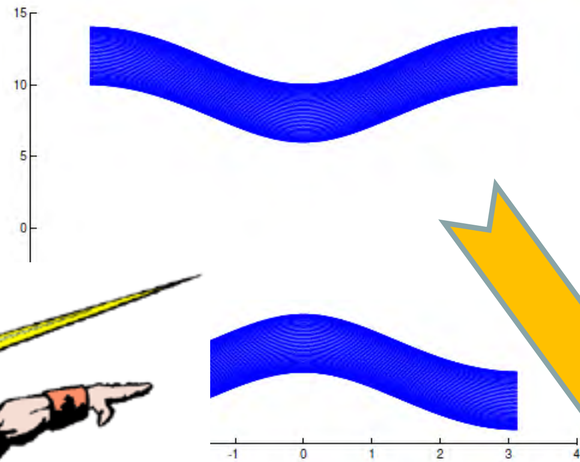
$$\hat{H} = (hopping) + (interactions) + \hat{F}(\cos \omega t)$$

Floquet topological phases?

Floquet topological phases

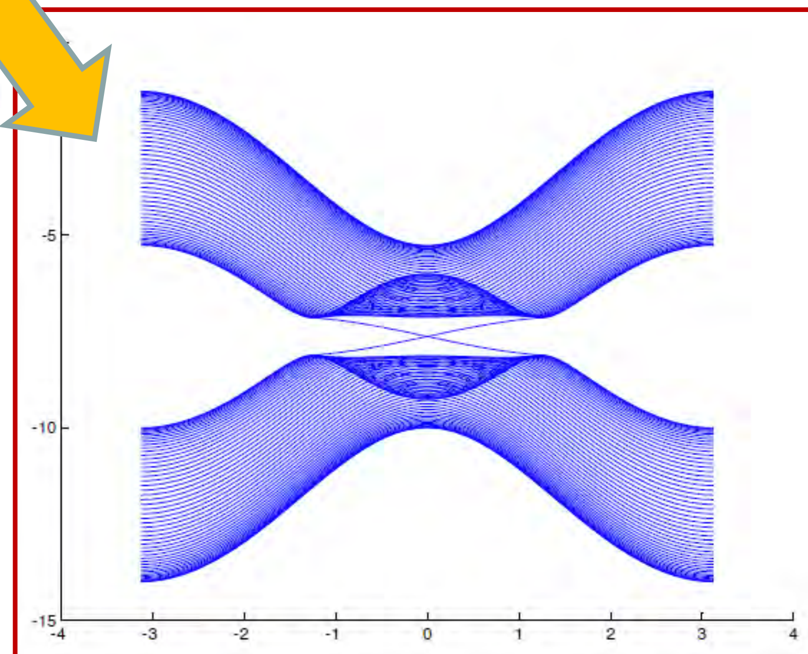


Floquet topological phases



Netanel Lindner
Technion

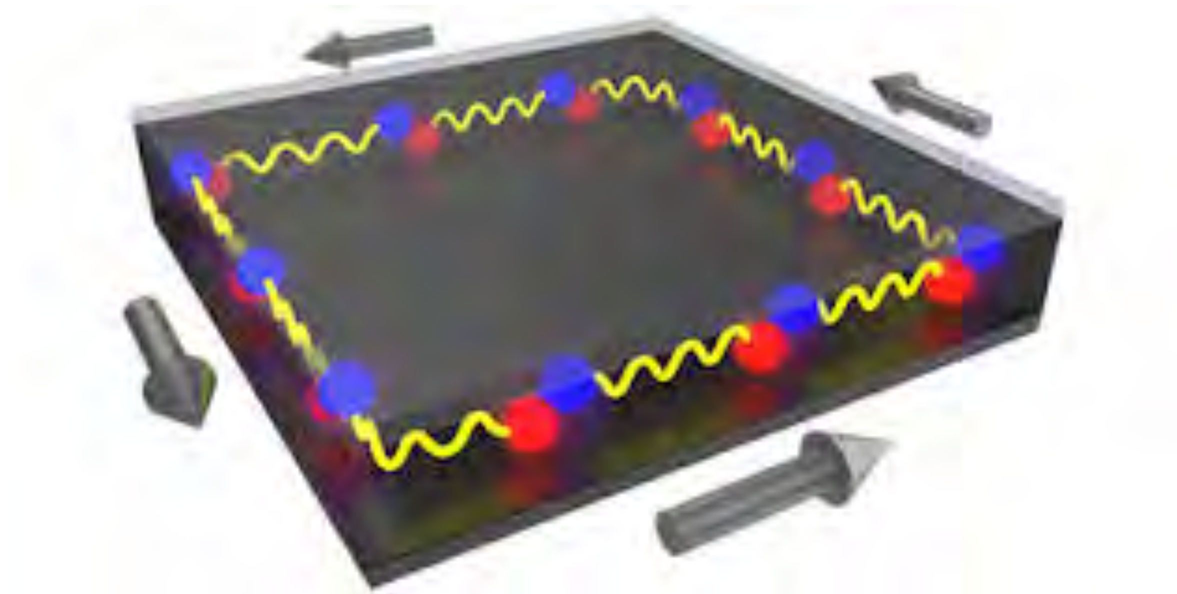
Victor Galitski
Maryland



Aoki, Oka (2009)
Lindner, Galitski, Refael (2010)
Kitagawa, Berg, Rudner, Demler (2010)
Jiang, Kitagawa,Akhmerov,Alicea,Refael, Pekker, Demler, Cirac, Zoller,Lukin (2011)
Grushin, Gomez-Leon, Neupert (2014)

Topological Polaritons

Karzig, Bardyn, Lindner, GR (2014)



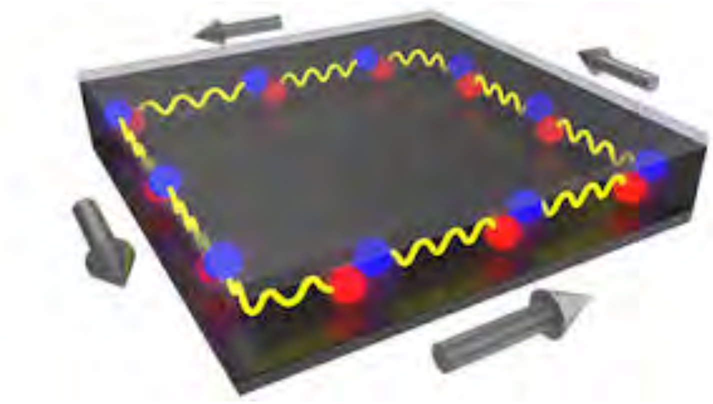
Torsten Karzig
Station Q

Charles Bardyn
Caltech

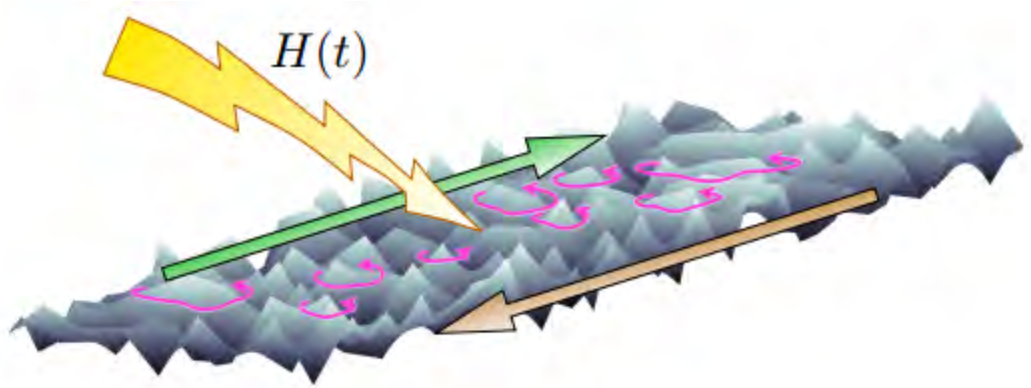
Netanel Lindner
Technion

Floquet-inspired new phenomena

- *Topolaritons:*



- *Anomalous Floquet Topological Anderson Insulator:*



Steady states?

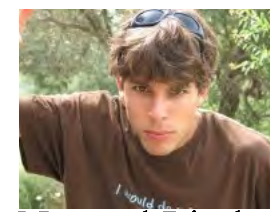
Driving → uncontrolled heating?

Steady states?

PRX, 2016

Karthik Seetharam
Caltech

Charles Bardyn
Caltech

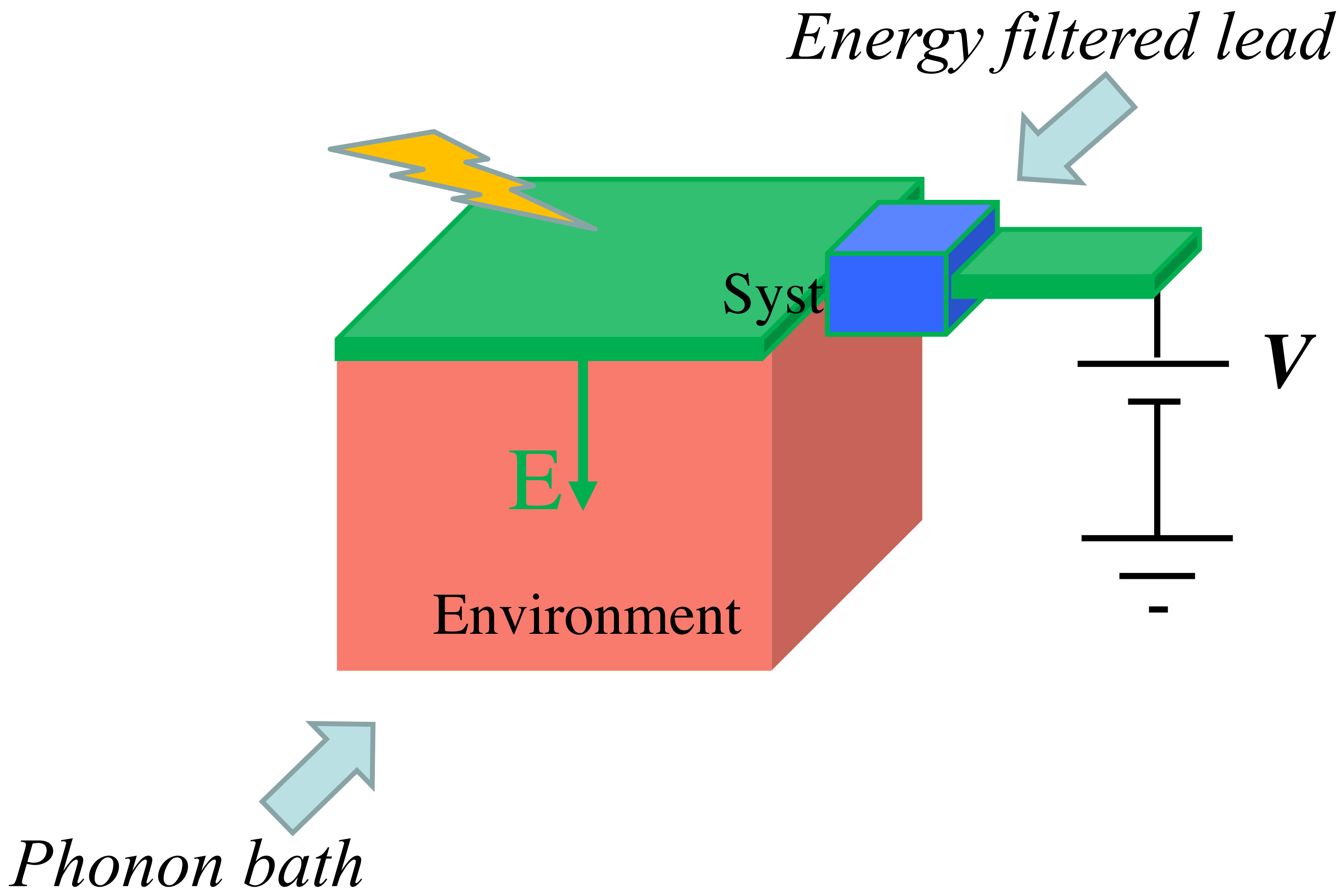


Netanel Lindner
Technion

Mark Rudner
Copenhagen

Dehghani, Oka, Mitra (2014)
Iadecola, Neupert, Chamon(2015)
Genske, Rosch (2015)
Bilitewski, Cooper (2014)
Chandran, Sondhi (2015)

Environment Engineering



Mathematical interlude: Floquet Hamiltonians

- Periodically varying Hamiltonian:

$$H = H_0 + V(t)$$

with $V(t) = V(t+T)$

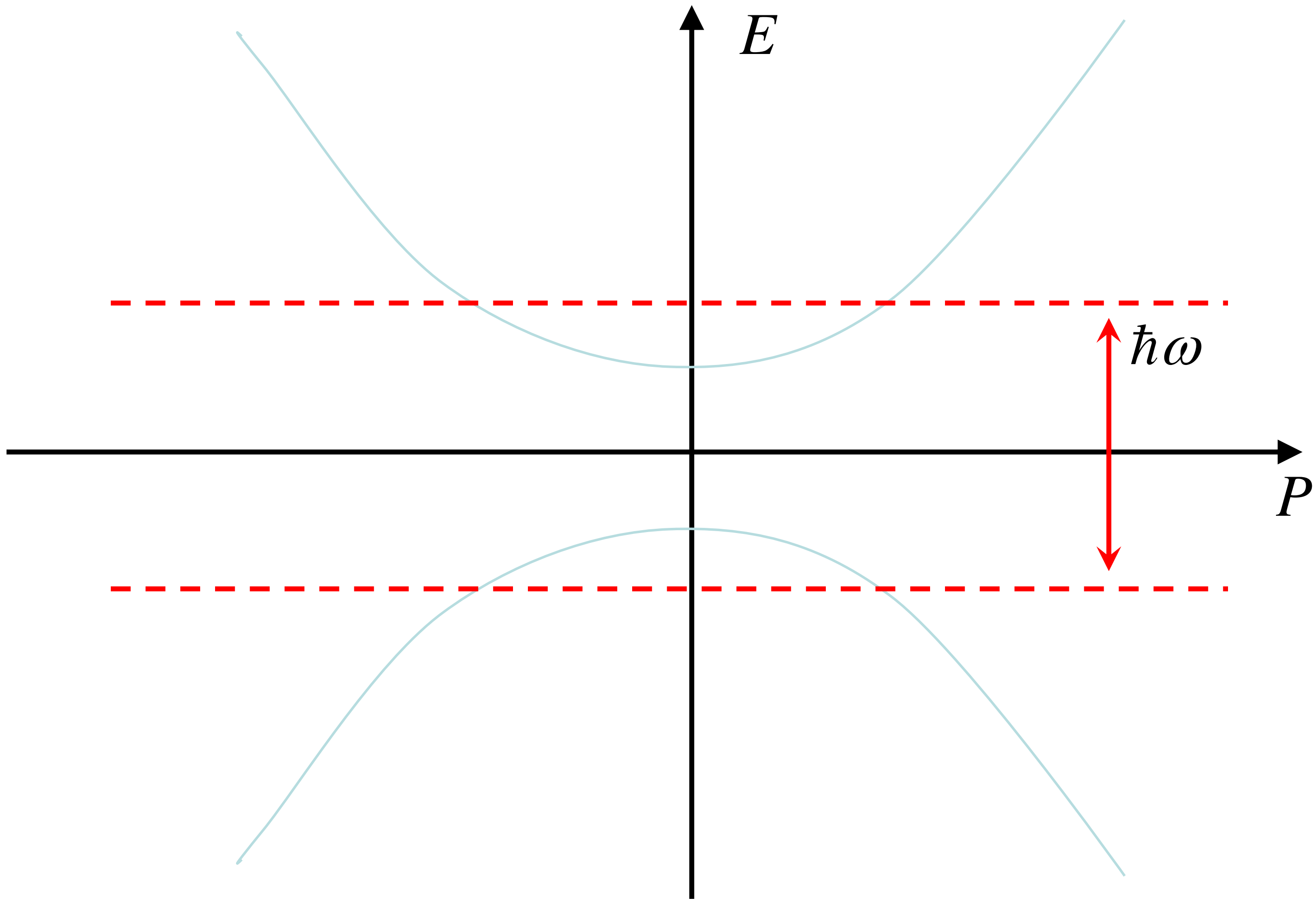
- Hamiltonian ➡ Floquet operator:

$$U(T) = \hat{\mathrm{T}} \exp\left(-i \int_0^T dt\, H(t) \right)$$

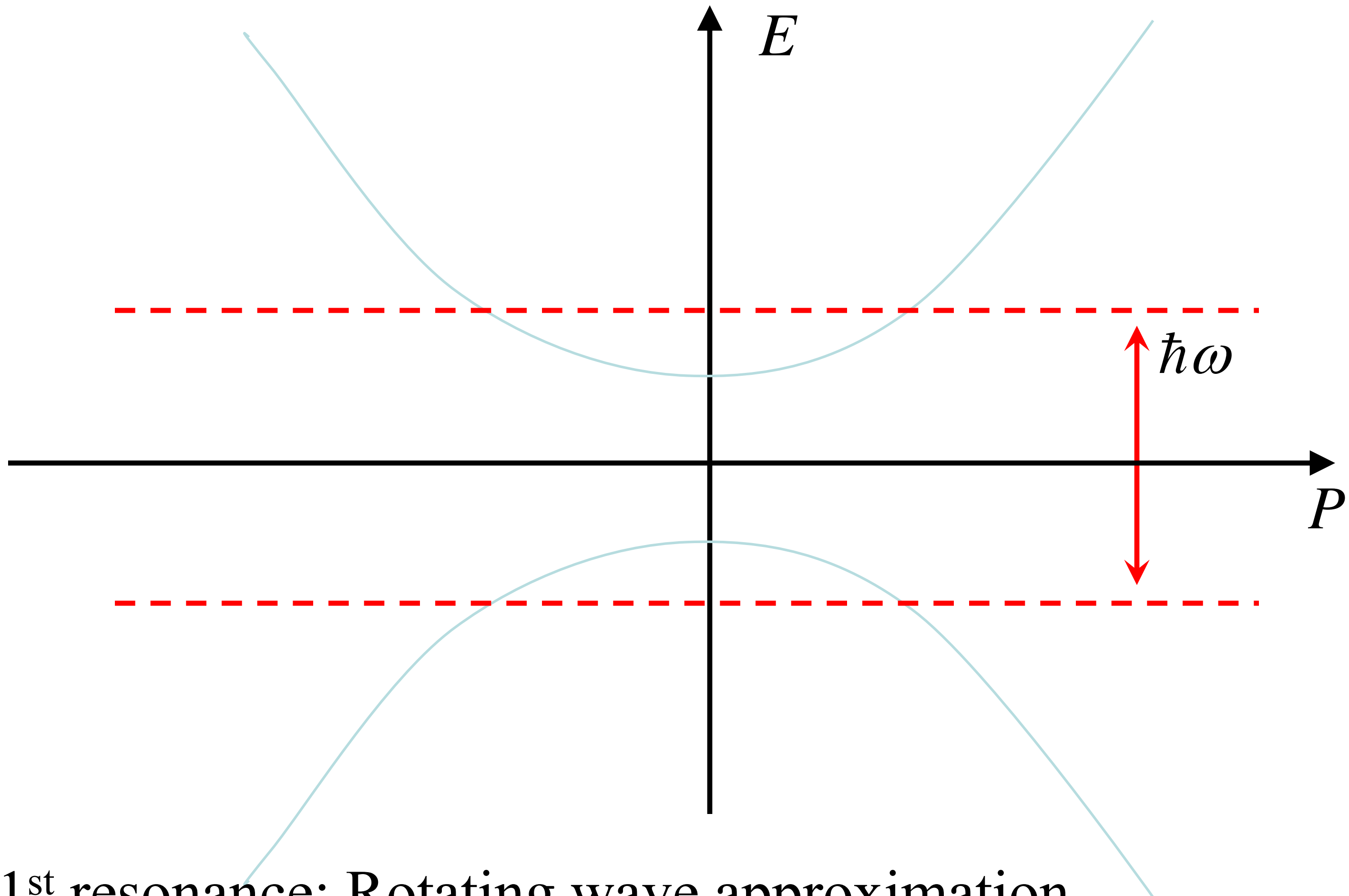
- Energy eigenstates ➡ Quasi-energy states:

$$U(T)\psi_n = e^{-i\varepsilon_n T}\psi_n \qquad \varepsilon_n \in [0,\ \omega] + \varepsilon_0 \qquad \omega = \frac{2\pi}{T}$$

Band structure folding and the Floquet zone

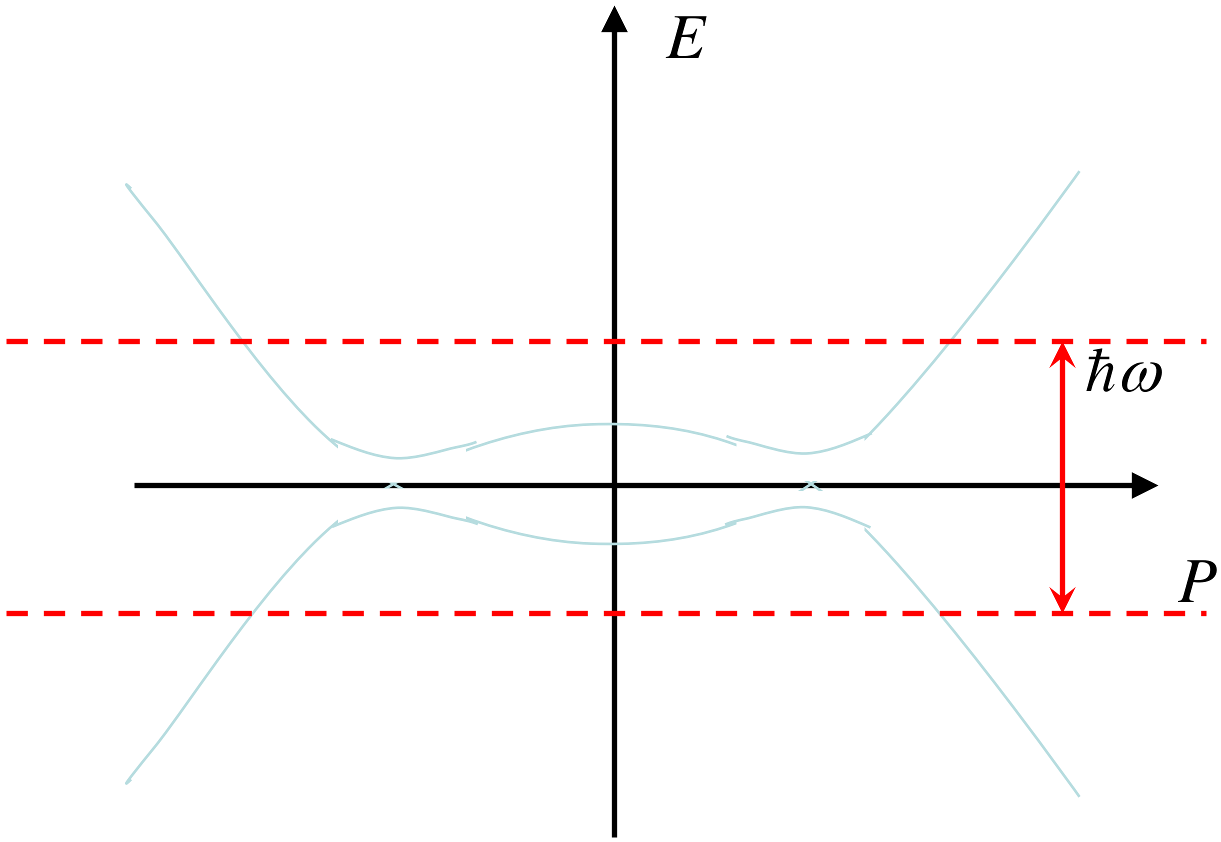


Band structure folding and the Floquet zone



(1) 1st resonance: Rotating wave approximation.

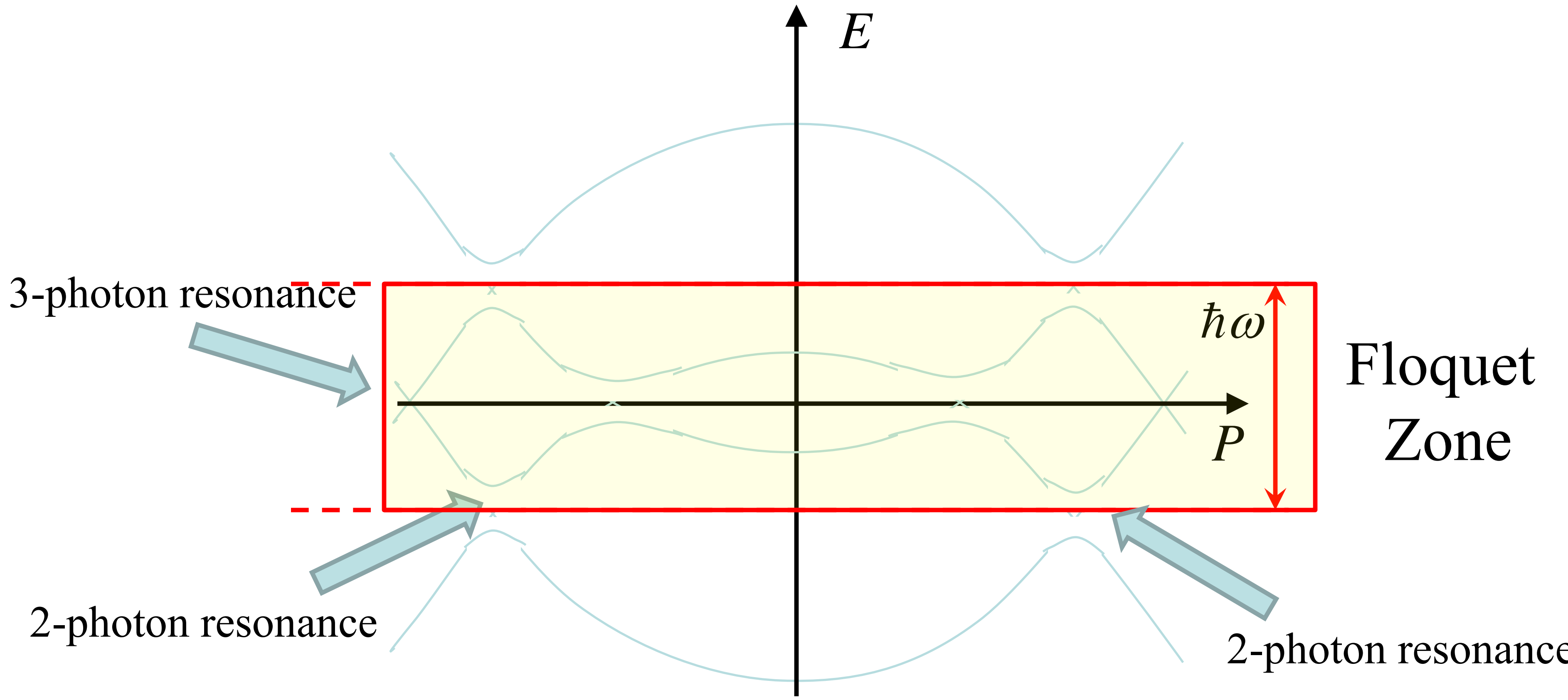
Band structure folding and the Floquet zone



(1) 1st resonance: Rotating wave approximation.

(2) Higher order resonances

Band structure folding and the Floquet zone

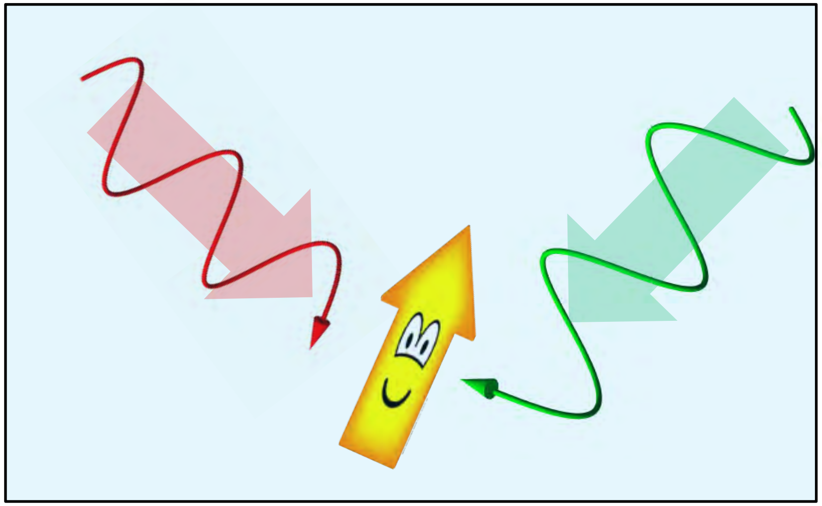


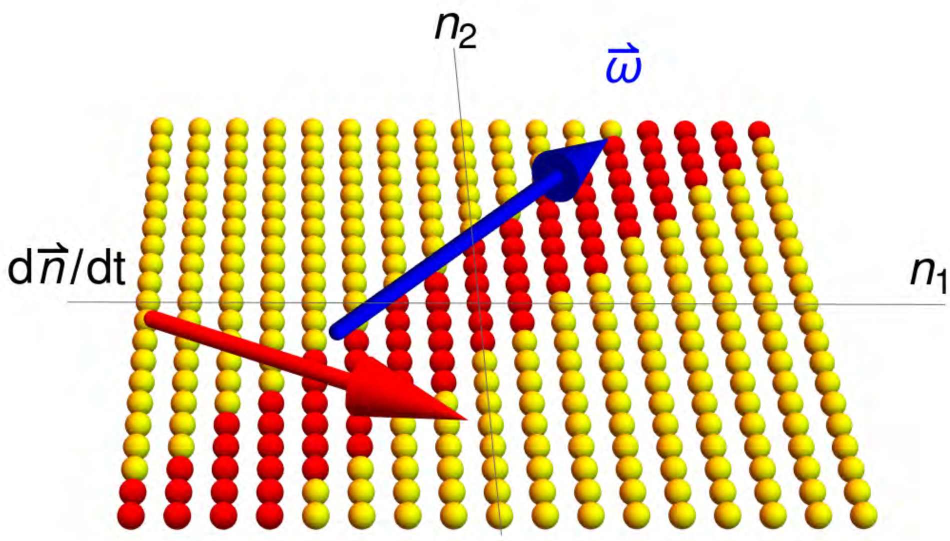
(1) 1st resonance: Rotating wave approximation.

(2) Higher order resonances

Floquet Double Drive:
2d Floquet and topological energy pumping

Ivar Martin (Argonne NL)
Frederik Nathan (Copenhagen)
Bertrand Halperin (Harvard)
Gil Refael (Caltech)

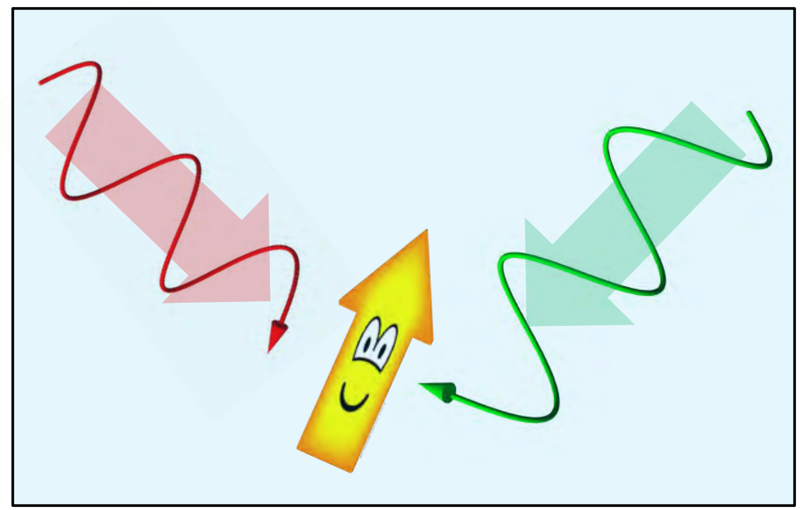




Overview

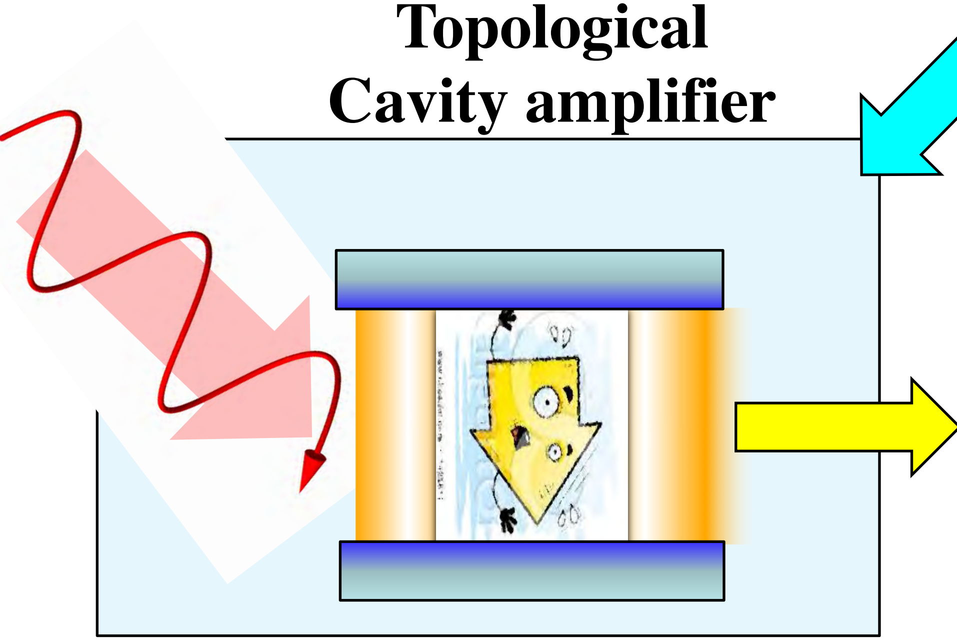
- Floquet intro *(Not again…)*
- topological phases

0-d topological drive:



**Quantized pumping
And frequency conversion**

**Topological
Cavity amplifier**



Floquet wave functions

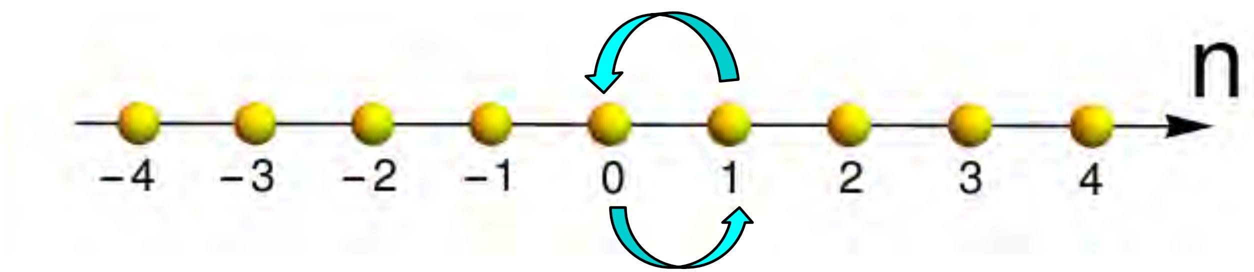
- Simple drive: $$H = \hat{H}_0 + \hat{V}\left(e^{i\omega t} + e^{-i\omega t}\right)$$

- Time evolving wave function:

$$|\psi(t)\rangle = \sum_n \exp(-in\omega t)|\psi_n(t)\rangle$$

Time dependent Schroedinger equation:

$$i\frac{\partial}{\partial t}|\psi_n(t)\rangle = \hat{H}_0|\psi_n\rangle + \hat{V}|\psi_{n+1}\rangle + \hat{V}|\psi_{n-1}\rangle - \omega n|\psi_n\rangle$$



Synthetic dimension – like a tight binding model

See also: Nathan Goldman; Shanhui Fan; Iacoppo Carussoto....

Photon space space vs. real space

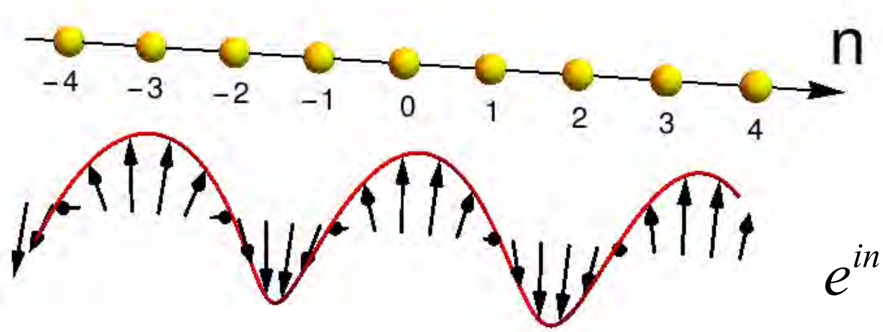
- Add phase to the drive:

$$H = \hat{H}_0 + \hat{V}\left(e^{i(\omega t+\phi)} + e^{-i(\omega t+\phi)}\right)$$

$$\left|\psi(t)\right\rangle = \sum_n \exp\left(in\omega t + in\phi\right)\left|\psi_n(t)\right\rangle$$

- Photon number operator?

$$\hat{n} = -i\frac{\partial}{\partial\phi} \quad \Longleftrightarrow \quad \hat{x} = -i\frac{\partial}{\partial k}$$



$e^{in\phi}$

Photon space space vs. real space

- Add phase to the drive:

$$H = \hat{H}_0 + \hat{V}\left(e^{i(\omega t+\phi)} + e^{-i(\omega t+\phi)}\right)$$

$$|\psi(t)\rangle = \sum_n \exp\left(in\omega t + in\phi\right)|\psi_n(t)\rangle$$

- Photon number operator?

$$\hat{n} = -i\frac{\partial}{\partial\phi} \quad \Longleftrightarrow \quad \hat{x} = -i\frac{\partial}{\partial k}$$

- Photon absorption rate?

$$\langle\dot{n}\rangle = \left\langle i\left[-i\frac{\partial}{\partial\phi}, H\right]\right\rangle \quad \Longleftrightarrow \quad \langle\dot{x}\rangle = i\left\langle\left[-i\frac{\partial}{\partial k}, H\right]\right\rangle$$

Phase vs. Momentum

- Why not identify momentum with drive-phase?

$$|\psi(t)\rangle = \sum_n \exp\left(ink_{(t)}\right)|\psi_n(t)\rangle \qquad k(t) = \omega t + \phi$$

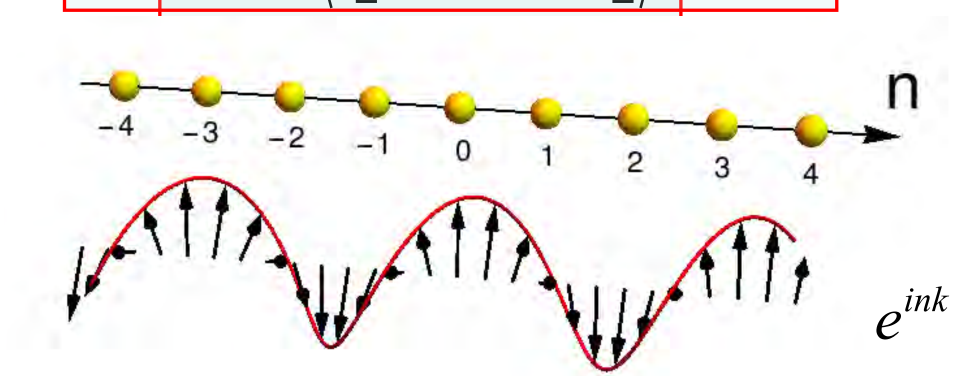
$$H = H_0 + V\left(e^{ik} + e^{-ik}\right) - \omega n$$

$$\hat{H} = \hat{H}_0 + \hat{V}\left(e^{ik} + e^{-ik}\right)$$

$$H(k)|\psi_k\rangle = \varepsilon_k|\psi_k\rangle$$

$$\langle \dot{n} \rangle = \left\langle i\left[\frac{\partial}{\partial k}, H(k)\right]\right\rangle \rightarrow \frac{\partial \varepsilon_k}{\partial k}$$

$$\langle \dot{n} \rangle = \left\langle i\left[-i\frac{\partial}{\partial \phi}, H\right]\right\rangle$$



Caveats

- Drive implies constant momentum change:

$$H = \hat{H}_0 + \hat{V}\left(e^{ik} + e^{-ik}\right) - \omega n \quad \Rightarrow \quad k(t) = \omega t + \phi$$

Can't access all momentum states.

- Dimension of Hilbert space = $\dim(H_0)$

Caveats

- Drive implies constant momentum change:

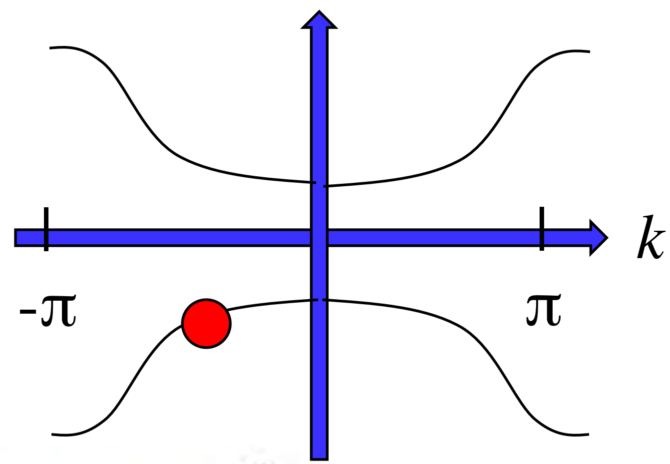
$$H = \hat{H}_0 + \hat{V}\left(e^{ik} + e^{-ik}\right) - \omega n \quad \Rightarrow \quad k(t) = \omega t + \phi$$

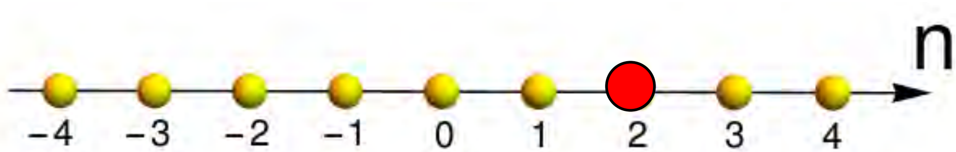
Can't access all momentum states.

- Dimension of Hilbert space = $\dim(H_0)=2$

$$H = h\sigma^z + V\left(e^{ik} + e^{-ik}\right)\sigma^x - \omega n$$

$$|\psi(t)\rangle = \psi_\uparrow(t)|\uparrow\rangle + \psi_\downarrow(t)|\downarrow\rangle$$





Caveats

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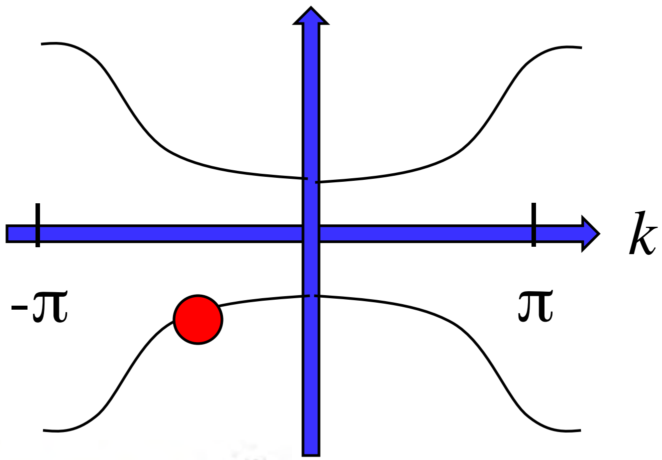
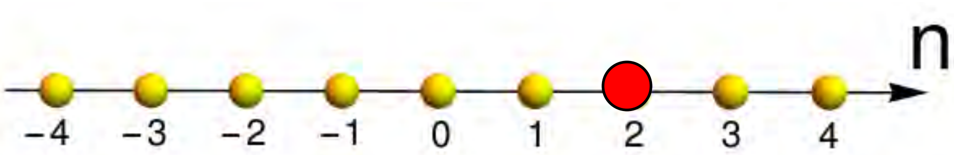




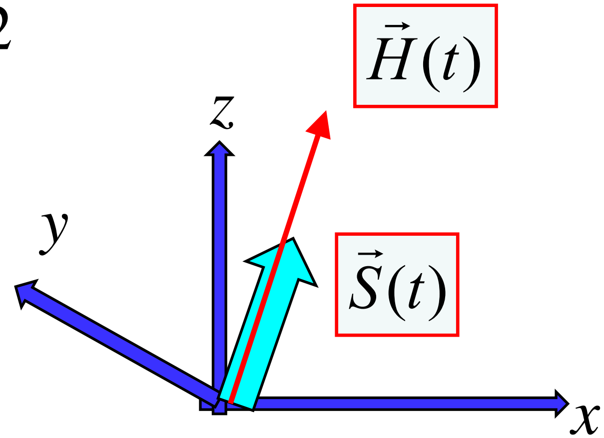
Caveats

- Drive implies constant momentum change:

$$H = h\sigma^z + V\sigma^x\left(e^{ik} + e^{-ik}\right) - \omega n \quad \Rightarrow \quad k(t) = \omega t + \phi$$

Can't access all momentum states.

- Dimension of Hilbert space = dim(H_0)=2
- Adiabatic evolution - if $\omega << h$



$$\left|\psi(0)\right\rangle = u_+\left|\psi_\phi,+\right\rangle + u_-\left|\psi_\phi,-\right\rangle$$

$$\left|\psi(t)\right\rangle = e^{-i\int dt\,\varepsilon_{k(t)}}u_+\left|\psi_{k(t)},+\right\rangle + e^{i\int dt\,\varepsilon_{k(t)}}u_-\left|\psi_{k(t)},-\right\rangle$$

Higher dimensional analogy?

- 2d and higher dimensional systems also possible:

$$H = H_0 + \hat{V}_1 \cos(\omega_1 t + \phi_1) + \hat{V}_2 \cos(\omega_2 t + \phi_2)$$

$$H = H(k_1, k_2) - n_1\omega_1 - n_2\omega_2$$

$$|\psi(t)\rangle = \sum_n \exp(in_1\omega_1 t + in_2\omega_2 t)|\psi_{n_1,n_2}(t)\rangle$$

And tight-binding effective Hamiltonian:

$$i\frac{\partial}{\partial t}|\psi_{n_1 n_2}(t)\rangle = \hat{H}_0|\psi_{n_1 n_2}\rangle + \hat{V}_1\left(|\psi_{n_1+1,n_2}\rangle e^{-i\phi_1} + |\psi_{n_1-1,n_2}\rangle e^{i\phi_1}\right)$$
$$+\hat{V}_2\left(|\psi_{n_1,n_2+1}\rangle e^{-i\phi_2} + |\psi_{n_1,n_2-1}\rangle e^{i\phi_2}\right) + (\omega_1 n_1 + \omega_2 n_2)|\psi_{n_1 n_2}\rangle$$

Higher dimensional analogy?

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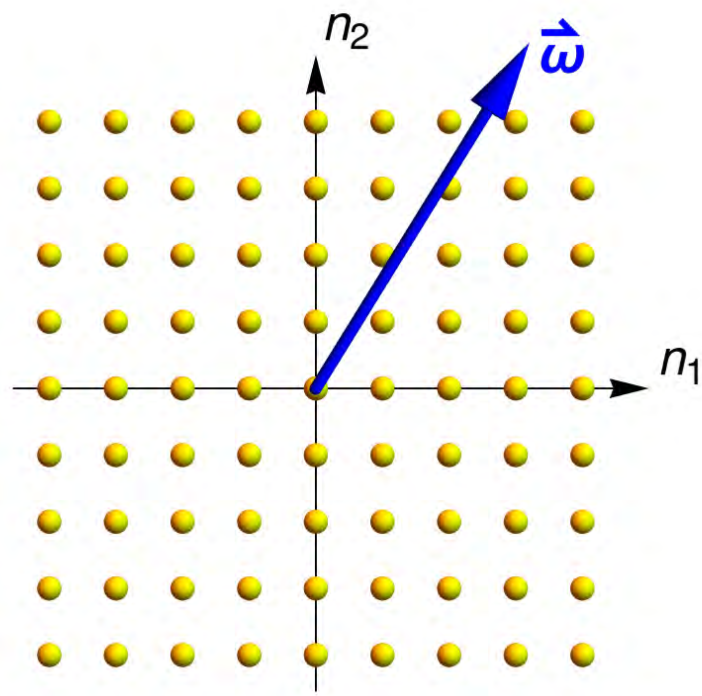
$$H = H_0 + \hat{V}_1 \cos(\omega_1 t + \phi_1) + \hat{V}_2 \cos(\omega_2 t + \phi_2)$$

$$H = H(k_1, k_2) - n_1\omega_1 - n_2\omega_2$$

$$|\psi(t)\rangle = \sum_n \exp(i n_1 \omega_1 t + i n_2 \omega_2 t) |\psi_{n_1,n_2}(t)\rangle$$

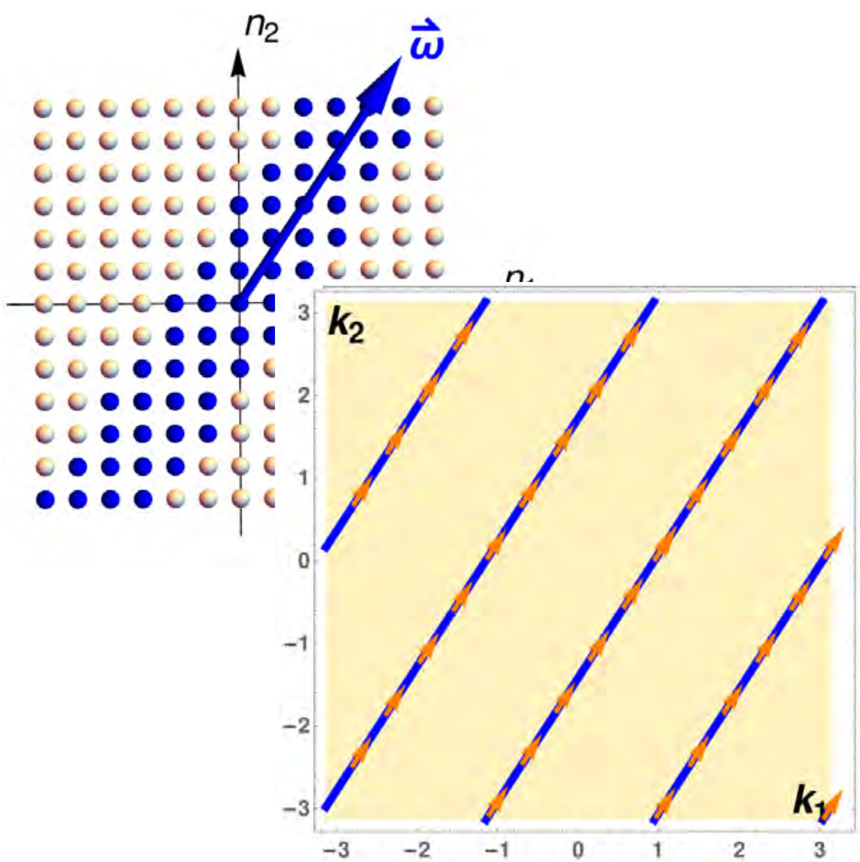


Commensurate vs incommensurate

$$H = H_0 + \hat{V}_1 \cos(\omega_1 t + \phi_1) + \hat{V}_2 \cos(\omega_2 t + \phi_2)$$

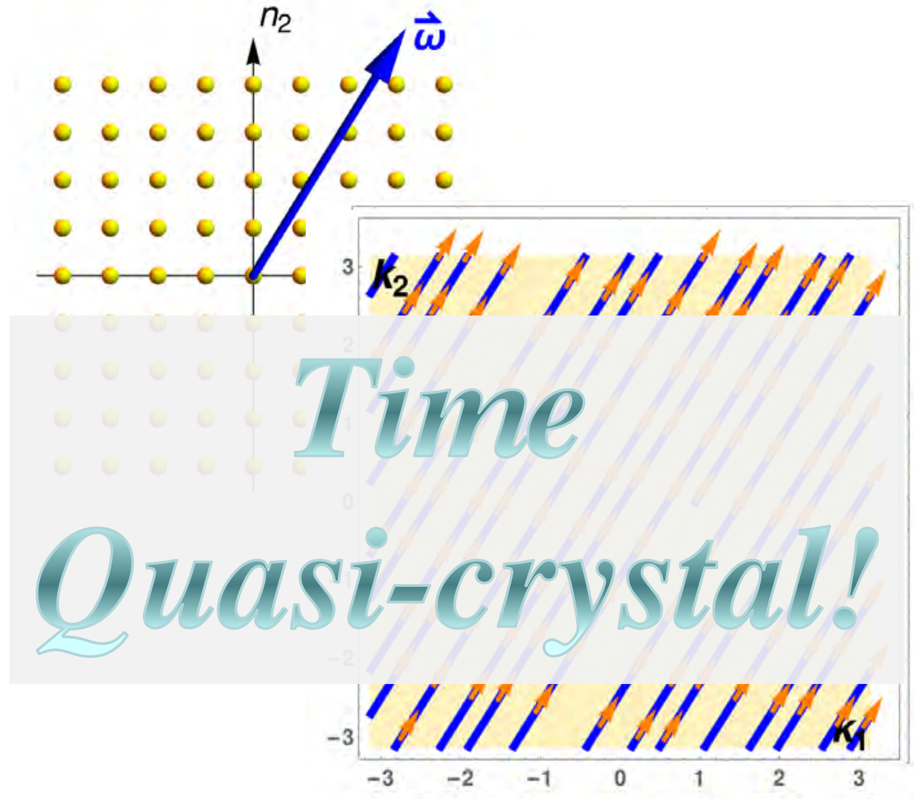
commensurate

$$\omega_1 / \omega_2 = p / q = 2/3$$



Incommensurate

$$\omega_2 = \omega_1 (\sqrt{5} + 1)/2$$



Time Quasi-crystal!

Topological synthetic Floquet phases

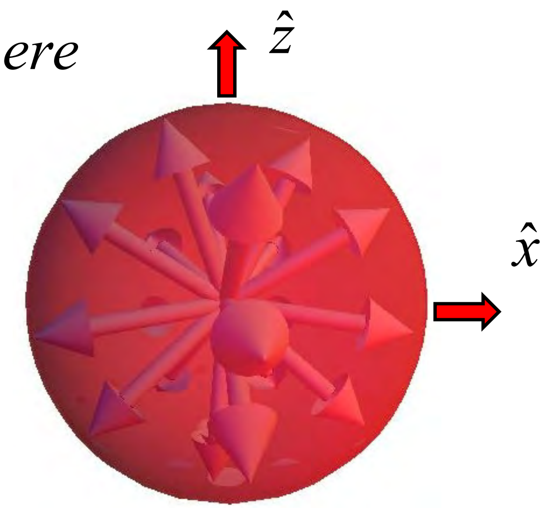
- Use BHZ band structure:

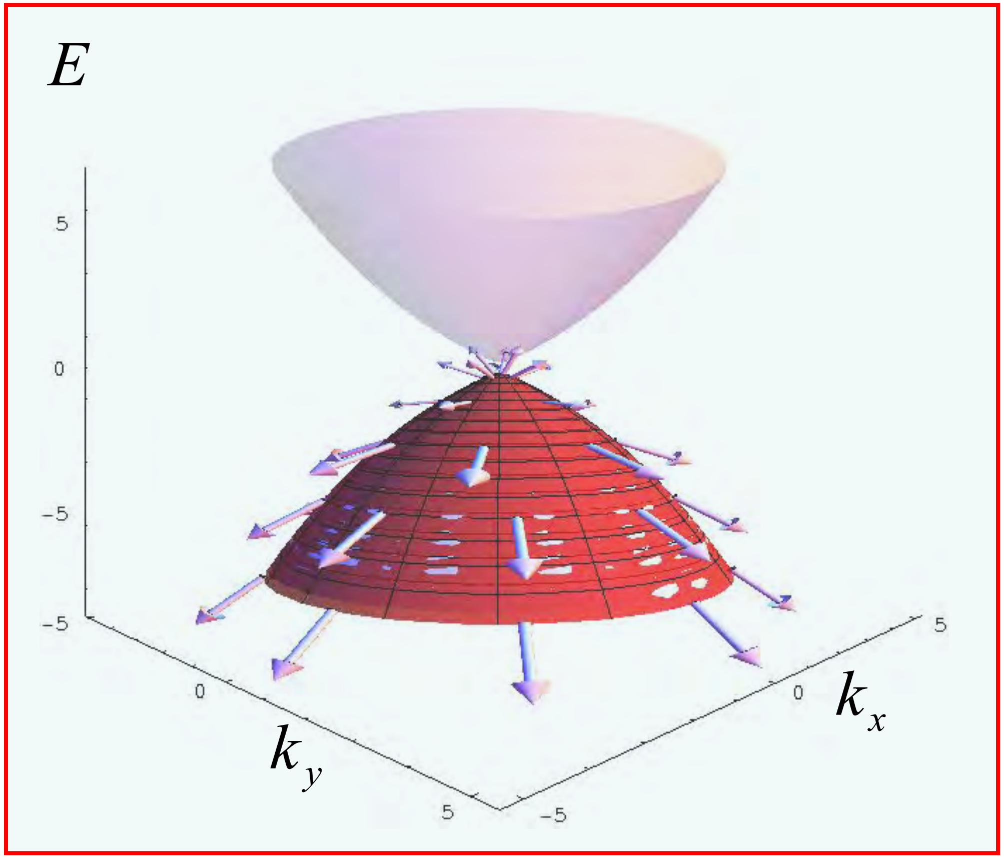
$$H = v_1\sigma^x \sin(k_1) + v_2\sigma^y \sin(k_2) + [m - b_1\cos(k_1) - b_2\cos(k_2)]\sigma^z$$

- BHZ magic: $H = \vec{\sigma}\cdot\vec{d}_{\vec{k}}$ $\hat{n} = \vec{d}/|\vec{d}|$

$$\sigma_{xy} = \frac{e^2}{4\pi h}\int_{p\in BZ} dk_x dk_y \;\hat{n}\cdot\left(\frac{\partial\hat{n}}{\partial k_x}\times\frac{\partial\hat{n}}{\partial k_y}\right)$$

$\hat{n} - sphere$





Topological synthetic Floquet phases

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$$H = v_1 \sigma^x \sin(k_1) + v_2 \sigma^y \sin(k_2) + \left[m - b_1 \cos(k_1) - b_2 \cos(k_2)\right]\sigma^z$$

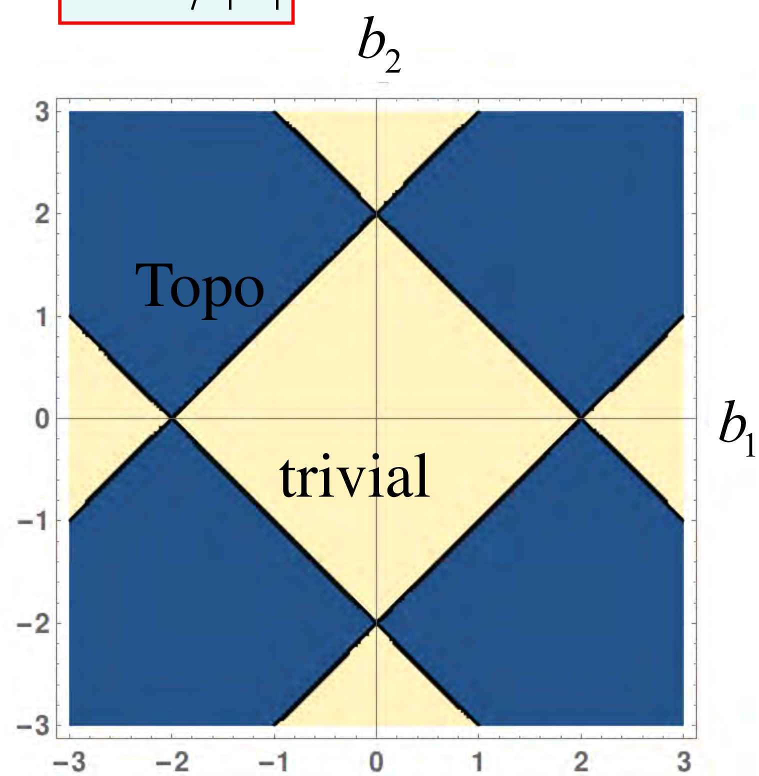
- BHZ magic: $H = \vec{\sigma} \cdot \vec{d}_{\vec{k}}$

$$\hat{n} = \vec{d} / \left|\vec{d}\right|$$

$$\sigma_{xy} = \frac{e^2}{4\pi h} \int_{k \in BZ} dk_x dk_y \; \hat{n} \cdot \left(\frac{\partial \hat{n}}{\partial k_x} \times \frac{\partial \hat{n}}{\partial k_y} \right)$$

$$\Omega(k_1, k_2) = \nabla_k \times A_k$$

Berry curvature



$m = 2$

Topological synthetic Floquet phases

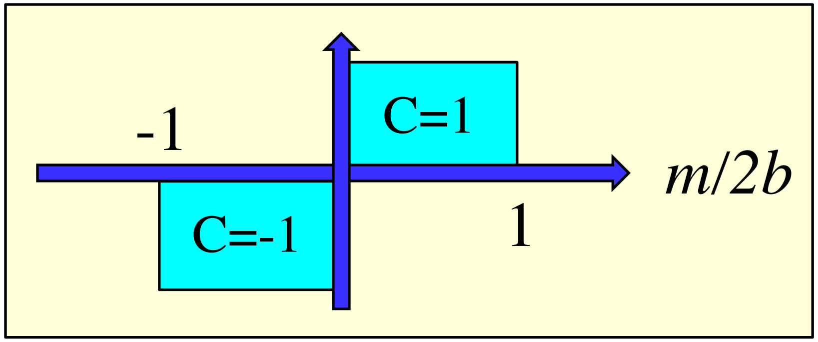
- Use BHZ band structure:

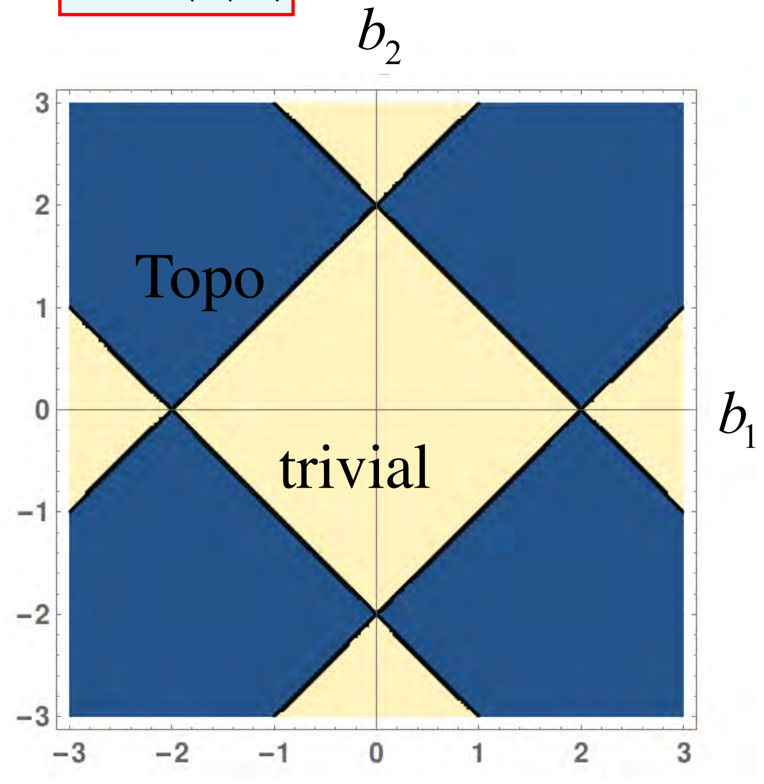
$$H = v_1\sigma^x \sin(k_1) + v_2\sigma^y \sin(k_2) + \left[m - b_1\cos(k_1) - b_2\cos(k_2)\right]\sigma^z$$

- BHZ magic: $H = \vec{\sigma}\cdot\vec{d}_{\vec{k}}$

$$\hat{n} = \vec{d}/\left|\vec{d}\right|$$

$$\sigma_{xy} = \frac{e^2}{4\pi h}\int_{k\in BZ} dk_x dk_y\ \hat{n}\cdot\left(\frac{\partial\hat{n}}{\partial k_x}\times\frac{\partial\hat{n}}{\partial k_y}\right)$$





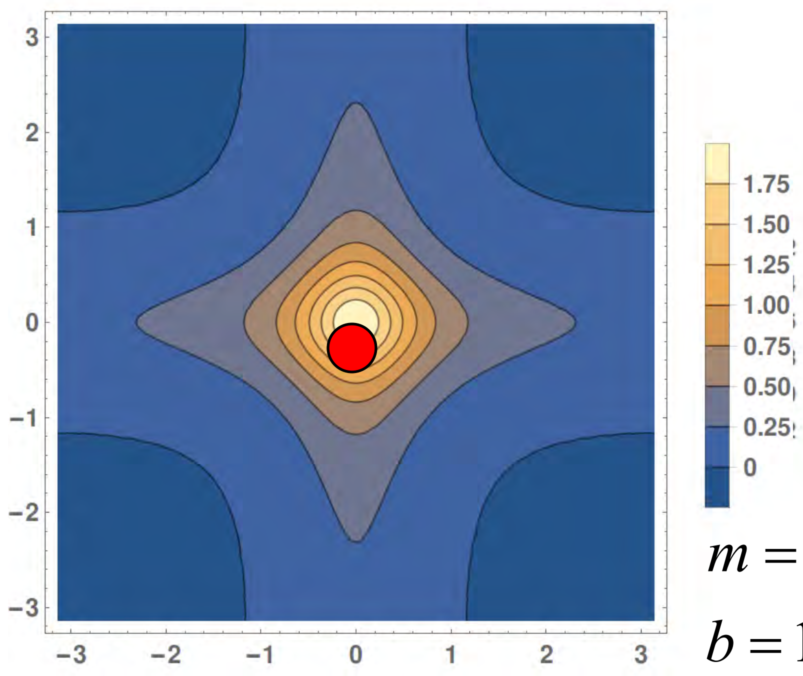
Semiclassical motion

- Berry curvature

$$\vec{A}_k = -i\langle \psi_k | \nabla_{\vec{k}} | \psi_k \rangle \qquad \Omega(k_1, k_2) = \nabla_k \times A_k$$

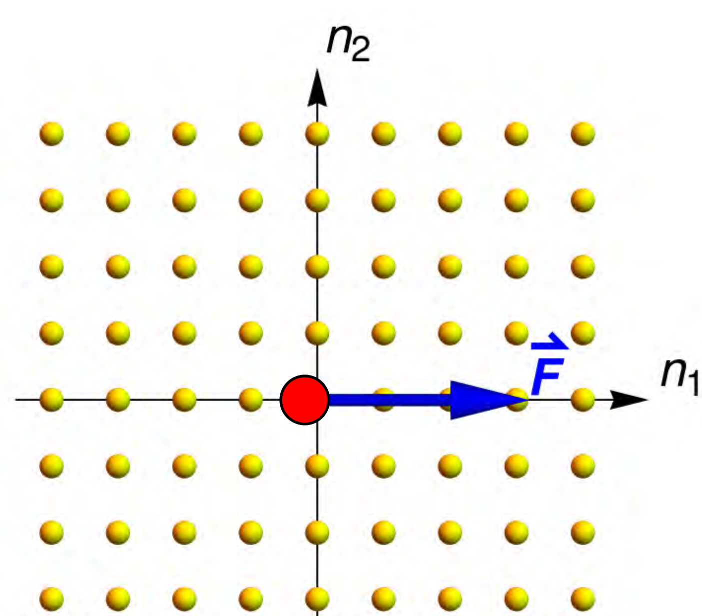
$$\frac{d\vec{r}}{dt} = \nabla_k \varepsilon_k + \Omega_{\vec{k}} \times \frac{d\vec{k}}{dt}$$

- BHZ Berry curvature and motion:



$m = 1.5$

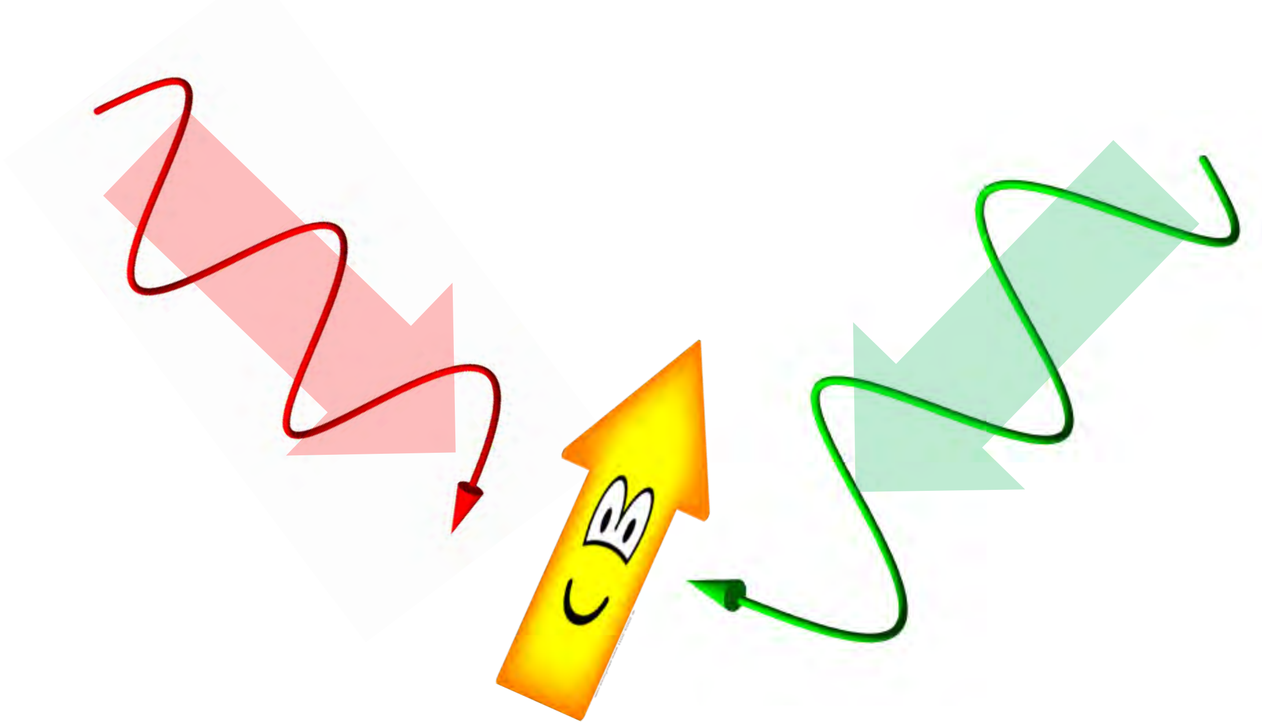
$b = 1$



Topological synthetic Floquet phases

- 2f Floquet BHZ:

$$H = v_1 \sigma^x \sin(\omega_1 t + \phi_1) + v_2 \sigma^y \sin(\omega_2 t + \phi_2) + \left[m - b_1 \cos(\omega_1 t + \phi_1) - b_2 \cos(\omega_2 t + \phi_2)\right]\sigma^z$$



Synthetic 2f BHZ

- 2f Floquet BHZ:

$$H = v_1\sigma^x \sin(\omega_1 t + k_1) + v_2\sigma^y \sin(\omega_2 t + k_2) + \left[m - b_1\cos(\omega_1 t + k_1) - b_2\cos(\omega_2 t + k_2)\right]\sigma^z$$

- Semiclassical motion:

$$\vec{A}_k = -i\langle\psi_k|\nabla_{\vec{k}}|\psi_k\rangle \qquad \Omega(k_1,k_2) = \nabla_k \times A_k \qquad \frac{d\vec{k}}{dt} = \vec{\omega}$$

$$\frac{dn_i}{dt} = \overline{\Omega}\varepsilon^{ij}\omega_j = \frac{C}{2\pi}\varepsilon^{ij}\omega_j$$

$$\frac{d\vec{r}}{dt} = \nabla_{\vec{k}}\varepsilon_k + \Omega_{\vec{k}} \times \frac{d\vec{k}}{dt}$$

Quantized σ_{xy}

$$\overline{\Omega} = \frac{1}{2\pi}C$$

Quantized energy pumping

$$\frac{dW_{1,2}}{dt} = \pm\overline{\Omega}\omega_1\omega_2 = \pm\frac{C}{2\pi^2}\omega_1\omega_2$$

Quantized energy pumping

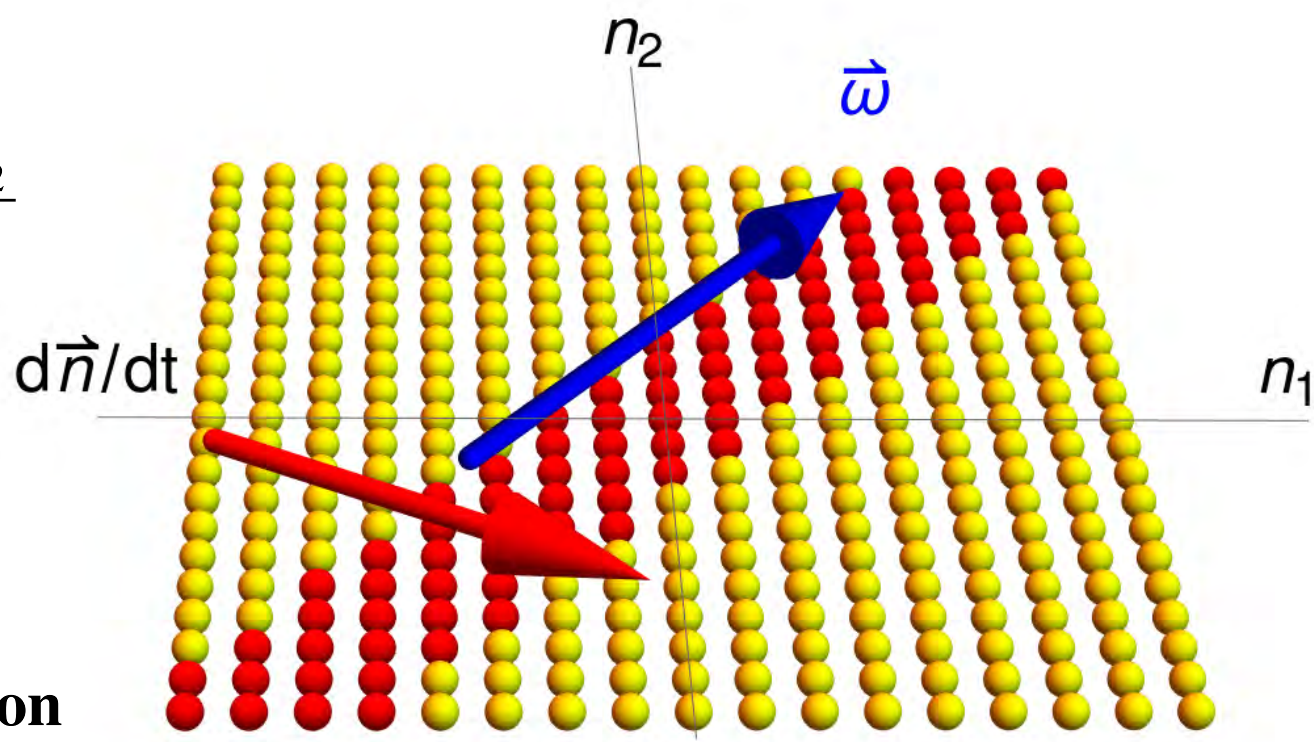
$$\overline{\Omega} = \frac{1}{2\pi} C$$

$$\frac{dE_{1,2}}{dt} = \pm \overline{\Omega}\, \omega_1 \omega_2 = \pm \frac{C}{2\pi} \omega_1 \omega_2$$

Quantized σ_{xy} → Quantized energy pumping

$$\omega_1 \frac{dn_1}{dt} = \omega_2 \frac{dn_2}{dt}$$

Energy pumped
between lasers:
Topological
frequency conversion



Quantization - if all BZ is covered…

Simulations

- Need to calculate:

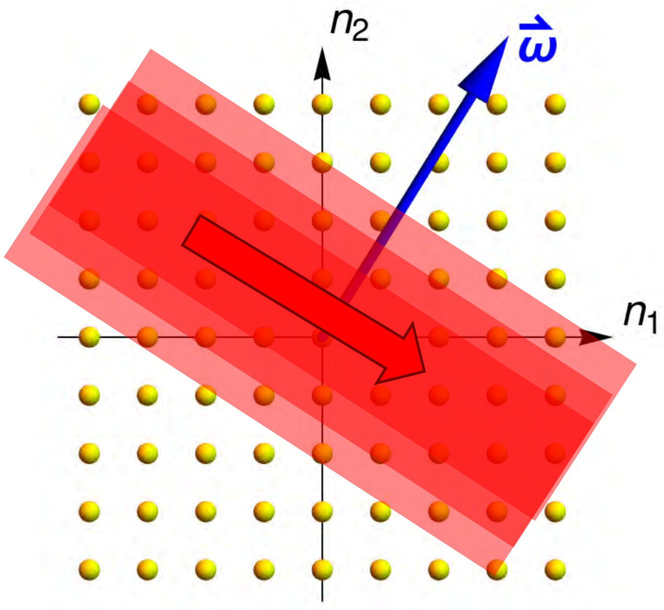
$$W_i = \int_0^t dt \langle \psi(t) | \frac{\partial H_i}{\partial t} | \psi(t) \rangle$$

$$|\psi(t)\rangle = T\left[e^{-i\int_0^t H(t)dt}\right]|\psi(0)\rangle$$

- Parameter regime?

Strong driving: $\omega_i << b, m$

Adiabatic evolution: $H(t)|\psi(t)\rangle = -|\psi(t)\rangle \varepsilon_{k(t)}$



- Initializing

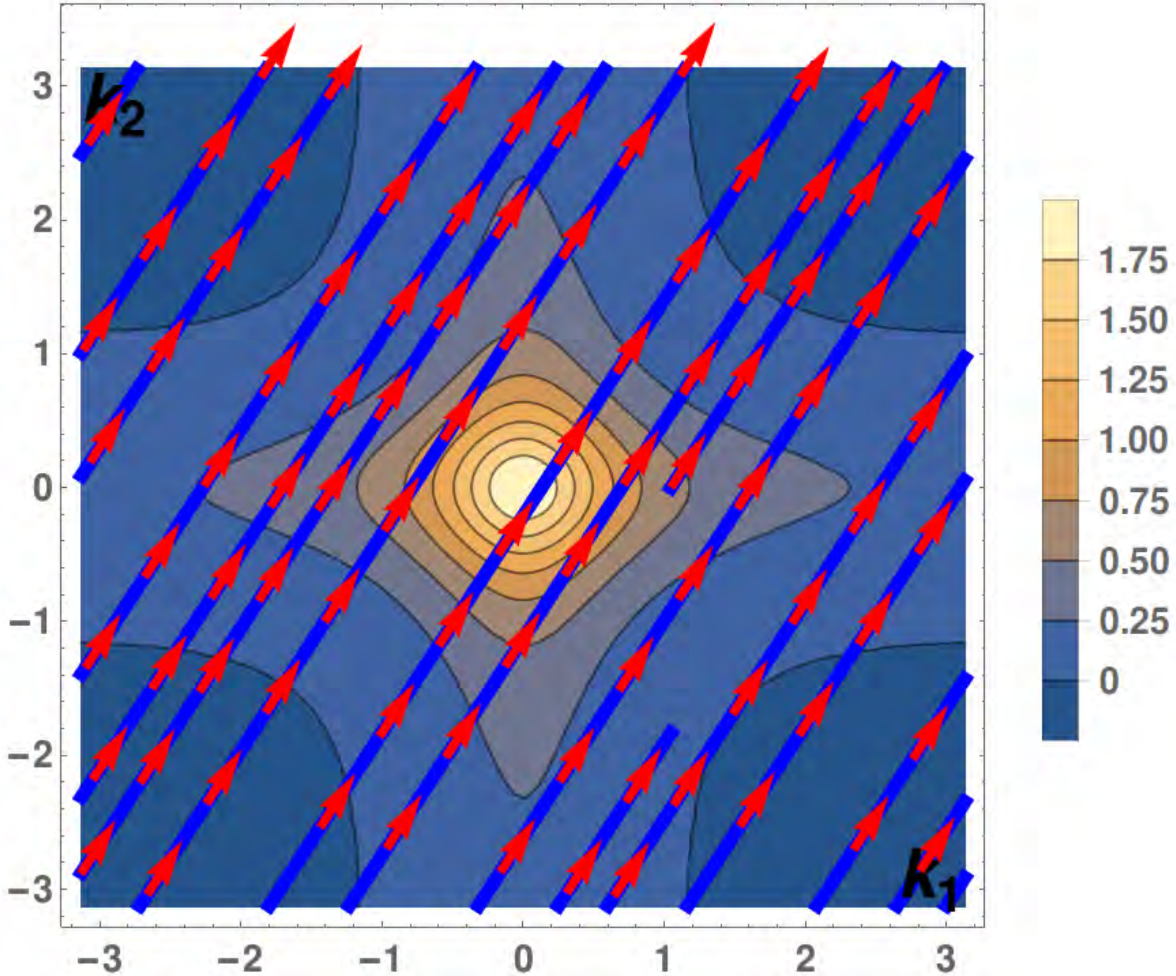
$$H(0)|\psi(0)\rangle = -\varepsilon_{k_0}|\psi(0)\rangle$$

Numerics I: Incommensurate Frequencies

$$\omega_1 = 0.1,\ \ \omega_2 = 0.1\frac{\sqrt{5}+1}{2},$$

- $\omega_1 / \omega_2 \neq p / q$:

$$m = 1.5,\ \ b = 1$$

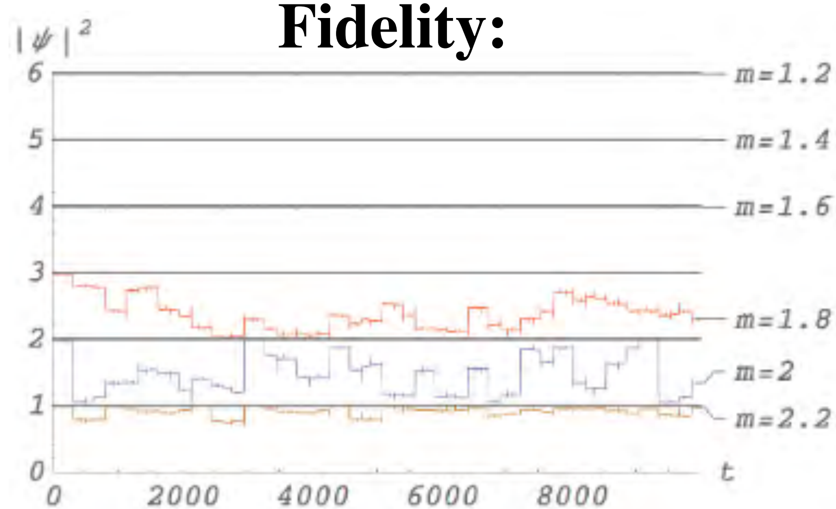


Numerics I: Incommensurate Frequencies [strong coupling]

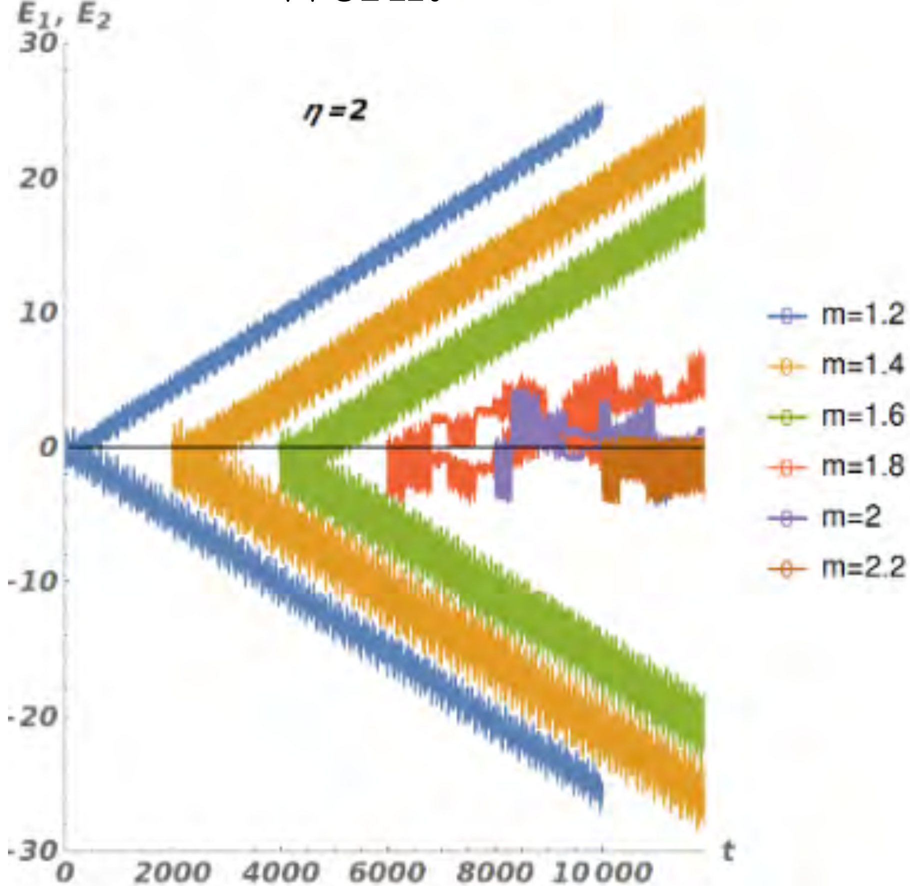
- $\omega_1 / \omega_2 \neq p/q$:

$$\omega_1 = 0.05,\ \omega_2 = \omega_1 \frac{\sqrt{5}+1}{2},\ b = 1$$

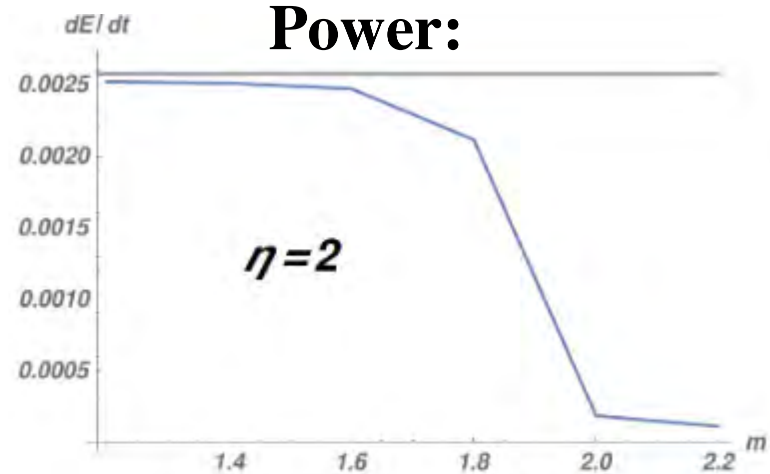
Fidelity:



Work:



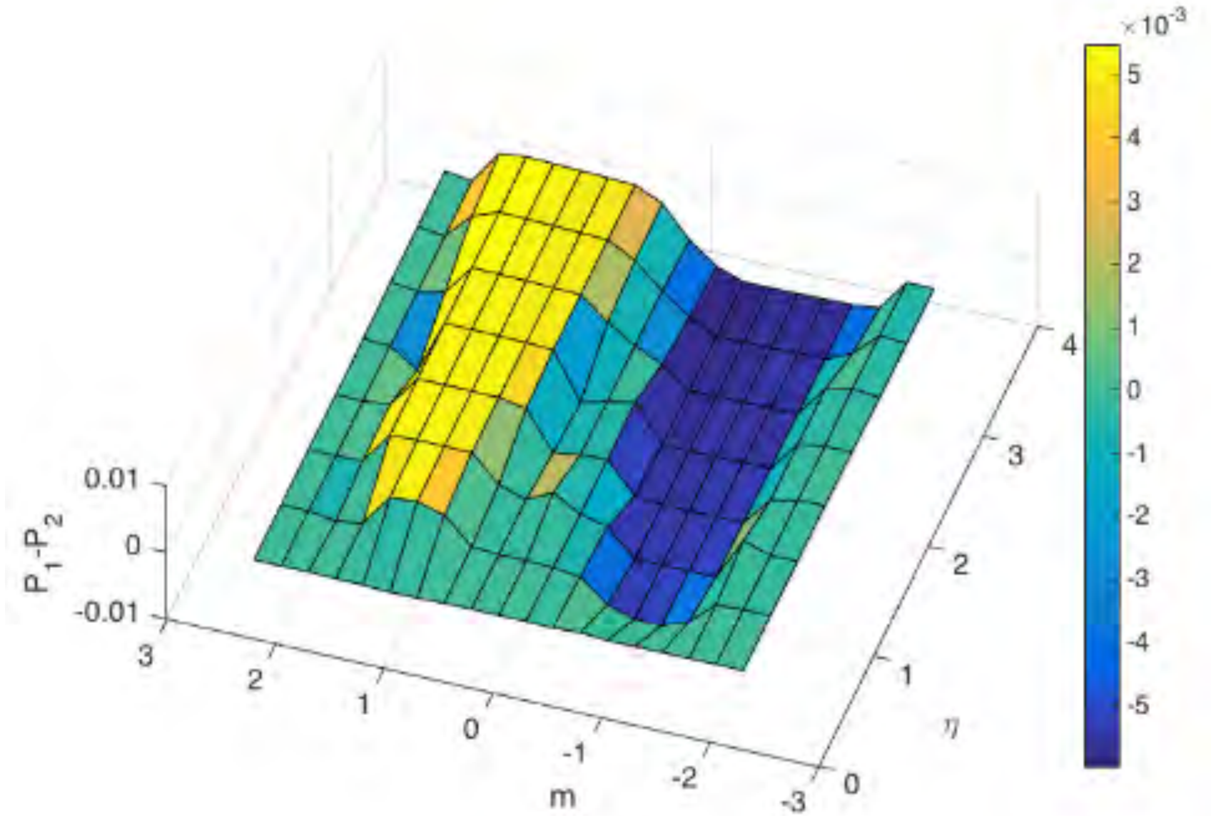
Power:

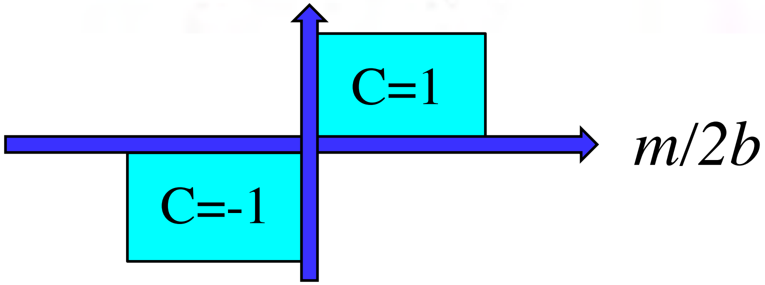


Numerics I: Incommensurate Frequencies [weak copling]

- $\omega_1 / \omega_2 \neq p / q$:

$$\omega_1 = 0.2, \;\; \omega_2 = \omega_1 \frac{\sqrt{5}+1}{2}, \;\; b = 1$$



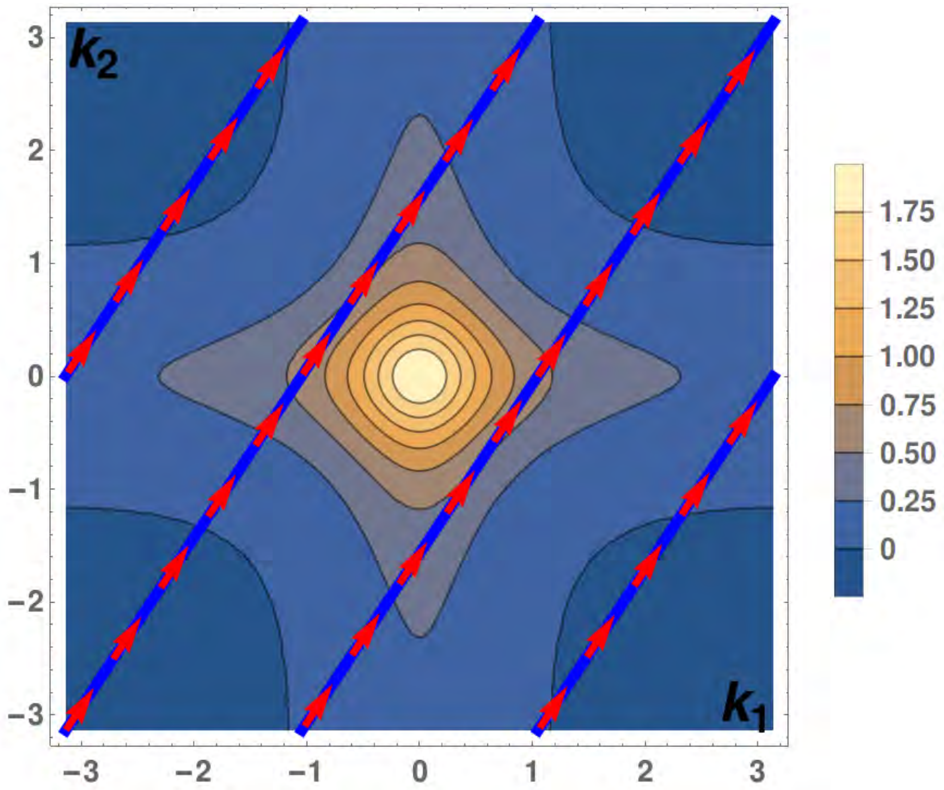


Numerics II: Commensurate Frequencies

- $\omega_1 / \omega_2 = p / q:$

$$T_{total} = T_2 q = T_1 p \qquad H(t) = H(t + T_{total})$$

Periodic Momentum paths

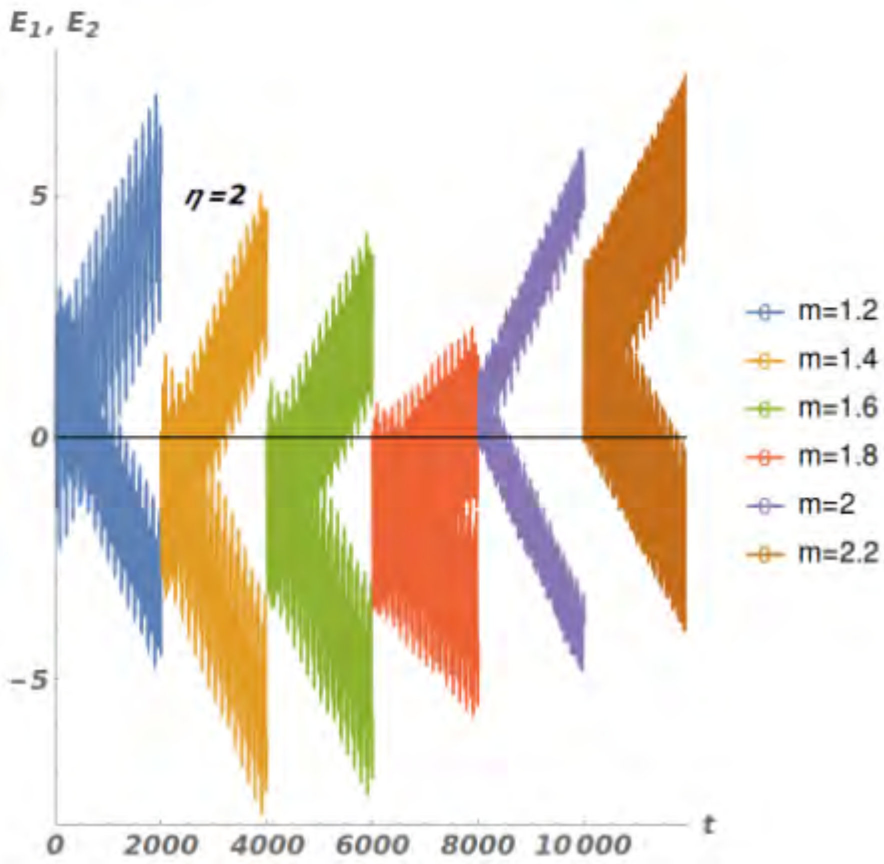


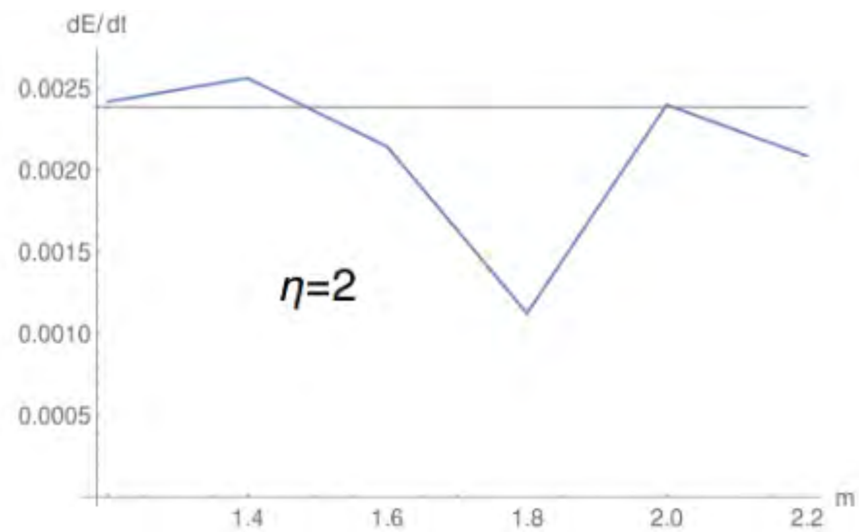
Initiate w/ Floquet eigenstates

Numerics II: Commensurate Frequencies [*m* dependence]

- $\omega_1 / \omega_2 = p / q$: $\omega_1 = 0.05,\ \omega_2 = 3\omega_1,\ b = 1, \phi_1 = \pi / 10,\ \phi_2 = 0$

$$T_{total} = T_2 q = T_1 p \qquad H(t) = H(t + T_{total})$$

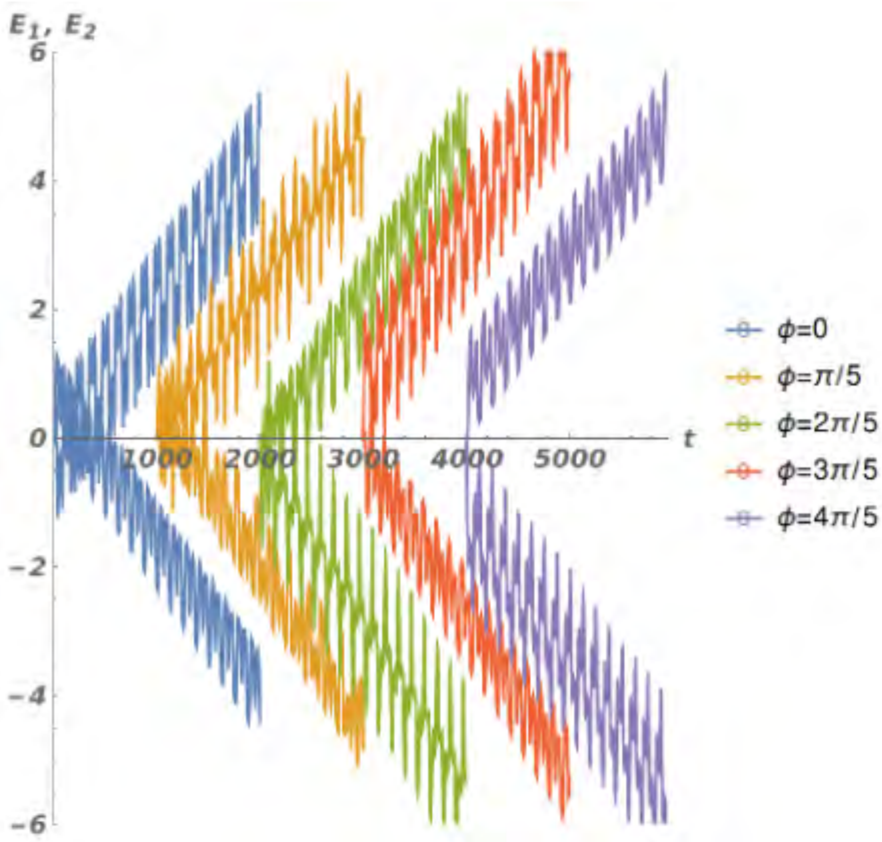


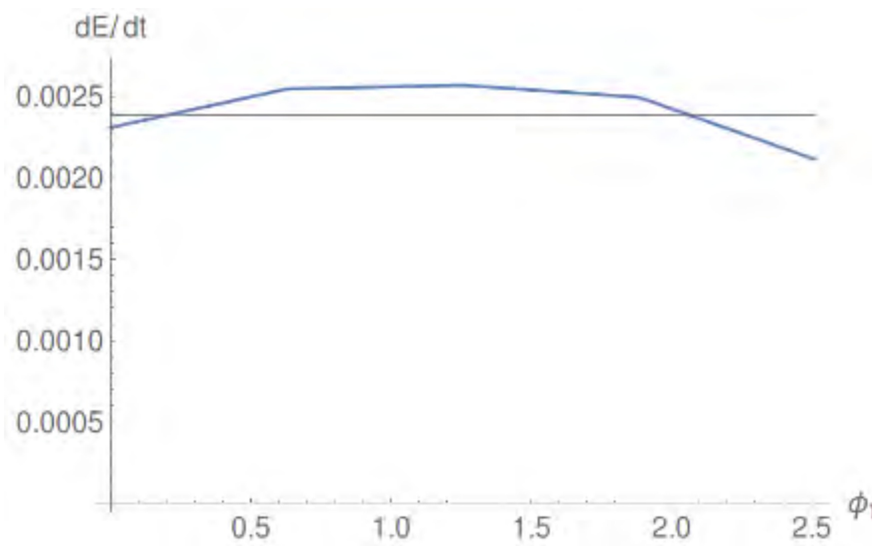


Numerics II: Commensurate Frequencies [phase dependence]

- $\omega_1 / \omega_2 = p / q$: $\omega_1 = 0.1,\ \omega_2 = 3\omega_1,\ m = 1, b = 1, \phi_2 = 0$

$$T_{total} = T_2 q = T_1 p \qquad H(t) = H(t + T_{total})$$





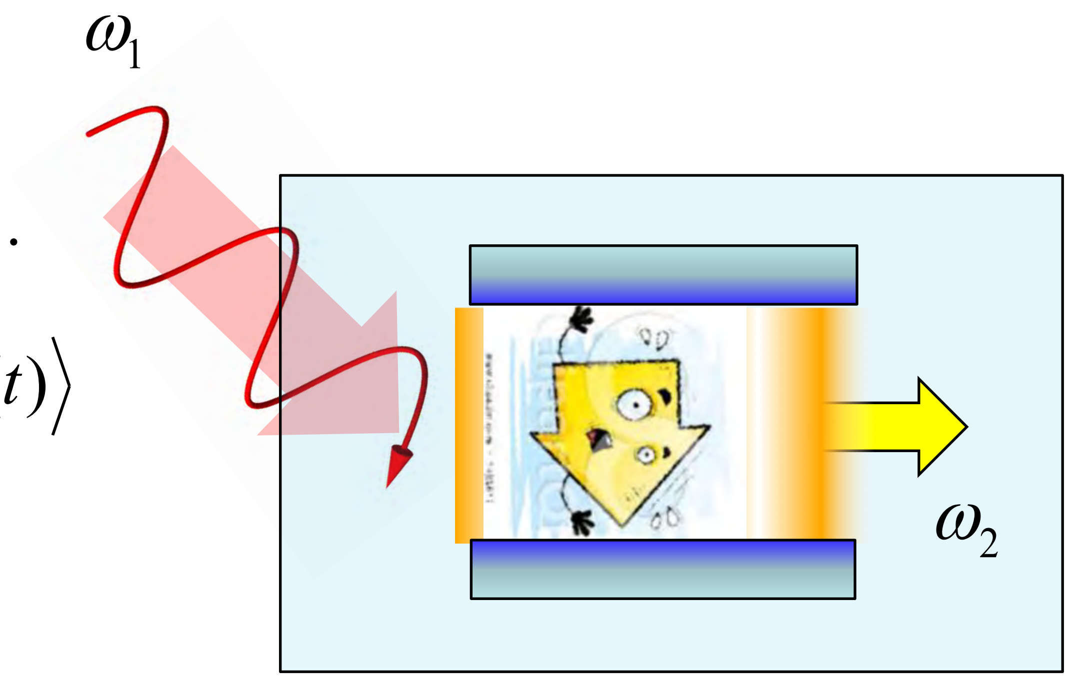
Quantum – Classical hybrid

- Replace one of the modes with a cavity:

$$H = v_1 \sigma^x \sin(\omega_1 t + k_1) + v_2 \sigma^y \left(a^{+} + a\right) + \left[m + b_1 \cos(\omega_1 t + k_1) + b_2 \left(a^{+} + a\right)\right] \sigma^z + \omega_2 a^{+} a$$

Much larger Hilbert space…

$$\left|\Psi(t)\right\rangle = \left|\psi_{spin}(t)\right\rangle \otimes \left|\psi_{cavity}(t)\right\rangle$$

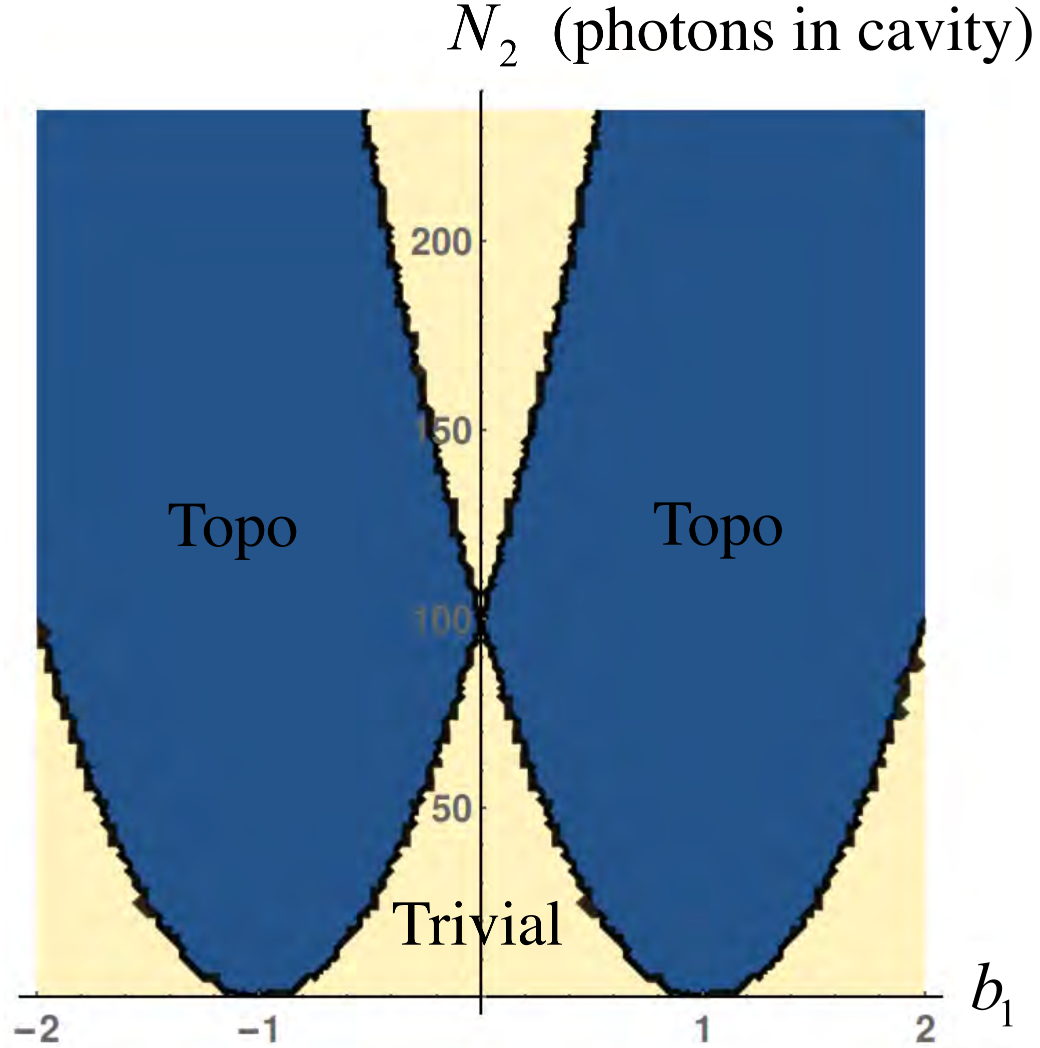


Quantum – Classical hybrid

- Phase diagram:

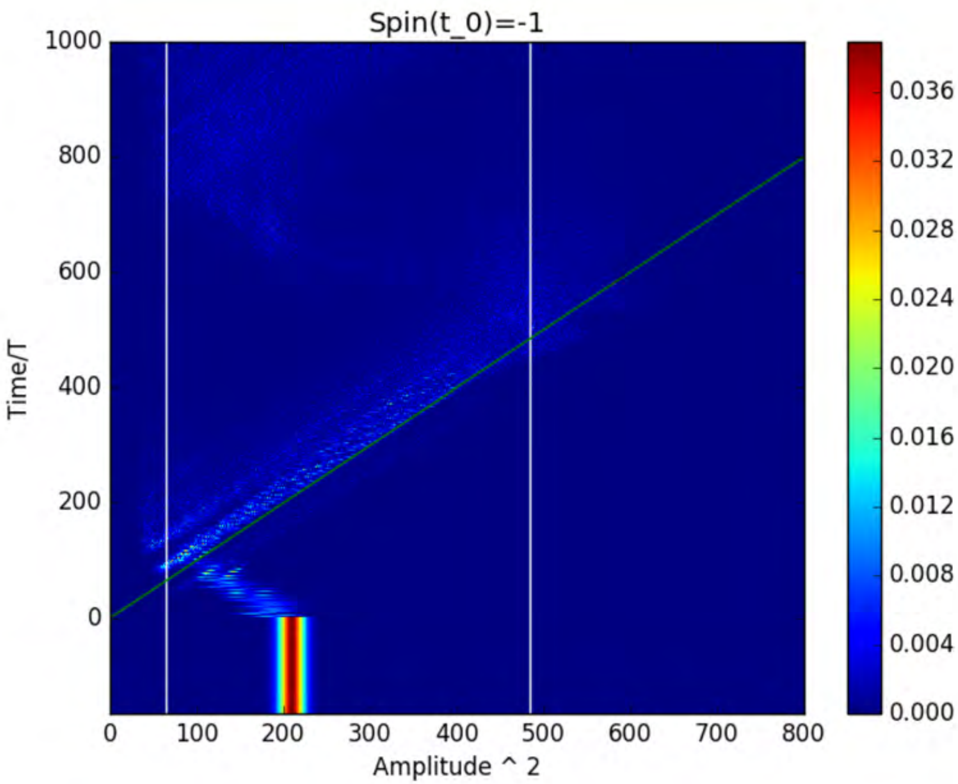
$$b_2^{\,eff} = \sqrt{N_2}\, b_2$$

$$[b_2 = 0.1]$$

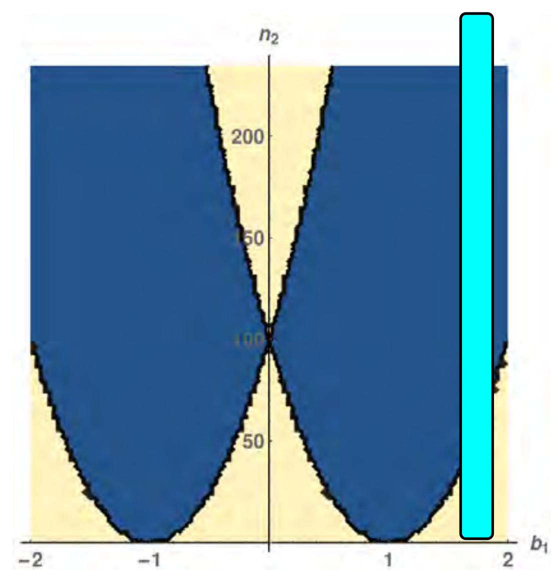


Quantum – Classical hybrid

- Pumping an occupied mode:

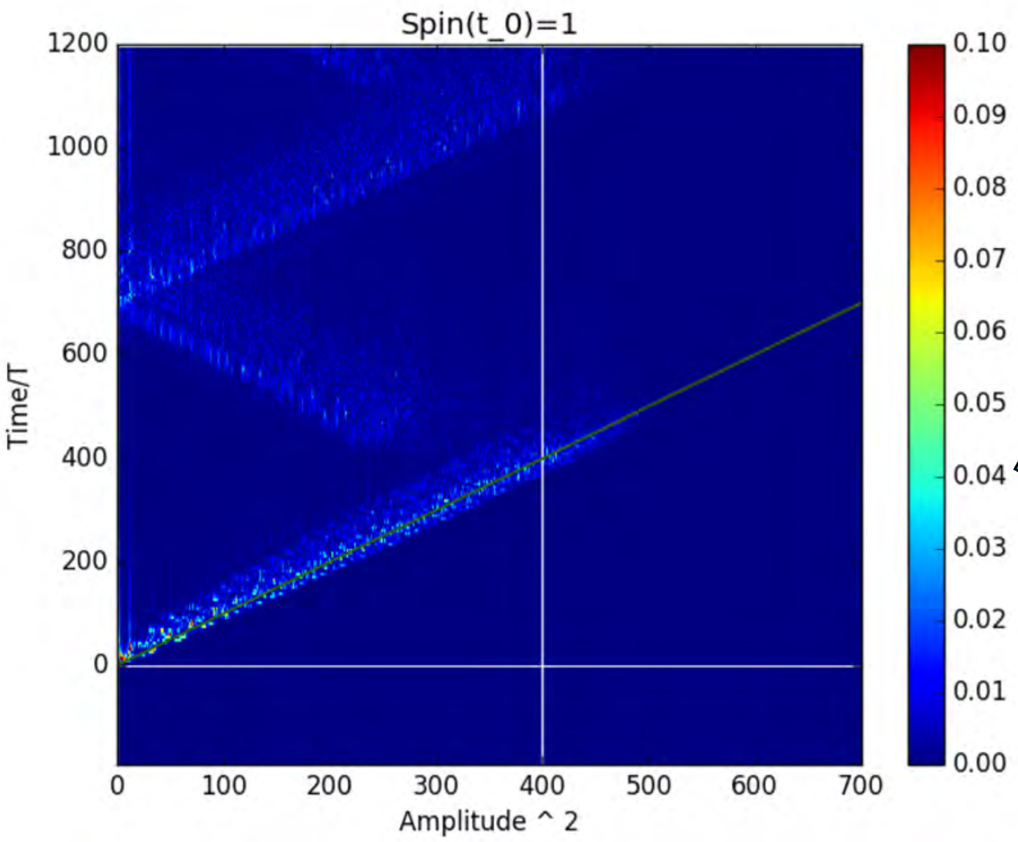


$$\left|\psi_{N_2}\right|^2 = \left\langle \psi_{cavity}(t) \middle| P[\hat{N}_2] \middle| \psi_{cavity}(t) \right\rangle$$

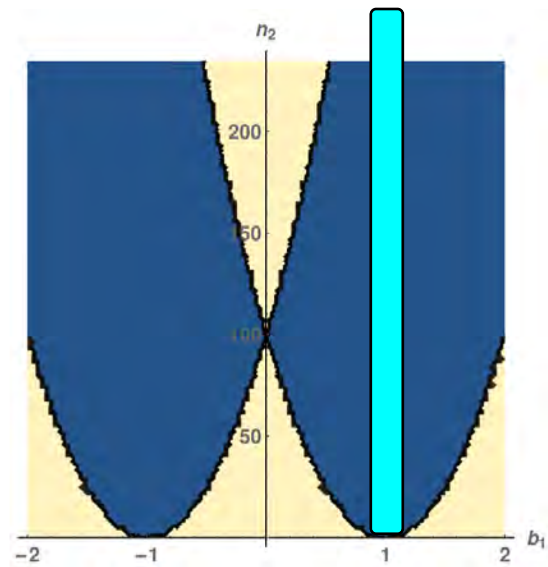


Quantum – Classical hybrid

•Pumping from zero:

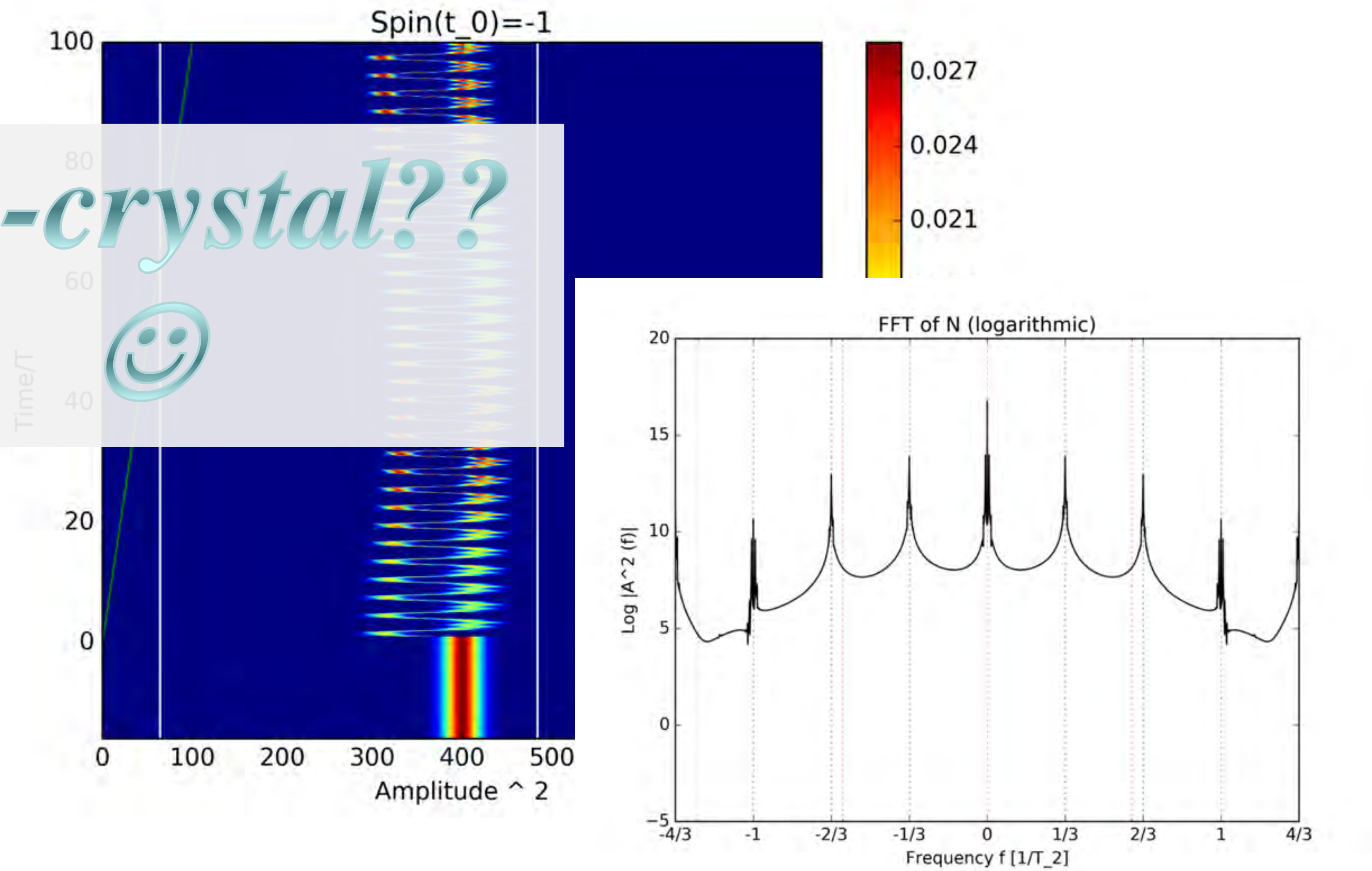


$$\left|\psi_{N_2}\right|^2 = \left\langle \psi_{cavity}(t) \middle| P[\hat{N}_2] \middle| \psi_{cavity}(t) \right\rangle$$



Period tripling

- Unexpected stable orbit:



Time-crystal?? ☺

Summary and conclusions

• Multiple drives could induce topological effects.

• Topological frequency conversion,
  quantized energy pumping, optical amplifier.

• Experimental realizations?
  Spins, qubits, cavities with rare Earth garnets?

• Open questions:
  Temporal disorder effects.
  Using dissipation to increase fidelity?
  Two cavity systems?

Outline

- Floquet review
- Double-drive Floquet
- Synthetic dimensions and Space-energy duality.
- Double-drive topological spin
- Topologically driven cavity

Energy pumping measurement

- How to measure n?

$$\frac{dn_i}{dt} = \left\langle \frac{\partial H}{\partial k_i} \right\rangle = \langle \psi(t) | \frac{\partial H}{\partial k_i} | \psi(t) \rangle$$

- Rate of work done:

$$\frac{dW_i}{dt} = \frac{dn_i}{dt}\omega_i = \omega_i \left\langle \frac{\partial H}{\partial k_i} \right\rangle$$

- Measurement:

$$H = H_1(\omega_1 t + k_1) + H_2(\omega_2 t + k_2)$$

$$\Rightarrow \quad \frac{dW_i}{dt} = \omega_i \left\langle \frac{\partial H}{\partial k_i} \right\rangle = \left\langle \frac{\partial H_i}{\partial t} \right\rangle$$