# Quantum spin dynamics, coherences and entanglement in trapped ion arrays

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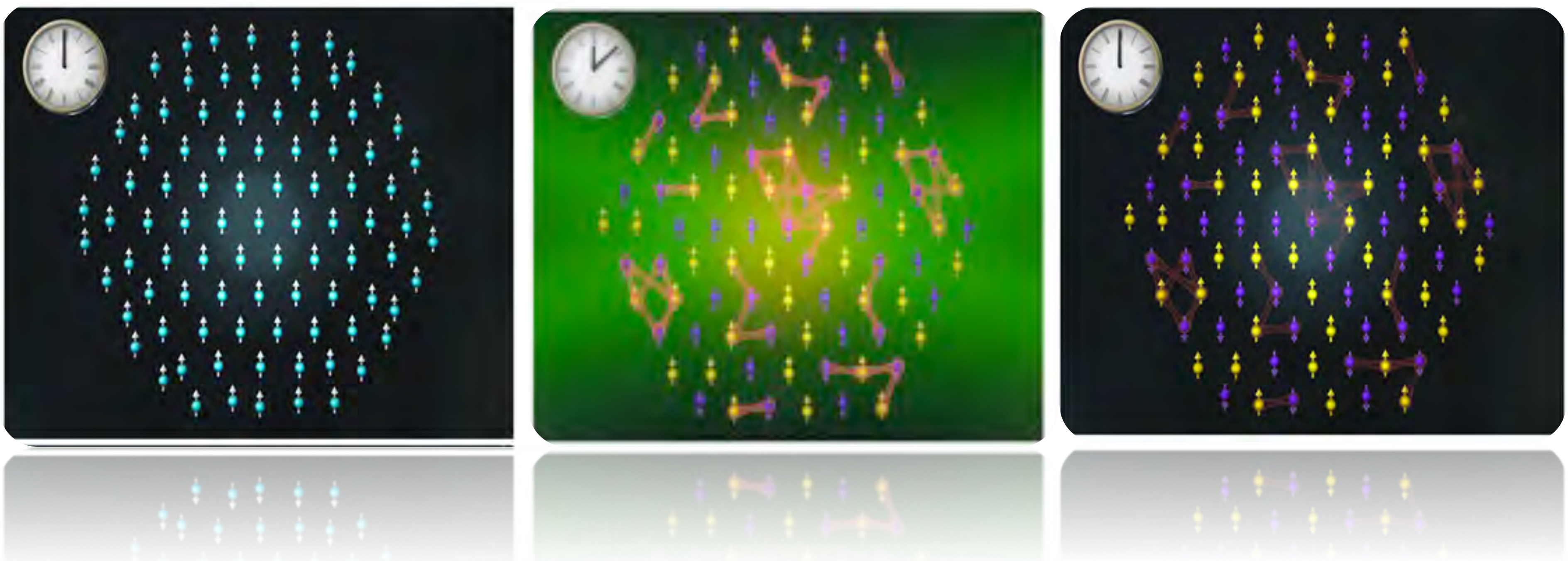
JILA NIST CU



Spice workshop: Non-equilibrium Quantum Matter, Mainz-Germany, June 2nd (2017)

# Theory

M. Wall

M. Gärttner

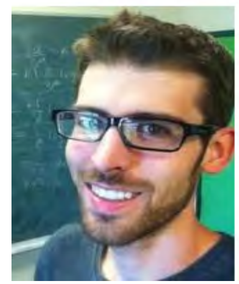
M. Foss-Feig

A. Safavi-Naini

R. Lewis-Swam

# Experiment

J. Bollinger



J. Bohnet

J. Britton

K. Gilmore

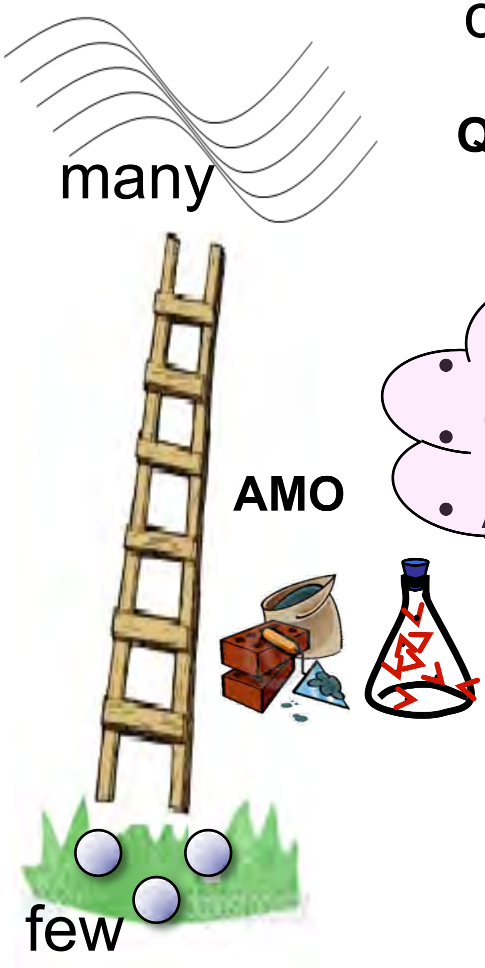
B. Sawyer

# Correlated Quantum Matter

## At the heart of modern quantum science:

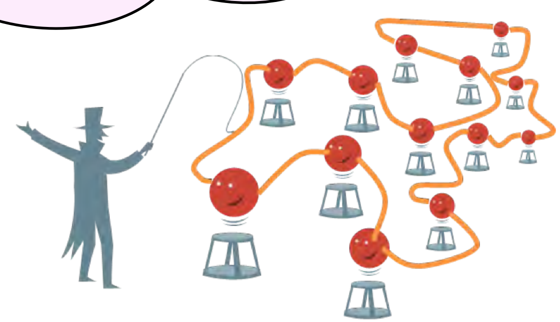
**Fundamental physics,Quantum technology, Metrology**

How do complex behaviors emerge from simple constituents and their interactions?




**QUANTUM SIMULATION**

- Well-understood microscopics
- Tunable interactions
- Access to quantum dynamics

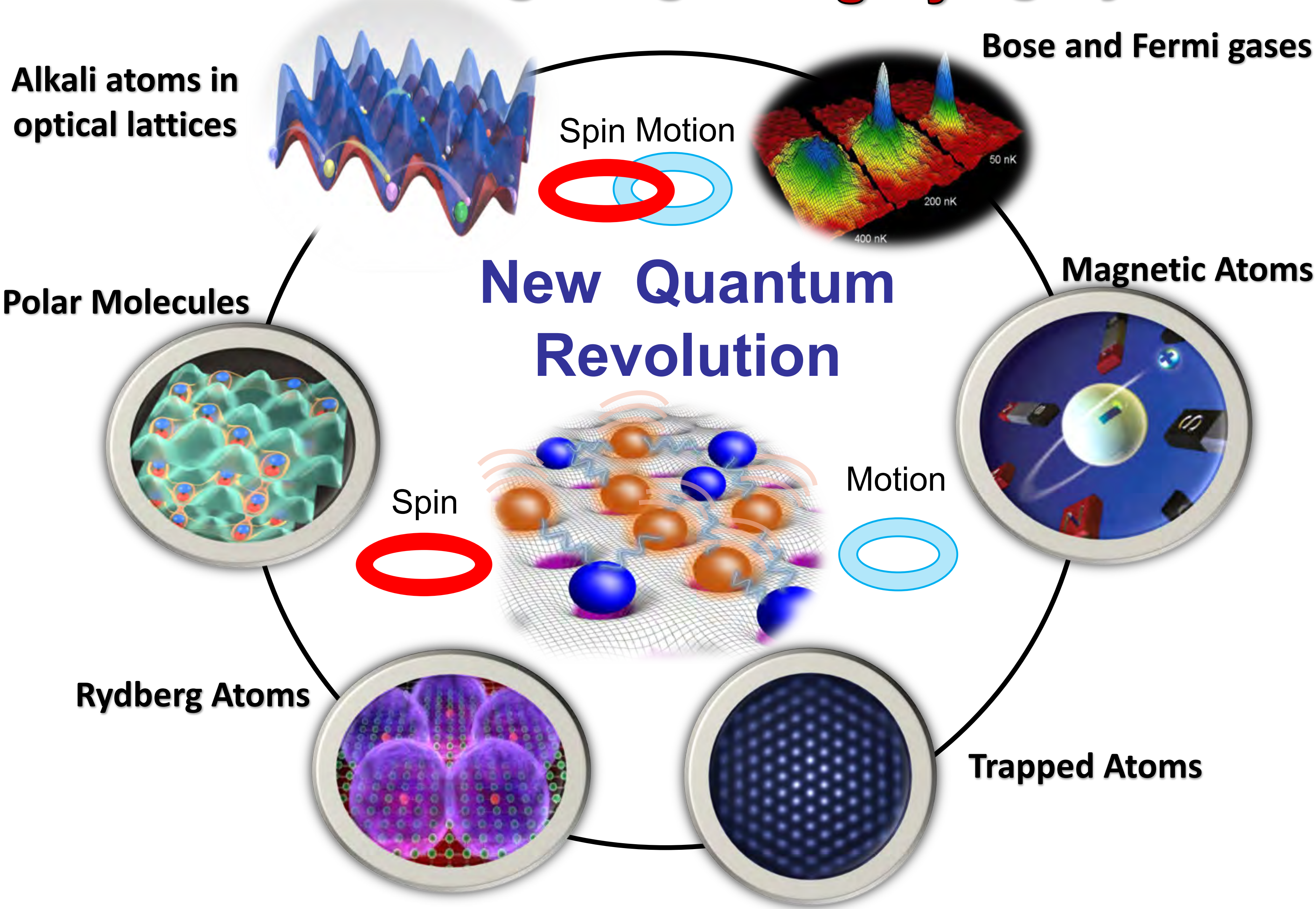


**Entanglement Generation**

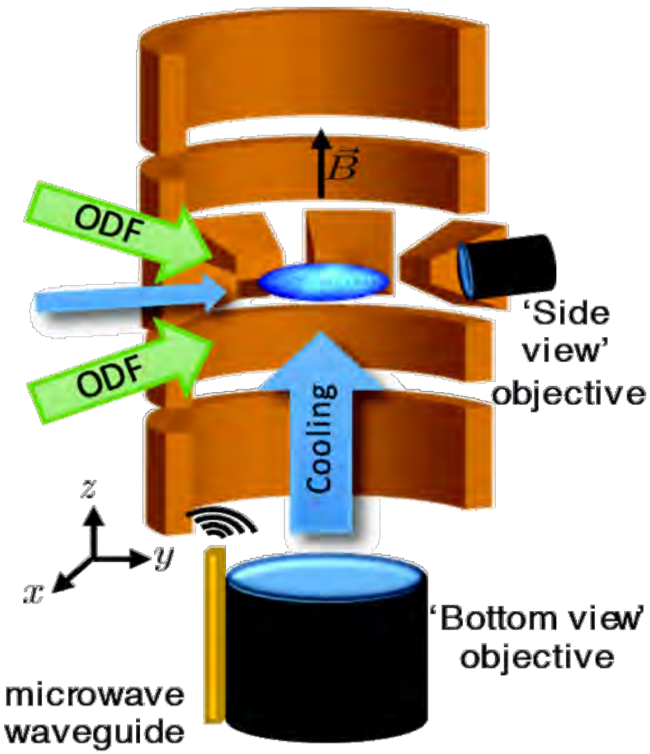
**Detection and Characterization**

**Control of Decoherence**

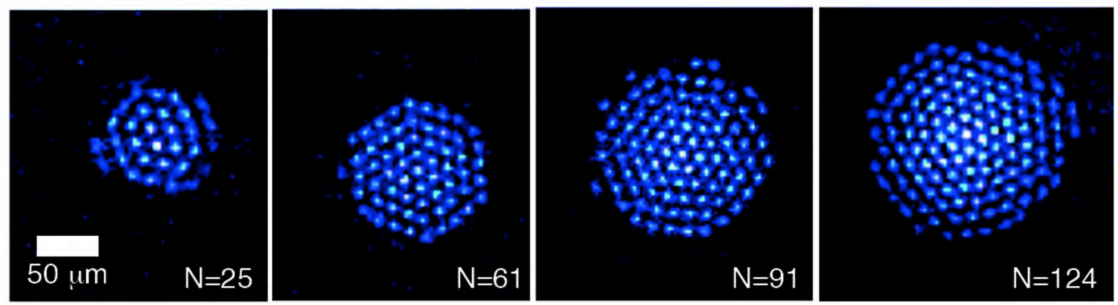
# Controllable Long-range Interacting Systems



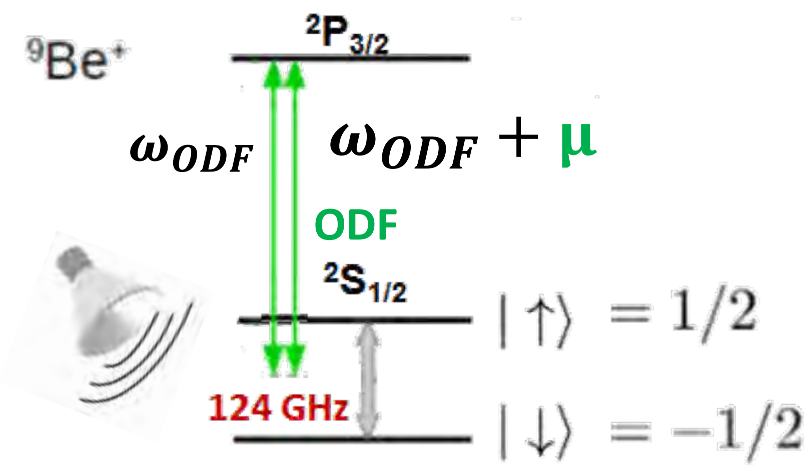
# Penning Trap Experiments: $^9Be^+$




- Penning trap: 2D triangular crystals of 20-300 ions




- Two hyperfine states used as spin ½ system




- **Spin–spin interactions by dependent force**

$$\hat{H}_{ODF}(t) = -F_0 \cos(\mu t) \sum_{j=1}^{N} \hat{z}_j \cdot \hat{\sigma}_j^z$$

# Phonons & Spin-spin interactions

$$\hat{H}_{ODF}(t) = -F_0 \cos(\mu t) \sum_{j=1}^{N} \hat{z}_j \cdot \hat{\sigma}_j^z \quad \xrightarrow{\text{Propagator}} \quad \hat{U}_{ODF}(t) = \hat{U}_{SP}(t)\, \hat{U}_{SS}(t)$$

$\hat{U}_{SP}(t)$ → spin-phonon

$\hat{U}_{SS}(t)$ → spin-spin

N drumhead eigenvalues $\omega_m$ and eigenvector $\vec{b}_m$

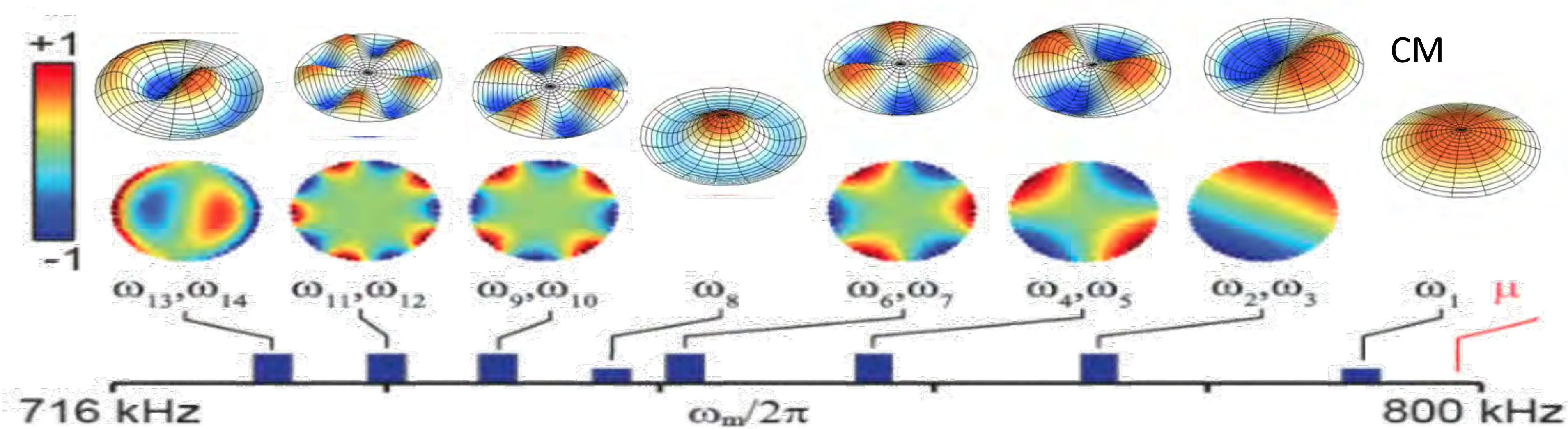
μ is selected to excite them

Dominant for large $|\delta| = |\mu - \omega_1|$

$$H_{SS} = \frac{1}{N} \sum_{i<j} J_{ij}(\mu) \sigma_i^z \sigma_j^z$$

$$J_{ij} \propto \frac{1}{r_{ij}^{\alpha(\mu)}} \qquad \alpha \in (0-3)$$

$$\hat{U}_{SS}(t) \sim e^{-i\hat{H}_{SS} t}$$



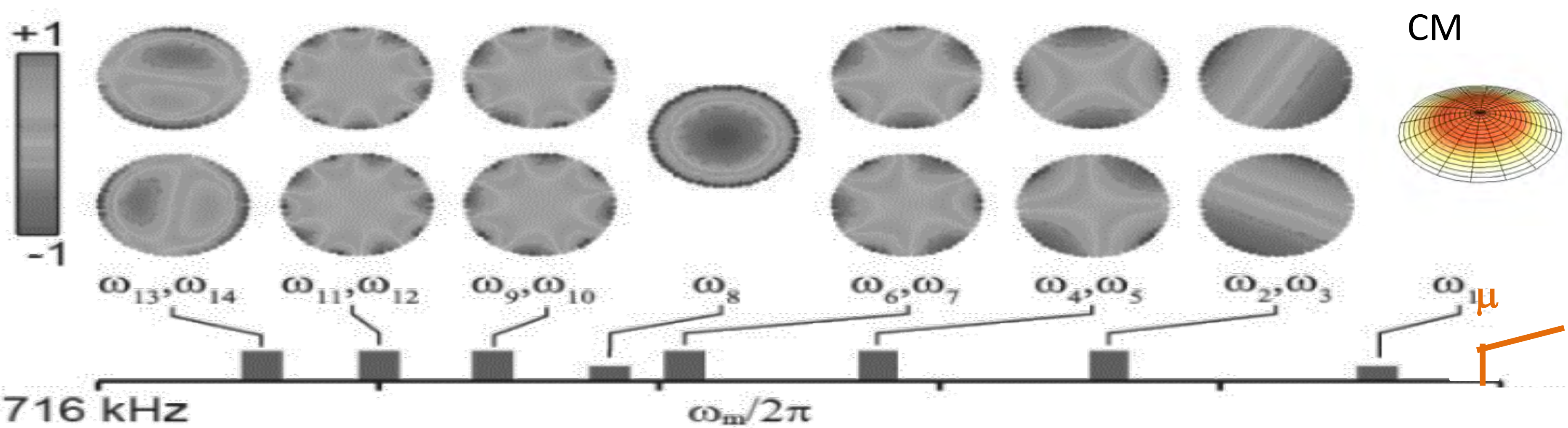

<a href="http://www.acs.psu.edu/drussell/Demos/waves/wavemotion.html">

# All-to-All Case: One Axis Twisting

$$H_{SS} \sim \frac{1}{N} \sum_{i<j} \frac{J_0}{\delta} \sigma_i^z \sigma_j^z = \chi\,(S^z)^2 \qquad J_{ij} \sim \frac{J_0}{\delta}$$

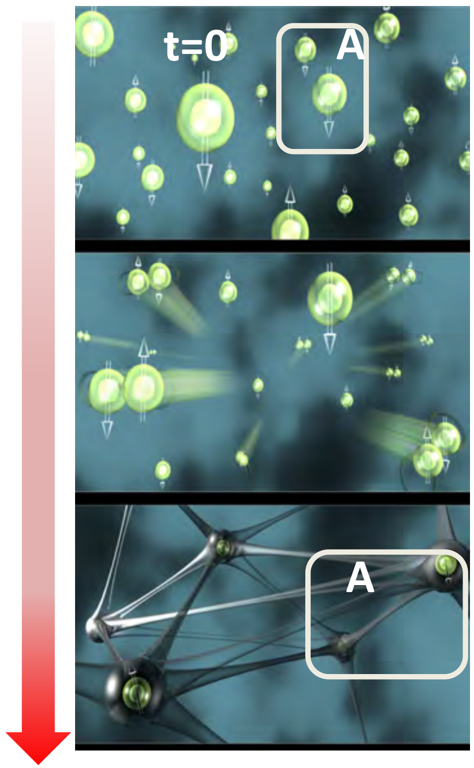
$$S^{x,y,z} = \frac{1}{2} \sum_i \sigma_i^{x,y,z} \qquad \boldsymbol{\delta = \mu - \omega_1}$$

**How we can measure & characterize build up of entanglement?**




<a href="http://www.acs.psu.edu/drussell/Demos/waves/wavemotion.html">

# Entanglement Entropy




Product state

$S_A = 0$

Global Unitary dynamics

$S_A \rightarrow L_A$

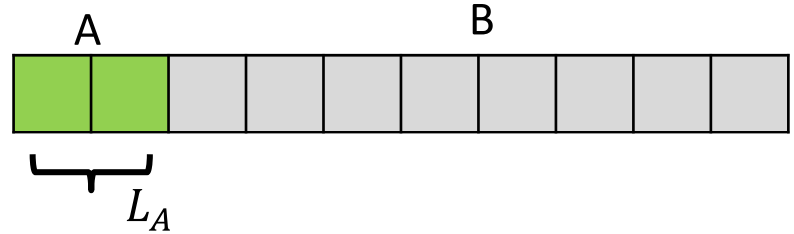
Highly Entangled

**Experiments**

A. Kaufman *et al* Science 353, 794 (2016)

$\hat{\rho}$ : Density Matrix of the close system

$$\ln[\mathrm{Tr}(\rho^2)] = 0$$




$$\hat{\rho}_A = \mathrm{Tr}_B\ \hat{\rho}$$

Reduce density Matrix of subsystem A

**Renyi entropy:** Purity of the A subsystem

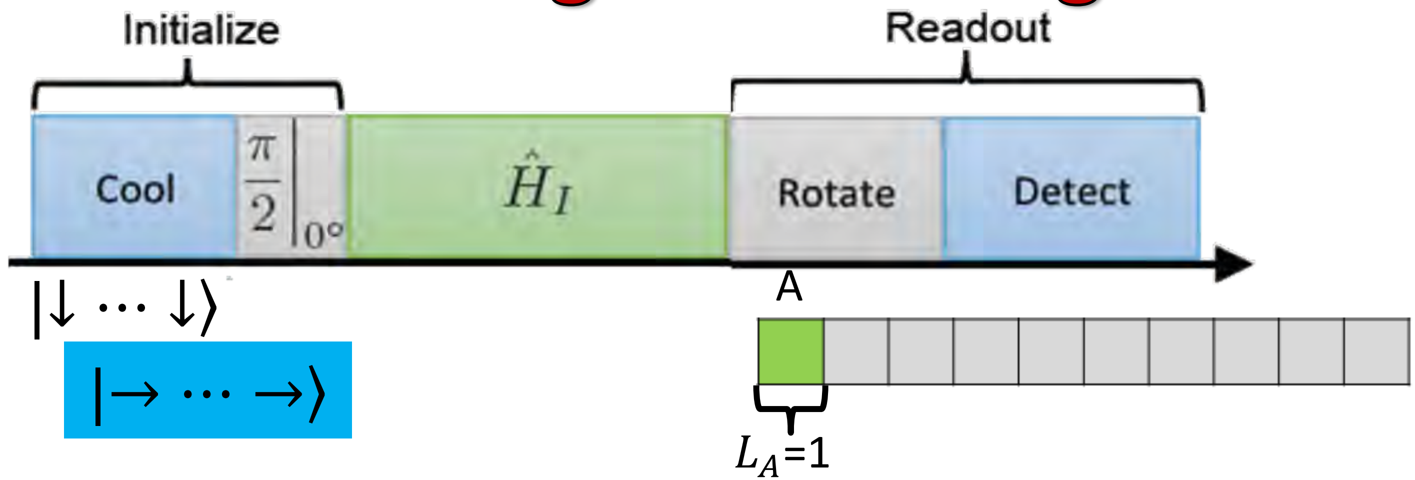
$$S_A = -\ln[\mathrm{Tr}(\rho_A^2)]$$

Product state $\hat{\rho} = \otimes_i \hat{\rho}_i$ $S_A = 0$

Entangled state $S_A > 0$ $S_A \rightarrow L_A$

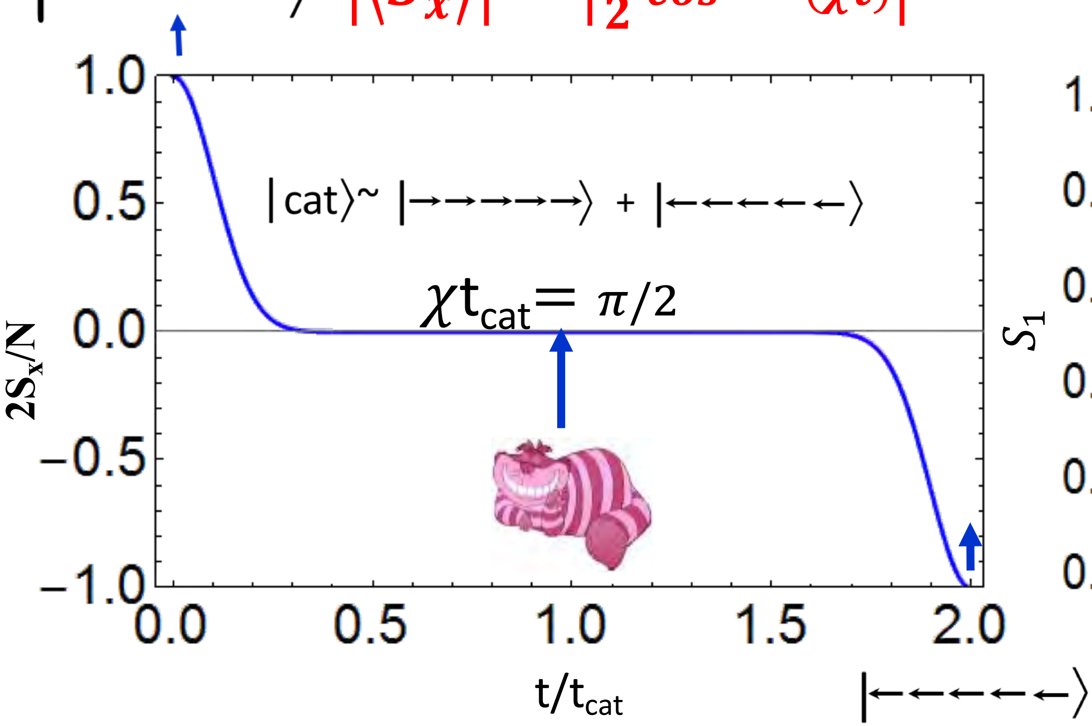
**Volume Law: Thermalization**

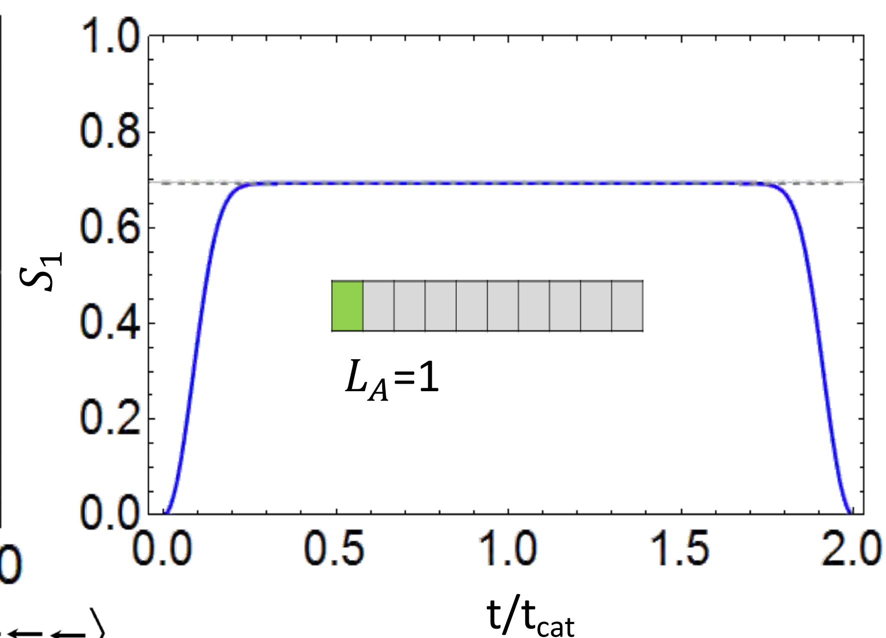
# ALL-to-All Ising local Entanglement?




$|\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rangle$ $\left|\langle\hat{S}_x\rangle\right| = \left|\frac{N}{2}\cos^{N-1}(\chi t)\right|$

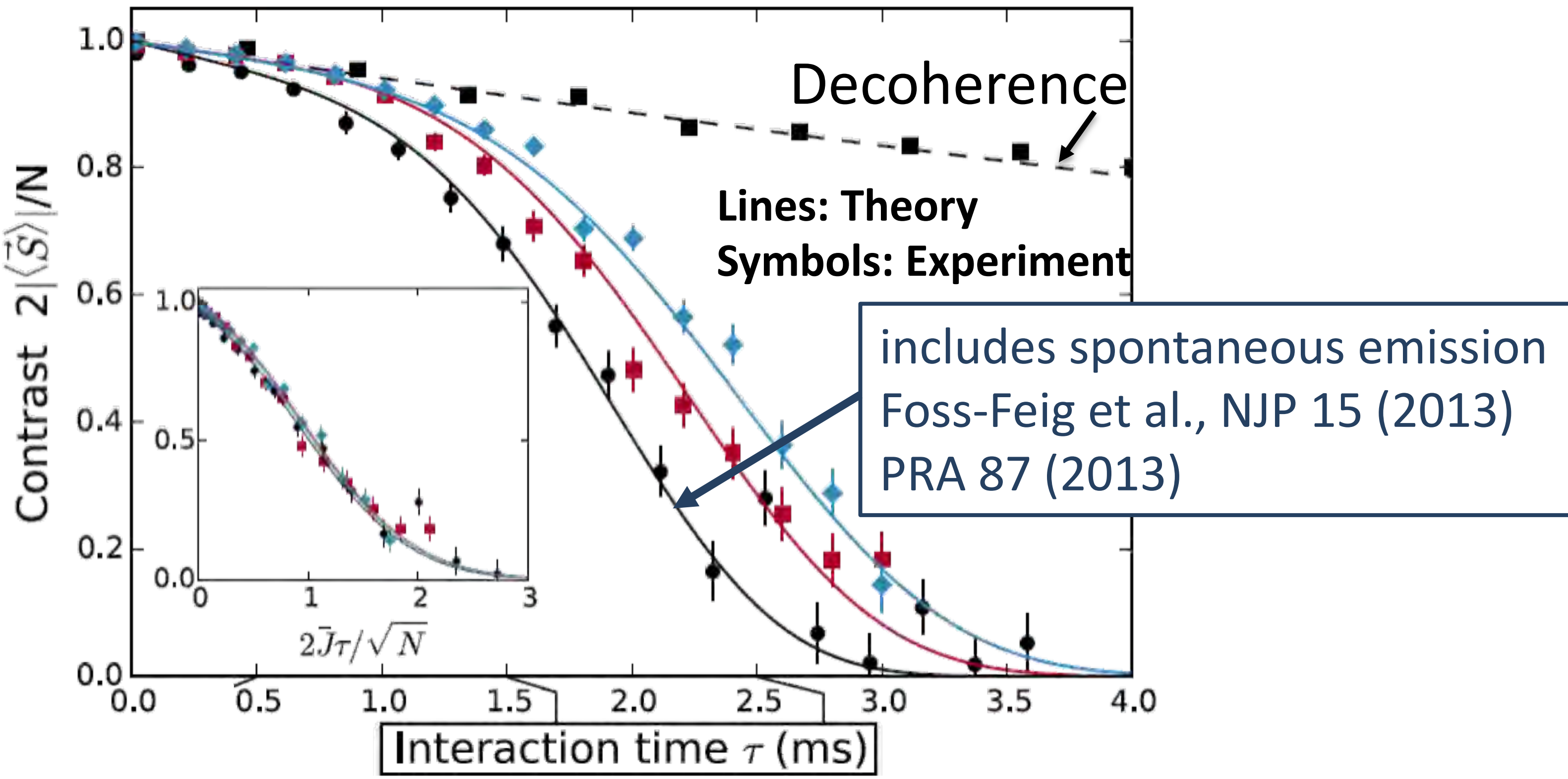
$S_1 = -\log\left[\frac{1}{2}\left(1 + \left(\frac{2}{N}\langle\hat{S}_x\rangle\right)^2\right)\right]$



# Comparison with experiment

- Coherent spin demagnetization: Bloch vector length $|\langle S_x \rangle|$ vs time

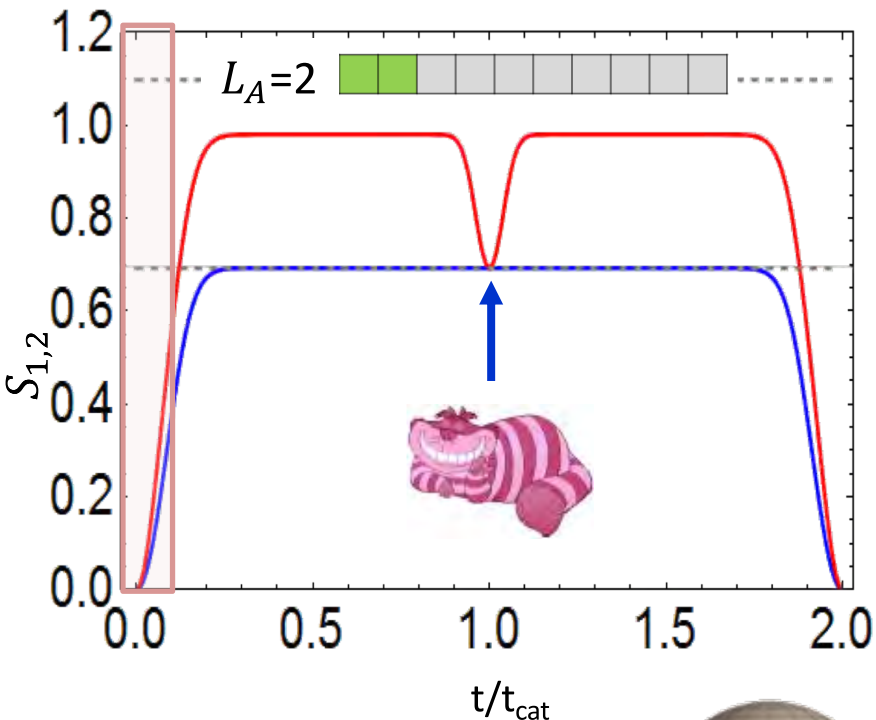




Quantum correlations!!

**Bohnet *et al.*, Science352,1297(2016).**

# Two point correlations & Spin Squeezing

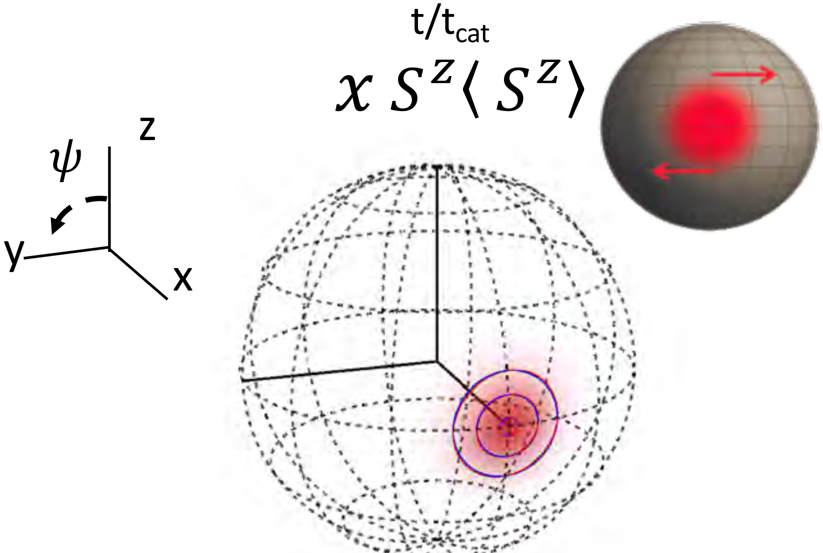


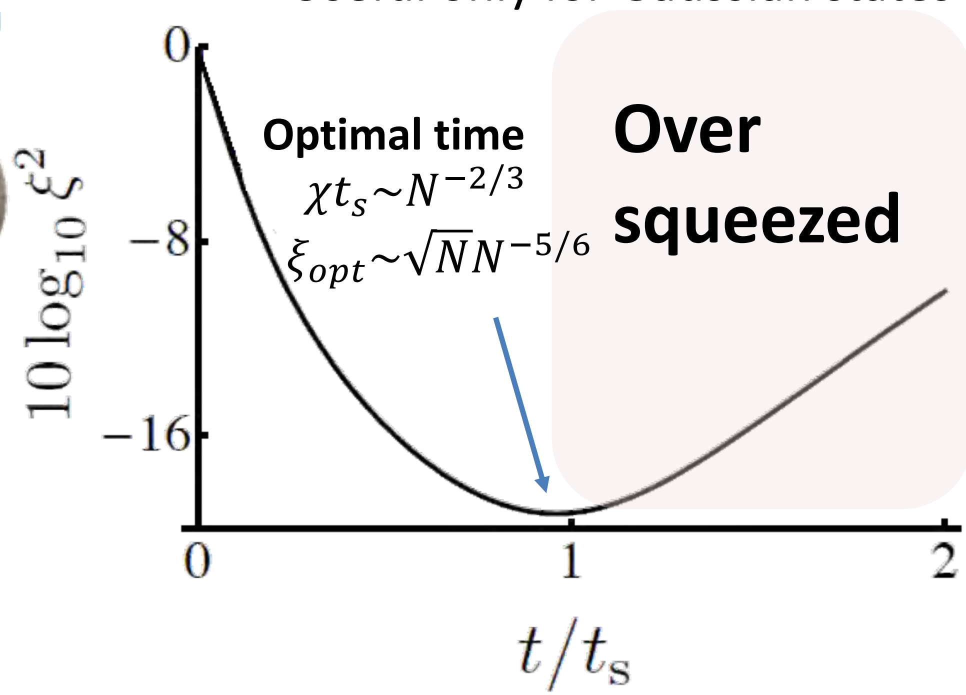

**Spin Squeezing Paremeter**

$$\xi(\psi) = \frac{\sqrt{N}\Delta S^{\psi}}{\langle \hat{S}^x \rangle} \qquad \xi^2 < 1$$

A. Sørensen *et al* Nature 409, 63 (2001)

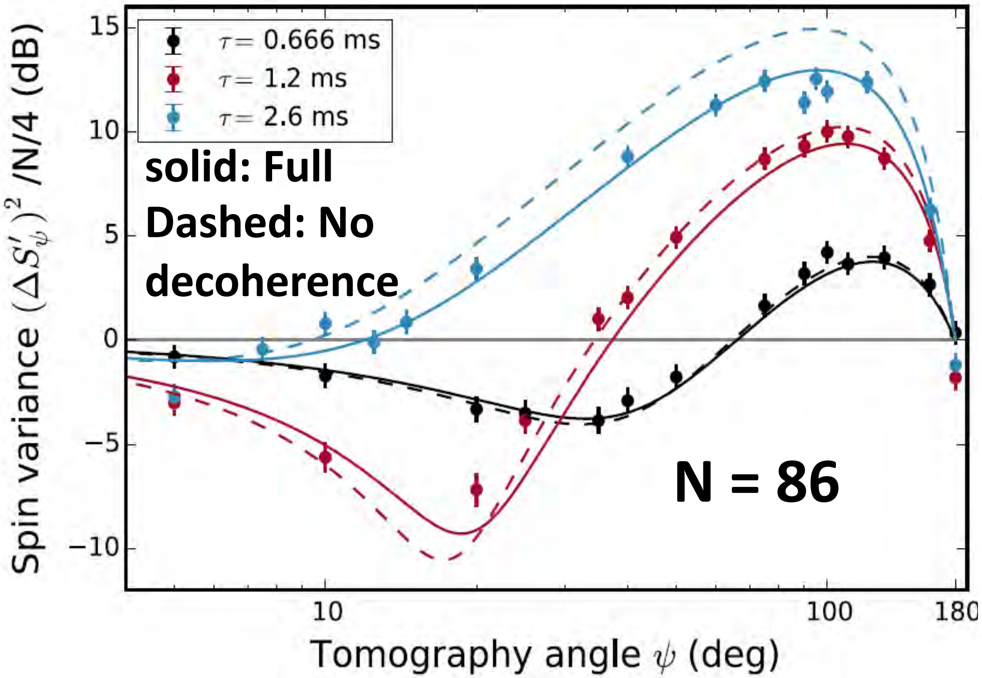
- Entanglement witness
- Enhanced sensitivity
- Useful only for Gaussian states

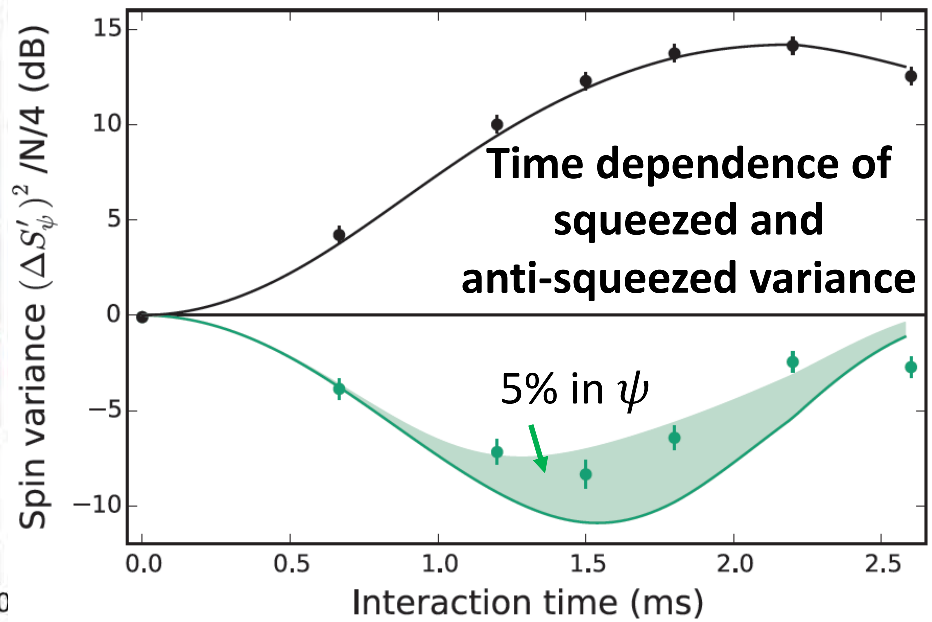


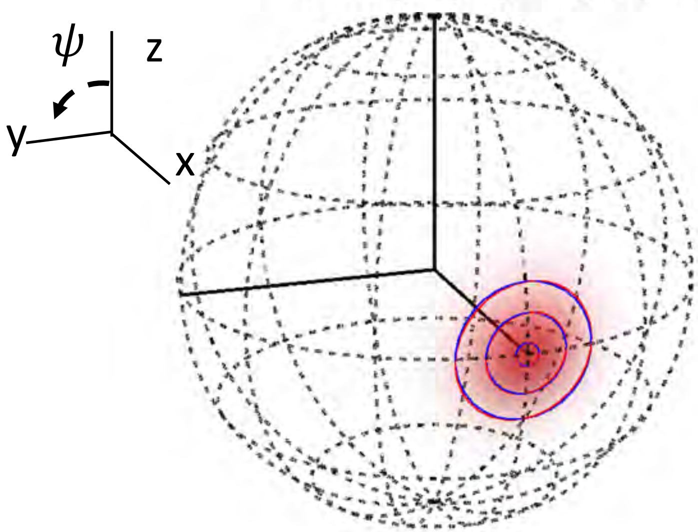



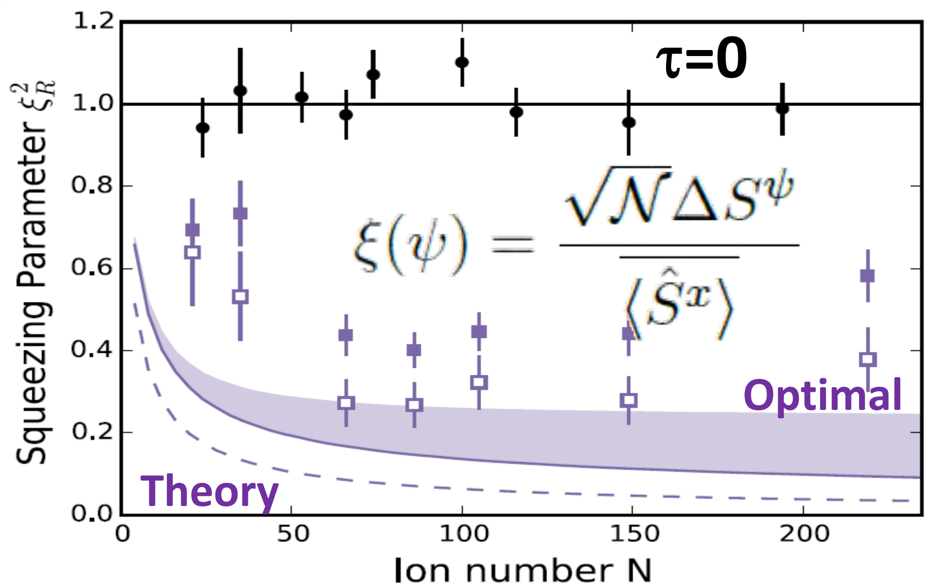

**Spin Squeezing**

# Comparison with experiment



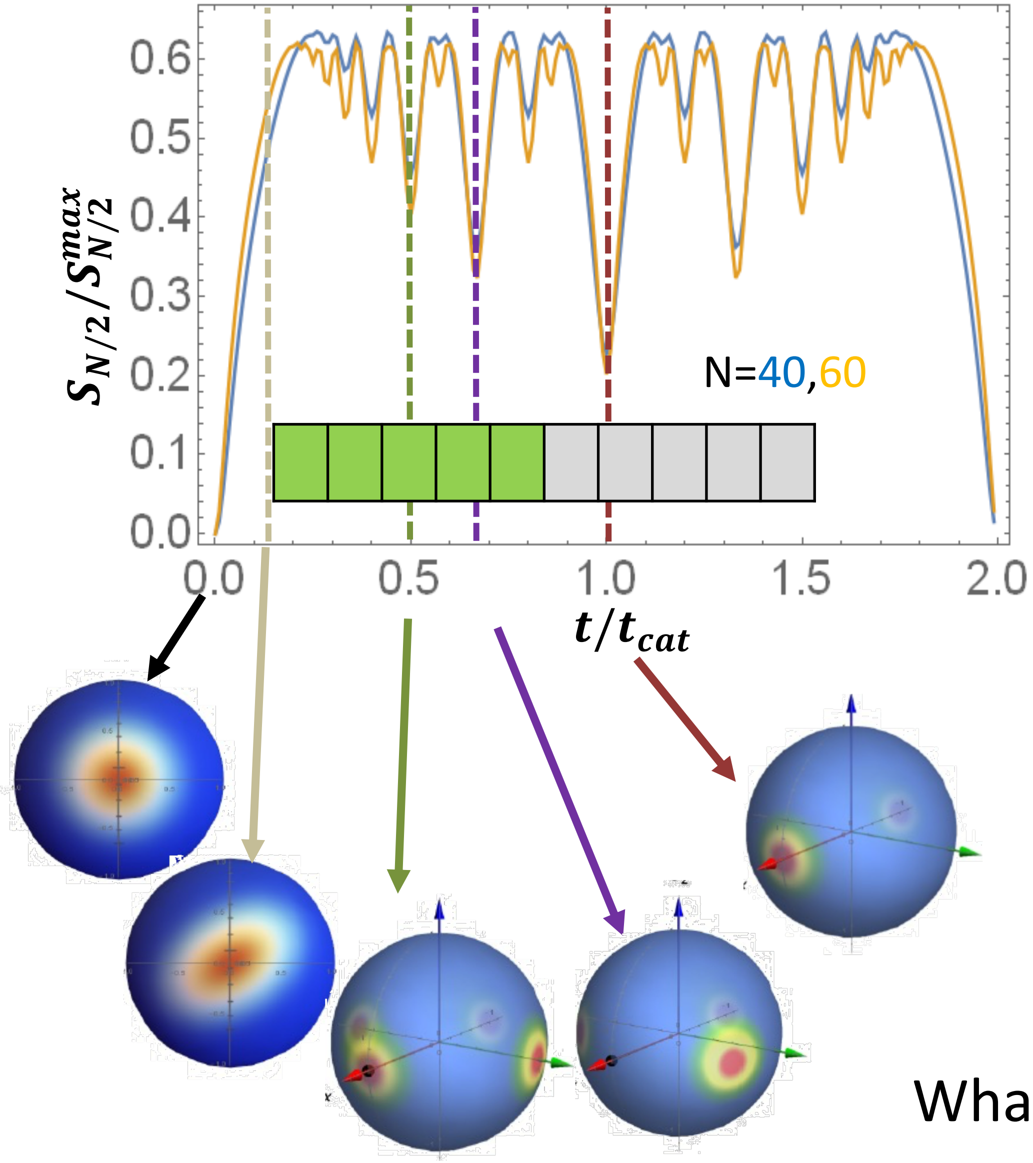




- largest inferred squeezing: -6.0 dB

# Entanglement in ALL-to-All Ising

## More Information




However entanglement entropy is hard to measure

Greiner group at Harvard : quantum gas microscope


What can we do with global probes?

# Multi-quantum Coherences

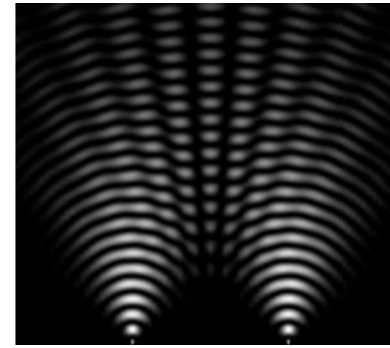
Multi-Quantum coherence spectrum (NMR)

M. Munowitz and M. Mehring , Sol. St. Com., 64, 605 (1987)

# Quantum Coherences

$$\hat{\rho} = \begin{bmatrix} \rho_{\uparrow\uparrow} & \rho_{\uparrow\downarrow} \\ \rho_{\downarrow\uparrow} & \rho_{\downarrow\downarrow} \end{bmatrix}$$

- **Phase sensitivity**
- **Quantum superposition**



$$\rho_{\uparrow\downarrow} = \langle \downarrow | \hat{\rho} | \uparrow \rangle$$

# Multi-quantum Coherences

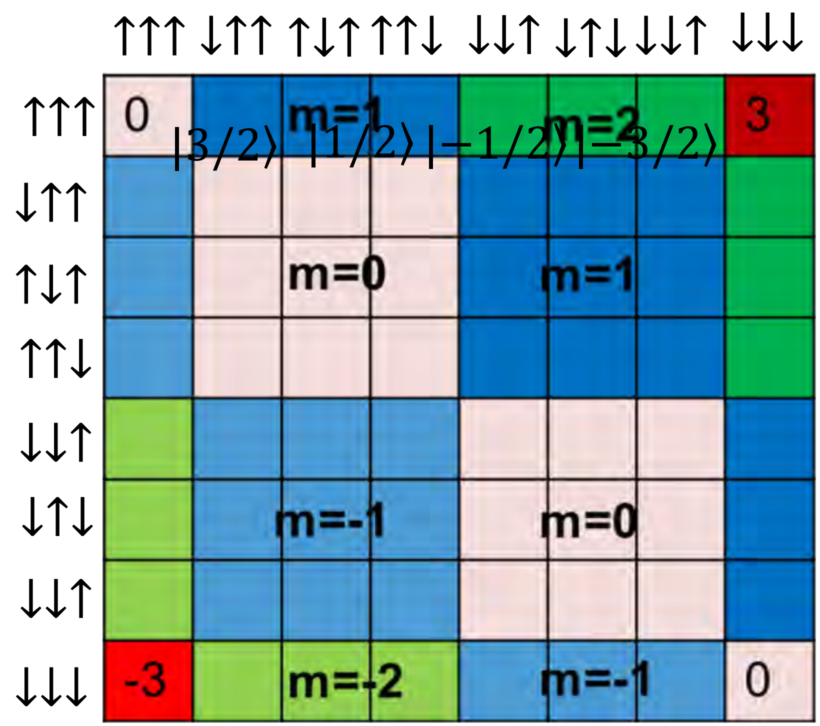
N=3

$\langle \uparrow\uparrow\uparrow | \rho | \downarrow\downarrow\downarrow \rangle$

$$\hat{\rho} = \sum_m \hat{\rho}_m$$

**$\hat{\rho}_m$:Contains elements**

$$\langle \{(n-m) \downarrow\} | \hat{\rho} | \{n \downarrow\} \rangle \neq 0$$




**Collective states**

$$\hat{\rho}_m = \langle M+m | \hat{\rho} | M \rangle \;\; |M+m\rangle\langle M|$$

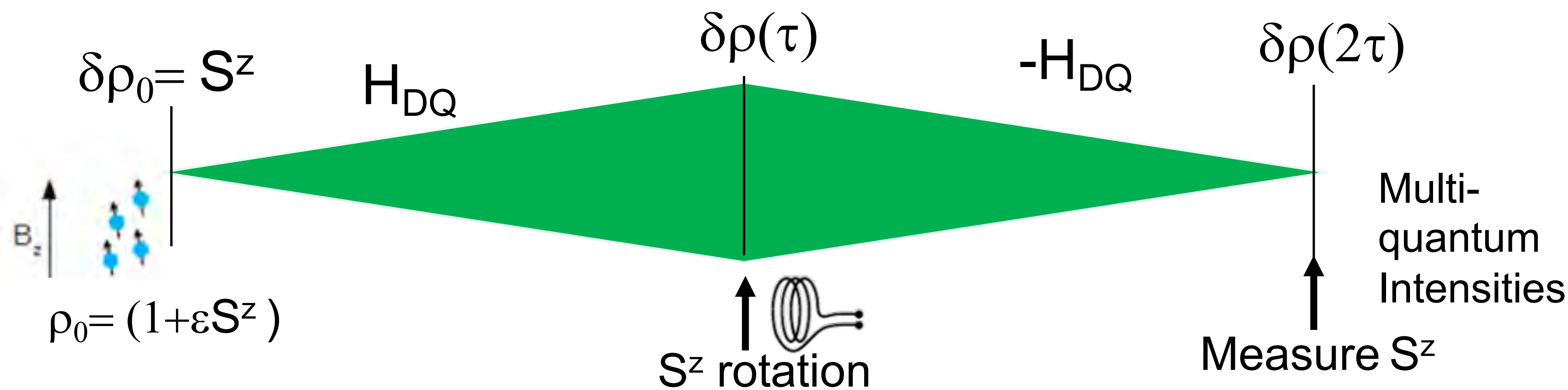
$$S_z |M\rangle = M |M\rangle$$

$$M = \{-\frac{N}{2}, \ldots, \frac{N}{2}\}$$

# Multi-quantum Coherences

Multi-Quantum coherence spectrum (NMR)

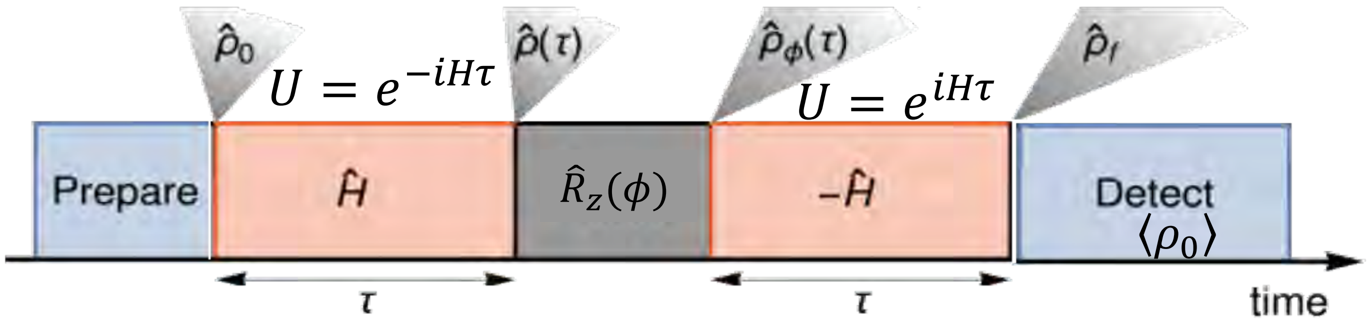
$$H_{DQ} \propto \sum J_{ij}(\sigma_i^+ \sigma_j^+ + \sigma_i^- \sigma_j^-)$$




$H_{DQ}$: Is obtained from the dipole-dipole $H_{ZZ}$ by pules:

M. Munowitz and M. Mehring , Sol. St. Com., 64, 605 (1987)

# Multi-quantum Coherences




Requirements:
1) Invert many-body time evolution.
2) Measure initial state.

$$\mathfrak{I}_\phi(\tau) = \langle \rho_0 \rangle = \mathrm{Tr}\left[\rho_0 \rho_f\right]$$

$$= \mathrm{Tr}\left[\rho_0 U^\dagger R_z(\phi) U \rho_0 U^\dagger R_z^\dagger(\phi) U\right]$$

$$= \mathrm{Tr}\left[U \rho_0 U^\dagger R_z(\phi) U \rho_0 U^\dagger R_z^\dagger(\phi)\right]$$

$$\rho(\tau) = U \rho_0 U^\dagger$$

$$\rho_\phi(\tau) = \left[R_z(\phi) \rho(t) R_z^\dagger(\phi)\right]$$

$$= \mathrm{Tr}\left[\rho(\tau) \rho_\phi(\tau)\right]$$ **Overlap of density Matrices**

**$I_m$=Multi-quantum intensities**

$$= \sum_{m=-N}^{N} \mathrm{Tr}[\rho_{-m}(\tau) \rho_m(\tau)] e^{-im\phi}$$

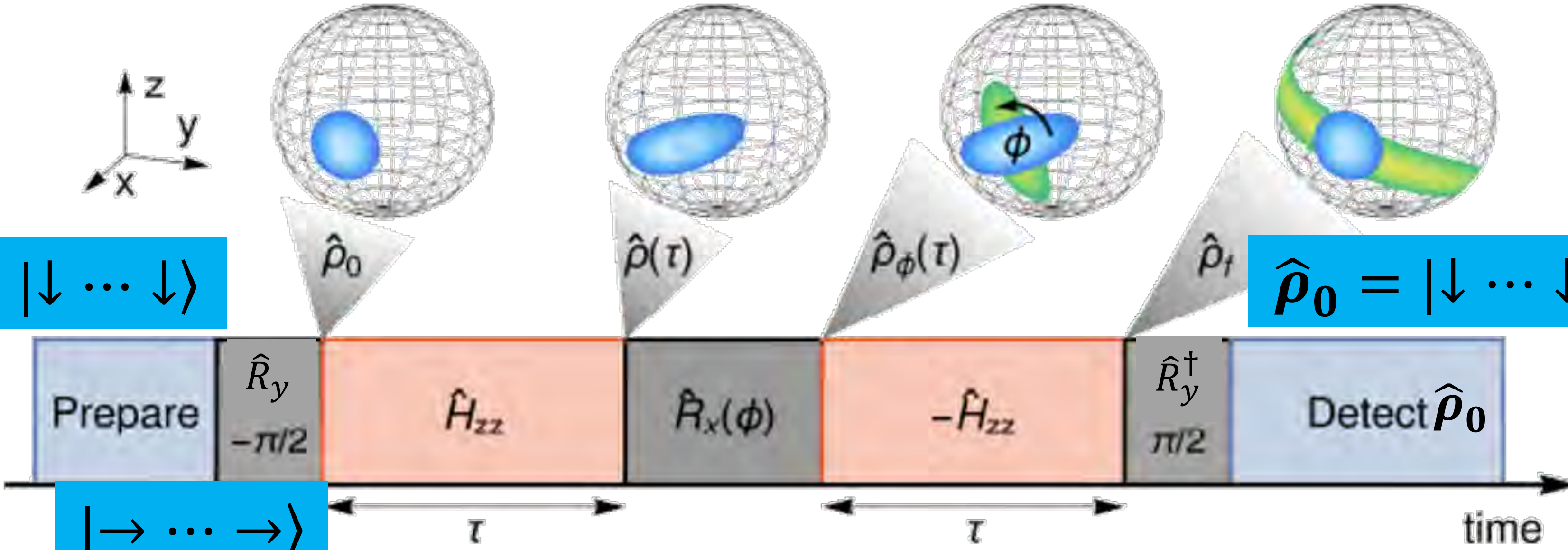
$$I_m = \mathrm{Tr}[\underbrace{\rho_{-m}(t) \rho_m(t)}_{\text{Type of purity}}]$$

$$I_0 = \mathrm{Tr}[\rho_0^2(t)]$$

$$= \sum_{m=-N}^{N} I_m e^{-im\phi}$$

**Fourier transform: $\phi$ gives the Multi-Quantum spectrum.**

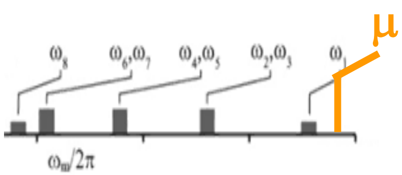
# Multi-quantum Coherences: Ions

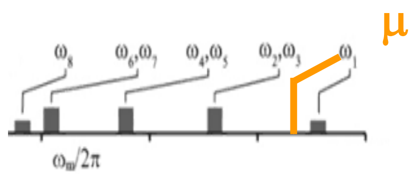



$|\downarrow \cdots \downarrow\rangle$

$|\rightarrow \cdots \rightarrow\rangle$

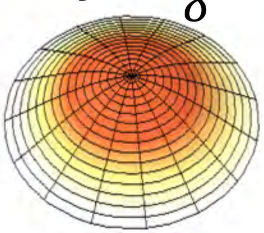
$\hat{\boldsymbol{\rho}}_0 = |\downarrow \cdots \downarrow\rangle\langle\downarrow \cdots \downarrow|$

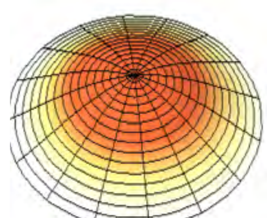



$J \sim \frac{J_0}{\delta}$

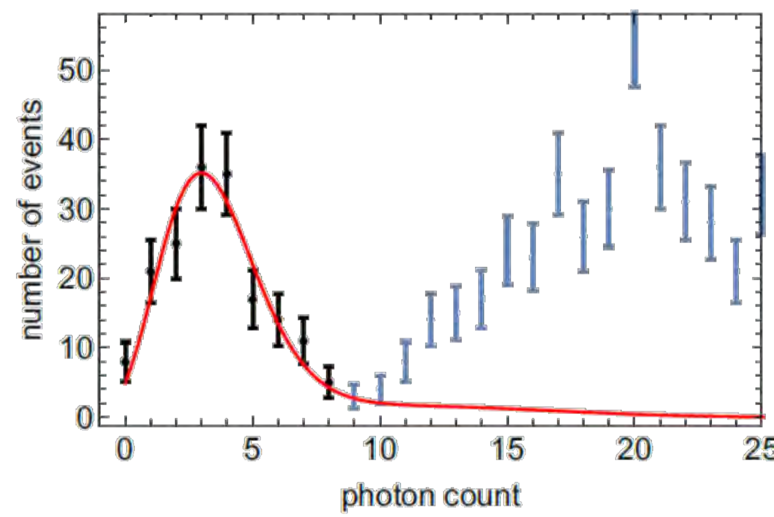
$J \sim -\frac{J_0}{\delta}$



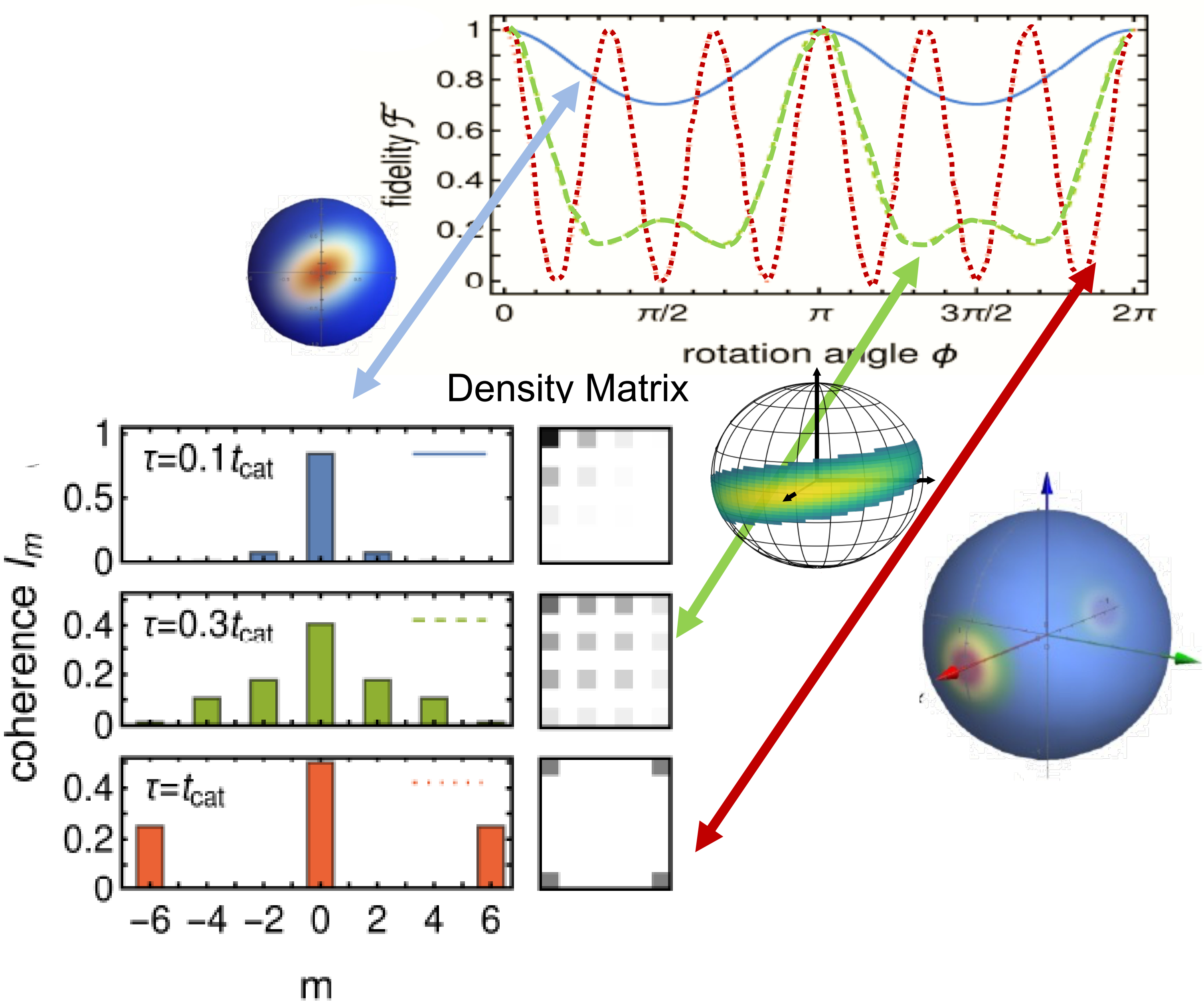


$S_z^2 = \frac{1}{4}(\boldsymbol{S_x^+ S_x^+} + \boldsymbol{S_x^- S_x^-} + S_x^+ S_x^- + S_x^- S_x^+)$

**Pure initial state -> Fidelity**
**Probability of all down**

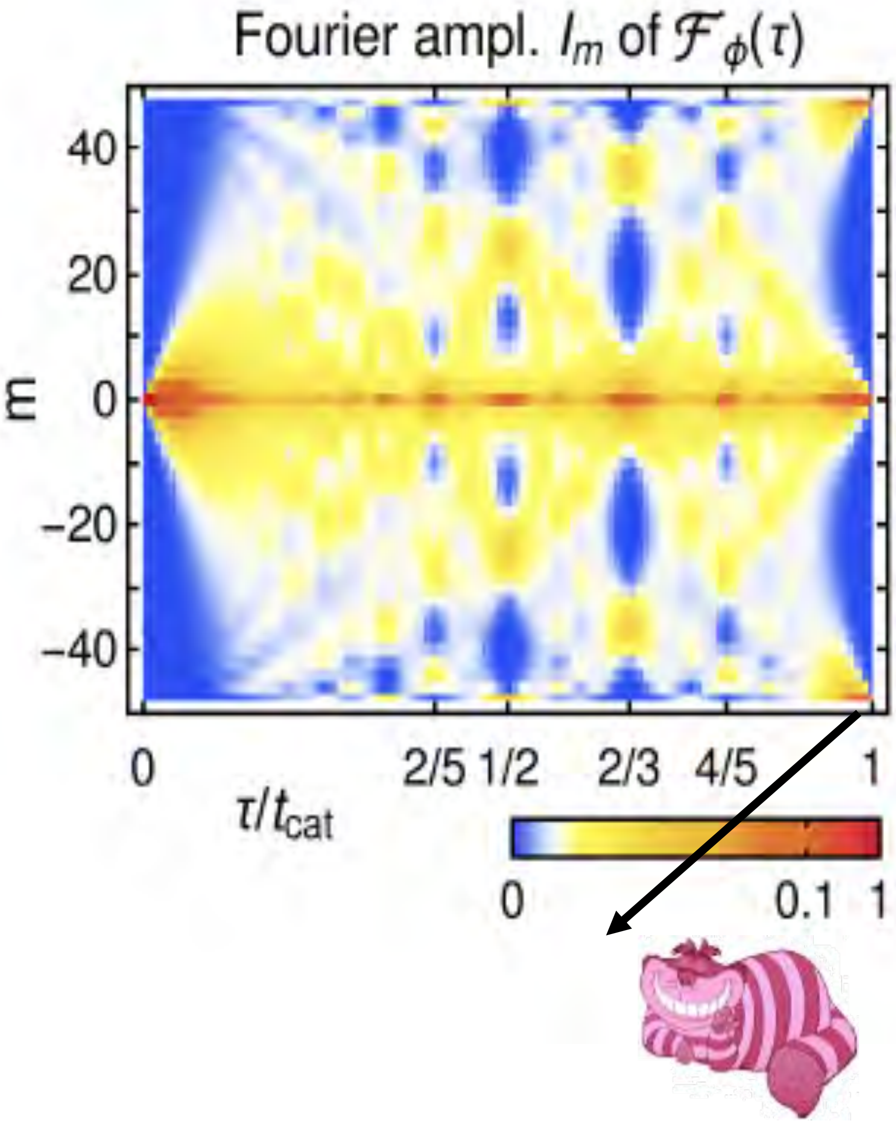


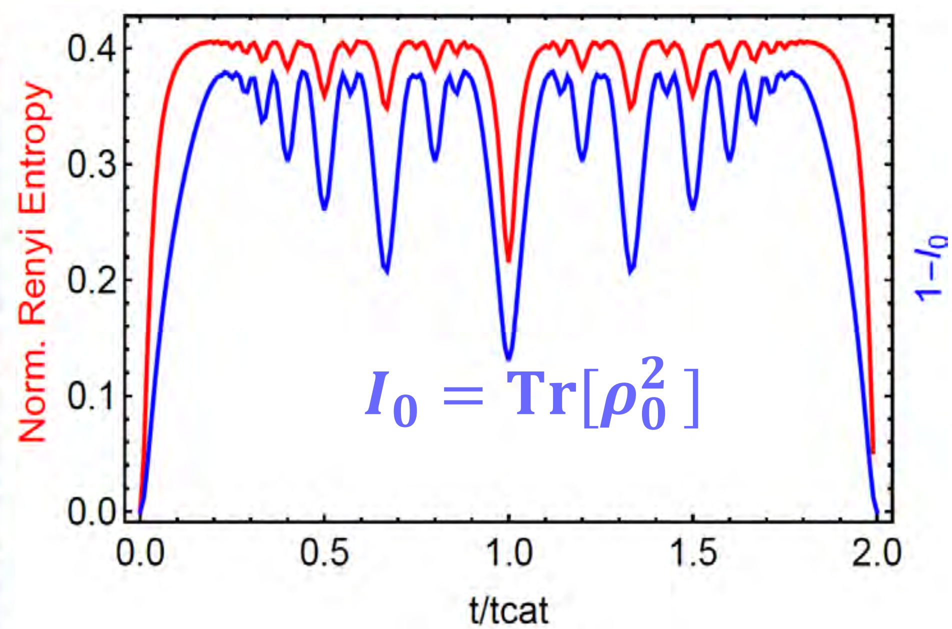
# MQC: Example N=6



# Multi-quantum Coherences: N=48

**Information stored in the initial (local) state is distributed , through the interactions, over many-body degrees of freedom of the system.**




**Detailed structure of the state …**

# Multiple quantum coherence an OTOC

$\mathfrak{I}_{\boldsymbol{\phi}}(t) = \langle \boldsymbol{\rho_0} \rangle$ $\qquad \widehat{\boldsymbol{\rho}}_{\mathbf{0}} = |\downarrow \cdots \downarrow\rangle\langle\downarrow \cdots \downarrow|$

$$= \left\langle \boldsymbol{\Psi_0} \right| e^{-itH_{xx}} R_z^{\dagger}(\phi)\, e^{-itH_{xx}} \widehat{\rho}_0\, e^{itH_{xx}} R_z(\phi) e^{-itH_{xx}} \left| \boldsymbol{\Psi_0} \right\rangle$$

$W = R_z(\phi)$ $\qquad V = \widehat{\rho}_0$ $\qquad$ Two commuting operators

$$= \langle \boldsymbol{\Psi_0} | \underbrace{e^{itH_{xx}} W^{\dagger} e^{-itH_{xx}}}_{W_t^{\dagger}} V^{\dagger} \underbrace{e^{itH_{xx}} W e^{-itH_{xx}}}_{W_t} V | \boldsymbol{\Psi_0} \rangle$$

$$= \langle W_t^{\dagger} V^{\dagger} W_t V \rangle$$ **Out-of-time order correlations (OTOCs)**

$\mathfrak{I}_{\boldsymbol{\phi}}(t)$=*1-C(t)* $\qquad C(t) = \langle [W_t, V]^{\dagger} [W_t, V] \rangle$

$\mathfrak{I}_{\boldsymbol{\phi}}(t)$ measures the degree of non-commutativity of V and the time evolved version of W

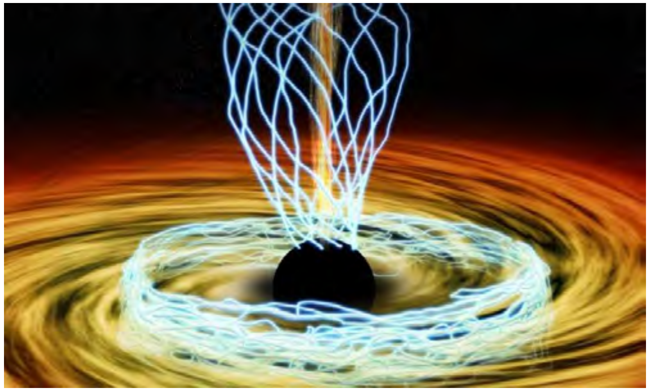
**Scrambling of quantum information**

# Quantum Scrambling

- **Scrambling occurs when local quantum information, is spread over all the degrees of freedom of a system, becoming inaccessible to local measurements**

- Link to entanglement entropy: thermalization
- Connections to quantum gravity: Black holes scramble quantum information as fast as possible: $C(t) \sim e^{\lambda t}$

  [Hayden-Preskill, Sekino-Susskind, Shenker-Stanford '13, Kitaev '14]



- Bound of growth of quantum chaos: $\lambda$ Lyapunov exponent

  [Maldacena-Shenker-Stanford][Martinis'16]

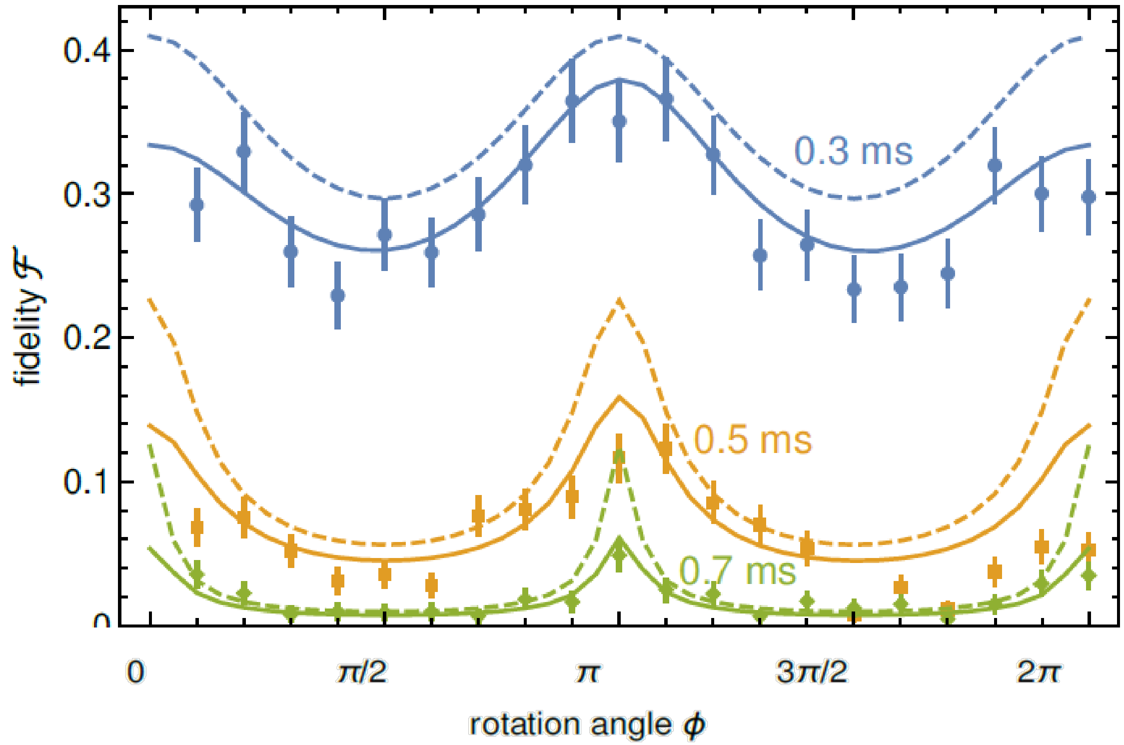
- Can we asses them?

[Swingle-Bentsen-Schleier-Smith-Hayden '16]
[Zhu-Hafezi-Grover '16]
[Yao-Grusdt-BGS-Lukin-StamperKurn-Moore-Demler '16]

(2017)
doi:10.1038/nphys4119
doi:10.1038/nphys4119

# Fidelity Measurements

**N=48**




**Solid lines: decoherence + phonons**

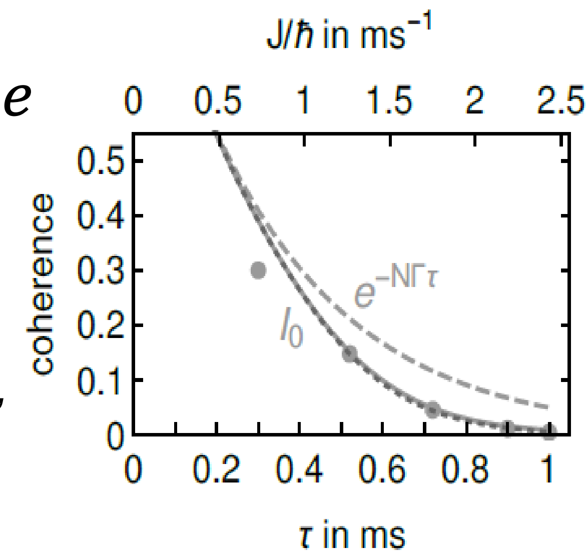
**Dashed Lines: decoherence**

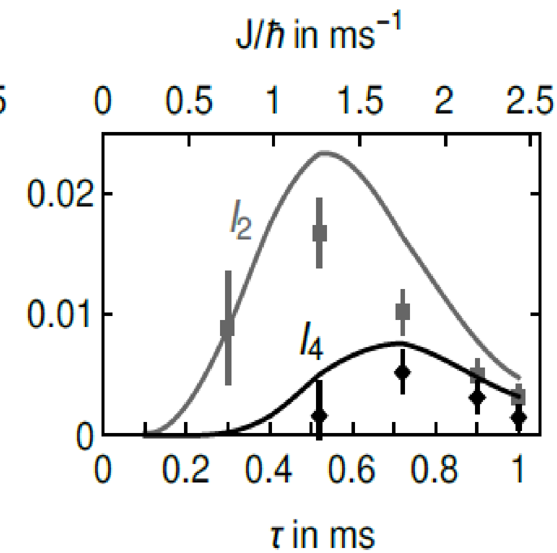
$\tau_{cat}$~15 ms

$$I_0(\tau) = e^{-\Gamma N \tau} I_0^{pure}$$

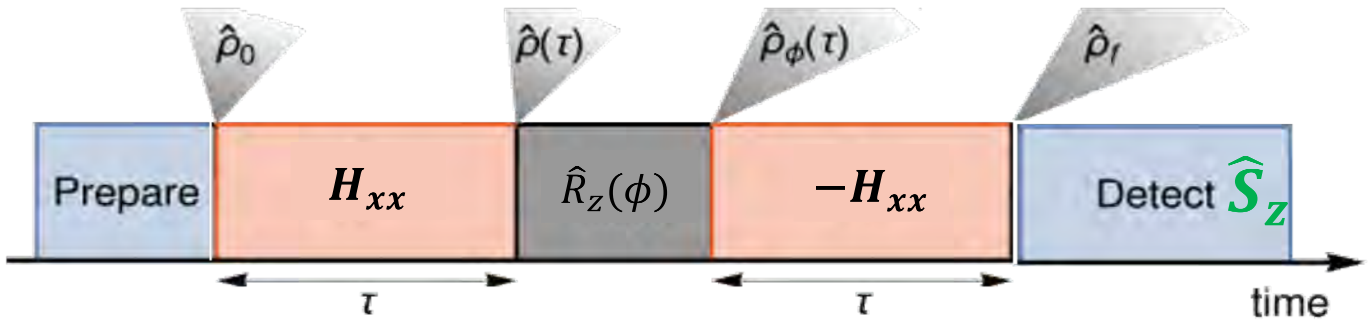
$$I_0^{pure}(\tau) = (1 + J^2\tau^2)^{-1}$$

Garttner et *al Nature Physics*, doi:10.1038/nphys4119



# Measuring OTOCS




**Measure Magnetization:** $\hat{S}_z$ $\quad W = R_z(\phi) \quad V = \hat{\sigma}_i^z$

$$F_\phi(\tau) = \frac{2}{N}\langle S_z\rangle$$

$$= \langle \Psi_0 |\, e^{-i\tau H_{xx}}\, R_z^\dagger(\phi)\, e^{-i\tau H_{xx}}\, \hat{\sigma}_i^z e^{i\tau H_{xx}} R_z(\phi) e^{-i\tau H_{xx}} |\Psi_0\rangle$$

$$= \frac{2}{N}\Big\langle \Psi_0 \Big|\, \underbrace{e^{i\tau H_{xx}}\, W\, e^{-i\tau H_{xx}}}_{W_t^\dagger}\, \underbrace{\hat{\sigma}_i^z}_{V}\, \underbrace{e^{i\tau H_{xx}}\, W e^{-i\tau H_{xx}}}_{W_t}\, \underbrace{\hat{\sigma}_i^z}_{V} \Big|\Psi_0 \Big\rangle$$

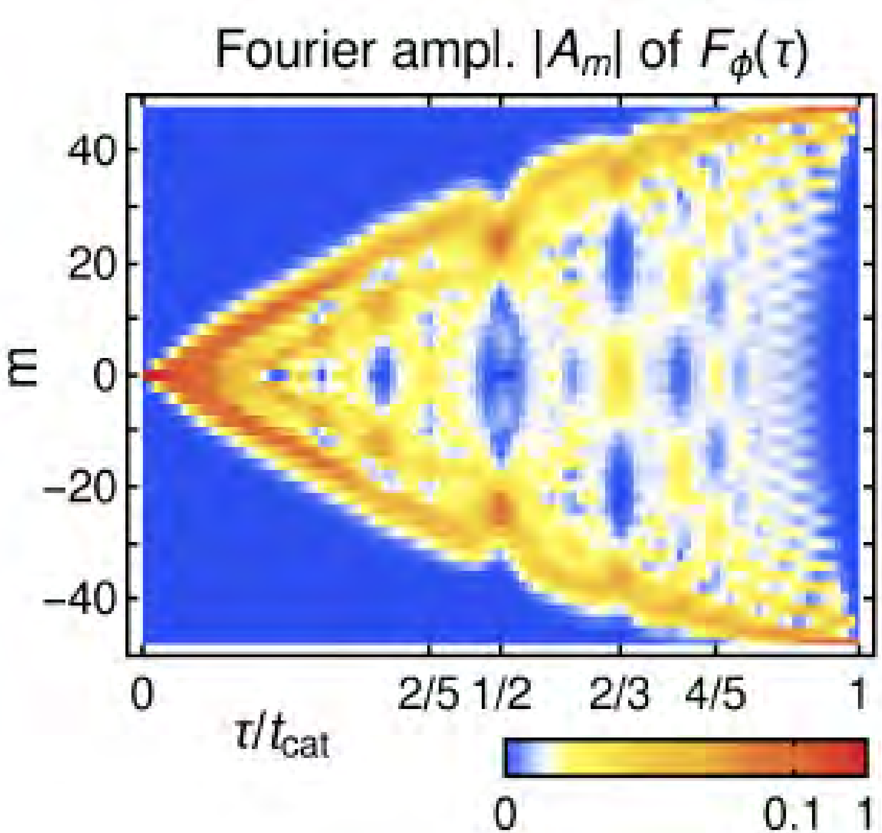
Same measurements done in NMR

# Magnetization OTOCs: N=48

$$F_\phi(\tau) = \frac{2}{N}\langle S_z \rangle = \sum_{m=-N}^{N} A_m(\tau) e^{-im\phi}$$

**Fourier component:**
$A_m$ →**m-body correlations**

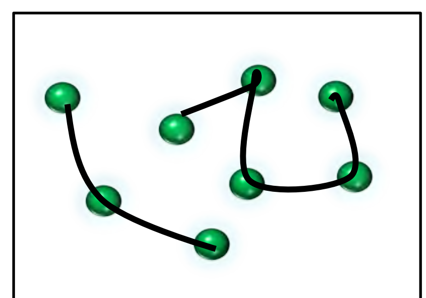
**A non-zero $A_m$ signals the buildup of at least m-body correlations.**




- In the case of the Ising model:

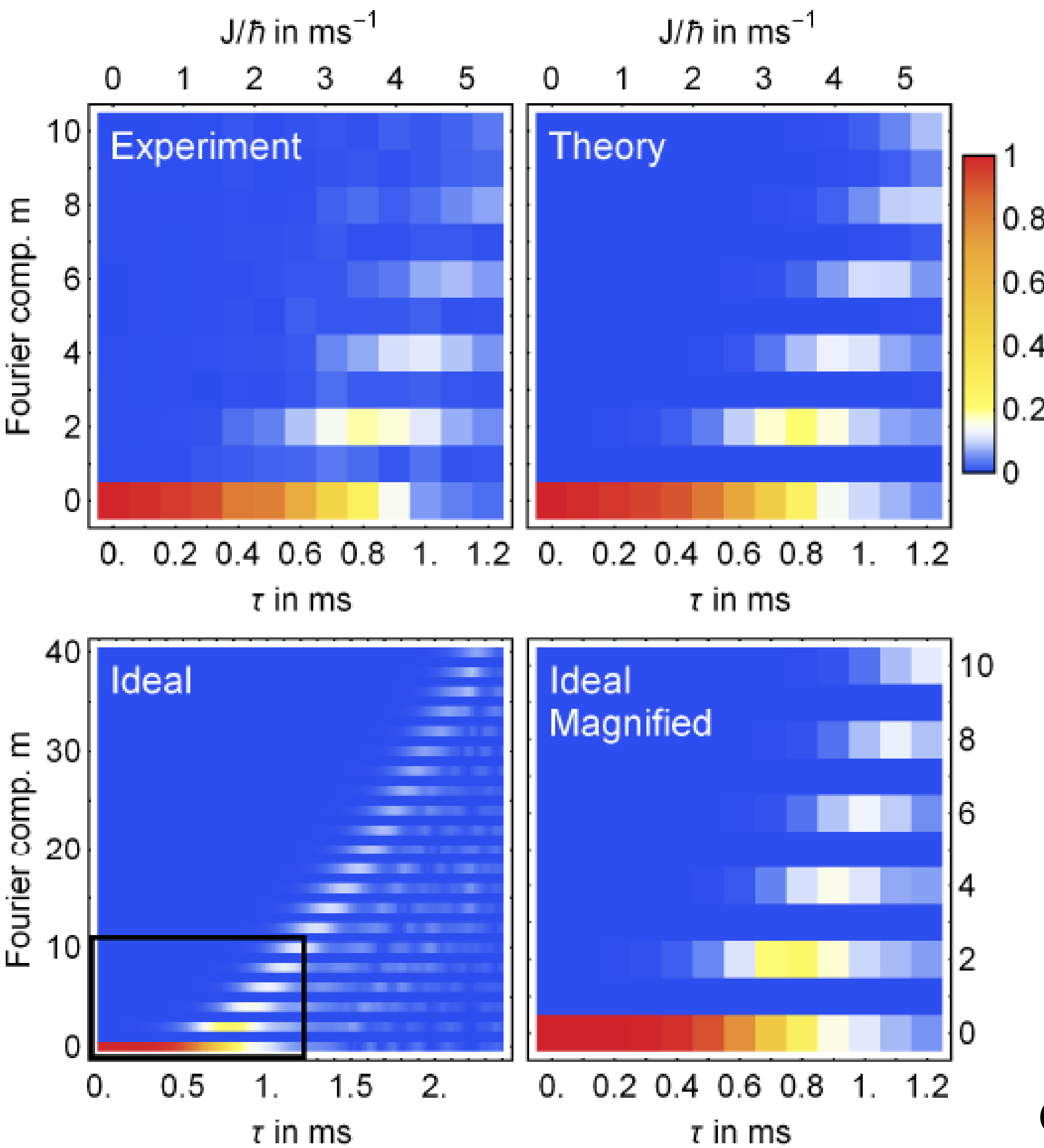
- A non-zero $A_m$ signals the existence of m spins directly coupled by the Hamiltonian.

- $A_{m+1}$ grows as $t^p$ with p>m



$A_{m>5}=0$

# Magnetization Measurements




**N=111**

**Solid lines: decoherence + phonons**

**Dasehd lines: decoherence**

$\tau_{cat}$=17 ms

**Up to m=8 significant correlations!!**

Garttner et *al Nature Physics*, doi:10.1038/nphys4119

# Penning trap simulator: A great vista ahead !

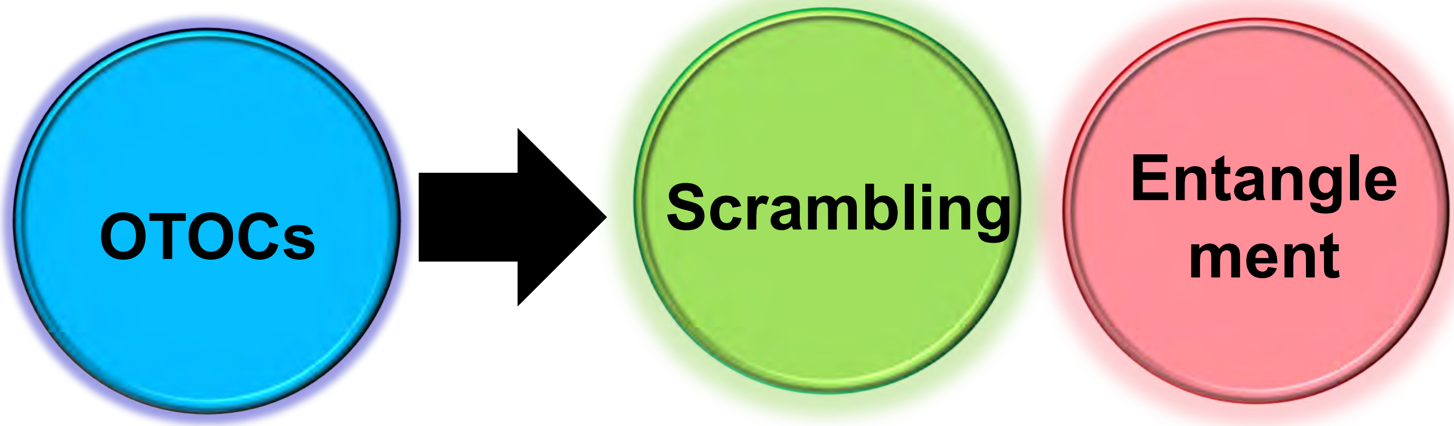
## Future Directions

Complexity

- **Transverse field**, and variable range
- **Mitigate decoherence : sub-Doppler**
- **Spatial correlations –single ion readout**

**Thank You!**

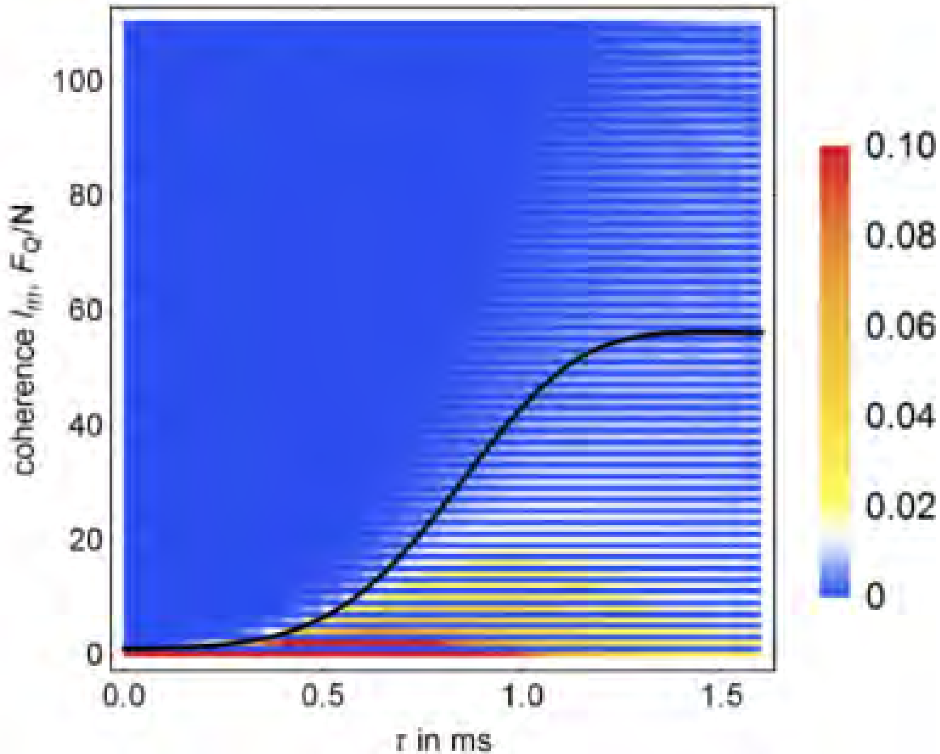
- **Measure OTOCS**
- Generate and observe **spin squeezed states**
- **Implement time-reversible Ising interactions in 2D arrays of 100’s of ions**

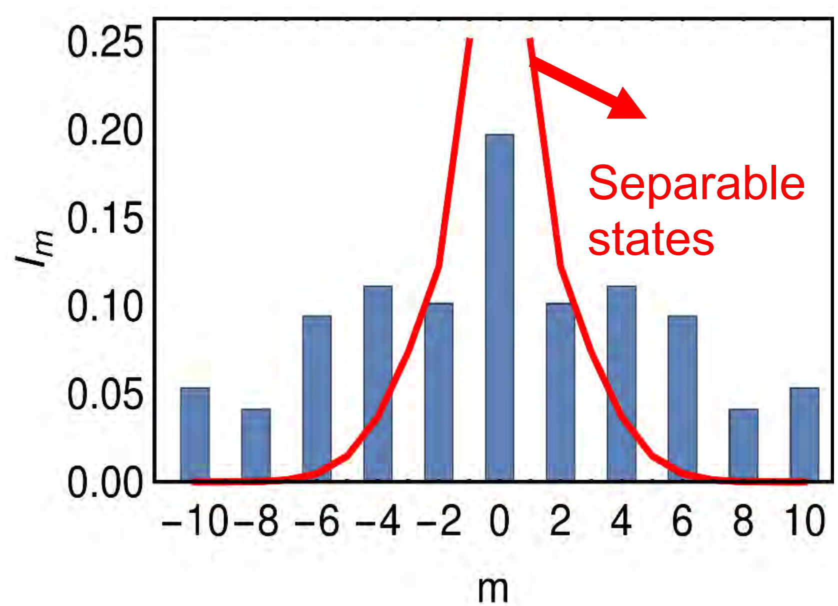



$$F_Q \geq 2\sum_{m=-N}^{m=N} m^2 I_m$$

Pure states saturate equality

Individual Im can detect entanglement too

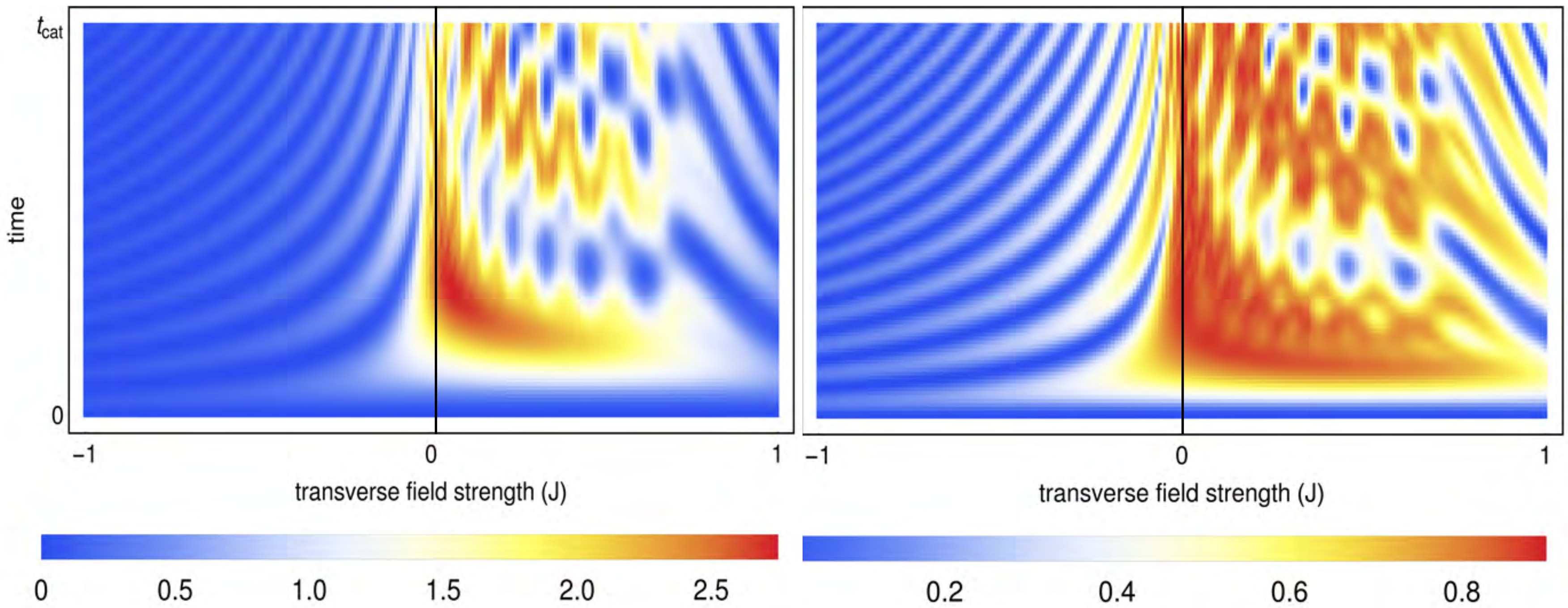


# Connection to Entanglement Entropy

$$H = -\frac{J}{N} S_z^2 - \Omega S_x$$

Renyi Entropy: $S_R$ 1-$I_0$ $N = 20$




Why? $I_0(\tau) = \frac{1}{2\pi}\int_0^{2\pi} d\phi\, \Im_\phi(\tau)$

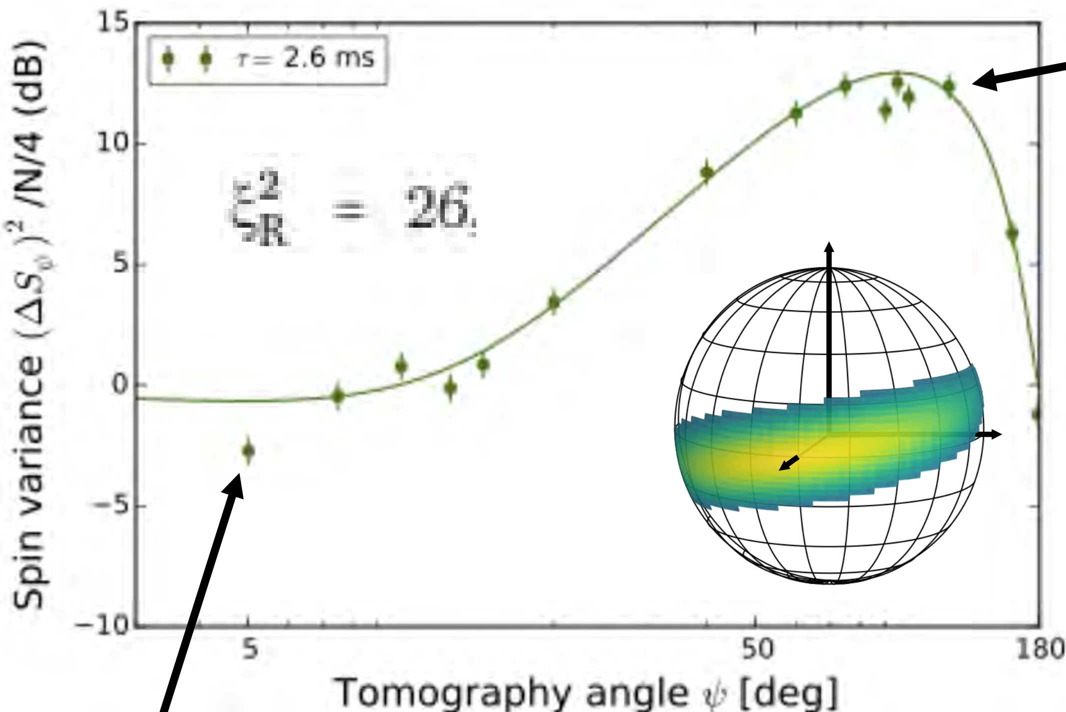
$$e^{-S_A} = \sum_{W \epsilon \bar{A}} \mathrm{Tr}\left[ W_t^\dagger \, \hat{O} \; e^{-\beta H} \, \hat{O}^\dagger W_t \, \hat{O} \, e^{-\beta H} \, \hat{O}^\dagger \right]$$

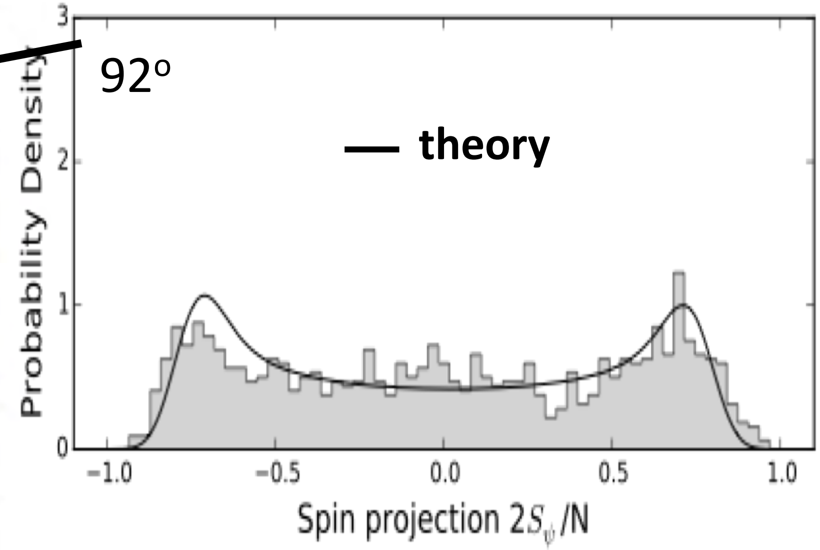
We sum over an incomplete set of operators

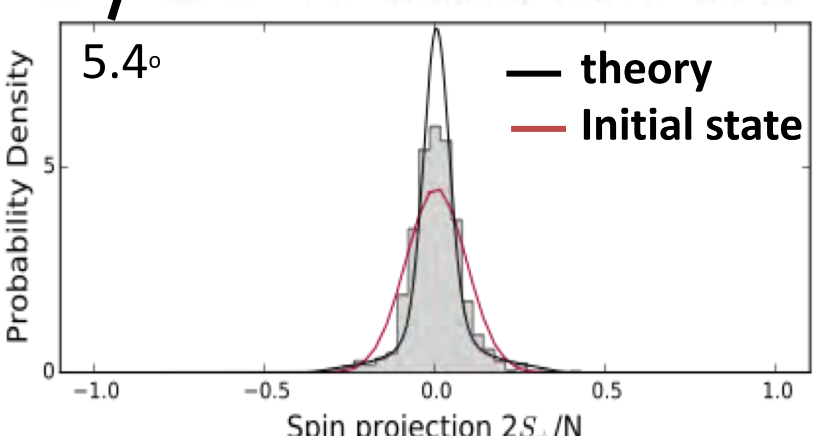
$$V = \hat{O} \, e^{-\beta H} \, \hat{O}^\dagger = \hat{\rho}_0$$

# Full Counting statistics: More information

N=127 $\pm$ 4



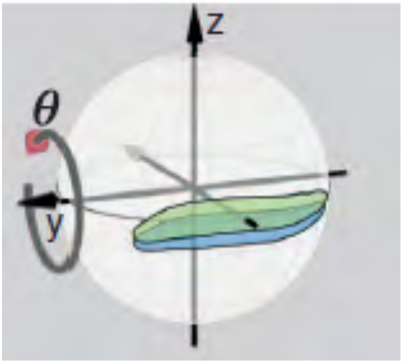




**Theory: need to compute N-point correlations**:

$$\langle \sigma_1^+ \sigma_2^+ \sigma_3^+ \cdots \sigma_n^+ \sigma_{n+1}^- \cdots \sigma_N^- \rangle$$

# Fisher Information $F_Q$:

Sensitivity of a quantum state with respect to an unitary transformation parametrized by a classical parameter, $\theta$:

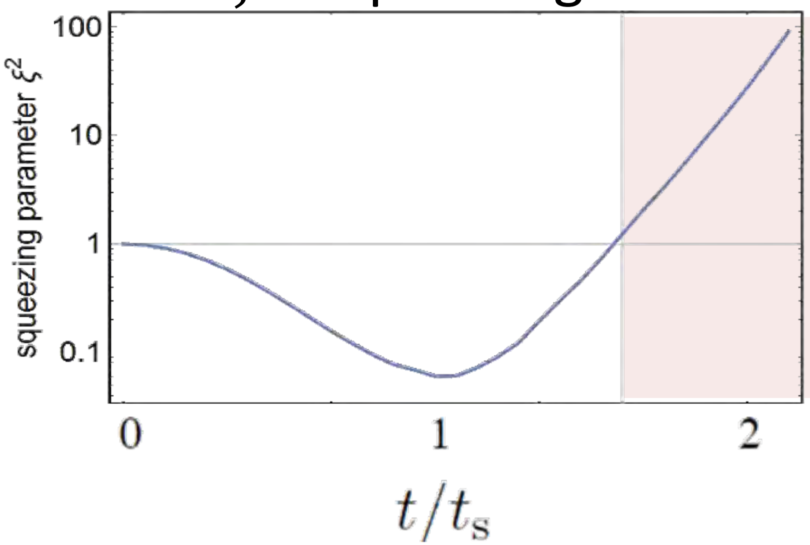



Strobel, et al., Science 345, 424 (2014).

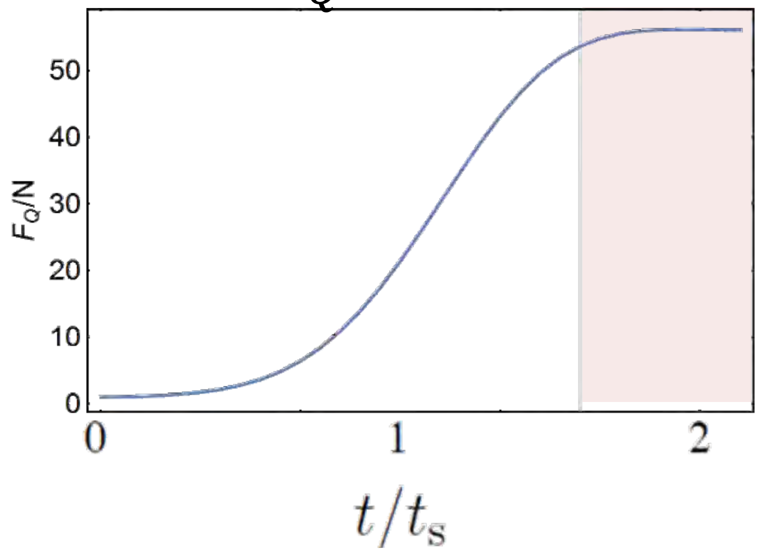
- Many-particle entanglement witness: $F_Q/N > k$ k-body entanglement
- Beyond Gaussian states
- Determines enhanced sensitivity

For the one axis twisting $F_Q = 4\langle(\Delta \hat{S}_y)^2\rangle$

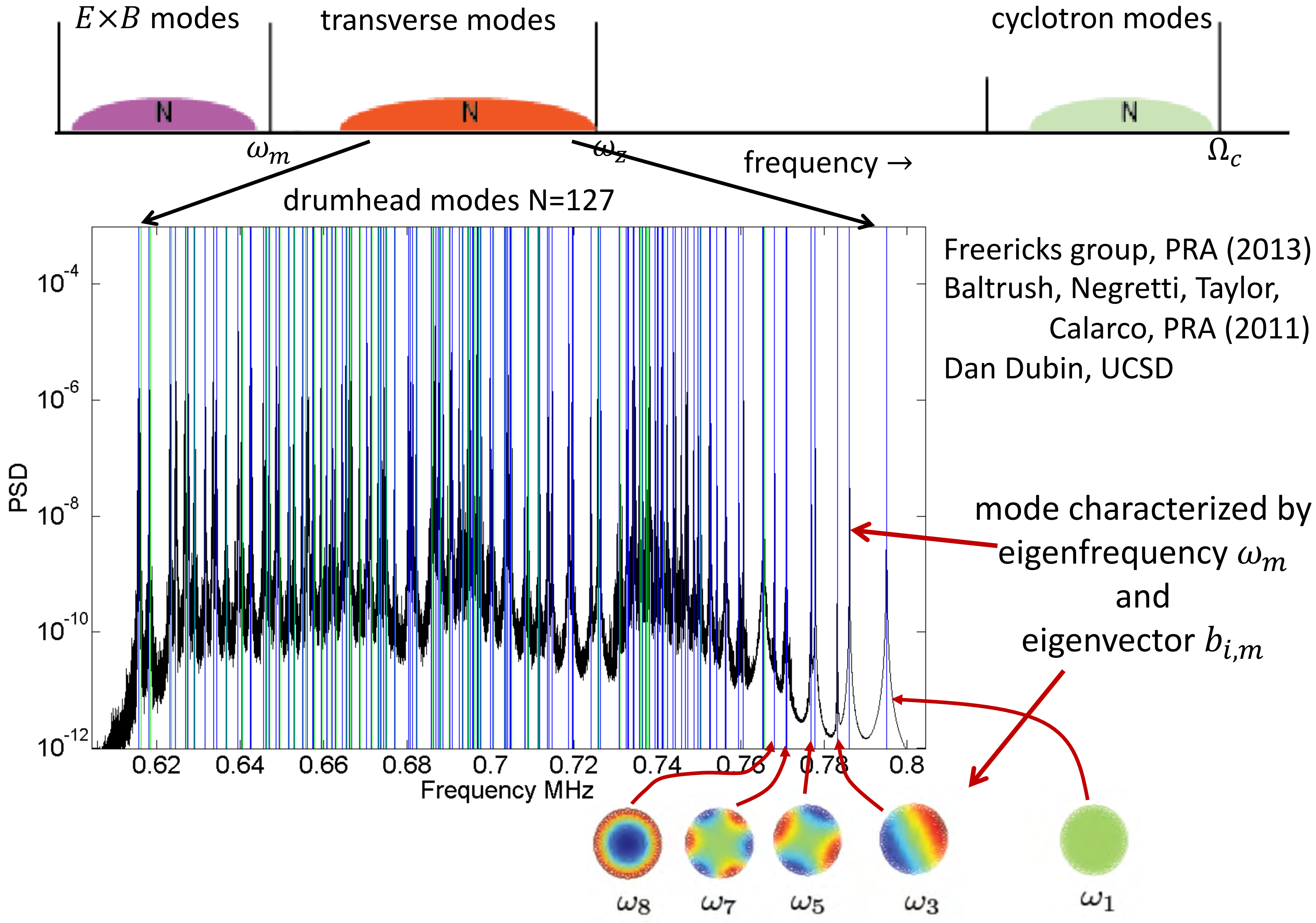
$\xi^2$: Squeezing




$F_Q$: Fisher



# Transverse (drumhead) modes




Freericks group, PRA (2013)

Baltrush, Negretti, Taylor, Calarco, PRA (2011)

Dan Dubin, UCSD