Non equilibrium quantum matter
Mainz, 30 May- 2 June 2017

SUPERFLUIDITY AND ROTATION OF A SPIN-ORBIT COUPLED BEC GAS

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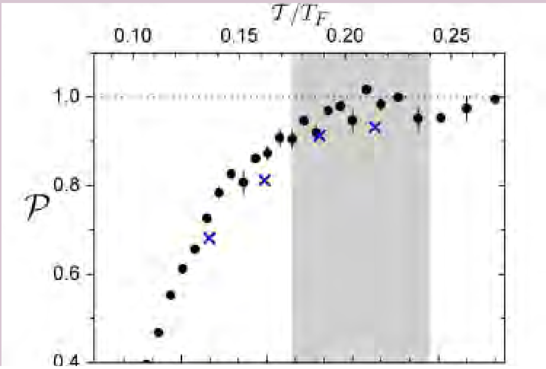
QUIC

PROVINCIA AUTONOMA DI TRENTO

IRROTATIONALITY OF VELOCITY FIELD IS DISTINCTIVE FEATURE OF ROTATING SUPERFLUIDS

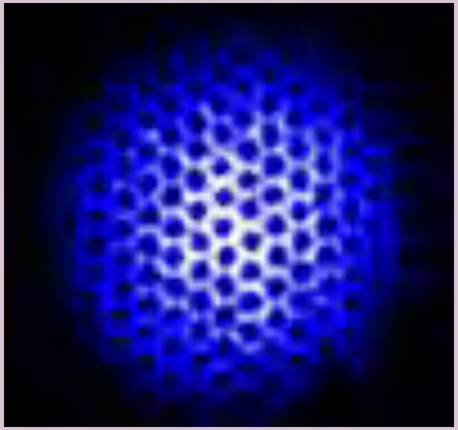
$$\vec{v}(\vec{r}) = \frac{\hbar}{m} \nabla \phi(\vec{r})$$

Role of the phase $\phi(\vec{r})$ of the order parameter

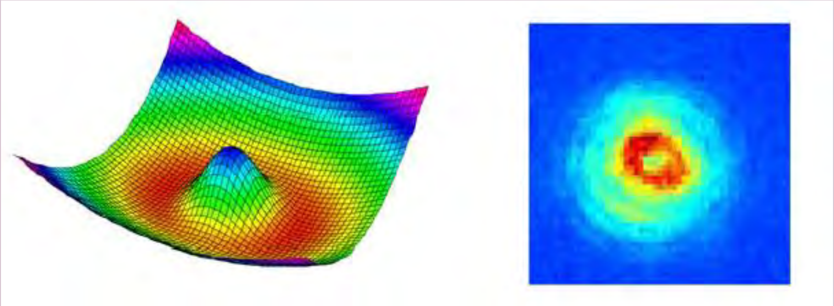


QUENCHING OF MOMENT OF INERTIA
(Innsbruck 2011)

- **Quantization of vortices**
  (ENS, JILA, MIT, 2000)



Persistent currents and Quantization of circulation



(Nist 2007)

Main message of this talk:

Spin-orbit coupling violates current-phase relation

$$\vec{j}(\vec{r}) = \frac{\hbar}{m} n(\vec{r}) \vec{\nabla} \phi(\vec{r})$$

and yields

violation of irrotationality of velocity field

with emergence of diffused vorticity

$$\nabla \times \vec{v} \neq 0$$

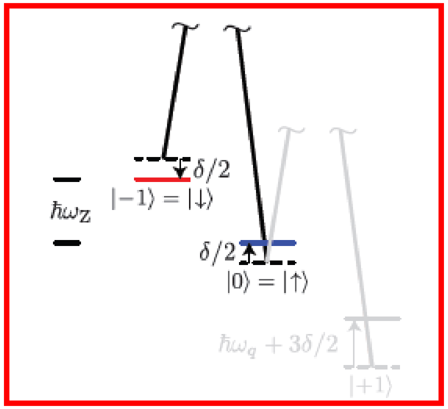
Questions addressed in this talk

- Calculation of **superfluid density** in uniform matter and quenching caused by spin-orbit coupling
- **Violation** of **irrotationality** constraint and emergence of **diffused vorticity:** consequences on **moment of inertia** and **quantum of circulation**
- **Anisotropic** expansion from **isotropic** trap

- Yi-Cai Zhang et al., Phys. Rev. A 94, 033635 (2016)
- S. Stringari, Phys. Rev. Lett. 118 145302 (2017)
- Chunlei Qu, Lev Pitaevskii and S.S., arXiv:1704.00677

Raman induced 1D spin-orbit Hamiltonian
Spielman (Nist 2009)

Two detuned and **polarized** laser beams + non linear Zeeman field provide Raman transitions between two spin states, giving rise to new s.p. Hamitonian



$$h_0 = \frac{1}{2m}[(p_x - \hbar k_0 \sigma_z)^2 + p_\perp^2]$$
$$-\frac{\hbar}{2}\Omega\sigma_x + \frac{\hbar}{2}\delta\sigma_z$$

$p_x = -i\hbar\partial_x$ is canonical momentum

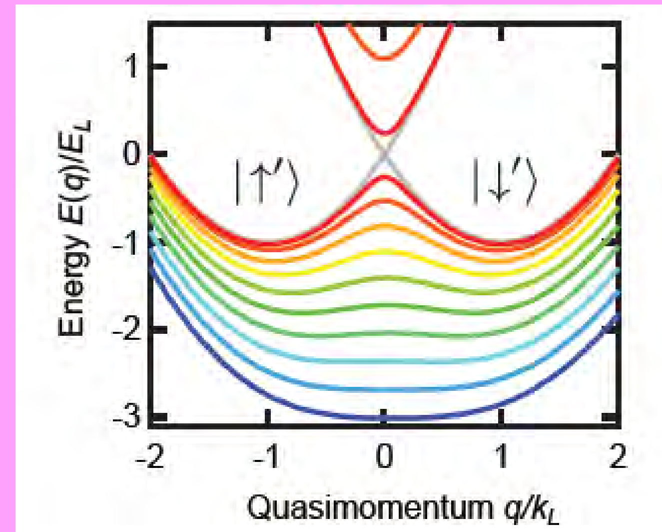
k_0 is laser wave vector difference

Ω Raman coupling, fixed by laser intensity

δ is effective detuning

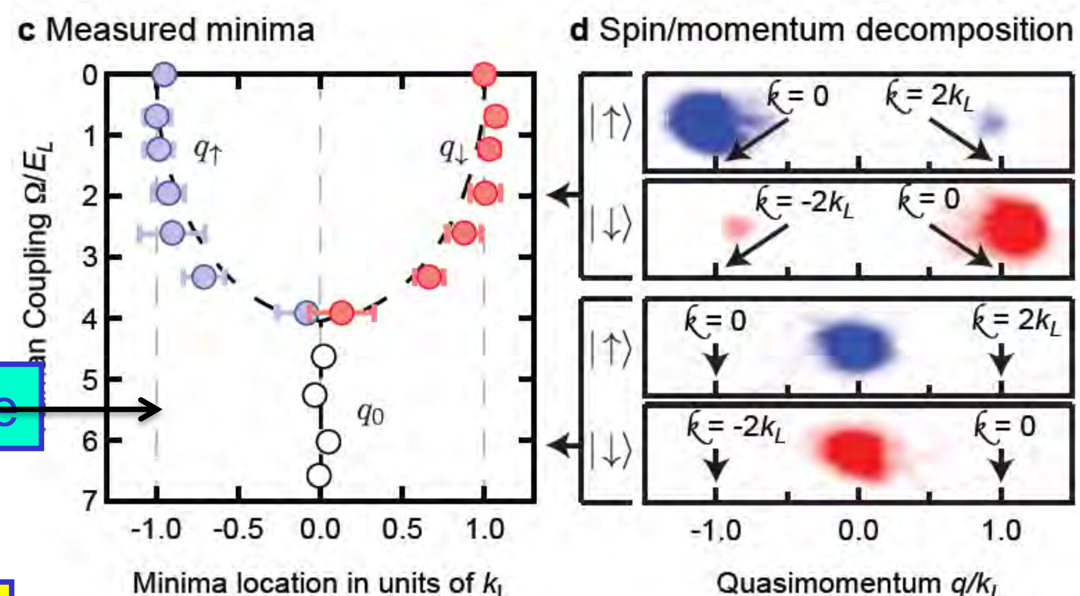
Single particle Hamiltonian gives rise to two band structure ($\delta = 0$)

- If $\Omega < 2\hbar k_0^2 / m$ lowest band exhibits **two degenerate minima** which can host a BEC
- If $\Omega > 2\hbar k_0^2 / m$ lowest band exhibits **single-minimum**



Plane wave phase

Single momentum phase



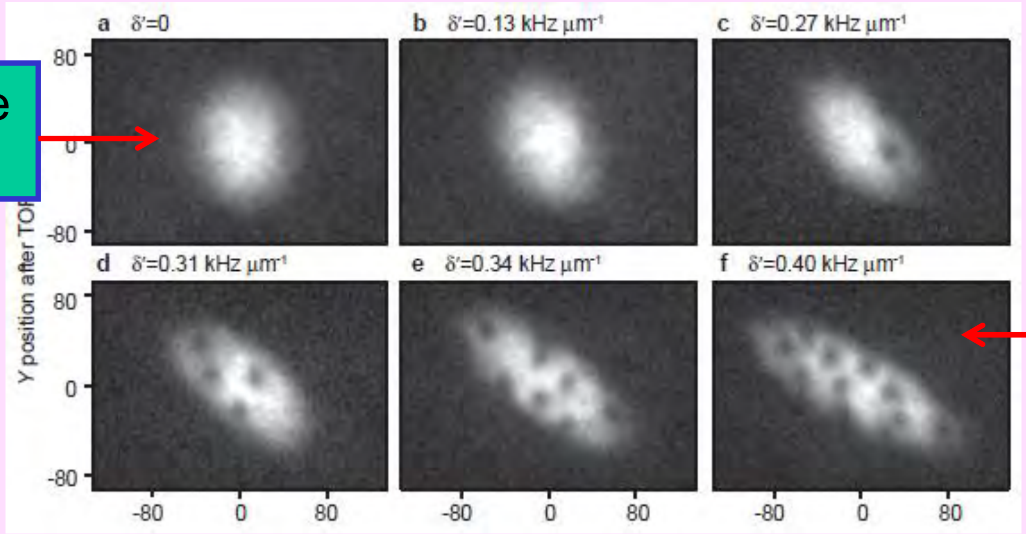
Second order phase transition at $\Omega = 2\hbar k_0^2 / m$

(Lin et al. Nature 2011)

Major motivation is generation of **synthetic magnetic** fields and **effective Lorentz force** in neutral atoms causing the emergence of **quantized vortices**. This is achieved by including **y-dependence** in **detuning field**

$$\delta \rightarrow \delta + \delta' y$$

No y-dependence In detuning



strong y-dependence In detuning

(Lin et al., Nature 2009)

Spin orbit Hamiltonian

$$h_0 = \frac{1}{2m}[\left(p_x - \hbar k_0 \sigma_z\right)^2 + p_\perp^2] - \frac{\hbar}{2}\Omega\sigma_x + \frac{\hbar}{2}\delta\sigma_z$$

is **translationally invariant.**

However it **breaks Galilean invariance,** as physical momentum $(p_x - \hbar k_0 \sigma_z)$ does not commute with h_0 .

SOC Hamiltonian yields result

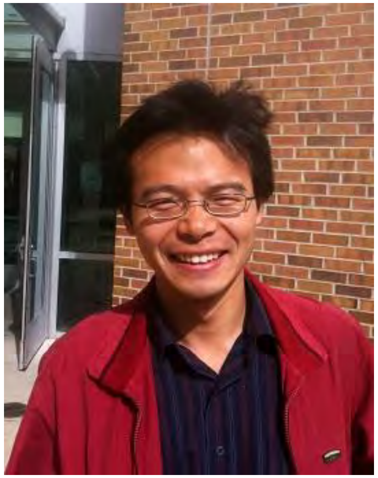
$$j_x(\vec{r}) = \frac{\hbar}{m}[n(\vec{r})\nabla_x \phi(\vec{r}) - k_0 s_z(\vec{r})]$$

for the **current** and can yield **violation of irrotational** constraint $\vec{v}(\vec{r}) = \vec{j}(\vec{r})/n(\vec{r})$ for velocity field.

Violation of Galilean invariance raises the question of breakdown of superfluidity and of consequences on the rotational properties

Suppression of superfluidity in SOC Bose-Einstein condesed gases

Collaboration with Lev Pitaevskii (Trento)
and Shizhong Zhang + collaborators (Hong Kong)
Yi-Cai Zhang et al., Phys. Rev. A 94, 033635 (2016)



Normal density $\rho_n = \rho - \rho_s$ is defined by static response to **transverse current** operator $J_x^T(q) = \sum_k P_{k,x} e^{iqy_k}$. At T=0 one finds

$$\frac{\rho_n}{\rho} = \frac{1}{N} \lim_{q \to 0} \sum_n \frac{|<0| J_x^T(q) | n>|^2}{E_n - E_0} + (q \to -q)$$ (Baym 1969).

At T=0 normal density vanishes in **Galilean invariant** superfluids (liquid Helium, usual BEC and Fermi gases) and system is **fully superfluid** ($\rho_s = \rho$) .

In SOC gases physical momentum P does not coincide with canonical momentum and current operator takes the form

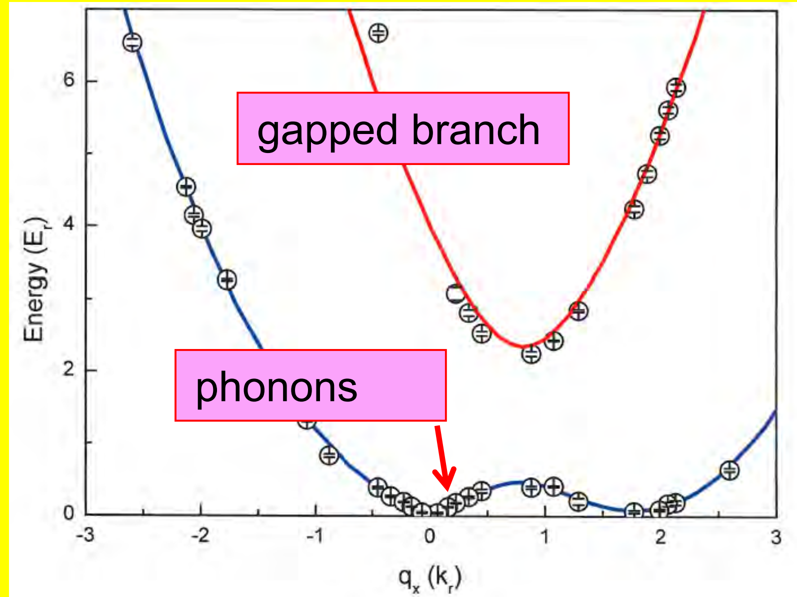
$$J_x^T(q) = \sum_k (p_{k,x} - \hbar k_0 \sigma_{k,z}) e^{iqy_k}$$

Normal density is **no longer expected to vanish** in SOC gases.

To calculate ρ_n one needs to know spectrum of **elementary excitations**

Spinor BEC's exhibit two branches in the excitation spectrum

Due to Raman coupling
only one branch is gapless
and exhibits phonon behavior at small q



Exp: Si-Cong Ji et al., PRL 2015;
Khamehchi et al, PRA 2014
Theory: Martone et al., PRA 2012

- **Phonons** are longitudinal excitations and **do not contribute** to transverse current response function.
- **Gapped branch** (caused by Raman coupling Ω) do instead contribute

Results for normal density in plane wave and single momentum phase ($\delta = 0$)

Plane wave phase

$$\Omega \leq \Omega_c$$

$$\frac{\rho_n}{\rho} = \frac{\Omega^2}{\Omega_c^2}$$

Single momentum phase

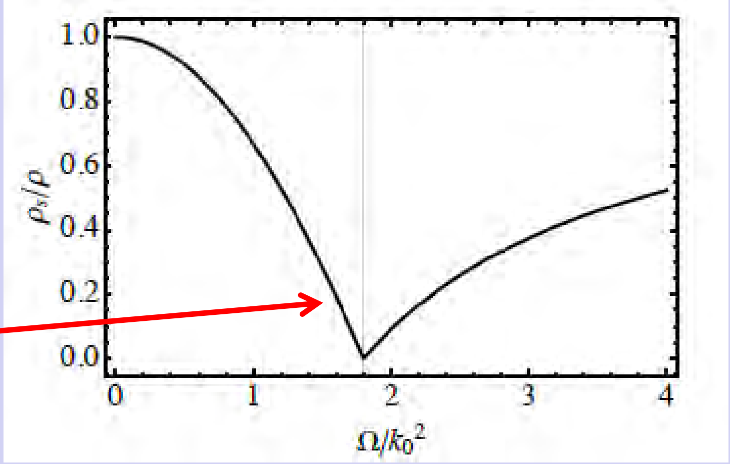
$$\Omega \geq \Omega_c$$

$$\frac{\rho_n}{\rho} = \frac{\Omega_c}{\Omega}$$

$$\Omega_C = 2\hbar k_0^2 / m$$

At the transition between the two phases one finds $\rho_n = \rho$ and superfluid density $\rho_S = \rho - \rho_n$ identically vanishes:

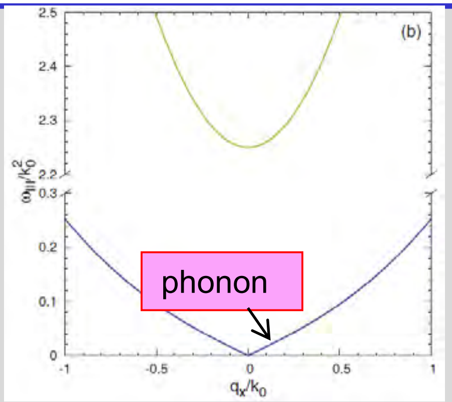
BEC without superfluidity !!



parameters of Rb87

CAN WE MEASURE ρ_s ?

Differently from Galilean invariant systems,



f-sum rule

$$\hbar^2 \int d\omega \omega S(q,\omega) = \hbar^2 \frac{q^2}{m}$$

is **not exhausted** by phonon branch ($\omega = cq$).
Contribution of **phonon** branch fixed by superfluid fraction ρ_s / ρ

Phonon branch instead exhausts compressibility sum rule

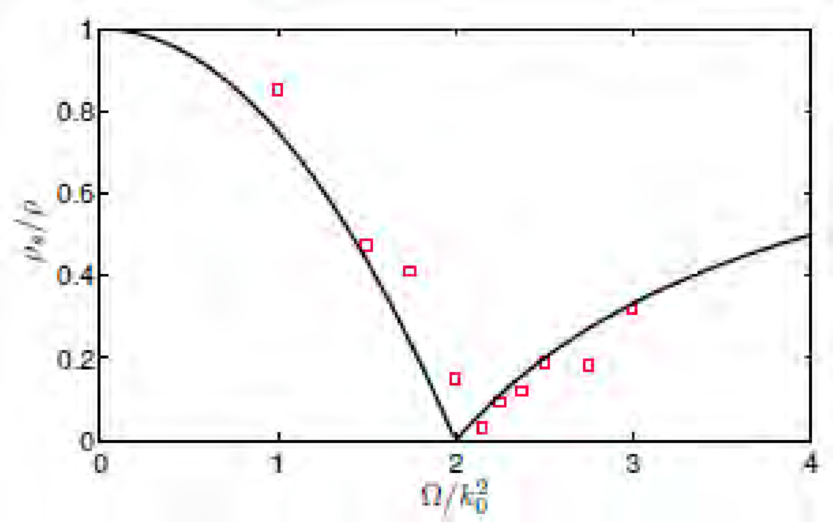
$$\lim_{q \to 0} \int \frac{d\omega}{\omega} S(q,\omega) = \kappa$$

Macroscopic relationship $\rho_s = \rho m c^2 \kappa$ holding at T=0

between **superfluid** density, **sound** velocity and **compressibility**.

- Equation of state (and hence compressibility) is not modified by SOC. **Velocity** of **sound** instead **exhibits strong suppression** (measured by Si-Cong ji et al., PRL 2015)

Superfluid density as a function of Raman coupling



$$\rho_s = \rho m c^2 \kappa$$

Yi-Cai Zhang et al., Phys. Rev. A 94, 033635 (2016)

- Full line is prediction of theory

- Experimental points are obtained using measured values of sound velocity

Consequences of reduced superfluidity on the **rotational properties** of a spin-orbit coupled BEC

Moment of inertia in the presence of spin-orbit coupling

(S.S., PRL 118 145302 (2017))

Moment of inertia is **linear response** to static **angular momentum** constraint $< L_z >_{\omega_{rot} \to 0} = \omega_{rot} \theta$

$$\mathrm{H} \to \mathrm{H} - \omega_{rot} \mathrm{L}_z$$

In Bose-Einstein condensates without **spin-orbit coupling** the rotational constraint induces irrotational velocity field of the form $\vec{v} \propto \vec{\nabla} xy$ yielding **irrotational** value for the moment of inertia:

$$\theta_{irr} = \delta^2 \theta_{rig}$$

$\delta = \dfrac{<(x^2 - y^2)>}{<(x^2 + y^2)>}$ is deformation of the atomic cloud

$\theta_{rig} = N < (x^2 + y^2) >$ is rigid value of moment of inertia

- For axi-symmetric trapping **moment of inertia vanishes at zero temperature** (effect of superfluidity)
- For **large** angular velocities **quantized vortices** are formed.

To calculate moment of inertia in a SOC system a useful description is provided by **hydrodynamic formalism**

For small angular velocities the **relative phase** of the order parameter of the two spin components is **locked** because of the gap caused by Raman coupling

$$\phi_1(\vec{r},t) = \phi_2(\vec{r},t) \equiv \phi(\vec{r},t)$$

The order parameter of s=1/2 spinor takes the form

$$\Psi(\vec{r},t) = \begin{pmatrix} \sqrt{n_1(\vec{r},t)} \\ \sqrt{n_2(\vec{r},t)} \end{pmatrix} e^{i\phi(\vec{r},t)}$$

and one should look for equations for total density (n), spin density (s_z) and the phase (ϕ)

Equations of **spinor hydrodynamics** (Martone PRA12)

$$\frac{\partial}{\partial t}n+\frac{\hbar}{m}\nabla\cdot(n\nabla\phi)-\frac{\hbar}{m}k_0\nabla_x s_z=0$$

$$\hbar\frac{\partial}{\partial t}\phi+gn+V_{ho}-\mu=0$$

$$-\frac{\hbar}{m}k_0 n\nabla_x\phi+\frac{1}{2}\Omega s_z=0$$

hold for low frequency dynamics

- **Equation of continuity affected** by SOC (new definition of the current: **spin** contribution

- **Equation for the phase** is not affected by SOC (EoS and density profiles are **unaffected** by SOC)

- **Relationhip** between **spin density** and **gradient** of the **phase** holds also **out** of **equilibrium**

In the presence of spin-orbit coupling angular momentum takes **additional spin** contribution

$$\mathrm{L}_z = \sum_k [x_k p_{k,y} - y(p_{k,x} - k_0 \sigma_{k,z})]$$

and HD equations at equilibrium with $H = H_{SOC} - \omega_{rot} L_z$ take the form

$$\nabla \cdot [n(\nabla\phi - \vec{\omega}_{rot} \times \vec{r})] - k_0 \nabla_x s_z = 0$$

$$-\frac{\hbar k_0}{m} \nabla_x \phi + \frac{1}{2} \Omega \frac{s_z}{n} - \omega_{rot} k_0 y = 0$$

For isotropic trapping HD eqs. admit solution

$$\phi = \alpha xy$$

$$k_0 s_z = 2\alpha y n$$

$$\alpha = \frac{\omega_{rot} k_0^2}{\Omega - \Omega_{cr}/2}$$

where $\Omega_{cr} = 2\hbar \frac{k_0^2}{m}$

Behavior of velocity field

$$\vec{v} = \frac{\hbar}{m}\nabla\phi - \frac{\hbar}{m}\vec{k}_0\frac{s_z}{n}$$

$$v_x = \frac{\hbar}{m}\nabla_x\phi - \frac{\hbar}{m}k_0\frac{s_z}{n} = \frac{\hbar}{m}(\alpha y - 2\alpha y) = -\frac{\hbar}{m}\alpha y$$

$$v_y = \frac{\hbar}{m}\nabla_y\phi \quad = \frac{\hbar}{m}\alpha x$$

Behavior of velocity field

$$\vec{v} = \frac{\hbar}{m}\nabla\phi - \frac{\hbar}{m}\vec{k}_0\frac{s_z}{n}$$

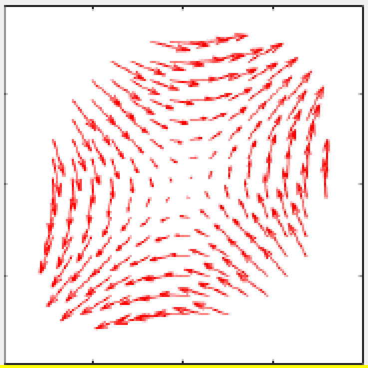
$$v_x = \frac{\hbar}{m}\nabla_x\phi - \frac{\hbar}{m}k_0\frac{s_z}{n} = \frac{\hbar}{m}(\alpha y - 2\alpha y) = -\frac{\hbar}{m}\alpha y$$

$$v_y = \frac{\hbar}{m}\nabla_y\phi = \frac{\hbar}{m}\alpha x$$

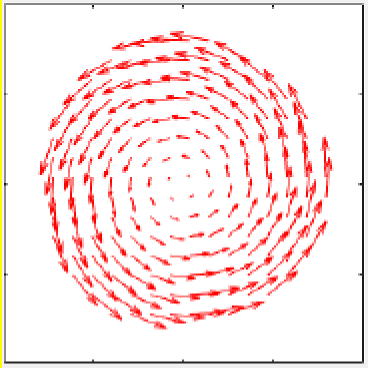
$$\Rightarrow \quad \vec{v} = \vec{\omega}_{rot} \times \vec{r}$$

at $\Omega = \Omega_{cr}$

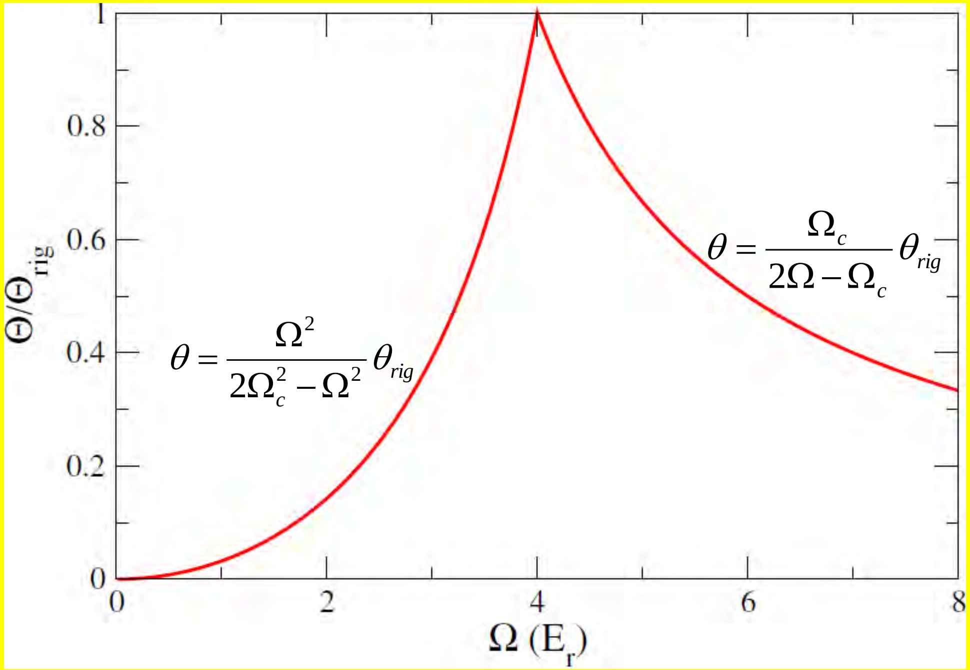
$$\vec{v} = \frac{\hbar}{m}\nabla\phi$$



$$\vec{v} = \vec{\omega}_{rot} \times \vec{r}$$



Behavior of moment of inertia



SOC BEC exhibits rigid value $\theta = \theta_{rig}$ at the transition between plane wave and single momentum phase.
Dramatic consequence of SOC on superfluid behavior

Since the rigid rotation is caused by the presence of spin contribution in angular momentum

$$\mathrm{L_z} = \sum_k [x_k p_{k,y} - y(p_{k,x} - \hbar k_0 \sigma_{k,z})]$$

one concludes that inclusion of **y-dependence in detuning** is the simplest (experimental) strategy to generate the rotation of the gas

$$h_0 = \frac{1}{2m}[\left(p_x - \hbar k_0 \sigma_z\right)^2 + p_\perp^2] - \frac{\hbar}{2}\Omega\sigma_x - \frac{\hbar}{2}\beta k_0 y \sigma_z$$

No need to rotate full apparatus.

β corresponds to **effective** rotational **frequency**

For **small** values of β solution corresponds to **rigid** rotation. **Large** values of β result in the formation of **vortices** (Nist 2009).

From **rigid rotation** to the formation of **vortices**

Collaboration (in progress) with Chunlei Qu

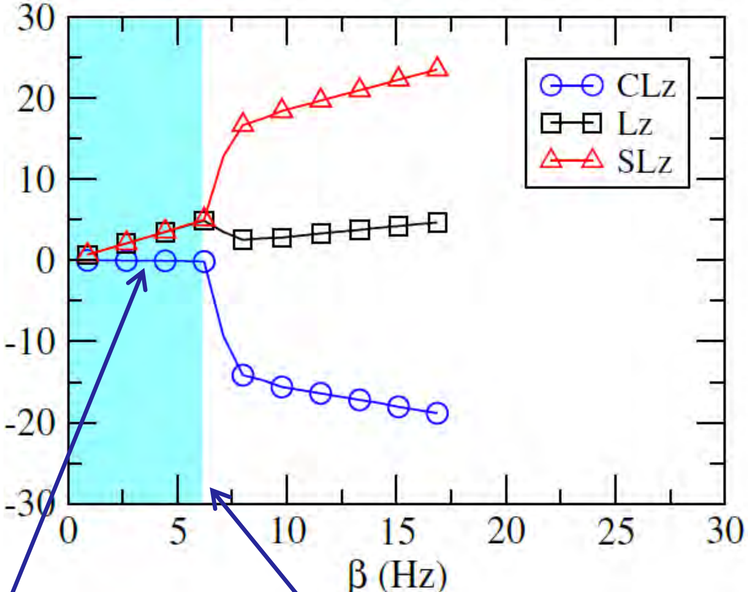
Results based on GP simulation at $\Omega = \Omega_c$ (transition between plane wave and single minimum phase)

Effect of y-dependence of detuning

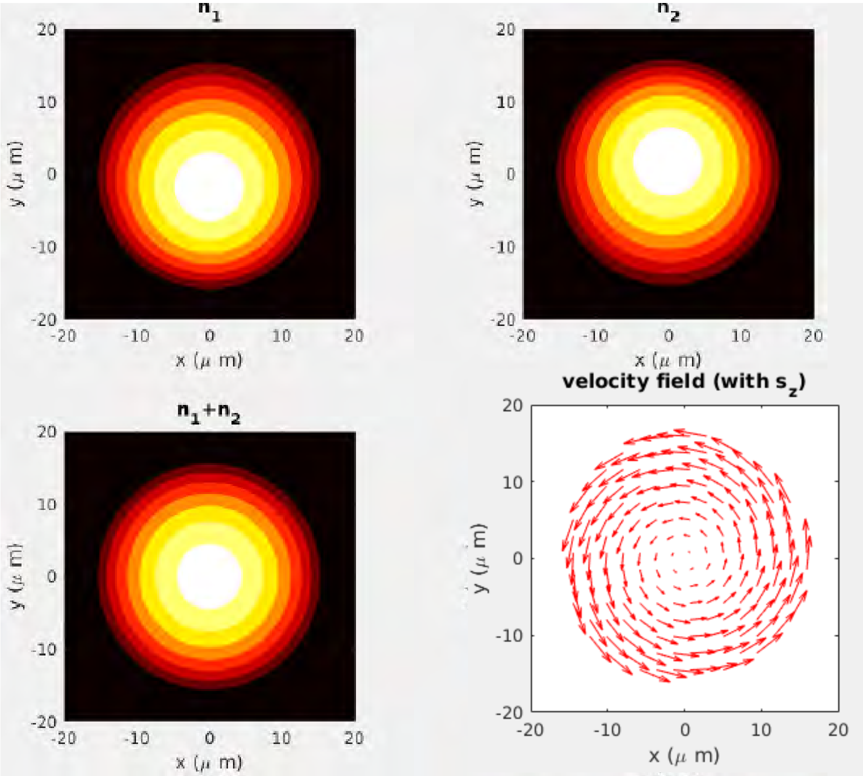
$$H = H_{SOC} - \beta k_0 y \sigma_z$$

Angular momentum
Δ spin contribution
○ canonical contribution

For **small** values of β vortices are not formed. System rotates like a **rigid** body confirming HD scenario



Onset of vortical formation $\beta_c \approx 6\ Hz$



$\beta = 4.5\,Hz$

- Canonical contribution vanishes for $\beta < \beta_c$
- Rotation is **rigid**

Large y-dependence in detuning

$$H = H_{SOC} - \beta k_0 y \sigma_z$$

Scenario investigated by Radic et al. (PRA 2011)

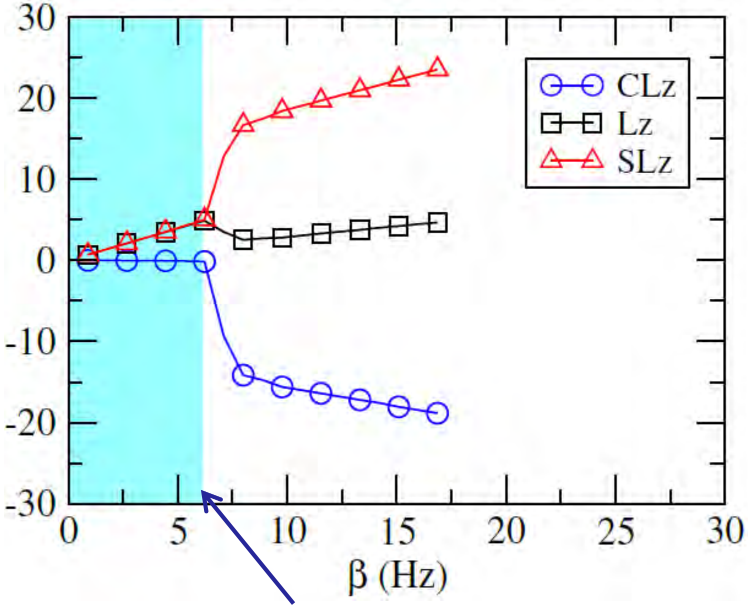
For larger values of β the two spin clouds separate and the upper and lower regions acquire opposite phase modulation $\nabla_x \phi = k_0 s_z / n \approx \pm k_0$

Vortices with **negative** circulation are formed at the **interface**

Angular momentum

△ spin contribution

○ canonical contribution



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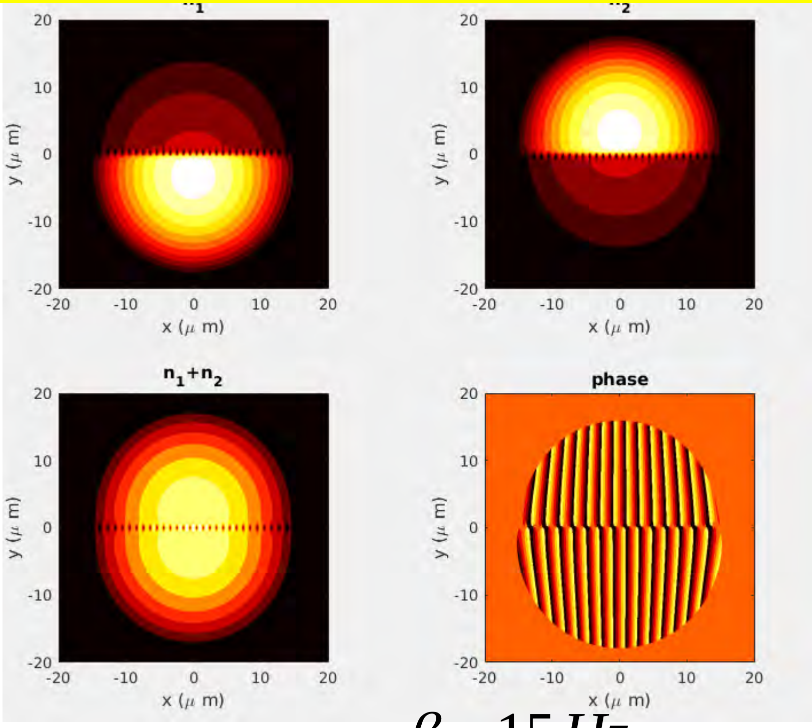
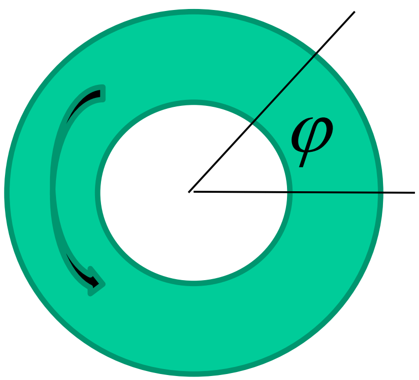


$\beta = 15\,Hz$

Effect of spin-orbit coupling on the quantization of circulation in toroidal configurations

Using **Laguerre-Gauss laser beams** transferring angular momentum $\hbar l$ it is possible to generate **spin-orbit coupling** in **toroidal** configuration.



In the absence of SOC the quantum of circulation is determined in units of

$$\oint d\vec{l} \cdot \vec{v}(\vec{r}) = \frac{h}{\mathrm{m}}$$

Persistent currents and quanta of circulation already **measured** in BECs confined in **toroidal traps** (Nist 2013)

SOC solution in the **ring** can be **mapped** into **1D spin orbit problem** with **periodic** boundary conditions

Relevant equation for the spin density

$$-n\frac{\hbar l}{mR^2}\partial_\varphi \phi + \frac{1}{2}\Omega s_z = 0$$

with the choice $\phi = \varphi$ (azimuthal angle)

yields new rule for **quantization of circulation**

$$\oint d\vec{l} \cdot \vec{v}(\vec{r}) = \frac{h}{\mathrm{m}}(1 - \frac{\Omega_c}{\Omega})$$

$$\Omega_c = 2\hbar l^2 / mR^2$$

SOC gives rise to reduction of quantum of circulation (reduction of angular momentum)

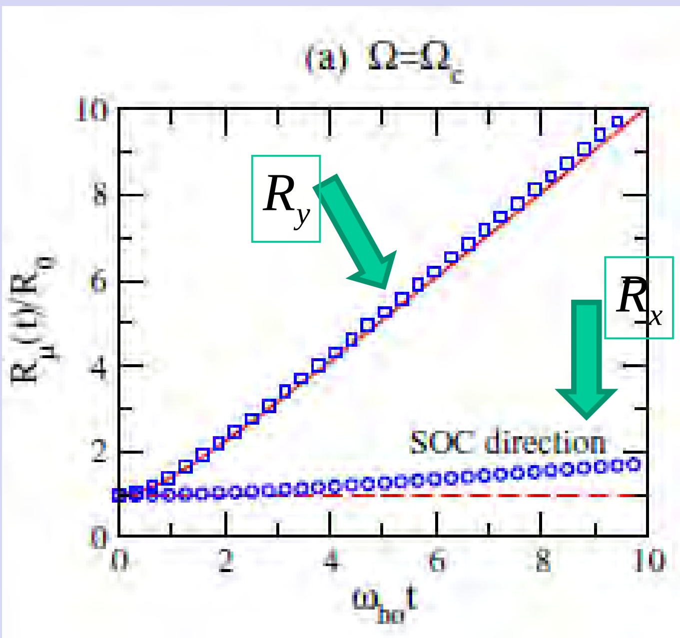
Anisotropic expansion from isotropic trap: consequence of quenched superfluidity

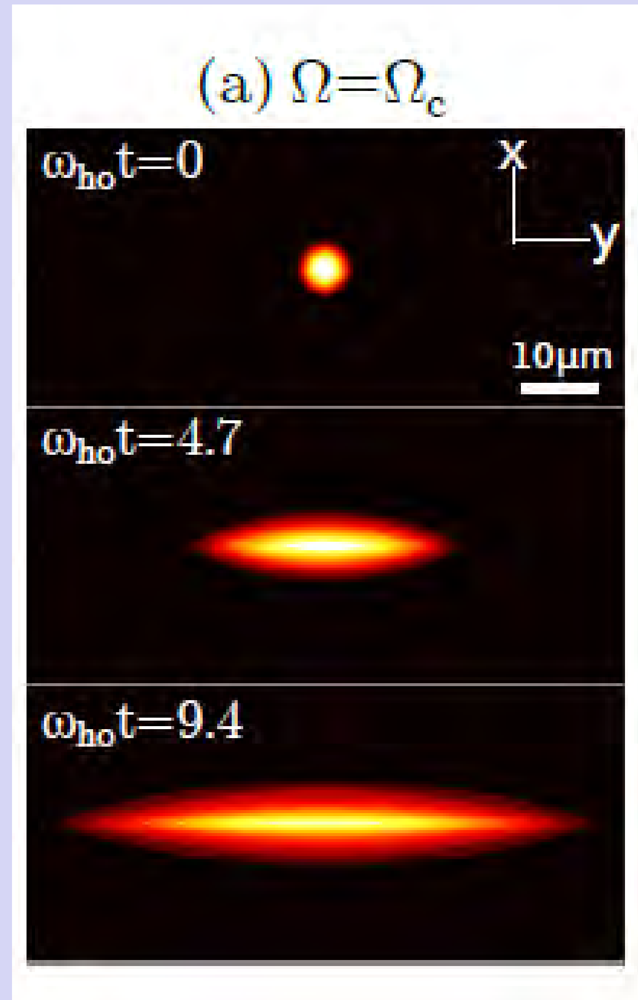
(Cunei Qu, Lev Pitaevskii and SS. arXiv:1704.00677)

Quenching of superfluidity causes **slowing down of expansion** along the direction of spin-orbit coupling

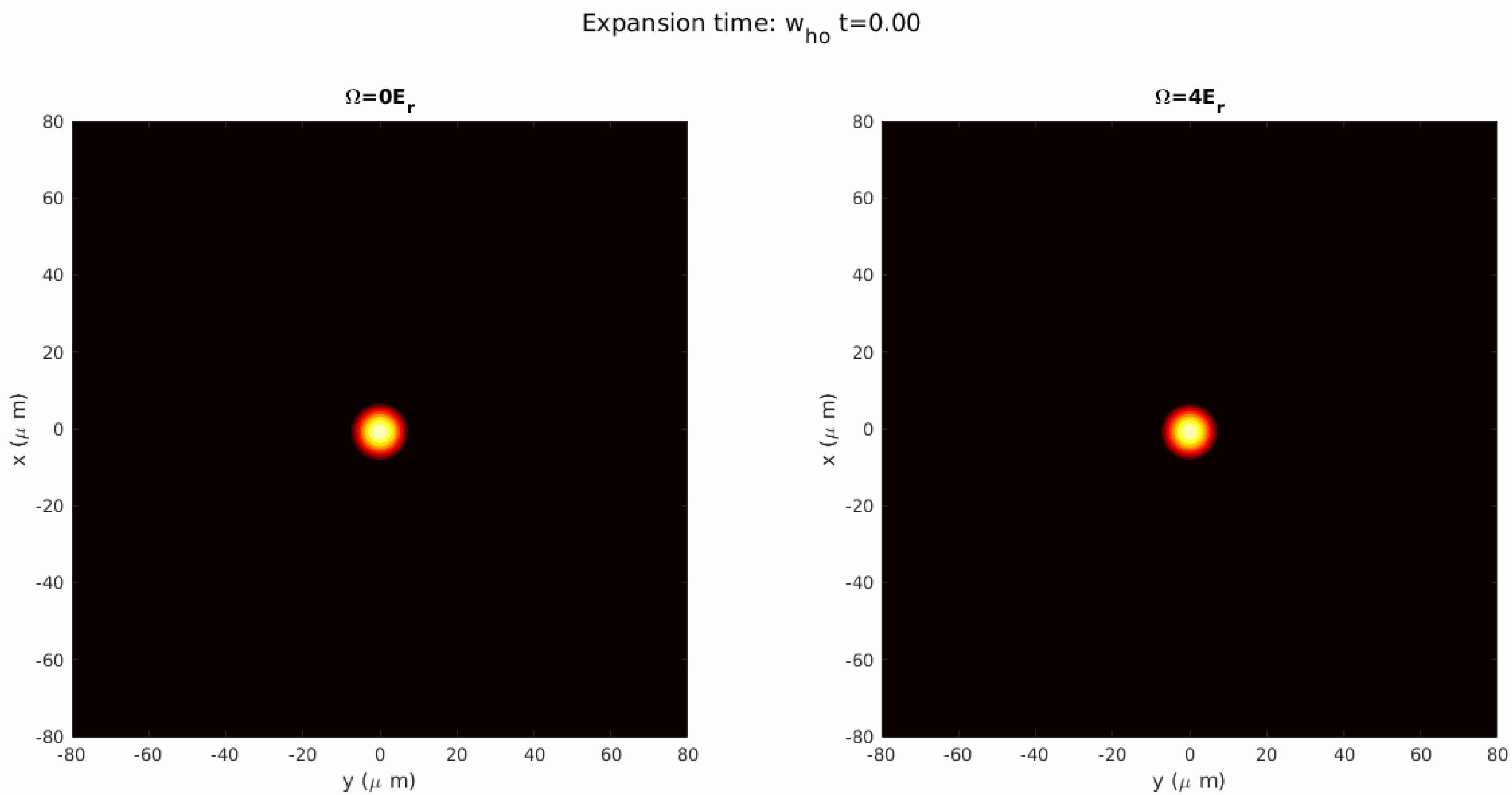
Behavior of anisotropic expansion at $\Omega = \Omega_{cr}$
(spin orbit coupling is kept on during the expansion)

Quenching of superfluid flow along the spin-orbit direction (x-axis)





Gross-Pitaevskii simulation
(Qu et al. arXiv:1704.00677)

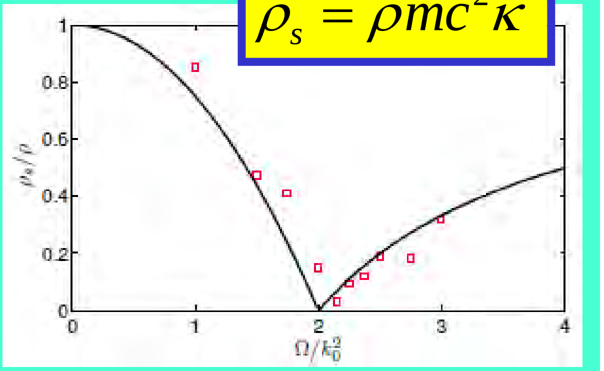


Expansion time: w_{ho} t=0.00
Ω=0E$_r$
Ω=4E$_r$
80
60
40
20
0
-20
-40
-60
-80
x (μ m)
-80
-60
-40
-20
0
20
40
60
80
y (μ m)

MAIN CONCLUSIONS

- Spin-orbit coupling deeply affects **superfluid** behavior of a BEC (consequence of **breaking of Galilean** invariance)
  Strong quenching of superfluidity at $\Omega = \Omega_c$

$$\rho_s = \rho m c^2 \kappa$$

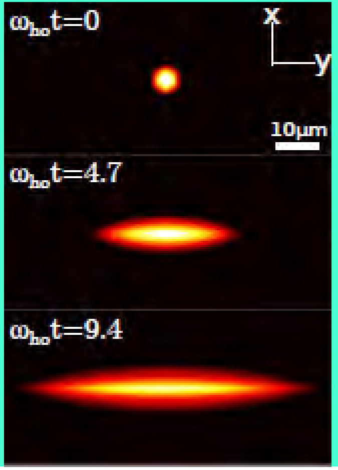


- superfluid density can be extracted from **measurement** of **sound velocity**

- Violation of irrotationality constraint:
  Moment of **inertia** acquires **rigid value** at the phase transition between plane wave and single momentum phase

- **Quantization of circulation** is affected by SOC

$$\oint d\vec{l} \cdot \vec{v}(\vec{r}) = \frac{h}{m}\left(1 - \frac{\Omega_c}{\Omega}\right)$$



- **Anistropic expansion** from **isotropic trap**

SOME RUNNING PROJECTS

- **Transition from rigid rotation to formation of quantized vortices (collaboration with Chunlei Qu)**

- **How to test rigid rotation of the gas. Consequences on frequency of density oscillations : precession of dipole oscillation caused by y-dependent detuning**

- **Superfluidity of Rashba Hamiltonian (SOC with non Abelian gauge fields)**

$$h_0 = \frac{1}{2m}[\left(p_x - \hbar k_x \sigma_x\right)^2 + \left(p_y - \hbar k_y \sigma_y\right)^2] + \frac{1}{2m} p_z^2$$

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Equations of hydrodynamics, in the presence of SOC, can be recasted in the form:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0$$

$$m^* \frac{\partial v_x}{\partial t} + \nabla_x \left(\frac{m^*}{2} v_x^2 + gn + V_{\text{ext}} \right) = 0$$

$$m \frac{\partial v_{y,z}}{\partial t} + \nabla_{y,z} \left(\frac{m}{2} v_{y,z}^2 + gn + V_{\text{ext}} \right) = 0$$

value of effective mass is strongly enhanced by SOC

$$m^* = m \left(1 - \Omega_c/\Omega\right)^{-1}$$

Enhancement of m* is directly related to **quenching of superfluidity** and **violation of irrotationality**

Breaking of Galilean invariance

Galilean transformation $e^{imvx/\hbar}$ transforms

$$h_0 = \frac{1}{2m}[(p_x - \hbar k_0 \sigma_z)^2 + p_\perp^2] - \frac{\hbar}{2}\Omega\sigma_x$$

into

$$h_0^{'} = U^{-1} h_0 U = \frac{1}{2m}[(p_x - \hbar k_0 \sigma_z + mv)^2 + p_\perp^2] - \frac{\hbar}{2}\Omega\sigma_x +$$

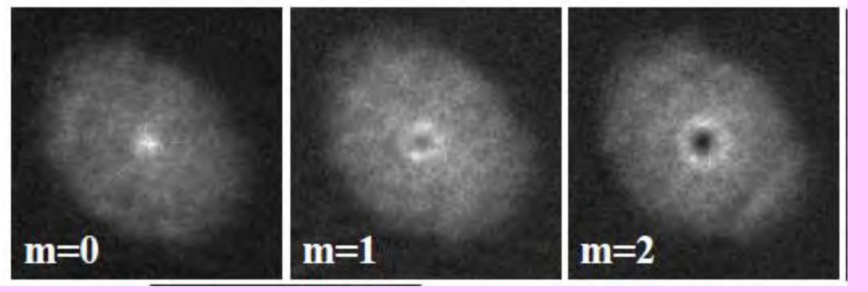
$$= h_0 + \frac{1}{2}mv^2 + mv(p_x - \hbar k_0 \sigma_z)$$

Physical momentum $(p_x - \hbar k_0 \sigma_z)$ does NOT commute with h_0 , because of Raman coupling

Hamiltonianas $h_0^{'}$ and h_0 do NOT describe equivalent dynamics !

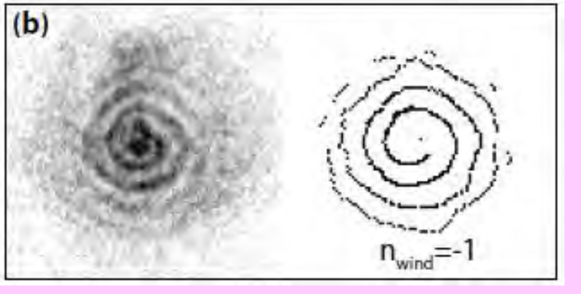
Measurement of quantization of circulation in ring geometry

- **Expansion** of the condensate



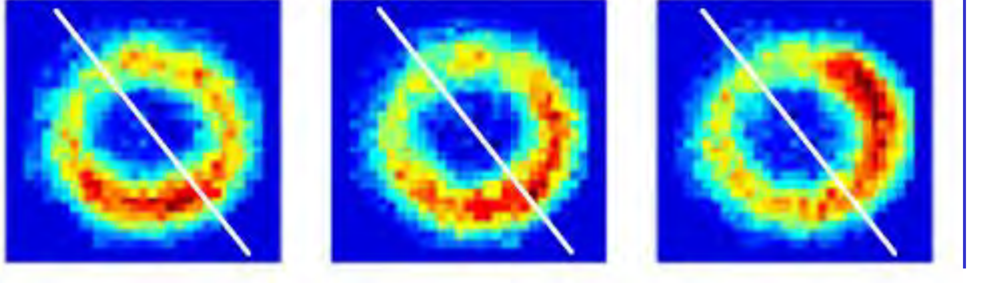
Murray et al PRA 2013

- **Interference** patterns



Corman et al. PRL 2014

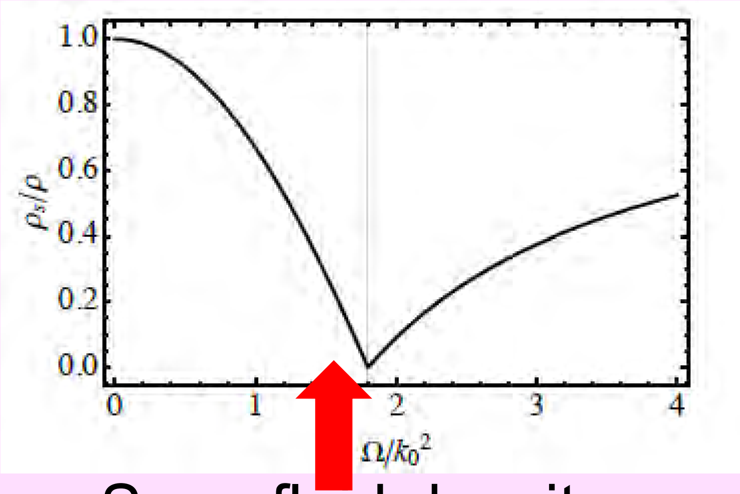
- **Doppler** measurement of angular momentum



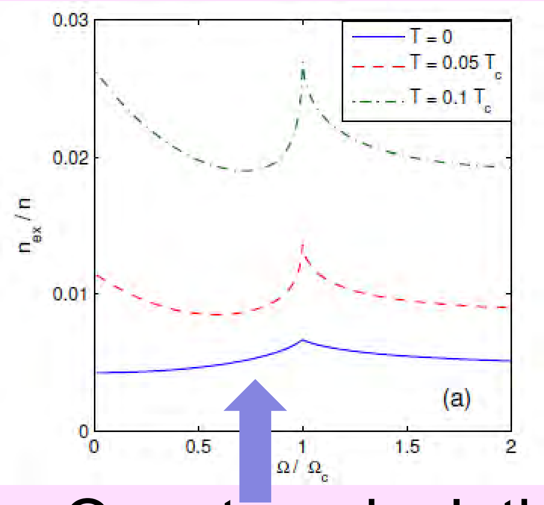
Kumar et al. NJP 2016

Superfluid density is strongly **suppressed** near the phase transition between the plane wave and zero-momentum phase

BEC fraction is instead practically unperturbed (**quantum depletion** always remains **very small in 3D gas**, less than 1%)



Superfluid density
(present work)

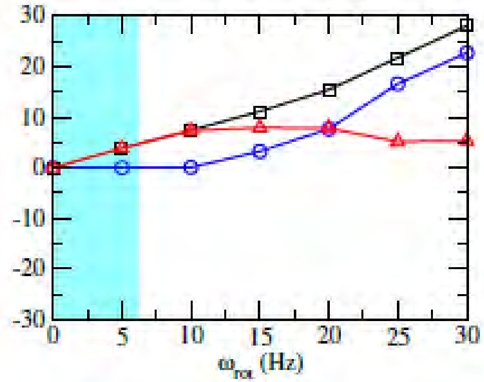


Quantum depletion
(W. Zheng et al. JPhysB 2013)

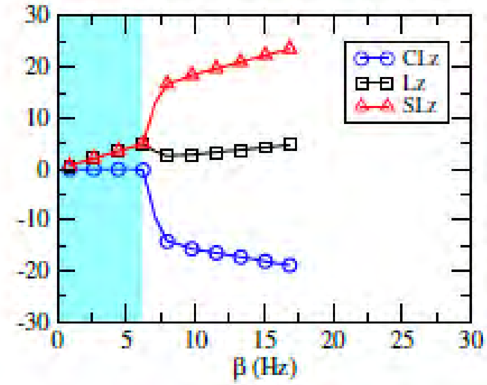
At the transition:

Bose-Einstein condensation without superfluidity !

$H_{rot}=H_{soc}-\omega_{rot}(L_z-k_0 y\sigma_z)$



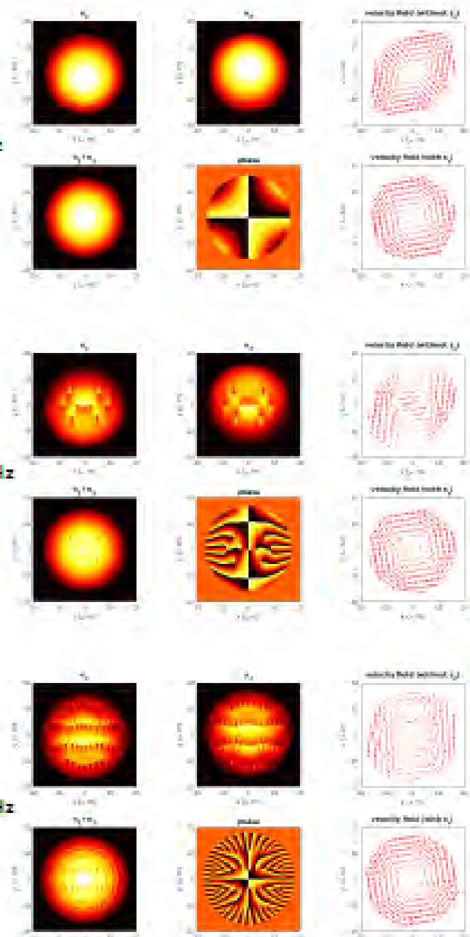
$H_\beta=H_{soc}+\beta k_0 y\sigma_z$



ω_{rot}=5Hz

ω_{rot}=20Hz

ω_{rot}=30Hz



β=4.44Hz

β=10.65Hz

β=15.09Hz