from analogue general relativity to cosmology

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Mainz, June 1, 2017

superfluid ^{4}He & phenomenological theories of hydrodynamic type

Landau two-fluid hydrodynamics

Einstein general relativity

liquid ^{3}He & effective theories from p-space topology

Landau theory of Fermi liquid

Standard Model + gravity

from helium liquids to dynamics of quantum vacuum

quantum vacuum as self-sustained medium,
why cosmological constant is small

European Research Council
Established by the European Commission

European Commission

Horizon 2020
European Union funding
for Research & Innovation

3+1 sources of effective theories of quantum liquids ^{4}He & ^{3}He
& relativistic 3+1 Quantum Feild Theories

I think it is safe to say that no one understands **Quantum Mechanics**

Richard Feynman

Thermodynamics is the only physical theory of universal content

Albert Einstein

Symmetry: conservation laws, translational invariance,
Galilean principle of relativity, ...

Topology: you can't comb the hair on a ball smooth

missing ingredient
in Landau theories

1. superfluid ^{4}He & effective theories of hydrodynamic type

Landau two-fluid hydrodynamics

Einstein general relativity

classical low-energy property
of quantum liquids

classical low-energy property
of quantum vacuum

Landau equations

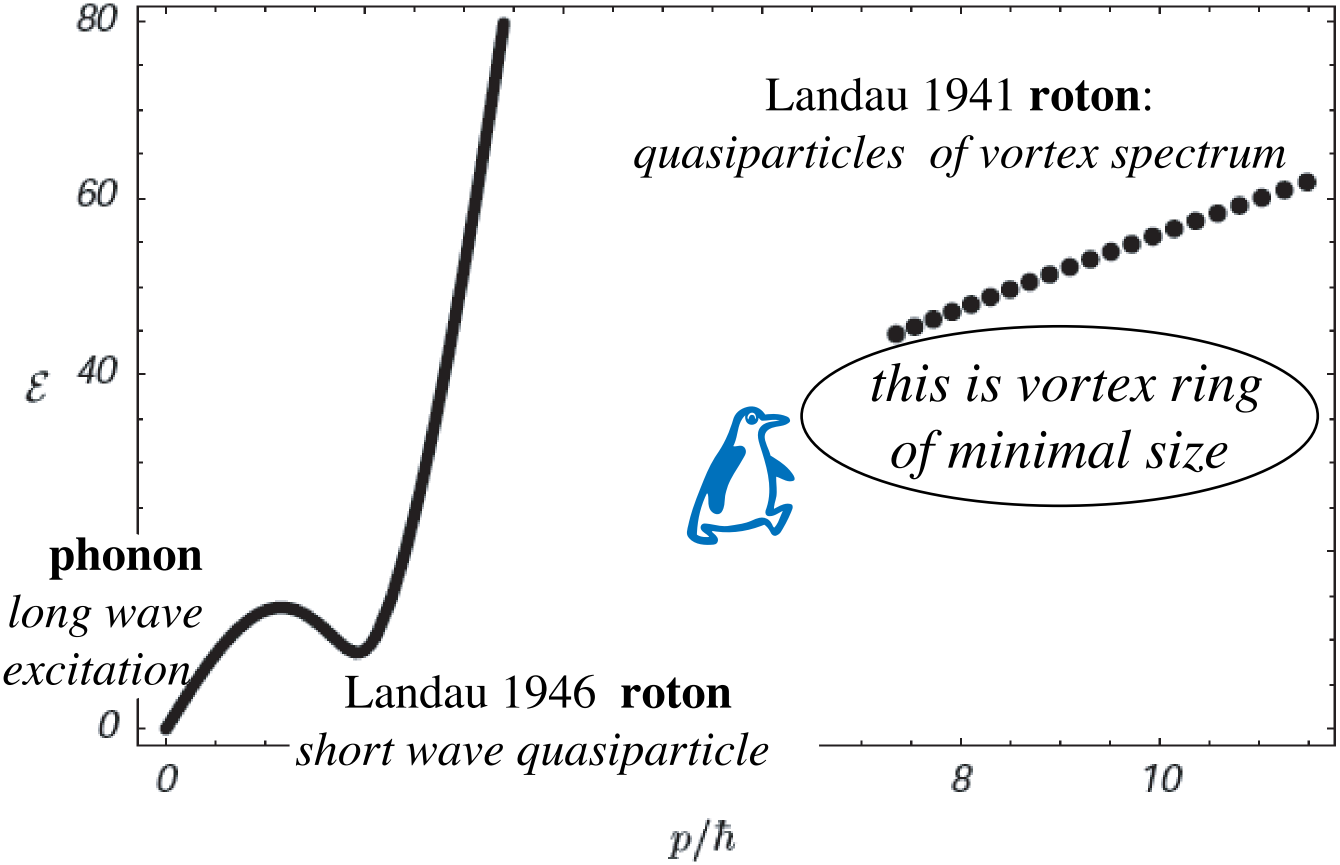
Einstein equations

Landau equations & Einstein equations
are **effective theories**
describing dynamics of
metric field + **matter** (quasiparticles)

Landau quasiparticles

weakly excited state of liquid *(of superfluid or of quantum vacuum)*
can be considered as system of "elementary excitations"

Landau, 1941



Landau 1941 **roton**:
quasiparticles of vortex spectrum

this is vortex ring of minimal size

phonon
long wave excitation

Landau 1946 **roton**
short wave quasiparticle

Landau gap in vortex spectrum

$$\boxed{\Delta \sim h^2 \rho^{2/3} m^{-5/3}}$$

this is energy of vortex ring of minimal size

Landau estimate of vortex gap is correct !

N.G. Berloff & P.H. Roberts
Nonlocal condensate models of superfluid helium
J. Phys. **A32**, 5611 (1999)

droplet of superfluid ^{4}He

Universe

superfluid ground state + elementary excitations

ground state = vacuum

vacuum + elementary particles

quasiparticles = elementary particles

long-wave quasiparticles: `relativistic' phonons

$E(\mathrm{k}) = ck$

c – speed of sound or light

long-wave quasiparticles: relativistic fermions & bosons

effective metric

$$g^{\mu\nu} k_\mu k_\nu = 0$$

general relativity

Effective metric in Landau two-fluid hydrodynamics

Doppler shifted phonon spectrum in moving superfluid

review:

Barcelo, Liberati & Visser,
Analogue Gravity
Living Rev. Rel. **8** (2005) 12

$$E = cp + \mathbf{p}\cdot\mathbf{v}_s$$

c speed of sound

$\mathbf{v}_s$ superfluid velocity

move $\mathbf{p}\cdot\mathbf{v}_s$ *to the left*

$$E - \mathbf{p}\cdot\mathbf{v}_s = cp$$

take square

$$(E - \mathbf{p}\cdot\mathbf{v}_s)^2 - c^2 p^2 = 0 \longrightarrow g^{\mu\nu} p_\mu p_\nu = 0$$

$$p_\nu = (-E,\ \mathbf{p})$$

effective metric

$$g^{00} = -1 \qquad g^{0i} = -v_s^{\,i} \qquad g^{ij} = c^2\delta^{ij} - v_s^{\,i} v_s^{\,j}$$

inverse metric $g_{\mu\nu}$ determines effective spacetime
in which phonons move along geodesic curves

$$ds^2 = -\,c^2dt^2 + (d\mathbf{r} - \mathbf{v}_s dt)^2 \longleftarrow ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$

reference frame for phonon is dragged
by moving liquid

Landau critical velocity = black hole horizon

superfluid ^{4}He

Painleve-Gulstrand metric

$$\boxed{ds^2 = -\,dt^2\,(c^2 - v^2) \;+\; 2\,v\,dr\,dt \;+\; dr^2 \;+\; r^2 d\Omega^2}$$

g_{00} g_{0r}

Gravity

acoustic horizon

black hole horizon

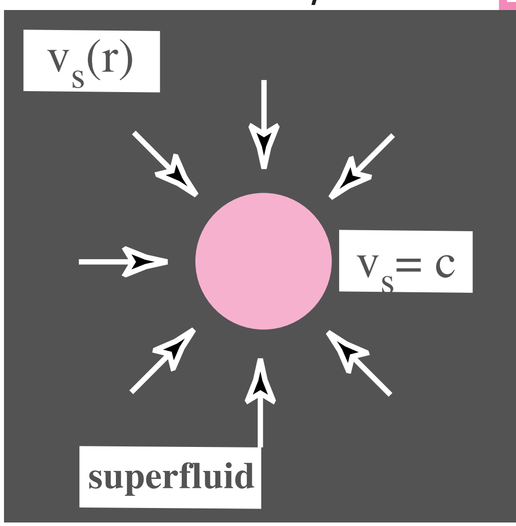
Schwarzschild metric

$$v^2(r) = c^2\,\frac{r_h}{r}$$

$$\boxed{ds^2 = -\,dt^2\,(c^2 - v^2) \;+\; dr^2/(c^2 - v^2) \;+\; r^2 d\Omega^2}$$

g_{00} g_{rr}

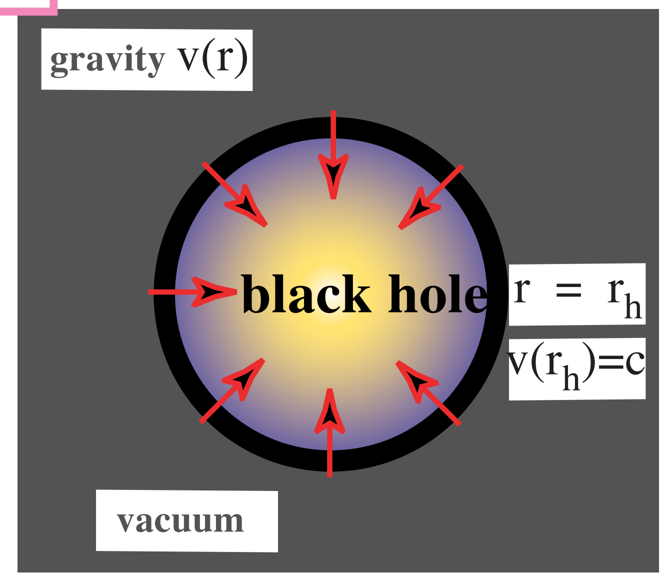
$$v^2(r) = \frac{2GM}{r} = c^2\,\frac{r_h}{r}$$



(*Unruh, 1981*)

horizon at $g_{00} = 0$
where flow velocity reaches
Landau critical velocity

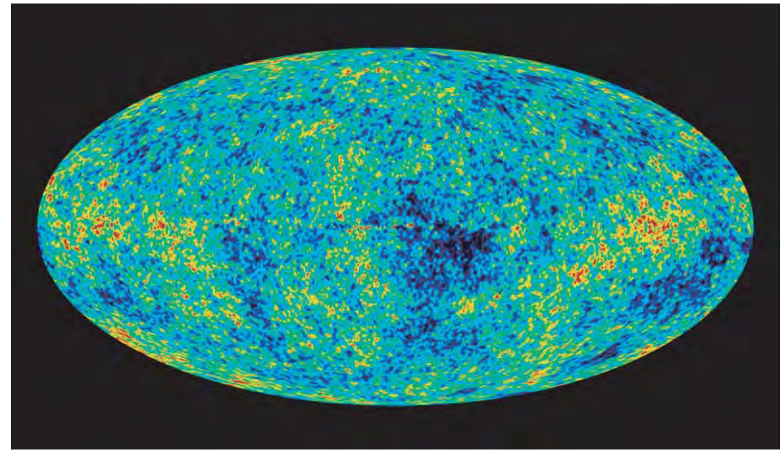
$$\boxed{v(r_h) = v_{Landau} = c}$$



Hawking radiation is phonon/photon creation
above Landau critical velocity

Superfluids

Universe



acoustic gravity **metric theories of gravity** **general relativity**

geometry of effective space time for quasiparticles (phonons)	$g_{\mu\upsilon}$	geometry of space time for matter
geodesics for phonons	$ds^2 = g_{\mu\upsilon} dx^\mu dx^\nu = 0$	geodesics for photons

Landau two-fluid equations

$$\boxed{\begin{aligned} &\dot{\rho} + \nabla\cdot(\rho \mathbf{v}_s + \mathbf{P}^{\rm Matter}) = 0 \\ &\dot{\mathbf{v}}_s + \nabla(\mu + \mathbf{v}_s^2/2) = 0 \end{aligned}}$$

equations for superfluid component

dynamic equations for metric field $g_{\mu\upsilon}$

Einstein equations of GR

$$\boxed{\frac{1}{8\pi G}\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) = T_{\mu\nu}^{\rm Matter}}$$

equation for normal component

$$\boxed{T^{\mu\nu}_{;\nu\,\rm Matter} = 0}$$

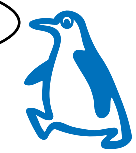
equation for matter

1/2 of GR

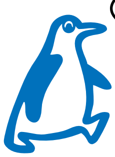
message from: **liquid helium** to: **gravity**

is gravity fundamental ?

it may emerge as classical output of underlying quantum system



as hydrodynamics ?



superfluid helium

↓

classical 2-fluid hydrodynamics

underlying microscopic quantum system at high energy

↓

emergent low-energy effective theory

quantum vacuum

↓

classical general relativity

$$\dot{\rho} + \nabla\cdot\left(\rho \mathbf{v}_s + \mathbf{P}^{\mathrm{Matter}}\right) = 0$$

$$\dot{\mathbf{v}}_s + \nabla\left(\mu + \mathbf{v}_s^2/2\right) = 0$$

$$\nabla_\nu T^{\mu\nu}_{\mathrm{Matter}} = 0$$

$$\frac{1}{8\pi G}\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) - \Lambda g_{\mu\nu} = T^{\mathrm{Matter}}_{\mu\nu}$$

2. Liquid ^{3}He & effective theories from p-space topology

Landau theory of Fermi liquid

Standard Model + gravity

two major universality classes of fermionic vacua:
vacuum with Fermi surface & vacuum with Fermi point

gravity emerges from
Fermi point
analog of
Fermi surface

$$\boxed{g^{\mu\nu}(p_\mu - eA_\mu - e\tau\cdot\mathbf{W}_\mu)(p_\nu - eA_\nu - e\tau\cdot\mathbf{W}_\nu) = 0}$$

crossover from Landau 2-fluid hydrodynamics to Einstein general relativity

they represent two different limits of hydrodynamic type equations

equations for $g^{\mu\nu}$ depend on hierarchy of ultraviolet cut-off's:
Planck energy scale $E_{\rm Planck}$ vs Lorentz violating scale $E_{\rm Lorentz}$

$E_{\rm Planck} >> E_{\rm Lorentz}$	$E_{\rm Planck} << E_{\rm Lorentz}$
Landau two-fluid hydrodynamic equations	**Einstein equations of general relativity**

^{3}He-A with Fermi point	**Universe**
$E_{\rm Lorentz} << E_{\rm Planck}$	$E_{\rm Lorentz} >> E_{\rm Planck}$
$E_{\rm Lorentz} \sim 10^{-3} E_{\rm Planck}$	$E_{\rm Lorentz} > 10^{15} E_{\rm Planck}$

what is common between cold liquid & Universe ???

high-energy physics and cosmology are extremely ultra-low temperature physics

characteristic high-energy scale in our vacuum
(analog of atomic scale in cond-mat) is Planck energy

$$E_P = (hc^5/G)^{1/2} \sim 10^{19}\ \mathrm{GeV} \sim 10^{32}\mathrm{K}$$

highest energy in accelerators

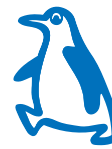
$$E_{ew} \sim 1\ \mathrm{TeV} \sim 10^{16}\mathrm{K}$$

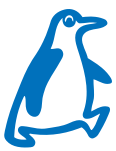
$$E_{ew} \sim 10^{-16} E_{Planck}$$

T of cosmic background radiation

$$T_{CMBR} \sim 1\ \mathrm{K}$$

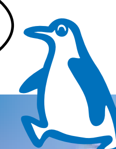
$$T_{CMBR} \sim 10^{-32} E_{Planck}$$



we live
at exremely low temperature?

this is natural for any cond-mat
if no energy flux from enviroment




How can we survive
in such a freezer ?

Right question !

cosmology is
extremely ultra-low frequency physics

B. L. Hu
New View on Quantum Gravity and the Origin of the Universe

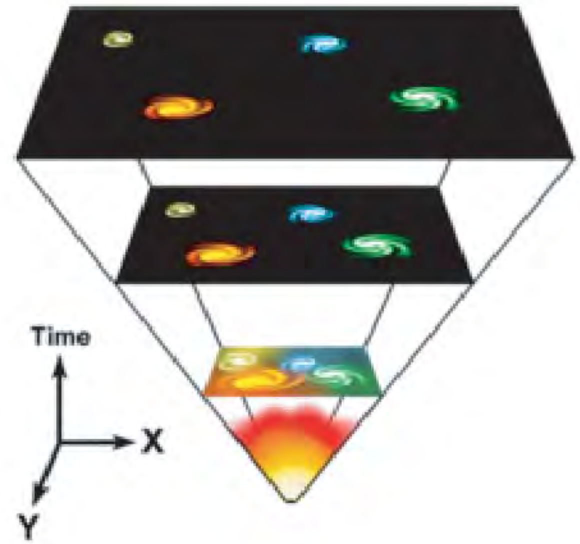
gr-qc/0611058

$$v(r) = Hr$$

expansion of Universe (Hubble parameter)

$$H \sim 10^{-60} E_{\mathrm{Planck}}$$

$$E_{\mathrm{P}} \sim 10^{19}\ \mathrm{GeV} \sim 10^{32}\mathrm{K}$$



our Universe is extremely close to equilibrium ?

this is also natural for any cond-mat if no energy flux from enviroment

Why no freezing at low T?

natural masses of elementary particles
are of order of characteristic energy scale
the Planck energy

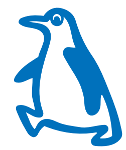
$$\boxed{m \sim E_{\text{Planck}} \sim 10^{19}\ \text{GeV} \sim 10^{32}\text{K}}$$

even at highest temperature
we can reach

$$\boxed{T \sim 1\ \text{TeV} \sim 10^{16}\text{K}}$$

everything should be completely frozen out

$$\boxed{e^{-m/T} = 10^{-10^{16}} = 0}$$



10^{-123}, $10^{-10^{16}}$
another great challenge?

main hierarchy puzzle

$$\boxed{m_{\text{quarks}}\,,\; m_{\text{leptons}} <<<\; E_{\text{Planck}}}$$

momentum-space topology solution:

$$\boxed{m_{\text{quarks}} = m_{\text{leptons}} = 0}$$

reason:

quasiparticles leaving near Fermi surface have no gap

quasiparticles leaving near Fermi point have no mass

cosmological constant puzzle

$$\boxed{\Lambda \;<<<<<\; E^4_{\text{Planck}}}$$

effective theory solution:

$$\boxed{\Lambda = 0}$$

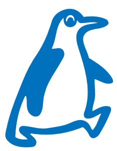
reason:

equilibrium condition in theories of hydrodynamic type:

Landau 2-fluid hydrodynamics & hydrodynamics of vacuum

Why no freezing at low T?

massless particles & gapless excitations
are not frozen out



who protects massless excitations?

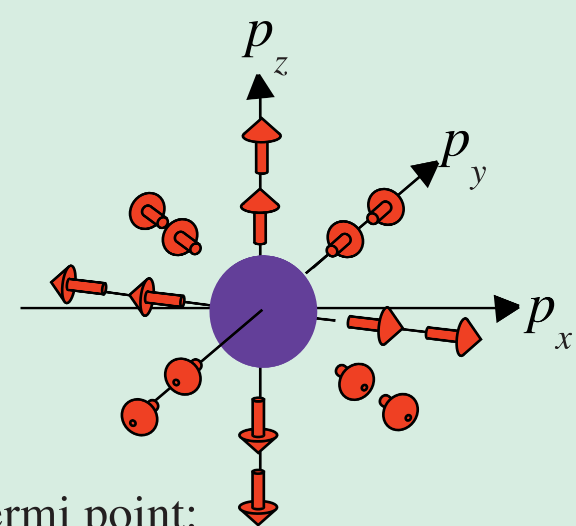
who protects massless fermions?

Topology

gapless fermions live near Fermi surface & Fermi point

no life without topology ?

we live because
Fermi point is the hedgehog
protected by topology



Fermi point:
hedgehog in momentum space

Life protection

hedgehog is stable:
one cannot comb the hair on a ball smooth

Universality classes of quantum vacua

physics at low T is determined by low-lying excitations

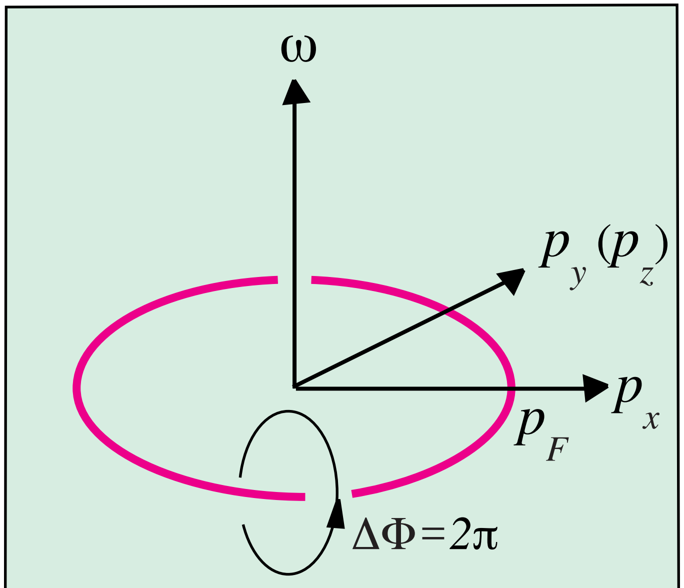
low-lying excitations live near Fermi surface

in metals

or near Fermi point

in our vacuum

Fermi surface class

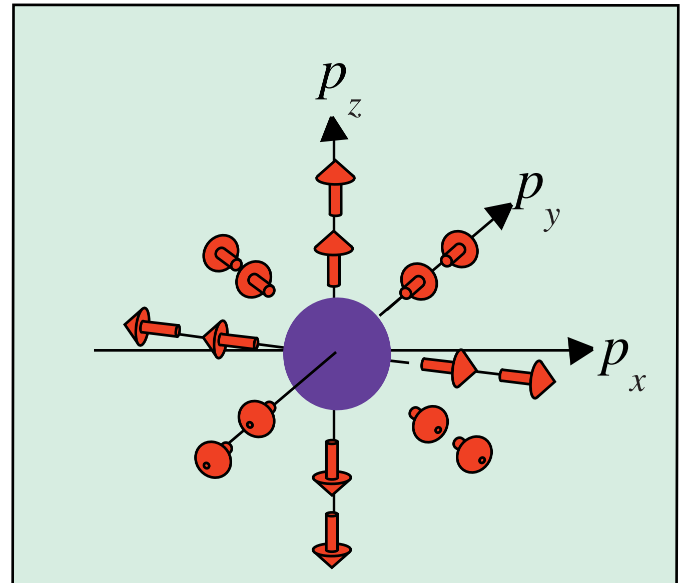


Fermi surface:
vortex in **p**-space

Topology in momentum space

P. Horava,
Stability of Fermi surfaces
& K-theory
PRL **95**, 016045 (2005)

Fermi point class



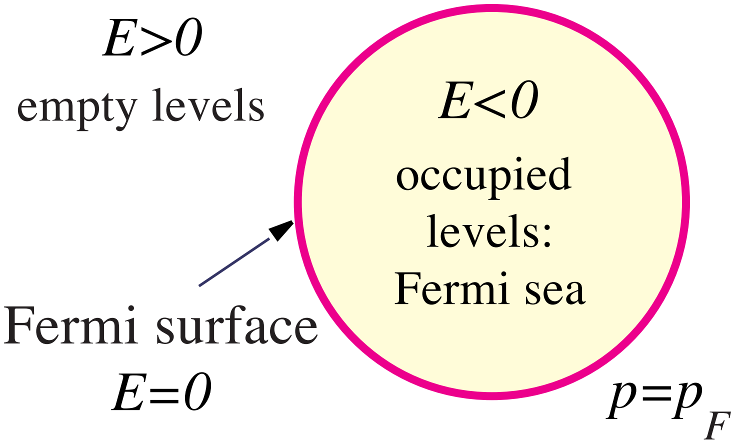
Fermi point:
hedgehog in **p**-space

topology *protects vortices & hedgehogs:*
one cannot comb the hair on a ball smooth

Topological stability of Fermi surface

Energy spectrum of
non-interacting gas of fermionic atoms

$$E(p) = \frac{p^2}{2m} - \mu = \frac{p^2}{2m} - \frac{p_F^2}{2m}$$

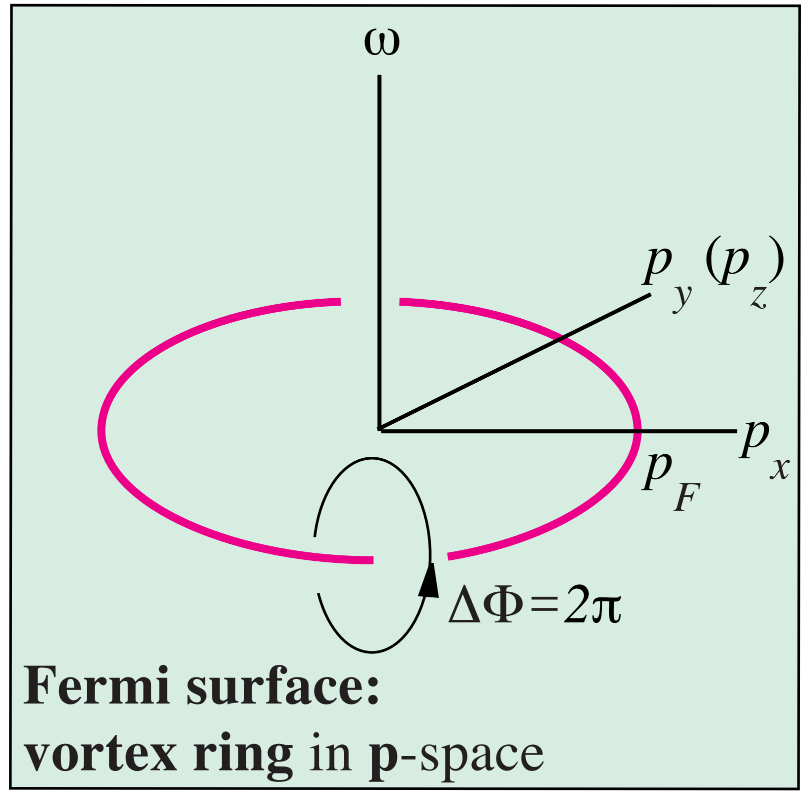


is Fermi surface a domain wall in momentum space?

no!
it is a **vortex ring**

Green's function

$$G^{-1} = i\omega - E(p)$$



phase of Green's function

$$G(\omega,\mathbf{p}) = |G| e^{i\Phi}$$

has winding number $N = 1$

Route to Landau Fermi-liquid

is Fermi surface robust to interaction ?

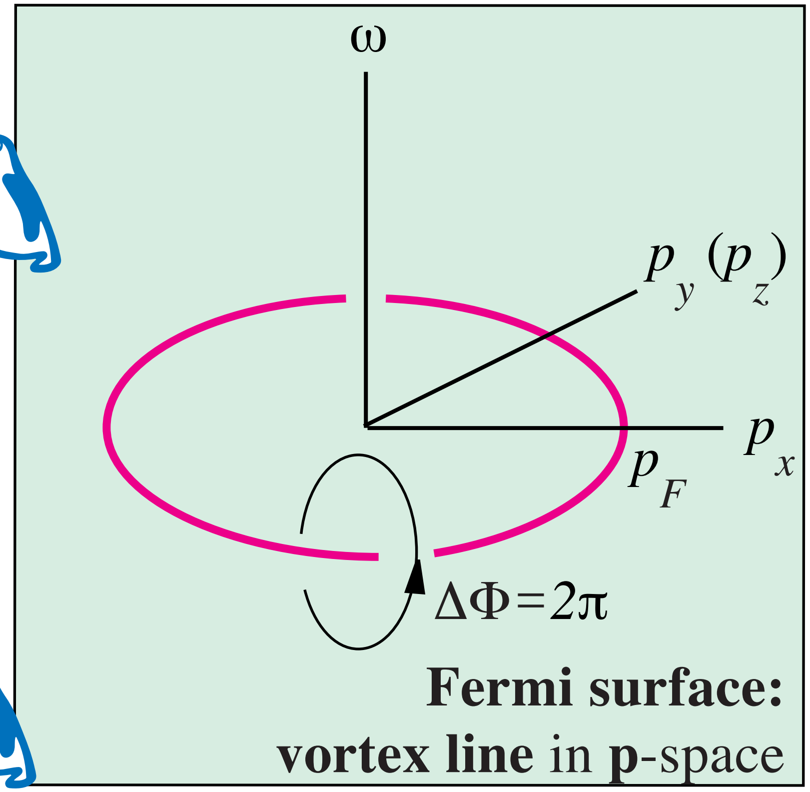
Sure! Because of topology:
winding number N=1 cannot change continuously,
interaction cannot destroy singularity

then Fermi surface survives in Fermi liquid ?

Landau theory of Fermi liquid
is topologically protected & thus is universal

all metals have Fermi surface ...

Not only metals.
Some superconductore too!



$$G(\omega,\mathbf{p})=|G|e^{\,i\Phi}$$

Stability conditions & Fermi surface topologies in a superconductor
Gubankova-Schmitt-Wilczek, *Phys.Rev.* **B**74 (2006) 064505

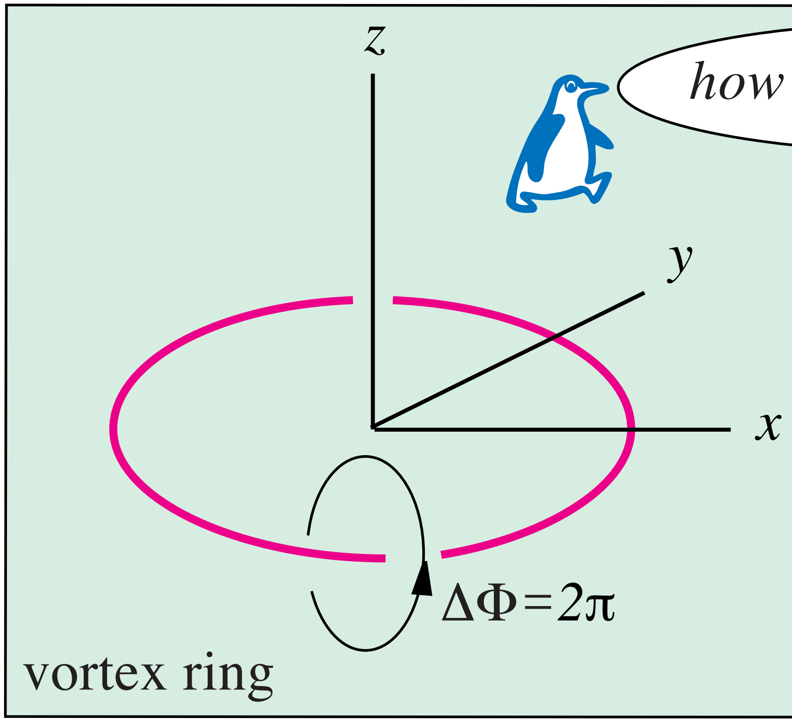
quantized vortex in **r**-space ≡ **Fermi surface** in **p**-space

$\boxed{\text{homotopy group } \pi_1}$

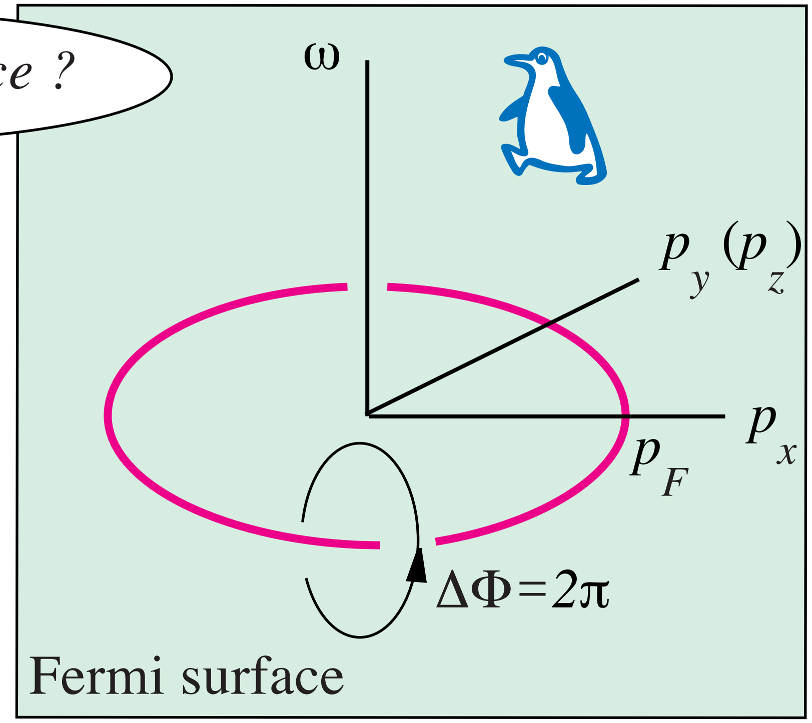
Topology in **r**-space

Topology in **p**-space

how is it in ***p****-space ?*



winding number $N_1 = 1$



$\Psi(\mathbf{r}) = |\Psi|\, e^{i\Phi}$

scalar order parameter
of superfluid & superconductor

$G(\omega,\mathbf{p}) = |G| e^{i\Phi}$

Green's function (propagator)

classes of mapping $S^1 \to U(1)$

manifold of
broken symmetry vacuum states

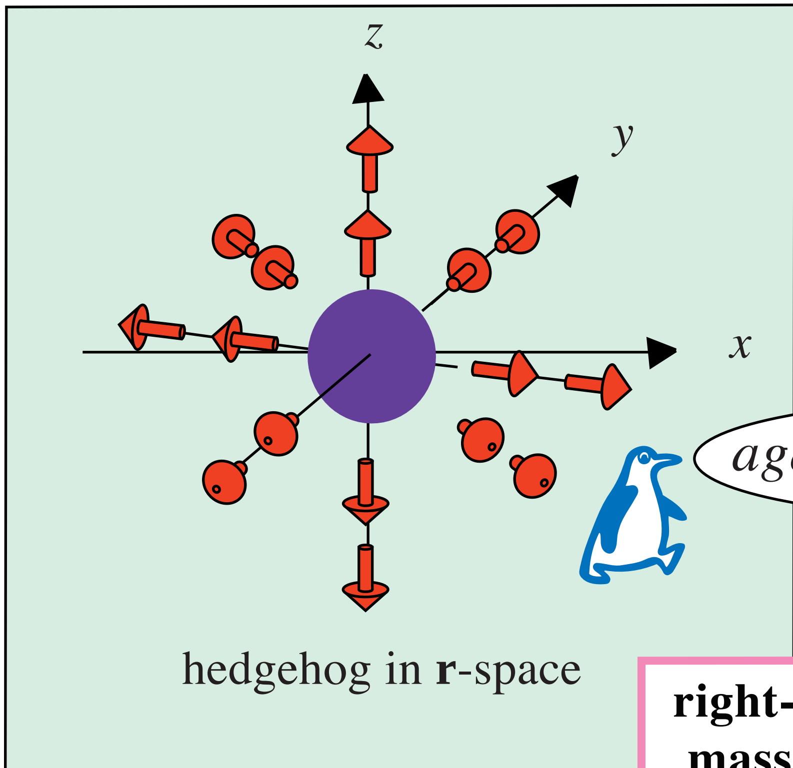
classes of mapping $S^1 \to GL(\mathrm{n,C})$

space of
non-degenerate complex matrices

3. Superfluid ^{3}He-A & Standard Model

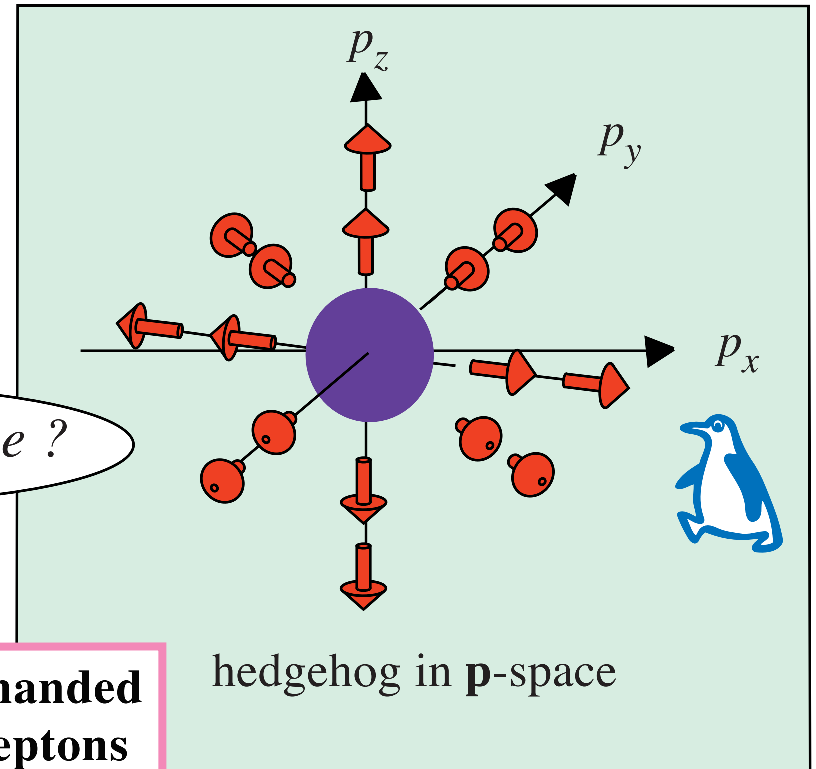
From Fermi surface to Fermi point

magnetic hedgehog vs right-handed electron



$\sigma(\mathbf{r}) = \hat{\mathbf{r}}$

again no difference ?



$\sigma(\mathbf{p}) = \hat{\mathbf{p}}$

right-handed and left-handed massless quarks and leptons are elementary particles in Standard Model

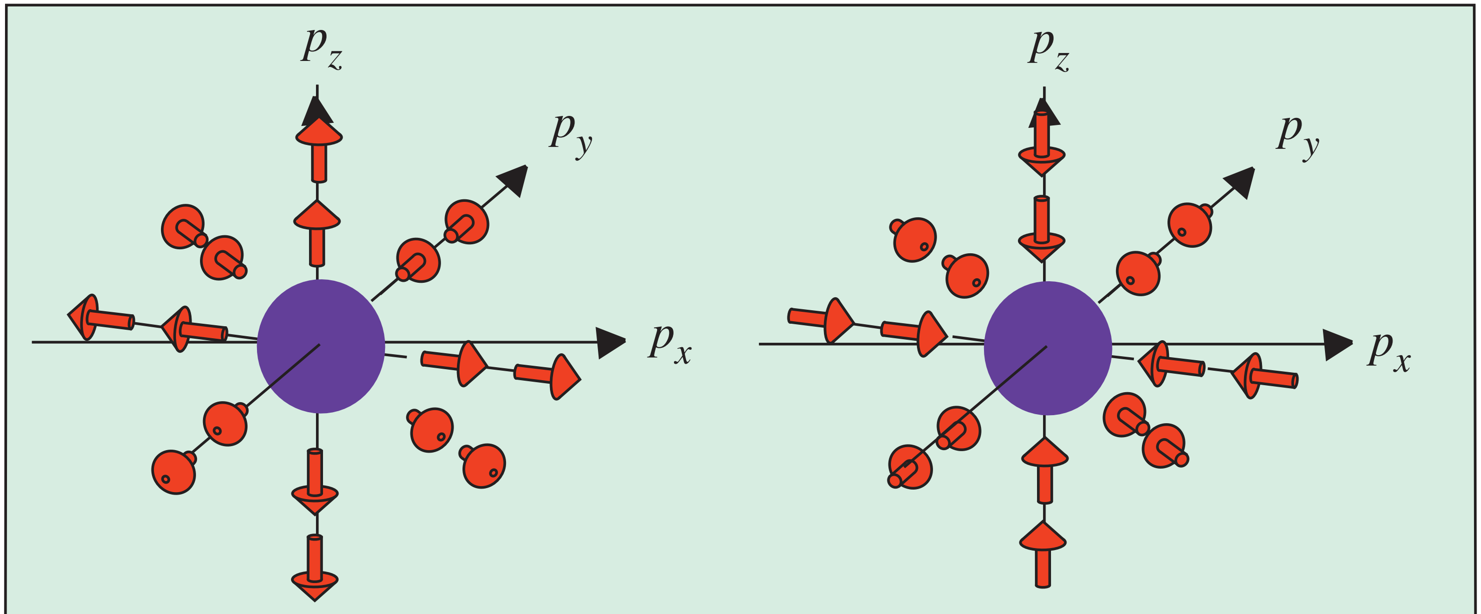
Landau CP symmetry is emergent

close to Fermi point

$$H = + c\, \sigma \cdot \mathbf{p}$$

right-handed electron =
hedgehog in **p**-space with spines = spins

Topological invariant for right-handed and left-handed elementary particles



hedgehog with spines (spins) outward ($N_3 = +1$)

hedgehog with spines (spins) inward ($N_3 = -1$)

right neutrino

$$H = +c\,\sigma \cdot \mathbf{p}$$
$$\mathbf{g}(\mathbf{p}) = +c\mathbf{p}$$

$$\boxed{H = \sigma \cdot \mathbf{g}(\mathbf{p})}$$

$$H = -c\,\sigma \cdot \mathbf{p}$$
$$\mathbf{g}(\mathbf{p}) = -c\mathbf{p}$$

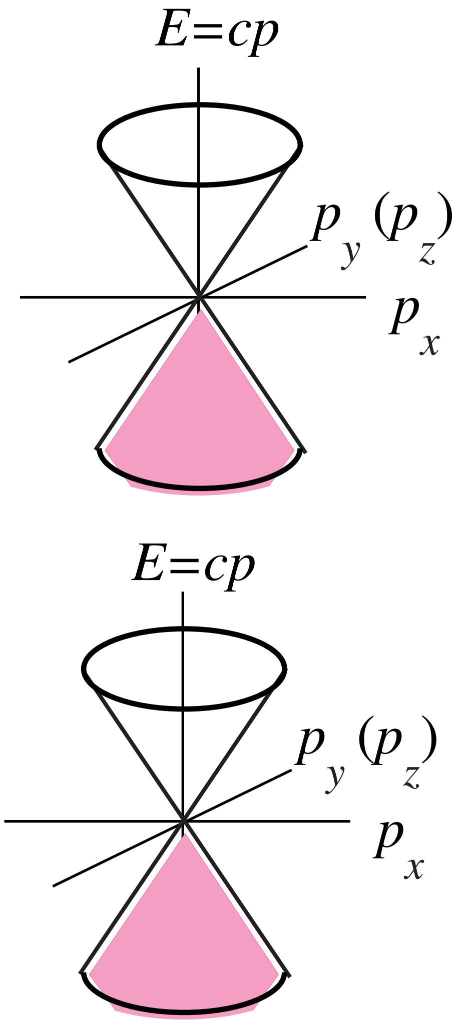
left neutrino

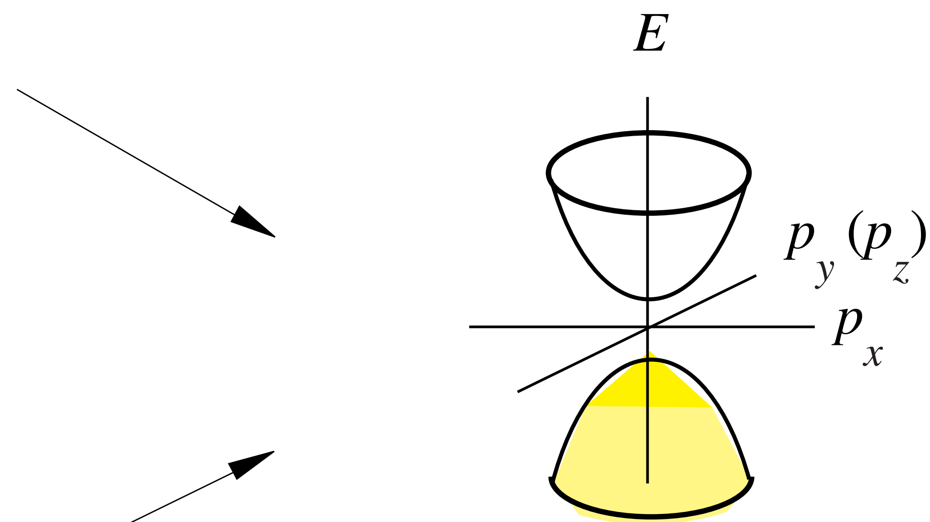
$$N_3 = \frac{1}{8\pi}\, e_{ijk} \int dS^i\, \hat{\mathbf{g}} \cdot (\partial^j \hat{\mathbf{g}} \times \partial^k \hat{\mathbf{g}})$$

over 2D surface around Fermi point

where are Dirac particles?

Dirac particle - composite object made of left and right particles





$$E^2 = c^2p^2 + m^2$$

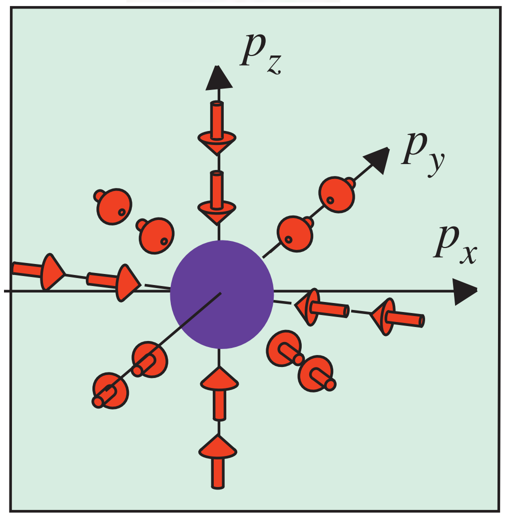
mixing of left and right particles is secondary effect, which occurs at extremely low temperature

$$T_{\mathrm{ew}} \sim 1\ \mathrm{TeV} \sim 10^{16}\mathrm{K}$$

Chiral fermions in Standard Model

Family #1 of quarks and leptons

left particles



hedgehog with spines (spins) inward $(N_3 = -1)$

quarks ($SU(3)_C$, $SU(2)_L$)

u_L (+2/3, +1/6)	d_L (-1/3, +1/6)	u_R (+2/3, +2/3)	d_R (-1/3, -1/3)
u_L (+2/3, +1/6)	d_L (-1/3, +1/6)	u_R (+2/3, +2/3)	d_R (-1/3, -1/3)
u_L (+2/3, +1/6)	d_L (-1/3, +1/6)	u_R (+2/3, +2/3)	d_R (-1/3, -1/3)

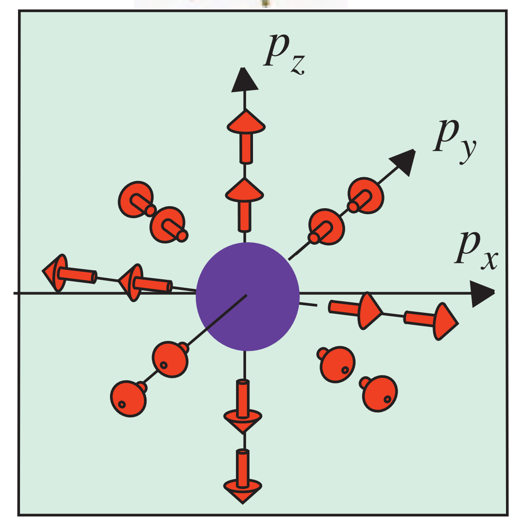
leptons

ν_L (0, -1/2)	e_L (-1, -1/2)	ν_R (0, 0)	e_R (-1, -1)

$$H = -\,c\,\sigma\cdot\mathbf{p} \qquad N_3 = -1$$

$$H = +\,c\,\sigma\cdot\mathbf{p} \qquad N_3 = +1$$

right particles

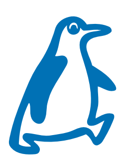


hedgehog with spines (spins) outward $(N_3 = +1)$

$$N_3 = \frac{1}{24\pi^2}\, e_{\mu\nu\lambda\gamma}\, \mathbf{tr} \int dS^\gamma\, \mathbf{G}^\mu\, \mathbf{G}^{-1}\, \mathbf{G}^\nu\, \mathbf{G}^{-1} \mathbf{G}^\lambda\, \mathbf{G}^{-1}$$

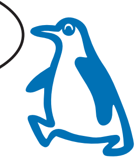
over 3D surface S in 4D momentum space

general topological invariant in terms of Green's function



life exists at low T because Fermi points are stable ?

right !



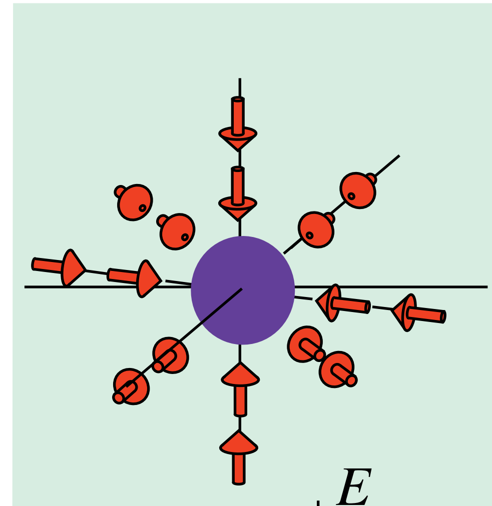
examples of Fermi points in condensed matter

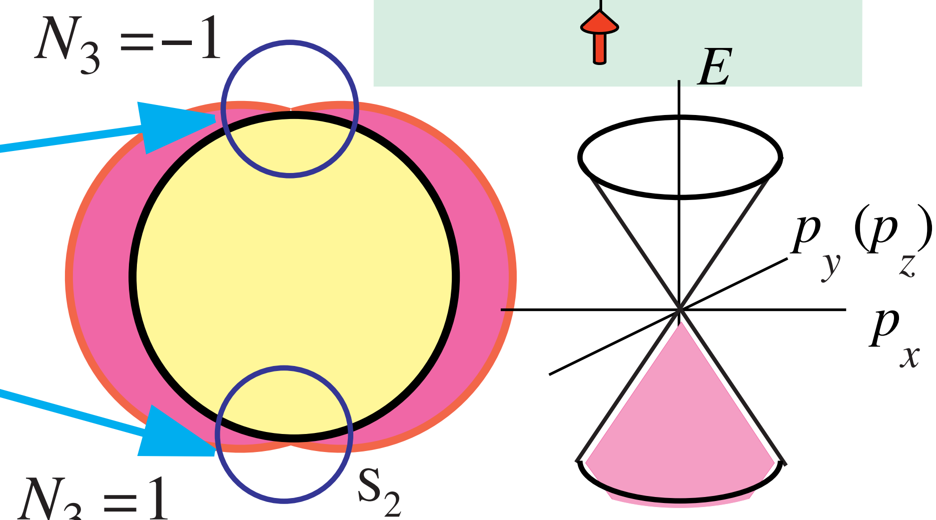
superfluids & superconductors with point nodes in gap:
superfluid ^{3}He-A, chiral superconductor Sr_2RuO_4,
triplet cold Fermi gases

$$N_3 = \frac{1}{8\pi} e_{ijk} \int dS^k \; \hat{\mathbf{g}} \cdot (\partial_{p_i} \hat{\mathbf{g}} \times \partial_{p_j} \hat{\mathbf{g}})$$

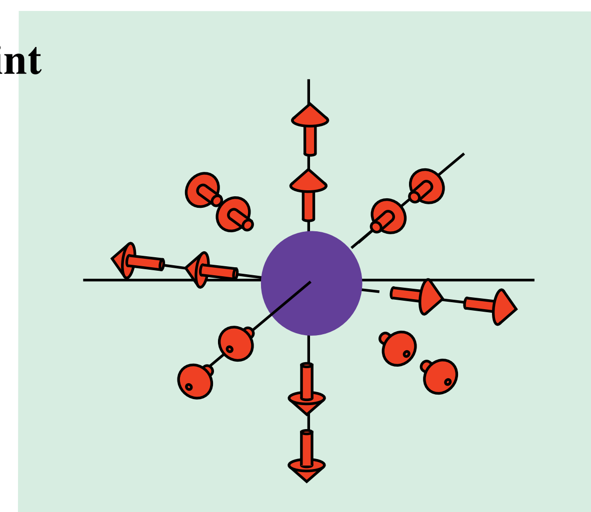
over 2D surface S in 3D p-space

Gap node - Fermi point (anti-hedgehog)





Gap node - Fermi point (hedgehog)



emergence of relativistic particles

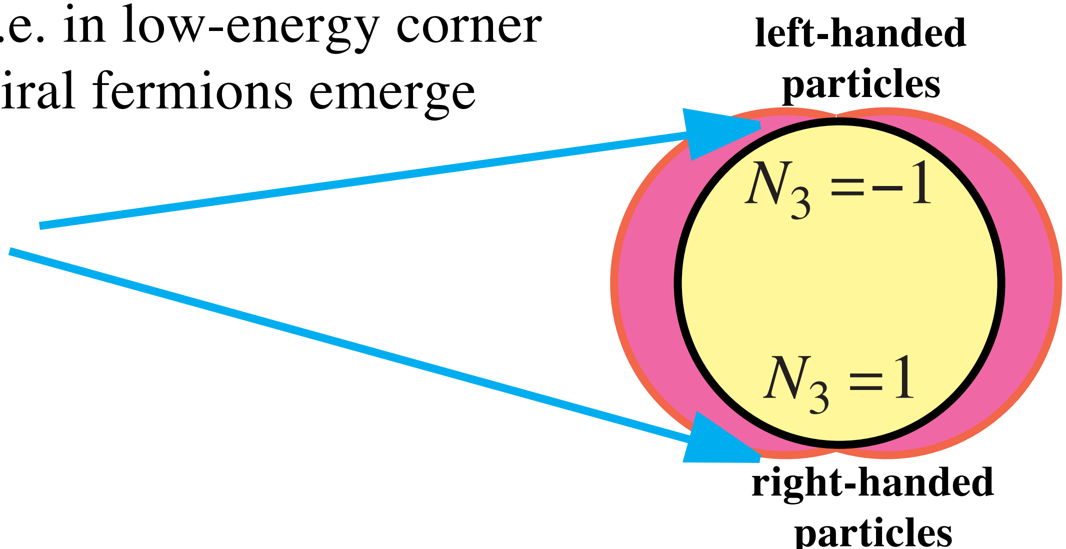
original non-relativistic Hamiltonian

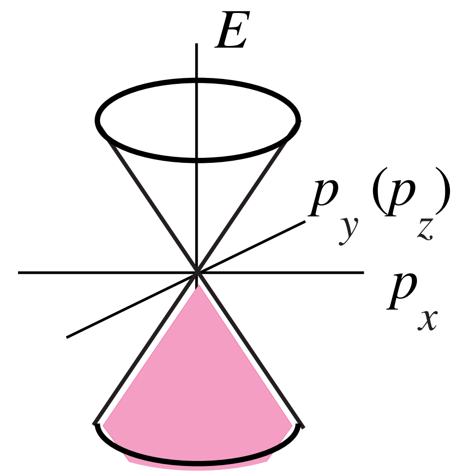
$$\mathrm{H} = \begin{pmatrix} \frac{p^2}{2m} - \mu & c(p_x + ip_y) \\ c(p_x - ip_y) & -\frac{p^2}{2m} + \mu \end{pmatrix} = \begin{pmatrix} g_3(\mathbf{p}) & g_1(\mathbf{p}) + i\, g_2(\mathbf{p}) \\ g_1(\mathbf{p}) - i\, g_2(\mathbf{p}) & -g_3(\mathbf{p}) \end{pmatrix} = \boldsymbol{\tau} \cdot \mathbf{g}(\mathbf{p})$$

close to nodes, i.e. in low-energy corner
relativistic chiral fermions emerge

$$\mathrm{H} = N_3\, c\, \boldsymbol{\tau} \cdot \mathbf{p}$$

$$E = \pm\, cp$$





top. invariant determines chirality in low-energy corner

chirality is emergent ??

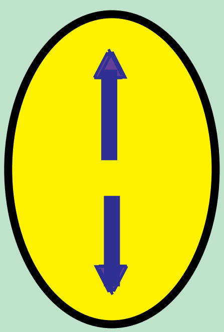
relativistic invariance as well

what else is emergent ?

bosonic collective modes in two generic fermionic vacua

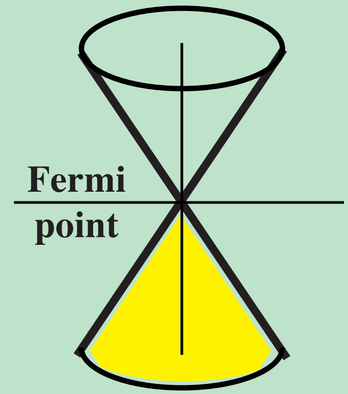
Landau theory of Fermi liquid

collective Bose modes
of fermionic vacuum:
propagating
oscillation of **shape**
of Fermi surface



Landau, ZhETF **32**, 59 (1957)

Standard Model + gravity

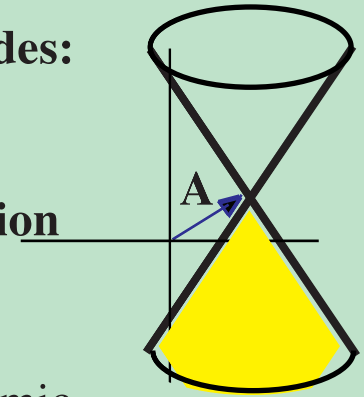


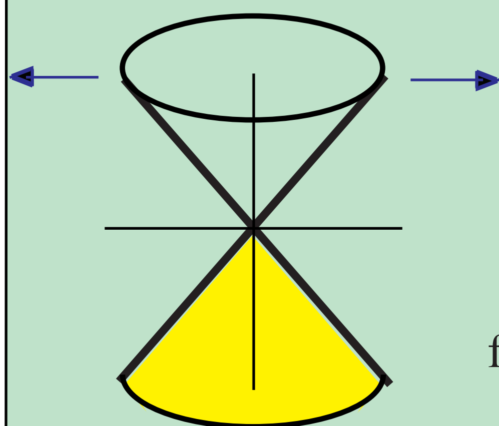
collective Bose modes:

propagating
oscillation of **position**
of Fermi point

$$\mathbf{p} \rightarrow \mathbf{p} - e\mathbf{A}$$

form effective dynamic
electromagnetic field

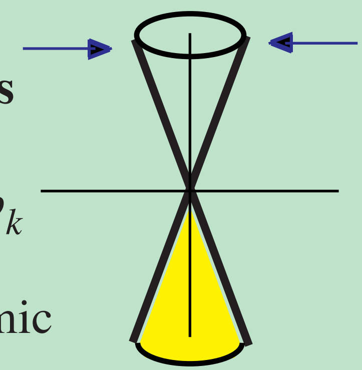




propagating
oscillation of **slopes**

$$E^2 = c^2p^2 \rightarrow g^{ik}p_i p_k$$

form effective dynamic
gravity field



two generic quantum field theories of interacting bosonic & fermionic fields

relativistic quantum fields and gravity emerging near Fermi point

Atiyah-Bott-Shapiro construction:
linear expansion of Hamiltonian near the nodes in terms of Dirac Γ-matrices

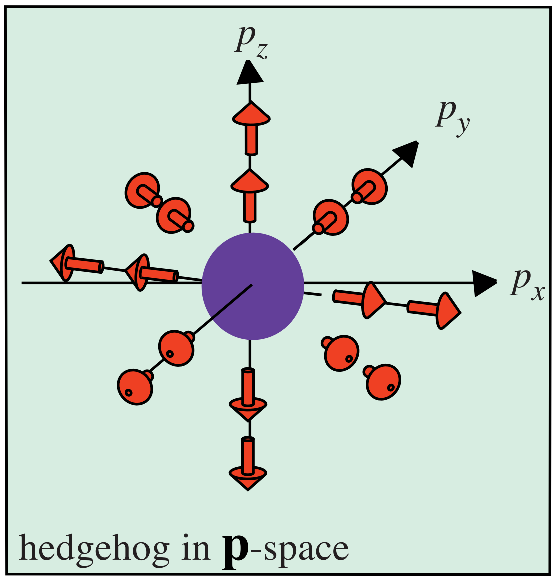
$$\boxed{E = v_F\,(p - p_F)}$$

linear expansion near
Fermi surface

emergent relativity

$$\boxed{H = e_i^k\,\Gamma^i \cdot (p_k - p_k^0)}$$

linear expansion near
Fermi point



hedgehog in **p**-space

$$\boxed{g^{\mu\nu}(p_\mu - eA_\mu - e\tau\cdot\mathbf{W}_\mu)(p_\nu - eA_\nu - e\tau\cdot\mathbf{W}_\nu) = 0}$$

effective metric:
emergent gravity

effective
electromagnetic
field

effective
$SU(2)$ gauge
field

effective
electric charge

$e = +1$ or -1

effective
isotopic spin

all ingredients of Standard Model :
chiral fermions & gauge fields
emerge in low-energy corner
together with spin, Dirac Γ−matrices, gravity & physical laws:
Lorentz & gauge invariance, equivalence principle, etc

gravity & gauge fields
are collective modes
of vacua with Fermi point

Extension of Landau idea:
electric charge purely from vacuum polarization

Landau proposal for QED

Landau, 1955

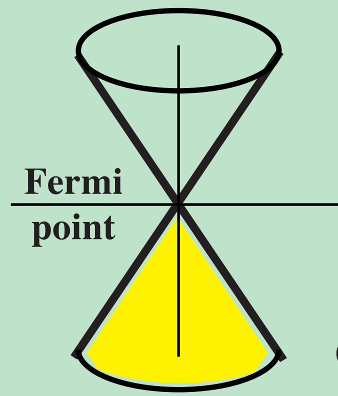
$$\boxed{\frac{1}{e^2} = \frac{\nu}{3\pi} \ln\left(E_{\rm Planck}^2 / m^2\right)}$$

ν number of charged particles
$E_{\rm Planck}$ Planck energy
m electron mass

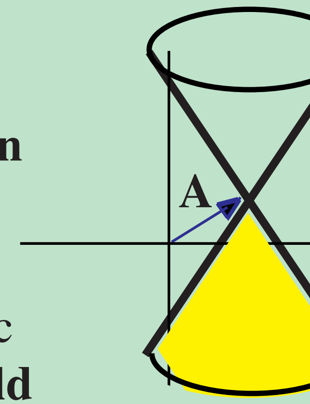
$$\boxed{\nu = 12}$$

$$\boxed{N_F = 4}$$

extension to Standard Model which as effective theory has two different cut-off for bosons & fermions



vacuum mode
which shifts **position**
of Fermi point
$\mathbf{p} \rightarrow \mathbf{p} - e\mathbf{A}$
is effective dynamic
electromagnetic field



$$\boxed{\frac{1}{e^2} = \frac{8N_F}{9\pi} \ln\left(E_{\rm UV}^2 / m_Z^2\right) - \frac{11}{6\pi} \ln\left(E_{\rm Planck}^2 / m_Z^2\right)}$$

N_F number of families
$E_{\rm UV}$ ultraviolet cutoff
m_Z mass of Z-boson

Klinkhamer-Volovik
JETP Lett. **81** (2005) 551

physics at the intermediate Planck scale is Lorentz invariant

Lorentz violating scale

breaking physical laws

anti GUT

emerging physical laws

emerging symmetry

maximum symmetry

Planck energy scale 10^{19} GeV

symmetry restoration

GUT

symmetry breaking

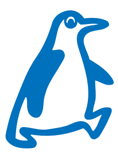
energy scale in accelerators 10^{3} GeV

Theory of quantum vacuum at Planck scale should be Lorentz invariant

crossover from Landau 2-fluid hydrodynamics to Einstein general relativity

they represent two different limits of hydrodynamic type equations

equations for $g^{\mu\nu}$ depend on hierarchy of ultraviolet cut-off's:
Planck energy scale $E_{\rm Planck}$ vs Lorentz violating scale $E_{\rm Lorentz}$

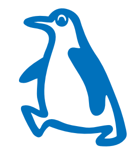


$E_{\rm Planck} >> E_{\rm Lorentz}$

emergent Landau two-fluid hydrodynamics

$E_{\rm Planck} << E_{\rm Lorentz}$

emergent general covariance & general relativity



^{3}He-A with Fermi point

$E_{\rm Lorentz} << E_{\rm Planck}$

$E_{\rm Lorentz} \sim 10^{-3}\, E_{\rm Planck}$

Universe

$E_{\rm Lorentz} >> E_{\rm Planck}$

$E_{\rm Lorentz} > 10^{9}\, E_{\rm Planck}$

4. self-sustained quantum liquids & quantum vacuum:

vacuum as Lorentz invariant medium

Thermodynamics & dynamics of quantum vacuum

superfluid He

$N = 10^{23}$ **atoms**

Galilean principles

quantum vacuum & superfluid helium are macroscopic systems; they obey macroscopic thermodynamic & hydrodynamic laws which follow from their symmetry

quantum vacuum

$N = 10^{180}$

"Planck units"

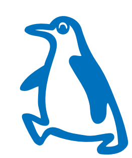
Lorentz invariance

Cosmological constant - weight of ether (vacuum vacuum)

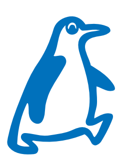
"ether should not be thought of as endowed with the quality characteristic of ponderable media ... The idea of motion may not be applied to it."

Einstein, 1920

zero cosmological constant ???



wait !



dark energy problem

recipe to cook Universe

Dark Energy	70%
Dark Matter	30%
Baryonic Matter	4%
Visible Matter	0,4%

Dark and Dark!
What is the difference?

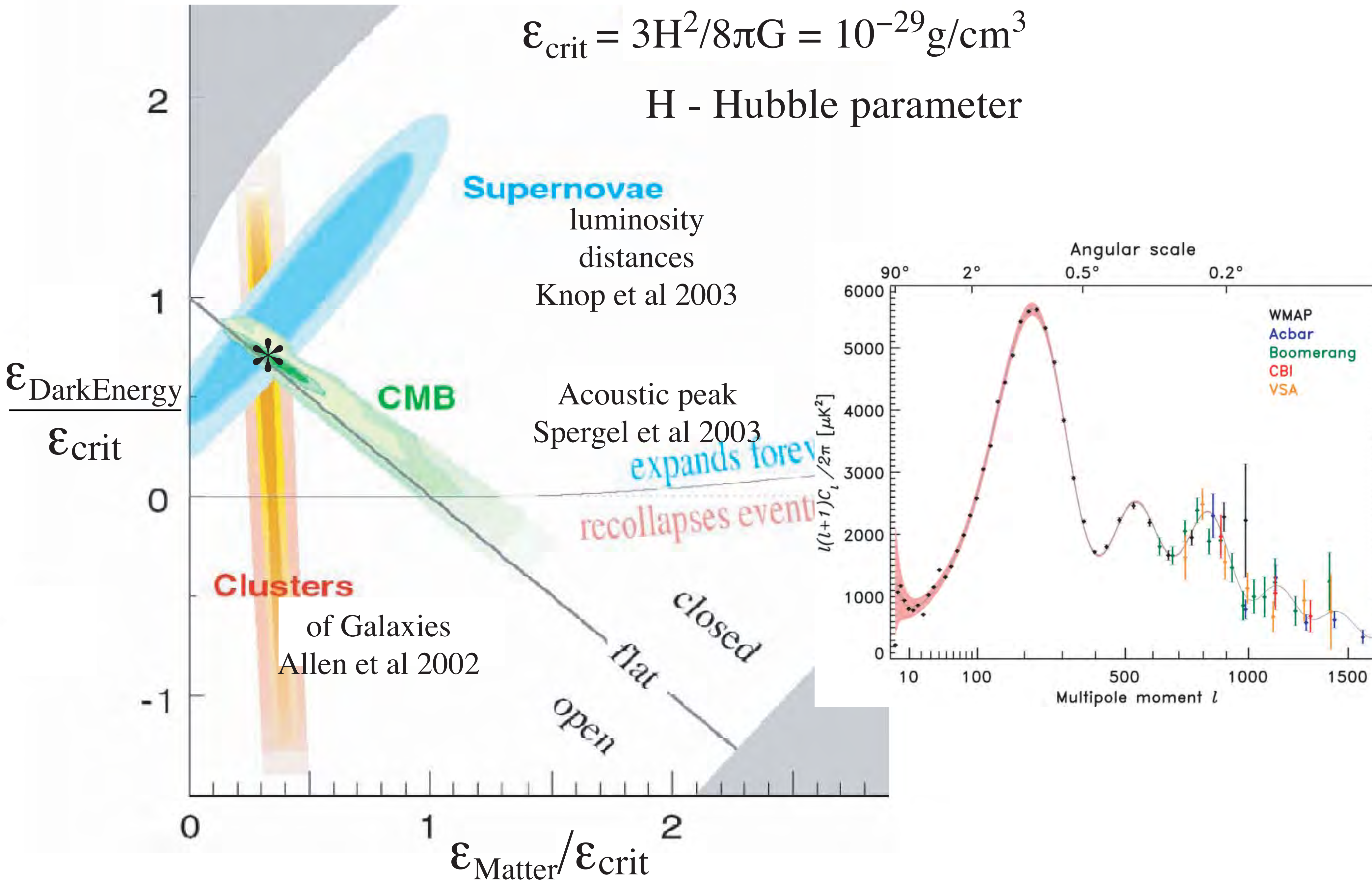
dark matter forms clusters
like ordinary matter

cosmological constant Λ is possible candidate for dark energy

$$\Lambda = \varepsilon_{\text{Dark Energy}}$$

observational cosmology:

dark energy vs **dark matter**



Cosmological Term

* **Original Einstein equations**

$$\boxed{\frac{1}{8\pi G}\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) = T_{\mu\nu}^{\mathrm{Matter}}}$$

* **1917: Einstein added the cosmological term**

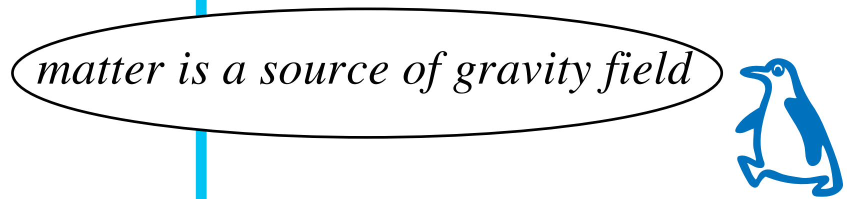
$$\boxed{\frac{1}{8\pi G}\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) - \Lambda g_{\mu\nu} = T_{\mu\nu}^{\mathrm{Matter}}}$$

cosmological constant

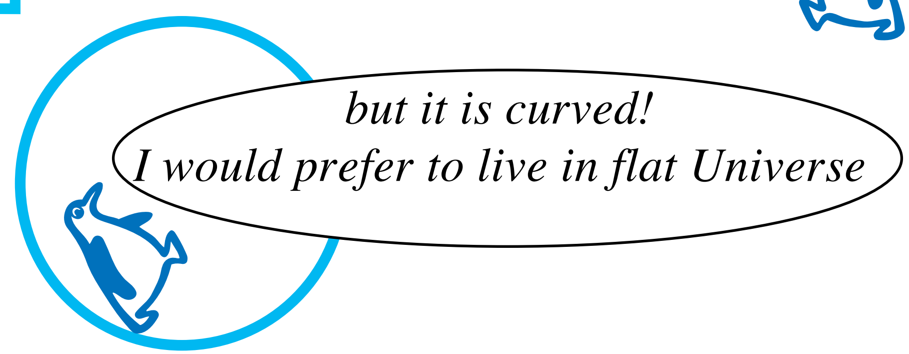
& obtained static solution: Universe as 3D sphere

$$\boxed{\Lambda = 0.5\ \varepsilon_{\mathrm{Matter}}} = 1/(8\pi G R^2)$$

energy density of matter **radius of 3D sphere**



$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = R^2$$



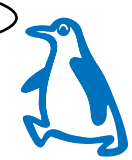
Estimation of cosmological constant

when I was discussing cosmological problems with Einstein, he remarked that the introduction of the cosmological term was the biggest blunder of his life

-- George Gamow, My World Line, 1970

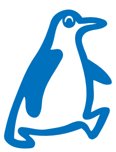
$\boxed{\Lambda = 0.5\ \varepsilon_{\mathrm{Matter}}} = 1/(8\pi G R^2)$

compare with observed $\Lambda = 2.3\ \varepsilon_{\mathrm{Matter}}$

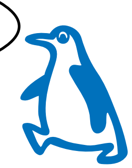


order of magnitude is OK, why blunder?

this was correct estimation of Λ



the first and the last one:
after 90 years nobody could improve it

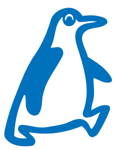


arguments against Λ !

* **1917: de Sitter found stationary solution of Einstein equations without matter**

$$\frac{1}{8\pi G}\,(R_{\mu\nu} - g_{\mu\nu}R/2) - \Lambda g_{\mu\nu} = 0$$

no source of gravity field !



What?
empty space gravitates !?

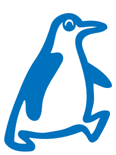
1923: expanding version of de Sitter Universe:

$$R = \exp\,(Ht) \qquad H^2 = 3/(8\pi\Lambda G)$$

* **1924: Hubble : Universe is not stationary**
* **1929: Hubble : recession of galaxies**

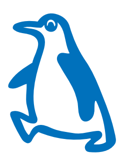
Wenn schon keine quasi-statische Welt, dann fort mit dem kosmologischen Glied.

A. Einstein $\longrightarrow$ H. Weyl, 23 Mai 1923



Away with curvature !
and with cosmological term !

Wait !



the question arises whether it is possible to represent the observed facts without introducing a curvature at all.

Einstein & de Sitter, PNAS **18** (1932) 213

Epoch of quantum mechanics: Λ *as vacuum energy*

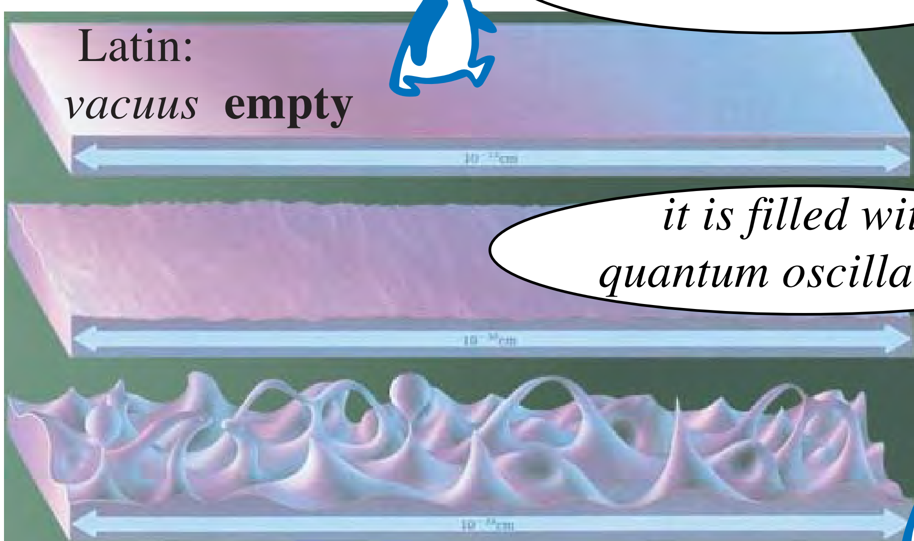
$$\frac{1}{8\pi G}\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) - \Lambda g_{\mu\nu} = T_{\mu\nu}^{\mathrm{Matter}}$$

move Λ to the right

$$\frac{1}{8\pi G}\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) = \Lambda g_{\mu\nu} + T_{\mu\nu}^{\mathrm{Matter}}$$

physical vacuum as source of gravity

vacuum is not empty ?



it is filled with quantum oscillations

zero point energy has weight ?

you do not believe Einstein? energy must gravitate !

quantum field is set of oscillators

$$E_n = h\nu\left(n + 1/2\right)$$

$n = 0$: **zero point energy of quantum fluctuations**

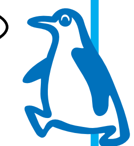
Equation of state of quantum vacuum

$$\frac{1}{8\pi G}\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) = T^{\mathrm{Vacuum}}_{\mu\nu} + T^{\mathrm{Matter}}_{\mu\nu}$$

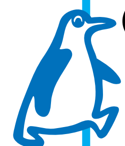
$$T^{\mathrm{Vacuum}}_{\mu\nu} = \Lambda g_{\mu\nu}$$

$$\Lambda = \varepsilon_{\mathrm{vac}} = -p_{\mathrm{vac}}$$

energy density of vacuum **pressure of vacuum**



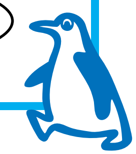
what is vacuum?



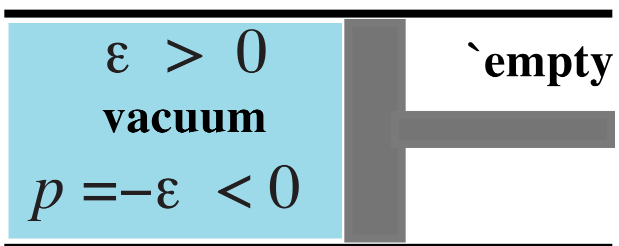
vacuum is medium with equation of state

$$\boxed{\varepsilon = -p}$$

right !



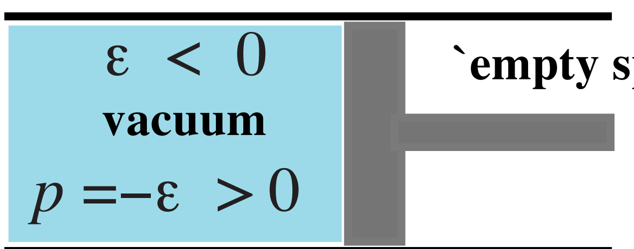
pumping the vacuum by piston



\`empty space'

applied external force

$F = \varepsilon A$, **negative pressure**



\`empty space'

applied external force

$F = \varepsilon A$, **positive pressure**

$$E_{\mathrm{vac}} = \varepsilon_{\mathrm{vac}} V$$

$$p_{\mathrm{vac}} = - dE/dV = - \varepsilon_{\mathrm{vac}}$$

How heavy is aether?

Luminiferous aether (photons)

$\Sigma_{\mathbf{k}}\, (n(\mathrm{k}) + 1/2)\, ck$

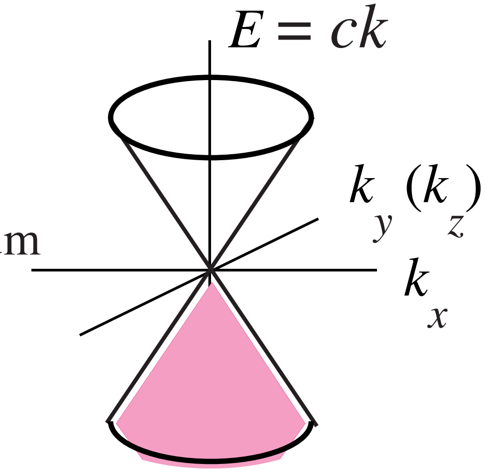
photonic vacuum:

$n(\mathrm{k})=0$

weight of photon vacuum

weight of Dirac vacuum

Dirac vacuum



occupied negative energy levels

$$\boxed{\varepsilon_{\text{zero point}} + \varepsilon_{\text{Dirac}} = (1/2)\sum_{\text{bosons}} E(\mathrm{k}) \;-\; \sum_{\text{fermions}} E(\mathrm{k}) \;= (\nu_{\mathbf{b}} - \nu_{\mathbf{f}})\, c k^4_{\text{Planck}}}$$

zero-point energy

energy of Dirac vacuum

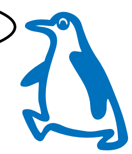
number of bosonic fields

number of Dirac fields

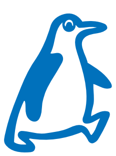
Planck scale

$$\boxed{\varepsilon_{\text{zero point}} \sim 10^{120} \Lambda_{\text{upper limit}}}$$

vacuum is too heavy !



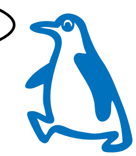
maybe $\Lambda = 0$ **?!**



supersymmetry:
symmetry between fermions and bosons

$\nu_{\mathbf{b}} = \nu_{\mathbf{f}}$

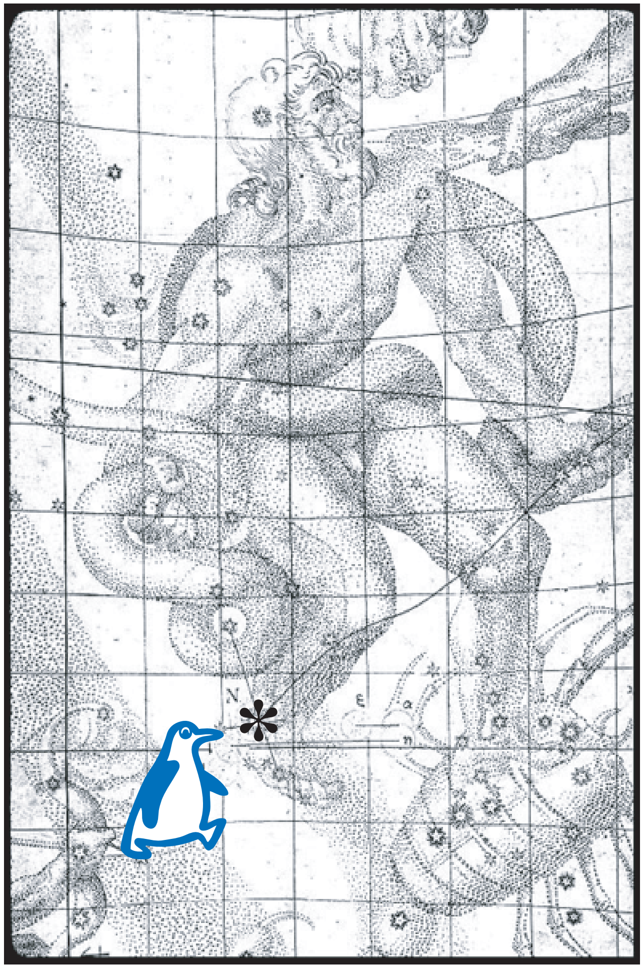
there is no supersymmetry below TeV



Λ in supernova era

Kepler's Supernova 1604

from `*De Stella Nova in Pede Serpentarii*'



distant supernovae: accelerating Universe

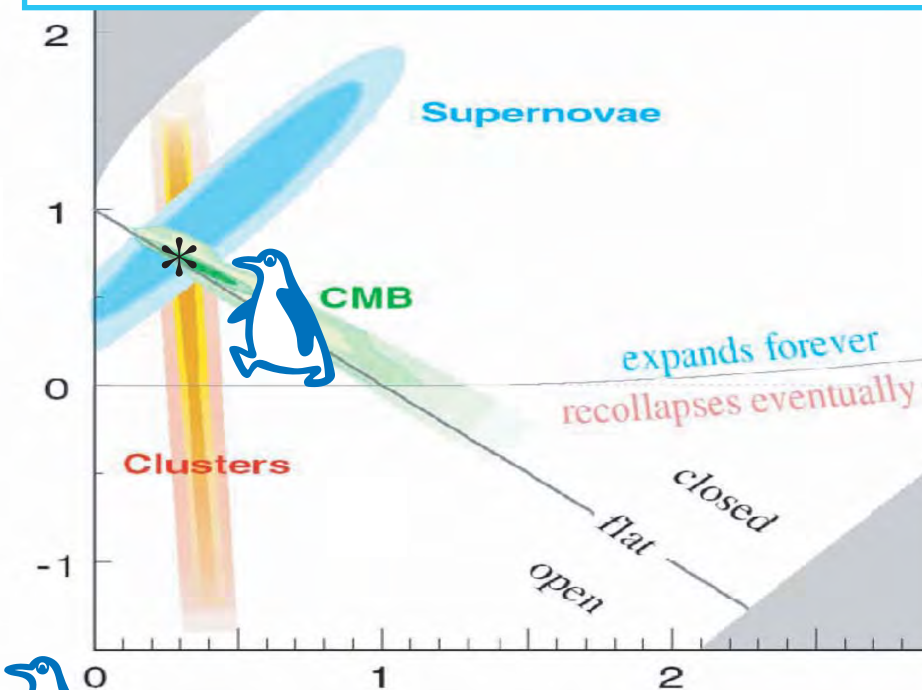
(Perlmutter et al., Riess et al.)

$$\Lambda_{exp} = 2\text{-}3\ \varepsilon_{\text{Dark Matter}} = 10^{-123}\varepsilon_{\text{zero point}}$$

$$\Lambda = \varepsilon_{vac} = \varepsilon_{\text{Dark Energy}} = 70\%$$

$$\varepsilon_{\text{Dark Matter}} = 30\%$$

$$\varepsilon_{\text{Visible Matter}} = 0{,}4\%$$



Universe is flat !

you are right,
but Λ is not zero

$$\Lambda_{exp} = 2\text{-}3\ \varepsilon_{\text{Dark Matter}} = 10^{-123}\varepsilon_{\text{zero point}}$$

it is easier to accept that $\Lambda = 0$
than 123 orders smaller

magic word: *regularization*

wisdom of particle physicist: $\boxed{\dfrac{1}{0} = 0}$

What can condensed matter physicist say on Λ ?

Why condensed matter ??!

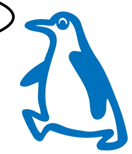
Cosmological constant paradox

$$\Lambda_{\text{observation}} = \varepsilon_{\text{Dark Energy}} \sim 2\text{–}3\ \varepsilon_{\text{DM}} \sim 10^{-47}\ \text{GeV}^4$$

$$\Lambda_{\text{theory}} = \varepsilon_{\text{zero point energy}} \sim E^4_{\text{Planck}} \sim 10^{76}\ \text{GeV}^4$$

$$\Lambda_{\text{observation}} \sim 10^{-123}\,\Lambda_{\text{Theory}}$$

too bad for theory

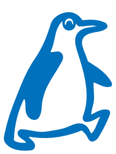


problems:

*** Why is vacuum not extremely heavy?**

*** Why is vacuum gravitating? Why is Λ non-zero?**

*** Why is vacuum as heavy as (dark) matter ?**



it is easier to accept that Λ=0
than 123 orders of magnitude smaller

what is natural value of cosmological constant ?

$$\Lambda = E_{\mathrm{Planck}}^{4}$$

$$\Lambda = 0$$

time dependent cosmological constant

$$\boxed{\Lambda \sim E_{\mathrm{Planck}}^4}$$

could be in early Universe

$$\boxed{\Lambda \sim 0}$$

should be in old Universe

vacuum as self-sustained system

self-sustaining:
opposing exterior influence
or independent

what is self-sustained system ?

medium characterized by conserved extensive quantity
whose total value $Q=qV$ determines the volume V of the system;
medium can exist as isolated system i.e. at external pressure $P=0$

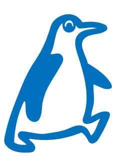
examples of self-sustained systems:

superfluid ^4He; liquid ^3He; solid ^4He; solid ^3He ...

$Q=N$ number of helium atoms, $q=n$ density of atoms

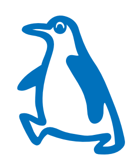
examples of non-self-sustained systems:

quantum gases, BEC, plasma, ... $Q=N$, but P is nonzero



relativistic quantum vacuum
is not a liquid or solid !

right! but let us start with
liquid helium



vacuum energy in self-sustained condensed matter

(microscopic theory of helium liquid)

liquid ^{4}He

N **= 10^{23} atoms**

Schrödinger quantum mechanics

for N atoms

$$\boxed{\hat{H} = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\Delta_i + \sum_{i=1}^{N}\sum_{j=i+1}^{N} U(\mathbf{r}_i - \mathbf{r}_j)}$$

liquid ^{3}He

N **= 10^{23} atoms**

too many atoms
for calculations

at large N
quantum field theory emerges

liquid ^{4}He

N **= 10 ^{23} atoms**

Quantum field theory of condensed matter

liquid ^{3}He

N **= 10 ^{23} atoms**

Abrikosov, Gor'kov & Dzyaloshinskii
Quantum Field Theoretical Methods
in Statistical Physics

second quantization

$$\boxed{H_{QFT} = H - \mu N = \int d^3x\, \psi^+\left(-\frac{\Delta}{2m} - \mu\right)\psi + \iint d^3x d^3y\, U(x-y)\, \psi^+(x)\psi^+(y)\psi(y)\psi(x)}$$

operator ψ bosonic in **^{4}He,** fermionic in **^{3}He**

we shall see later that it is

$$E - \mu Q$$

which is gravitating in the quantum vacuum

not E

Vacuum energy in cond-mat Quantum Field Theory

$$H_{QFT} = \int d^3x\, \psi^+\left(-\frac{\Delta}{2m} - \mu\right)\psi + \iint d^3x\, d^3y\, U(x-y)\, \psi^+(x)\psi^+(y)\psi(y)\psi(x)$$

Macroscopic (thermodynamic) approach

$$\boxed{\varepsilon_{vac} V = \langle H_{QFT} \rangle_{vac} = \langle H \rangle_{vac} - \mu \langle N \rangle_{vac}}$$

thermodynamic Gibbs-Duhem relation

$$\boxed{\langle H \rangle - \mu \langle N \rangle - TS = -pV}$$

$T = 0$

$$\boxed{\varepsilon_{vac} = -\, p_{vac}}$$

all quantum vacua
obey the same equation of state

$$\varepsilon_{vac} = - p_{vac}$$

all quantum vacua
obey the same equation of state

relativistic and non-relativistic ?

thermodynamics is more general
than Lorentz symmetry

liquid He

$N = 10^{23}$ **atoms**

quantum vacuum & condensed matter
are macroscopic systems;
both must obey macroscopic
thermodynamic laws

Universe

$N = 10^{180}$
"Planck atoms"

handle with care !!!

"ether should not be thought of as endowed with
the quality characteristic of ponderable media ...
The idea of motion may not be applied to it."

Einstein, 1920

energy of free vacuum as energy of the self-sustained system

self-sustained system
may exist without environment

no environment - no force

empty space	self-sustained vacuum	empty space
no external force	$p = -\varepsilon = 0$	no external force

$$\boxed{\varepsilon_{vac} = -p_{vac} = 0}$$

energy of equilibrium self-sustained
vacuum is zero

liquid ^{4}He: ground state of $N = 10^{23}$ atoms

isolated droplet of quantum liquid:
no external pressure

$$\boxed{\varepsilon_{vac} = -p_{vac} = 0}$$

liquid ^{3}He: ground state of $N = 10^{23}$ atoms

what about surface tension?

when $N \Rightarrow \infty$ surface tension is neglected

self-sustained relativistic quantum vacuum

helium liquids & solids are self-sustained
due to conservation of number N of helium atoms,
its density n is non-zero in equilibrium

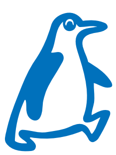
$$\boxed{n = n_{eq}}$$

relativistic quantum vacuum

$N = ?$

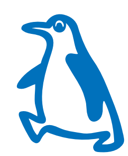
what is n
in relativistic vacuum ?

J. Bjorken



can relativistic vacuum be a self-sustained system ?

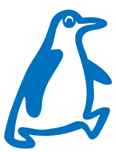
?



charge density of relativistic quantum vacuum

relativistic quantum vacuum

$n = ?$



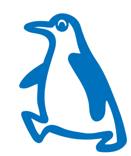
n cannot be charge density: charge density is not invariant under Lorentz transformation

$$n = J^0$$

$$\nabla_\mu J^\mu = 0$$

in Lorentz invariant vacuum

$$J^\mu = 0$$



otherwise preferred reference frame

Lorentz invariance is guiding principle

> *"ether should not be thought of as endowed with the quality characteristic of ponderable media ... The idea of motion may not be applied to it."*

Einstein, 1920

Lorentz invariant vacuum has no momentum

$$\mathbf{P}_{\rm vac} = 0$$

$$\mathbf{P}' = \gamma\,(\mathbf{P} + \mathbf{v}\,E)$$

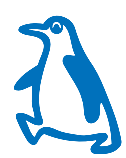
$$E' = \gamma\,(E + \mathbf{v}\cdot\mathbf{P})$$

γ - *Lorentz factor*

$$P^{\mu}_{\rm vac} = 0$$

$$E_{\rm vac} = 0$$

it has no energy too !



Zero energy of Lorentz invariant vacuum

$\boxed{E_{vac} = 0}$

$\boxed{\mathbf{P}_{vac} = \mathrm{E}_{vac}\mathbf{v}_{vac} = 0}$

in perfect vacuum
vacuum energy $= 0$

$\boxed{\mathrm{E}_{vac} = \int dV\, \varepsilon_{vac} = 0} \longrightarrow \boxed{\Lambda = \varepsilon_{vac} = 0}$

and cosmological constant
$\Lambda = 0$

perfect vacuum has no weight

self-sustained equilibrium vacuum
is
Minkowski vacuum

vacuum has weight in presence of matter

matter violates Lorentz invariance
(preferred frame)
vacuum becomes ponderable

matter

$$\varepsilon_{\rm m}(\mathbf{v}) = \gamma^2 \left(\varepsilon_{\rm m} + \mathbf{v}^2 P_{\rm m}\right)$$

$$\mathbf{p}_{\rm m}(\mathbf{v}) = \mathbf{v}\, \gamma^2 \left(\varepsilon_{\rm m} + P_{\rm m}\right)$$

vacuum

$$\varepsilon_{\rm v}(\mathbf{v}) = \varepsilon_{\rm v} = -P_{\rm v}$$

$$\mathbf{p}_{\rm v}(\mathbf{v}) = 0$$

pressure of isolated system matter + vacuum

$$P_{\rm total} = P_{\rm v} + P_{\rm m} = 0$$

$$\varepsilon_{\rm v}(\mathbf{v}) = \varepsilon_{\rm v} = -P_{\rm v} = P_{\rm m}$$

vacuum has weight: its mass depends on matter

$$\mathbf{p}_{\rm total} = \mathbf{p}_{\rm v}(\mathbf{v}) + \mathbf{p}_{\rm m}(\mathbf{v}) = \mathbf{p}_{\rm m}(\mathbf{v}) = \mathbf{v}\, \gamma^2 \left(\varepsilon_{\rm m} + P_{\rm m}\right)$$

$$\varepsilon_{\rm total} = \varepsilon_{\rm v}(\mathbf{v}) + \varepsilon_{\rm m}(\mathbf{v}) = \gamma^2 \left(\varepsilon_{\rm m} + P_{\rm m}\right)$$

$$\mathbf{p}_{\rm total} = \mathbf{v}\, \varepsilon_{\rm total}$$

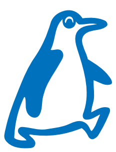
vacuum + matter obey Lorentz invariance, while separately they do not

three assumptions on quantum vacuum

1. quantum vacuum is a Lorentz-invariant state

2. quantum vacuum is a self-sustained medium at zero external pressure

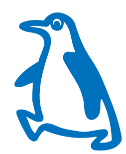
3. quantum vacuum is characterized by a conserved Lorentz-invariant charge q (analog of particle density n), which is constant over spacetime

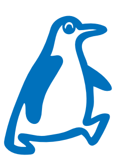


q cannot be conventional charge density n:
n is not Lorentz invariant

$$\boxed{n' = \gamma(n + \mathbf{v}\cdot\mathbf{j})}$$

$$\boxed{q' = q}$$





do such q exist ?

relativistic charges q

possible

$$\boxed{\nabla_\alpha q^{\alpha\beta} = 0}$$

$$\boxed{\nabla_\alpha q^{\alpha\beta\mu\nu} = 0}$$

$$\boxed{q^{\alpha\beta} = q\, g^{\alpha\beta}}$$

$$\boxed{q^{\alpha\beta\mu\nu} = q\, e^{\alpha\beta\mu\nu}}$$

impossible

$$\boxed{\nabla_\alpha q^{\alpha} = 0}$$

$$\boxed{q^{\alpha} = ?}$$

thermodynamics in flat space

the same as in cond-mat

$$F_{\kappa\lambda\mu\nu} = \nabla_{[\kappa} A_{\lambda\mu\nu]}$$

$$F_{\kappa\lambda\mu\nu} = q\, e_{\kappa\lambda\mu\nu}$$

conserved charge Q

$$Q = \int dV\, q$$

thermodynamic potential

$$\Omega = E - \mu Q = \int dV\, (\varepsilon\,(q) - \mu q)$$

Lagrange multiplier or chemical potential μ

pressure

$$P = -\, dE/dV = -\varepsilon + q\, d\varepsilon/dq$$

$$E = V\, \varepsilon(Q/V)$$

$$d\Omega/dq = 0$$

equilibrium vacuum

$$d\varepsilon/dq = \mu$$

equilibrium self-sustained vacuum

$$d\varepsilon/dq = \mu$$

$$\varepsilon - q\, d\varepsilon/dq = -P = 0$$

vacuum energy & cosmological constant

equilibrium self-sustained vacuum

$$\boxed{\begin{aligned} &d\varepsilon/dq = \mu \\ &\varepsilon - q\, d\varepsilon/dq = -P = 0 \end{aligned}}$$

$$\boxed{q \sim \mu \sim E^2_{\rm Planck}}$$

vacuum variable in equilibrium self-sustained vacuum

$$\boxed{\varepsilon\,(q) \sim E^4_{\rm Planck}}$$

energy of equilibrium self-sustained vacuum

pressure

$$\boxed{P = -\varepsilon + q\, d\varepsilon/dq = -\Omega}$$

$$\boxed{\Lambda = \Omega = -P = \varepsilon - \mu\, q}$$

cosmological constant

$$\boxed{\Lambda = \underset{\displaystyle E^4_{\rm Planck}}{\varepsilon} - \underset{\displaystyle E^4_{\rm Planck}}{\mu q} = 0}$$

cosmological constant in equilibrium self-sustained vacuum

***self-tuning*: two Planck-scale quantities cancel each other in equilibrium self-sustained vacuum**

dynamics of q in flat space

$$q^2 = -\frac{1}{24} F_{\kappa\lambda\mu\nu} F^{\kappa\lambda\mu\nu}$$

$$F_{\kappa\lambda\mu\nu} = \nabla_{[\kappa} A_{\lambda\mu\nu]}$$

$$F^{\kappa\lambda\mu\nu} = q\, e^{\kappa\lambda\mu\nu}$$

action $$S = \int dV\, dt\, \varepsilon(q)$$

Maxwell equation $$\nabla_\kappa \left(F^{\kappa\lambda\mu\nu} q^{-1} d\varepsilon/dq\right) = 0 \longrightarrow \nabla_\kappa (d\varepsilon/dq) = 0$$

solution $$d\varepsilon/dq = \mu$$

integration constant μ in dynamics
becomes chemical potential in thermodynamics

4-form field $F_{\kappa\lambda\mu\nu}$ is an example of conserved charge q in relativistic vacuum

dynamics of q in curved space

$$\boxed{q^2 = -\frac{1}{24} F_{\kappa\lambda\mu\nu} F^{\kappa\lambda\mu\nu} \qquad F_{\kappa\lambda\mu\nu} = \nabla_{[\kappa} A_{\lambda\mu\nu]}}$$

action

$$\boxed{S = \int d^4x \, (-g)^{1/2} \left[\, \varepsilon(q) + K(q) R \, \right] + S_{\text{matter}}}$$

gravitational coupling $K(q)$ is determined by vacuum and thus depends on vacuum variable q

Maxwell equations

$$\nabla_\kappa \left(F^{\kappa\lambda\mu\nu} \, q^{-1} \left(d\varepsilon/dq + R \, dK/dq \right) \right) = 0 \longrightarrow d\varepsilon/dq + R \, dK/dq = \underbrace{\mu}_{\text{integration constant}}$$

Einstein equations

$$\underbrace{K\left(R g_{\mu\nu} - 2R_{\mu\nu}\right)}_{\text{Einstein tensor}} + \underbrace{\left(\varepsilon - \mu q\right) g_{\mu\nu}}_{\text{cosmological term} \neq \varepsilon g_{\mu\nu}} - 2\left(\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla^\lambda \nabla_\lambda \right) K = \underbrace{T_{\mu\nu}}_{\text{matter}}$$

$$\boxed{\nabla_\mu T^{\mu\nu} = 0}$$

Einstein tensor

cosmological term $\neq \varepsilon \, g_{\mu\nu}$

matter

integration constant

case of $K = const$

Maxwell equations

$$\nabla_\kappa \left(F^{\kappa\lambda\mu\nu} \; q^{-1} \left(d\varepsilon/dq + R\, dK/dq \right) \right) = 0 \longrightarrow d\varepsilon/dq + R\, dK/dq = \mu$$

modified Einstein equations

$$K\left(Rg_{\mu\nu} - 2R_{\mu\nu}\right) + \left(\varepsilon - \mu q\right) g_{\mu\nu} - 2\left(\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla^\lambda \nabla_\lambda \right) K = T_{\mu\nu}$$

Einstein tensor

cosmological term

$$\boxed{\nabla_\mu T^{\mu\nu} = 0}$$

matter

integration constant

$$K = const = \frac{1}{16\pi G}$$

Einstein equations

$$\boxed{\frac{1}{16\pi G}\left(Rg_{\mu\nu} - 2R_{\mu\nu}\right) + \Lambda\, g_{\mu\nu} = T_{\mu\nu}}$$

Minkowski solution

Maxwell equations

$$d\varepsilon/dq + R\, dK/dq = \mu$$

Einstein equations

$$K\left(Rg_{\mu\nu} - 2R_{\mu\nu}\right) + \left(\varepsilon - \mu q\right)g_{\mu\nu} - 2\left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^\lambda\nabla_\lambda\right)K = T_{\mu\nu}$$

Einstein tensor **cosmological term** $\nabla_\mu T^{\mu\nu} = 0$ **matter**

Minkowski vacuum solution

$$R = 0 \qquad d\varepsilon/dq = \mu$$

$$\Lambda = \varepsilon(q) - \mu q = 0$$

vacuum energy in action: $\varepsilon\,(q) \sim E^4_{\mathrm{Planck}}$

thermodynamic vacuum energy: $\varepsilon - \mu q = 0$

Model vacuum energy $\varepsilon\,(q)$

$$\varepsilon\,(q) = \frac{1}{2\chi}\left(-\frac{q^2}{q_0{}^2} + \frac{q^4}{3q_0{}^4}\right)$$

$$q^2 = -\frac{1}{24}\,F_{\kappa\lambda\mu\nu}F^{\kappa\lambda\mu\nu}$$

$$F_{\kappa\lambda\mu\nu} = \nabla_{[\kappa}A_{\lambda\mu\nu]}$$

Minkowski vacuum solution

$$d\varepsilon/dq = \mu$$

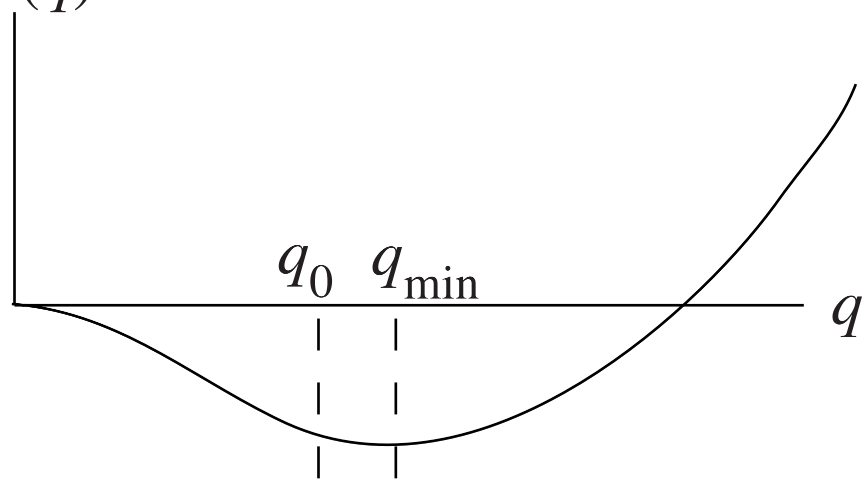
$$\varepsilon - \mu q = 0$$

$\longrightarrow$

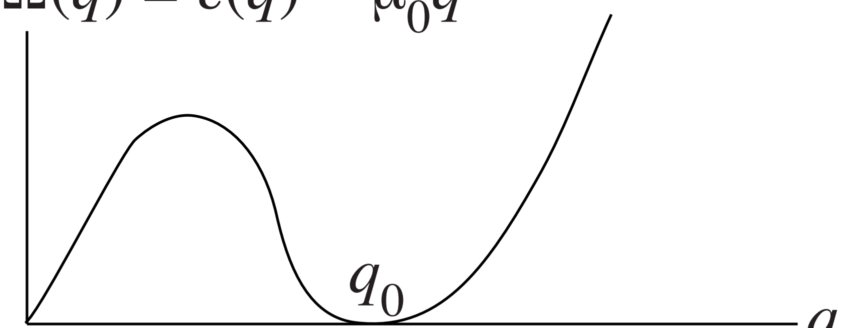
$$q = q_0$$

$$\mu = \mu_0 = -\frac{1}{3\chi q_0}$$

$\varepsilon(q)$



$\Omega(q) = \varepsilon(q) - \mu_0 q$



empty space $q = 0$

self-sustained vacuum $q = q_0$

vacuum compressibility

$$\chi = -\frac{1}{V}\frac{dV}{dP}$$

$$\frac{1}{\chi} = \left(q^2\, d^2\varepsilon/dq^2\right)_{q=q_0} > 0$$

vacuum stability

Minkowski vacuum (q-independent properties)

$$\Lambda = \Omega_{vac} = -P_{vac}$$

energy density of vacuum — **pressure of vacuum**

$$P_{vac} = -\, dE/dV = -\, \Omega_{vac}$$

$$\chi_{vac} = -(1/V)\, dV/dP$$

compressibility of vacuum

$$<(\Delta P_{vac})^2> = T/(V\chi_{vac})$$

$$<(\Delta\Lambda)^2> = <(\Delta P)^2>$$

pressure fluctuations

natural value of Λ *determined by macroscopic physics*

$$\boxed{\Lambda = 0}$$

natural value of χ_{vac} *determined by microscopic physics*

$$\boxed{\chi_{vac} \sim E^{-4}_{\mathbf{Planck}}}$$

volume of Universe is large:

$$\boxed{V > T_{CMB}/(\Lambda^2\chi_{vac})}$$

$$V > 10^{28}\, V_{hor}$$

dynamics of q in curved space: relaxation of Λ

Maxwell equations

$$d\varepsilon/dq + R\, dK/dq = \mu$$

Einstein equations

$$K\left(R g_{\mu\nu} - 2R_{\mu\nu}\right) + g_{\mu\nu}\Lambda(q) - 2\left(\nabla_\mu \nabla_\nu - g_{\mu\nu}\nabla^\lambda \nabla_\lambda\right) K = T^{\text{matter}}_{\mu\nu}$$

$$\Lambda(q) = \varepsilon(q) - \mu_0 q$$

dynamic solution

$$q(t) - q_0 \sim q_0 \frac{\sin \omega t}{t}$$

$$\Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2}$$

$$H(t) = \frac{\dot a(t)}{a(t)} = \frac{2}{3t}\left(1 - \cos \omega t\right)$$

$$\omega \sim E_{\text{Planck}}$$

similar to scalar field with mass $M \sim E_{\text{Planck}}$

A.A. Starobinsky, Phys. Lett. **B 91**, 99 (1980)

Relaxation of Λ (generic q-independent properties)

$$\Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2}$$

cosmological "constant"

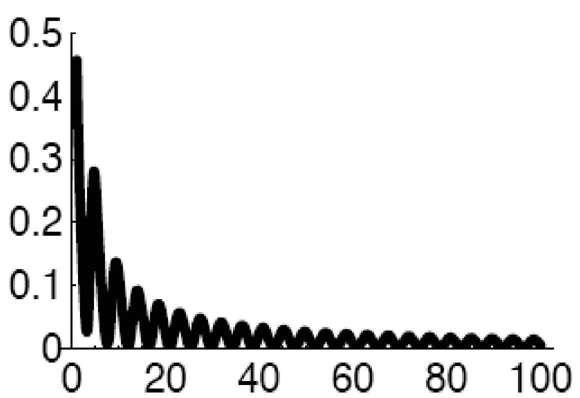
$$H(t) = \frac{\dot a(t)}{a(t)} = \frac{2}{3t}\left(1 - \cos \omega t\right)$$

Hubble parameter

$$G(t) = G_N \left(1 + \frac{\sin \omega t}{\omega t}\right)$$

Newton "constant"

$\omega \sim E_{\mathrm{Planck}}$



$$<\Lambda(t_{\mathrm{Planck}})> \sim E^4_{\mathrm{Planck}}$$

$$\Lambda(t = \infty) = 0$$

natural solution of the main cosmological problem ?

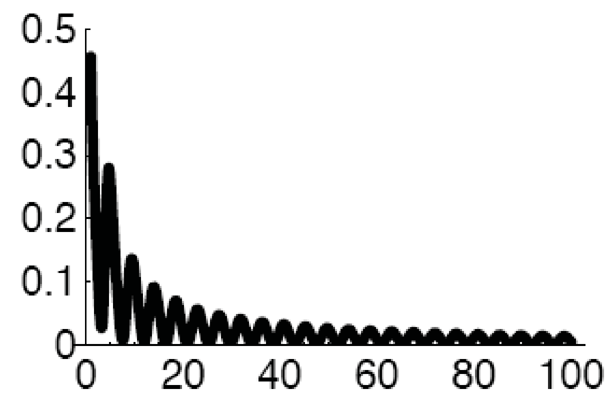
Λ relaxes from natural Planck scale value to natural zero value

present value of Λ

$$\Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2}$$

$$\omega \sim E_{Planck}$$

dynamics of Λ:
from Planck to present value

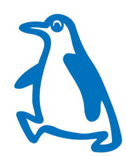


$$<\Lambda(t_{Planck})> \sim E^4_{Planck}$$

$$<\Lambda(t_{present})> \sim E^2_{Planck} \,/\, t^2_{present} \sim 10^{-120}\, E^4_{Planck}$$

coincides with present value of dark energy

something to do with coincidence problem ?



cold matter simulation

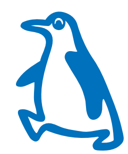
$$\boxed{\Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2}}$$

$$\boxed{H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{2}{3t}\left(1 - \cos \omega t\right)}$$

$$\boxed{<H(t)> = \frac{2}{3t}}$$

$$\boxed{<\Lambda(t)> \sim \frac{E^2{}_{\mathrm{Planck}}}{t^2} \sim E^4{}_{\mathrm{Planck}} \frac{a^3(t_{\mathrm{Planck}})}{a^3(t)}}$$

relaxation of vacuum energy mimics expansion of cold dark matter



$$\boxed{\rho(t)\, a^3(t) = \mathrm{const}}$$

though equation of state corresponds to Λ

$$\boxed{\Lambda = \Omega = -P}$$

$$\Omega = \varepsilon(q) - \mu q$$

another example of vacuum variable: from 4-vector

version of Ted Jacobson's Einstein-Aether theory

energy density $\boxed{\epsilon_{vac}(u^{\mu}{}_{\nu})}$ **of vacuum is function of** $\boxed{u^{\mu}{}_{\nu} = \nabla_{\nu} u^{\mu}}$

equilibrium vacuum is obtained from equation

$$\boxed{\delta\epsilon_{vac}/\delta u^{\mu} = \nabla_{\nu}(\delta\epsilon_{vac}/\delta u^{\mu}{}_{\nu}) = 0}$$

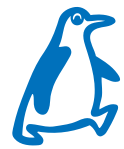
equilibrium solution**:**

$$\boxed{u_{\mu\nu} = q g_{\mu\nu}}$$

$q = const$

vacuum variable

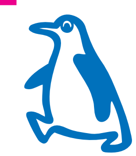
microscopic vacuum energy



$$\boxed{\epsilon_{vac}(q) \sim E^{4}_{\mathbf{Planck}}}$$

cosmological constant

$$\boxed{\Lambda = \Omega(q_0) = 0}$$



macroscopic thermodynamic vacuum energy:
from energy momentum tensor

$$\boxed{T_{\mu\nu} = \delta S/\delta g^{\mu\nu} = (\, \epsilon_{vac}(q) - q\, d\epsilon_{vac}/dq)\, g_{\mu\nu}}$$

It is $T_{\mu\nu}$ **which is gravitating, thus cosmological constant is**

$$\boxed{\Lambda = \Omega(q) = \epsilon_{vac}(q) - q\, d\epsilon_{vac}/dq}$$

relativistic quantum vacuum vs helium liquids

microscopic energy

natural Planck scale

$$\varepsilon_{\rm vac}(q) \sim E^4_{\rm Planck}$$

natural atomic scale

$$\varepsilon(n) \textit{ is atomic}$$

macroscopic energy

$$\Omega = \varepsilon(q) - q\, d\varepsilon/dq = -P_{\rm vac}$$

$$\Omega = \varepsilon(n) - n\, d\varepsilon/dn = -P$$

in the absence of environment

$$\Lambda = \Omega = -P_{\rm vac} = 0$$

$$\Omega = -P = 0$$

two microscopic quantities cancel each other without fine tuning

self tuning due to $\boxed{\textbf{thermodynamics}}$

Conclusion

Landau two-fluid hydrodynamics for **superfluid ^{4}He**
& **Einstein general relativity**
are effective hydrodynamic theories:
they are two different extreme limits
of parameters in underlying microscopic theory

Landau theory of Fermi liquid
& **Standard Model of electroweak & strong interactions**
are effective theories
for two major classes of fermionic vacua:
vacua with Fermi surface
(**normal ^{3}He** and metals)
& vacua with Fermi point
(relativistic quantum vacuum & **superfluid ^{3}He-A**)

Landau ideas first applied to **liquid ^{4}He and ^{3}He**
are applicable to **quantum vacuum** -
the modern aether