SPICE Workshop Non-Equilibrium Quantum Matter, Mainz, 5/30/2017

Sound and Solitonic Excitations in Fermionic Superfluids

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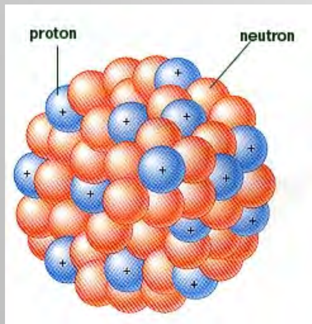
Massachusetts Institute of Technology
Center for Ultracold Atoms

Strongly Interacting Fermi Systems

A good place to search for exotic physics

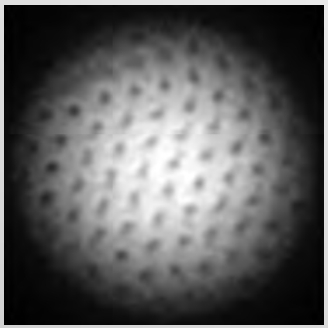
Length scales

10^{-15} m



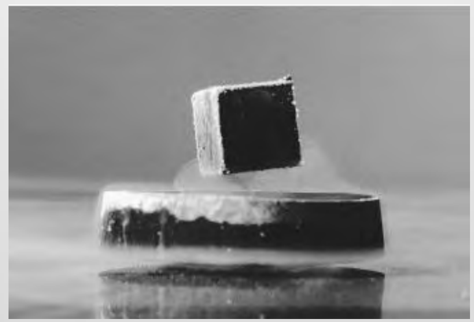
Nuclei

1 mm



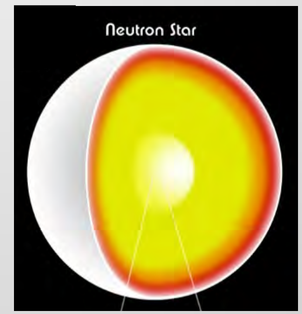
Ultracold Gases

1 m



High-T_c Superconductors

10^4 m



Neutron Star

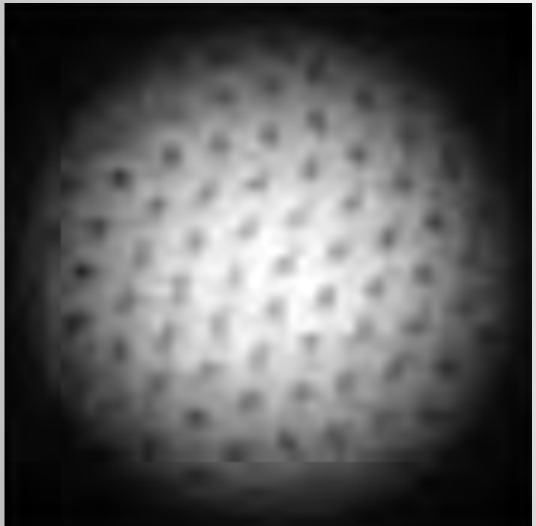
10^7 m

White dwarf

- **A wealth of unusual quantum phases**
- **Many open qualitative questions**
  (e.g. Pseudo-gap phase in High-Tc materials)
- **Highly challenging theoretically**
  ("Fermion Sign Problem")

Ultracold Atomic Fermi Gases

Ideal test-bed for Many-Body physics



Interactions

Geometry

Spin Composition etc...

Realize idealized models of many-body physics

Benchmarking the many-body problem

→ ***Unitary Fermi Gas, Fermi-Hubbard Model…***

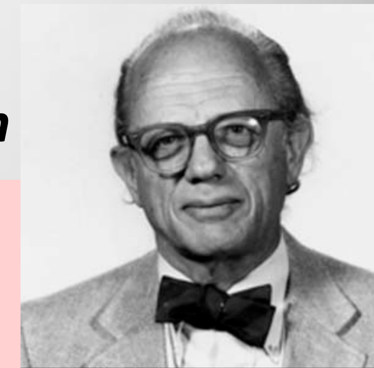
Create entirely new systems

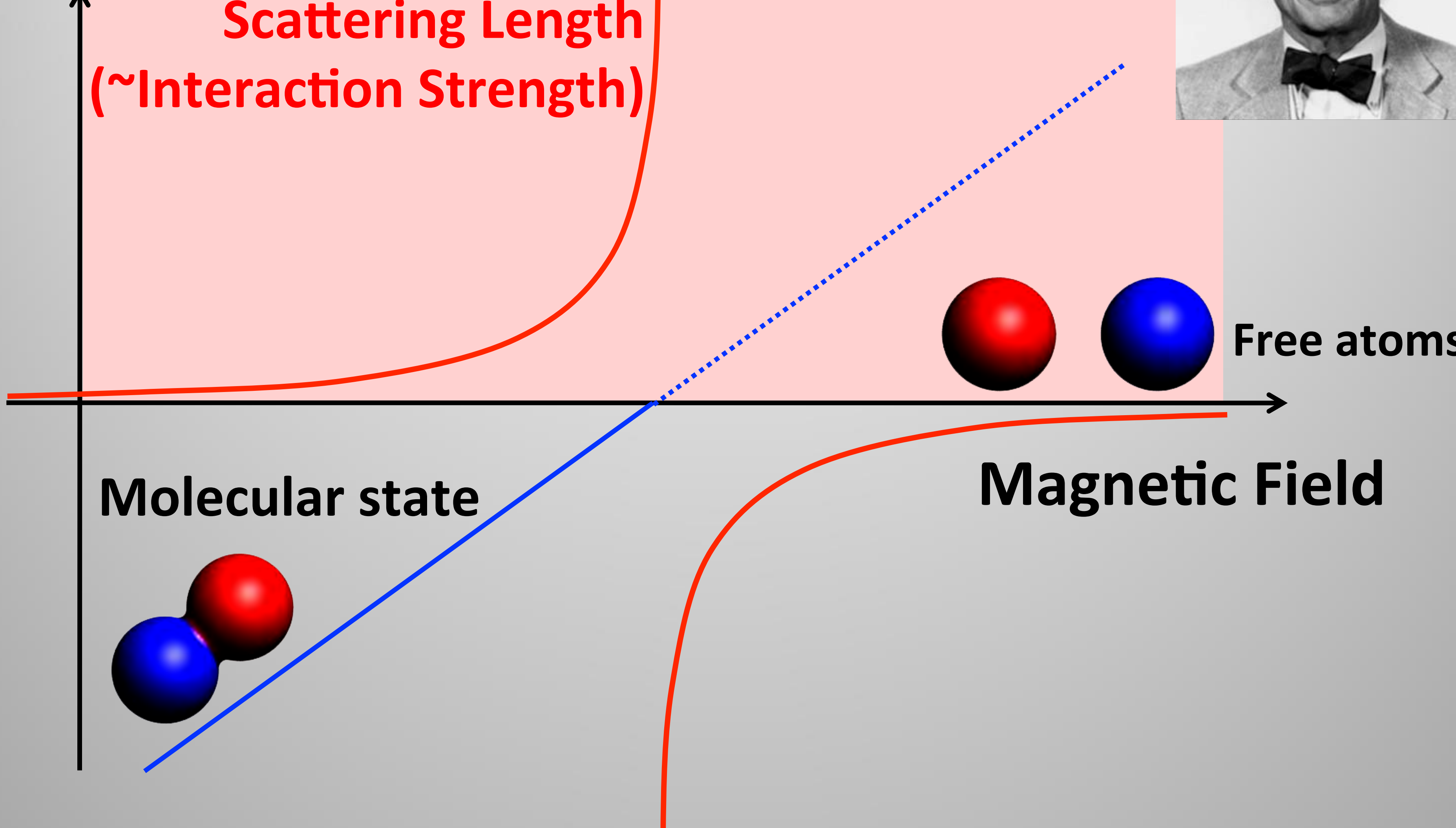
→ ***Dipolar Fermi gases***

→ ***Topological Superfluids?***

Strong interactions via Feshbach resonances

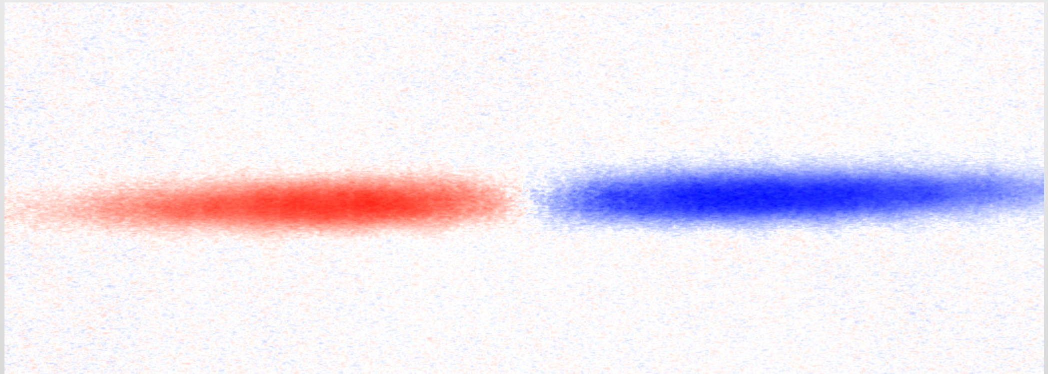
Herman Feshbach



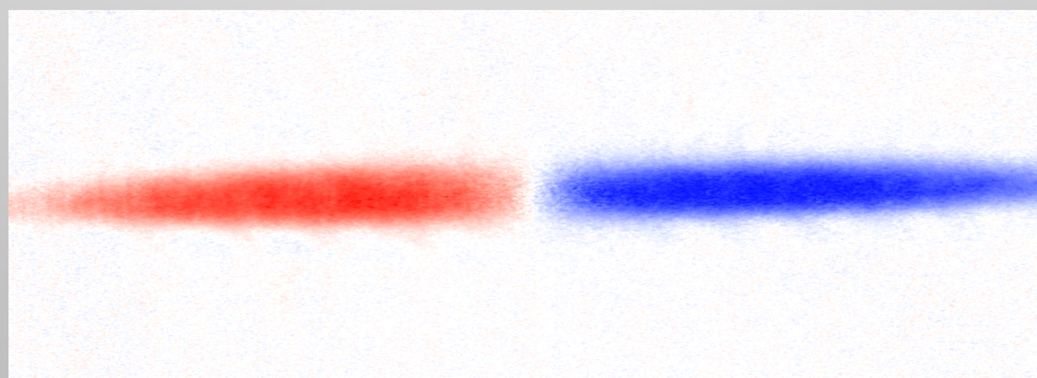


Little Fermi Collider (LFC)

Without Interactions

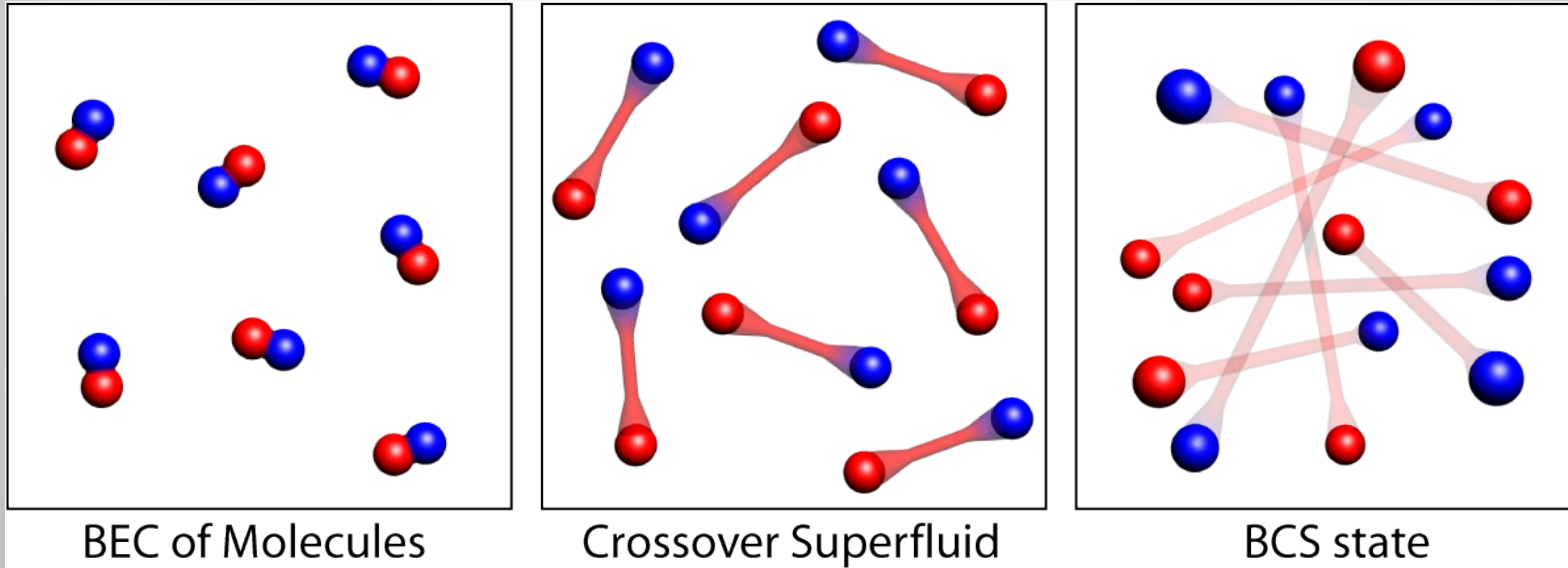


Resonant Interactions



A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)

From BEC to BCS



? 1   1   0 -1 = -1

$$\left(k_F a\right)^{-1} = \frac{\text{Interparticle Distance}}{\text{Scattering Length}}$$

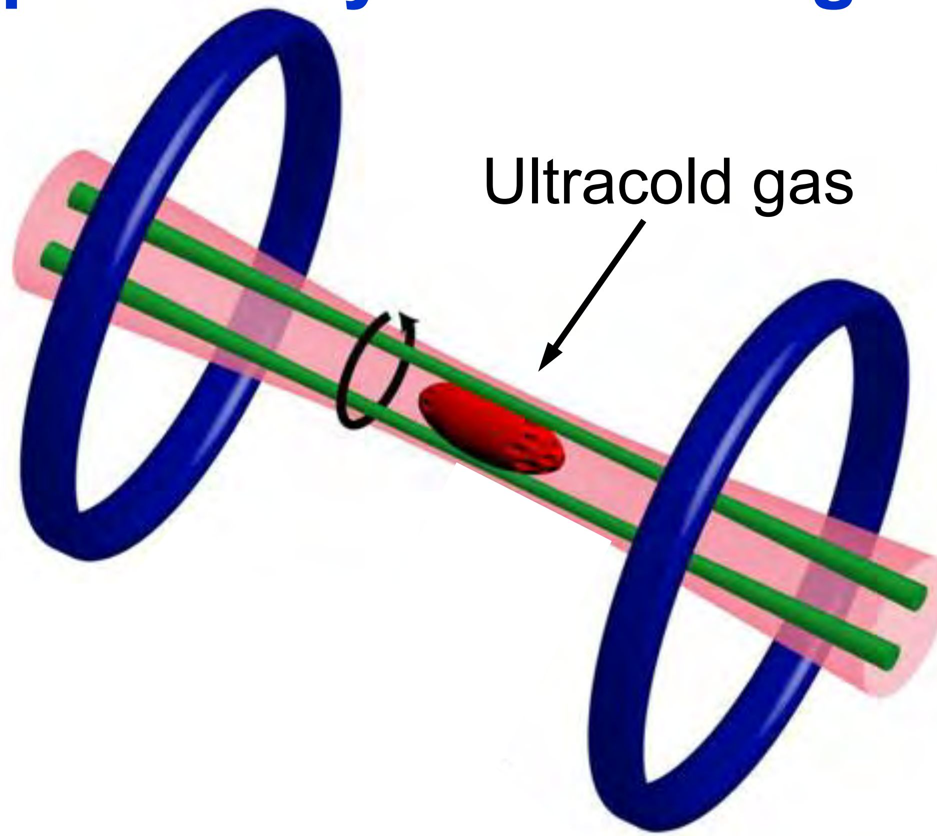
Weakly Interacting Bosons

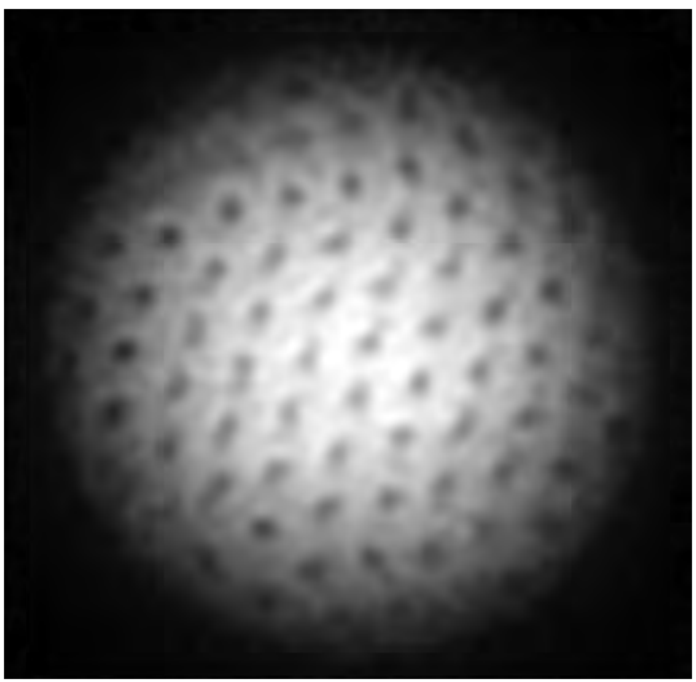
→ Strongly Interacting Bosons

→ Strongly Interacting Fermions

→ Weakly Interacting Fermions

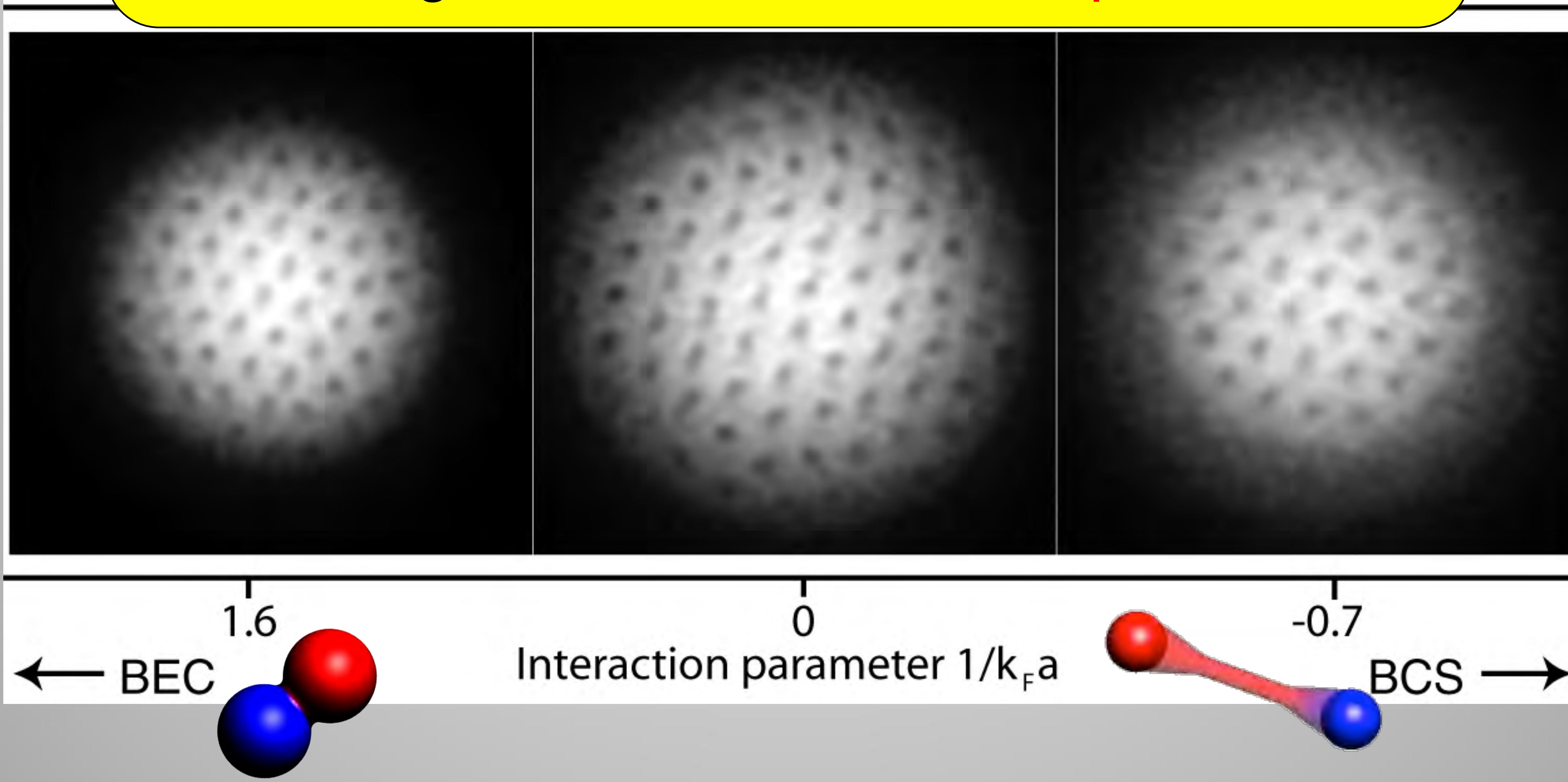
Demonstration of superfluidity in a Fermi gas





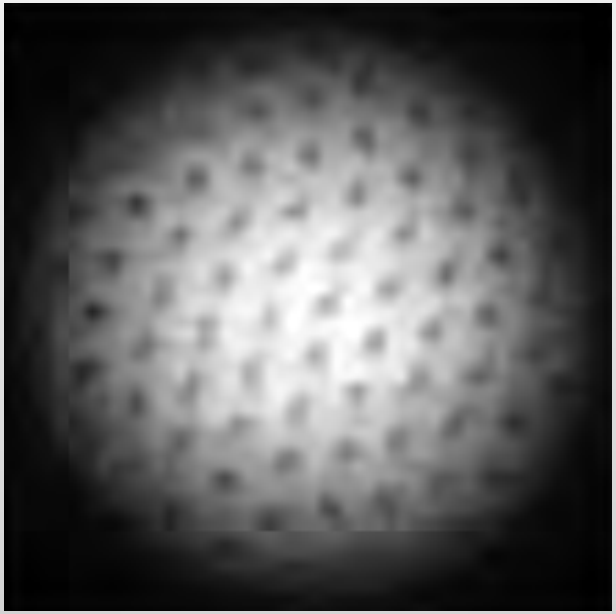
Vortex lattices in the BEC-BCS crossover

Establishes *superfluidity* and *phase coherence* in gases of fermionic atom pairs



M.W. Zwierlein, J.R. Abo-Shaeer, A. Schirotzek, C.H. Schunck, W. Ketterle, Nature 435, 1047-1051 (2005)

Do we understand (s-wave) strongly interacting Fermi gases?

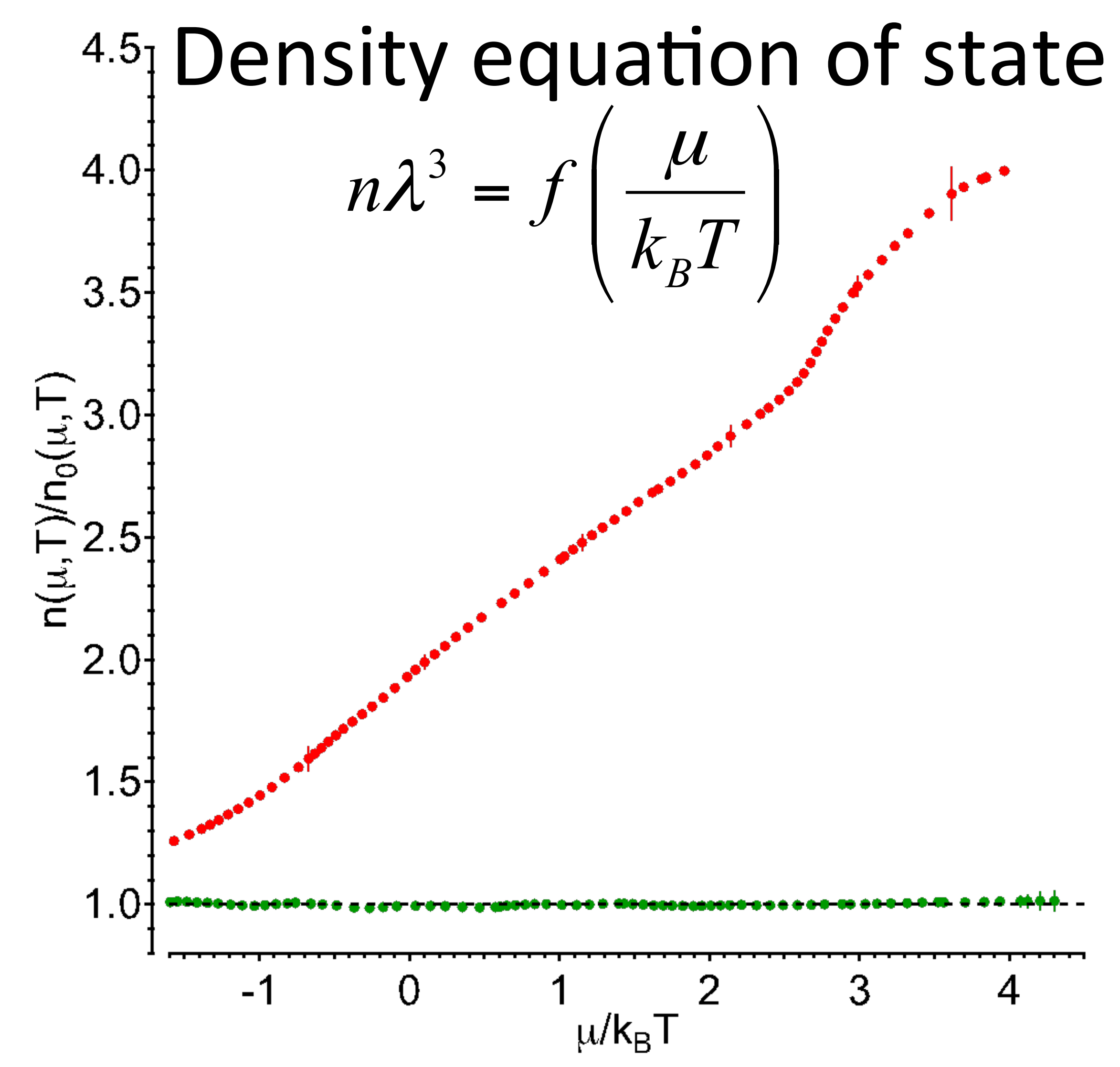


e.g.: ground-state energy: $$E = \xi \frac{3}{5} N E_F$$

$$\xi_{\text{Mean-Field}} = 0.59 \qquad \xi_{\text{Experiment}} = 0.37(1)$$

Mark Ku, Ariel Sommer, Lawrence Cheuk, MWZ, Science **335**, 563-567 (2012)

Equation of State of a Strongly Interacting Fermi Gas



Unitary Gas (Expt.)

Meanfield

Non-Interacting Gas

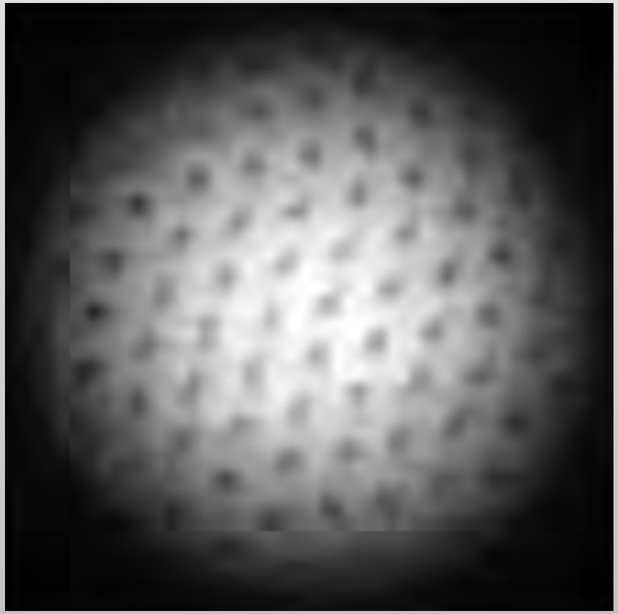
Mark Ku, Ariel Sommer, Lawrence Cheuk, MWZ, Science **335**, 563-567 (2012)
K. Van Houcke, F. Werner, E. Kozik, N. Prokofev, B. Svistunov,
M. Ku, A. Sommer, L. Cheuk, A. Schirotzek, MWZ, Nature Physics **8**, 366 (2012)

How about excitations?

Vast body of work: Collective excitations, first sound, second sound, pair breaking excitations (PA, RF, PES), polarons (Innsbruck, Duke/NCSU, Rice, JILA, ENS, Swinburne, Heidelberg, MIT,…)

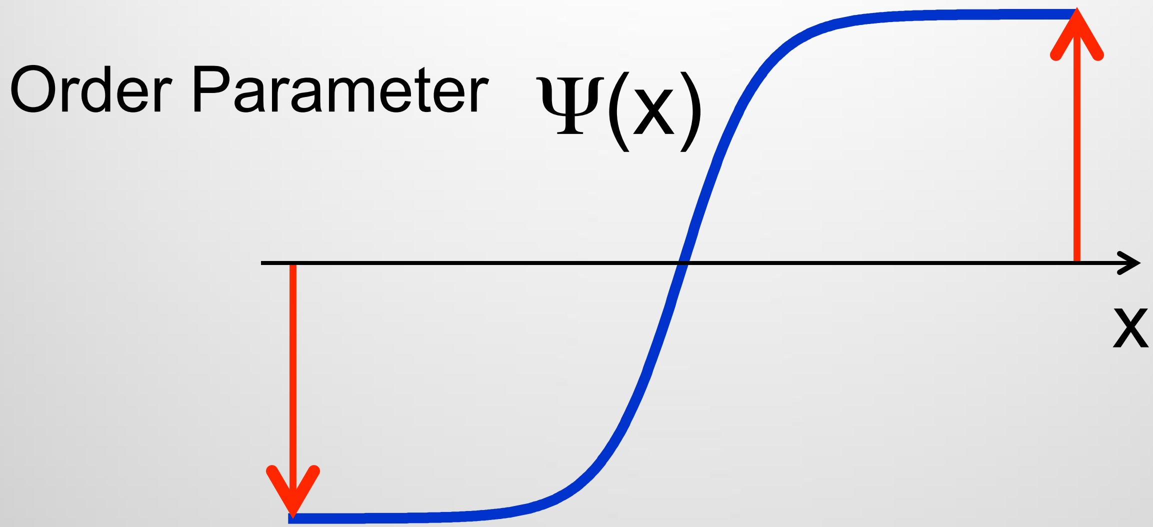
Regarding the superfluid wavefunction:

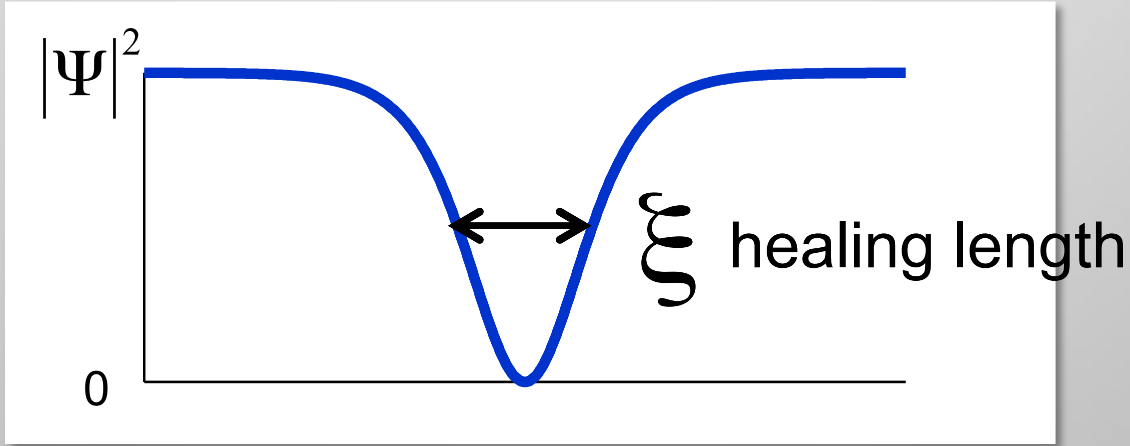
We know we have matter waves…



But we do not know the wave equation

Solitary Waves as Microscopic Probe



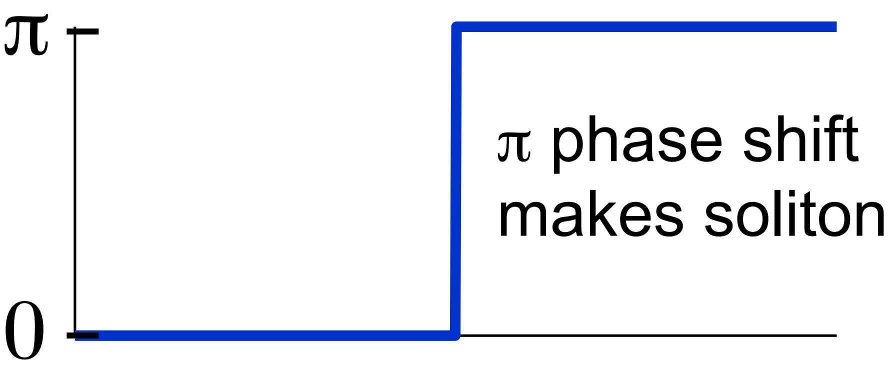


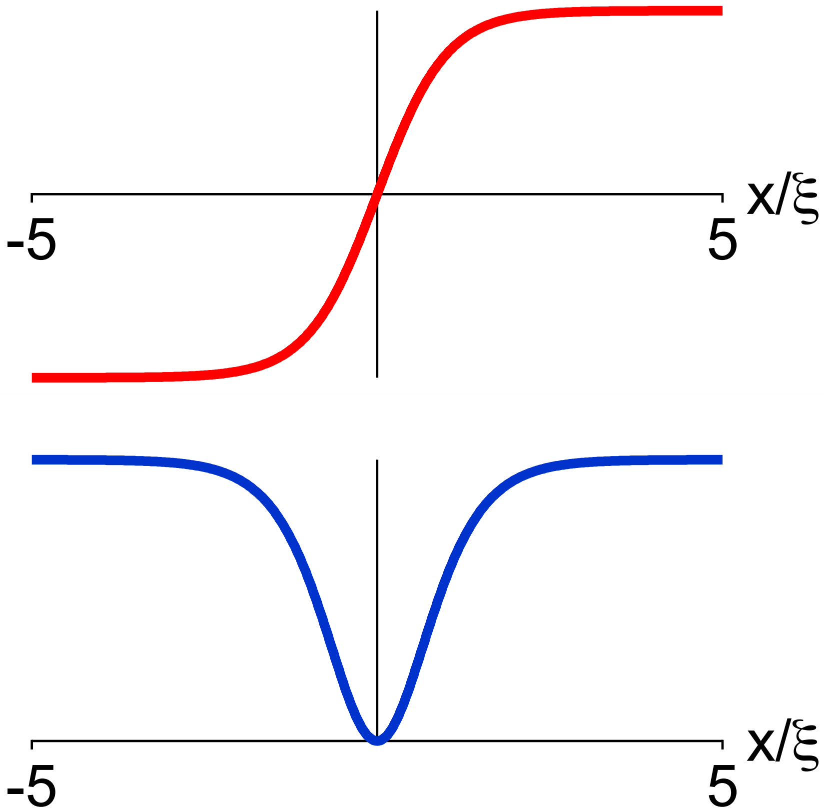
A localized, highly non-linear excitation

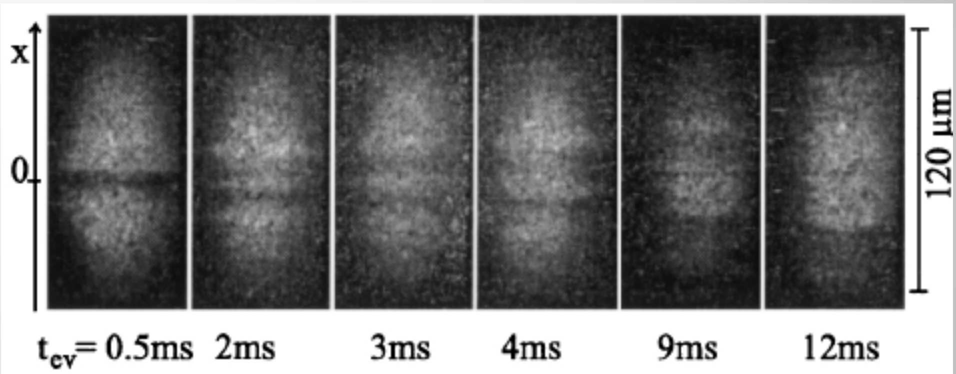
An excellent probe for the medium in which it propagates

Ex: Fiber optics, BEC, Dirac Fields, Holographic Theories

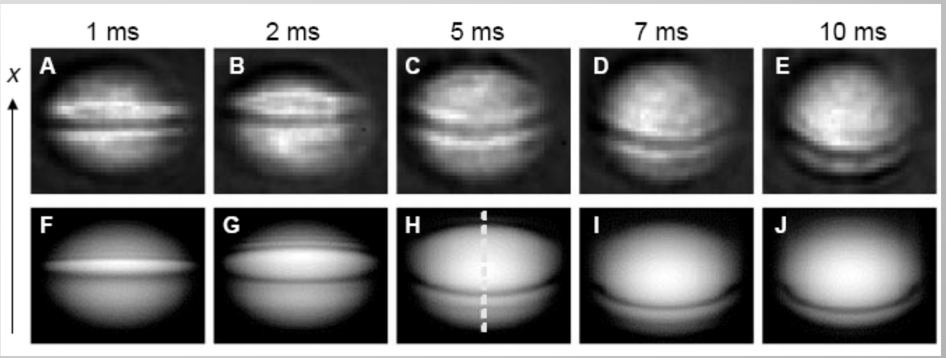
Dark Solitons in Bose-Einstein Condensates







Burger et al., PRL 83, 5198 (1999)



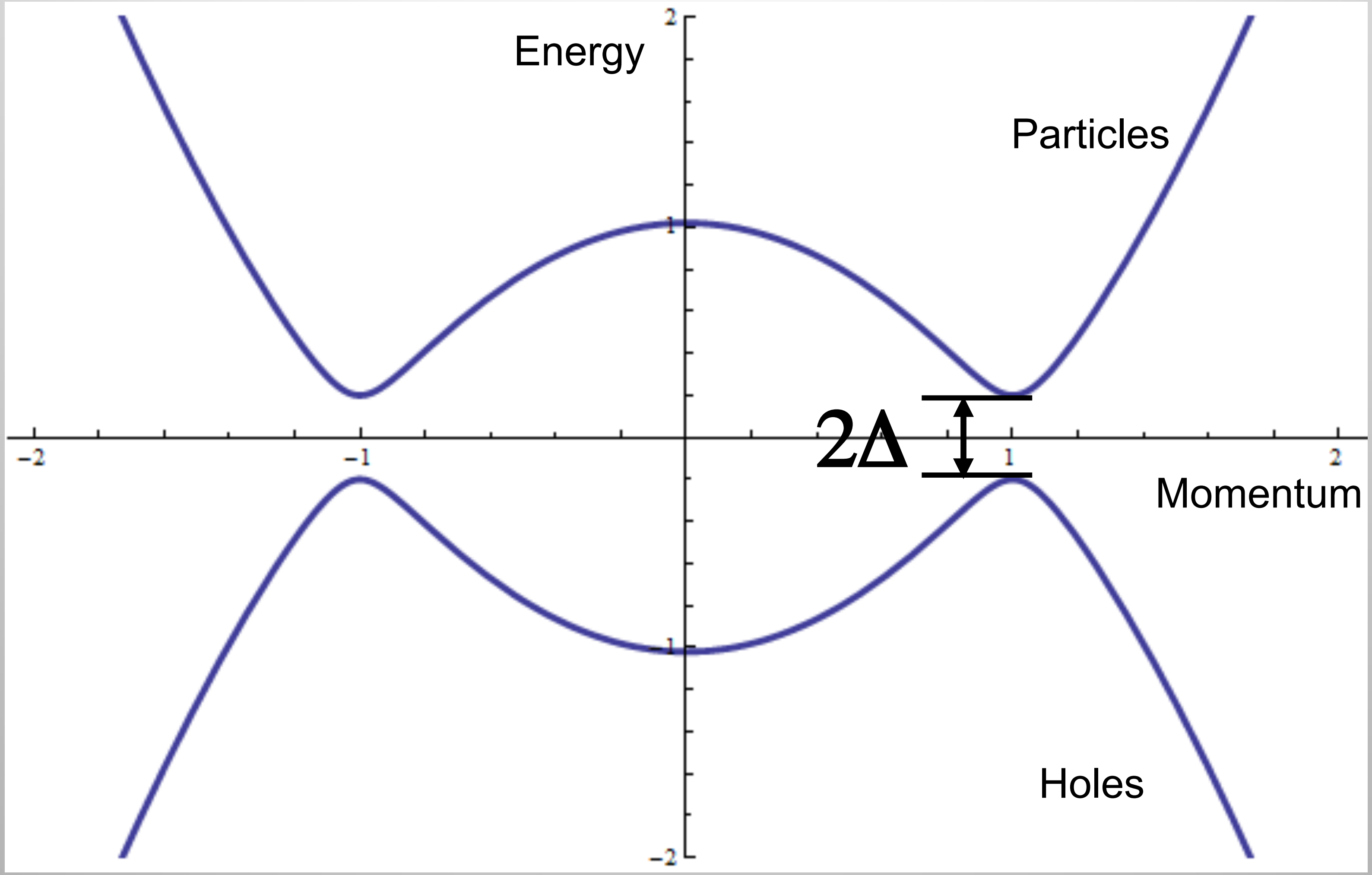
J. Denschlag et al., Science 287, 97 (2000)

Solitons in a Fermionic Superfluid

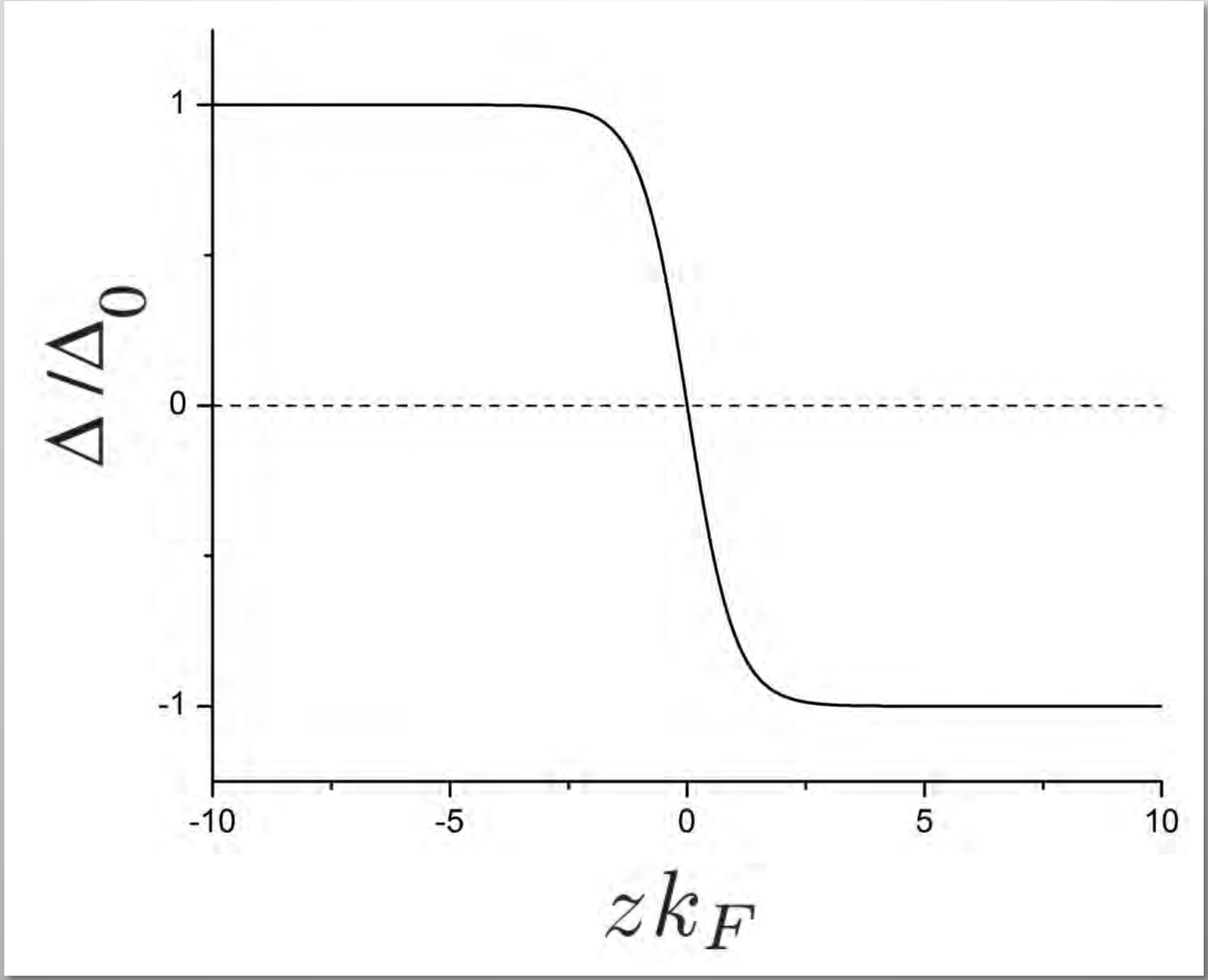
T. Yefsah, A. Sommer, M. J.-H. Ku, L. Cheuk, W. Ji, W. Bakr, MWZ,
Nature **499**, 426–430 (2013)
M.J.H. Ku, W. Ji, B. Mukherjee, E. Guardado-Sanchez, L.W. Cheuk, MWZ,
PRL 113, 065301 (2014)
Mark J.H. Ku, Biswaroop Mukherjee, Tarik Yefsah, MWZ,
PRL **116**, 045304 (2016)

BCS Pairing

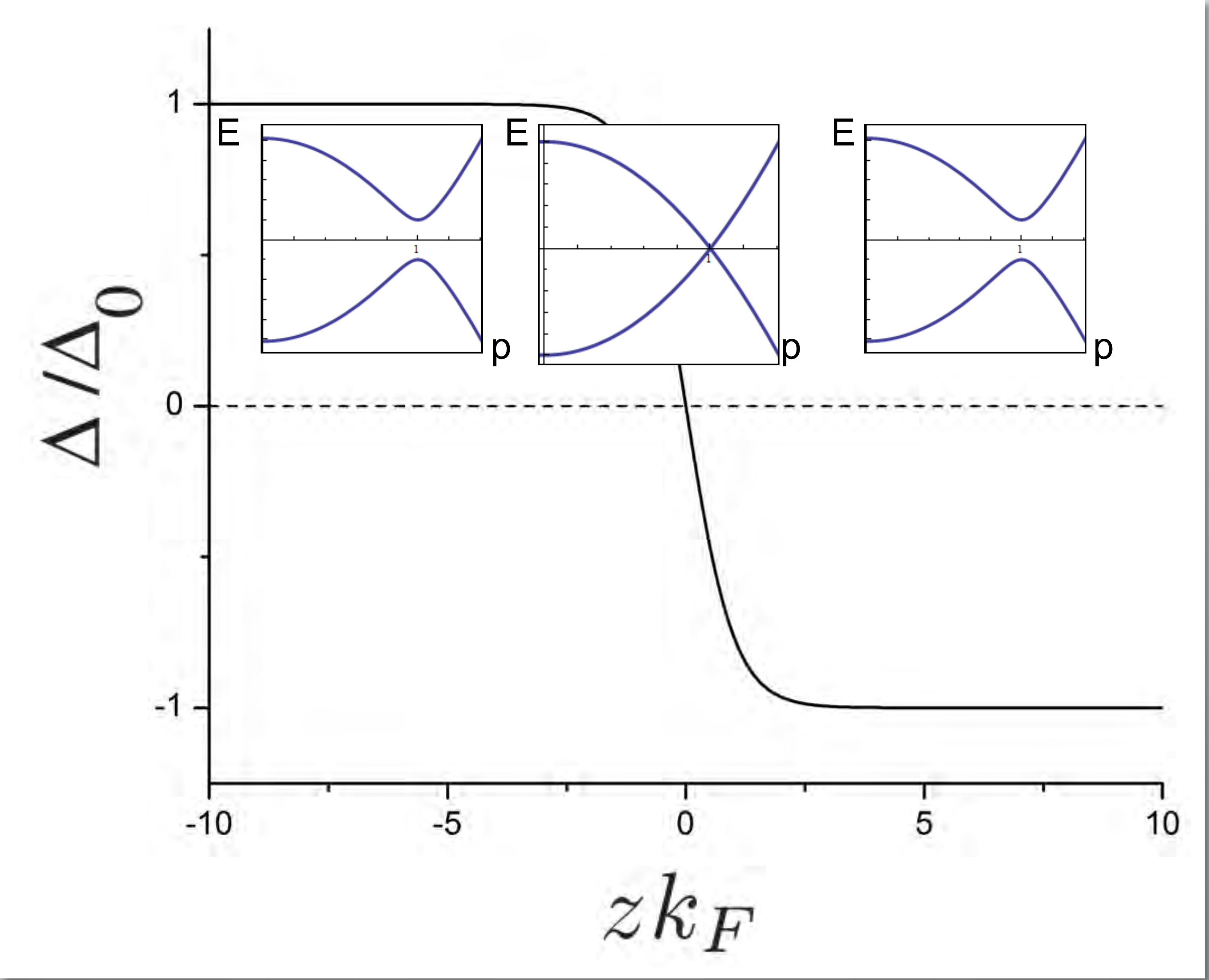
BCS-Bogoliubov Quasi-particles: $\gamma_{\vec{k}1}^{\dagger} = u_{\vec{k}} c_{\vec{k}\uparrow}^{\dagger} + v_{\vec{k}} c_{-\vec{k}\downarrow}$



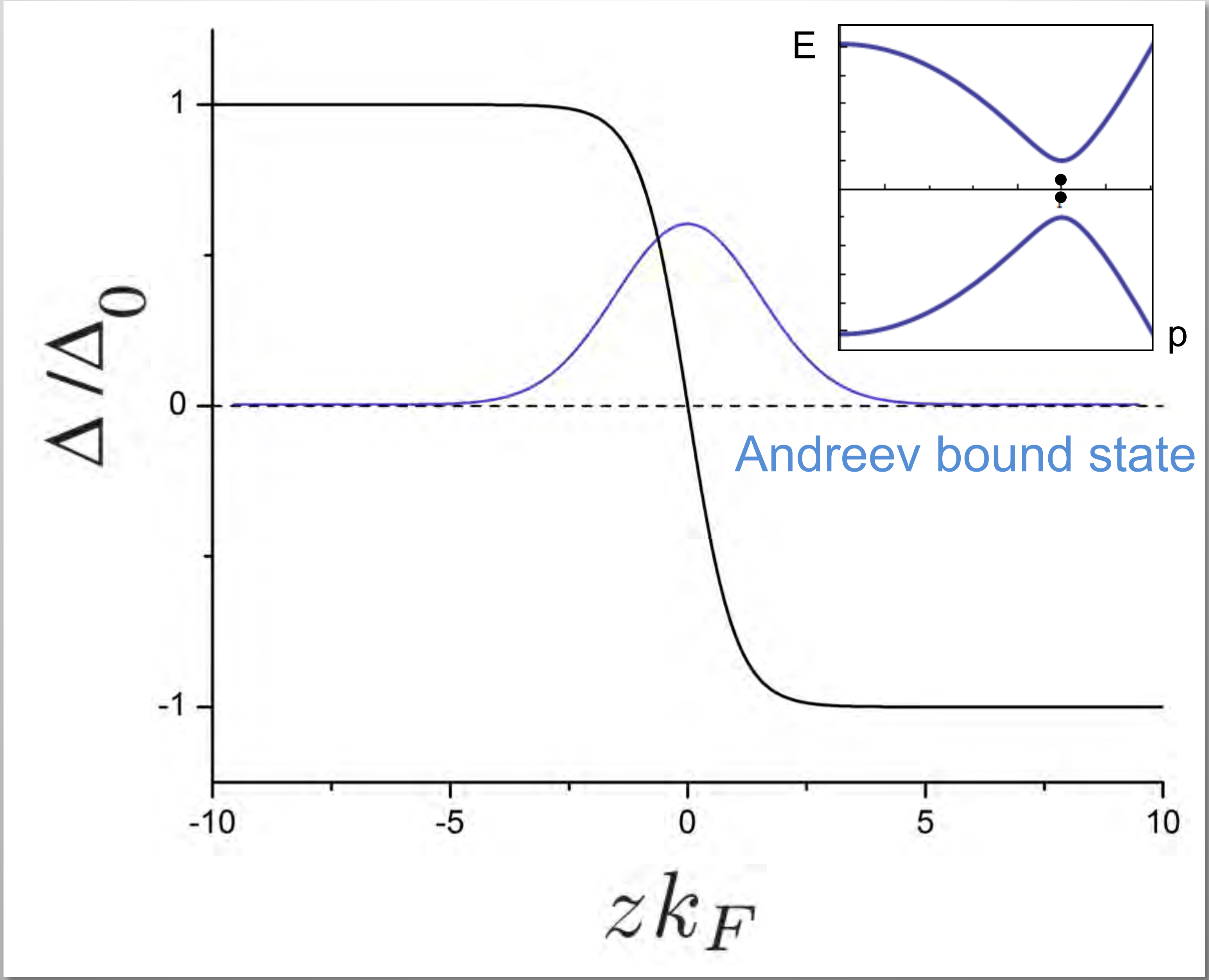
Solitons in Fermionic Superfluids



Solitons in Fermionic Superfluids



Solitons in Fermionic Superfluids



Dark Solitons in a Fermionic Superfluid

Limit of small gap: Andreev equation

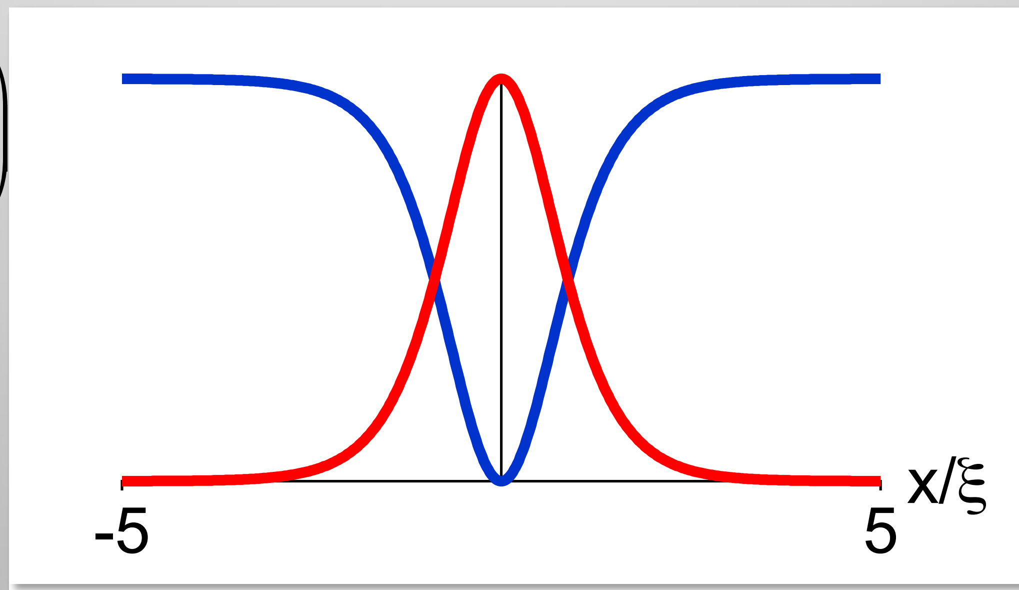
$$\left(-i\hbar v_F \frac{\partial}{\partial z}\sigma_z + \Delta(z)\sigma_x\right)\begin{pmatrix} u_n \\ v_n \end{pmatrix} = E_n \begin{pmatrix} u_n \\ v_n \end{pmatrix}$$

Dirac equation with spatially varying mass

Solitons with fermion number ½, Jackiw, Rebbi 1976

= continuum version of Su-Schrieffer-Heeger model 1979/80

$$|\Delta|^2 = \Delta_0^2 \tanh^2\left(\frac{z}{\xi}\right)$$



Andreev Bound State

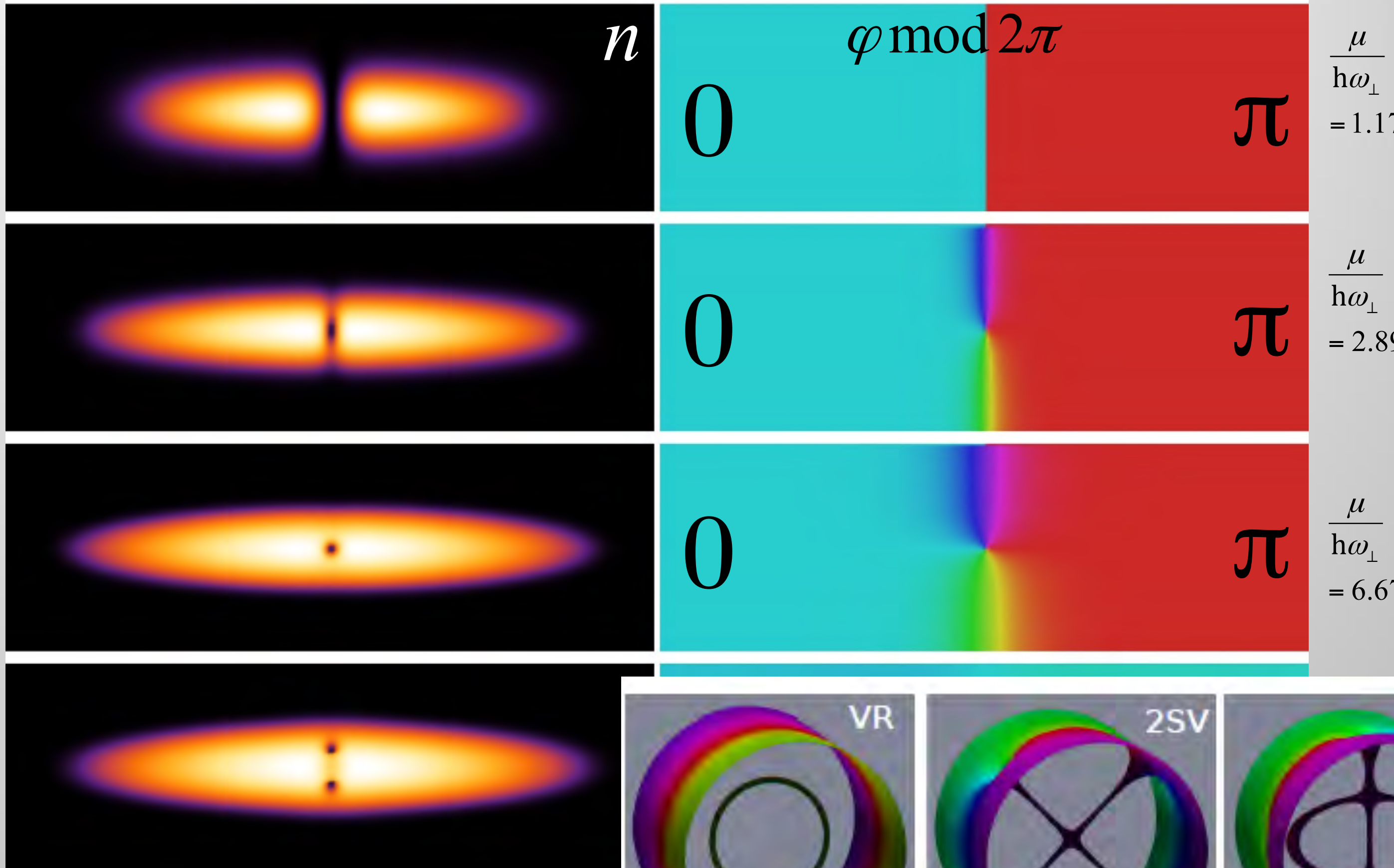
Recent solution for finite velocity soliton: V. Galitski, D.K. Efimkin, PRA **91**, 023616 (2015)

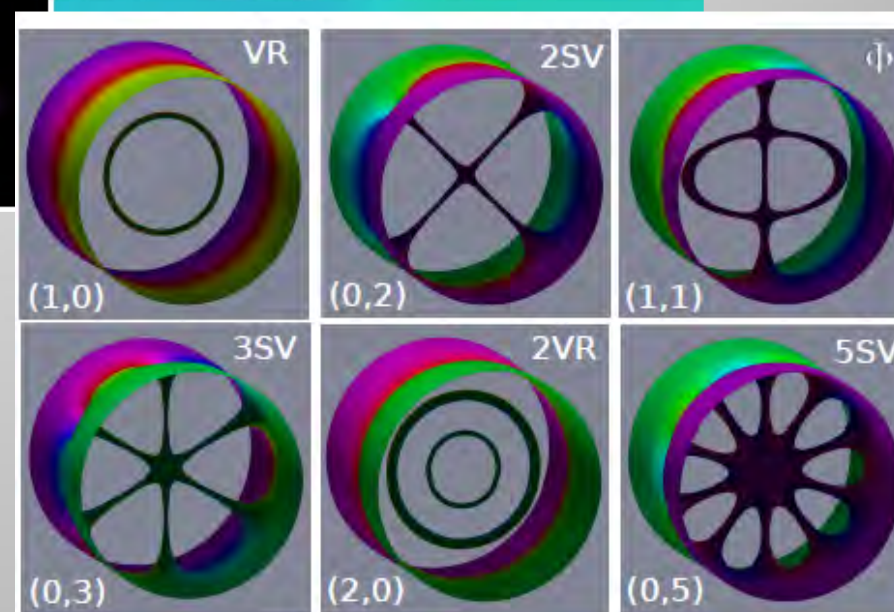
Generalization in 3D: Solitary Waves

Soliton

Solitonic Vortex

Vortex Ring





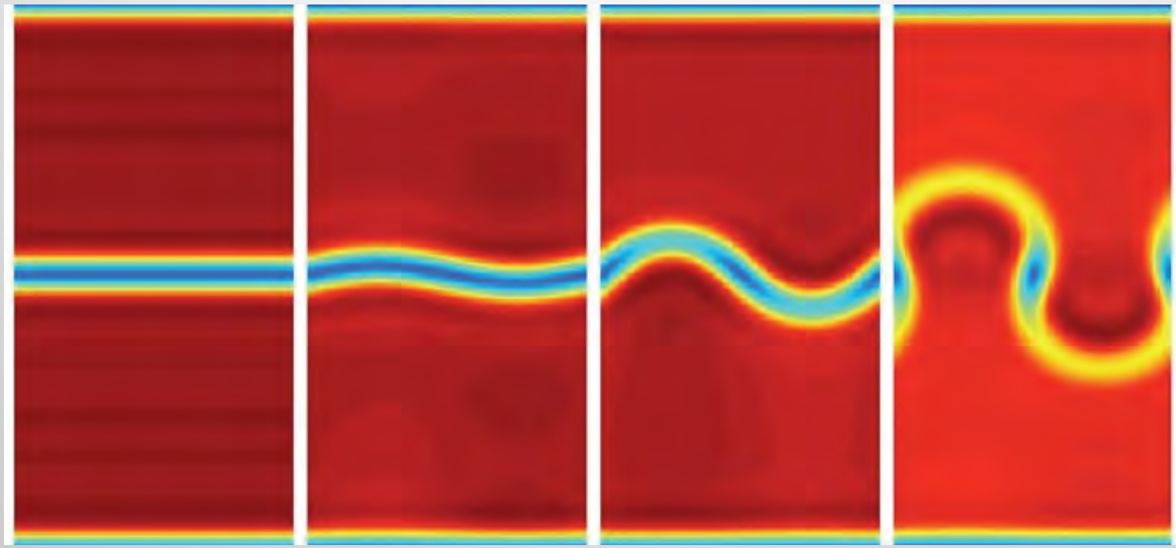
After
Brand, Reinhardt
PRA 65, 043612 (2002)

All examples of

Chladni Solitons

Mateo, Brand, PRL 113, 255302 (2014)

Snake Instability in a Fermionic Superfluid

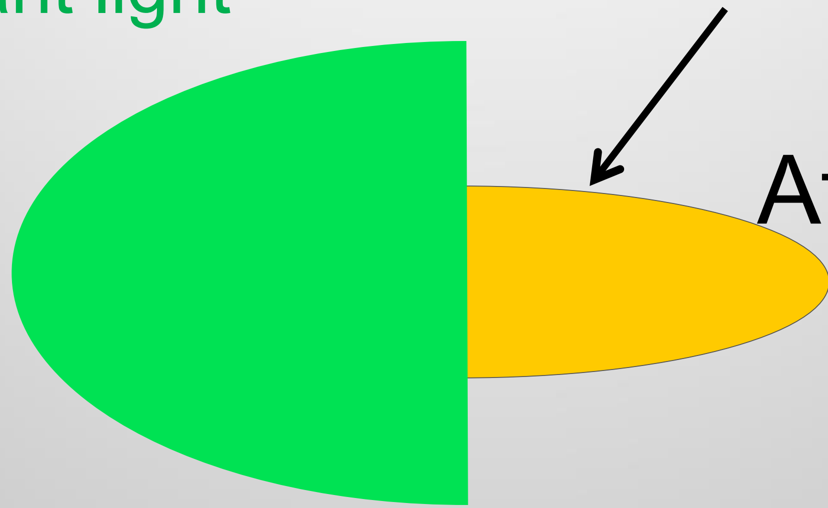


Theory:
Cetoli, Brand, Scott, Dalfovo, Pitaevskii
Phys. Rev. A 88, 043639 (2013)

Making Solitons by phase imprinting

Pulse
off-resonant light

superfluid



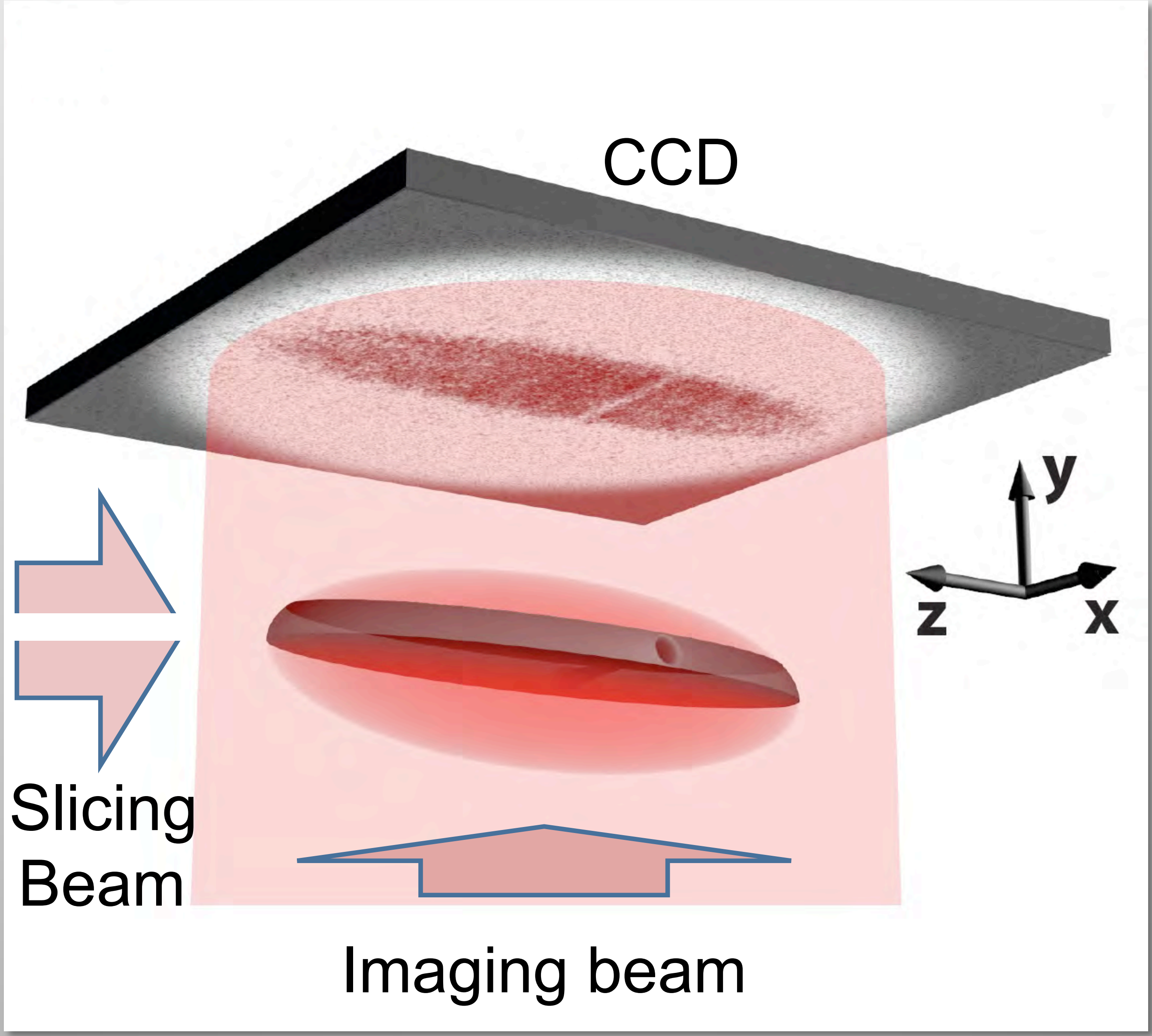
After the pulse:

$$\Delta\varphi = \frac{2Ut}{\hbar}$$

Needs to be fast enough: $t < \frac{\hbar}{\mu} \sim 100\,\mu s$

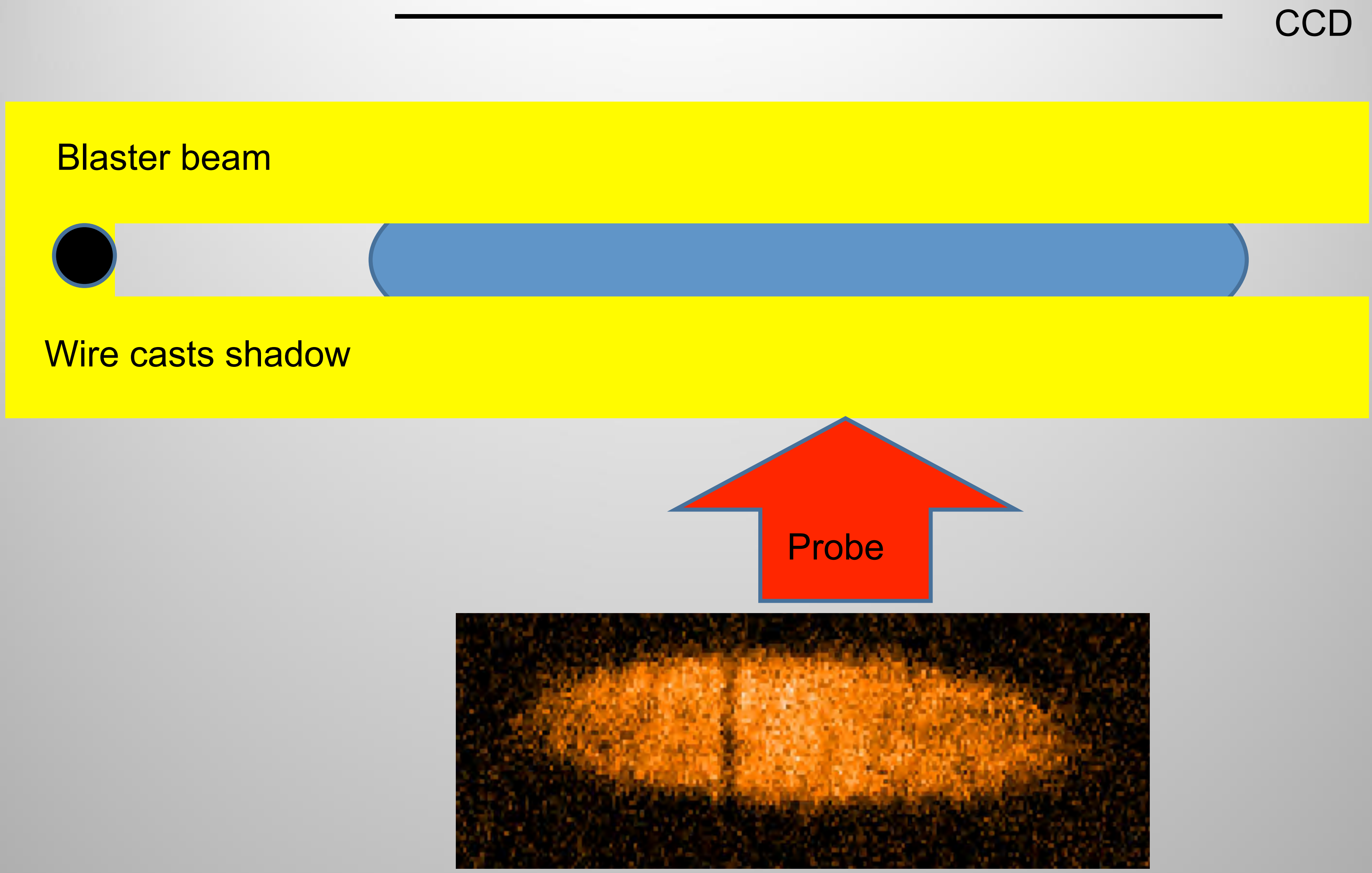
Solitons in BECs by phase imprinting: Hannover, Hamburg, NIST,...

Tomography: Slicing the Cloud



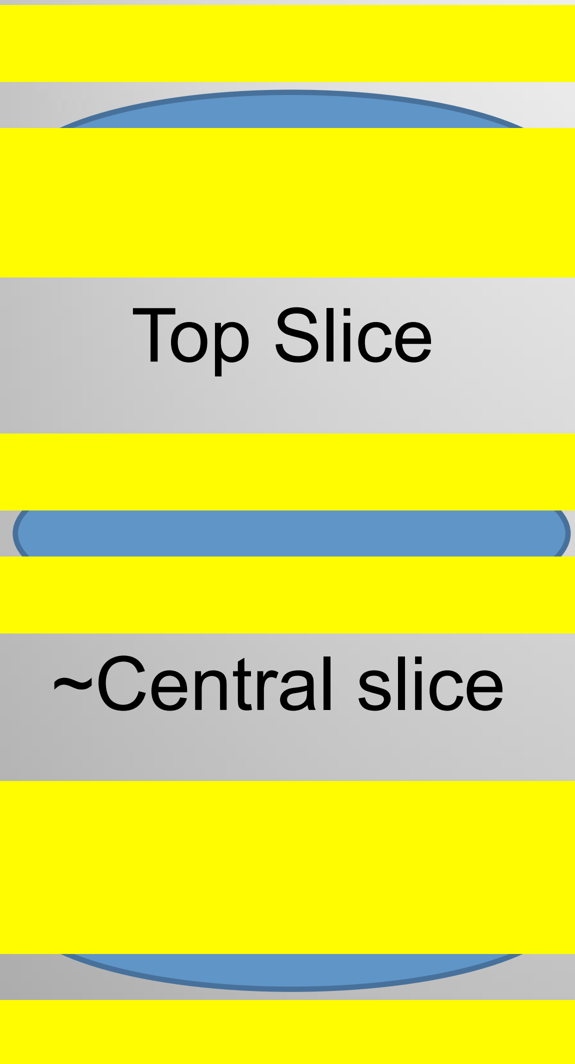
M.J.H. Ku, W. Ji, B. Mukherjee, E. Guardado-Sanchez, L.W. Cheuk, MWZ, PRL 113, 065301 (2014)

Tomography: Slicing the Cloud



Tomography: Slicing the Cloud

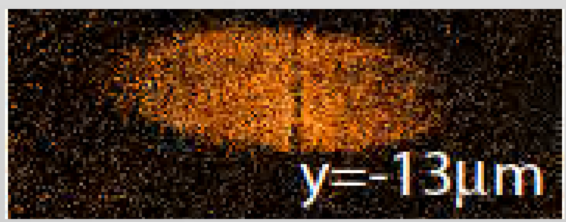
“Solitonic Vortex”
Brand, Reinhardt
PRA 65, 043612 (2002)



Top Slice

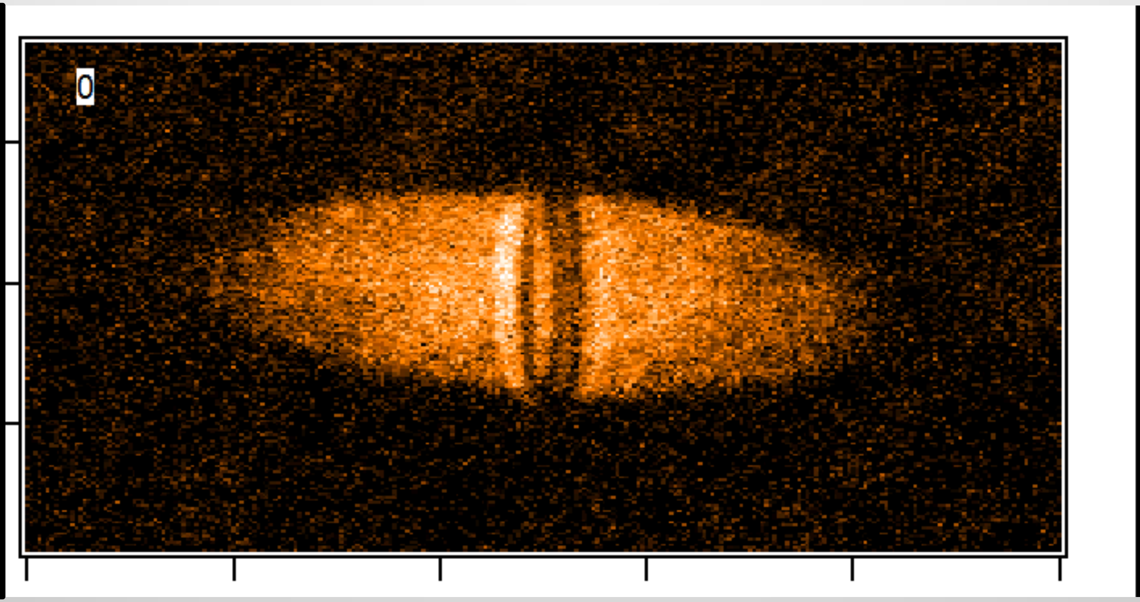
~Central slice

Bottom Slice



Mark J.-H. Ku, W. Ji,
B. Mukherjee,
E. Guardado-Sanchez,
L. W. Cheuk, T. Yefsah,
MWZ, PRL 113, 065301
(2014)

Cascade of Solitary Waves in a Unitary Fermi Gas

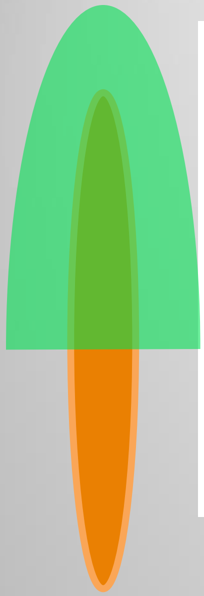


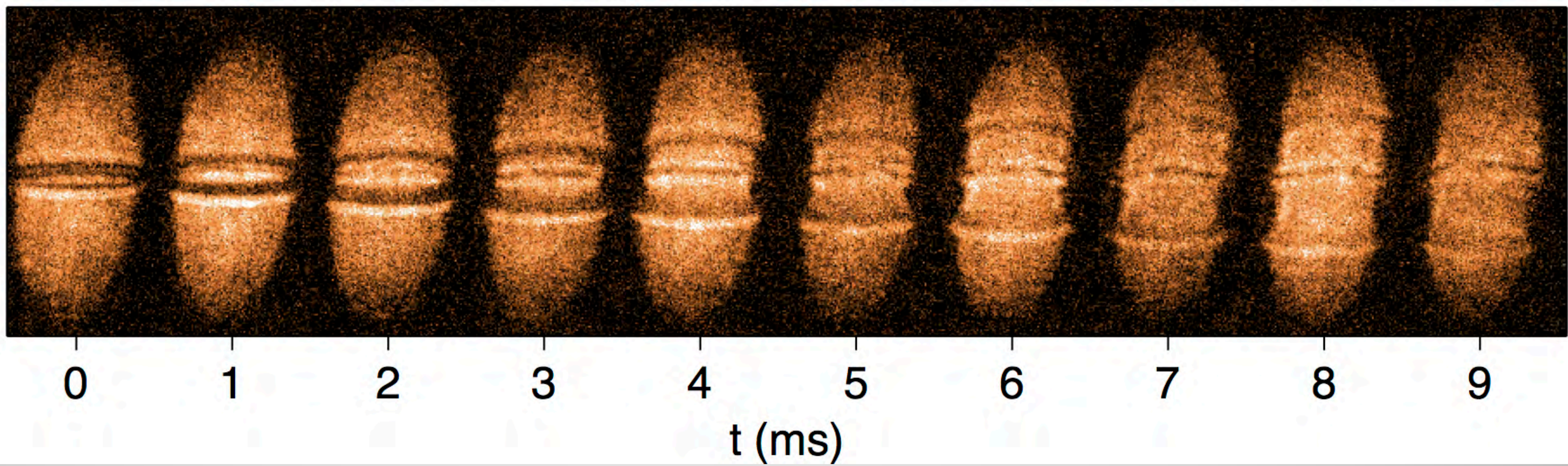
Planar soliton →
vortex ring →
vortex / anti-vortex pair →
solitonic vortex

Mark J.H. Ku, Biswaroop Mukherjee, Tarik Yefsah, Martin W. Zwierlein
PRL **116**, 045304 (2016)

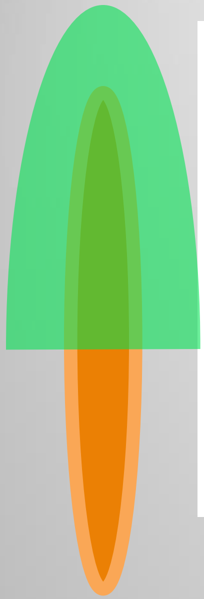
Instability Cascade of Solitary Waves in Unitary Fermi Gas

Early time dynamics after imprint (central slice)





Instability Cascade of Solitary Waves in Unitary Fermi Gas



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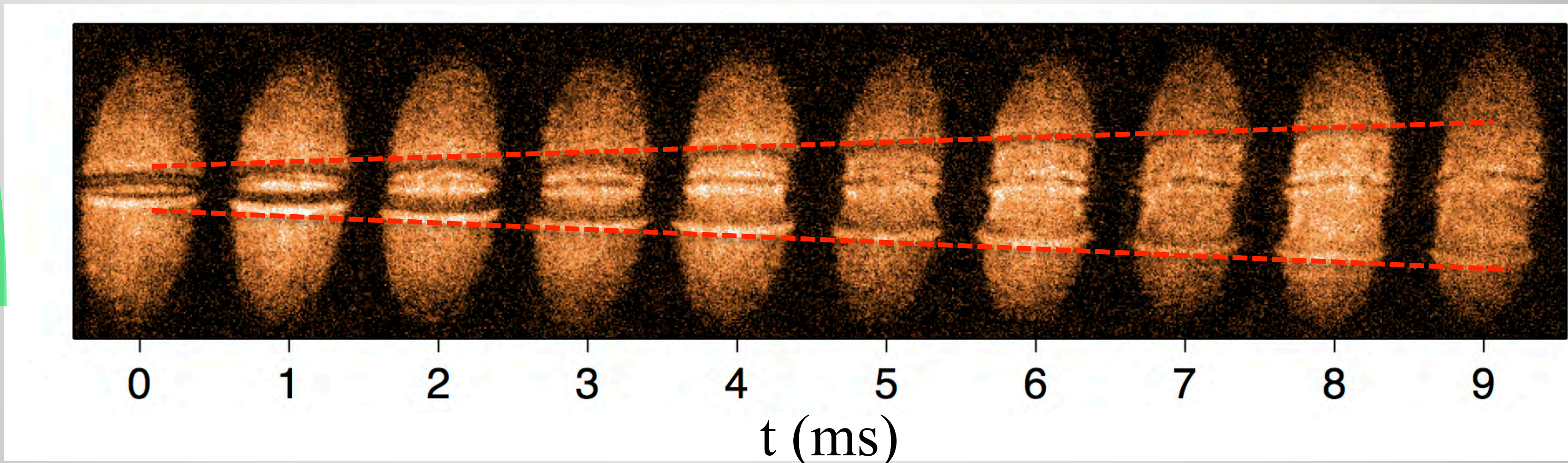
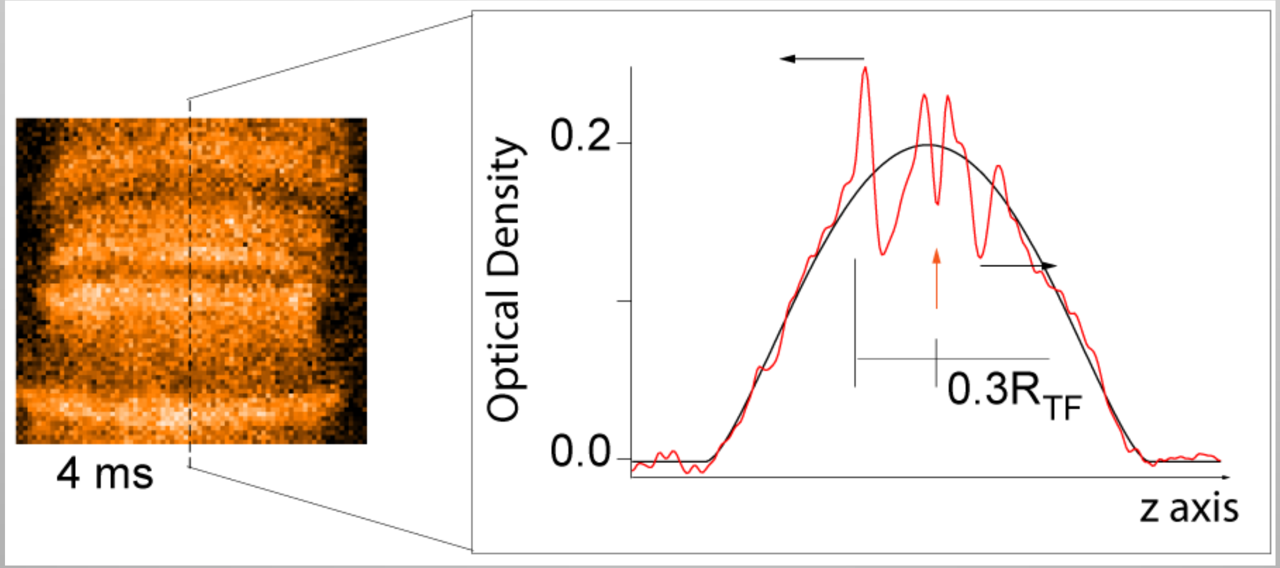


2 shock wave fronts +
1 slow solitary wave



Instability Cascade of Solitary Waves in Unitary Fermi Gas

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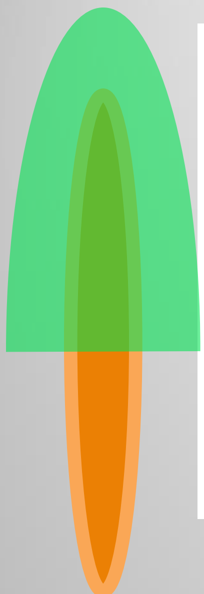
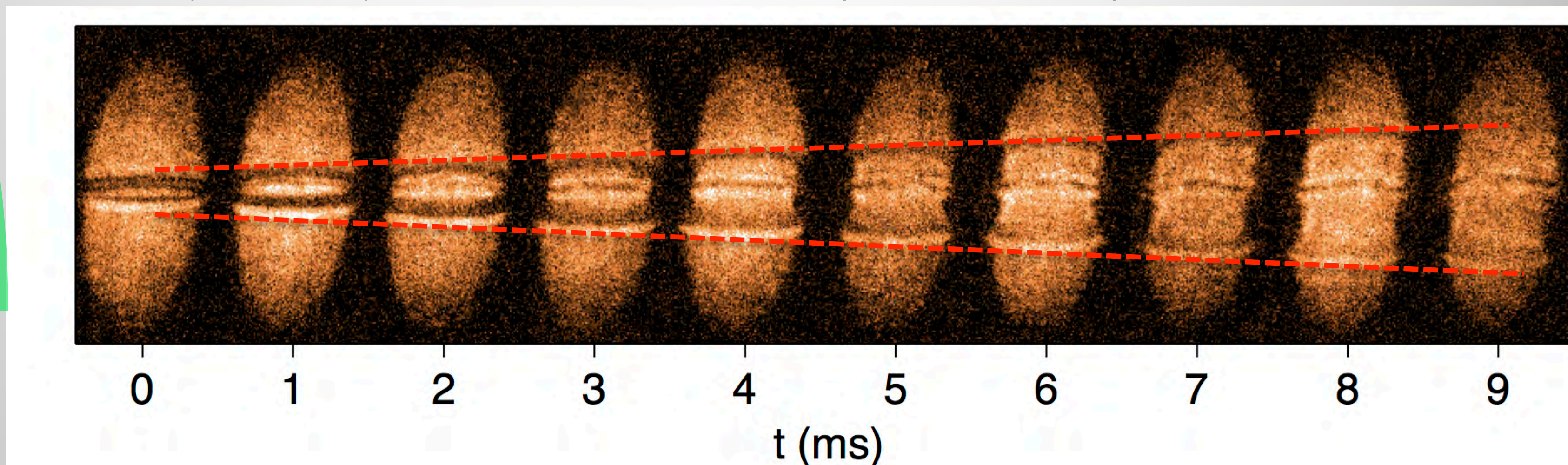
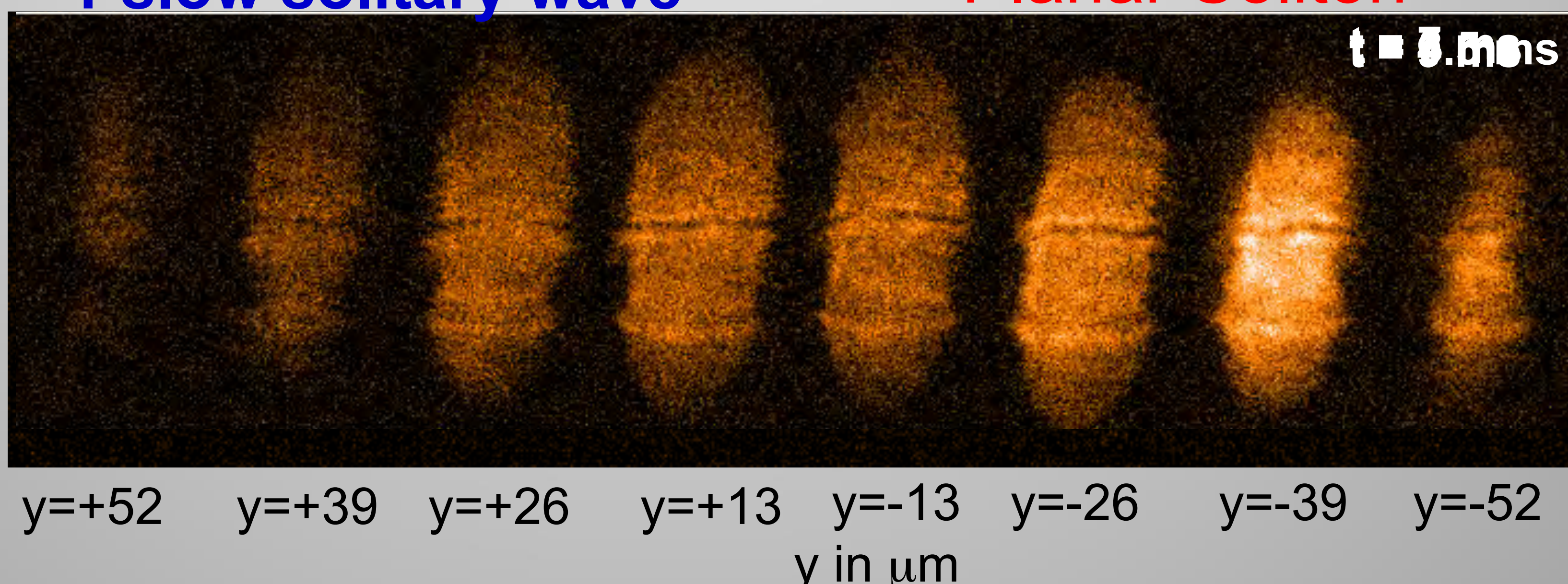


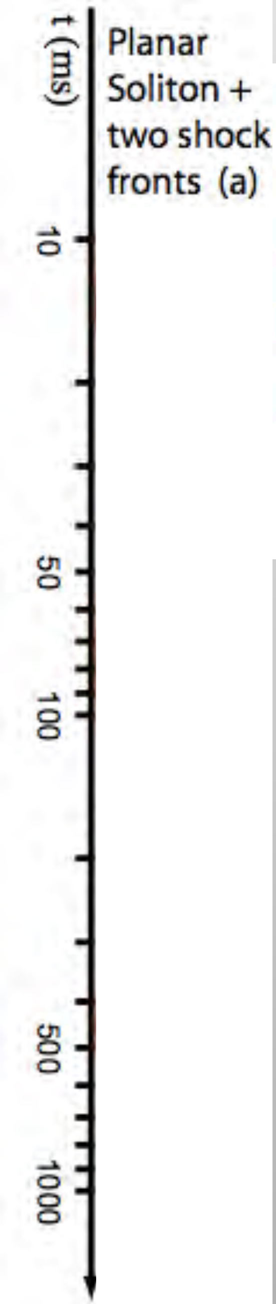


2 shock wave fronts +
1 slow solitary wave

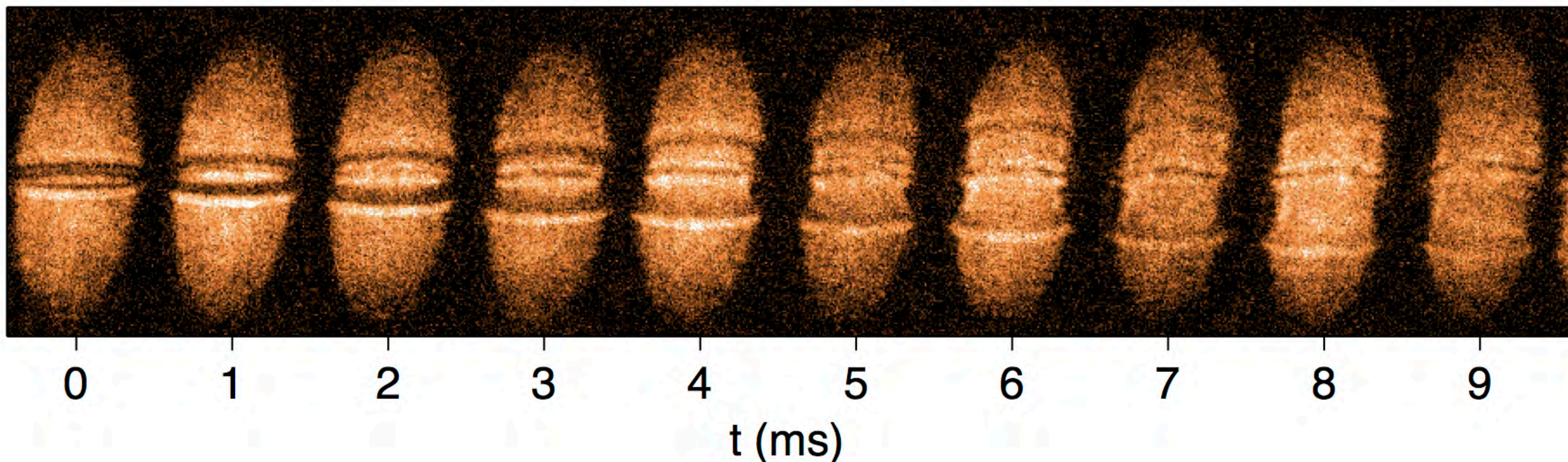
Tomography:
Planar Soliton



Instability Cascade of Solitary Waves in Unitary Fermi Gas



Early time dynamics after imprint (central slice)



2 shock wave fronts +
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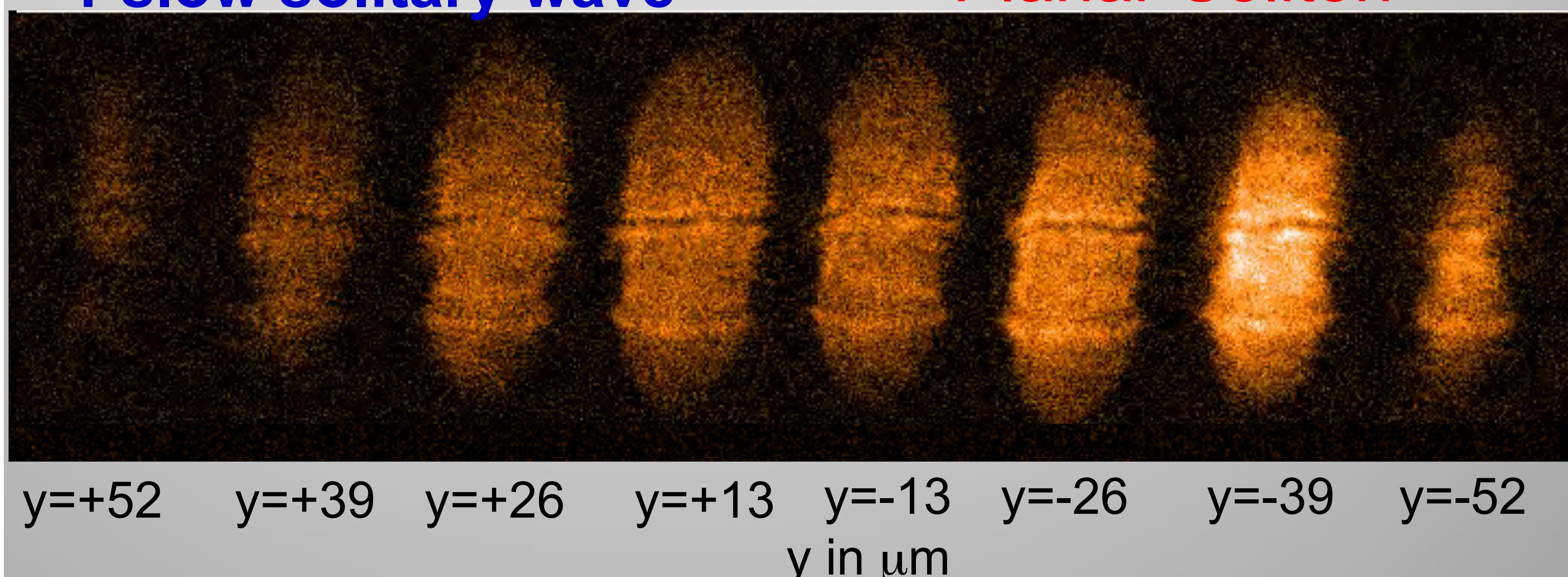
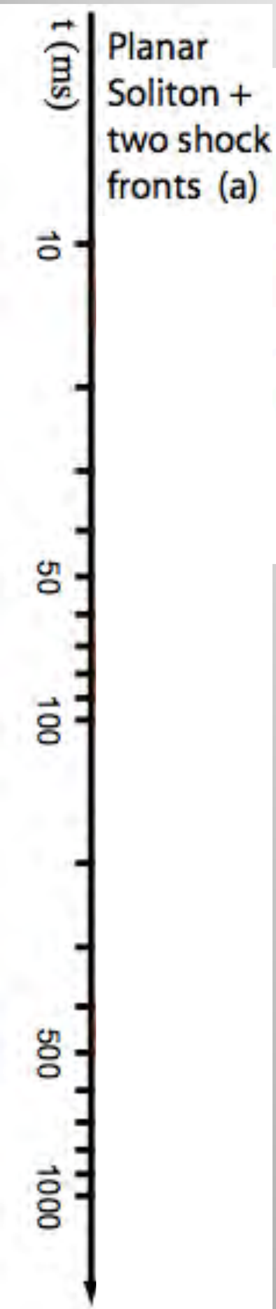
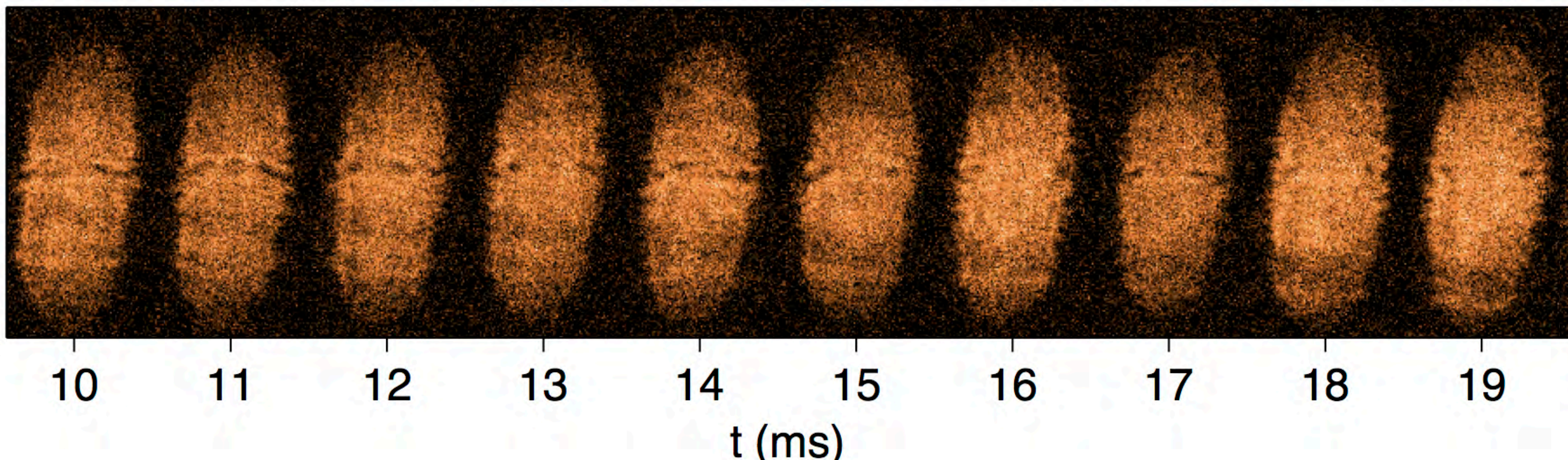


Instability Cascade of Solitary Waves in Unitary Fermi Gas

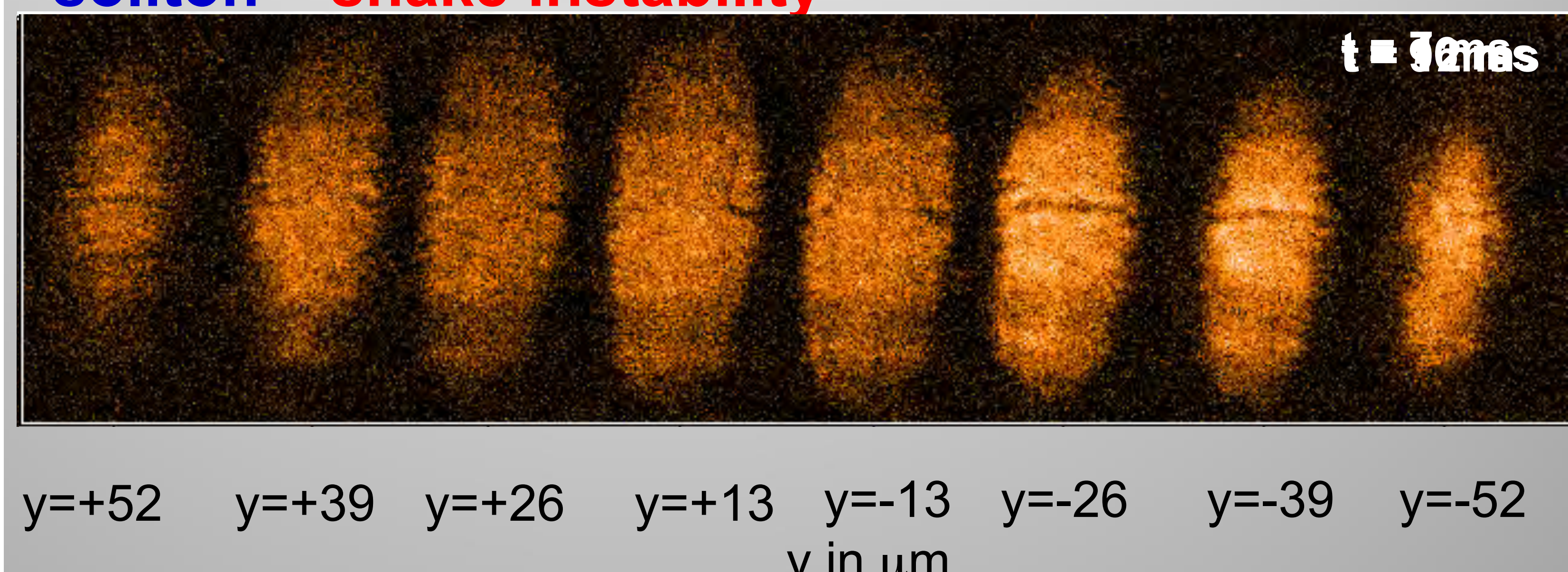


Early time dynamics after imprint (central slice)

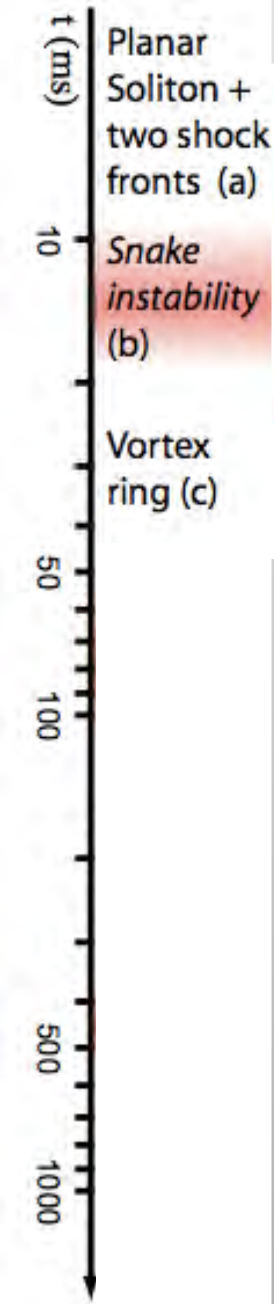


Bending & breaking of planar soliton – snake instability

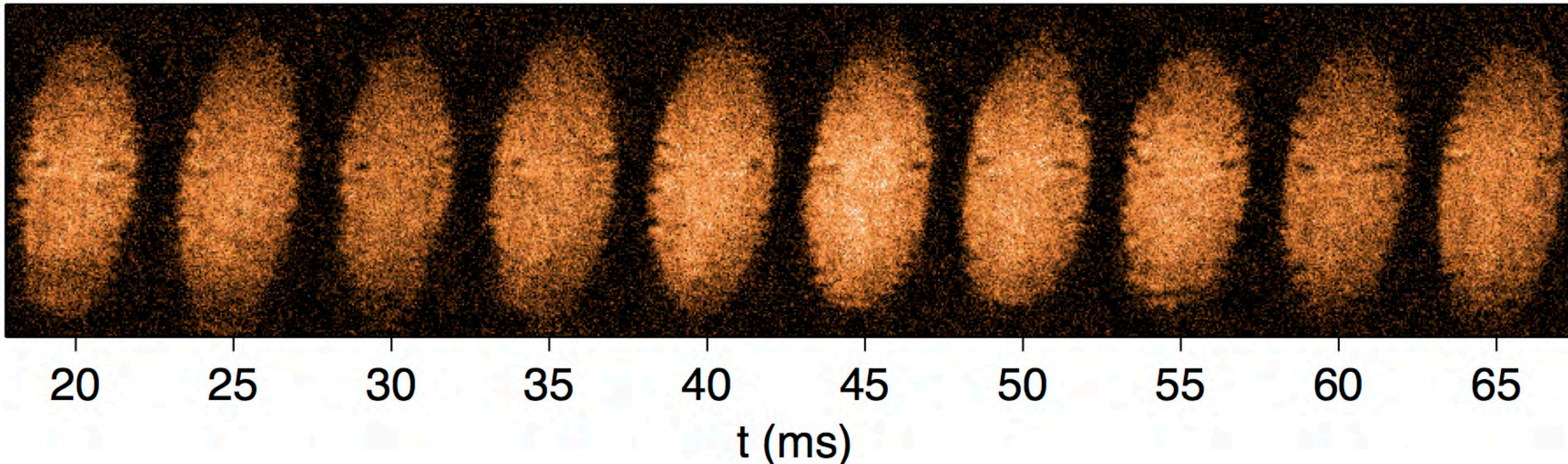
Tomography:



Instability Cascade of Solitary Waves in Unitary Fermi Gas

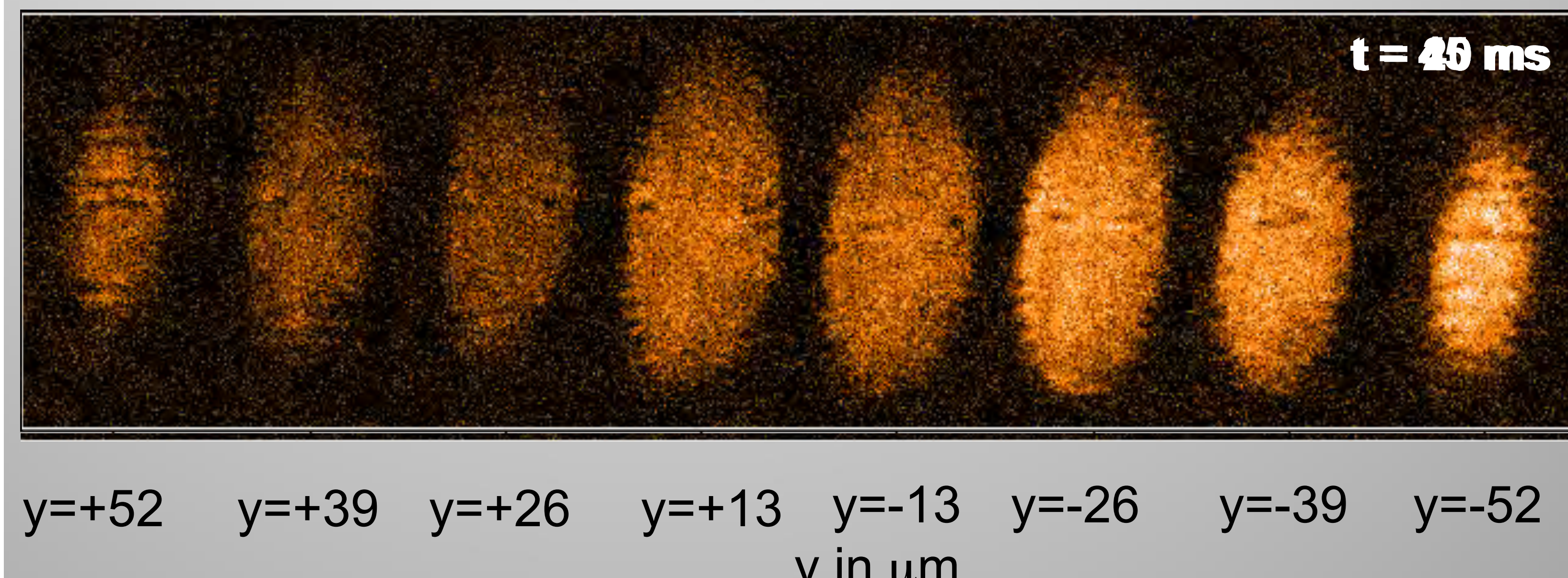


Early time dynamics after imprint (central slice)

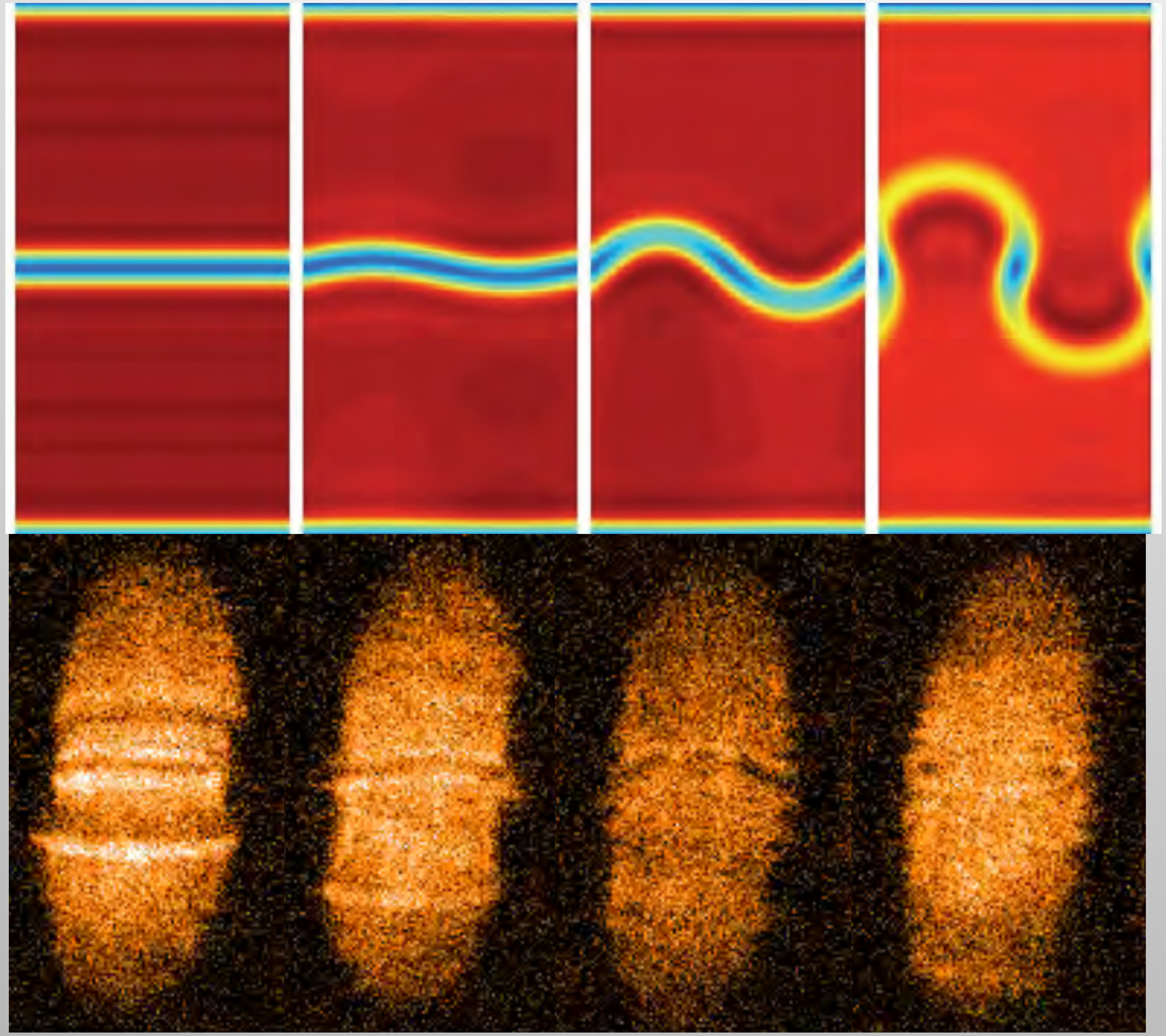


Vortex ring

Tomography:



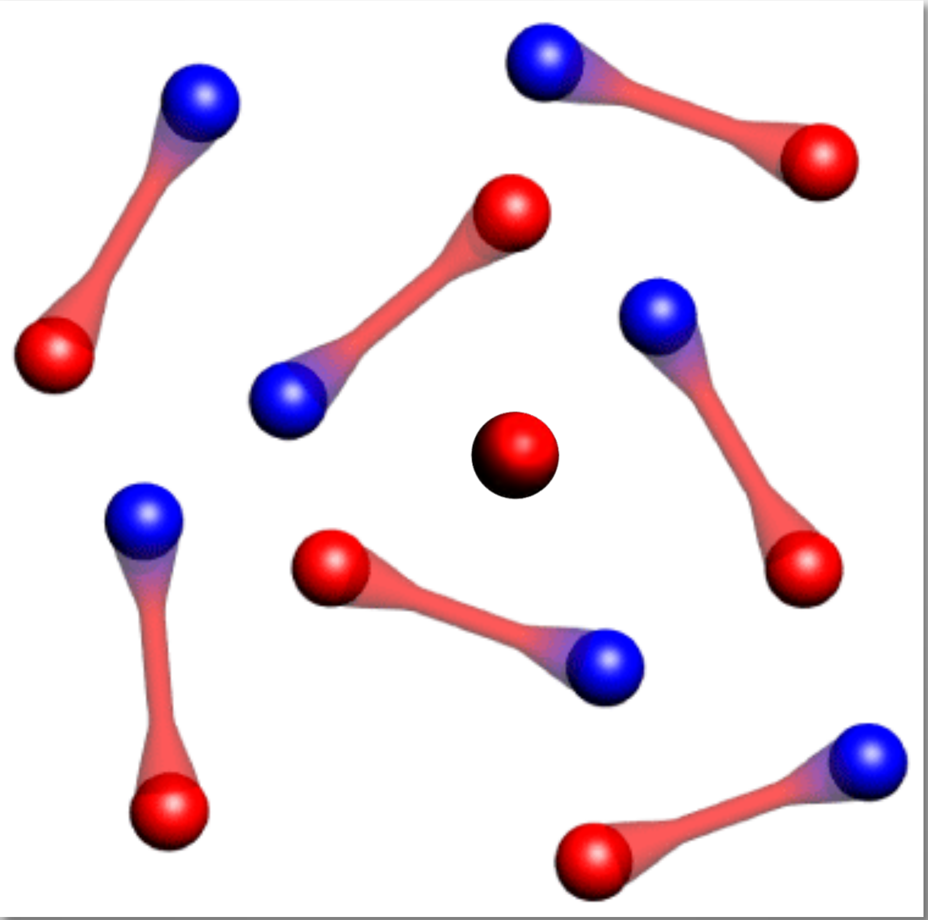
Snake Instability in a Fermionic Superfluid



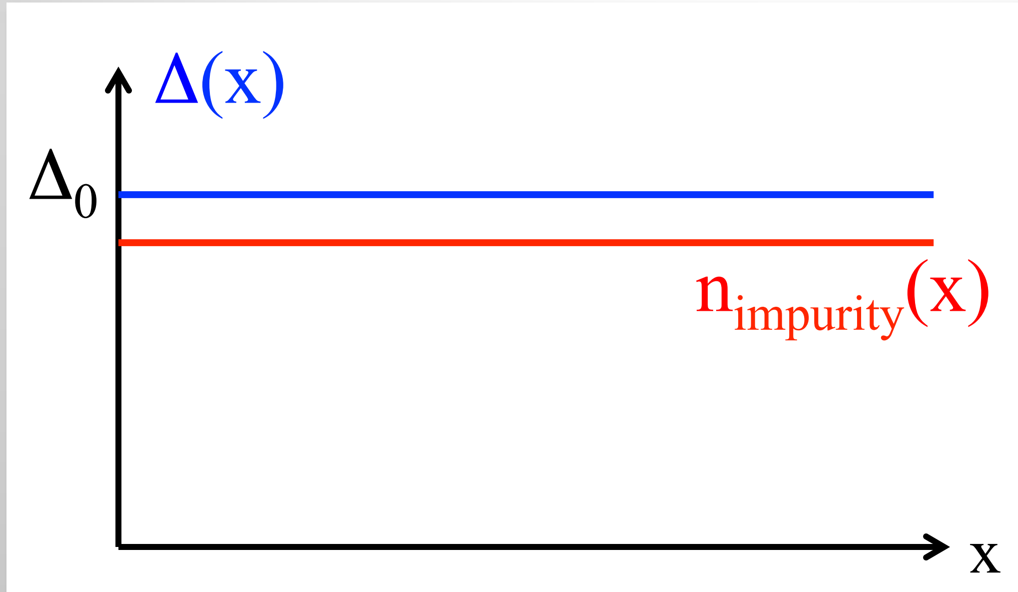
Theory:
Cetoli, Brand,
Scott, Dalfovo,
Pitaevskii
Phys. Rev. A 88,
043639 (2013)

Experiment:
M.J.H. Ku,
B. Mukherjee,
T. Yefsah, MWZ
PRL 116,
045304 (2016)

One excess fermion in the superfluid

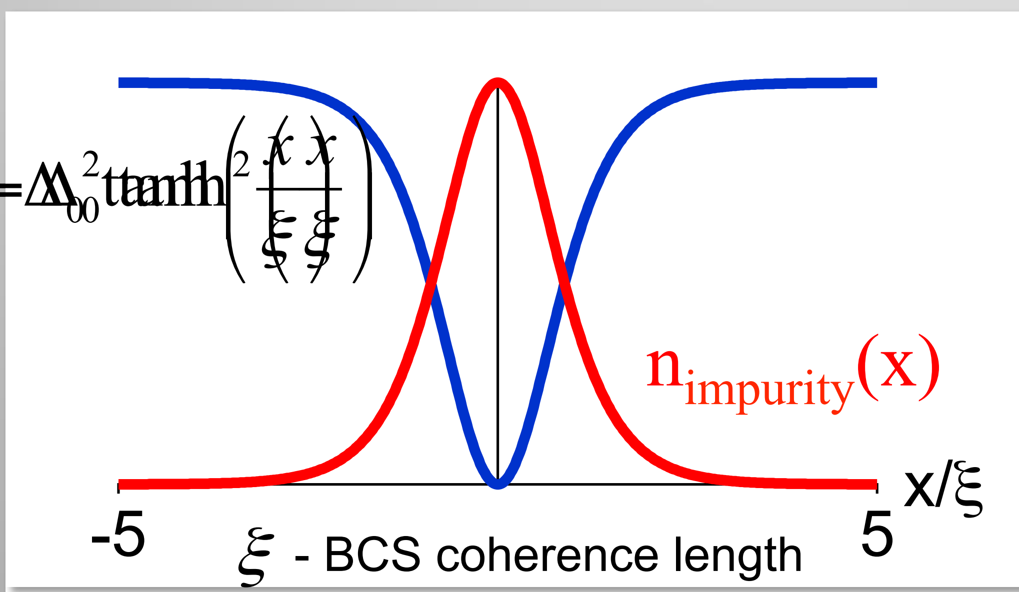


The fate of a single impurity in 1D: Stuck in a soliton



Energy cost:

$$\Delta_0$$



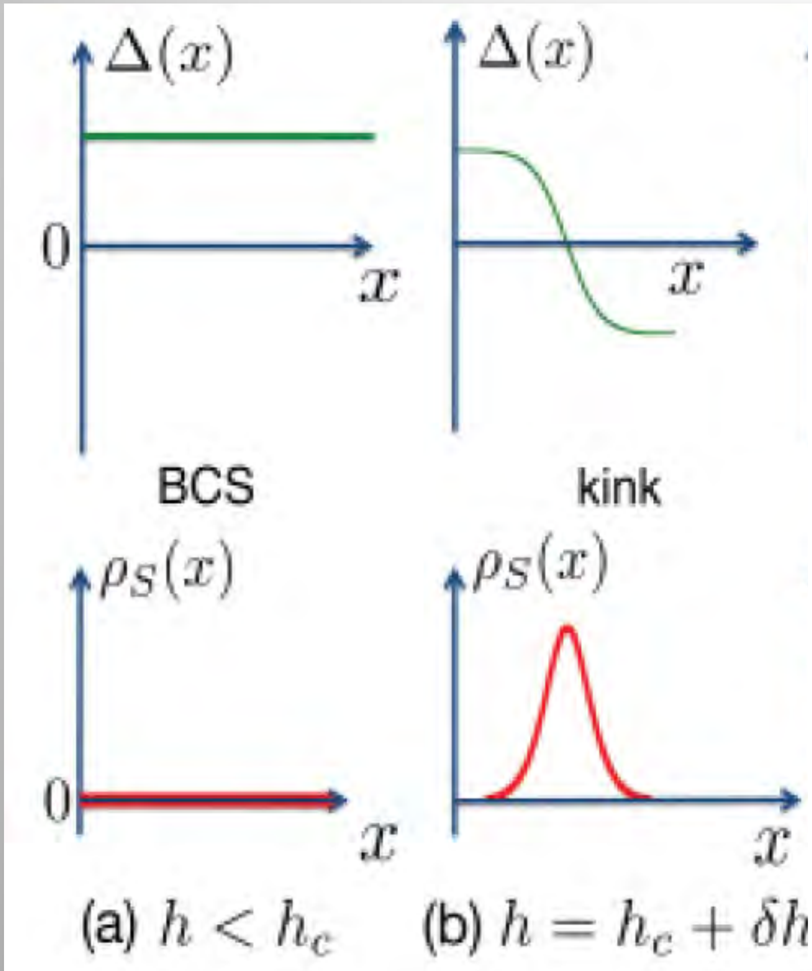
$$|\Delta|^2 = \Delta_0^2 \tanh^2\left(\frac{x}{\xi}\right)$$

Energy cost:

$$\frac{\Delta_0^2}{E_F} n_{1D}\xi \sim \frac{2}{\pi}\Delta_0$$

$$\xi \sim \frac{1}{k_F}\frac{E_F}{\Delta_0}$$

Solitons as one limit of the FFLO state

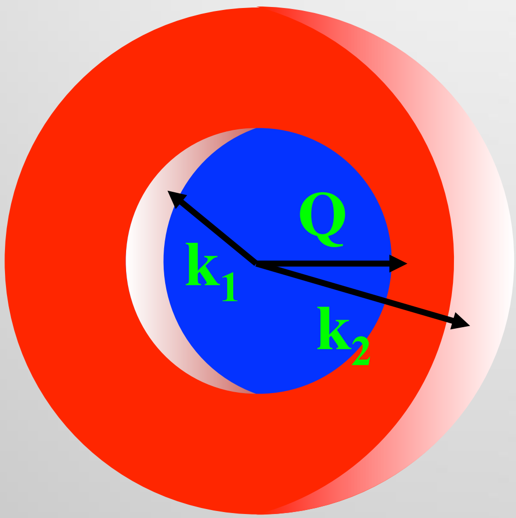


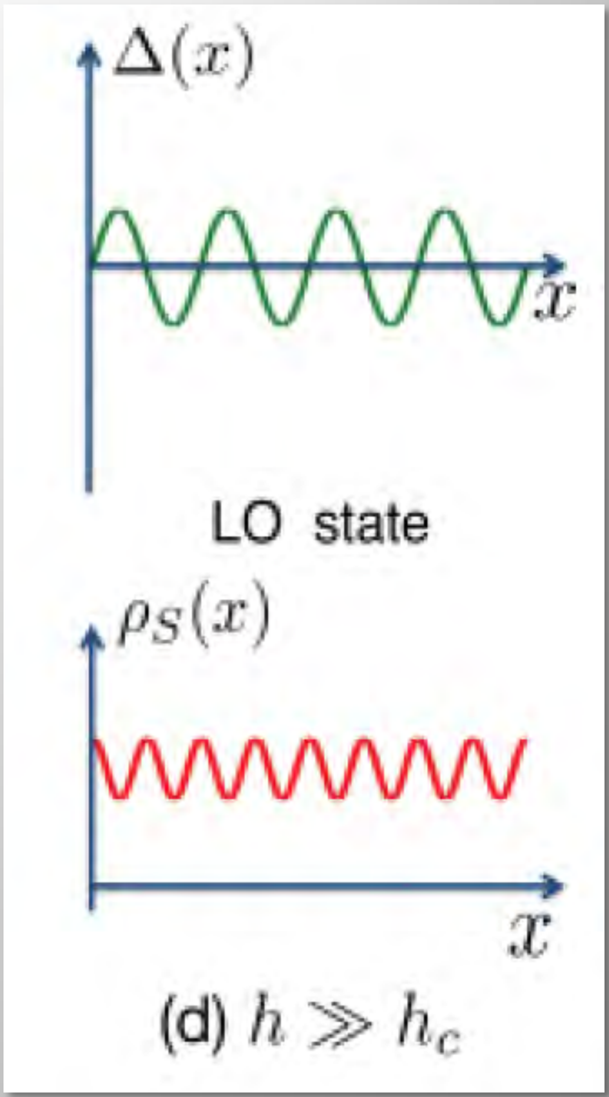
from Lutchyn, Dzero, Yakovenko, PRA 84, 033609 (2011)

See Yoshida, Yip, PRA 75, 063601 (2007), Radzihovsky, PRA 84, 023611 (2011)

Fulde-Ferrell-Larkin-Ovchinnikov State

Cooper pairs with non-zero momentum



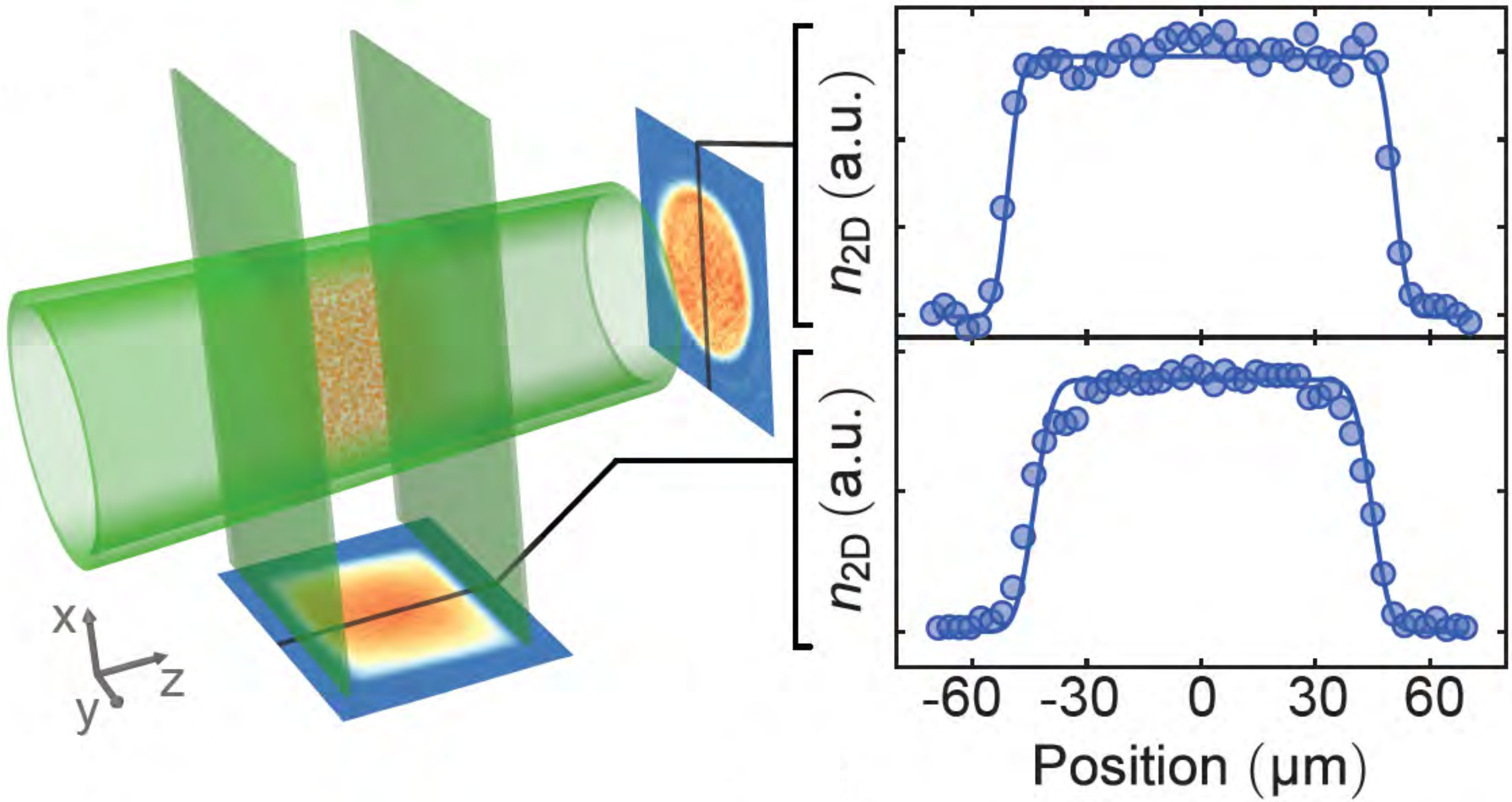


FF $$\Delta(x) = \Delta_0 e^{ikx}$$

LO $$\Delta(x) = \Delta_0 \sin(kx)$$

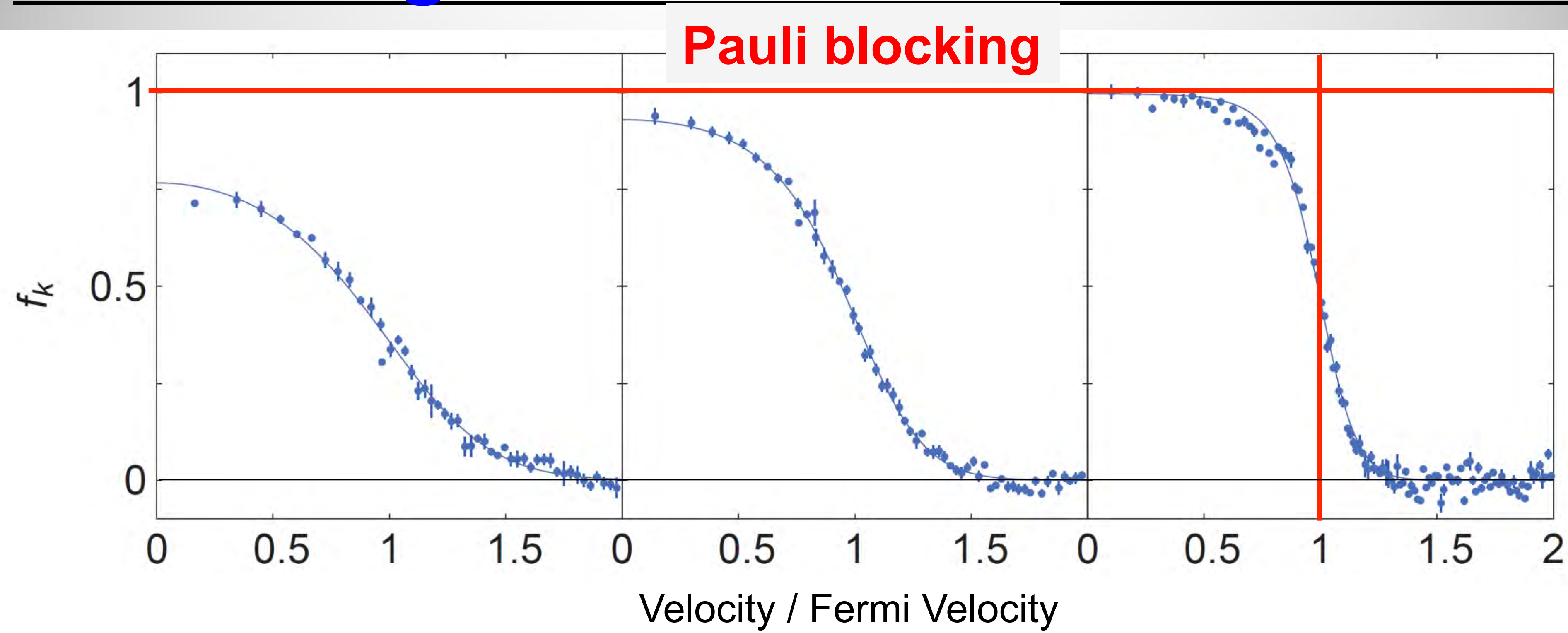
A. I. Larkin, Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. ***47****, 1136 (1964)*
P. Fulde, R. A. Ferrell, Phys. Rev. 135, A550 (1964)

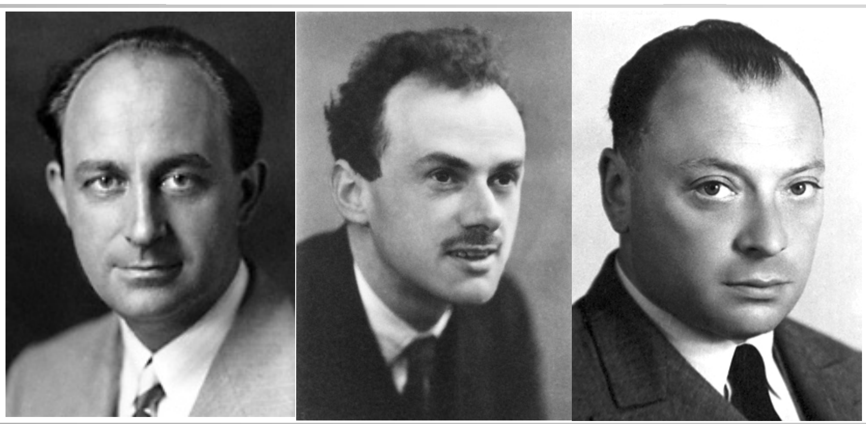
Fermions in a Box



Z. Yan, P. Patel, B. Mukherjee, Z. Hadzibabic, T. Yefsah, J. Struck, MWZ, PRL 2017

Measuring the Fermi-Dirac distribution





$$f_k = \frac{1}{e^{\beta(\varepsilon_k - \mu)} + 1}$$

90th anniversary

Fermi surface formation

See also: Drake et al., PRA 2012, selectively probe the central portion of an inhomogeneous gas.

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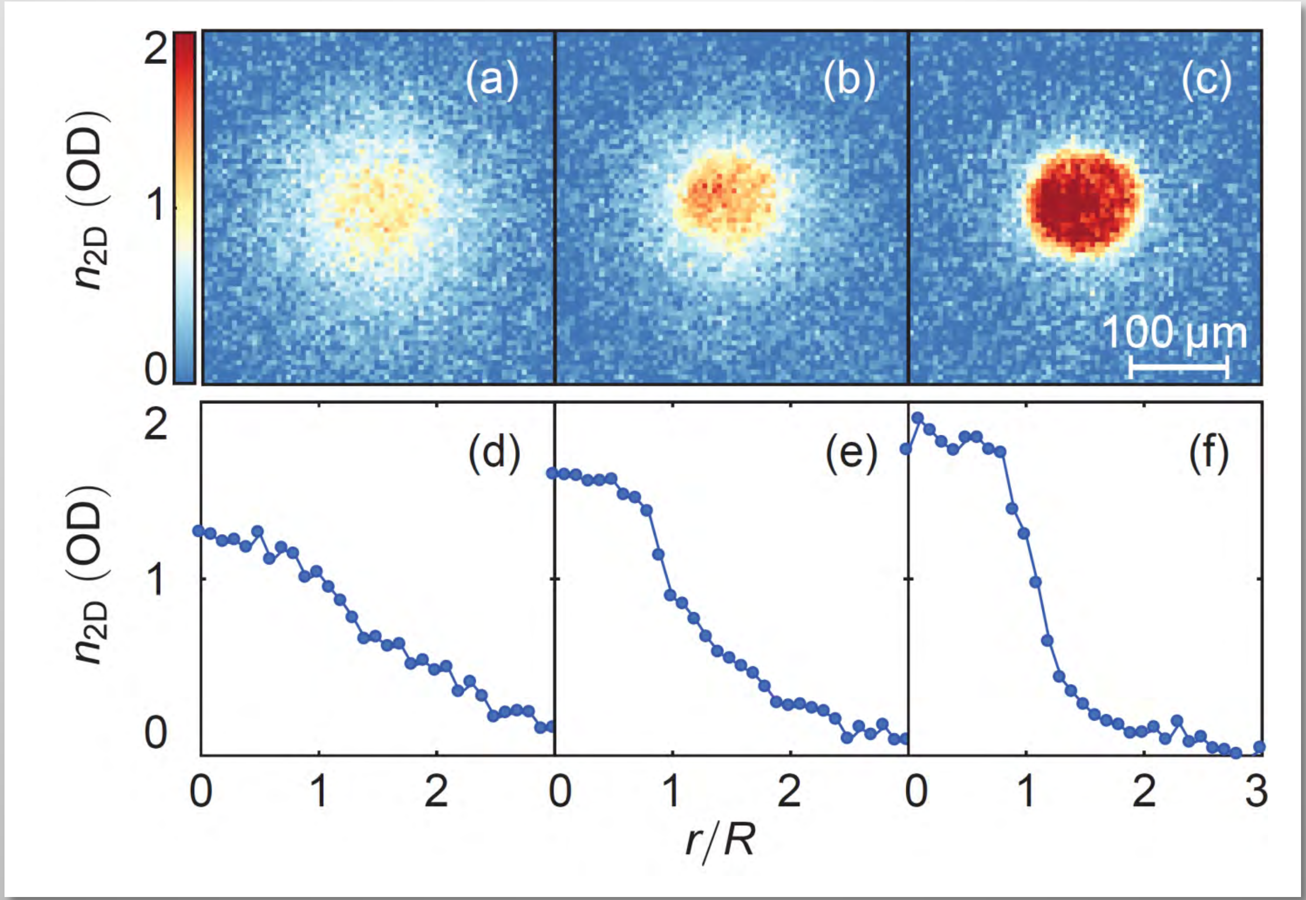
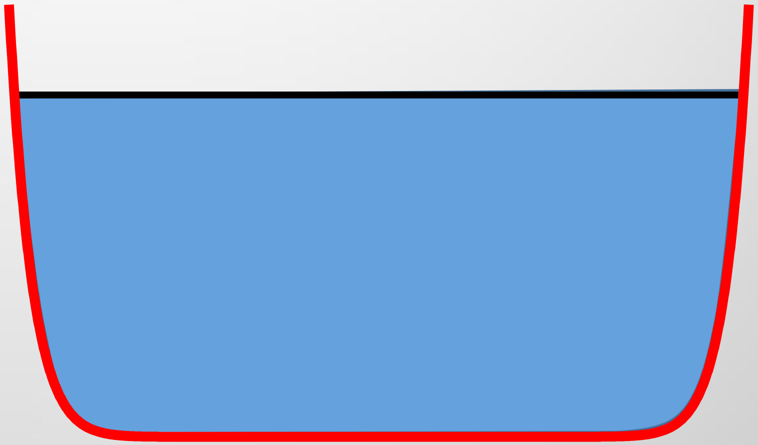
Fermi superfluids in a box

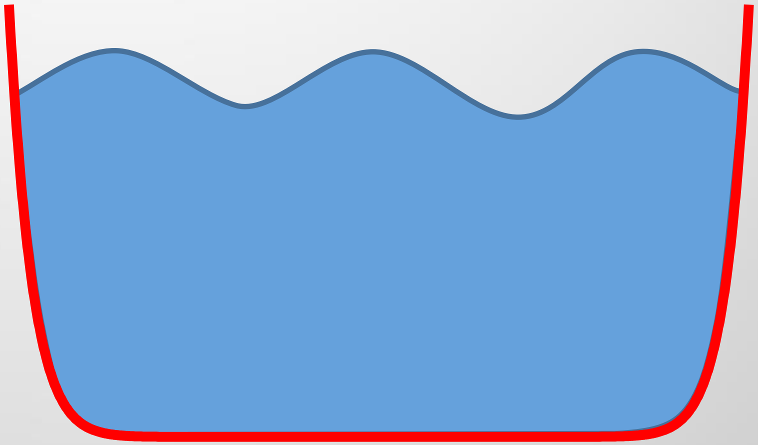


Z. Yan, P. Patel, B. Mukherjee, Z. Hadzibabic, T. Yefsah, J. Struck, MWZ, PRL 2017

Shaking the box

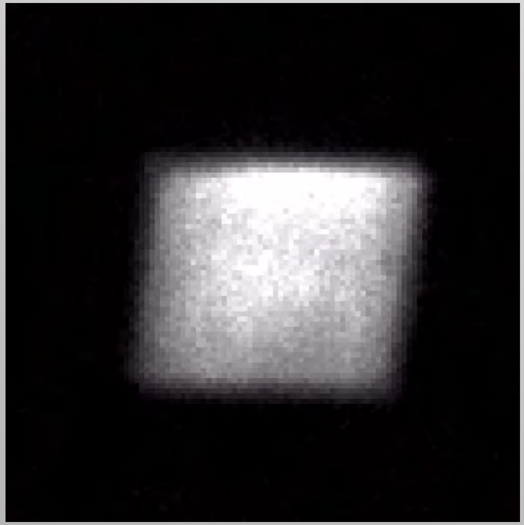


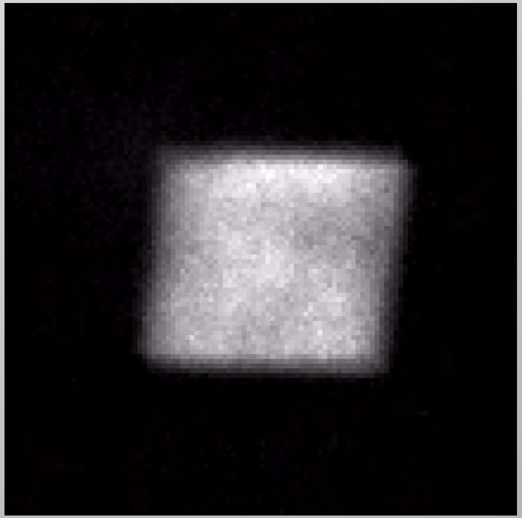
Shaking the box



Shaking with gradient

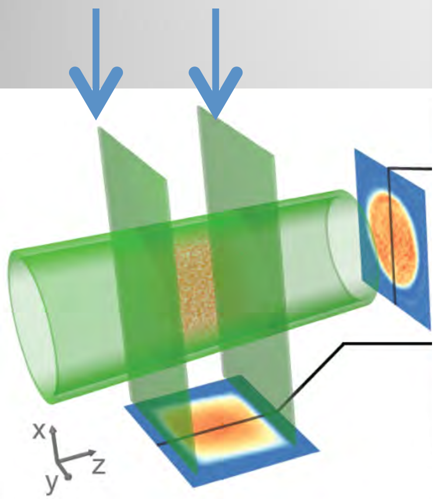
Sinusoidal modulation of confining "end-caps"





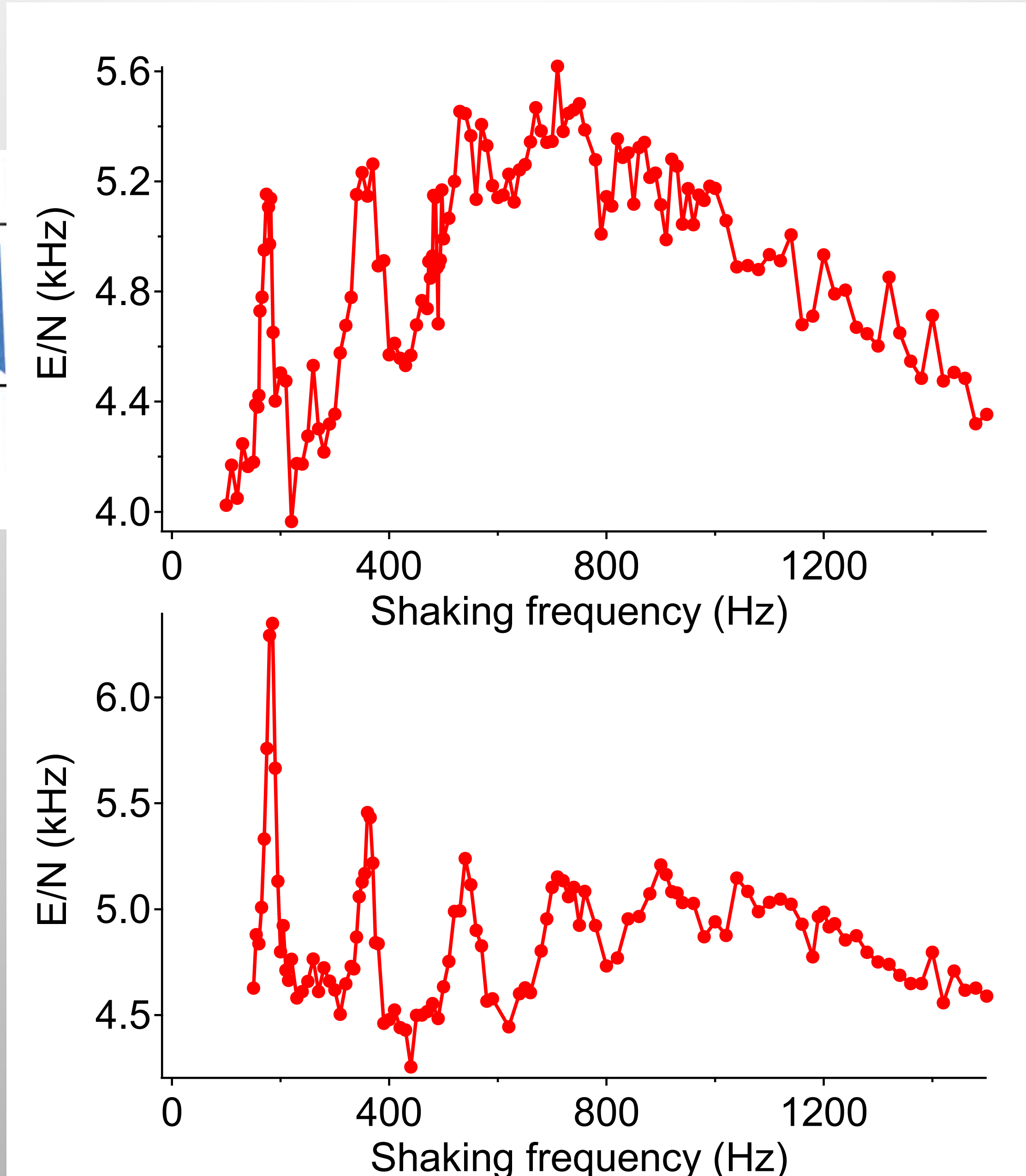
Energy Spectra

Modulate these



$$\dot{E} = \hbar\omega\,\Gamma(\omega)$$

Shaking with
Fixed # of cycles



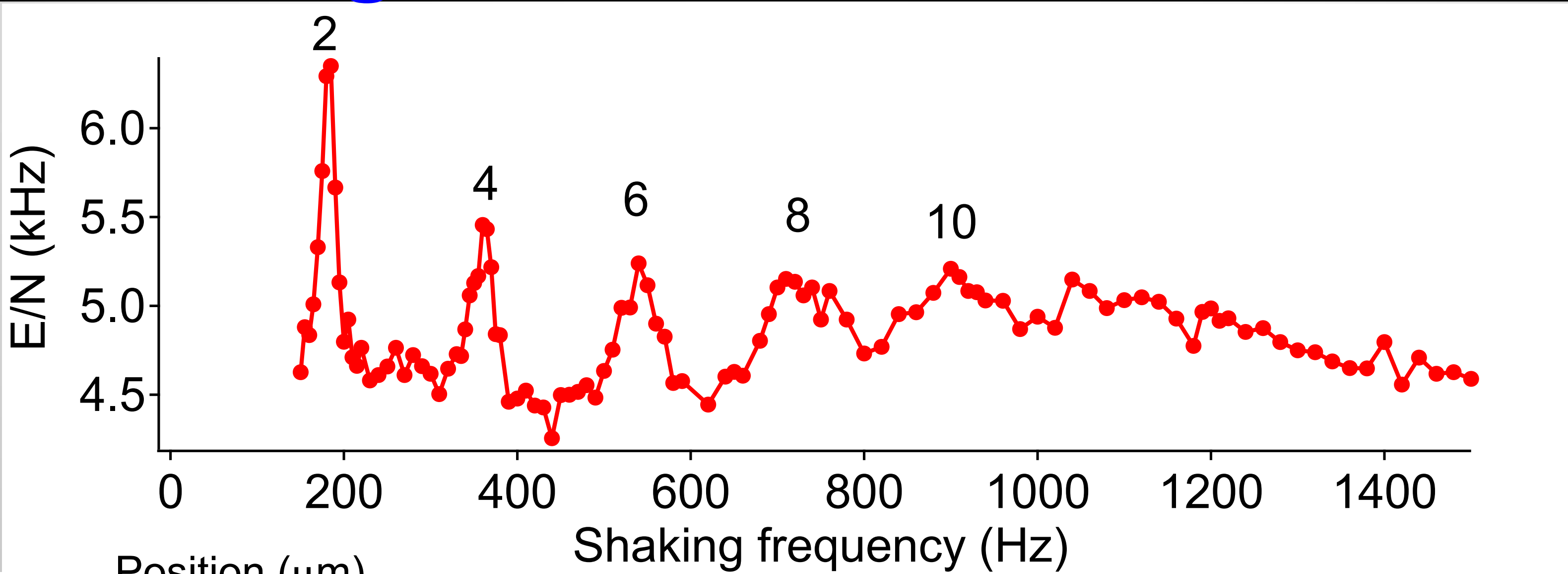
$T > T_C$

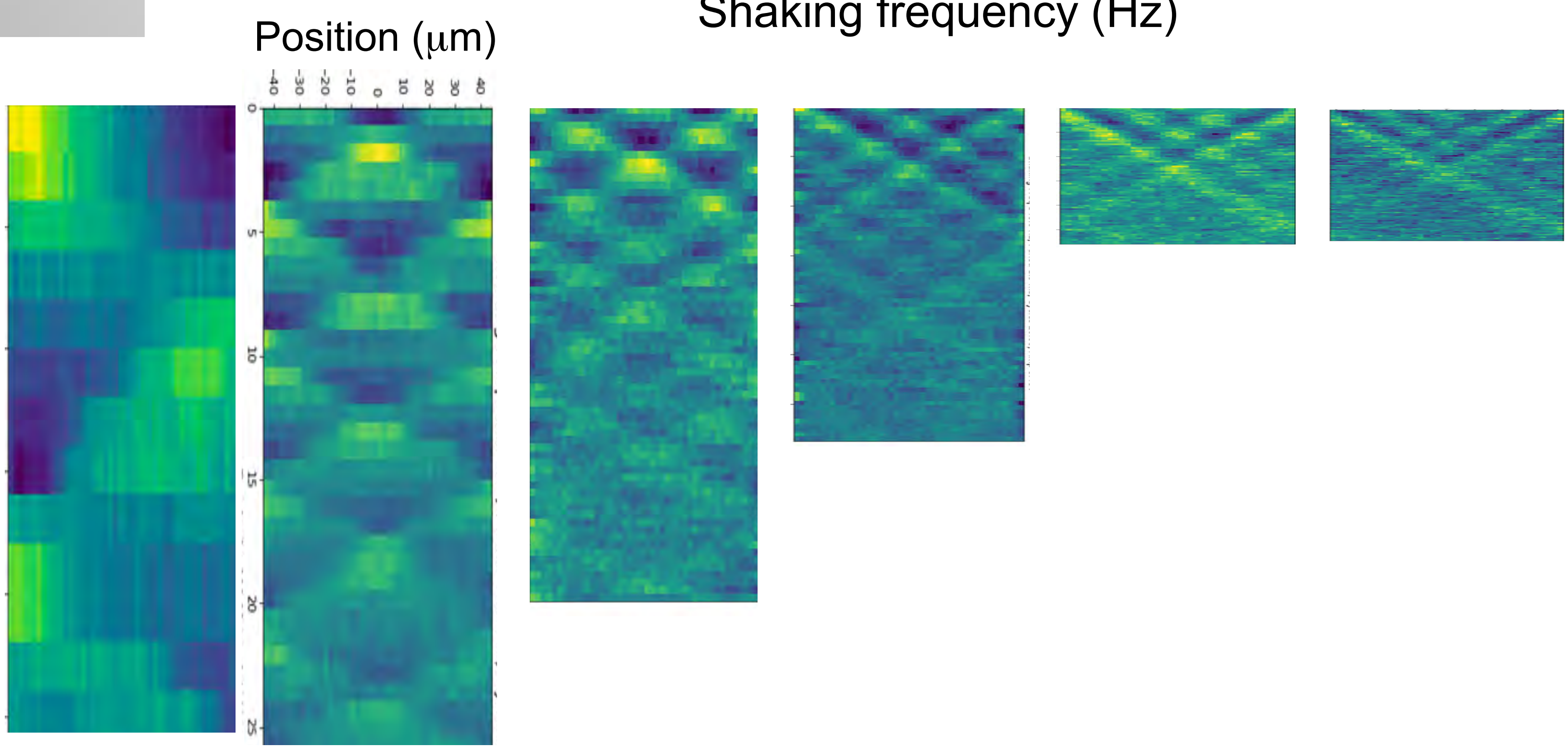
(E_F = 11 kHz)

$T < T_C$

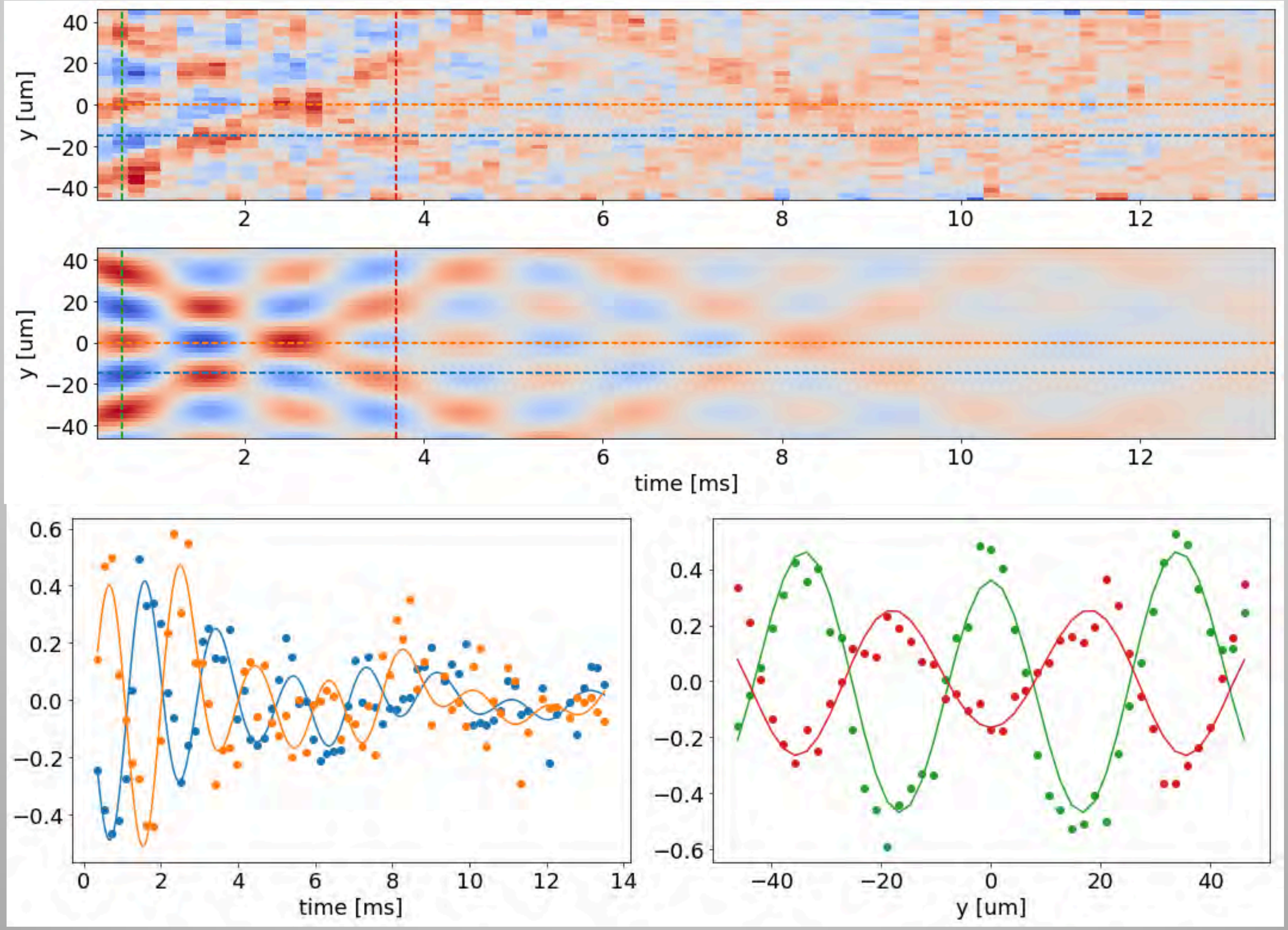
(E_F = 15 kHz)

Eigenmodes in the box

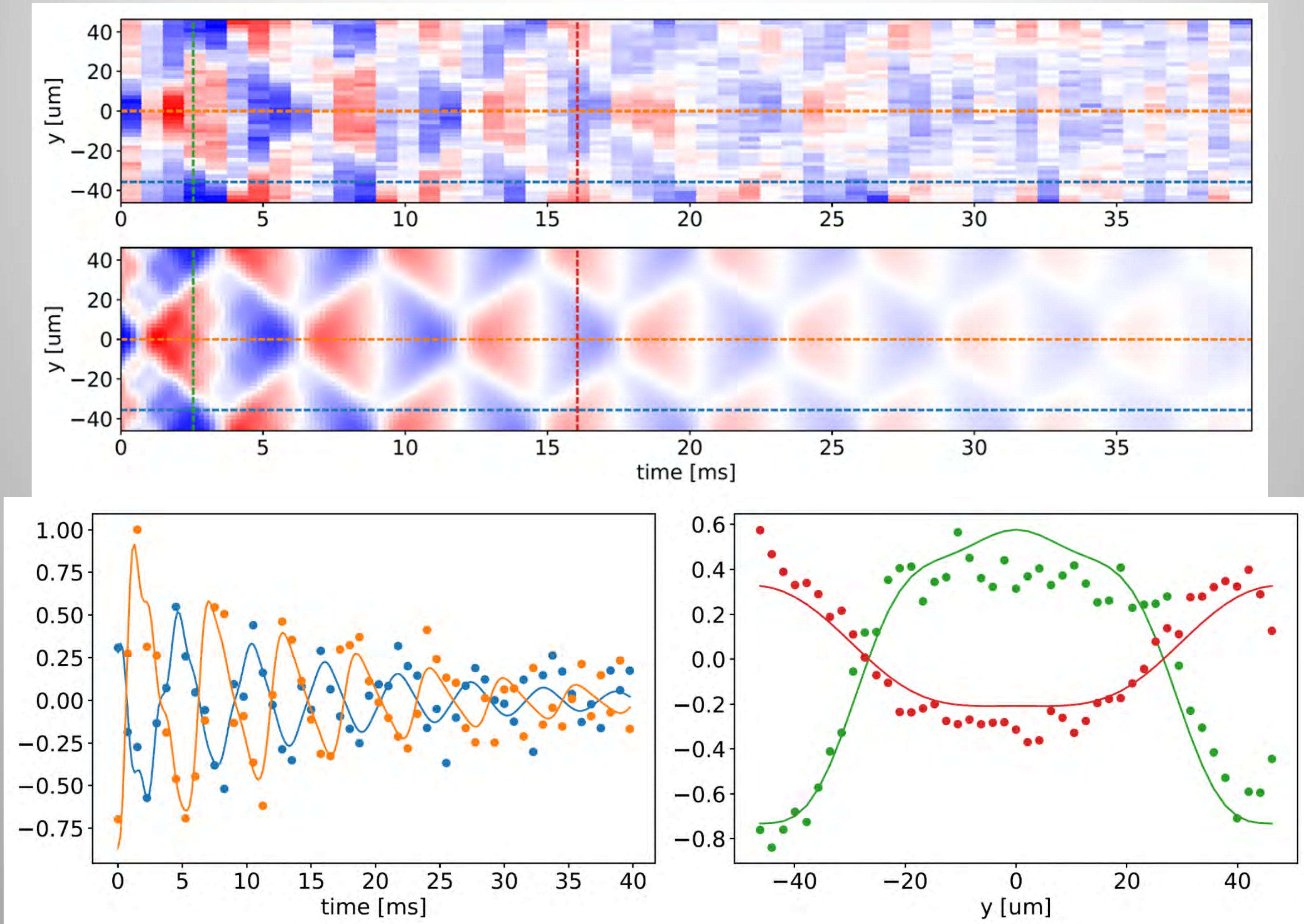




Example: n=6



Example: 2nd mode, non-linearities



Viscosity from Sound Attenuation

(see 70 years of work in Helium-4 and Helium-3…)

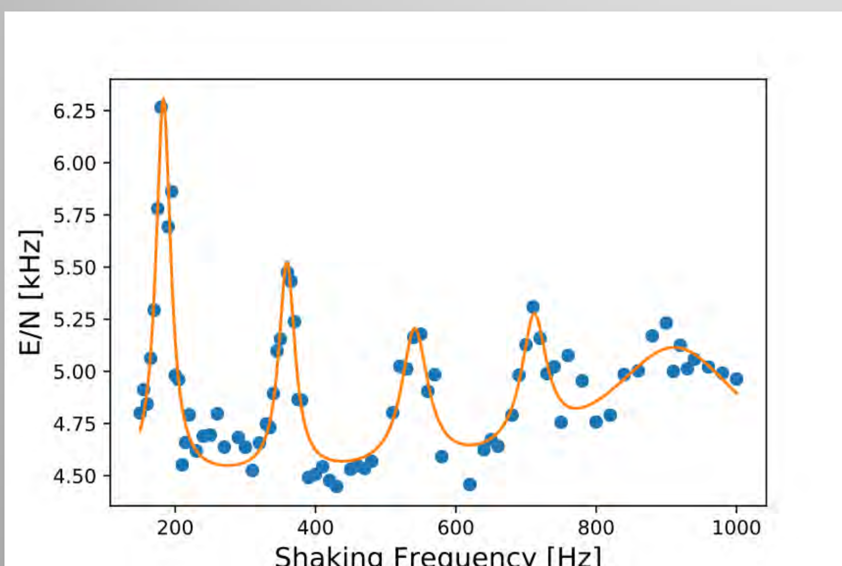
Equation for Sound:
(Bulk viscosity=0)

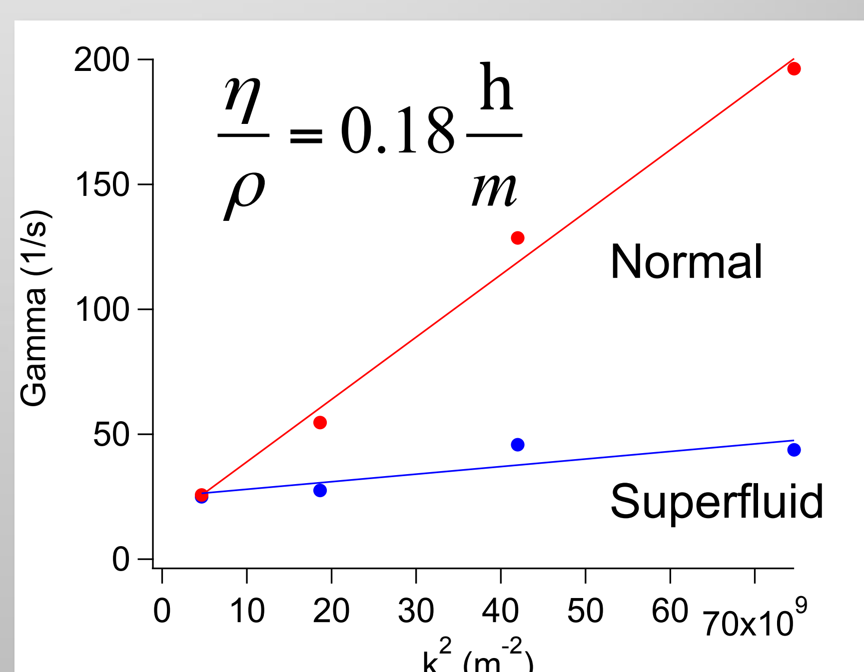
$$\frac{\partial j_z}{\partial t} + \frac{\partial p}{\partial z} = \frac{4}{3}\eta \frac{\partial^2 v_z}{\partial z^2}$$

Dispersion:

$$\omega^2 = \left.\frac{\partial p}{\partial \rho}\right|_S k^2 + i\omega \boxed{\frac{4}{3}\frac{\eta}{\rho}k^2}$$

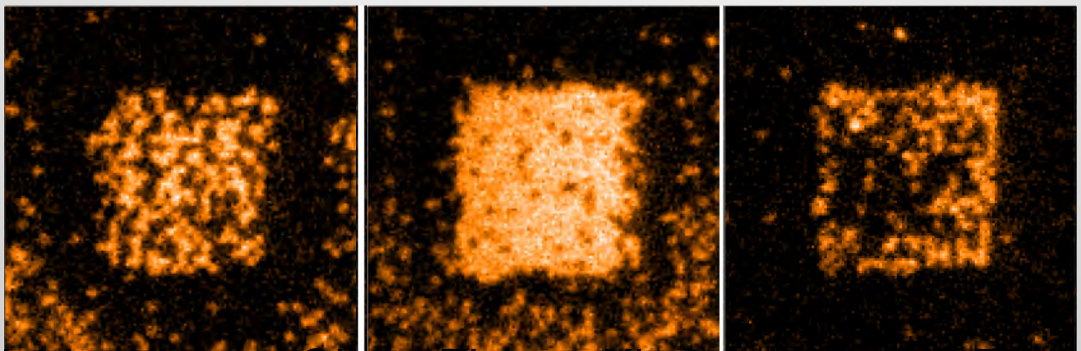
Damping rate





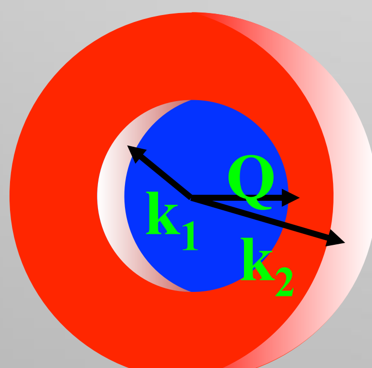
Outlook

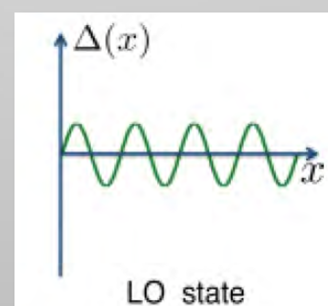
- Measurements of Viscosity across the Superfluid Transition
- Spin-Imbalanced Mixtures
- Shaking in the Fermi-Hubbard Model



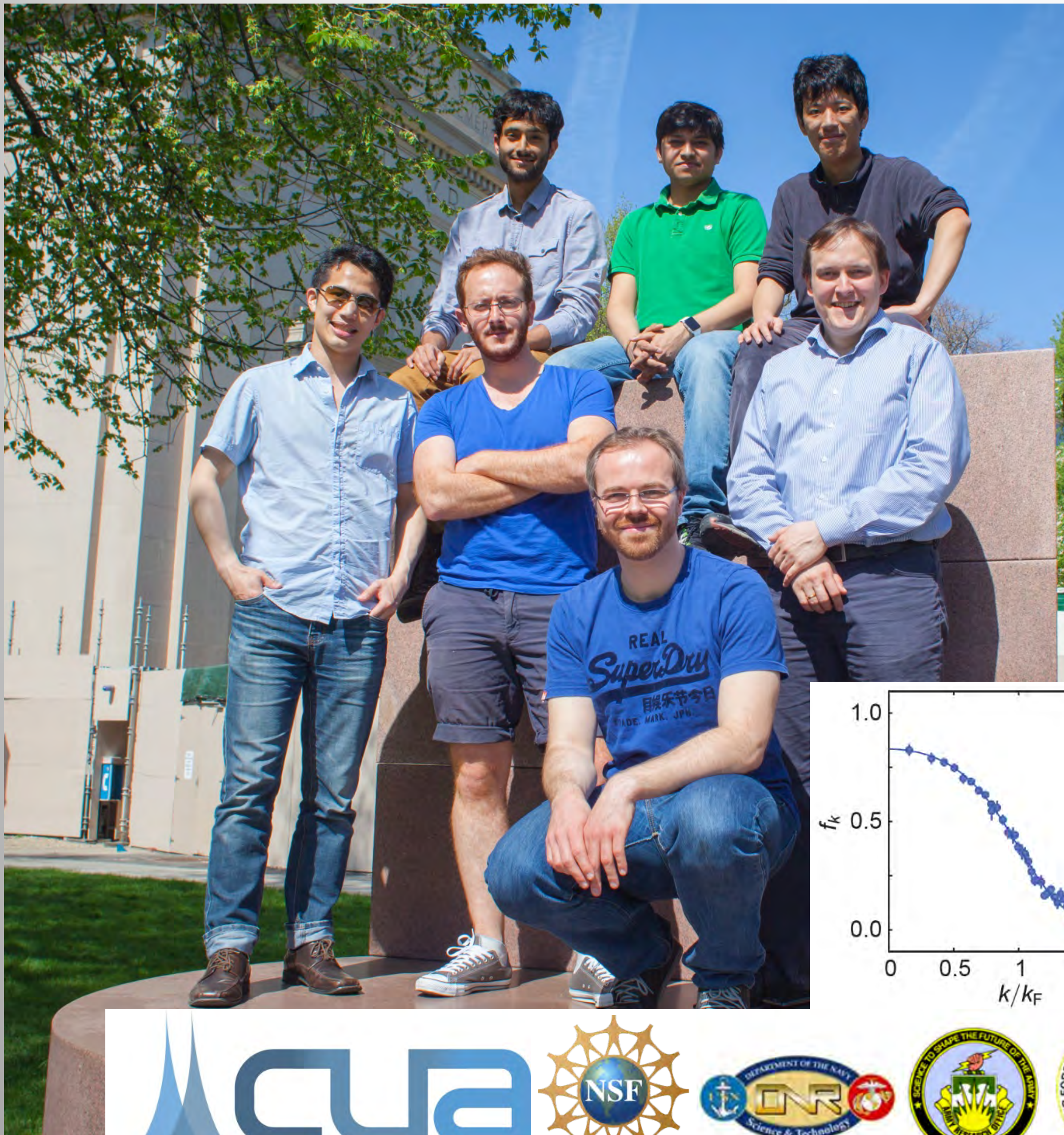
Cheuk, Nichols, Lawrence, Okan, Zhang, Khatami, Trivedi, Paiva, Rigol, MWZ, Science 2016,

- FFLO Superfluidity





Fermions in a Box



BEC 1

Fermions in a box

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Dr. Richard Fletcher
Visiting Professor:
Zoran Hadzibabic
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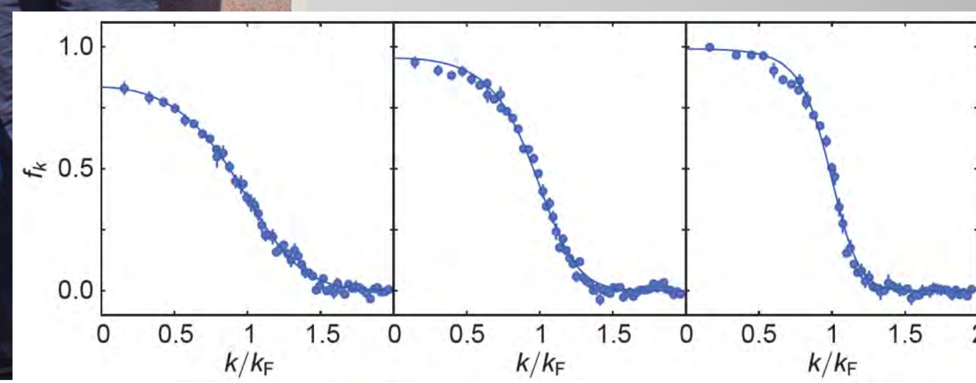
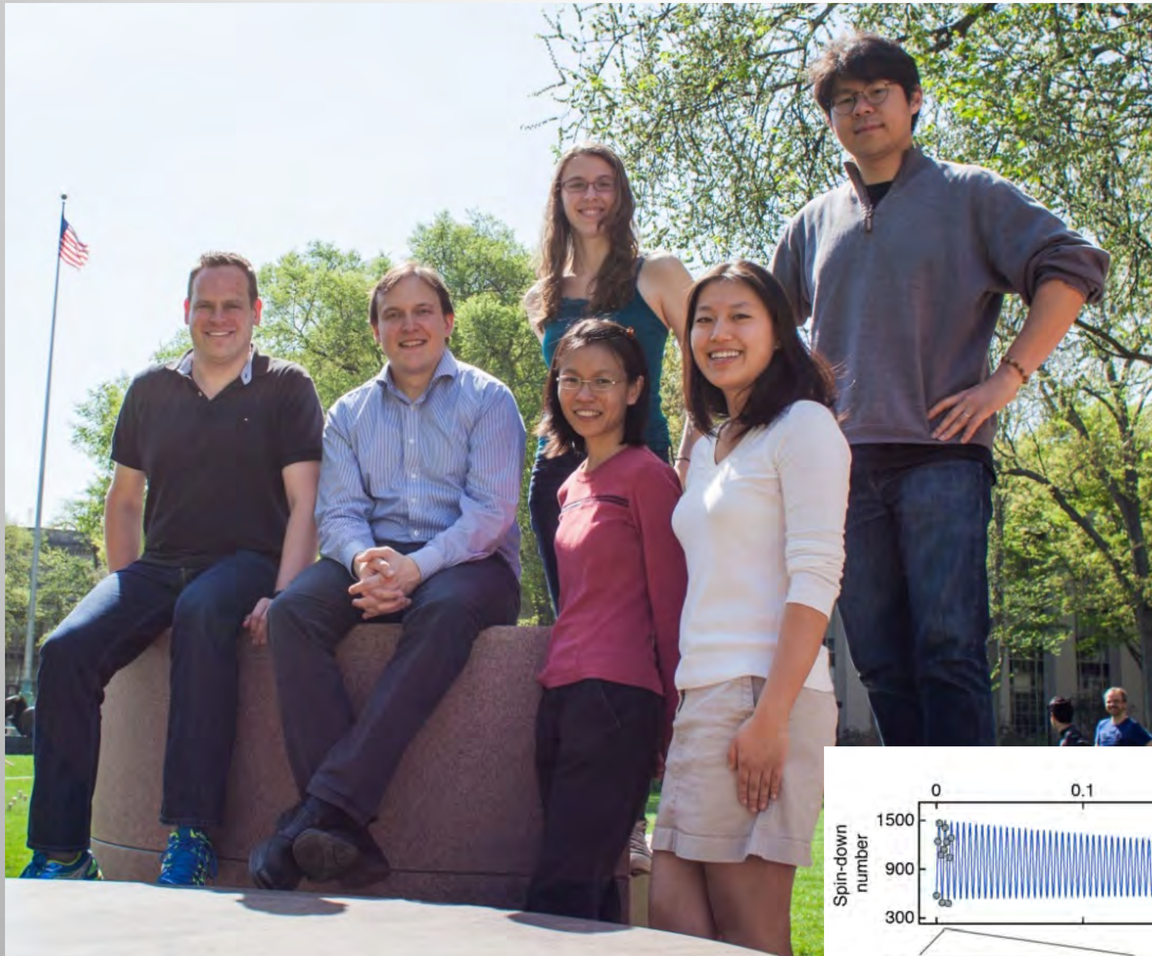


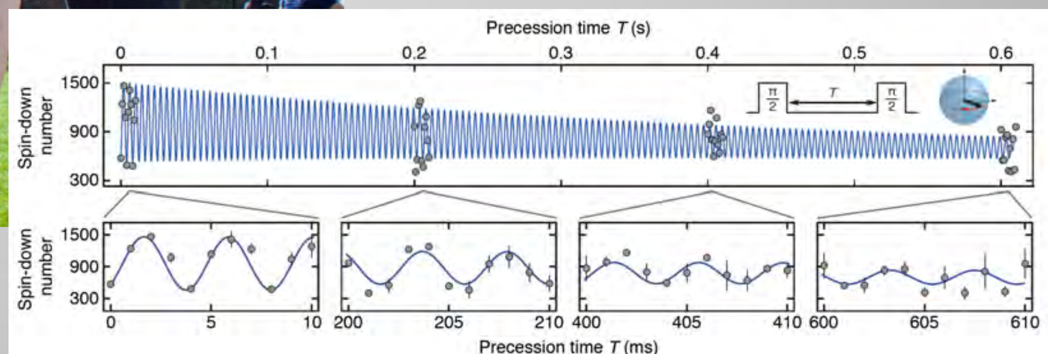
NaK Molecules



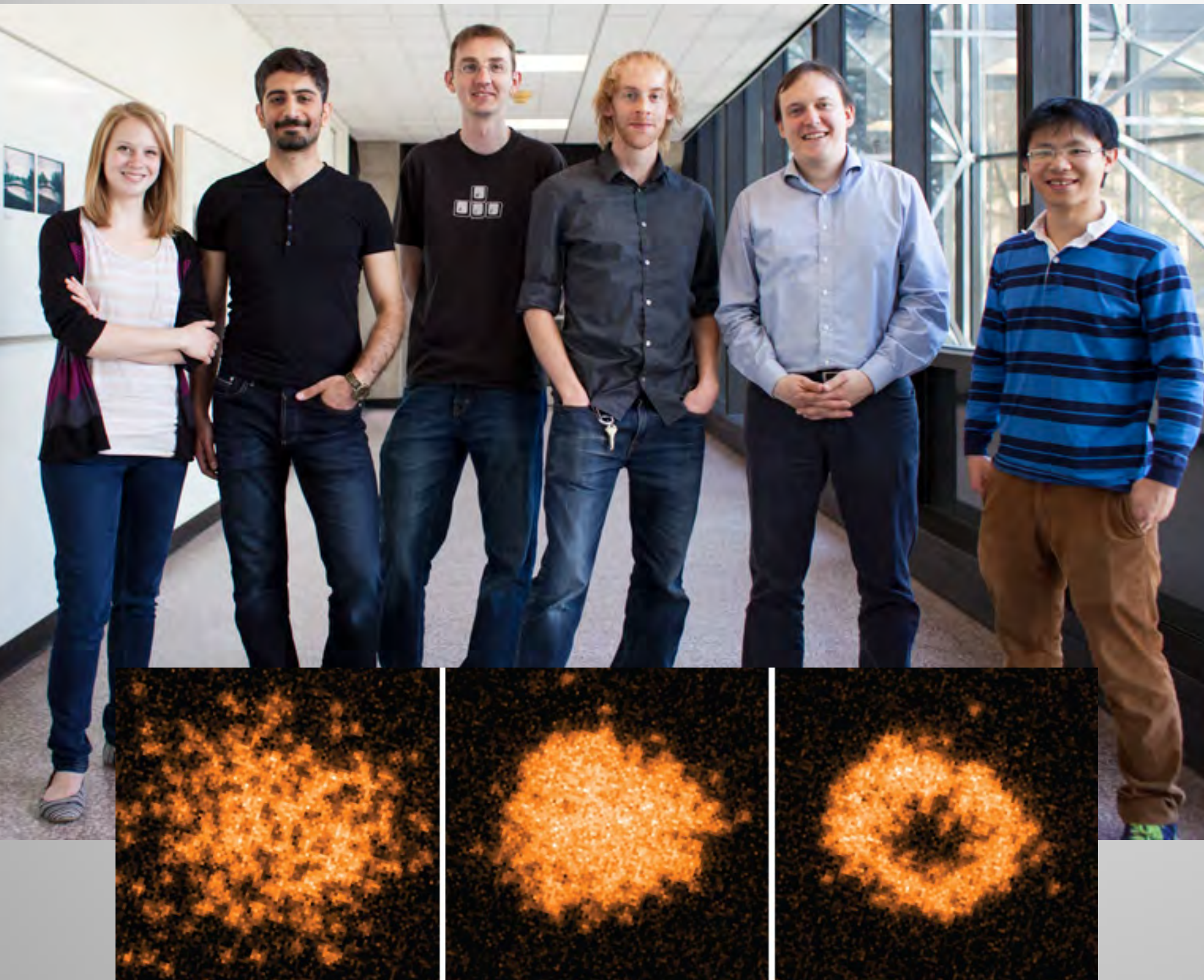
Fermi 1

NaK Dipolar Molecules

Jeewoo Peter Park (PhD 2016)
Zoe Yan
Yiqi Ni
Dr. Huanqian Loh
Dr. Sebastian Will
(→ Columbia U.)



Fermi-Hubbard Model under the Microscope



Fermi 2

Lawrence Cheuk
Melih Okan
Matthew Nichols
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